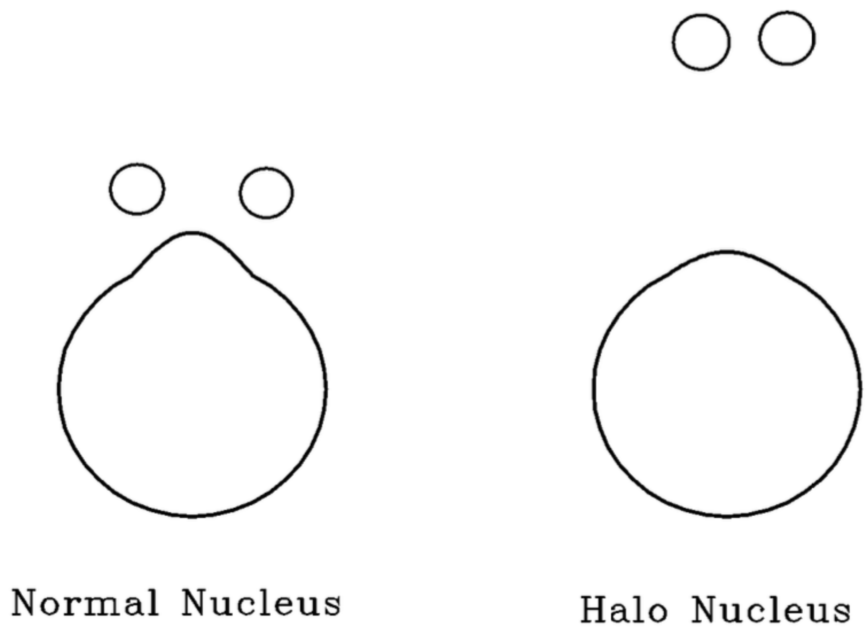
Enrico Vigezzi INFN Milano

NUCLEAR FIELD THEORY AND THE SPECTROSCOPY OF HALO NUCLEI

In collaboration with
F. Barranco (Sevilla)
G. Potel (LLNL)

ECT*, Trento, *Towards a consistent approach for nuclear structure and reactions: microscopic optical potentials, 19/6/2024*

In 1997 Kuo et al predicted that in halo nuclei core polarizaion would be supprressed, and that the fundamental nucleon-nucleon interaction could be probed in a clearer and more direct way in halo nuclei than in ordinary nuclei….

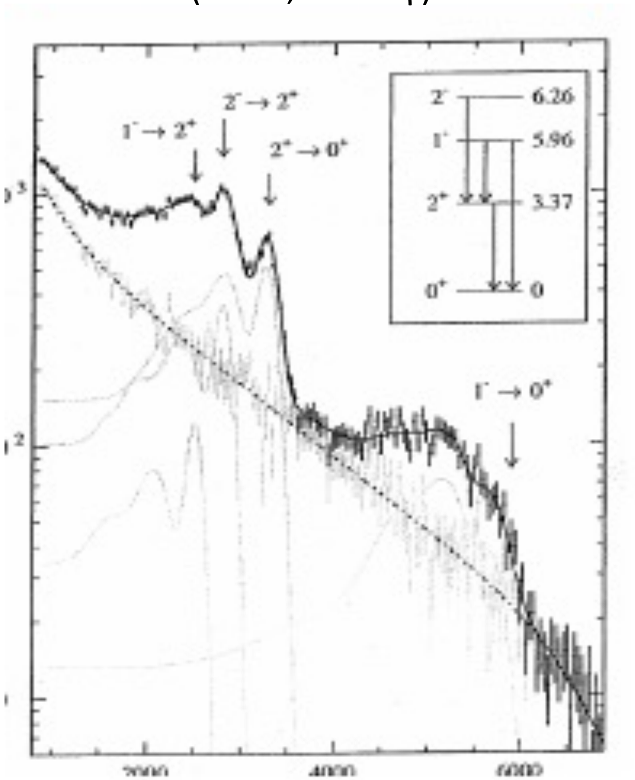


T.T.S. Kuo et al,
PRL 78 (1997) 2708

… But experiments demonstrated that the core dynamics plays an important role…

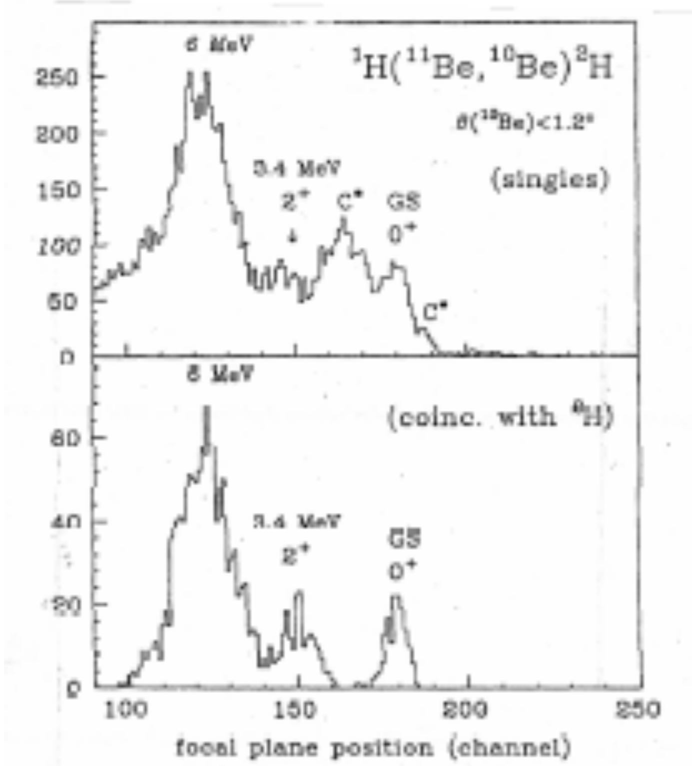
The admixture of $d_{5/2}$ x 2^+ configuration in the $1/2^+$ g.s. of ^{11}Be is about 15%

9Be(11Be,10Be+γ)X

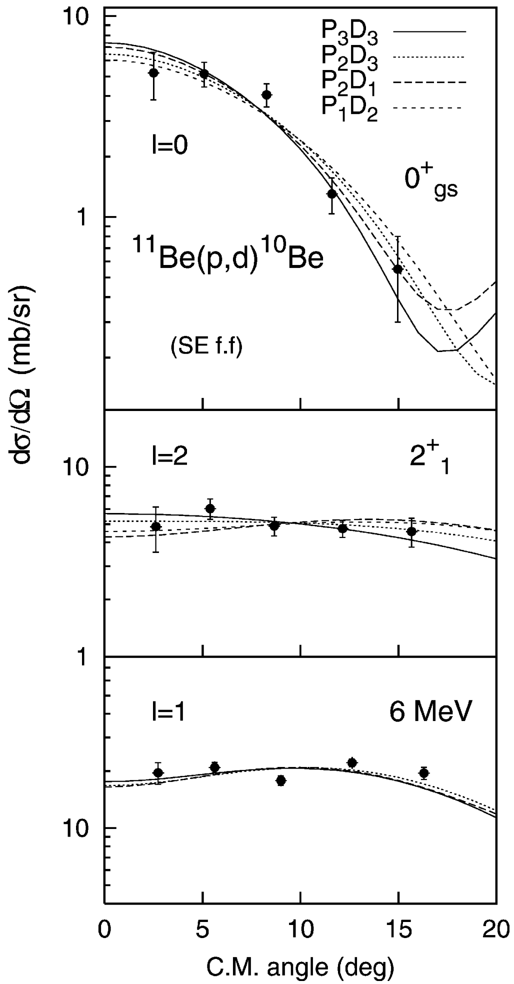


T. Aumann et al. PRL 84 (2000) 35

p(11Be,10Be)d



S. Fortier et al. Phys. Lett. B461 (1999)22
J.S. Winfield et al., Nucl.Phys. A683 (2001)48



J.S. Winfield et al.,
Nucl.Phys. A683 (2001)48

A careful analysis of transfer reactions is needed
to estimate phonon admixtures in the wavefunctions

$$\left|{}^{11}\mathrm{Be}_{gs}\right\rangle = \alpha \left|{}^{10}\mathrm{Be}(0^+) \otimes 2s\right\rangle + \beta \left|{}^{10}\mathrm{Be}(2^+) \otimes 1\mathrm{d}\right\rangle$$

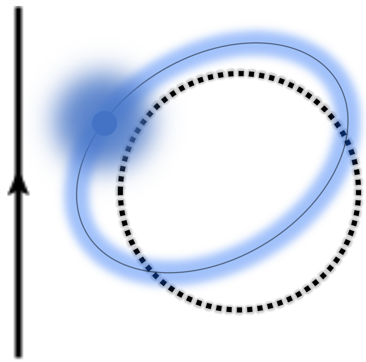
Good agreement with 2+ cross sections is obtained
in DWBA with β^2 = 0.17 considering the coupling effects on
the transfer from factor; using β as a simple spectroscopic
factor one finds β^2 = 0.28

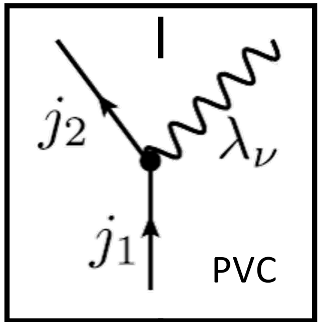
Can we obtain a description of thee systems in terms of elementary modes of excitation including some core degrees of freedom ?

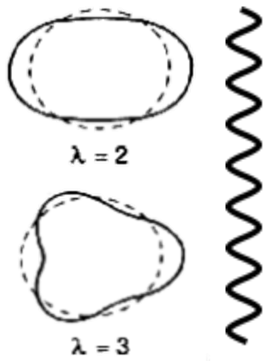
Independent Particles

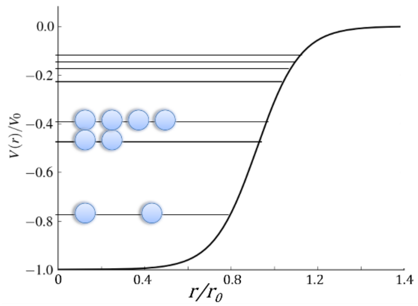
Collective Phonons

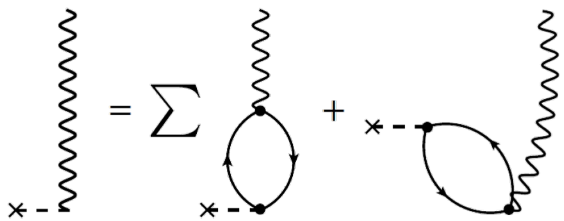
Particle-vibration coupling











Hartree-Fock mean Field

Random Phase Approximation

The Nuclear Field Theory shows how to construct an exact solution of the many body problem for a given two-body force acting between fermions,
in terms of a series expansion for each observable, and based on

1.Fermions: HF
2.Bosons: RPA phonons (particle-hole + pair addition/removal)
3. A linear Particle-Vibration Coupling

$$H_{NFT} = H_p + H_c + H_2 + H_{PVC}$$

$$H_p = \sum \epsilon_j c_j^+ c_j \qquad H_c = \sum_\lambda \hbar\omega_\lambda \Gamma_\lambda^+ \Gamma_\lambda$$

$$H_2 = \frac{1}{4} \sum_{j_1 j_2 j_3 j_4} < j_1 j_2 | V | j_3 j_4 > c_{j_1}^+ c_{j_2}^+ c_{j_4} c_{j_3}$$

$$H_{PVC} = \sum_{\lambda j_1 j_2} [h(j_1, j_2\lambda)\Gamma_\lambda^+ c_{j_2}^+ c_{j_1} + h(j_1, j_2\lambda)\Gamma_\lambda c_{j_1}^+ c_{j_2}]$$

$$h(j_1, j_2\lambda) = \sum_{jj'} < j_1 j | V | j_2 j' >< 0 | c_j^+ c_{j'} | \lambda >$$

D.R. Bes et al, NPA 260 (1976) 1
P.F. Bortignon et al, Phys. Rep. 30 (1977) 305
D.R. Bes Phys. Scr. 91 (2016) 063010

A set of diagrammatic rules is introduced to take into account the overcompleteness of the basis and the Pauli principle. The solution is then the same as for the original fermion problem.

The expansion parameter is $1/\Omega$, where Ω is the effective degeneracy available for the construction of the bosons.

The order of a given diagram is $\Omega^{N_f - N_\Lambda/2 - N_b}$

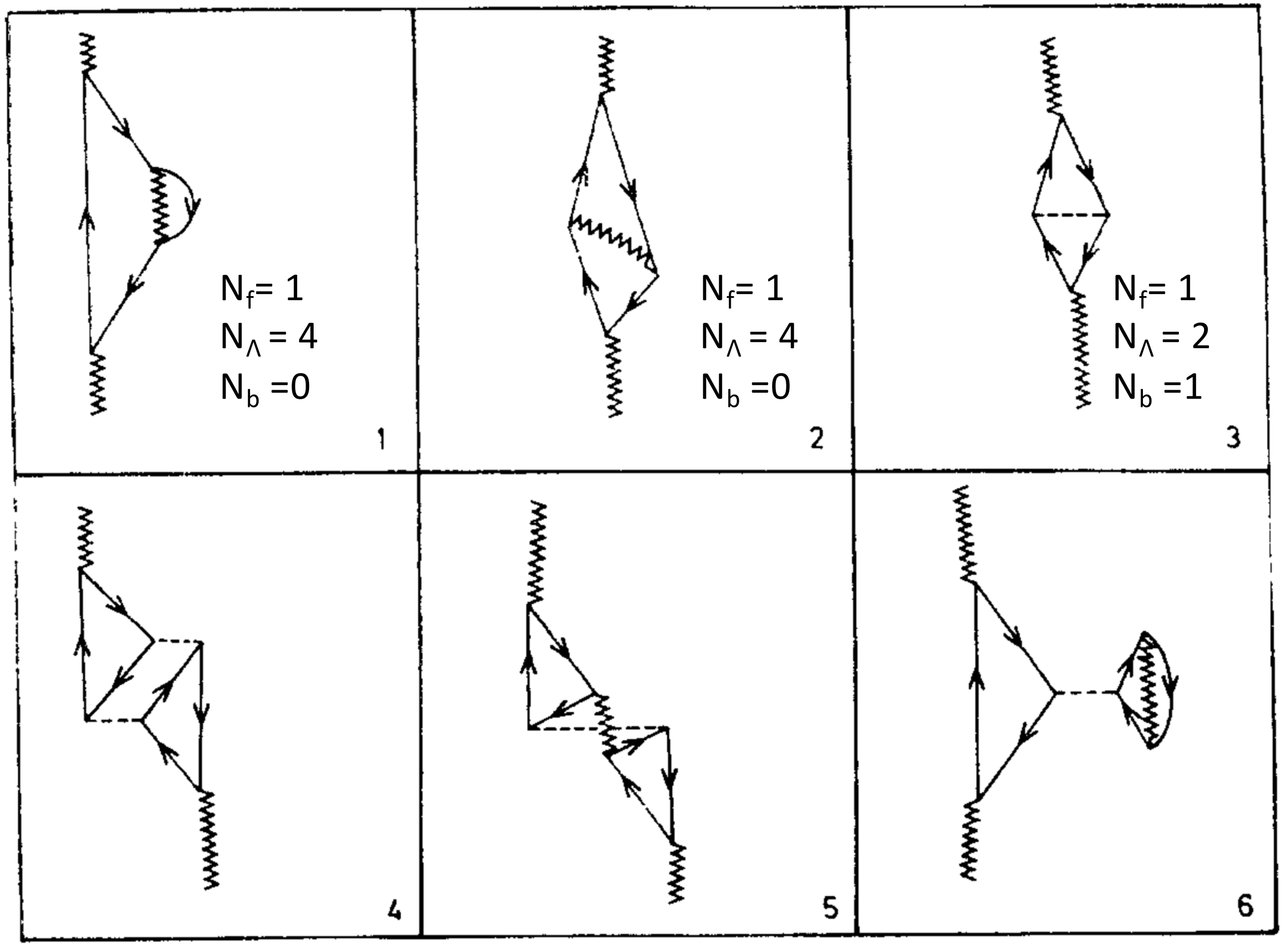
N_f = Number of fermion loops

N_Λ = Number of particle-phonon vertices

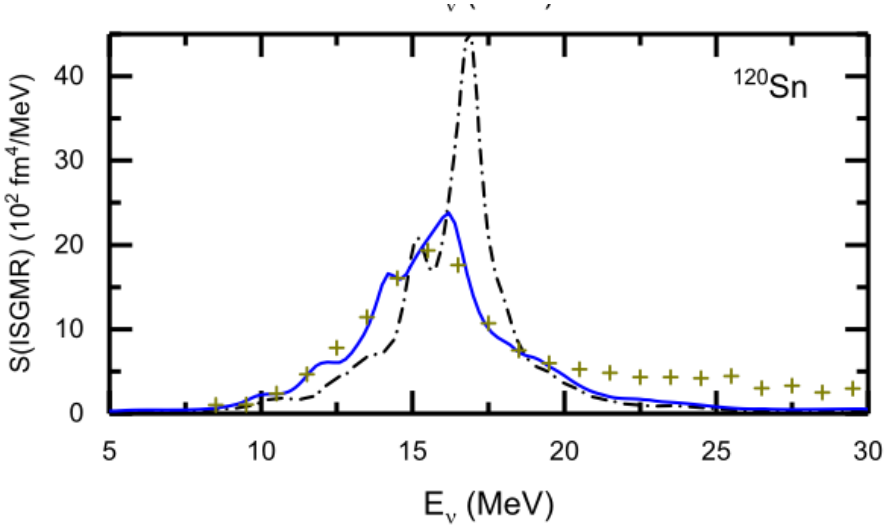
N_b = Number of two-body vertices

Example: renormalization of a phonon state at order Ω^{-1}

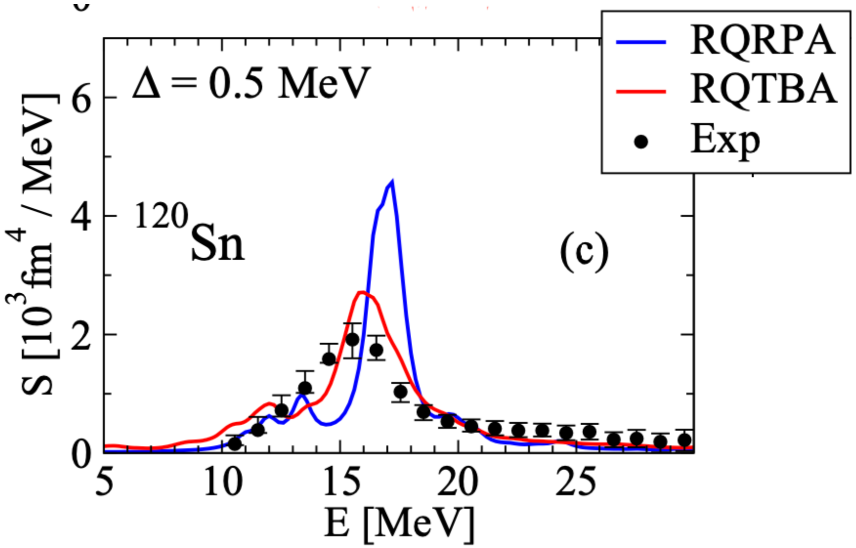
$$\Omega^{N_f - N_\Lambda/2 - N_b}$$



Particle-vibration coupling on top of self-consistent density functional calculations has been mostly applied to heavy nuclei near closed shells. It provides a successful reproduction of the width of giant resonance modes ….



Z.Z. Li, Y.F. Niu, G. Colò, PRL 131 (2023) 082501



E. Litvinova, PRC 107 (2023) L041302

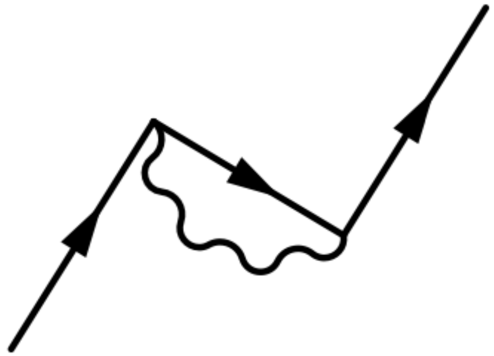
… although the situation is less clear concerning the centroids and the renormalization of single particle states

Renormalization of a particle at order Ω^{-1}

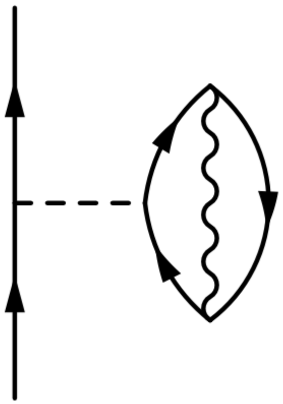
$$\Omega^{N_f - N_\Lambda - N_b}$$

$N_f = 0$
$N_\Lambda = 2$
$N_b = 0$

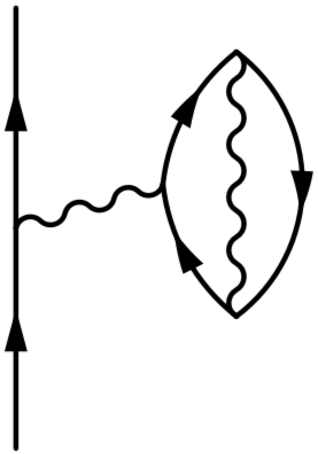
$N_f = 0$
$N_\Lambda = 2$
$N_b = 0$



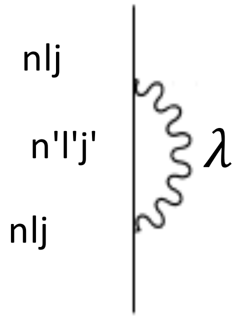
$N_f = 1$
$N_\Lambda = 2$
$N_b = 1$



$N_f = 1$
$N_\Lambda = 4$
$N_b = 0$

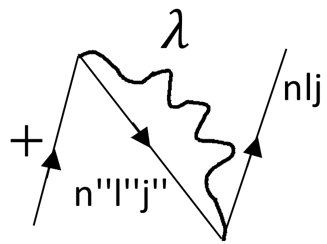


Basic effect of particle-vibration coupling on the single-particle energies close to the Fermi energy

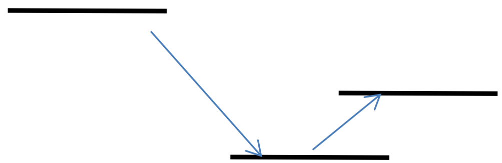


$$= \frac{h^2(j,j',\lambda)}{e_j - (e_{j'} + \hbar\omega_\lambda)} < 0$$

+



$$= \frac{h^2(j,j',\lambda)}{(e_j - e_{j''} + \hbar\omega_\lambda)} > 0$$



$$\Sigma_{\gamma\delta}(\omega) = \sum_{pp'h'p''h''} V_{pp''h''\gamma} \sum_f \frac{X^f_{p'h'} X^f_{p''h''}}{\omega - \epsilon_p - \hbar\omega_f} V_{pp'h'\delta}$$

A close connection with

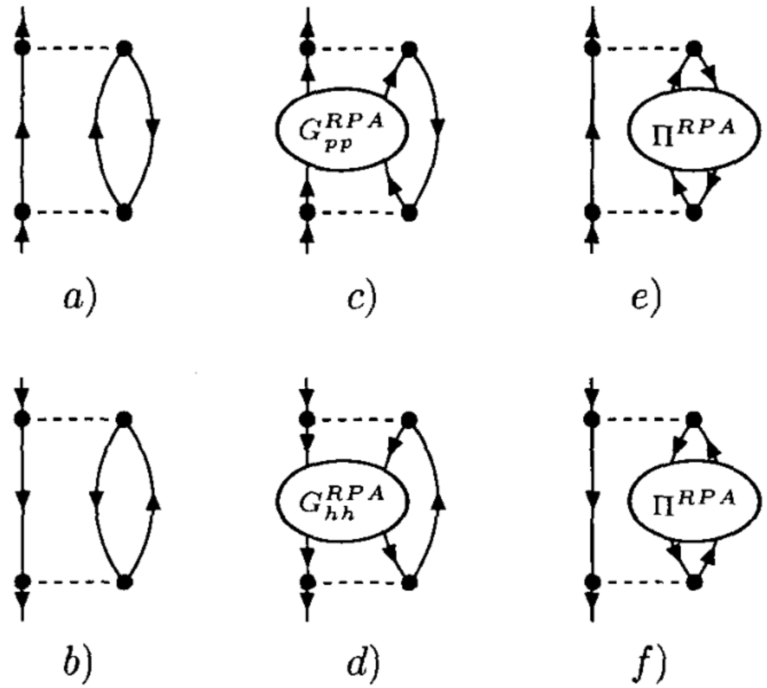
$$\Sigma^{RPA}(\alpha, \beta : E) = \frac{1}{2} \left\{ \sum_{\mu > F, n \neq 0} \frac{\Delta^{A+,n*}_{\alpha\mu} \Delta^{A+,n}_{\beta\mu}}{E - (\varepsilon_\mu + (E^A_n - E^A_0)) + i\eta} + \sum_{\mu < F, m \neq 0} \frac{\Delta^{A-,m}_{\alpha\mu} \Delta^{A-,m*}_{\beta\mu}}{E - (\varepsilon_\mu + (E^A_0 - E^A_m)) - i\eta} \right\}$$

with

$$\Delta^{A+,n}_{\alpha\mu} = \sum_{\nu > F, \kappa < F; \nu < F, \kappa > F} \langle \alpha\kappa | G | \mu\nu \rangle R^{A,n}_{\nu\kappa}$$

and

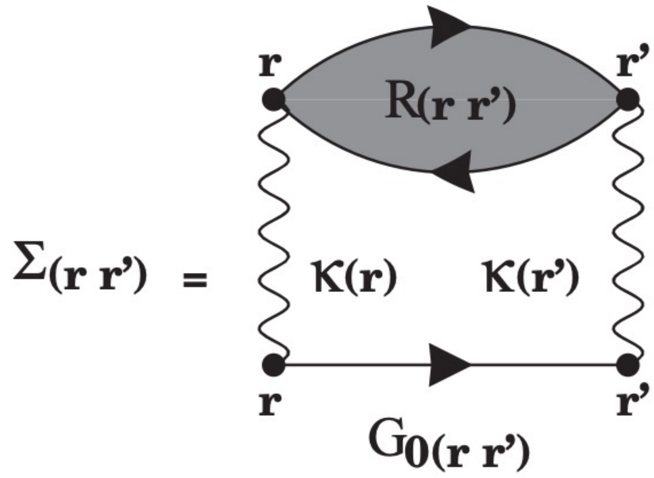
$$\Delta^{A-,m}_{\alpha\mu} = \sum_{\nu > F, \kappa < F; \nu < F, \kappa > F} \langle \alpha\kappa | G | \mu\nu \rangle R^{A,m}_{\kappa\nu}.$$



W. Dickhoff, D. Van Neck, Many-Body Theory Exposed!, p. 493

cPVC with effective Skyrme interaction SLy5

$$\Sigma_{lj}(rr';\omega) = \sum_{l'j',L} \frac{|\langle lj||Y_L||l'j'\rangle|^2}{2j+1} \times \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\kappa(r)}{r^2} G_{0,l'j'}(rr';\omega-\omega') \times \frac{\kappa(r')}{r'^2} i R_L(rr';\omega').$$

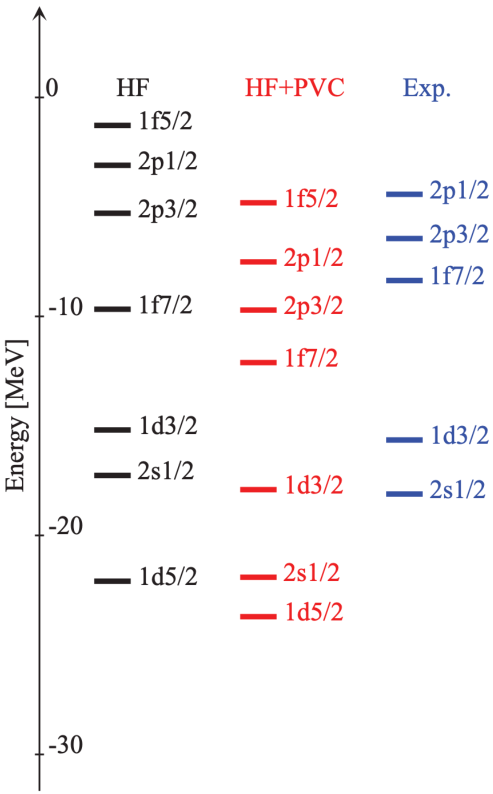


The response function R and the interaction κ are derived consistently from the SLy5 interaction

K. Mizuyama et al, PRC 86 (2012) 034318

G. Colò et al, PRC 82 (2010) 064307

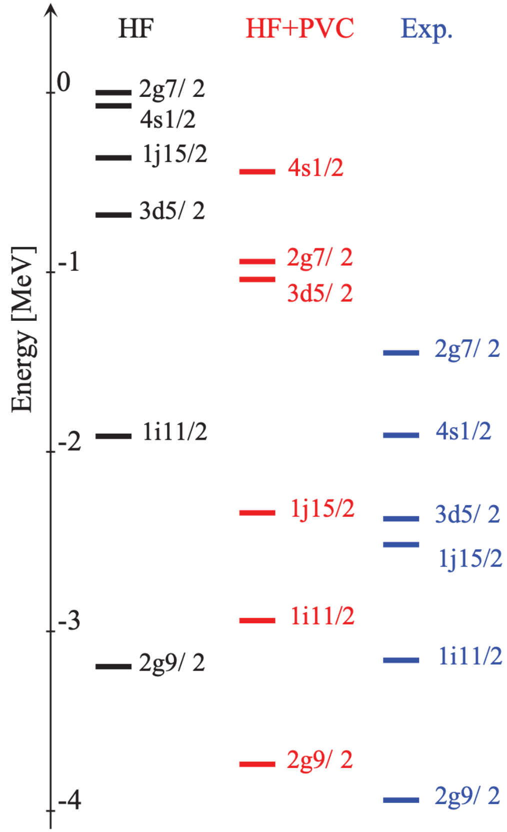
^{40}Ca



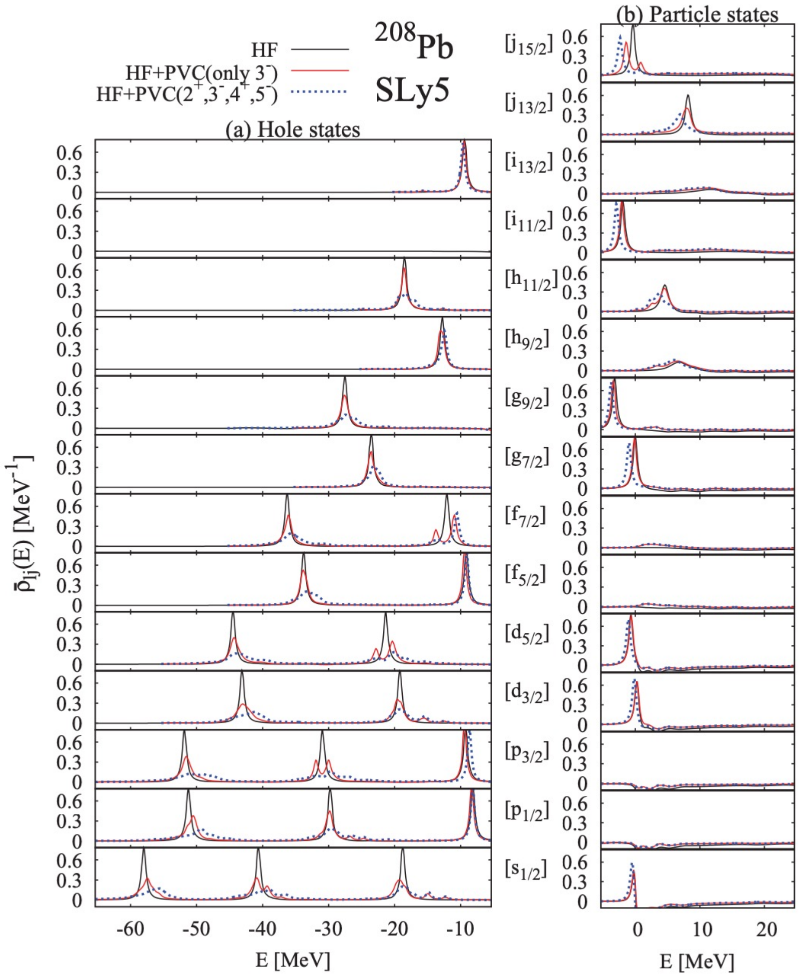
^{40}Ca					
Holes			Particles		
J^π	$S_{lj}(^{39}\text{Ca})$		J^π	$S_{lj}(^{41}\text{Ca})$	
	Exp.	Theory		Exp.	Theory
$d_{3/2}$	0.88	0.80	$f_{7/2}$	0.74	0.66
$s_{1/2}$	0.84	0.80	$p_{1/2}$	0.80	0.81
$p_{3/2}$	2.9×10^{-3}	0.05	$p_{3/2}$	0.73	0.79
$d_{5/2}$	0.73	0.75	$d_{5/2}$	0.11	0.04
			$f_{5/2}$	0.88	0.77
			$g_{9/2}$	0.28	0.36

^{208}Pb					
Holes			Particles		
J^π	$S_{lj}(^{207}\text{Pb})$		J^π	$S_{lj}(^{209}\text{Pb})$	
	Exp.	Theory		Exp.	Theory
$p_{1/2}$	1.07	0.82	$g_{9/2}$	0.76	0.77
$p_{3/2}$	1.50	0.84	$s_{1/2}$	0.87	0.47
$f_{5/2}$	1.07	0.84	$d_{3/2}$	0.93	0.52
$f_{7/2}$	1.02	0.84	$d_{5/2}$	0.85	0.75
$h_{9/2}$	1.06	0.86	$g_{7/2}$	0.90	0.74
$h_{11/2}$	0.39	0.39	$i_{11/2}$	0.82	0.82
$i_{13/2}$	0.90	0.87	$j_{15/2}$	0.54	0.71

^{208}Pb

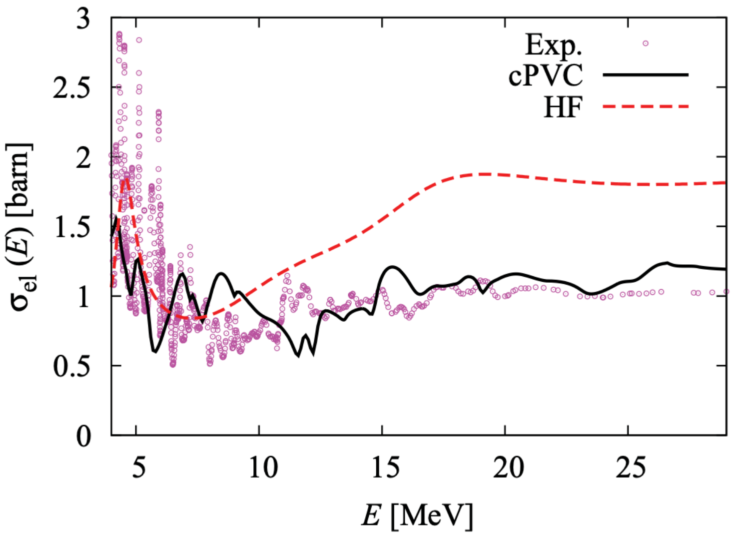


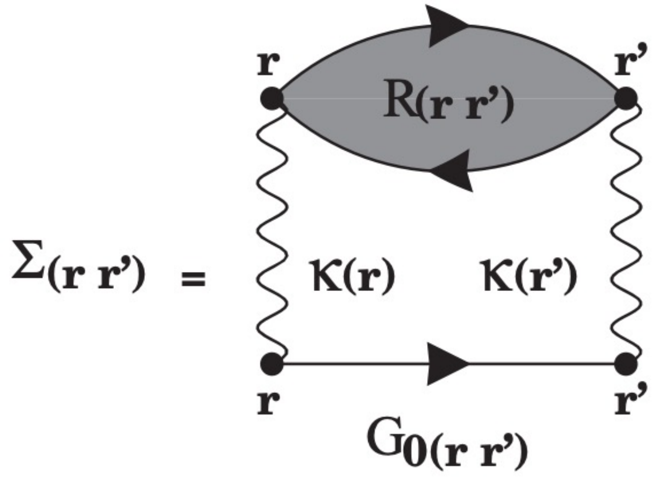
$$\bar{\rho}_{lj}(\omega) = \frac{\pm 1}{\pi} \int dr \,\mathrm{Im}[G_{lj}(rr,\omega) - G_{\mathrm{Free},lj}(rr,\omega)].$$

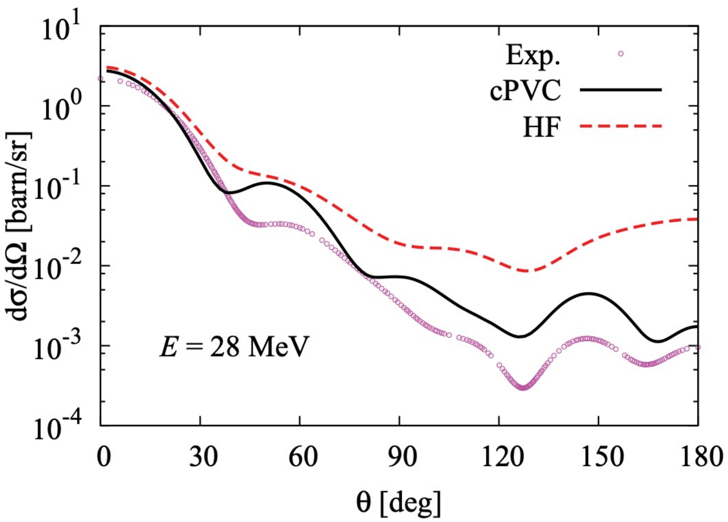


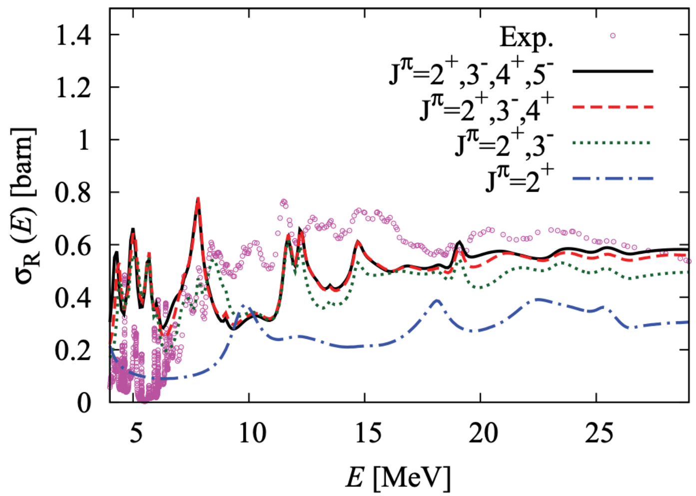
Nucleus	J^π	Theory (RPA)		Experiment	
		Energy (MeV)	$B(Q_J^{\tau=0})$ ($e^2\mathrm{fm}^{2J}$)	Energy (MeV)	$B(Q_J^{\tau=0})$ ($e^2\mathrm{fm}^{2J}$)
^{208}Pb	2^+	5.12	2.35×10^3	4.09	3.00×10^3
	3^-	3.49	7.08×10^5	2.62	6.11×10^5
	4^+	5.69	5.16×10^6	4.32	15.5×10^6
	5^-	4.49	4.96×10^8	3.20	4.47×10^8

n + ^{16}O



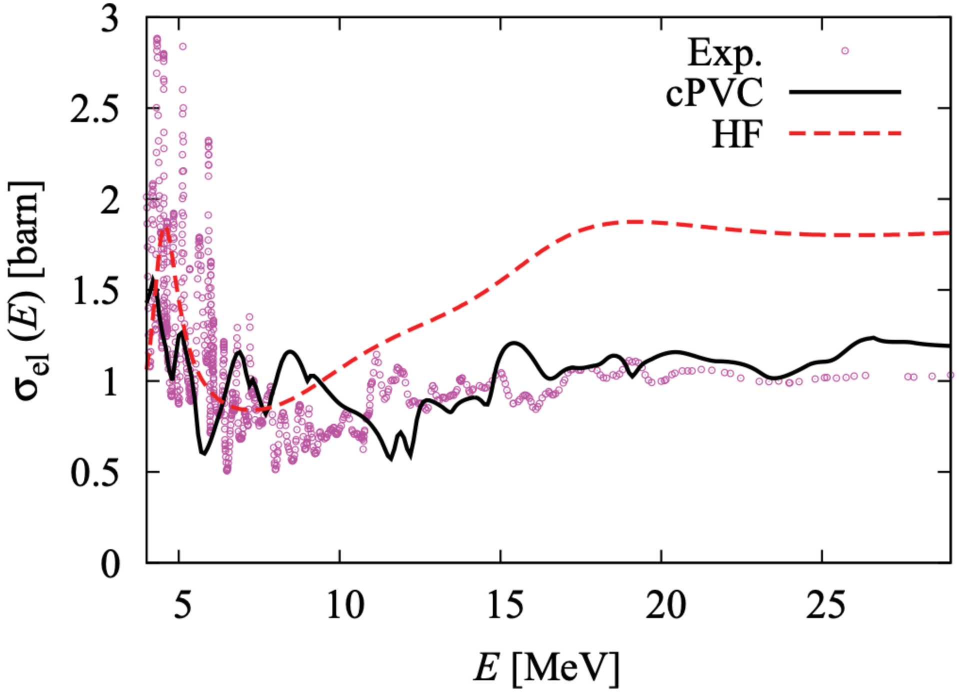






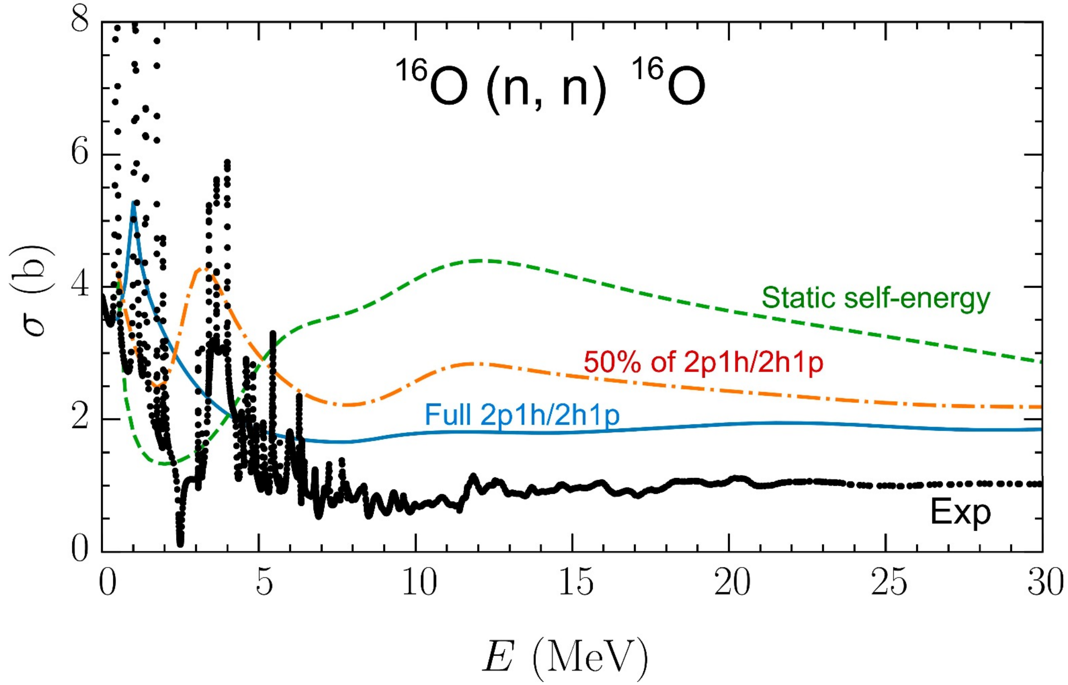
K. Mizuyama and K. Ogata
PRC 86 (2012) 041603

SkM*



K. Mizuyama and K. Ogata
PRC 86 (2012) 041603

$NNLO_{sat}$



A. Idini, C. Barbieri and Navrátil,
PRL 123 (2019) 092501

Adopting the effective separable interaction

$$V(\mathbf{r_1},\mathbf{r_2}) = -\kappa_{self} r_1 \frac{dV}{dr_1} r_2 \frac{dV}{dr_2} \sum_{\lambda\mu} \chi_\lambda Y^*_{\lambda\mu}(\theta_1) Y_{\lambda\mu}(\theta_2)$$

$$\kappa_{\text{self}} = -[\int r \frac{\partial\rho}{\partial r} r \frac{\partial U}{\partial r} r^2 dr]^{-1}$$

$$\beta_\lambda r \frac{\partial\rho}{\partial r} = \delta\rho$$

$$H = H_c + H_p + H_{PVC}$$

$$H_c = \sum_{\lambda\mu} \hbar\omega_\lambda [\Gamma^+_{\lambda\mu} \Gamma_{\lambda\mu} + 1/2]$$ → Collective ($\Gamma_{\lambda\mu}^+$ creates a phonon)

$$H_p = -\hbar^2/2m \; d^2/dr^2 + V(r) + V_{ls}(r)$$ → Single-particle

$$H_{PVC} = \sum_{\lambda\mu} -r dV/dr \beta_\lambda Y_{\lambda\mu} [\Gamma^+_{\lambda\mu} + (-1)^\mu \Gamma_{\lambda\mu}]$$ → Linear interaction

(β_λ ↓) Def. parameter

The coupled equations (no ground state correlations):

Approximate the odd nucleon wavefunction as

$$\Psi_a = [\psi_a^x + (\psi_b^C \otimes \Gamma_\lambda^+)_{j_a} + ...]\Phi_{GS}^A$$

$$\psi_a^x = (R_a^x(r)/r)\Theta_{j_a m_a}$$

(particle part)

$$(\psi_b^C \otimes \Gamma_\lambda^+)_{j_a} = (R_b^C(r)/r)(\Theta_{j_b} \otimes \Gamma_\lambda^+)_{j_a}$$

(phonon admixture)

Expanding over a HF basis in a box (only unoccupied levels, $e_{ai} > e_F$):

$$R_a^x(r) = \sum_i x_{a_i} R_{a_i}^{HF}(r) \quad .; \quad R_b^C(r) = \sum_i C_{b_i} R_{b_i}^{HF}(r)$$

one finds

$$\begin{cases} [-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + V_a(r) + 0\hbar\omega_\lambda]R_a^x(r) + \Xi_{a,b\lambda}(-\beta_\lambda r dV/dr)R_b^C(r) = ER_a^x(r) \\ \Xi_{a,b\lambda}(-\beta_\lambda r dV/dr)R_a^x(r) + [-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + V_b(r) + 1\hbar\omega_\lambda]R_b^C(r) = ER_b^C(r) \end{cases}$$

with the angular coupling :

$$\Xi_{a,b\lambda} = \langle \Theta_{j_a m_a} | \sum_{\lambda\mu} Y_{\lambda\mu}[\Gamma_{\lambda\mu}^+ + (-1)^\mu \Gamma_{\lambda\mu}] | [\Theta_{j_b} \otimes \Gamma_\lambda^+]_{j_a m_a} \rangle = -i^{l_a+\lambda-l_b}\left(\frac{1}{\sqrt{4\pi}}\right)^{1/2} < j_a \frac{1}{2} \lambda 0 | j_b \frac{1}{2} >$$

Expand over the unoccupied states of a Saxon-Woods basis:

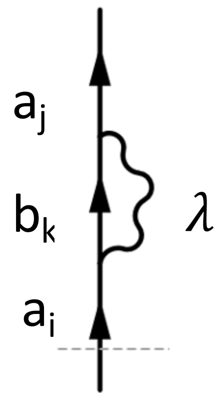
$$R_a^x(r) = \sum_i x_{ai} R_{ai}^{WS}(r) \qquad i \in non-occ.$$
$$R_b^C(r) = \sum_i C_{bi} R_{bi}^{WS}(r) \qquad i \in non-occ.$$

$$\begin{pmatrix} e_{a1} & 0\ldots & h_{a1,b1\lambda} & h_{a1,b2\lambda}\ldots \\ 0 & e_{a2}\ldots & h_{a2,b1\lambda} & h_{a2,b2\lambda}\ldots \\ h_{a1,b1\lambda} & h_{a2,b1\lambda}\ldots & e_{b1}+\hbar\omega & 0\ldots \\ h_{a1,b2\lambda} & h_{a2,b2\lambda}\ldots & 0 & e_{b2}+\hbar\omega\ldots \end{pmatrix} \begin{pmatrix} x_{a1} \\ x_{a2}\ldots \\ C_{b1} \\ C_{b2}\ldots \end{pmatrix} = \tilde{E} \begin{pmatrix} x_{a1} \\ x_{a2}\ldots \\ C_{b1} \\ C_{b2}\ldots \end{pmatrix}$$

$$\begin{pmatrix} e_{a1}+\Sigma_{11}(\tilde{E}) & \Sigma_{12}(\tilde{E})\ldots \\ \Sigma_{21}(\tilde{E}) & e_{a2}+\Sigma_{22}(\tilde{E})\ldots \end{pmatrix} \begin{pmatrix} x_{a1} \\ x_{a2}\ldots \end{pmatrix} = \tilde{E} \begin{pmatrix} x_{a1} \\ x_{a2}\ldots \end{pmatrix}$$

$$\Sigma_{ij}(\tilde{E}) = \sum_{bk>e_F} \frac{h_{ai,bk\lambda}\, h_{aj,bk\lambda}}{\tilde{E} - (E_{bk} + \hbar\omega_\lambda)}$$

$$h_{a,b\lambda} = \Xi_{a,b\lambda}\beta_\lambda \int dr\, r \frac{dV}{dr} \phi_a(r)\phi_b(r)$$



Include ground state correlations with proper antisymmetrization

$$\Psi_a = [\psi_a^x + [\psi_b^C \otimes \Gamma_\lambda^+]_{j_a} + \psi_a^y + [\psi_c^D \otimes \Gamma_\lambda]_{j_a} + ...]\Phi_{GS}$$

$$\psi_a^y = (R_a^y(r)/r)\Theta_{j_a} \qquad [\psi_c^D \otimes \Gamma_\lambda]_{j_a} = (R_c^D(r)/r)[\Theta_{j_b} \otimes \Gamma_\lambda]_{j_a}$$

Extended set of radial equations:

$$[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + V_a(r) + 0\hbar\omega]R_a^x(r) + \Xi_{a,b\lambda}(-rdV/dr)R_b^C(r) + \Xi_{a,c\lambda}(-rdV/dr)R_c^D(r) = ER_a^x(r)$$

$$\Xi_{a,b\lambda}(-rdV/dr)R_a^x(r) + [-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + V_b(r) + 1\hbar\omega]R_b^C(r) + \Xi_{a,b\lambda}(-rdV/dr)R_a^y(r) = ER_b^C(r)$$

$$\Xi_{a,b\lambda}(-rdV/dr)R_b^C(r) + [-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + V_a(r) + 0\hbar\omega]R_a^y(r) - \Xi_{a,c\lambda}(-rdV/dr)R_c^D(r) = ER_a^y(r)$$

$$\Xi_{a,c\lambda}(-rdV/dr)R_a^x(r) - \Xi_{a,c\lambda}(-rdV/dr)R_a^y(r) + [-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + V_c(r) - 1\hbar\omega]R_c^D(r) = ER_c^D(r)$$

These radial wavefunctions can be used as form factors
to calculate one-nucleon transfer reactions in DBWA

We now expand also over occupied states:

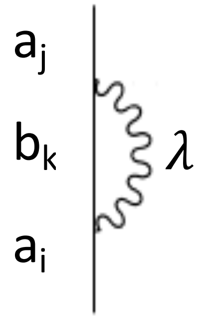
$$R^x{}_a(r) = \sum_i x_{ai} R^{WS}_{ai}(r); e_{ai} > 0$$

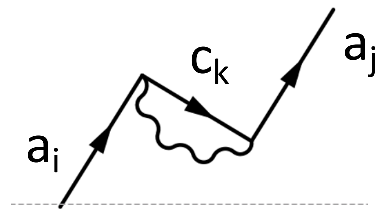
$$R^C{}_b(r) = \sum_i C_{bi} R^{WS}_{bi}(r); e_{bi} > 0$$

$$R^D{}_c(r) = \sum_i D_{ci} R^{WS}_{ci}(r); e_{ci} < 0$$

$$\left(\begin{array}{cc|cc|cc}
e_{a1}-e_F & 0\ldots & h_{a1,b1\lambda} & h_{a1,b2\lambda}\ldots & -h_{a1,c1\lambda} & -h_{a1,c2\lambda}\ldots \\
0 & e_{a2}-e_{F\ldots} & h_{a2,b1\lambda} & h_{a2,b2\lambda}\ldots & -h_{a2,c1\lambda} & -h_{a2,c2\lambda}\ldots \\
\hline
h_{a1,b1\lambda} & h_{a2,b1\lambda}\ldots & e_{b1}-e_F+\hbar\omega & 0\ldots & 0 & 0\ldots \\
h_{a1,b2\lambda} & h_{a2,b2\lambda}\ldots & 0 & e_{b2}-e_F+\hbar\omega\ldots & 0 & 0\ldots \\
\hline
-h_{a1,c1\lambda} & -h_{a2,c1\lambda}\ldots & 0 & 0 & e_{c1}-e_F-\hbar\omega & 0\ldots \\
-h_{a1,c2\lambda} & -h_{a2,c2\lambda}\ldots\ldots & 0 & 0 & 0 & e_{c2}-e_F-\hbar\omega\ldots
\end{array}\right)
\begin{pmatrix} x_{a1} \\ x_{a2}\ldots \\ C_{b1} \\ C_{b2\ldots} \\ D_{c1} \\ D_{c2}\ldots \end{pmatrix}
= \tilde{E}
\begin{pmatrix} x_{a1} \\ x_{a2}\ldots \\ C_{b1} \\ C_{b2\ldots} \\ D_{c1} \\ D_{c2}\ldots \end{pmatrix}$$

$$\Sigma_{ij}(\tilde{E}) = \sum_{bk>e_F} \frac{h_{ai,bk\lambda}\, h_{aj,bk\lambda}}{\tilde{E} - (E_{bk} + \hbar\omega_\lambda)} + \sum_{ck<e_F} \frac{h_{ai,ck\lambda}\, h_{aj,ck\lambda}}{\tilde{E} - E_{ck} + \hbar\omega_\lambda}$$





Expressing the self-energy in coordinate space,

$$(H_p - e_F)R^x_{ai}(r) + \int dr' \left[\sum_{bk>e_F} \frac{\Xi^2_{a,b\lambda}\beta^2_\lambda \, R^{WS}_{bk}(r')R^{WS}_{bk}(r)f(r)f(r')}{\tilde{E}_{ai} - (E_{bk} + \hbar\omega_\lambda)} + \sum_{ck<e_F} \frac{\Xi^2_{a,c\lambda}\beta^2_\lambda \, R^{WS}_{ck}(r')R^{WS}_{ck}(r)f(r)f(r')}{\tilde{E}_{ai} - E_{ck} + \hbar\omega_\lambda} \right] R^x_{ai}(r') = \tilde{E}_{ai}R^x_{ai}(r)$$

$$f(r) \equiv r dV/dr$$

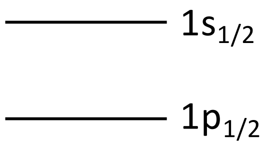
The self-energy is a non-local function which is then used as an optical potential to describe the neutron-core interaction.
See Potel's talk tomorrow, about its use in (d,p) reactions.

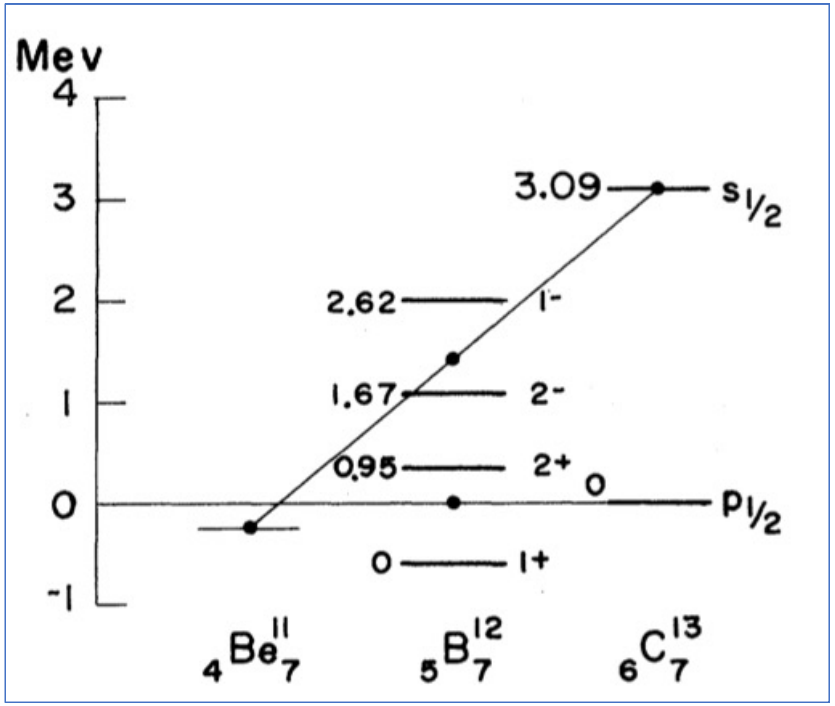
I will apply the NFT to build a model for the calculation low-lying excitations in light (A ≈ 10-15) weakly bound nuclei containing a few phenomenological parameters, trying to correlate explicitly different experimental results.

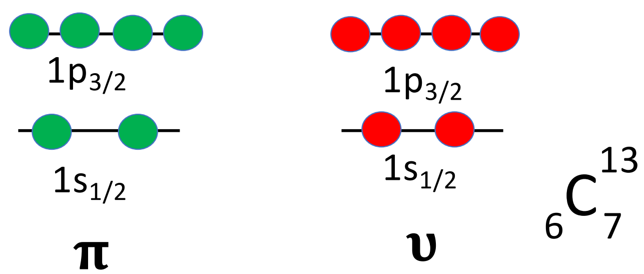
- Only the coupling to the low-lying 2+ excitation will be included, taking the energy and deformation parameter from experiment.

- The mean field will be taken as a Woods-Saxon potential with parameters fitted on experimental data INCLUDING effects beyond mean field

Parity inversion in N=7 isotones : the role of p-n interaction

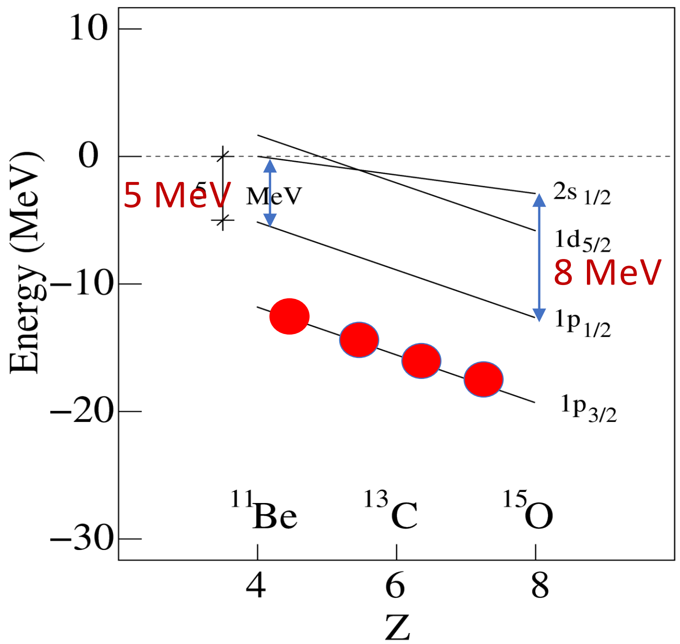






I. Talmi and I. Unna, PRL 4, 469 (1961)

Parity inversion in N=7 isotones is not reproduced by spherical mean field calculations, although the mean field includes the effect of the neutron-proton interaction



Typical spherical mean-field results with Skyrme forces (Sagawa,Brown,Esbensen PLB 309(93)1)

A possible explanation of parity inversion: dynamical coupling between the core and the loosely bound neutron

VOLUME 70, NUMBER 10 PHYSICAL REVIEW LETTERS 8 MARCH 1993

Structure of Exotic Neutron-Rich Nuclei

Takaharu Otsuka, Nobuhisa Fukunishi, and Hiroyuki Sagawa
Department of Physics, University of Tokyo, Hongo, Bunkyo-ku, Tokyo, 113, Japan
(Received 13 November 1992)

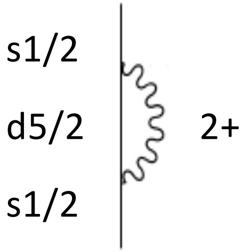
A new framework, the variational shell model, is proposed to describe the structure of neutron-rich unstable nuclei. An application to ^{11}Be is presented. Contrary to the failure of the spherical Hartree-Fock model, the anomalous $\frac{1}{2}^+$ ground state and its neutron halo are reproduced with the Skyrme (SIII) interaction. This state is bound due to dynamical coupling between the core and the loosely bound neutron, which oscillates between the $2s_{1/2}$ and the $1d_{5/2}$ orbits.

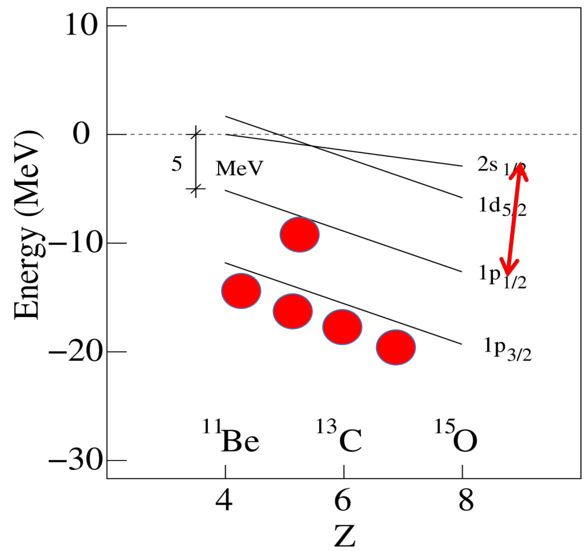
The core: spherical or deformed?
Important role of fluctuations expected
We propose a dynamical description

N. Vinh Mau, Nucl. Phys. A 592 (1995) 43
G.F. Esbensen and H. Sagawa, Phys. Rev C 51 (1995)1274
F.M. Nunes and I. Thompson, Nucl. Phys. A 703 (2002) 593
G. Blanchon et al., Phys Rev. C 82 (2010) 034313
Myo et al, PRC 86 (2012) 024318
I. Hamamoto and S. Shimoura, J. Phys. G 34 (2007) 2715

^{11}Be

Eshift = - 2.5 MeV

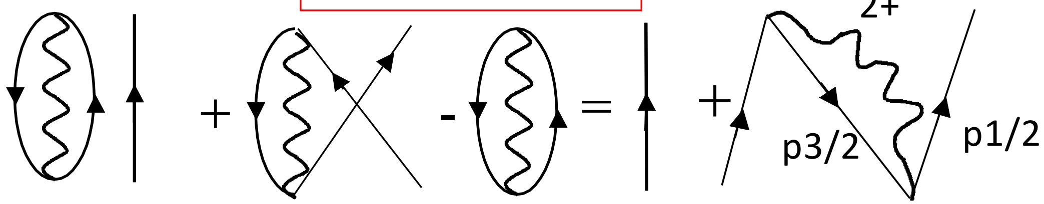




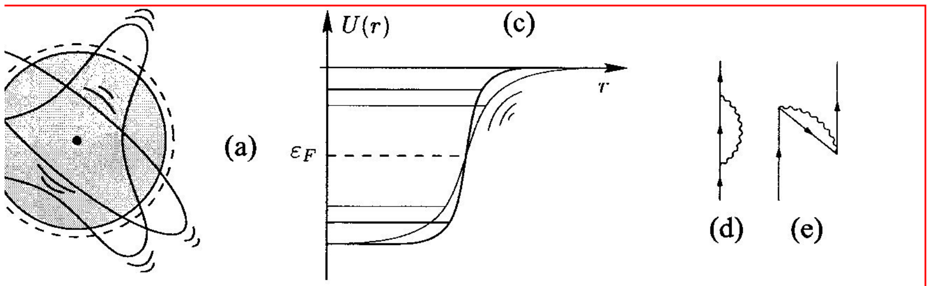
Self-energy

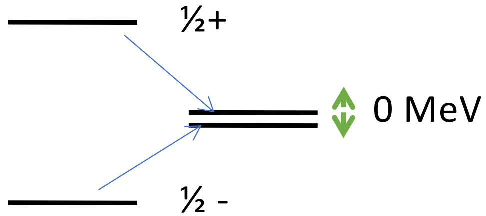
+

Eshift = + 2.5 MeV



Pauli blocking of core ground state correlations





↓

Level inversion

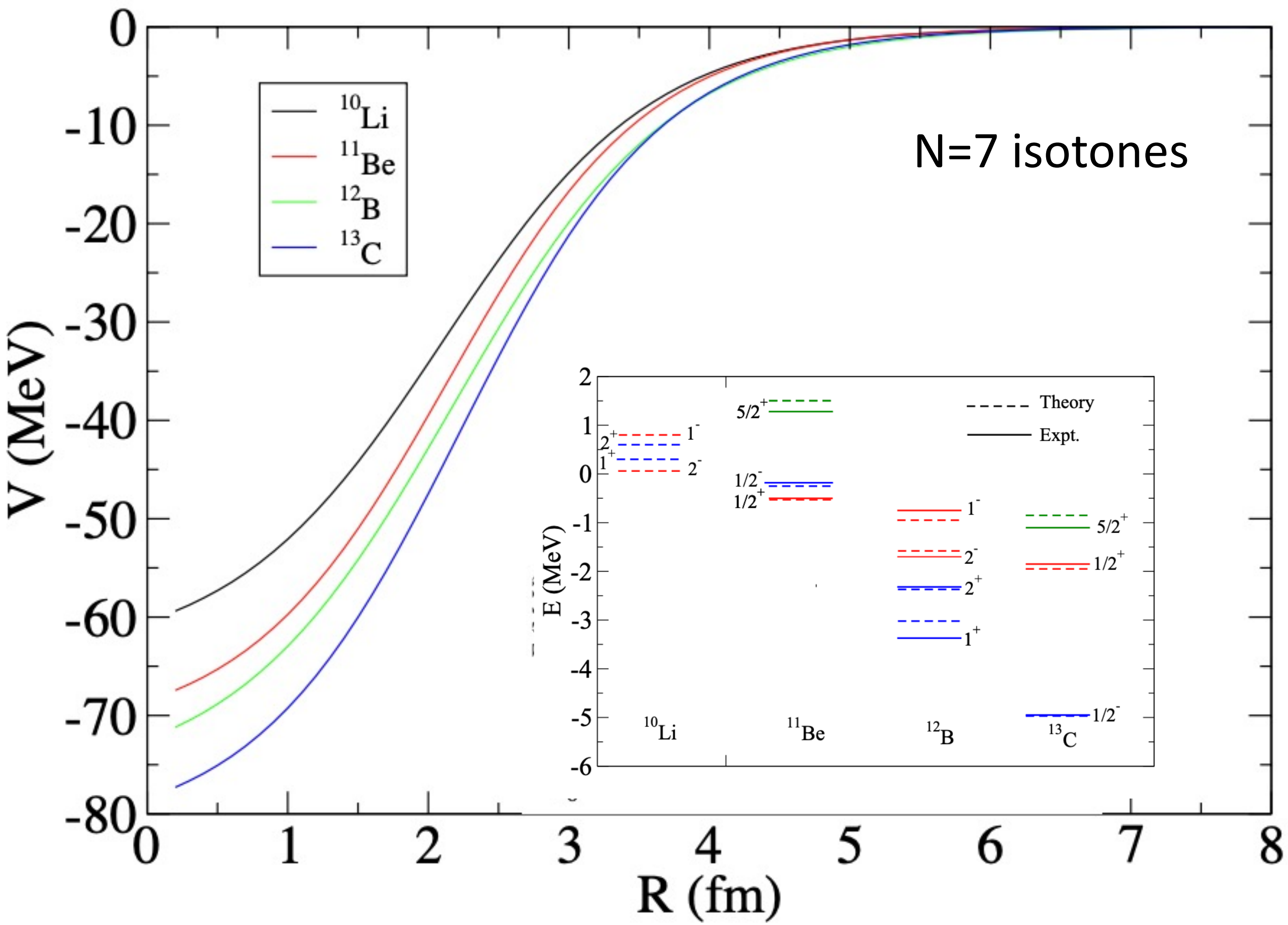
How do we determine the mean field? Parameter optimization

We perform the many-body calculation starting from a Woods-Saxon potential,

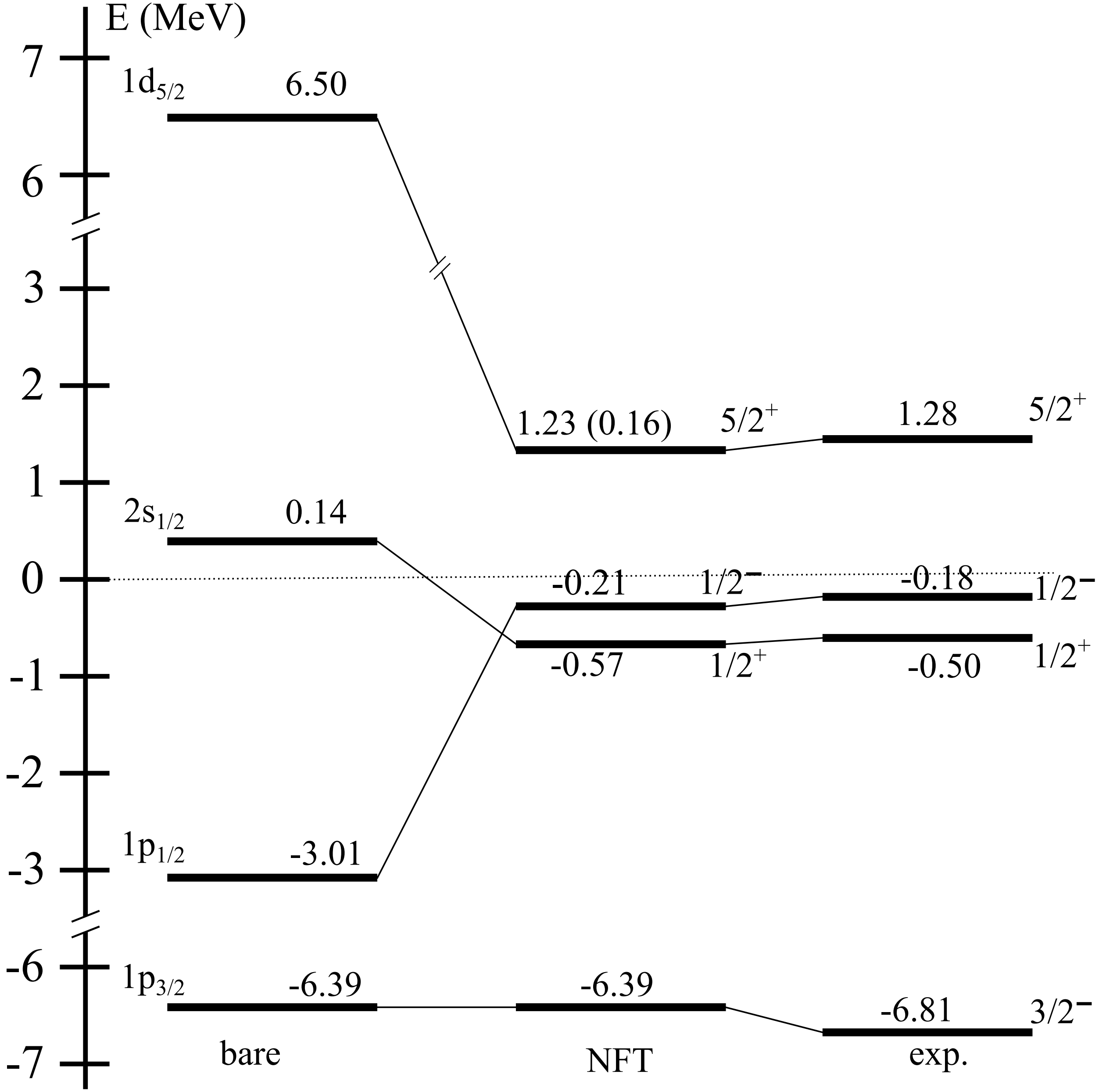
The following parameters are fitted to obtain the best agreement of the renormalized energies with the experimental $1/2^+$, $1/2^-$ and $5/2^+$ states in ^{11}Be and $3/2^-$ in ^{9}Be:

- Depth, diffuseness, radius, strength of spin-orbit coupling

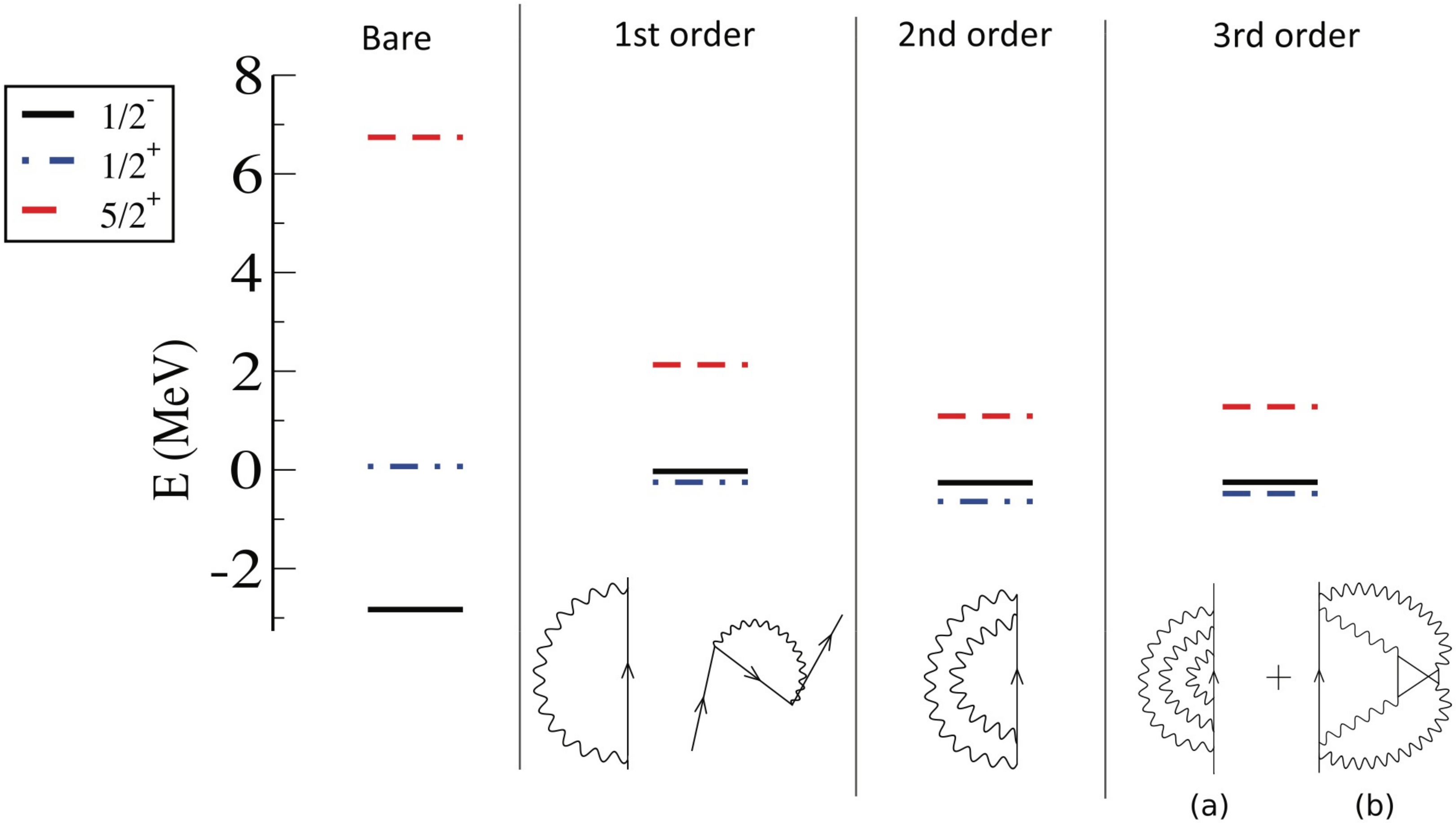
	$\hbar\omega_{2^+}$ (MeV)	β_2^n	V_{WS} (MeV)	V_{ls} (MeV)	a_{WS} (fm)	R_{WS} (fm)
^{10}Li	3.37	0.68	64	14	0.75	2.10
^{11}Be	3.37	0.71	72	18	0.72	2.14
^{12}B	3.80	0.57	77	22	0.78	2.18
^{13}C	4.4	0.46	82	27	0.73	2.23



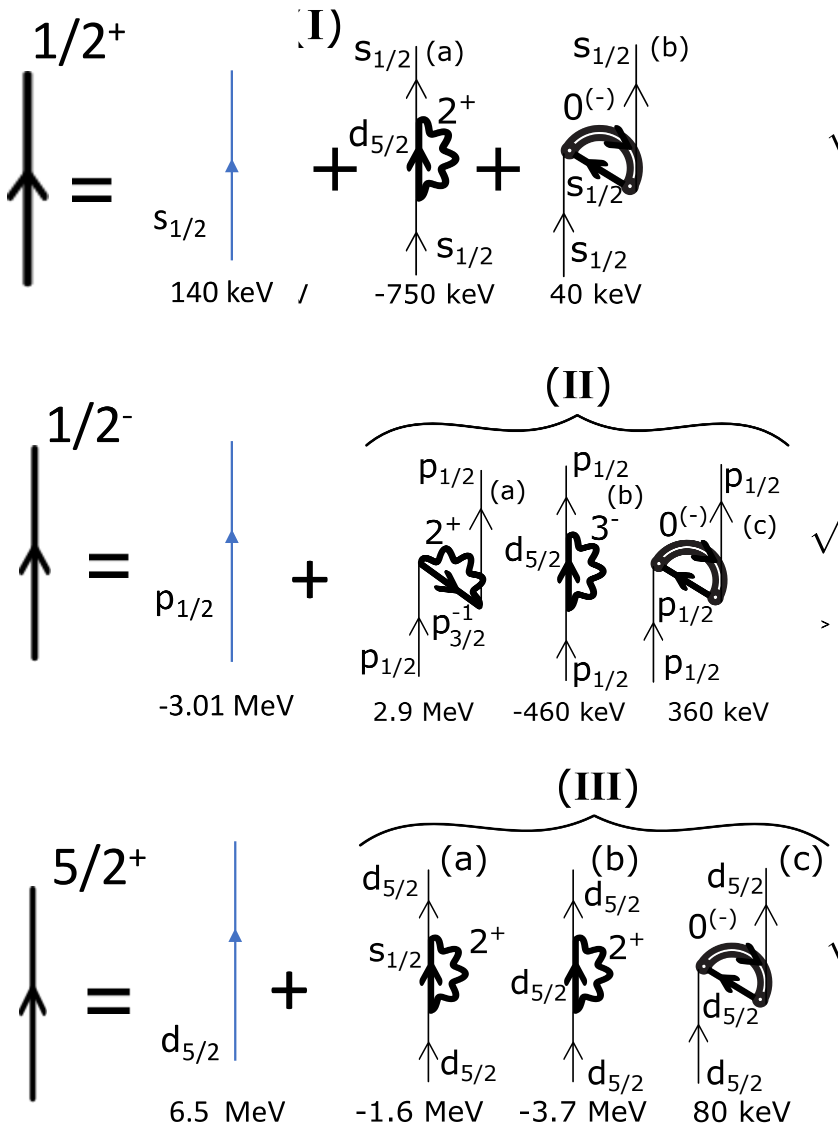
N=7 isotones
V (MeV)
R (fm)
0
-10
-20
-30
-40
-50
-60
-70
-80
0
1
2
3
4
5
6
7
8
10Li
11Be
12B
13C
E (MeV)
Theory
Expt.
5/2+
1/2-
1/2+
1-
2+
1+
2-
5/2+
1/2+
1/2-
2
1
0
-1
-2
-3
-4
-5
-6



E (MeV)
7
6
3
2
1
0
-1
-2
-3
-6
-7
1d5/2
6.50
2s1/2
0.14
1p1/2
-3.01
1p3/2
-6.39
bare
1.23 (0.16)
5/2+
-0.21
1/2−
-0.57
1/2+
-6.39
NFT
1.28
5/2+
-0.18
1/2−
-0.50
1/2+
-6.81
3/2−
exp.



Bare
1st order
2nd order
3rd order
1/2-
1/2+
5/2+
E (MeV)
8
6
4
2
0
-2
(a)
(b)

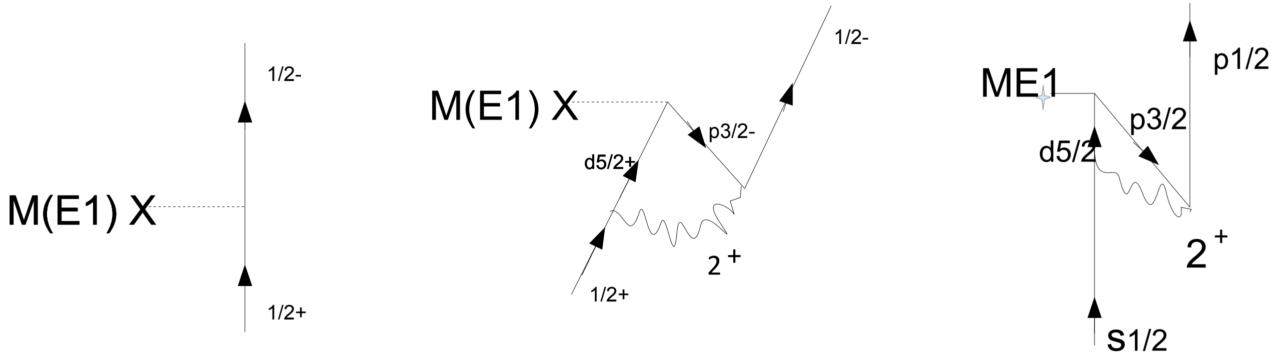


$$\sqrt{0.83}|s_{1/2}> + \sqrt{0.17}|(d_{5/2}\otimes 2^+)_{1/2^+}>$$

$$\sqrt{0.81}|p_{1/2}> + \sqrt{0.02}|(d_{5/2}\otimes 3^-)_{1/2^-}>$$
$$+\sqrt{0.15}|((p_{1/2}, 1p^{-1}_{3/2})_{2^+}\otimes 2^+)\ p_{1/2}>$$

$$\sqrt{0.34}|d_{5/2}> + \sqrt{0.32}|(s_{1/2}\otimes 2^+)_{5/2^+}>$$
$$+\sqrt{0.34}|(d_{5/2}\otimes 2^+)_{5/2^+}>$$

Strength of the dipole transition between ½+ and ½- states



1.95 e fm

-0.26 e fm

-0.19 e fm

B(E1) (th.) = 0.11 e^2 fm^2

B(E1) (exp.) = 0.102 ± 0.002 e^2 fm^2

This result is sensitive to the details of the mean field potential

Isotopic shift of the charge radius

$(\langle r^2 \rangle_{10Be})^{1/2} = 2.361 \pm 0.017$ fm $(\langle r^2 \rangle_{11Be})^{1/2} = 2.466 \pm 0.015$ fm

$(\langle r^2 \rangle)^{1/2}_{1s1/2} = 7.1$ fm $(\langle r^2 \rangle)^{1/2}_{1p1/2} = 3$ fm

Single-particle picture: S=1

Many-body picture: S=0.83

$(\langle r^2 \rangle)^{1/2}_{d5/2,coll} = 3$ fm

$$\langle r^2\rangle_{11Be} = \left(\langle r^2\rangle_{10Be} + \left(\frac{\langle r^2\rangle^{1/2}_{1s1/2}}{11}\right)^2\right)\times S^2 + (1-S^2)\times\left(\langle r^2\rangle_{10Be}\left(1+\frac{2}{4\pi}\beta_\pi^2\right) + \left(\frac{\langle r^2\rangle^{1/2}_{d5/2coll}}{11}\right)^2\right) = .$$

$$\langle r^2\rangle_{10Be} + \underline{\left(\frac{\langle r^2\rangle^{1/2}_{1s1/2}}{11}\right)^2 \times S^2 + (1-S^2)\times\left(\left(\frac{\langle r^2\rangle^{1/2}_{d5/2coll}}{11}\right)^2 + \langle r^2\rangle_{10Be}\frac{2}{4\pi}\beta_\pi^2\right)}$$

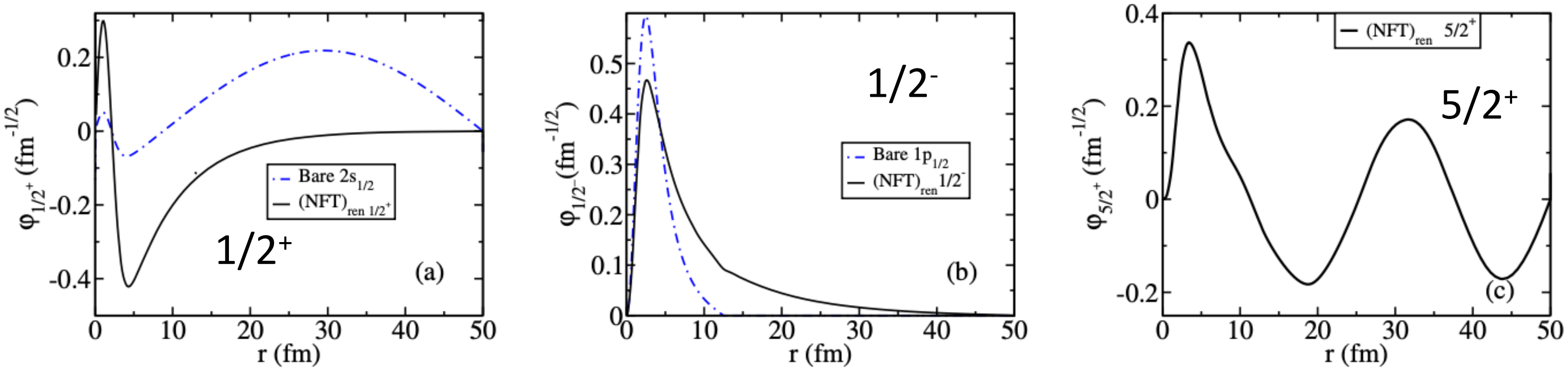
$\Delta \langle r^2 \rangle^{1/2}_{11Be}$ (th.) = 0.12 fm / 0.07 fm

$\Delta \langle r^2 \rangle^{1/2}_{11Be}$ (exp.)= 0.11 fm

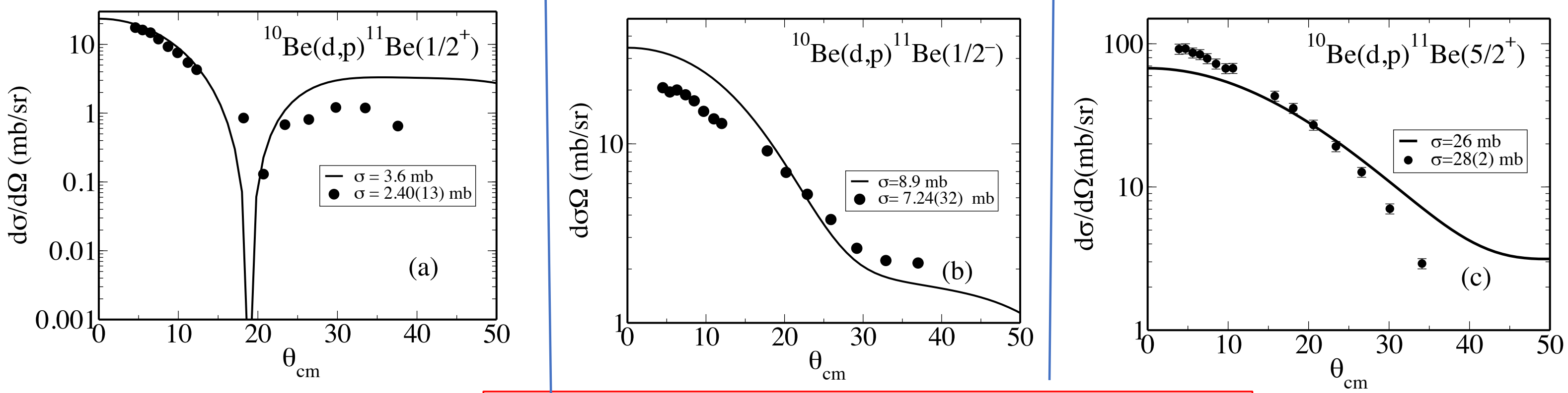
$^{10}Be(d,p)^{11}Be$ at E_d = 21.4 MeV

Test of the single-particle component of the many-body wavefunction

Form factors



Cross sections

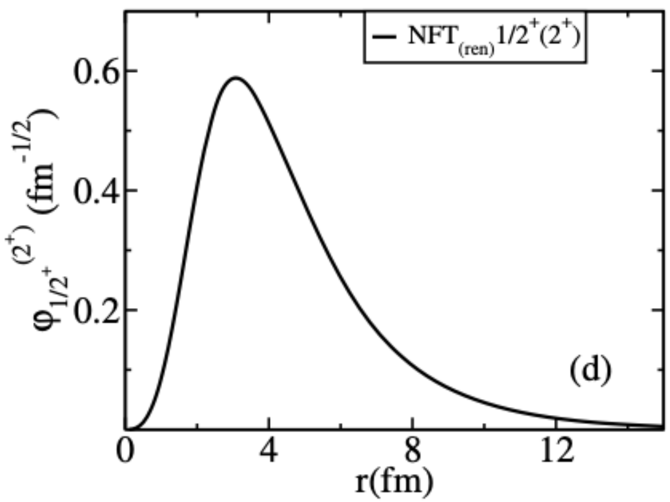


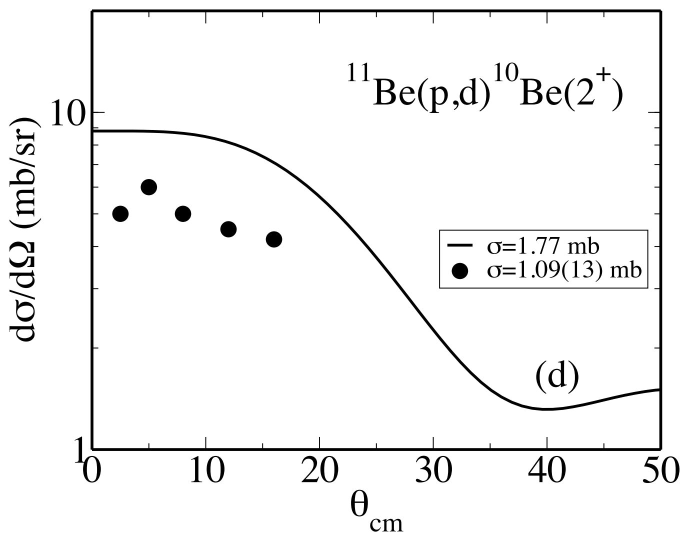
K.T. Schmitt et al., PRC88 (2012) 064612

$^{11}Be(1/2^+)(p,d)^{10}Be(2^+)$

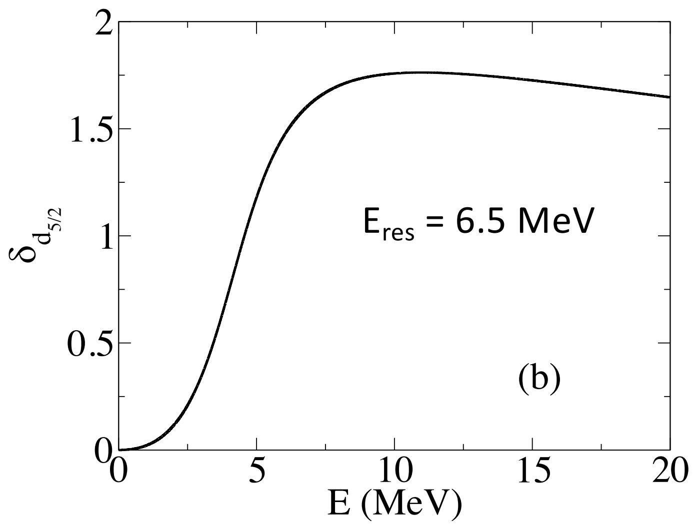
Test of the collective component $R^C_{d5/2}$ of the many-body wavefunction

$$(\psi_b^C \otimes \Gamma_\lambda^+)_{j_a} = (R_b^C(r)/r)(\Theta_{j_b} \otimes \Gamma_\lambda^+)_{j_a}$$

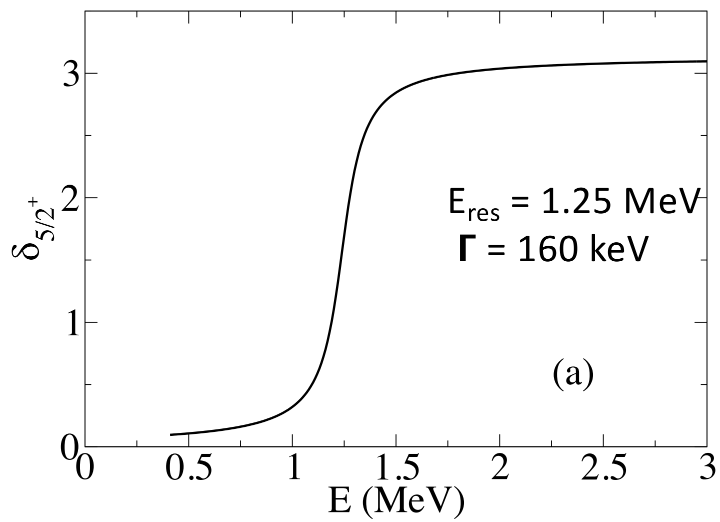




d5/2 phase shift in the bare potential

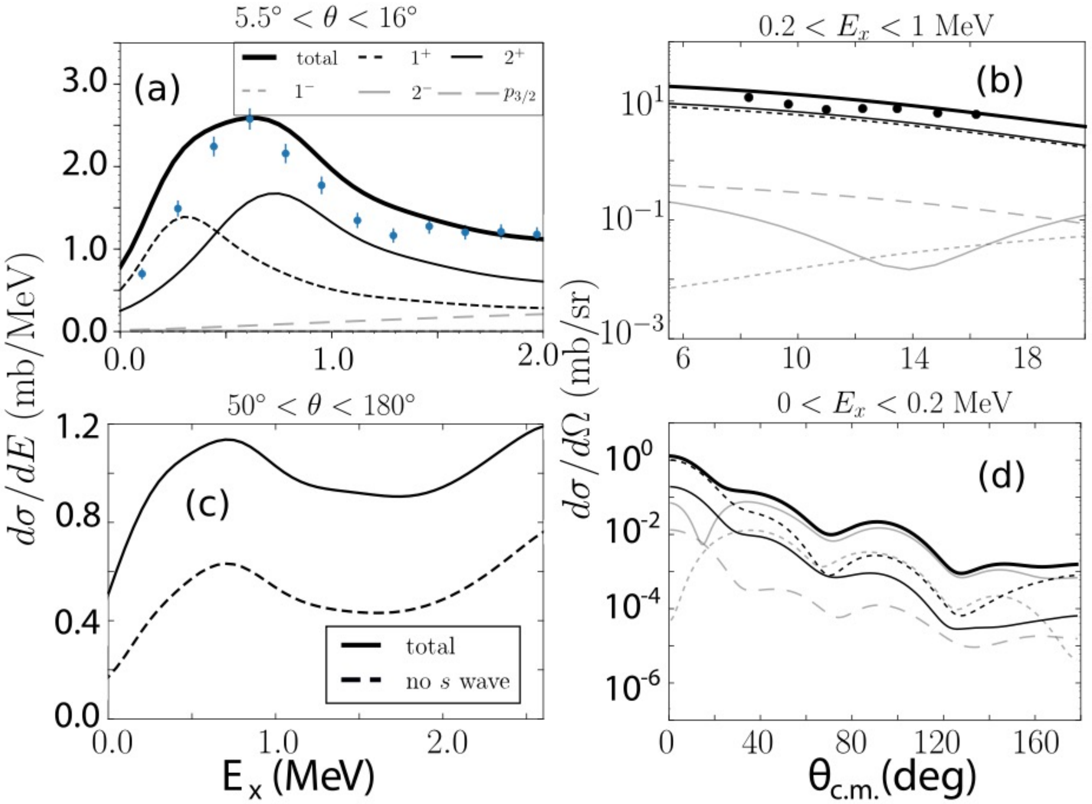


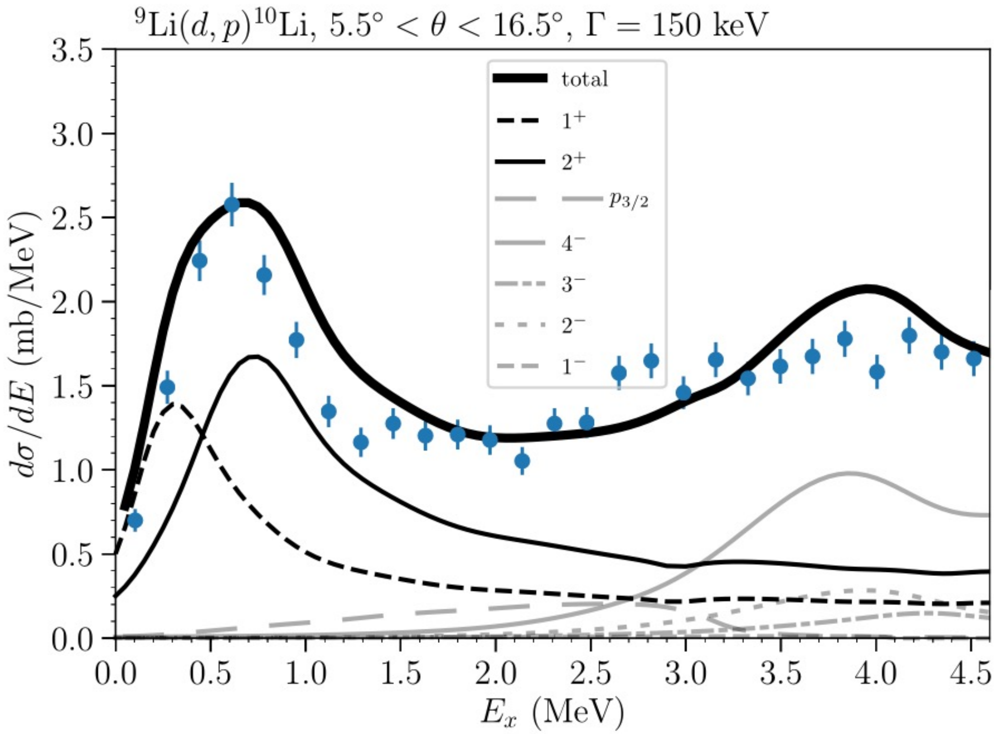
Renormalized 5/2+ phase shift



^{10}Li and the ^{9}Li(d,p)^{10}Li reaction

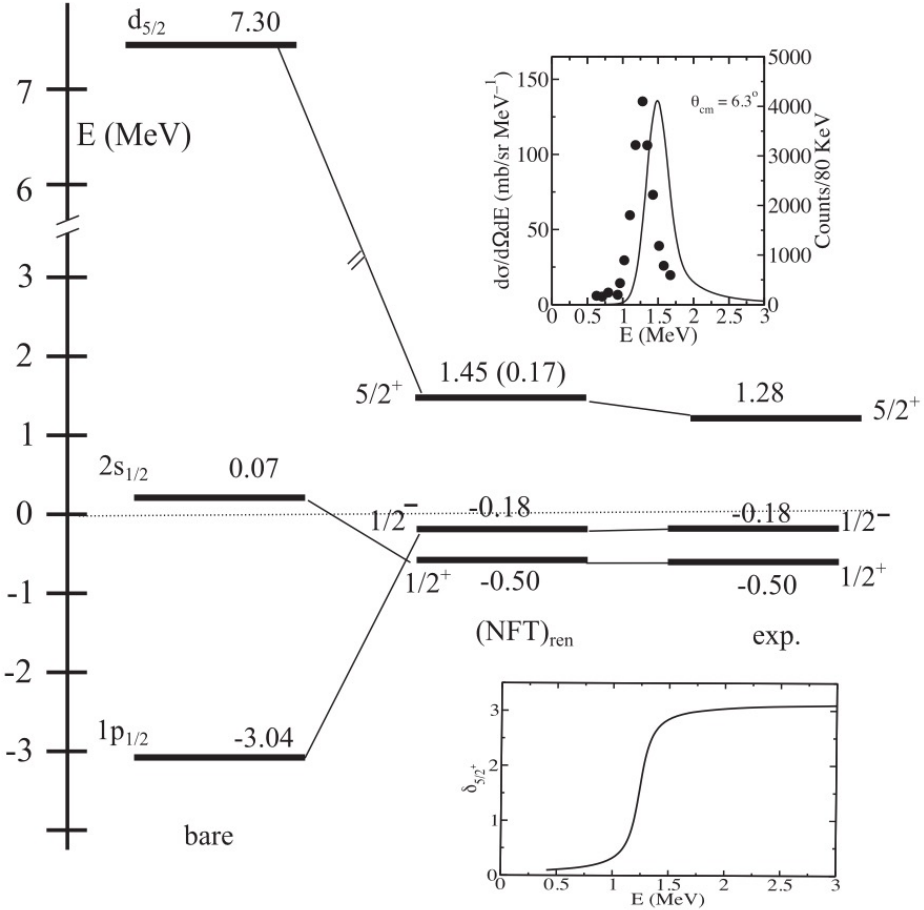
F. Barranco et al,
Phys. Rev. C 101 (2020) 031305(R)



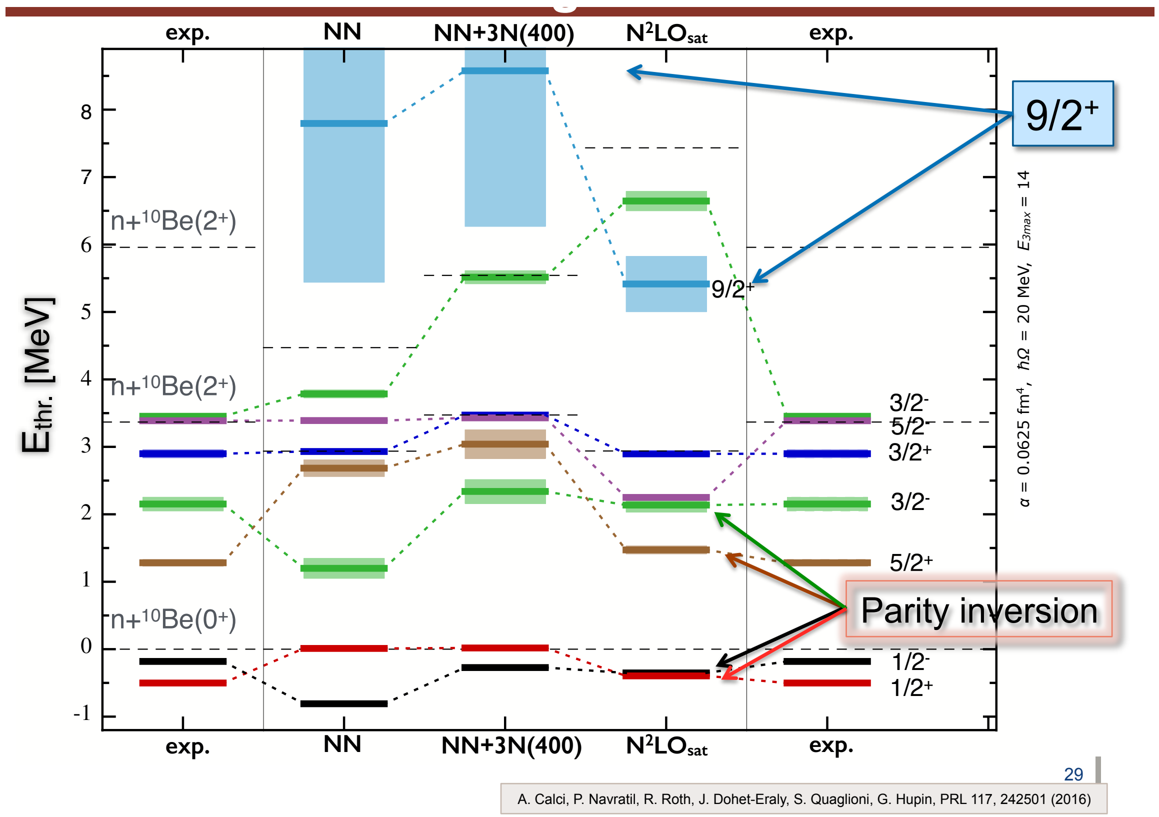


The critical description of the experimental results from complementary approaches can be of great interest

PVC



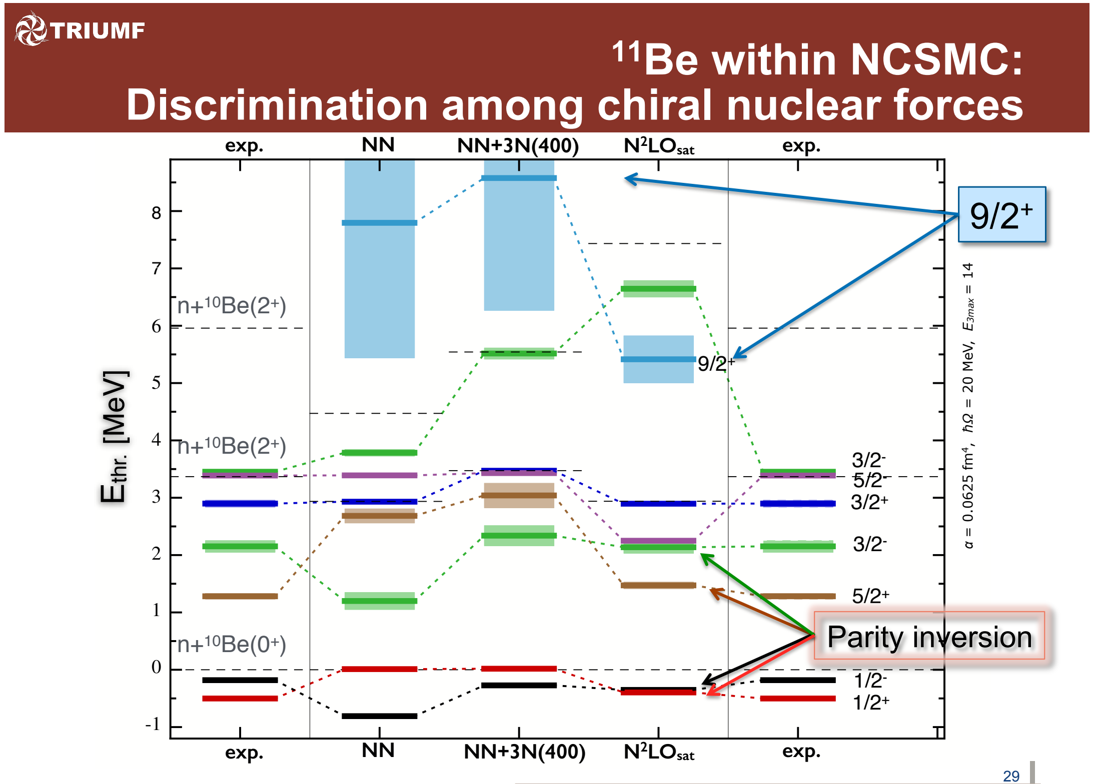
Ab Initio



… and ab initio calculation turned out to be quite challenging

PRL **117,** 242501 (2016) PHYSICAL REVIEW LETTERS week ending 9 DECEMBER 2016

Can *Ab Initio* Theory Explain the Phenomenon of Parity Inversion in ^{11}Be?



A. Calci, P. Navratil, R. Roth, J. Dohet-Eraly, S. Quaglioni, G. Hupin, PRL 117, 242501 (2016)

$$\Sigma_{\gamma\delta} = \sum_{pf} h(\gamma, pf) \frac{1}{e - e_p - \hbar\omega_f} h(\delta, pf)$$

$$h(\gamma, pf) = \sum_{p'h'} V_{\gamma h', pp'} X^f_{p'h'}$$

$$h(\delta, pf) = \sum_{p''h''} V_{\delta h'', pp''} X^f_{p''h''}$$

$$\Sigma_{\gamma\delta} = \sum_{pf} \left[\sum_{p'h'} V_{\gamma h', pp'} X^f_{p'h'} \right] \frac{1}{e - e_p - \hbar\omega_f} \left[\sum_{p''h''} V_{\delta h'', pp''} X^f_{p''h''} \right]$$

$$\Sigma_{\gamma\delta} = \sum_{p'h'p''h''} V_{\gamma h', pp'} \left[\sum_{pf} X^f_{p'h'} \frac{1}{e - e_p - \hbar\omega_f} X^f_{p''h''} \right] V'_{\delta h'', pp'}$$