GFT: Connecting the Optical Potential with reaction observables.

Gregory Potel Aguilar

Trento, June 20 2024

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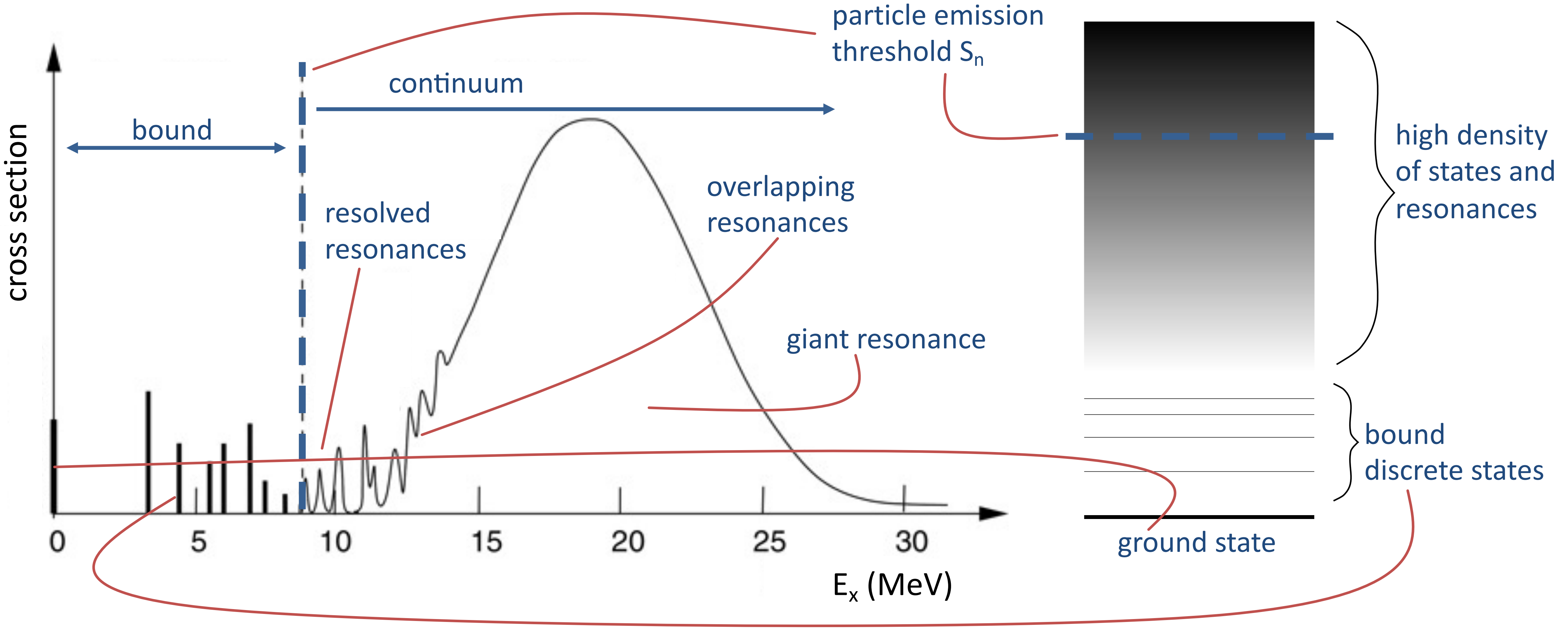
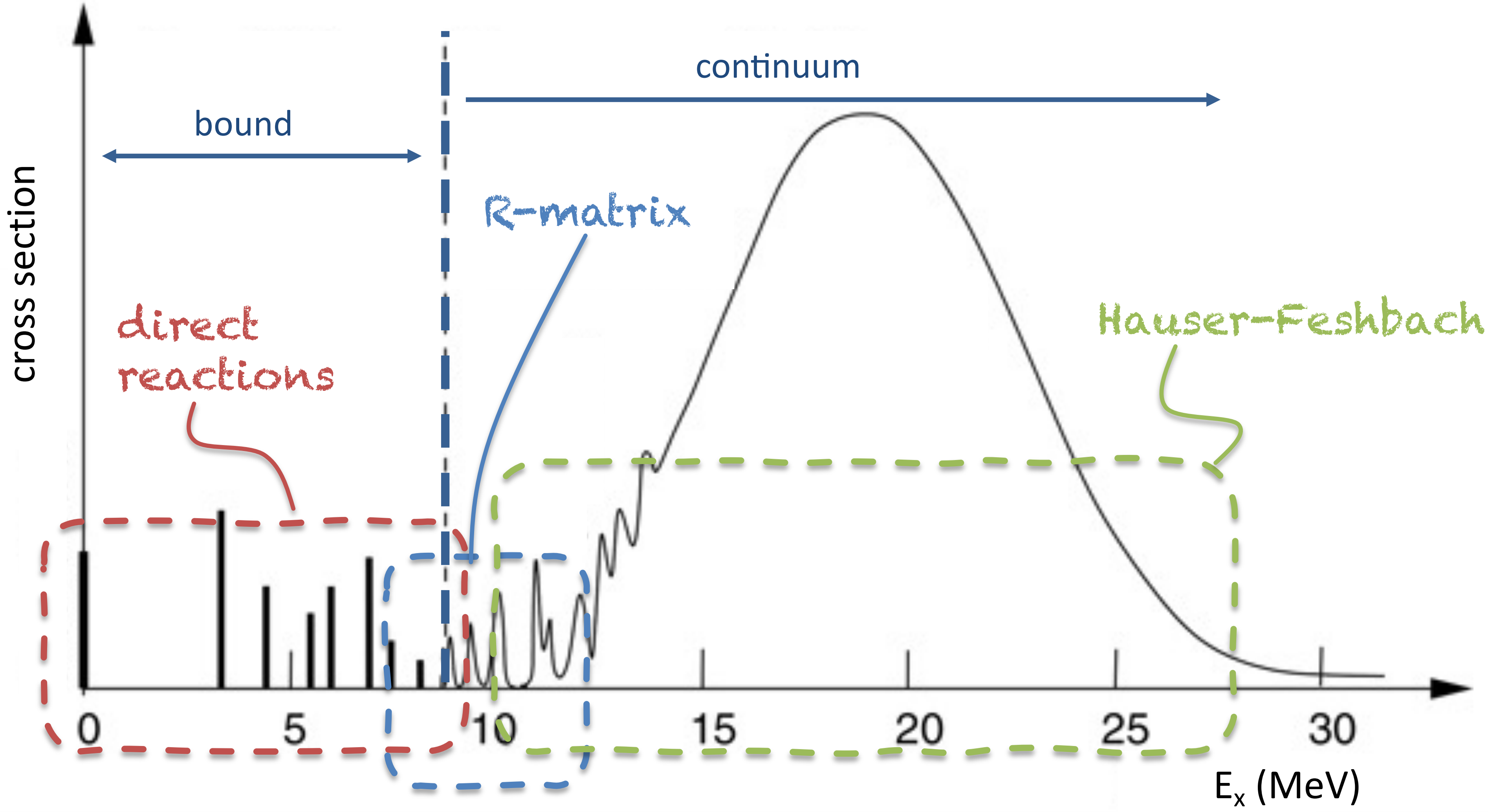
Lawrence Livermore National Laboratory

Features of nuclear spectra probed by nuclear reactions



Which reaction theories, and where?

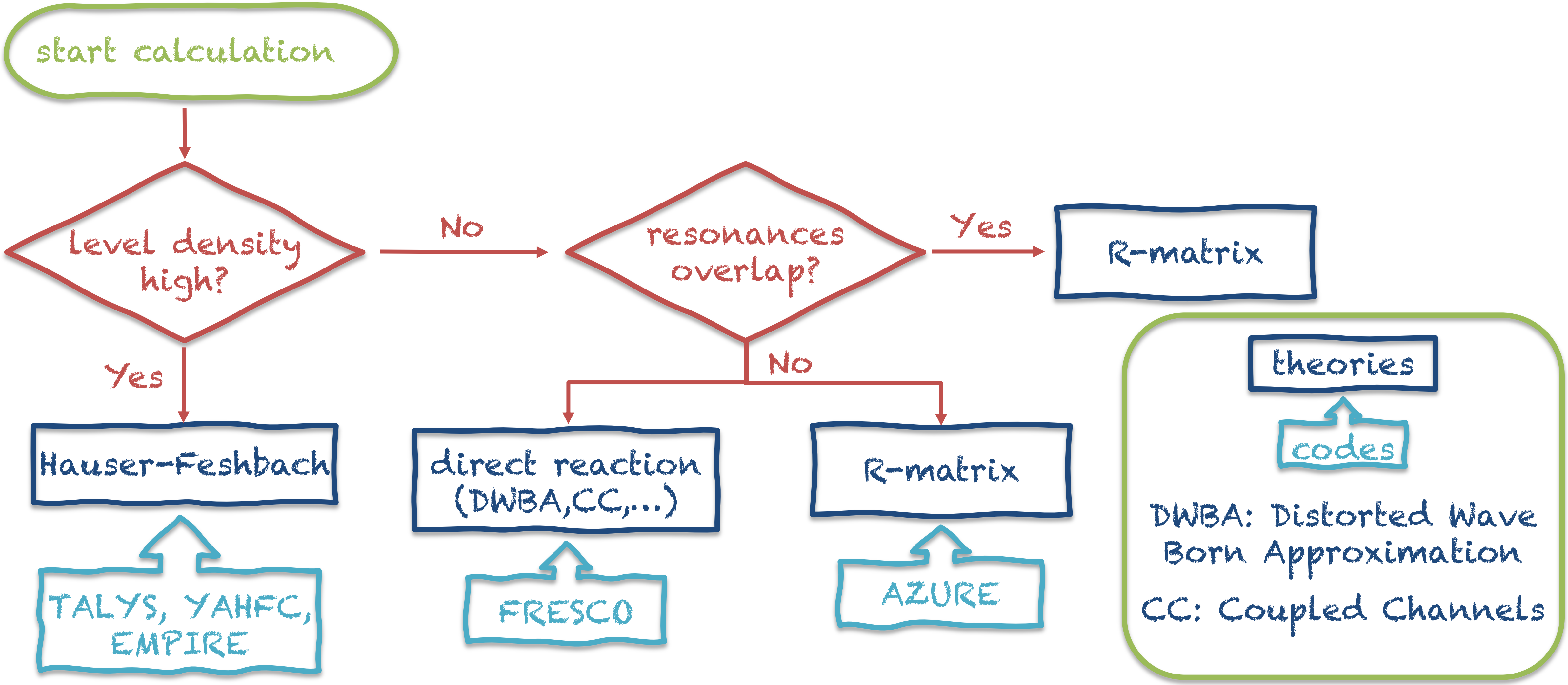


The Optical Potential is a projection of the many-body Hamiltonian on the elastic channel

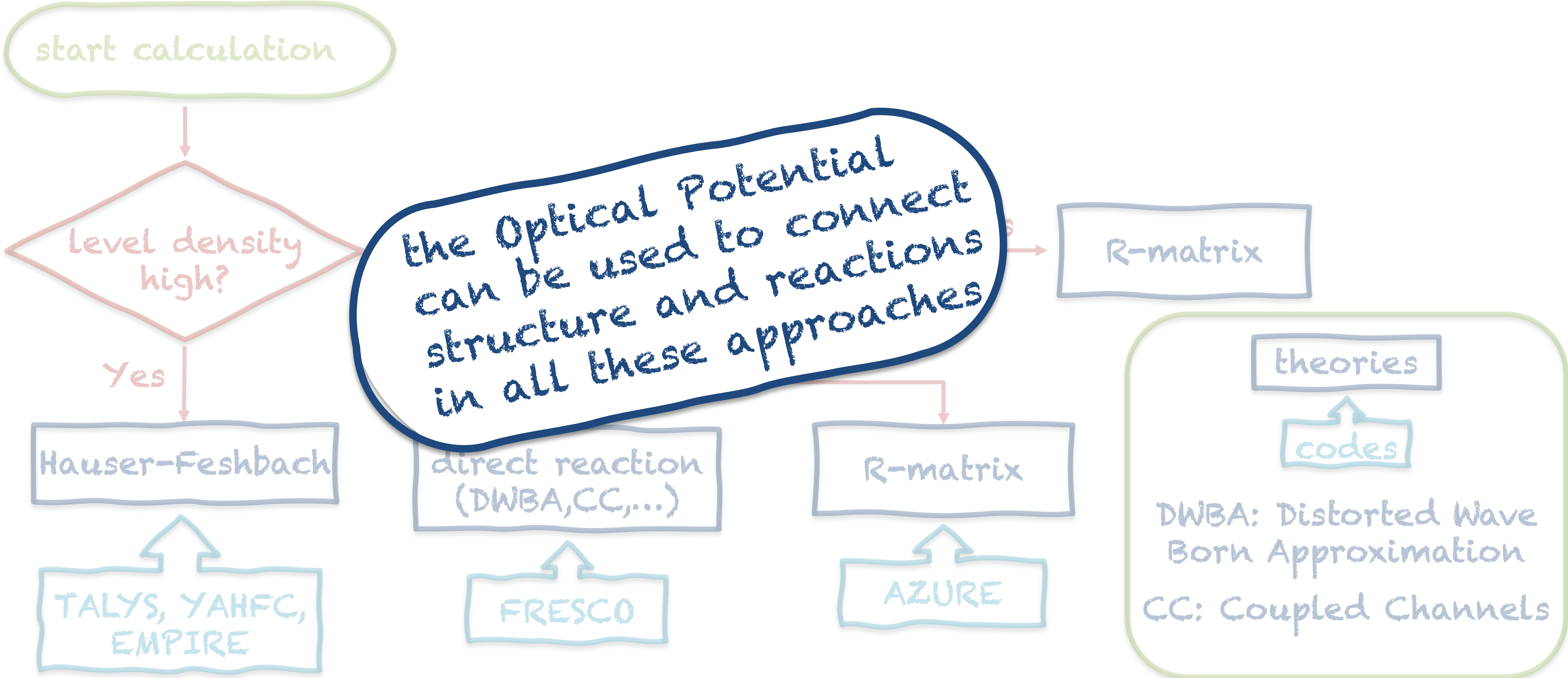
$$\begin{bmatrix} T+V_{00} & V_{01} & V_{02} & \cdots \\ V_{10} & T+V_{11} & V_{12} & \cdots \\ V_{20} & V_{21} & T+V_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \Rightarrow [T+V]$$

- The "optical reduction" transforms a many-body operator into a one-body operator
- It is a well-defined, in principle exact, mathematical operation

Nuclear reaction theorist's roadmap

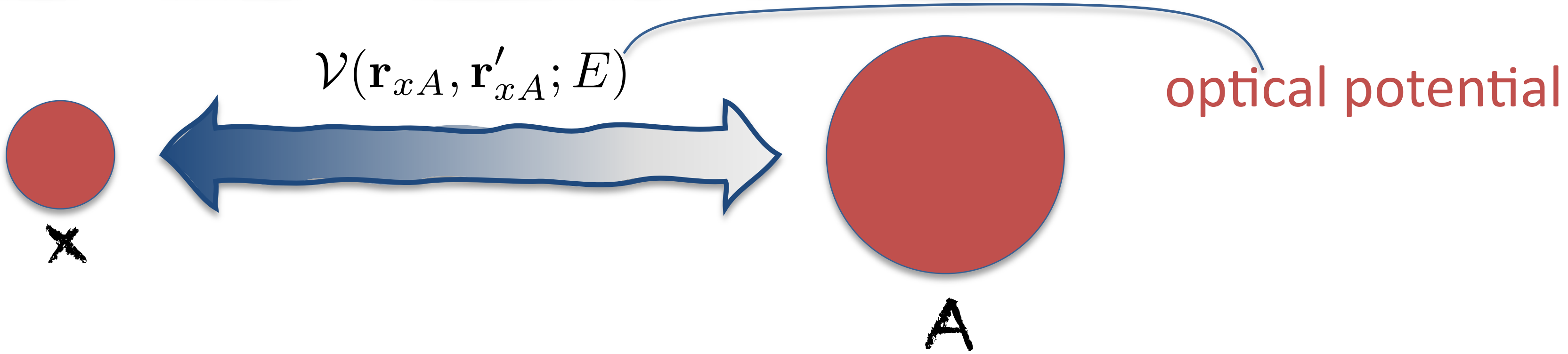


Nuclear reaction theorist's roadmap



The Green's Function Transfer (GFT) formalism

elastic scattering between 2 nuclei x and A



The Green's Function Transfer (GFT) formalism

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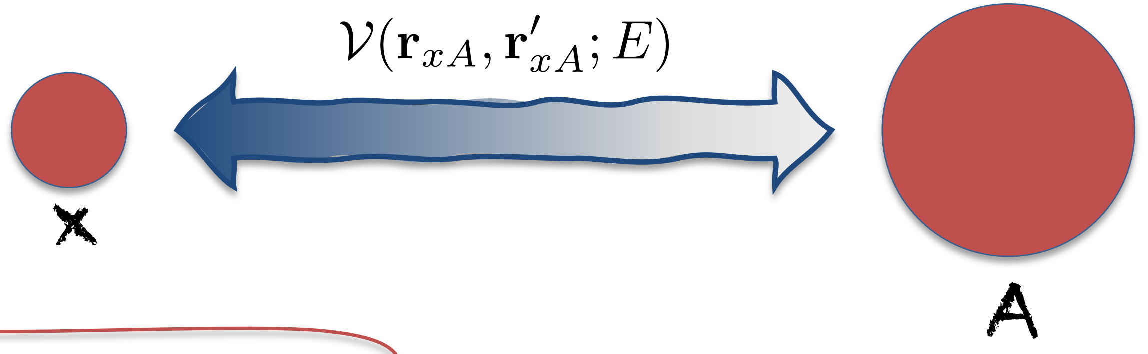


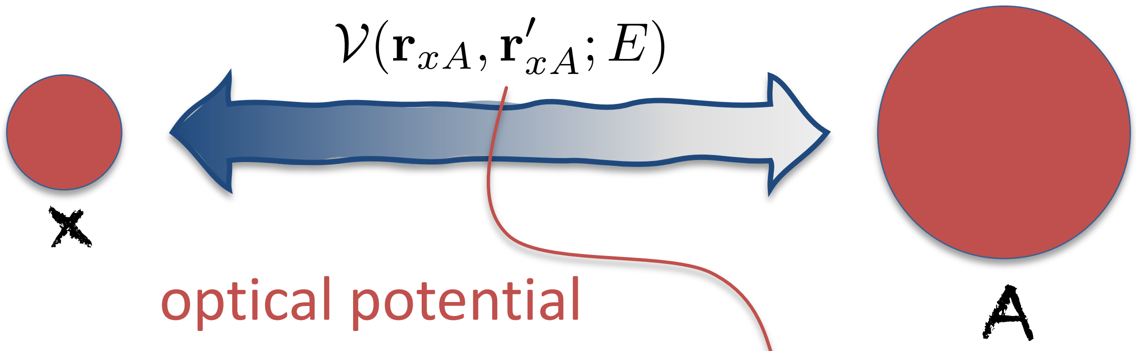
free wave

$$\psi_0(r_{xA}, E) = F(r_{xA}) + G(E)\,\mathcal{V}(E)\,F(r_{xA})$$

Lippmann-Schwinger equation

The Green's Function Transfer (GFT) formalism

elastic scattering between 2 nuclei x and A

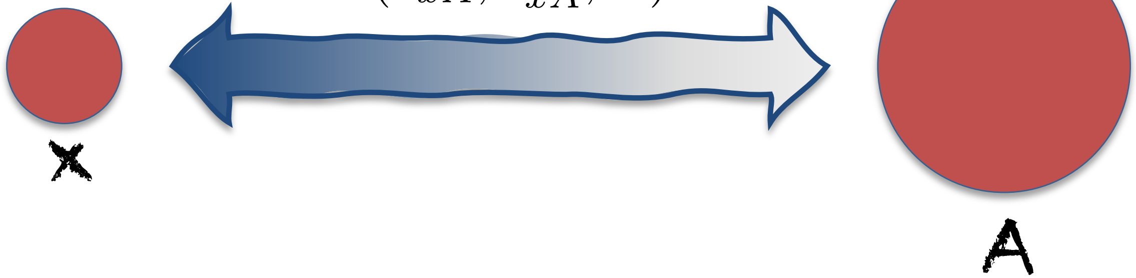


$$\psi_0(r_{xA}, E) = F(r_{xA}) + G(E)\,\mathcal{V}(E)\,F(r_{xA})$$

The Green's Function Transfer (GFT) formalism

elastic scattering between 2 nuclei x and A

$$\mathcal{V}(\mathbf{r}_{xA}, \mathbf{r}'_{xA}; E)$$



$$\psi_0(r_{xA}, E) = F(r_{xA}) + G(E)\,\mathcal{V}(E)\,F(r_{xA})$$

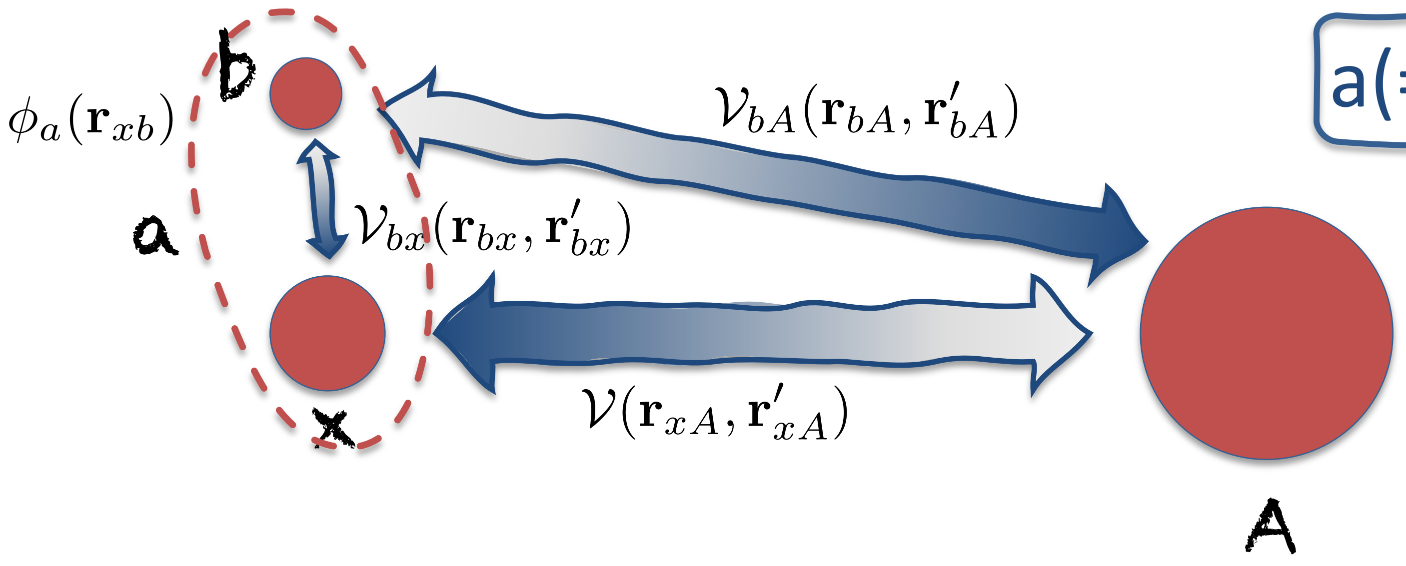
Green's function

$$G(E) = (E - T_x - \mathcal{V}(E))^{-1}$$

reaction cross section

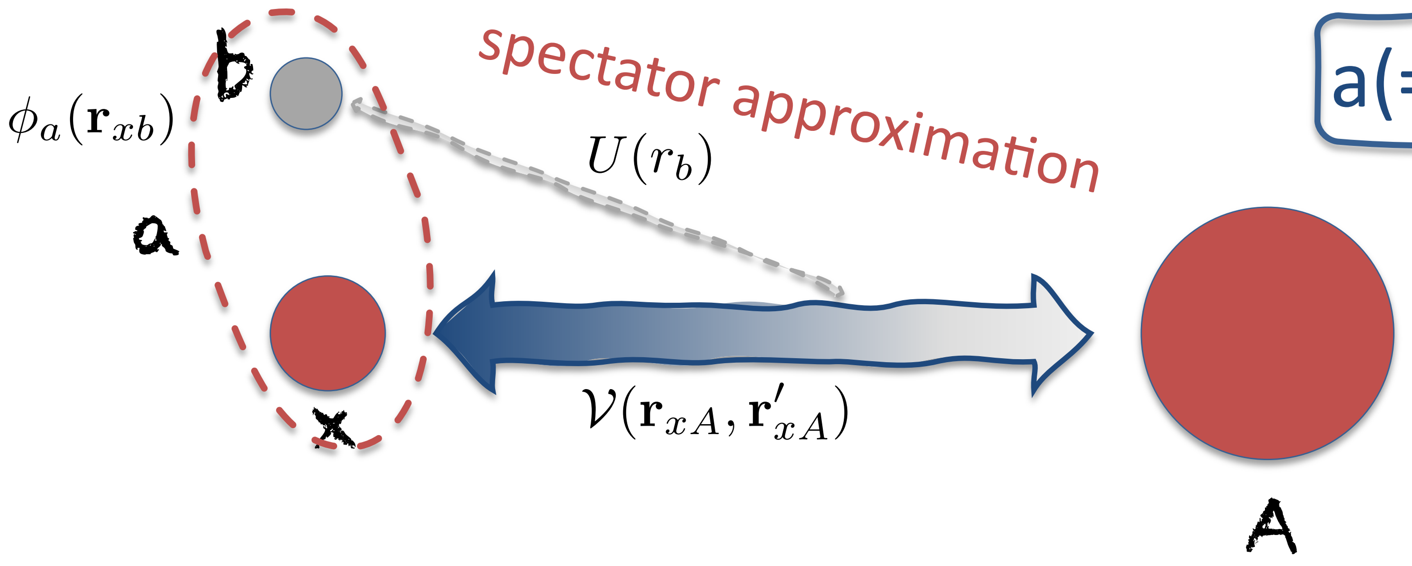
$$\sigma_R = \frac{2\mu}{\hbar k_x}\langle\psi_0|\mathrm{Im}\mathcal{V}|\psi_0\rangle$$

The Green's Function Transfer (GFT) formalism



a(=b+x)+A → b+B(=A+x)

The Green's Function Transfer (GFT) formalism



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The Green's Function Transfer (GFT) formalism

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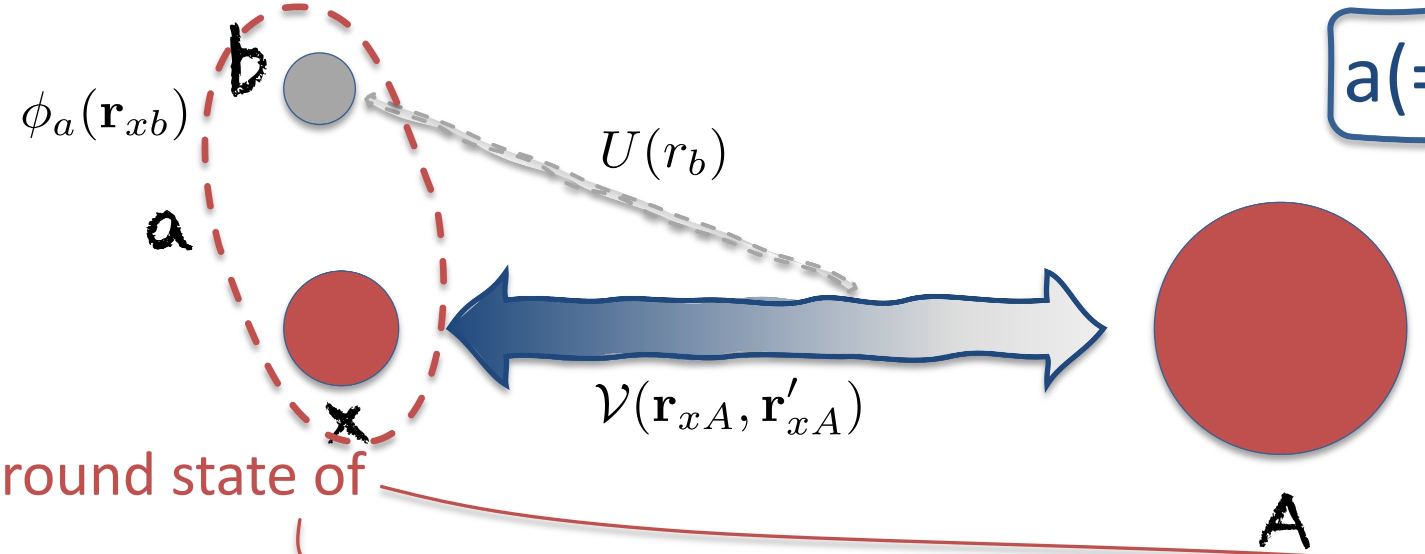
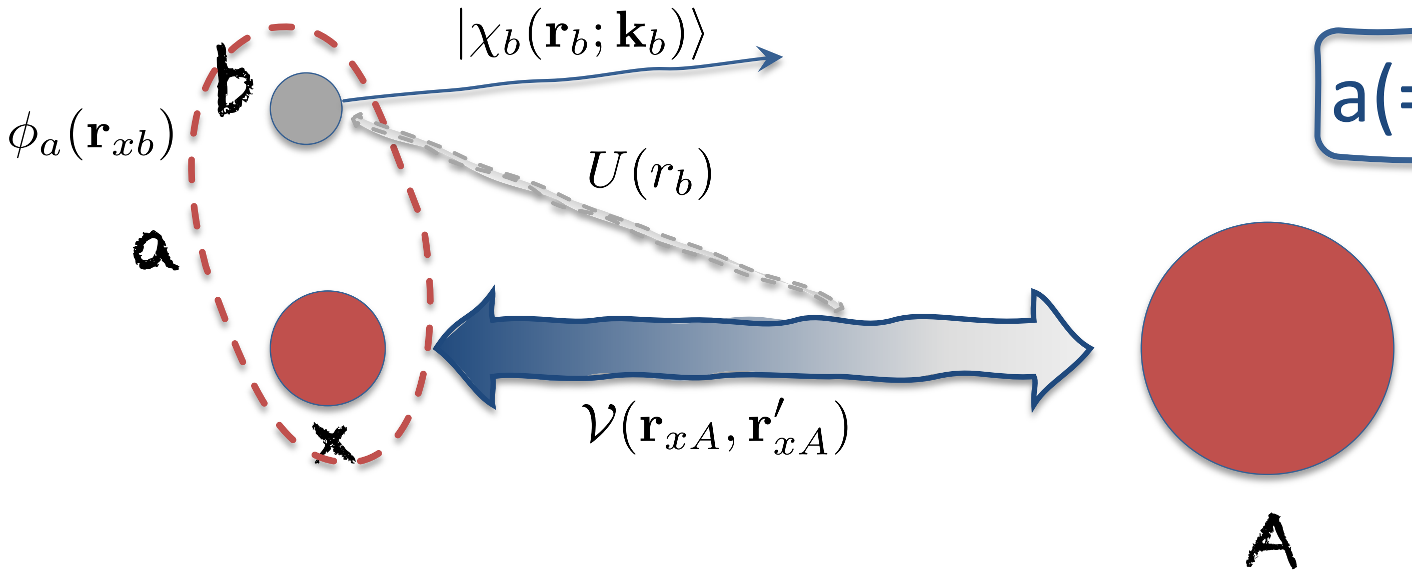


intrinsic ground state of nucleus a

$$\Psi_0(\mathbf{r}_{xA}, \mathbf{r}_b) = F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb}) + G(E - E_b)\mathcal{P}(\mathbf{r}_b)\left[\mathcal{V}(E - E_b) + U_b(\mathbf{r}_b)\right]F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb})$$

free wave for nucleus a

The Green's Function Transfer (GFT) formalism



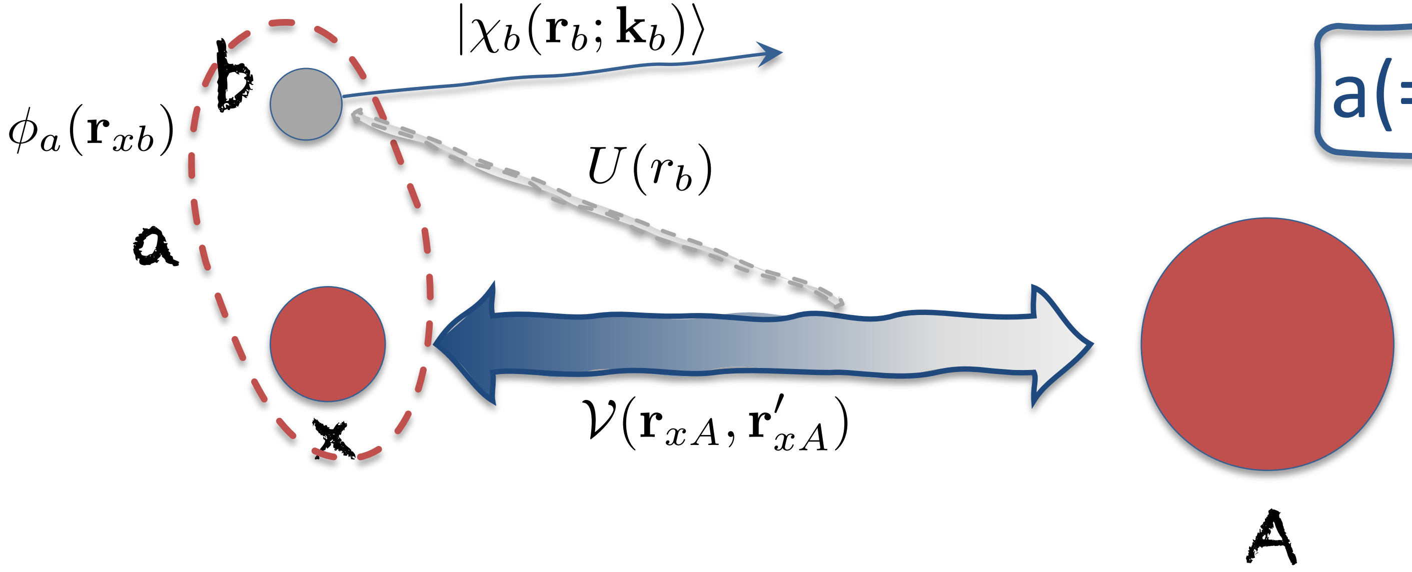
a(=b+x)+A ⟶ b+B(=A+x)

$$\Psi_0(\mathbf{r}_{xA},\mathbf{r}_b) = F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb}) + G(E-E_b)\mathcal{P}(\mathbf{r}_b)\big[\mathcal{V}(E-E_b) + U_b(r_b)\big]F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb})$$

projector over b states $\mathcal{P}(\mathbf{r}_b) = \int |\chi_b(\mathbf{r}_b;\mathbf{k}_b)\rangle\langle\chi_b(\mathbf{r}_b;\mathbf{k}_b)|\, d\mathbf{k}_b$

$$E_b = \frac{\hbar^2 k_b^2}{2\mu_b}$$

The Green's Function Transfer (GFT) formalism



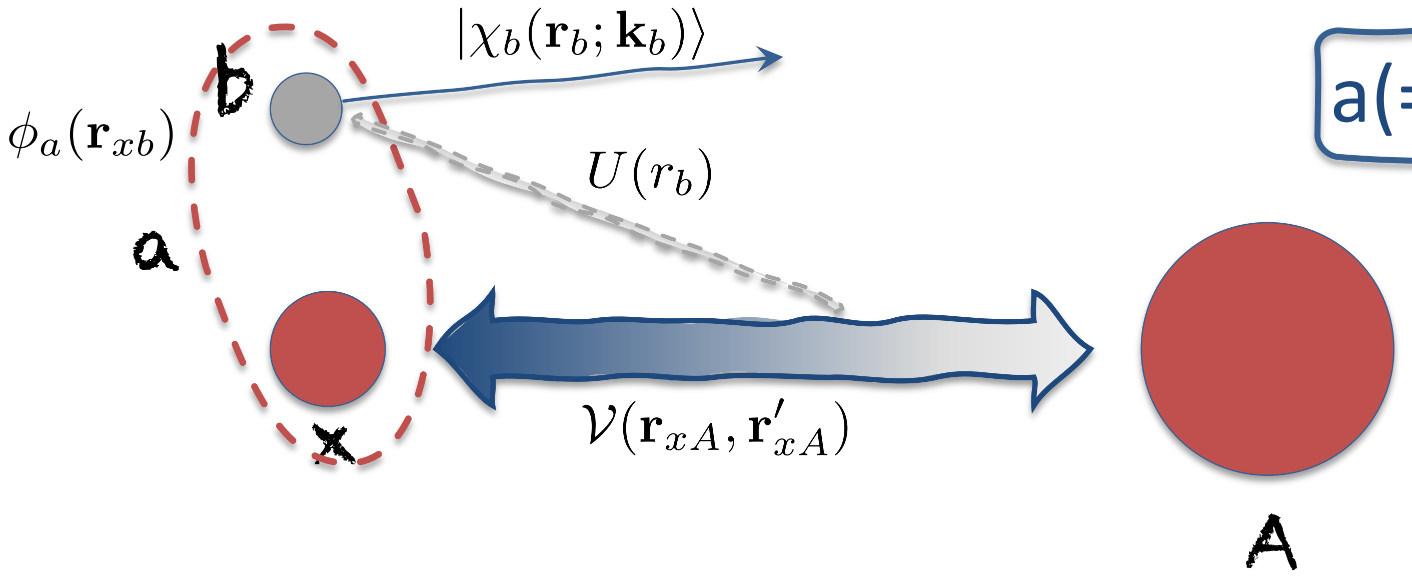
a(=b+x)+A ⟶ b+B(=A+x)

$$\Psi_0(\mathbf{r}_{xA},\mathbf{r}_b) = F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb}) + G(E-E_b)\mathcal{P}(\mathbf{r}_b)\left[\mathcal{V}(E-E_b)+U_b(\mathbf{r}_b)\right]F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb})$$

Green's function $G(E) = (E - T_x - \mathcal{V}(E))^{-1}$

Same as for x-A scattering!

The Green's Function Transfer (GFT) formalism

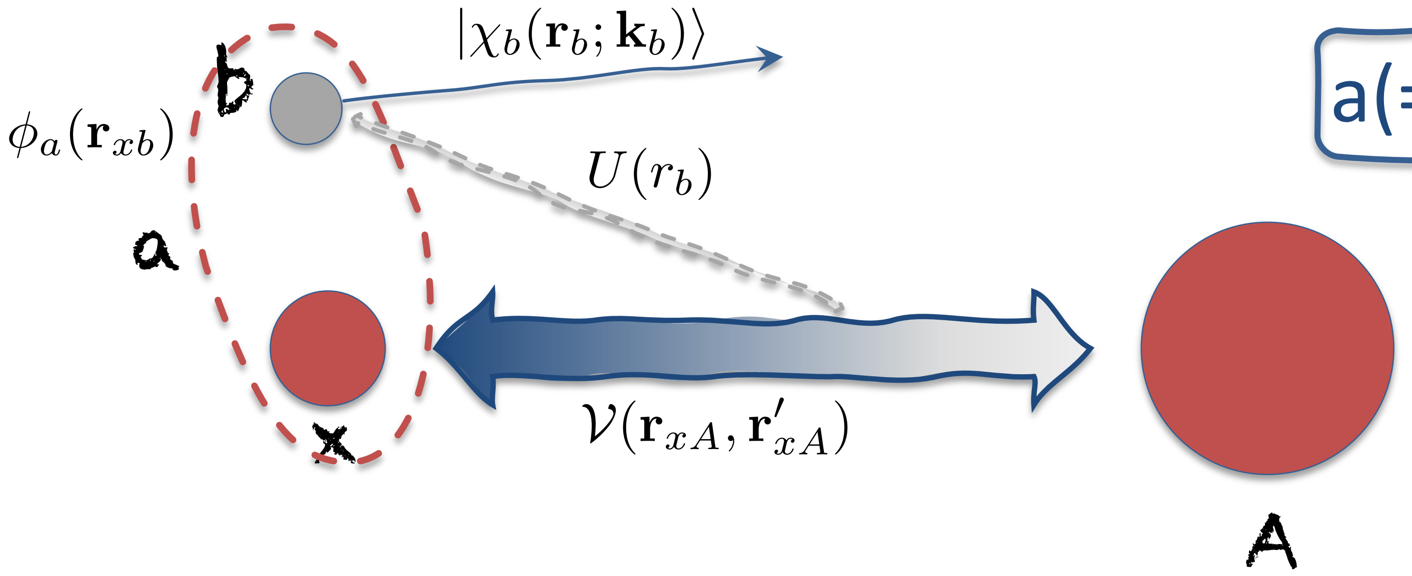


a(=b+x)+A ⟶ b+B(=A+x)

$$\Psi_0(\mathbf{r}_{xA},\mathbf{r}_b) = F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb}) + G(E-E_b)\mathcal{P}(\mathbf{r}_b)\left[\mathcal{V}(E-E_b)+U_b(\mathbf{r}_b)\right]F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb})$$

factorized propagator

The Green's Function Transfer (GFT) formalism

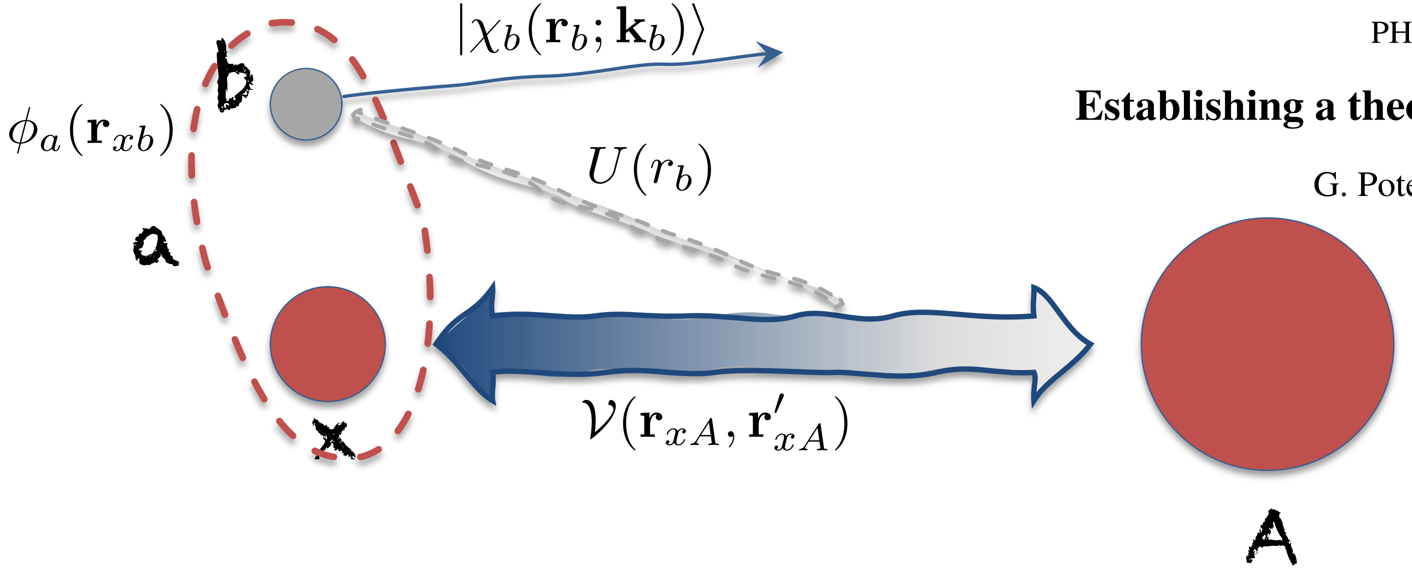


a(=b+x)+A ⟶ b+B(=A+x)

$$\langle\chi_b(\mathbf{r}_b;\mathbf{k}_b)|\Psi_0(\mathbf{r}_{xA},\mathbf{r}_b) = \langle\chi_b(\mathbf{r}_b;\mathbf{k}_b)|\left(F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb}) + G(E-E_b)\mathcal{P}(\mathbf{r}_b)\left[\mathcal{V}(E-E_b)+U_b(\mathbf{r}_b)\right]F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb})\right)$$

project over b state to get x-A wavefunction

The Green's Function Transfer (GFT) formalism



PHYSICAL REVIEW C **92**, 034611 (2015)

Establishing a theory for deuteron-induced surrogate reactions

G. Potel,[1,2] F. M. Nunes,[1,3] and I. J. Thompson[2]

$$\psi_0^I(\mathbf{r}_{xA},E) = \psi^{HM}(\mathbf{r}_a) + G(E-E_b)\left[\mathcal{V}(E-E_b)\psi^{HM} + \langle\chi_b|U_b(\mathbf{r}_b)\right]F(\mathbf{r}_a)\phi_a(\mathbf{r}_{xb})$$

$$\psi^{HM}(\mathbf{r}_{xA}) = \int \chi_b^*(\mathbf{r}_{bB},\mathbf{k}_b)\phi_a(\mathbf{r}_{xb})F(\mathbf{r}_{aA})\,d\mathbf{r}_{xb}$$

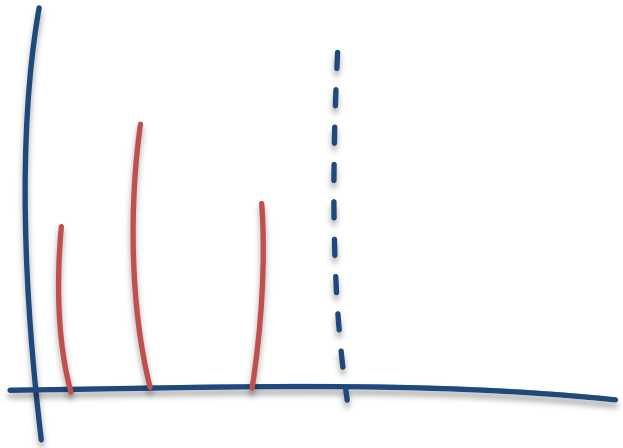
Hussein-McVoy term

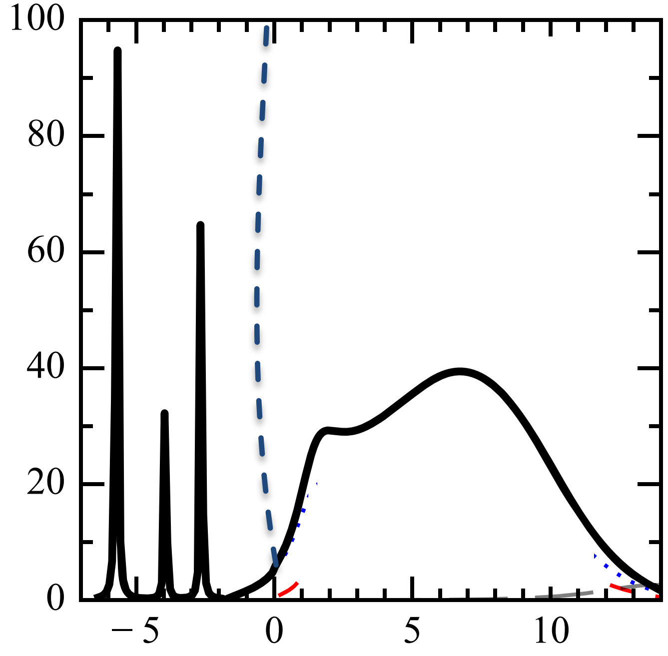
$$\sigma_R^I(E,E_b) = \frac{2\mu}{\hbar k_x}\langle\psi_0^I|\mathrm{Im}\mathcal{V}(E-E_b)|\psi_0^I\rangle$$

inclusive x–A cross section

DWBA vs GFT

$$\sigma_{i0}^{DWBA} \sim |\langle \psi_i | V | \psi_0 \rangle|^2 \qquad \sigma_R^{GFT}(E) \sim \langle G(E)\left(\mathcal{V}(E) + U_b\right)\psi^{HM} | \mathrm{Im}\mathcal{V}(E) | G(E)\left(\mathcal{V}(E) + U_b\right)\psi^{HM} \rangle$$





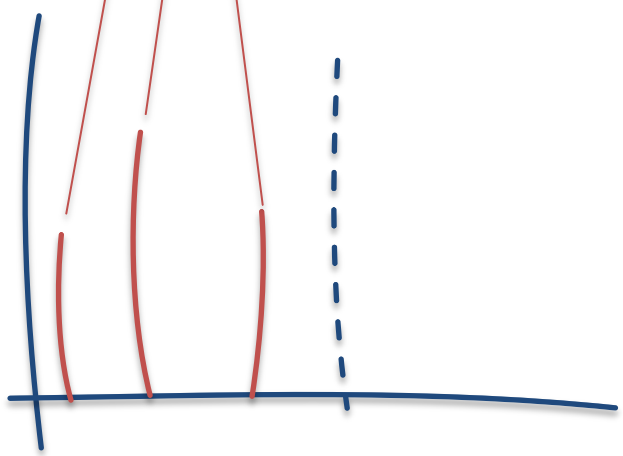
DWBA vs GFT

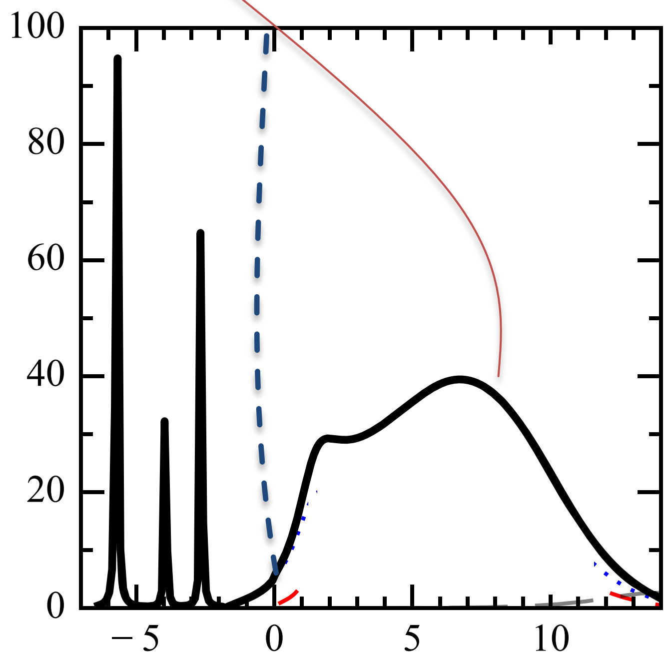
$$\sigma_{i0}^{DWBA} \sim |\langle\psi_i|V|\psi_0\rangle|^2$$

$$\sigma_R^{GFT}(E) \sim \langle G(E)\,(\mathcal{V}(E)+U_b)\,\psi^{HM}|\mathrm{Im}\mathcal{V}(E)|G(E)\,(\mathcal{V}(E)+U_b)\,\psi^{HM}\rangle$$

discrete final states

continuous function of E





DWBA vs GFT

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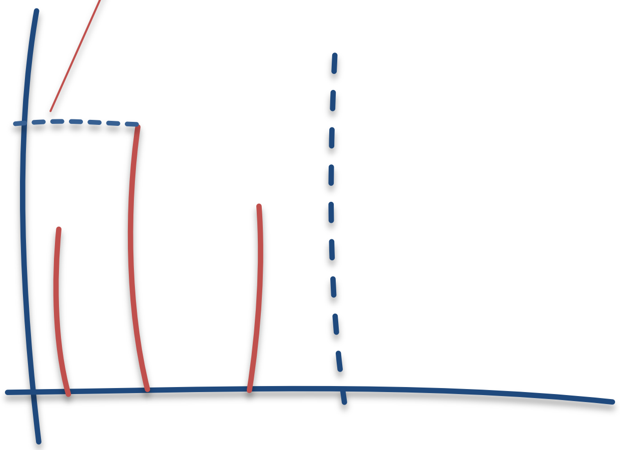
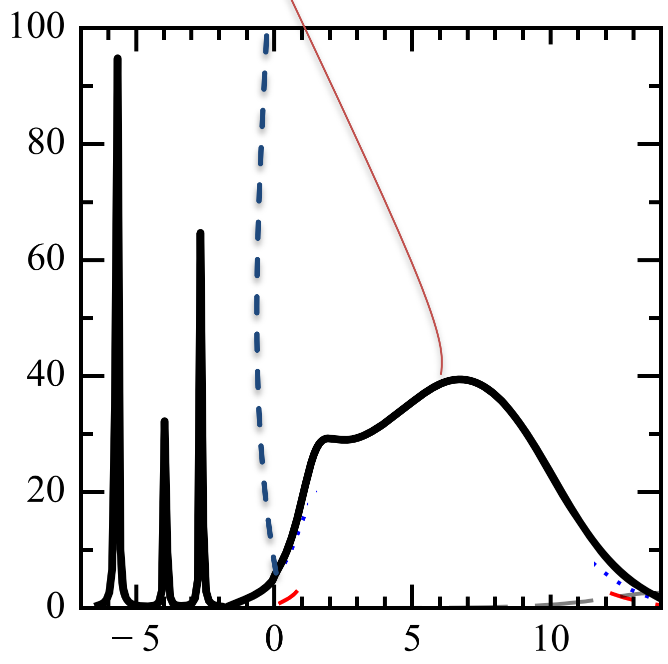
consistent normalization

extracted S=σ_{exp}/σ_{th}





DWBA vs GFT

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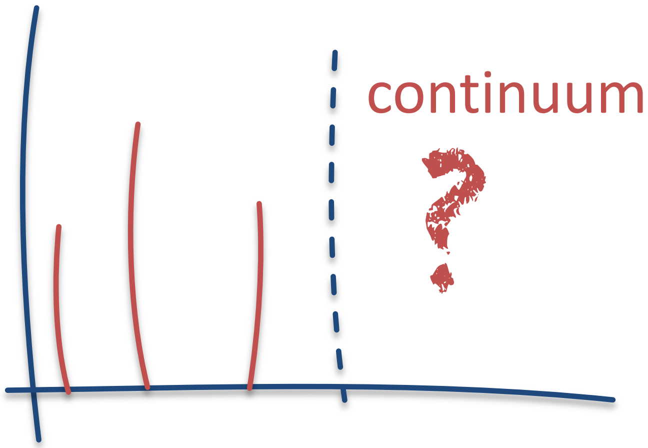
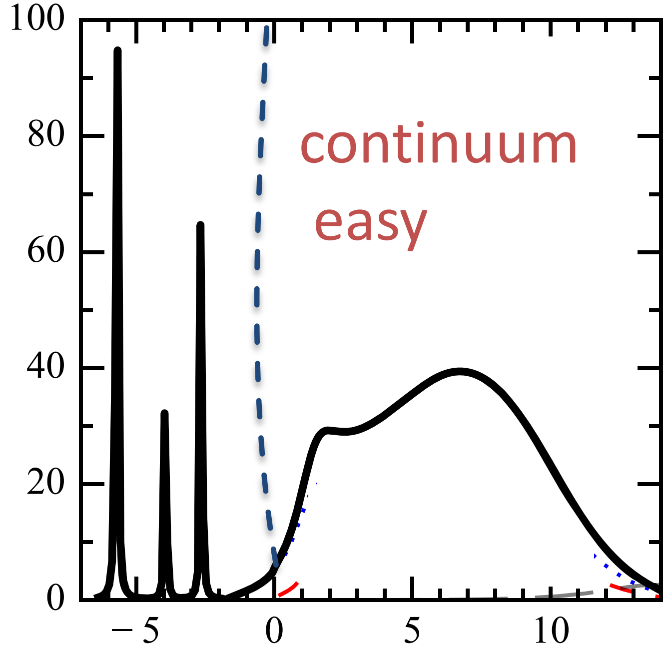
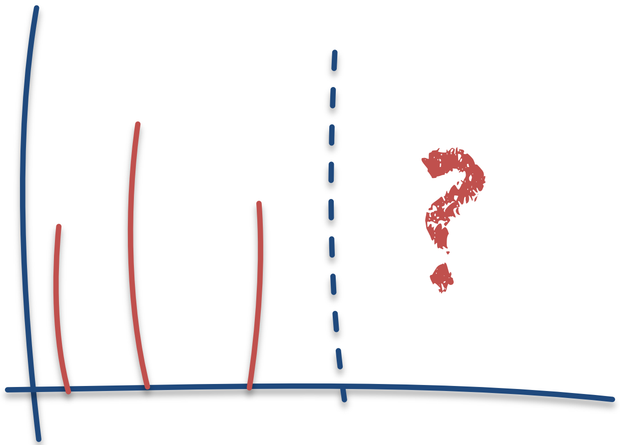




DWBA vs GFT

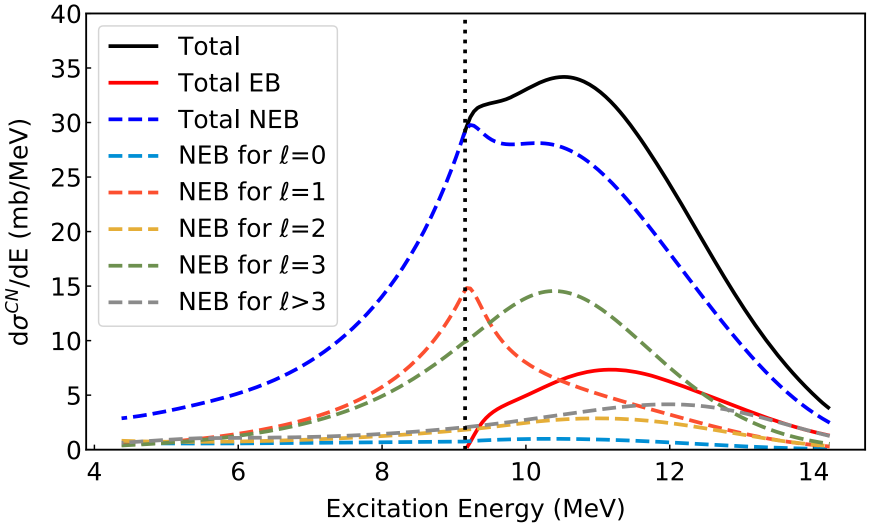
$$\sigma_{i0}^{DWBA} \sim |\langle \psi_i | V | \psi_0 \rangle|^2 \qquad \sigma_R^{GFT}(E) \sim \langle G(E)\left(\mathcal{V}(E) + U_b\right)\psi^{HM} | \mathrm{Im}\mathcal{V}(E) | G(E)\left(\mathcal{V}(E) + U_b\right)\psi^{HM} \rangle$$



$$G(E) = \left(E - T - \mathcal{V}(E)\right)^{-1}$$

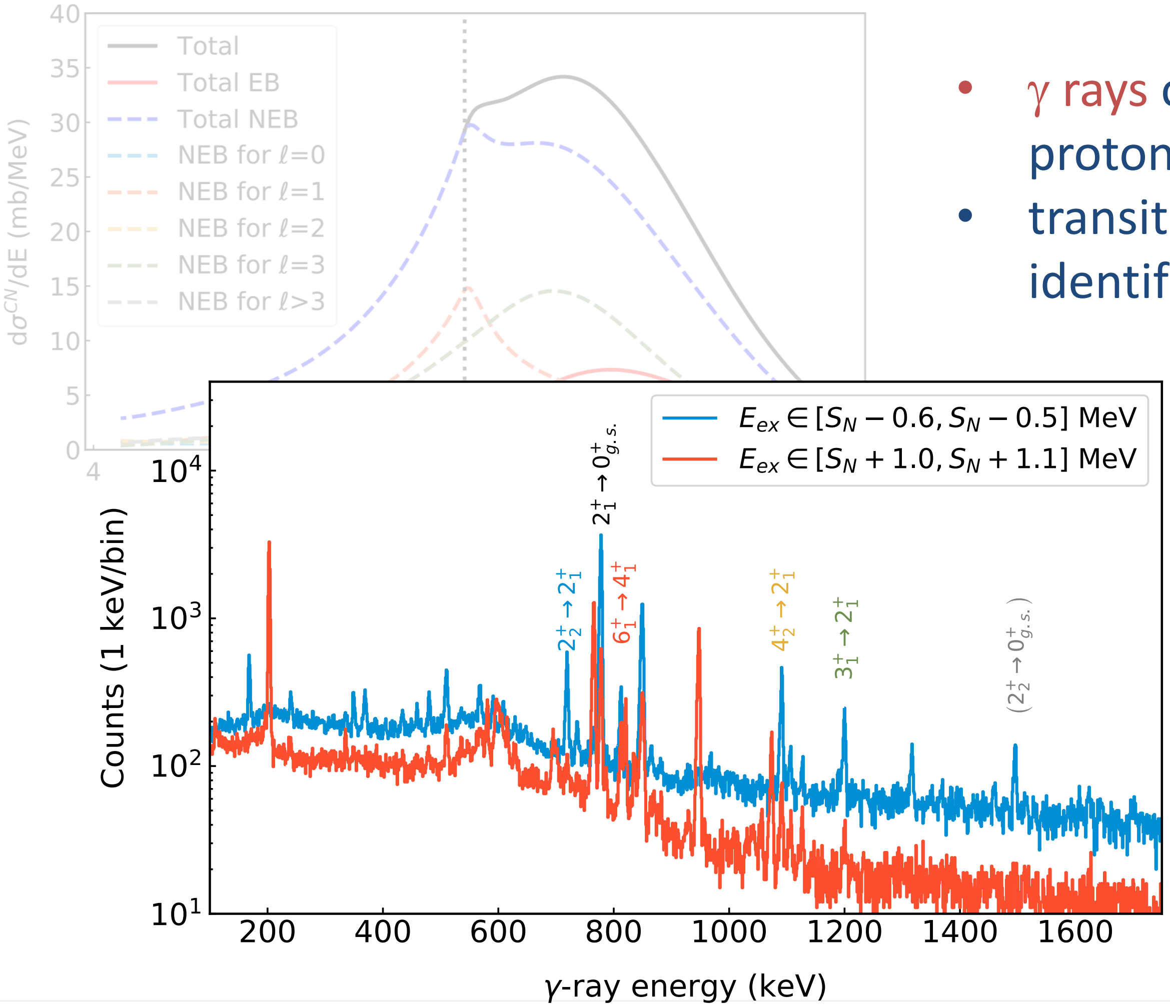
- Consistency between structure and reactions
- Same ingredients as x–A scattering
- Need for tools for inverting Hamiltonians with non-local potentials

Applications Koning-Delaroche: $^{95}Mo(d,p\gamma)$ for (n,γ)



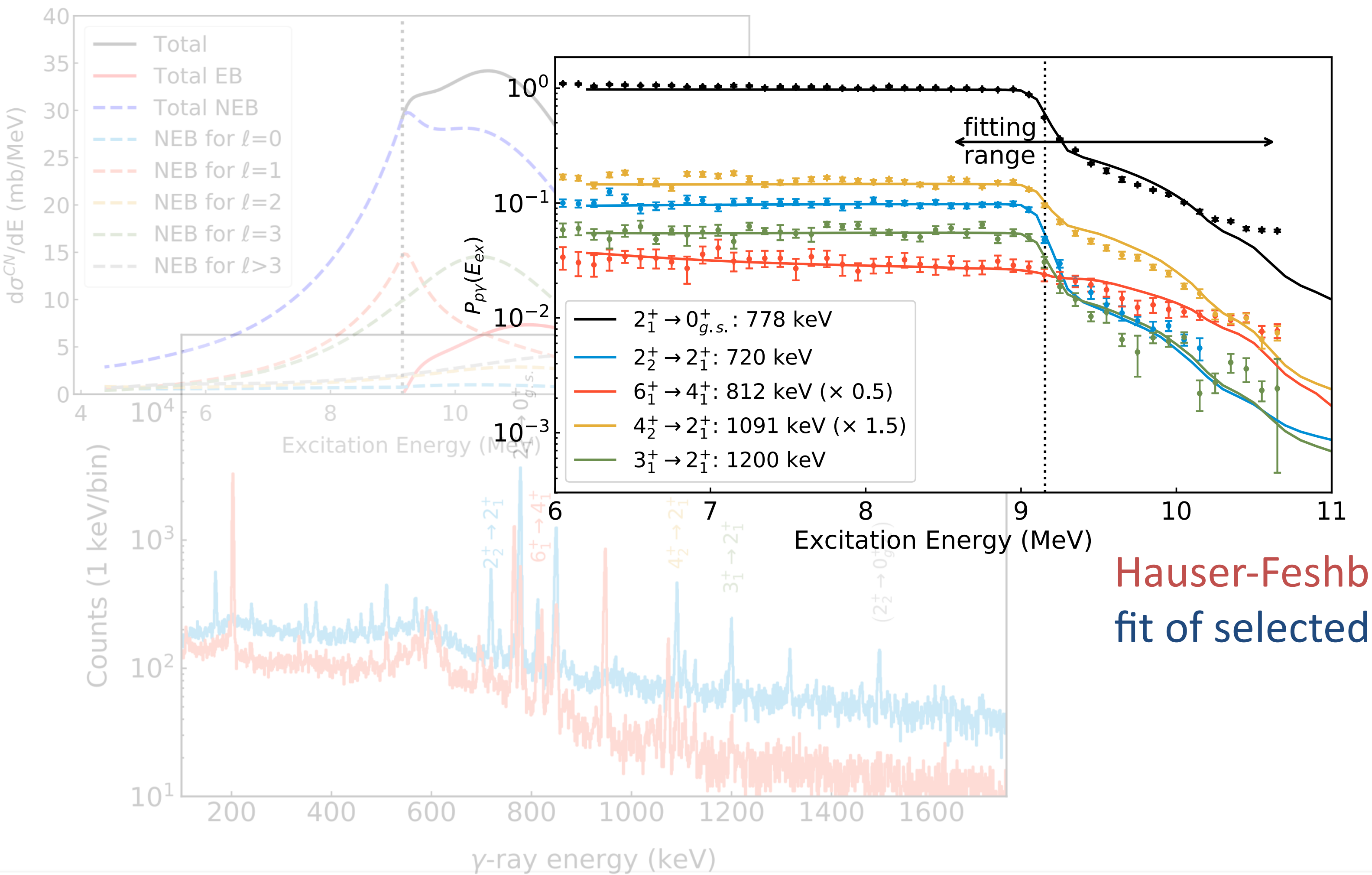
- Absorption of the neutron as a function of excitation energy and spin computed with GFT formalism
- We used the phenomenological Koning-Delaroche OP

Applications Koning-Delaroche: ^{95}Mo(d,pγ) for (n,γ)



- γ rays observed in coincidence with protons
- transitions from both ^{95}Mo and ^{96}Mo are identified

Applications Koning-Delaroche: ^{95}Mo(d,pγ) for (n,γ)



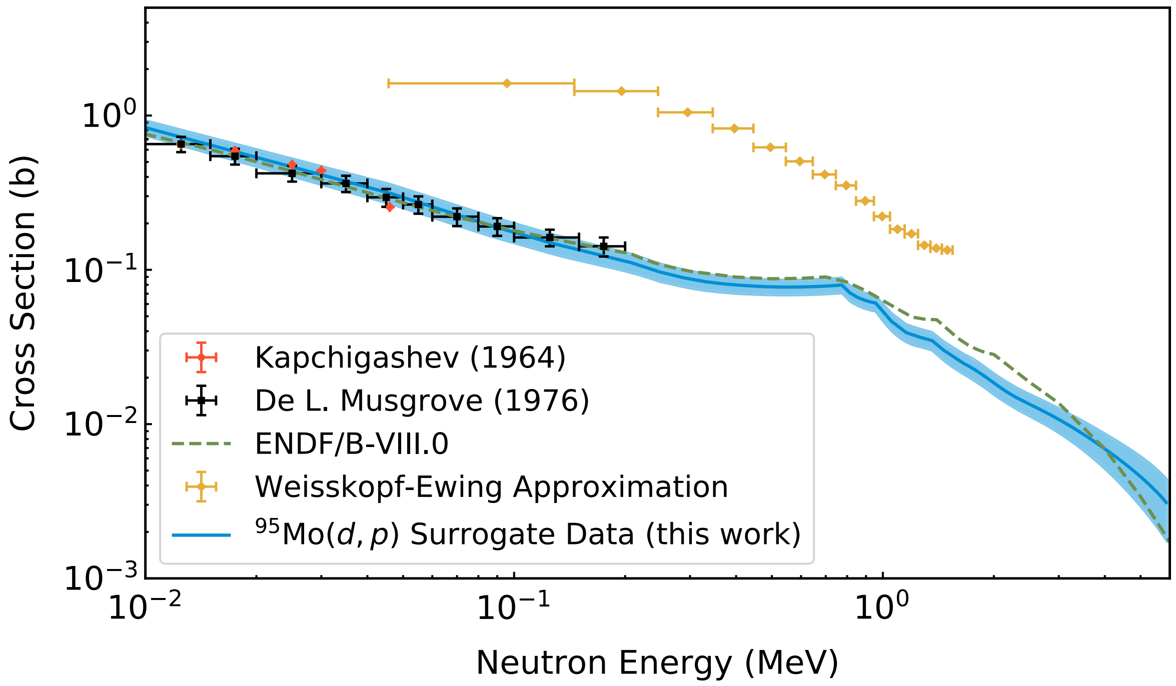
Hauser-Feshbach parameters determined by fit of selected γ lines (J. Escher)

Applications Koning-Delaroche: ^{95}Mo(d,pγ) for (n,γ)

PHYSICAL REVIEW LETTERS **122,** 052502 (2019)

Towards Neutron Capture on Exotic Nuclei: Demonstrating (*d,pγ*) as a Surrogate Reaction for (*n,γ*)

A. Ratkiewicz,[1,2,*] J. A. Cizewski,[2] J. E. Escher,[1] G. Potel,[3,4] J. T. Burke,[1] R. J. Casperson,[1] M. McCleskey,[5] R. A. E. Austin,[6] S. Burcher,[2] R. O. Hughes,[1,7] B. Manning,[2] S. D. Pain,[8] W. A. Peters,[9] S. Rice,[2] T. J. Ross,[7] N. D. Scielzo,[1] C. Shand,[2,10] and K. Smith[11]



- The obtained Hauser-Feshbach parameters are used to calculate (n,γ)
- We found an excellent agreement with the direct measurement.

Applications Koning-Delaroche: ^{95}Mo(d,pγ) for (n,γ)

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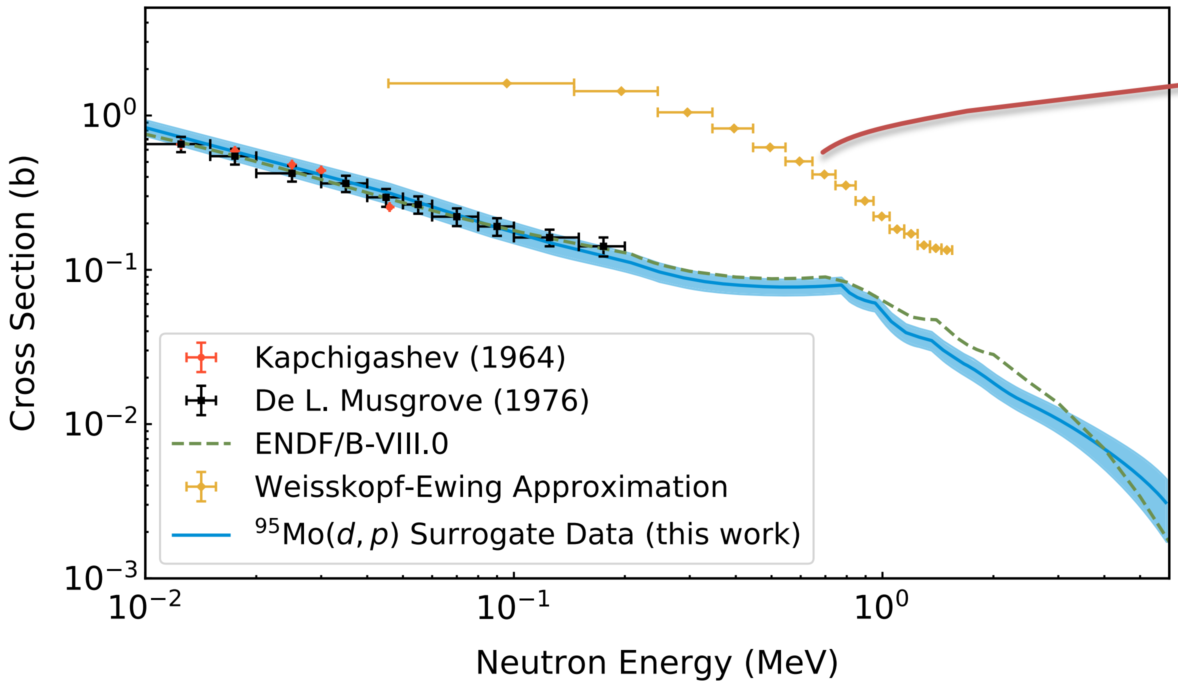


a failure to account for the initial spin distribution (Weisskopf-Ewing approximation) leads to poor results!

- The obtained Hauser-Feshbach parameters are used to calculate (n,γ)
- We found an excellent agreement with the direct measurement.

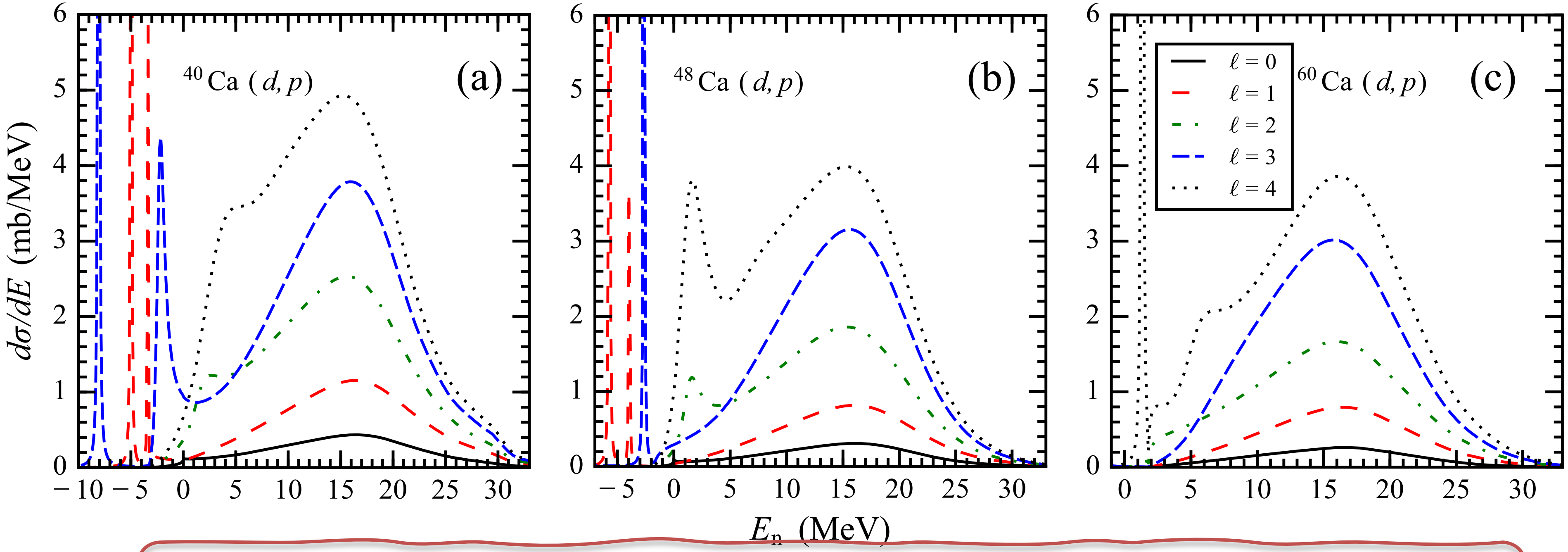
Applications: Dispersive Optical Model (DOM); Ca(d,p)

Eur. Phys. J. A (2017) **53**: 178

THE EUROPEAN PHYSICAL JOURNAL A

Toward a complete theory for predicting inclusive deuteron breakup away from stability

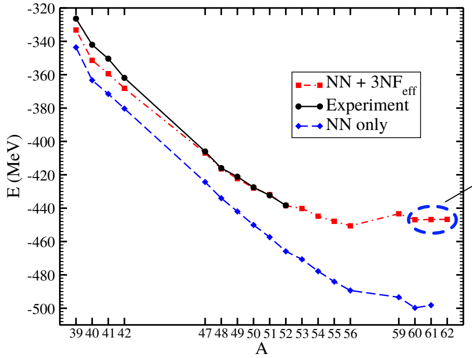
G. Potel[1,a], G. Perdikakis[1,2,3,b], B.V. Carlson[4,c], M.C. Atkinson[5], W.H. Dickhoff[5], J.E. Escher[6], M.S. Hussein[4,7,8], J. Lei[9,d], W. Li[1], A.O. Macchiavelli[10], A.M. Moro[9], F.M. Nunes[1,11], S.D. Pain[12], and J. Rotureau[1]



- Dispersive: reproduction of positive and negative energy cross section
- Controlled extrapolation to exotic nuclei

Applications: Dispersive Optical Model (DOM); Ca(d,p)

is ^{61}Ca bound?

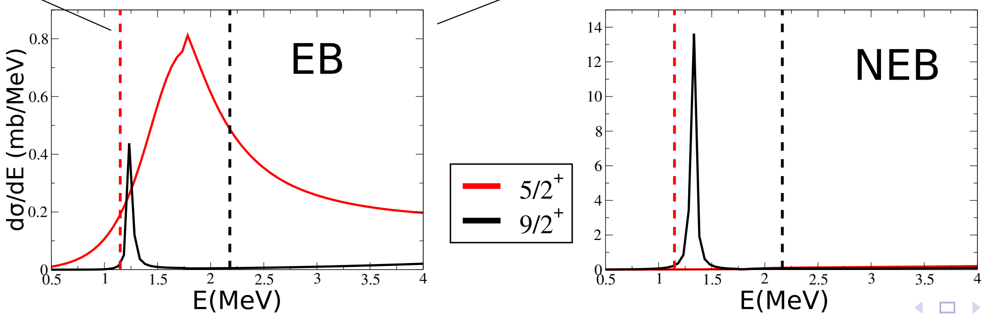


NO (but too close to be conclusive)

	^{53}Ca		^{55}Ca		^{61}Ca	
J^{π}	Re[E]	Γ	Re[E]	Γ	Re[E]	Γ
$5/2^+$	1.99	1.97	1.63	1.33	1.14	0.62
$9/2^+$	4.75	0.28	4.43	0.23	2.19	0.02

coupled cluster
Hagen et al., PRL **109**, 032502 (2012)

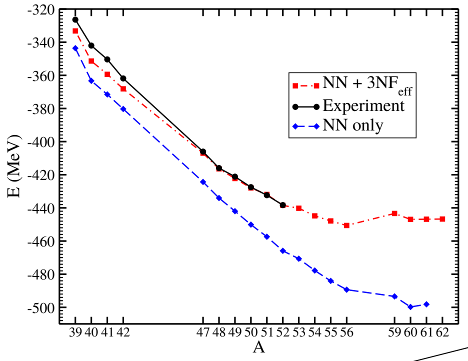
NO



^{60}Ca(d,p) with DOM

Applications: Dispersive Optical Model (DOM); Ca(d,p)

inversion of 9/2+ and 5/2+ orbitals?



YES

	^{53}Ca		^{55}Ca		^{61}Ca	
J^π	Re[E]	Γ	Re[E]	Γ	Re[E]	Γ
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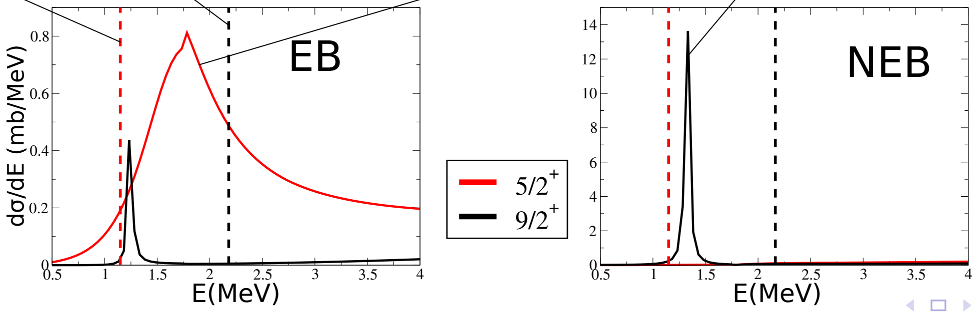
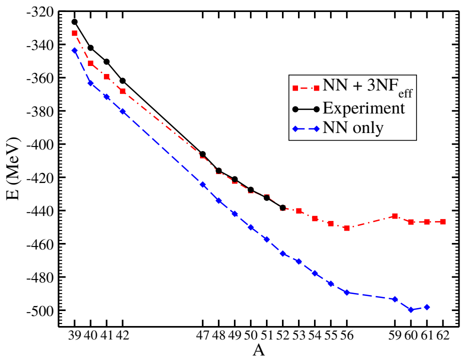


coupled cluster
Hagen et al., PRL **109**, 032502 (2012)

^{60}Ca(d,p) with DOM

Applications: Dispersive Optical Model (DOM); Ca(d,p)



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J^π	Re[E]	Γ	Re[E]	Γ	Re[E]	Γ
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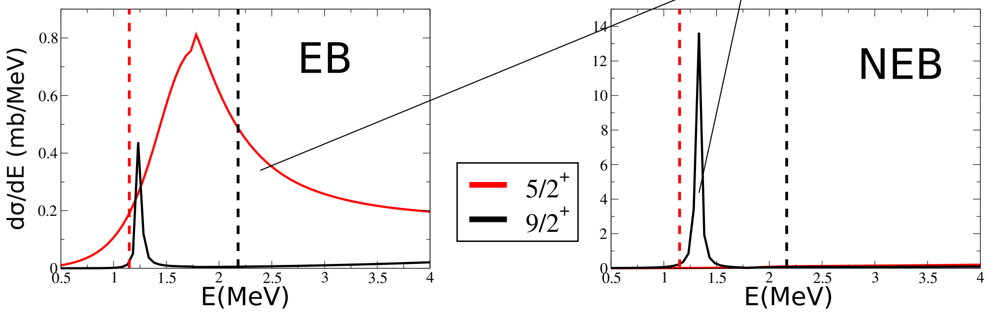
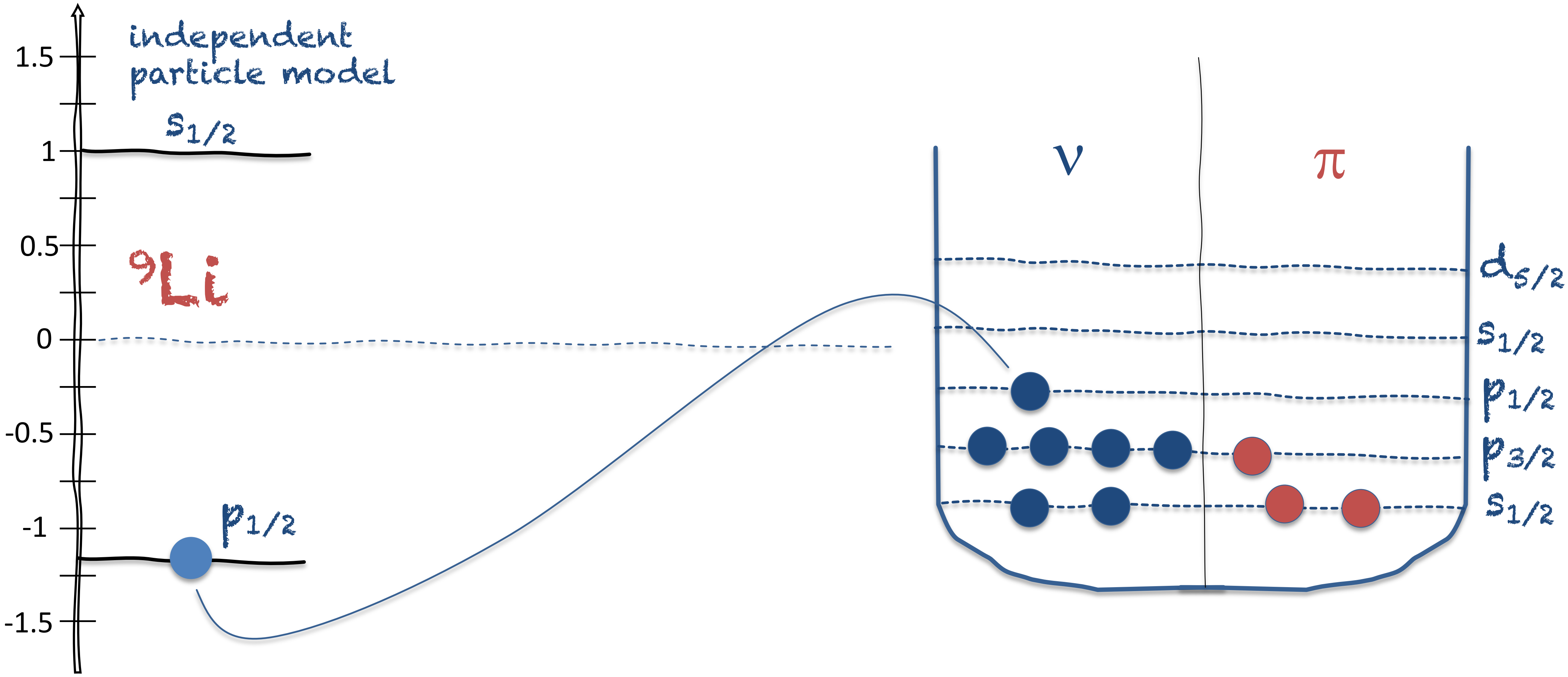
transfer strength function informs about widths

coupled cluster
Hagen et al., PRL **109**, 032502 (2012)

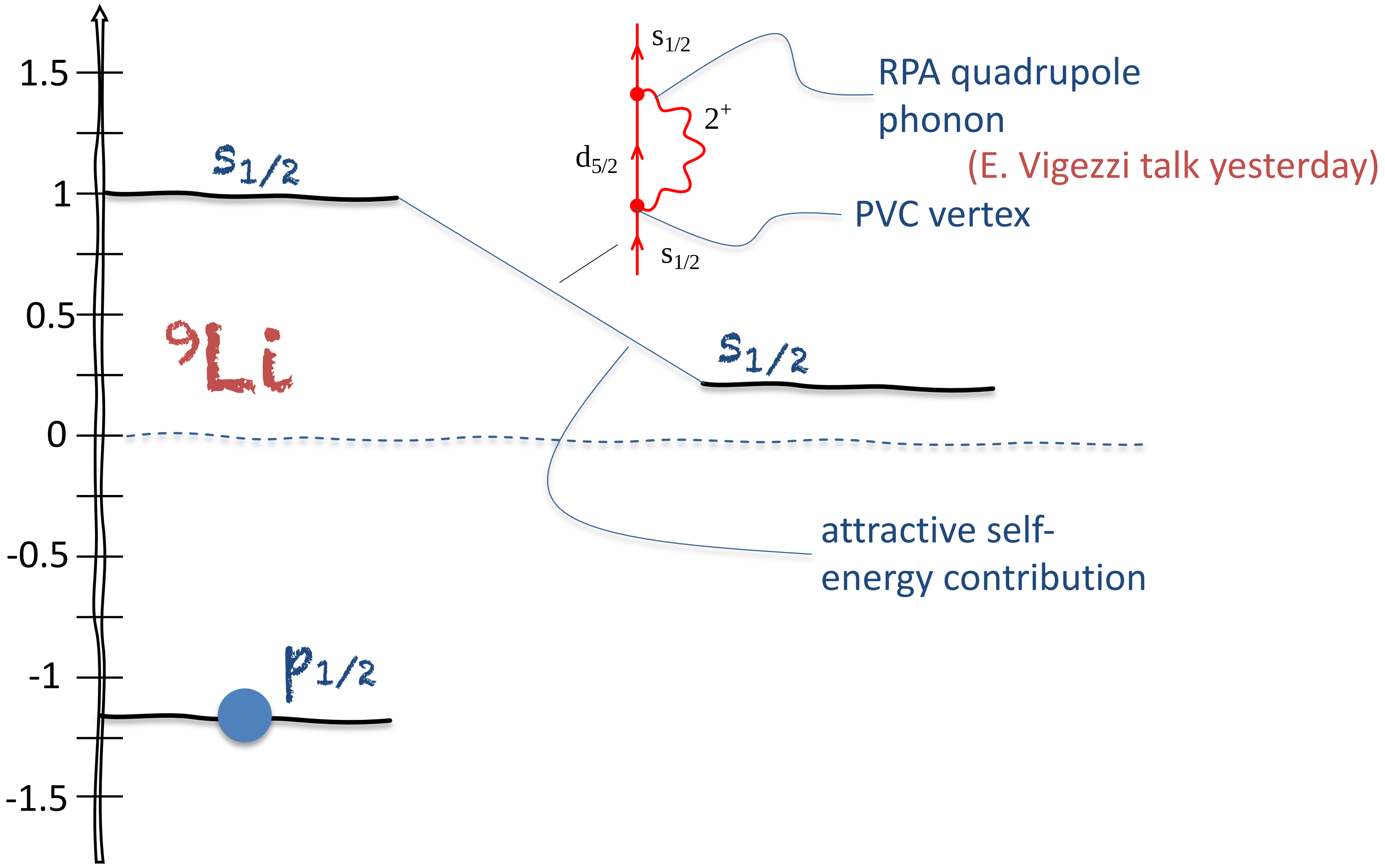


^{60}Ca(d,p) with DOM

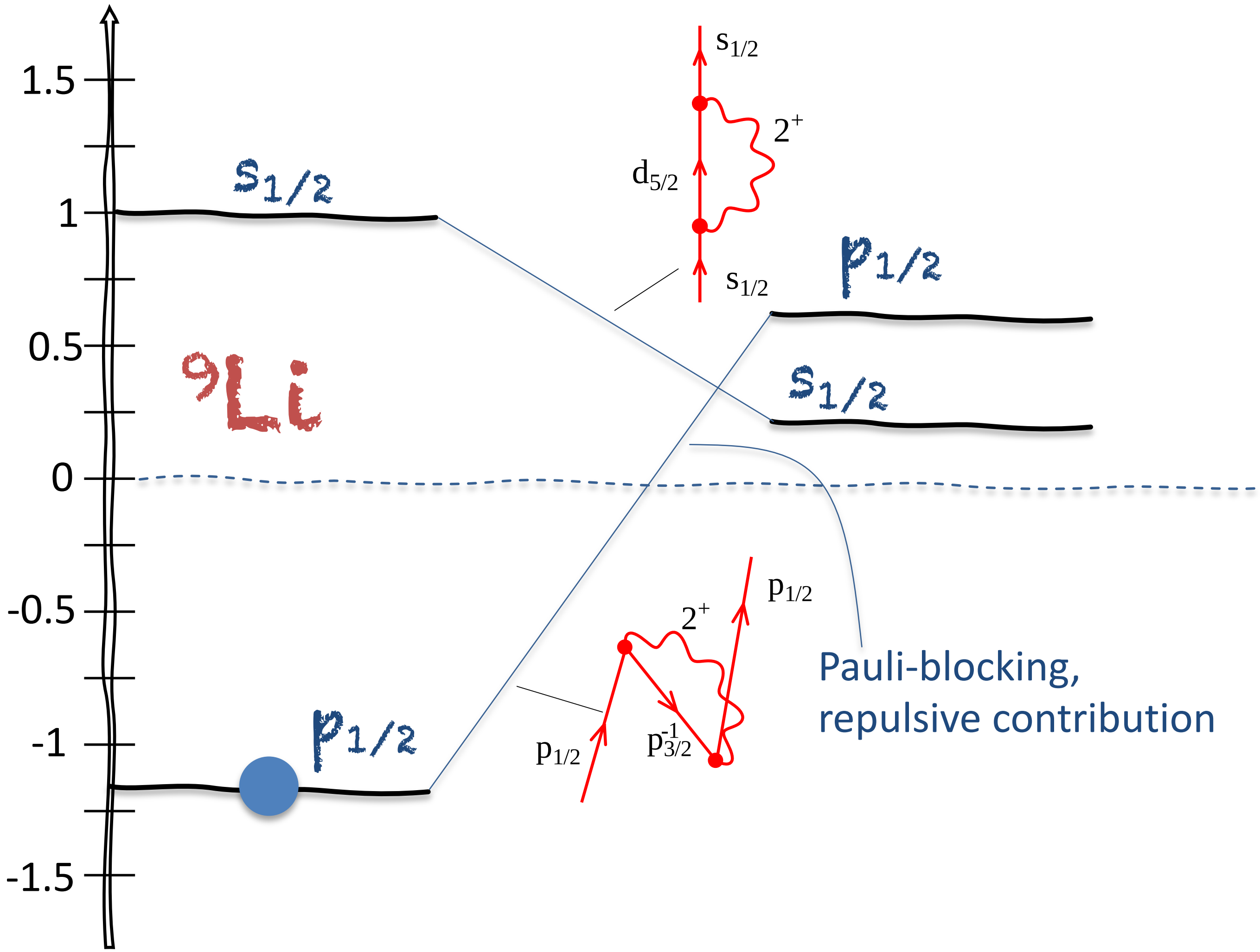
Applications: Nuclear Field Theory (NFT); ^{9}Li(d,p)



Applications: Nuclear Field Theory (NFT); ^{9}Li(d,p)

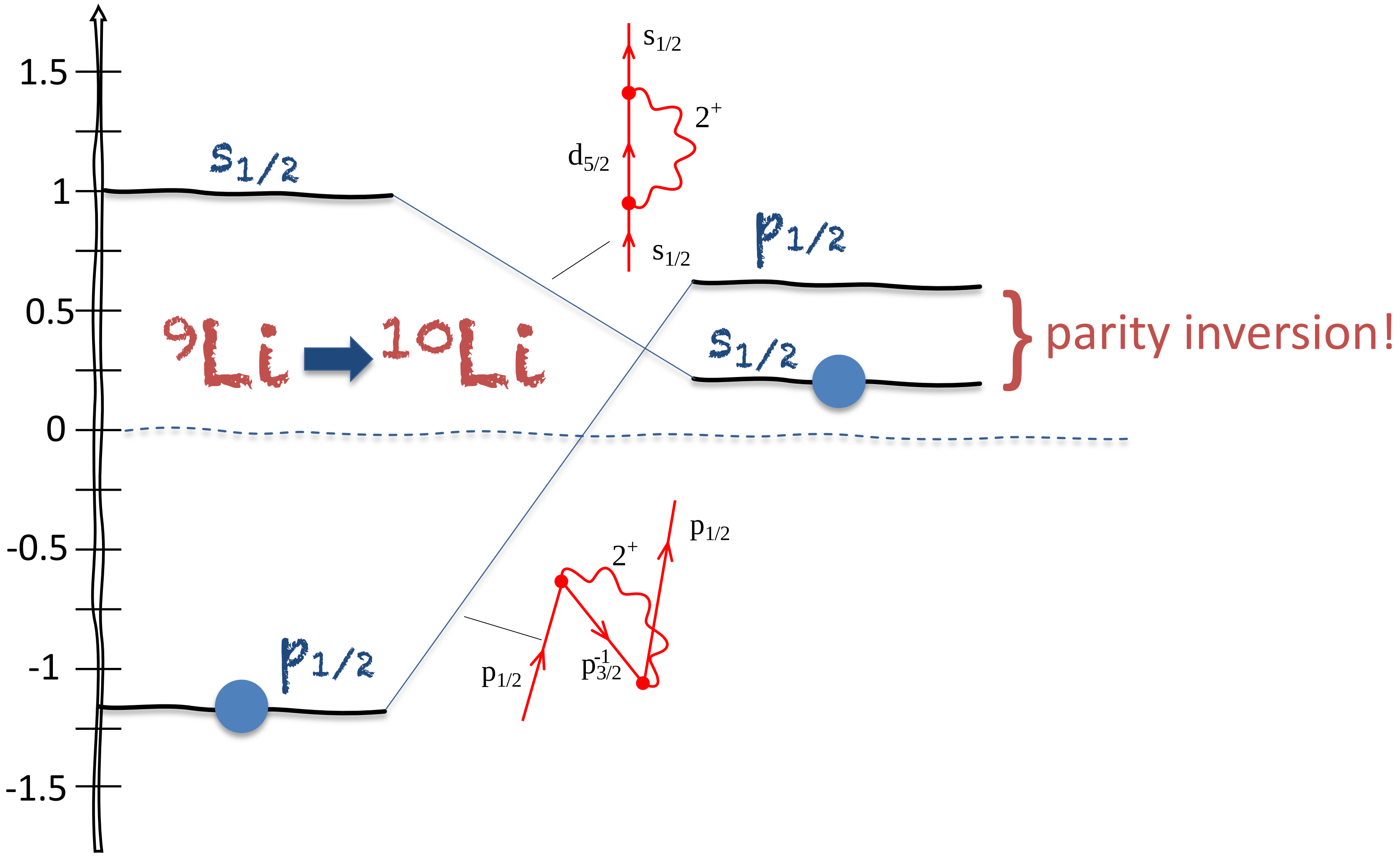


Applications: Nuclear Field Theory (NFT); ^{9}Li(d,p)

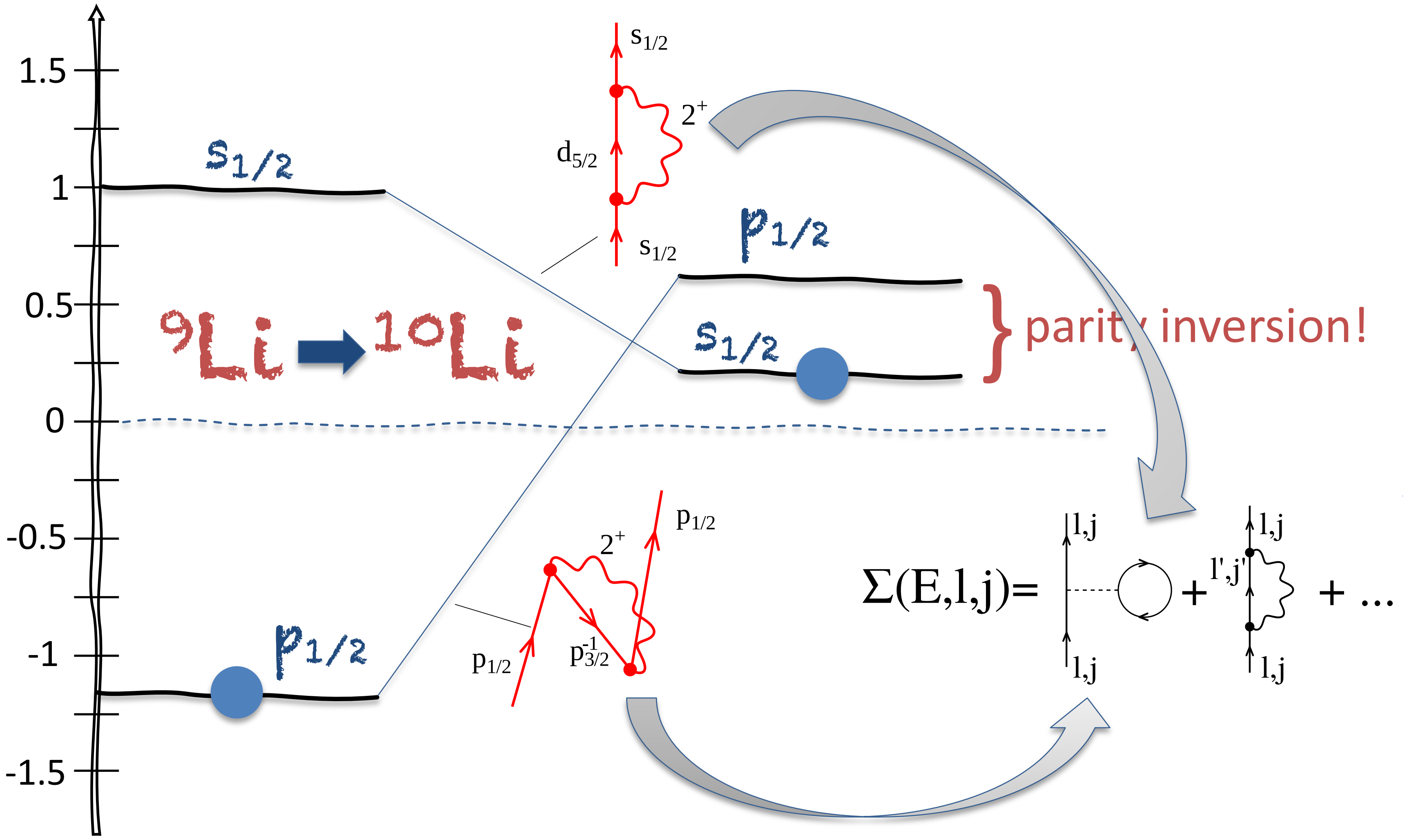


Pauli-blocking, repulsive contribution

Applications: Nuclear Field Theory (NFT); ^{9}Li(d,p)



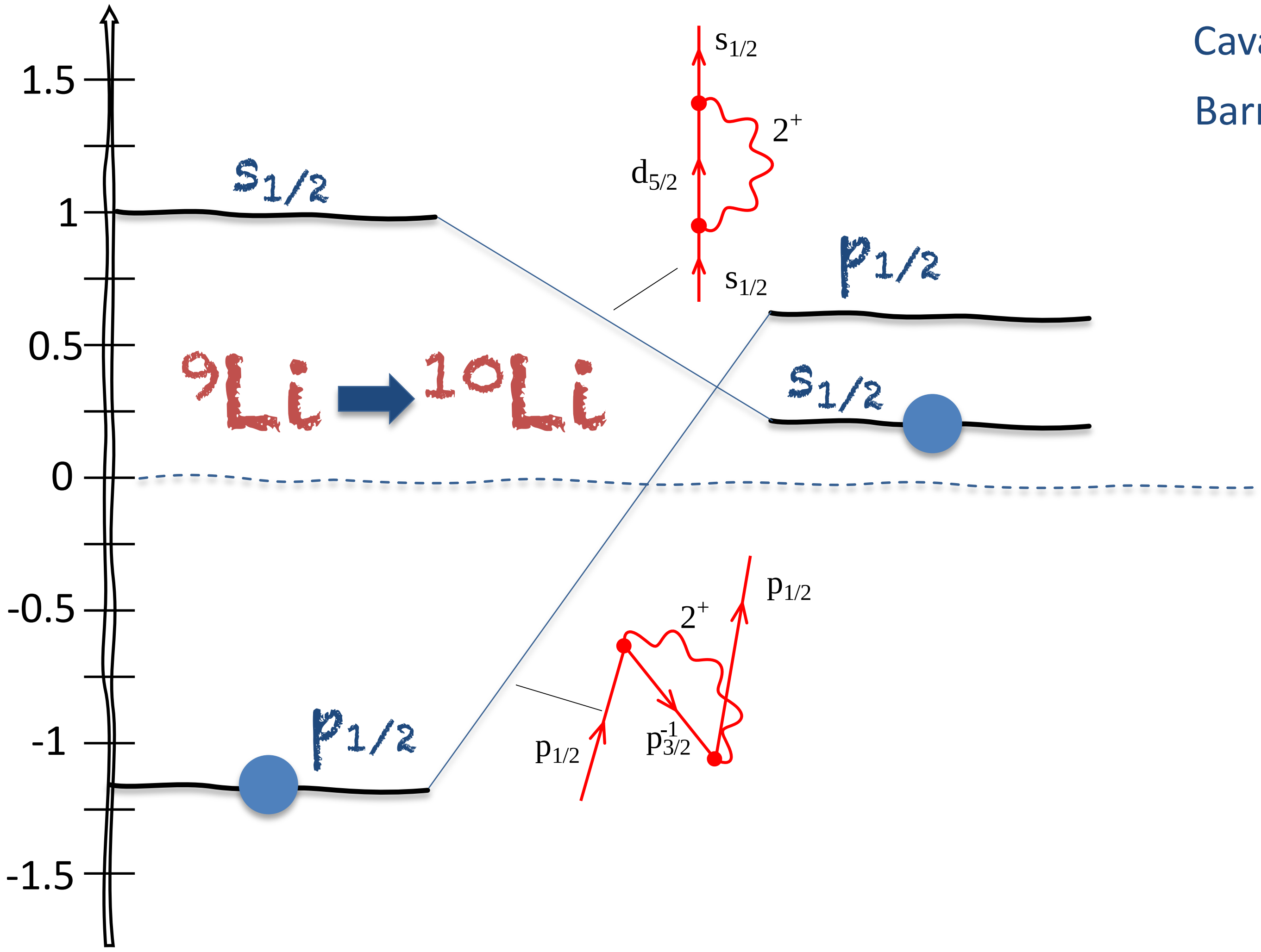
Applications: Nuclear Field Theory (NFT); ^{9}Li(d,p)

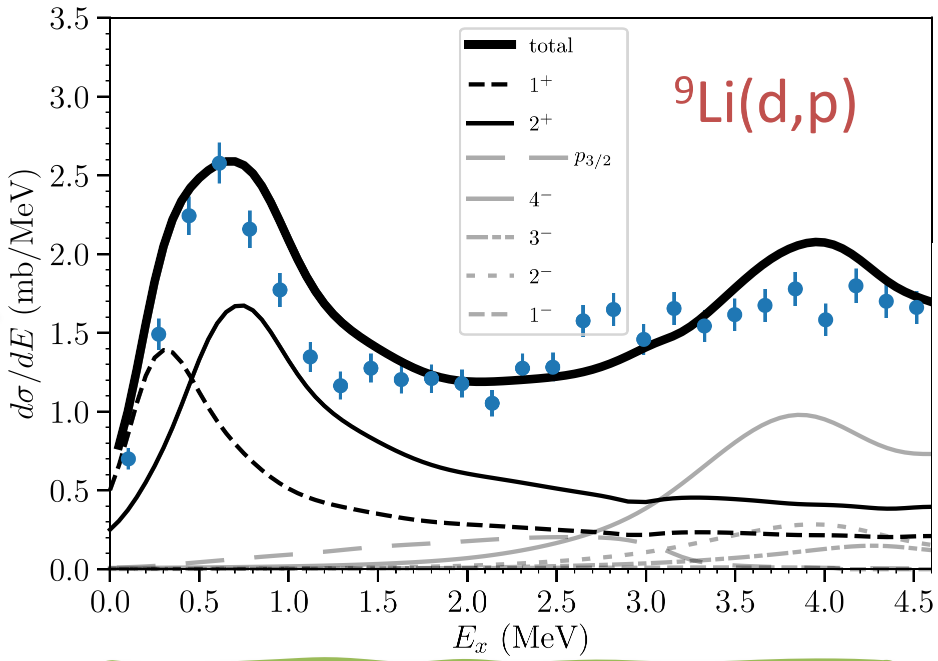


Applications: Nuclear Field Theory (NFT); ^{9}Li(d,p)

Cavallaro *et al.*, PRL **118**, 012701 (2017)

Barranco, GP, Vigezzi, Broglia PRC **101**, 031305(R) (2020)





theoretical description validated by experiment

^{24}Mg+n with shell model: connection with the statistical model

excitation energy E_i

angular momentum

parity

spectroscopic factor S_i

0	0	2	1	0.584066
0.701	0	0	1	-0.716831
1.169	0	2	1	0.488705
2.033	0	2	1	-0.288311
2.529	0	0	1	0.318332
2.701	0	2	1	0.542416
3.859	0	2	1	0.0495903
3.926	0	1	-1	-0.0298132
4.118	0	1	-1	-0.584623
4.226	0	3	-1	-0.651056
4.46	0	2	1	0.100777
4.816	0	2	1	0.0975601
4.945	0	1	-1	0.516883
5.065	0	1	-1	-0.0511968
5.416	0	0	1	-0.283037
5.638	0	3	-1	-0.219064
5.785	0	2	1	-0.0103979
5.792	0	2	1	0.132477
5.935	0	3	-1	0.069005
5.936	0	2	1	0.094658
6.033	0	1	-1	0.0353684
6.12	0	1	-1	-0.159821
6.243	0	2	1	-0.174362
6.35	0	3	-1	0.122727
6.385	0	0	1	0.182001
6.417	0	2	1	0.115995
6.609	0	2	1	0.100457
6.739	0	3	-1	0.157325
6.771	0	1	-1	0.419452
6.801	0	3	-1	0.160889

~600 states from E_i=0 to E_i=14.6 MeV

Shell model calculations by K. Kravvaris with PSDPF interaction M Bouhelal, *et al.*, Nucl. Phys. A **864** (2011)

(G. Sargsyan talk yesterday)

$$V(\mathbf{r},\mathbf{r}';E) = U_0(r) + \sum_i U_{0i}(\mathbf{r})G(E-E_i,\mathbf{r},\mathbf{r}')U_{i0}(\mathbf{r}')$$

$$G(\mathbf{r},\mathbf{r}',E) = (E - T - V(\mathbf{r},\mathbf{r}';E))^{-1}$$

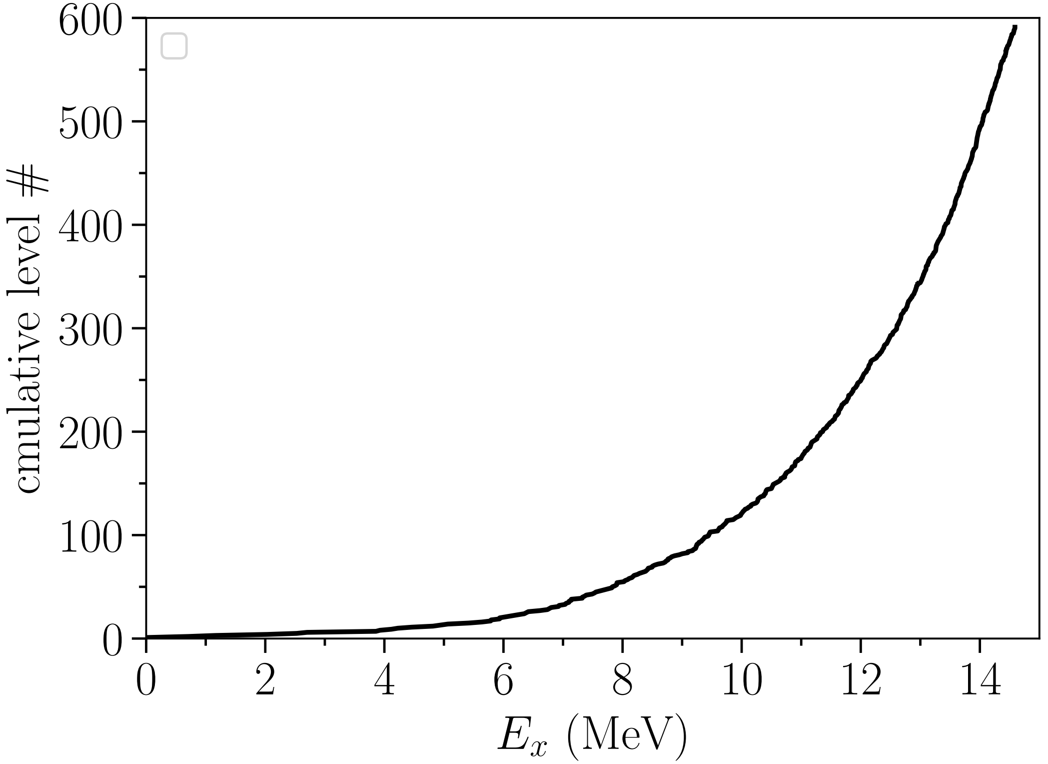
- Iterate until convergence is achieved
- Consistency between potential and Green's function is achieved, as expressed by Dyson's equation:

$$G(\mathbf{r},\mathbf{r}';E) = G_0(\mathbf{r},\mathbf{r}';E) + G_0(\mathbf{r},\mathbf{r}';E)V(\mathbf{r},\mathbf{r}';E)G(\mathbf{r},\mathbf{r}';E)$$

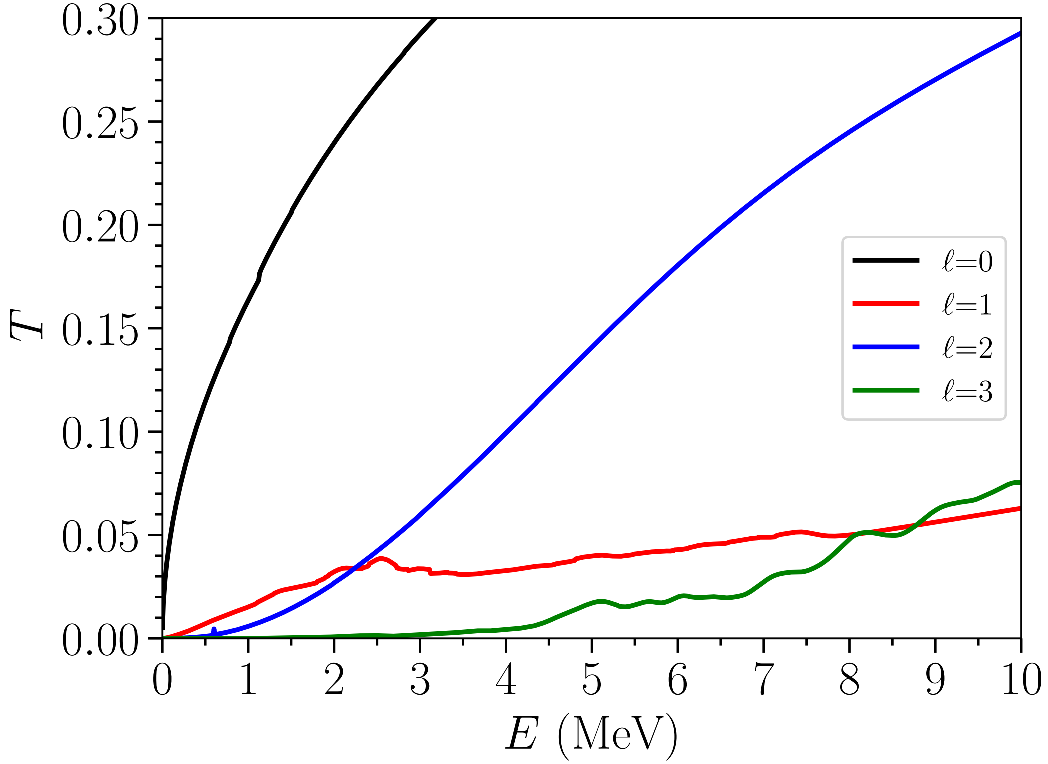
$$G_0(\mathbf{r},\mathbf{r}';E) = (E - T - U_0(r))^{-1}$$

^{24}Mg+n with shell model: connection with the statistical model

cumulative number of levels

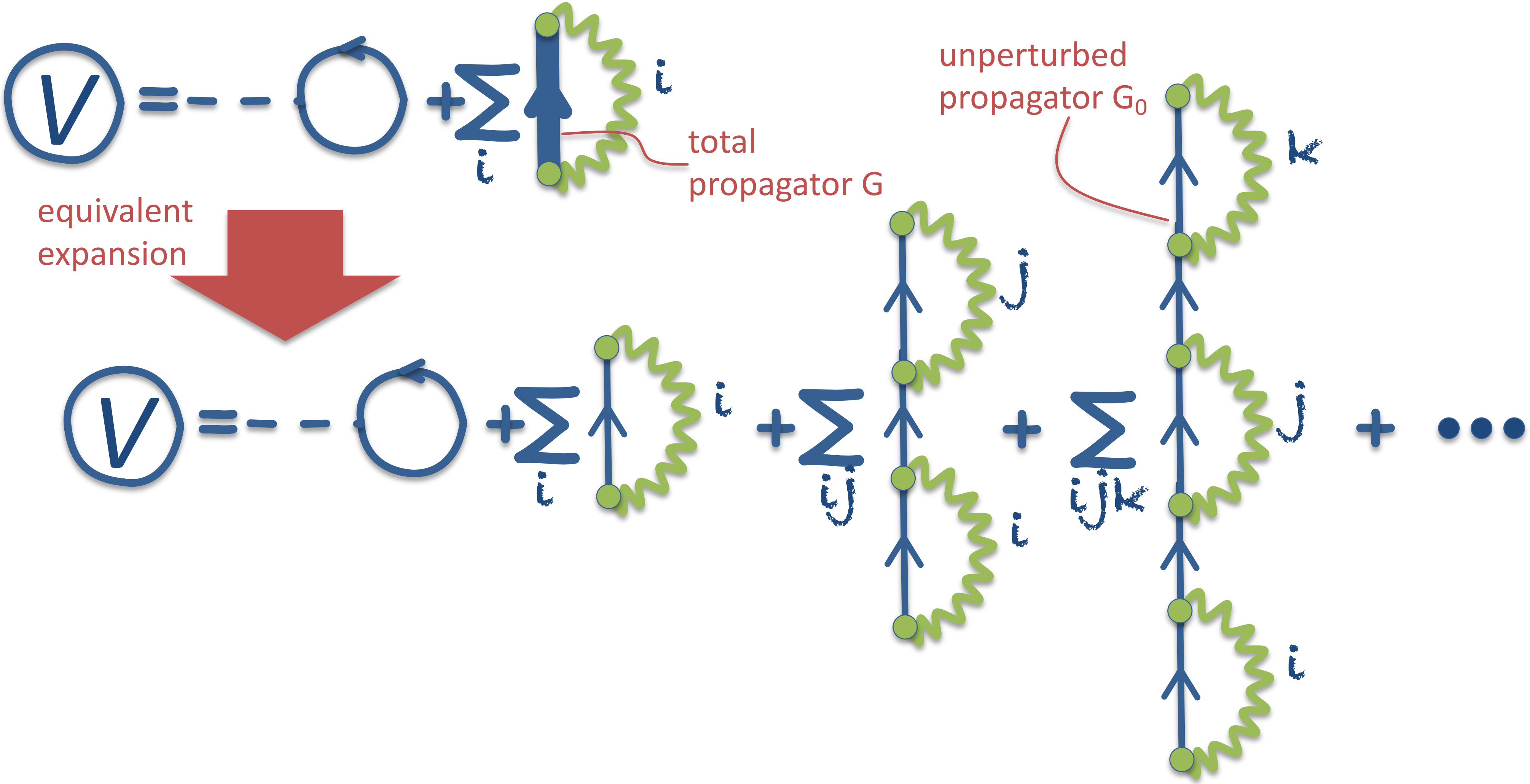


transmission coefficients

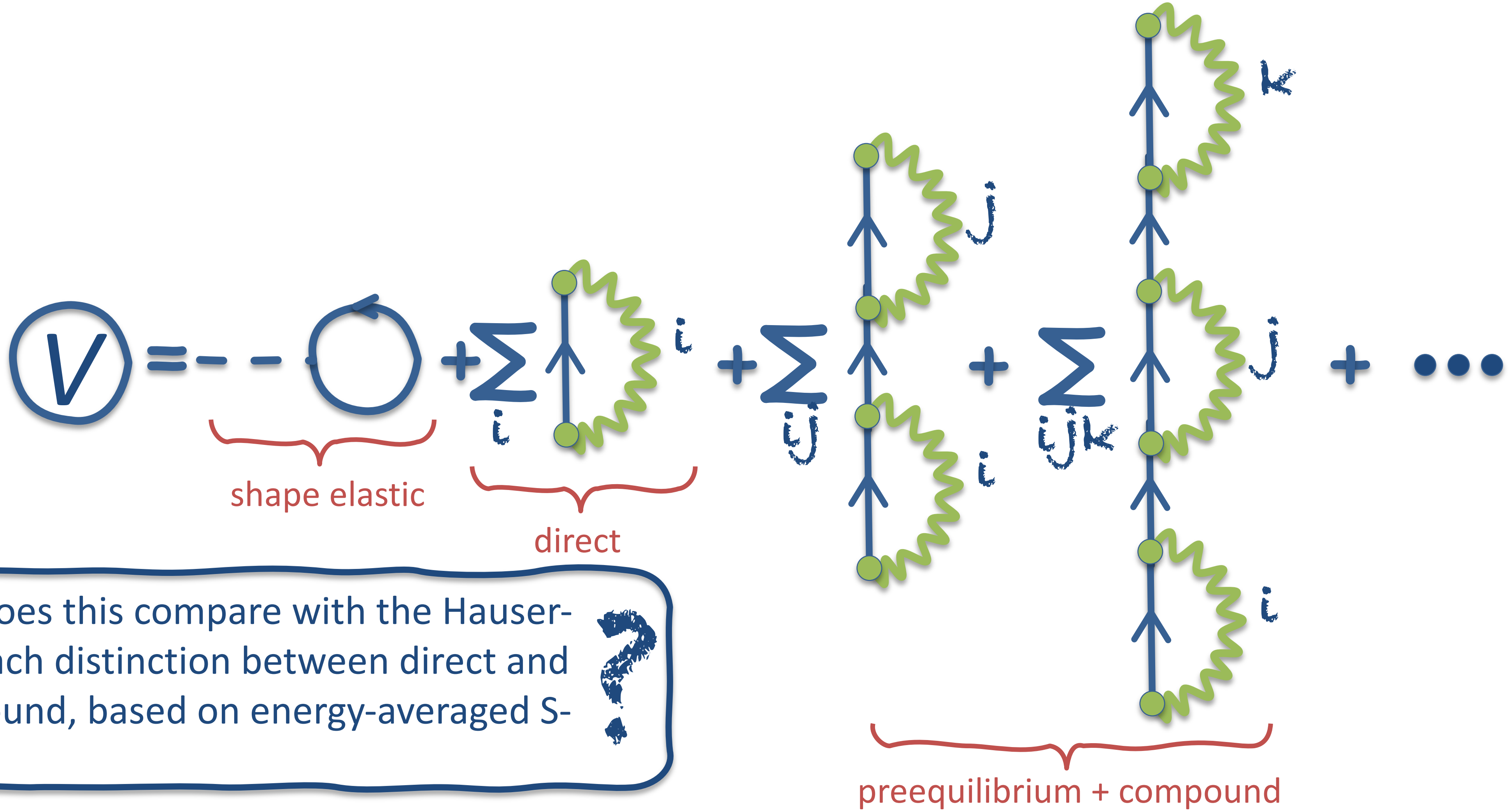


We can explicitly check the limits of the statistical model (Hauser-Feshbach approach)

An equivalent expansion in powers of the couplings can shed light on direct, preequilibrium, and compound processes

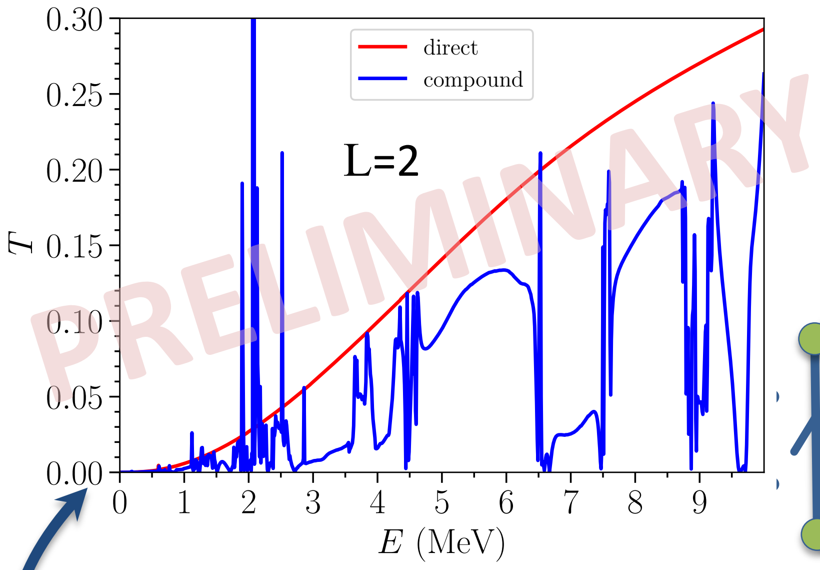


An equivalent expansion in powers of the couplings can shed light on direct, preequilibrium, and compound processes

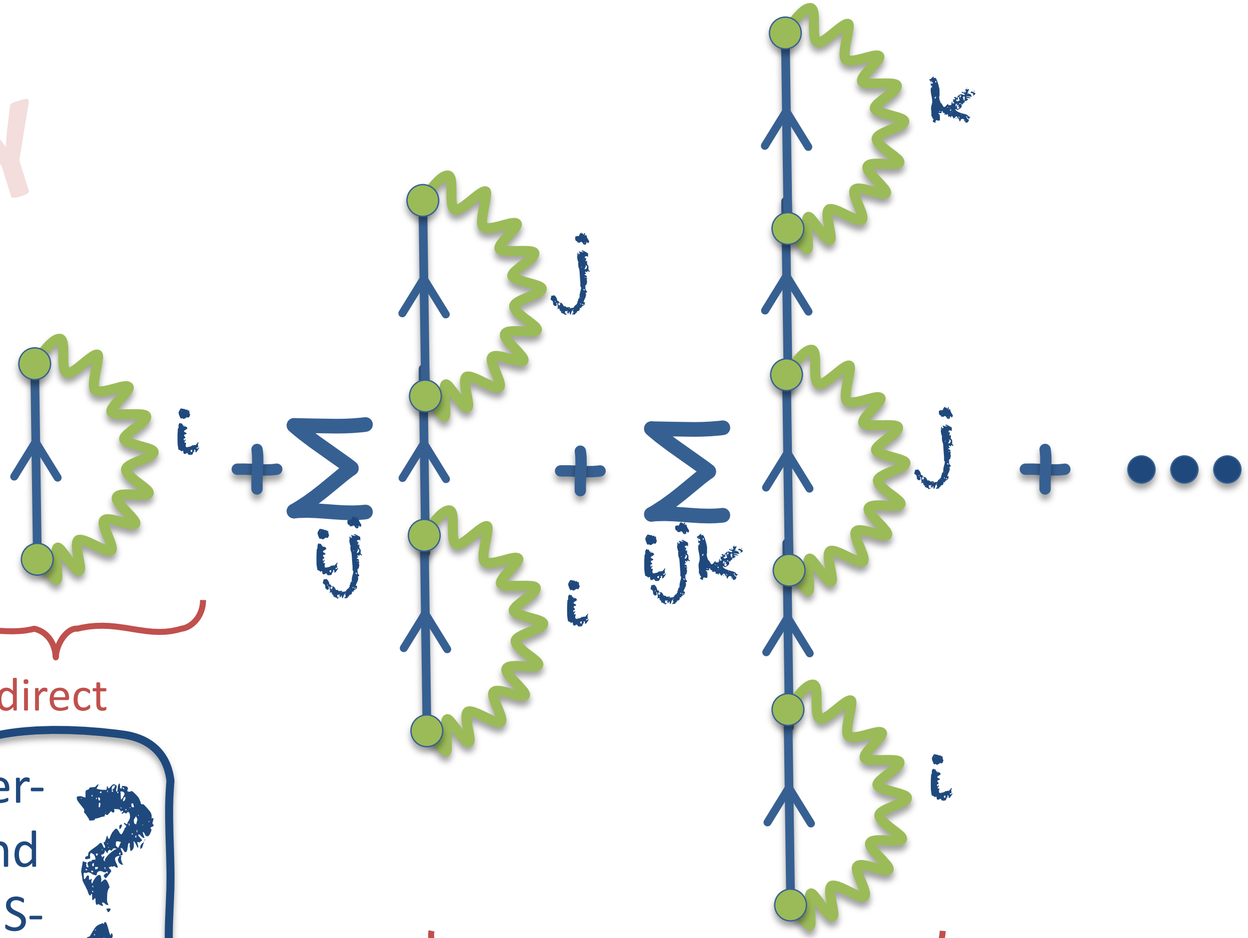


How does this compare with the Hauser-Feshbach distinction between direct and compound, based on energy-averaged S-matrix ?

An equivalent expansion in powers of the couplings can shed light on direct, preequilibrium, and compound processes



shape elastic

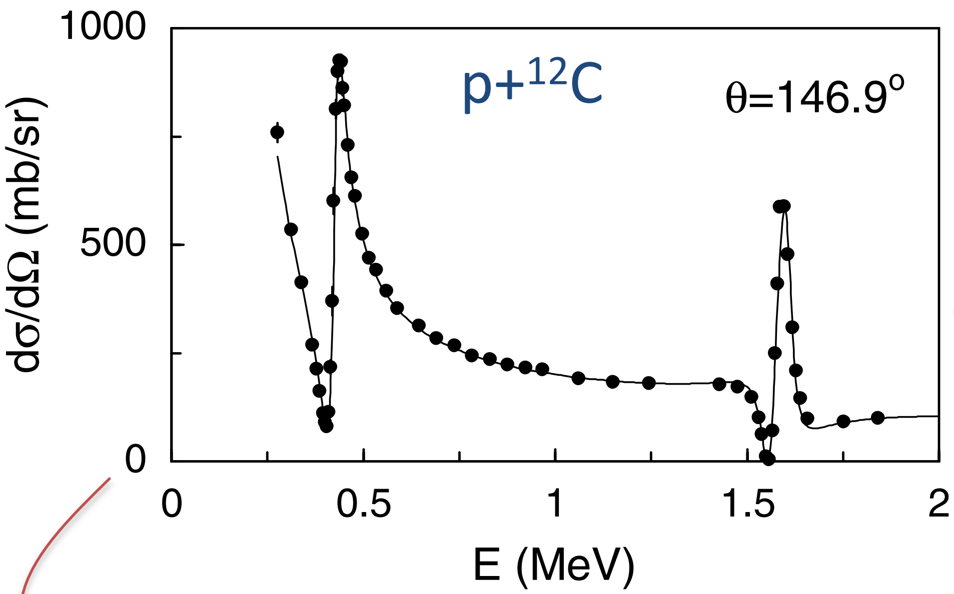


direct

preequilibrium + compound

How does this compare with the Hauser-Feshbach distinction between direct and compound, based on energy-averaged S-matrix?

An *R*-matrix parametrization for the indirect cross section



Rep. Prog. Phys. **73** (2010) 036301 (44pp)

The *R*-matrix theory

P Descouvemont[1] and D Baye[1,2]

$$\frac{d\sigma(E)}{d\sigma} \propto |T_{i0}|^2$$

$$T_{i0} = \sqrt{P_i(E)P_0(E)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E) + iP_c(E))}$$

T-matrix partial widths and energy parameters fitted from data

An *R*-matrix parametrization for the indirect cross section

connection between direct and indirect R-matrix parameters

example:

- direct: α scattering
- indirect: (^{6}Li,d)

indirect T-matrix

$$T^I_{i0} = \int \sqrt{P_i(E_k)P_0(E_k)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E_k)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E_k) + iP_c(E_k))} g(\mathbf{k})\, d\mathbf{k}.$$

$$E_k = \frac{\hbar^2 k^2}{2\mu}$$

broadening factor

$$g(\mathbf{k}) = \int \psi^{HM}(\mathbf{r}_{xA})\, F^*(\mathbf{r}_{xA}, \mathbf{k})\, d\mathbf{r}_{xA}$$

An *R*-matrix parametrization for the indirect cross section

connection between direct and indirect R-matrix parameters

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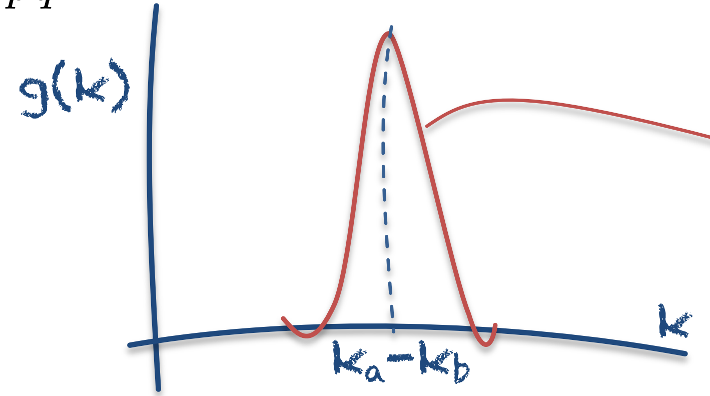
indirect T-matrix

$$T^I_{i0} = \int \sqrt{P_i(E_k)P_0(E_k)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E_k)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E_k) + iP_c(E_k))} \, g(\mathbf{k}) \, d\mathbf{k}.$$



$$g(\mathbf{k}) = \int \psi^{HM}(\mathbf{r}_{xA}) \, F^*(\mathbf{r}_{xA}, \mathbf{k}) \, d\mathbf{r}_{xA}$$

An *R*-matrix parametrization for the indirect cross section

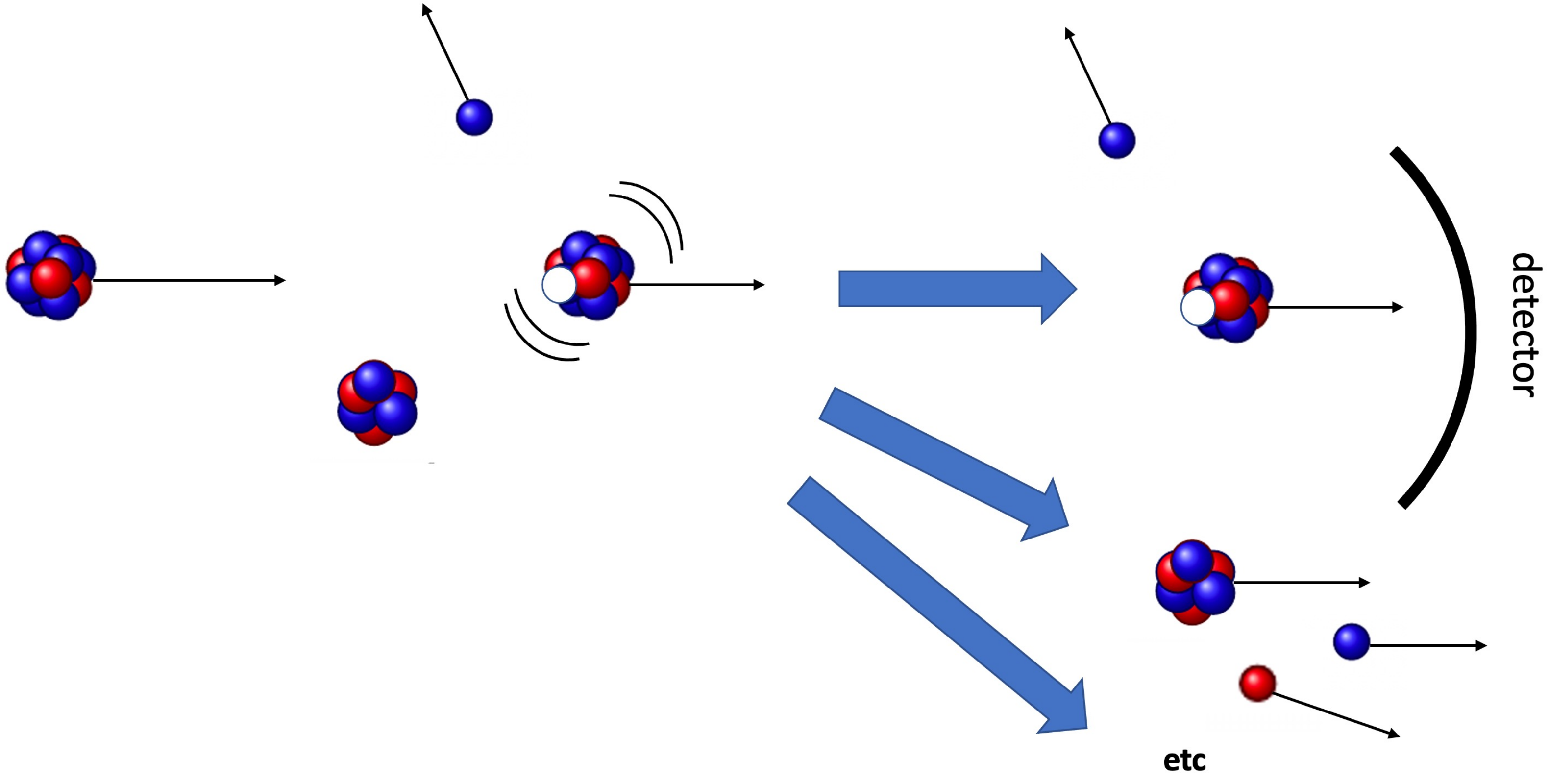
- If the broadening distribution is narrow, the T-matrix can be evaluated at the peak
- This is essentially the approximation made by Barker in *Aust. J. Phys*. **20** (341) 1967 for isolated resonances

$$T^I_{i0} = \int \sqrt{P_i(E_k)P_0(E_k)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E_k)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E_k) + iP_c(E_k))} \, g(\mathbf{k}) \, d\mathbf{k}.$$

$$T^I_{i0} \approx \sqrt{P_i(E^{max}_k)P_0(E^{max}_k)} \sum_{pq} \frac{\gamma_{ip}\gamma_{0q}}{(E_p - E^{max}_k)\delta_{pq} - \sum_c \gamma_{ic}\gamma_{jc}(S_c(E^{max}_k) + iP_c(E^{max}_k))} \int g(\mathbf{k}) \, d\mathbf{k}.$$

An extension to holes: The Green's function knockout (GFK) and the asymmetry plot

(talk by J. Gómez Camacho)



An extension to holes: The Green's function knockout (GFK) and the asymmetry plot

PHYSICAL REVIEW C **107**, 014607 (2023)

Green's function knockout formalism

C. Hebborn[1,2,*] and G. Potel[2,†]

(talk by J. Gómez Camacho)

$$\frac{d\sigma}{dE_{f_{cT}}d\Omega} = -\frac{2\mu_{PT}}{\hbar^2 k_{PT}}\rho(E_{f_{cT}}) \times \sum_{E_h=-S_x^{(c)}-S_N^{(P)}}^{-S_N^{(P)}} \left\langle \phi_h^{(f_{NT})} \middle| \mathrm{Im}\,\hat{U}_h(E_h) \middle| \phi_h^{(f_{NT})} \right\rangle$$

hole optical potential

sum over energies below particle threshold

$$\phi_h^{(f_{NT})}(\boldsymbol{r}) = \hat{G}_h^{\mathrm{opt}}(E_h)\,\rho_h^{(f_{NT})}(\boldsymbol{r}).$$

hole Green's function

Thank you!

Lawrence Livermore National Laboratory