Inclusive cross-section of deformed ^{24}Mg from the Generator Coordinate Method

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Modeling optical potential

Phenomenological methods widely used to interpret experiment

- Lacks proper account of correlations
- Must be fitted to experiment

Possibilites of microscopic methods

- Predictive power
- Exotic nuclei
  - Neutron-rich
  - Radioactive beams
- Shell effects
  - Spectroscopic factor shows the degree of single-particle behaviour

Using microscopic methods

- Nuclear structure calculation
- Calculate spectra
- Construct optical potential

Method summary

- Method includes
  - Collective modes
  - Correlations
  - Particle-hole excitations
- Explores the huge complete Hilbert space in a systematic way
- Gives wavefunction from which observables can be calculated
- Can be extended

Generator Coordinate Method

- Generate a basis from one or more generator coordinates

$$|\phi(x_1, x_2, \ldots)\rangle$$

- Use a linear combination as ansatz

$$\int f(x_1, x_2, \ldots)\,|\phi(x_1, x_2, \ldots)\rangle\, \mathrm{d}\vec{x}$$

  (discretized)

- Solve the Hill-Wheeler equation

$$Hh = EOh$$

Overview of method

- Fit an effective Hamiltonian to the results of an Energy Density Functional (SLy4)
- Solve the effective Hamiltonian in mean field with pairing (HFB) with constraints as generator coordinates
- Introduce randomized particle-hole excitations (similar to temperature)
- Project the resulting HFB states to good quantum numbers
- Solve the resulting Hill-Wheeler equation

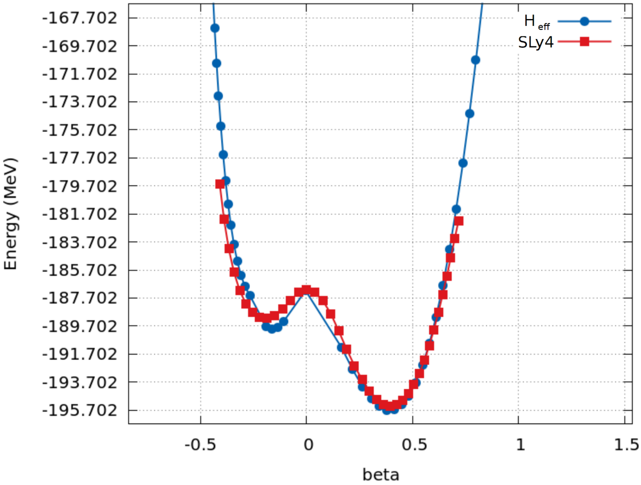
Ljungberg et al. Phys. Rev. C 106, 014314 (2022)

Effective Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_Q + \hat{H}_P$$

- $\hat{H}_0$ – Single particle part
- $\hat{H}_Q$ – Generalized quadrupole interaction
- $\hat{H}_P$ – Uniform seniority pairing

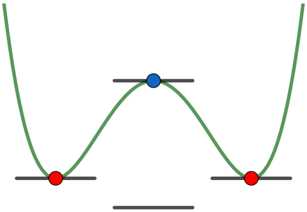
Effective Hamiltonian fit (^{24}Mg)



Generator coordinates

- We proceed by solving a constrained HFB equation
- Generator coordinates used:
  - deformation β, γ
  - pairing strengths G_n, G_p
  - cranking j_x
- Results in a basis of HFB states
- Randomized particle-hole excitations

Projection



- HFB breaks symmetries, e.g.
  - Particle number
  - Angular momentum
- Restore using projection
- Particle number

$$P^N = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{e}^{i\left(\hat{\mathrm{N}}-N\right)\theta} \, \mathrm{d}\theta$$

- Angular momentum

$$P^I_{MK} = \frac{2I+1}{8\pi^2} \int D^I_{MK}{}^* \left(\Omega\right) \hat{\mathrm{R}} \left(\Omega\right) \, \mathrm{d}\Omega$$

Hill-Wheeler equation

- Finally, the Hill-Wheeler equation is solved

$$\sum_j H_{ij} h_j^n = E_n \sum_j O_{ij} h_j^n$$

- Which gives the final wavefunctions

$$\left|\Psi_n^A\right\rangle = \sum_{a,K} h_{aK}^n P_{MK}^I P^Z P^N \left|\phi_a\right\rangle$$

- We can now evaluate matrix elements between these states

Method summary

- Solution in terms of linear combination of projected HFB states
- Includes
  - Collective modes through generator coordinates
  - Correlations through projection and mixing
  - Particle-hole excitations through temperature
- Explores the Hilbert space through choice of generator coordinates
- Gives wavefunction from which observables can be calculated

Odd case

- Single quasiparticle excitation on each HFB state
- Then do the same thing:
  - Project the resulting HFB+1qp states
  - Same effective Hamiltonian
  - Solve the resulting Hill-Wheeler equation

  Gives $\left|\Psi_k^{\pm}\right\rangle$ for $A \pm 1$
- Spectroscopic factors[1]

$$\left\langle \Psi_k^{+} \middle| a_\alpha^\dagger \middle| \Psi_0 \right\rangle$$

[1] Boström et al. J. Phys.: Conf. Ser. 2586 012080 (2023)

Green's function

Calculate the Green's function

$$G^{I}_{\alpha,\beta}(E) = \sum_i \frac{\left\langle \Psi_0 \middle| a_\alpha \middle| \Psi^{+}{}^{I}_{i} \right\rangle \left\langle \Psi^{+}{}^{I}_{i} \middle| a^\dagger_\beta \middle| \Psi_0 \right\rangle}{E - \left(E^{+}{}^{I}_{i} - E_0\right) + i\eta} + \text{holes}$$

Derived by inserting $\mathrm{I} = \sum_i \left| \Psi^{\pm}{}^{I}_{i} \right\rangle \left\langle \Psi^{\pm}{}^{I}_{i} \right|$

Completeness

To insert $\sum_i \left|\Psi^{\pm}{}_i^I\right\rangle \left\langle\Psi^{\pm}{}_i^I\right|$

$$a_\alpha^\dagger \left|\Psi_0\right\rangle = \sum_i \left|\Psi^{+}{}_i^I\right\rangle \left\langle\Psi^{+}{}_i^I\middle|a_\alpha^\dagger\middle|\Psi_0\right\rangle$$

No guarantee that this holds

Completing

At $E \to \infty$, correlations vanish $\to$ should approach HF

$$G_{\alpha,\beta}(E) = \sum_i \frac{{\sigma_{i,\alpha}}^* \ \sigma_{i,\beta}}{E - \epsilon_i + i\eta} + \sum_i^M \frac{{c_{i,\alpha}}^* \ c_{i,\beta}}{E - \epsilon'_i + i\eta}$$

$(\sigma_{i,\alpha} = \left\langle \Psi_i^{\pm} \middle| a_\alpha^\dagger \middle| \Psi_0 \right\rangle)$

As few as possible while still giving HF at $E \to \infty$

- Completely determines spectroscopic factors and energies

Dyson equation

- Dyson equation for self-energy $\Sigma(E)$

$$G(E) = G_0(E) + G_0(E)\,\Sigma(E)\,G(E)$$

- Solved for $\Sigma(E)$ as

$$\Sigma(E) = G_0(E)^{-1} - G(E)^{-1}$$

Non-local optical potential

- Construct the potential

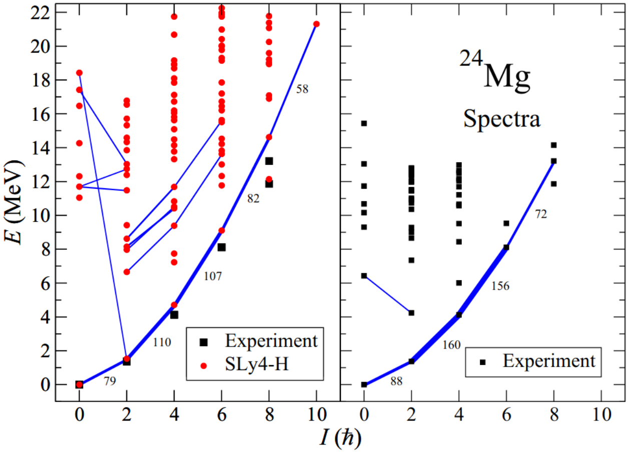
$$V_{a,b}\left(E\right) = \Sigma^{\infty}_{a,b} + \Sigma_{a,b}\left(E\right)$$

- Expressed in momentum-space

$$V\left(k,k'\right) = \sum_{a,b}^{N} V_{a,b}\psi_a\left(k\right)\psi_b^*\left(k'\right)$$

- Lippmann-Schwinger equation $T = V + VG_{\text{free}}T$
- Phase shifts and cross-sections

^{24}Mg spectra



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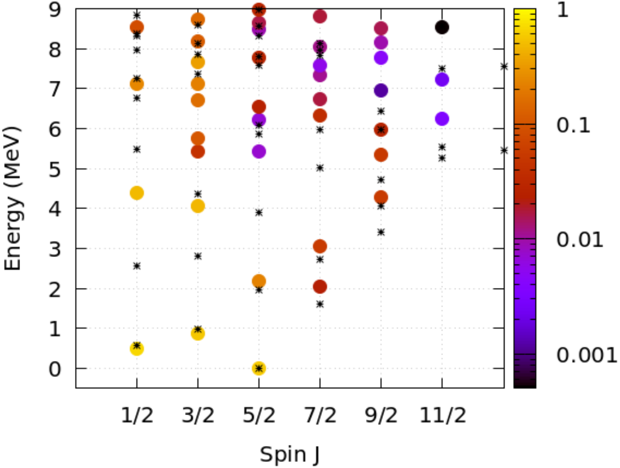
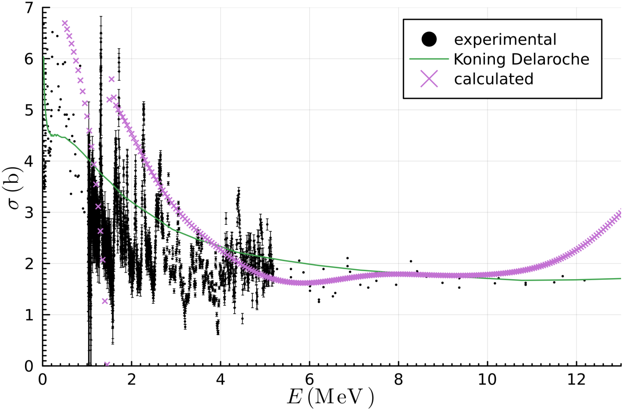
^{25}Mg spectroscopic factors (positive parity)



Preliminary ^{24}Mg inclusive neutron cross-section



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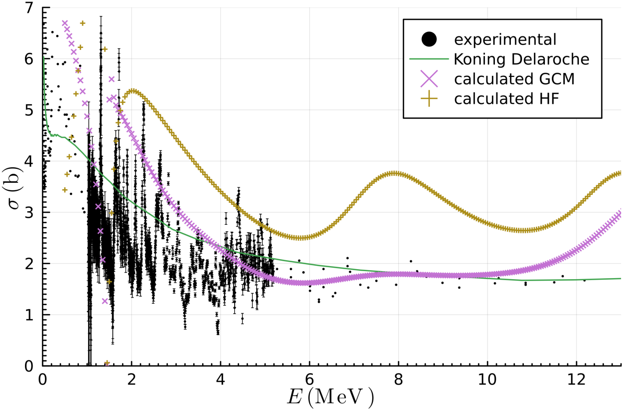
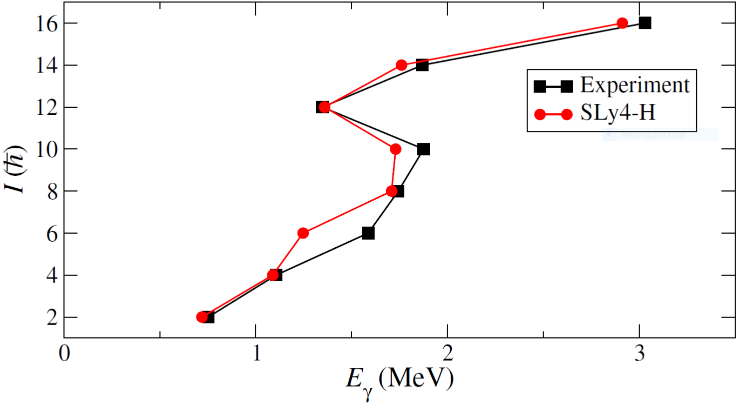


Summary

- Lack of important correlations in many cross-section calculations
- Major step towards including many-body correlations in deformed nuclei
- Paper in progress

Backbending for ^{48}Cr



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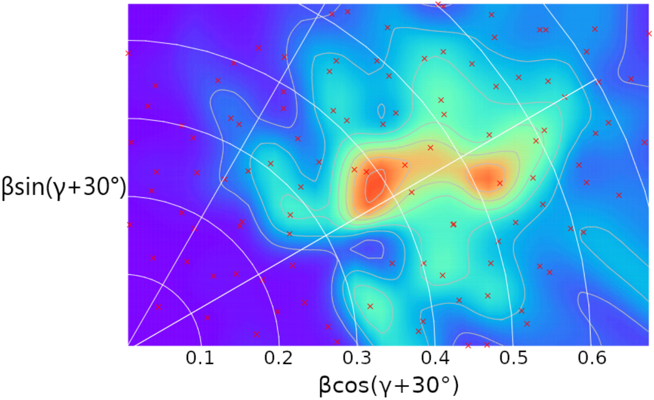
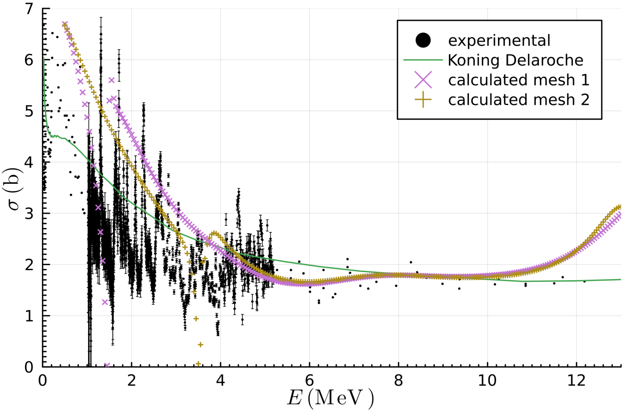
^{24}Mg 0^{+} wavefunction beta-gamma plane



Related to probability amplitude

Preliminary ^{24}Mg inclusive neutron cross-section



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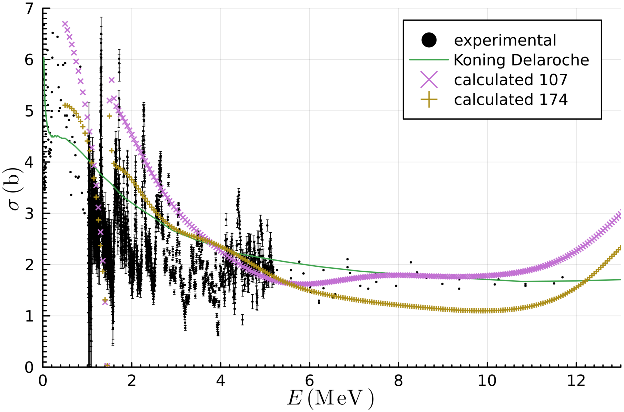

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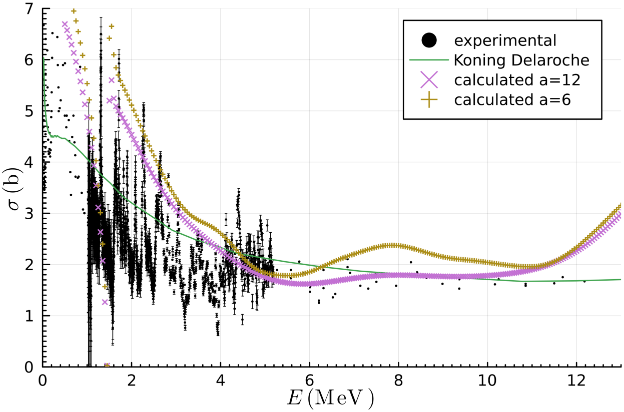

Imaginary part

$$\eta\left(E\right) = \frac{a}{\pi}\frac{\left(E - E_{\mathrm{F}}\right)^2}{\left(E - E_{\mathrm{F}}\right)^2 + b^2}$$

Spectroscopic factors

$$\left\langle \Psi_k^+ \middle| a_\alpha^\dagger \middle| \Psi_0 \right\rangle =$$

$$\sum_{a\,b\,x\,K} \left(h_{a\,x\,K}\right)^* h_b \left\langle \Phi_a \middle| \beta_x P^{A+1} P^I_{KM} a_\alpha^\dagger P^0_{00} P^A \middle| \Phi_b \right\rangle =$$

$$\sum_{a\,b\,x\,K} \left(h_{a\,x\,K}\right)^* h_b \left\langle \Phi_a \middle| \beta_x a^\dagger_{\alpha\,K} P^0_{00} P^A \middle| \Phi_b \right\rangle$$

$$a^\dagger_{\alpha\,K} = \sum_l \left(U^a_{\alpha\,K,l}\right)^* \beta_l^\dagger + V^a_{\alpha\,K,l} \beta_l$$