Microscopic optical potentials: achievements and challenges

Matteo Vorabbi

Towards a consistent approach for nuclear structure and reactions: microscopic optical potentials

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UNIVERSITY OF SURREY

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- Carlotta Giusti
- Michael Gennari
- Paolo Finelli
- Petr Navrátil
- Vittorio Somà

Outline

- Motivations
- The nucleon-nucleus optical potential within the multiple scattering theory
- Application to light nuclei with NCSM densities
- Application to medium-mass nuclei with SCGF densities
- Application to inelastic scattering & future challenges
- The nucleus-nucleus optical potential within the multiple scattering theory
- Summary & outlook

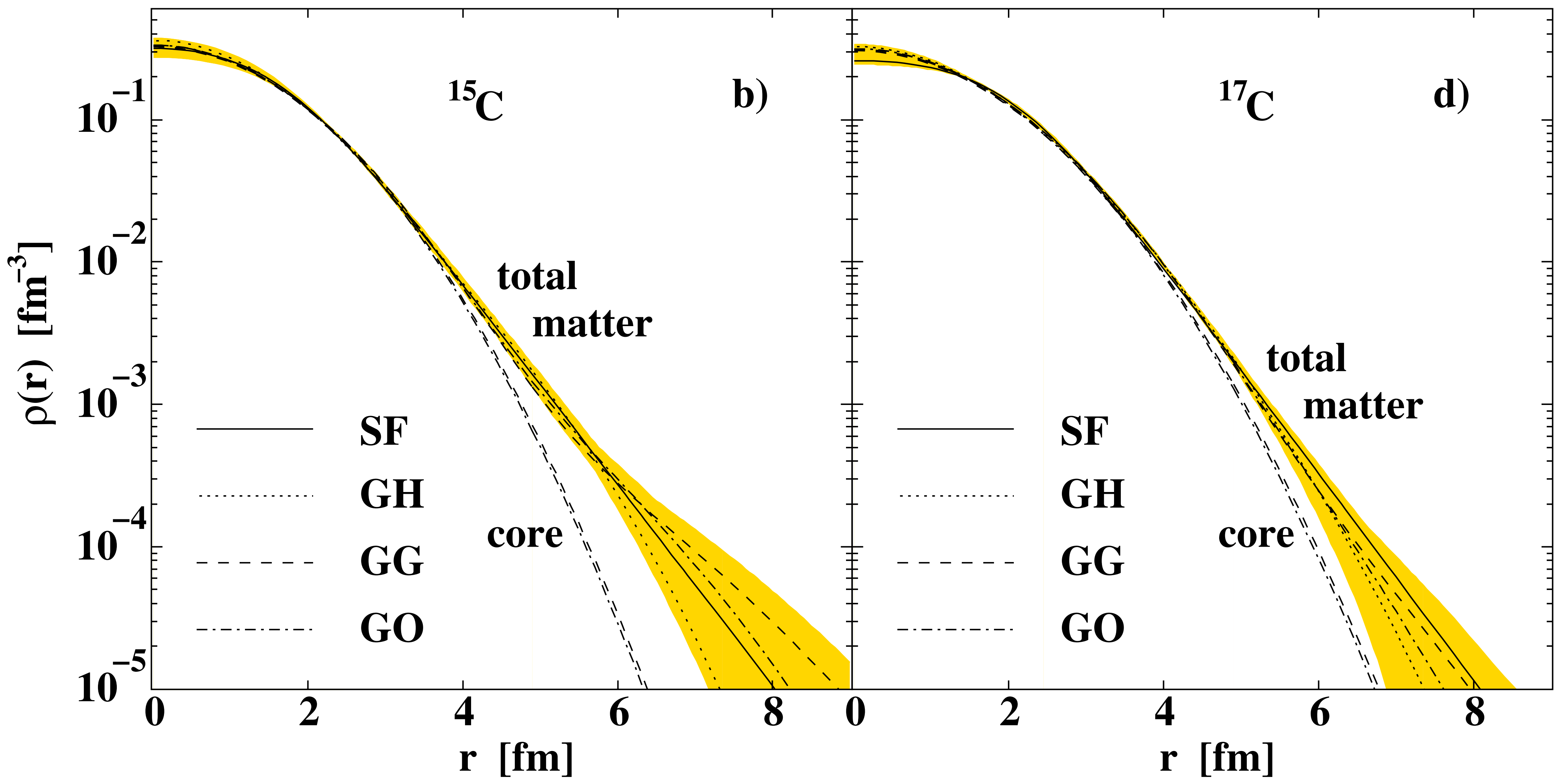
Motivations

- Increasing experimental efforts to develop the technologies necessary to study the elastic proton scattering in inverse kinematics
- Attempts to use such experiments to determine the matter distribution of nuclear systems at intermediate energies

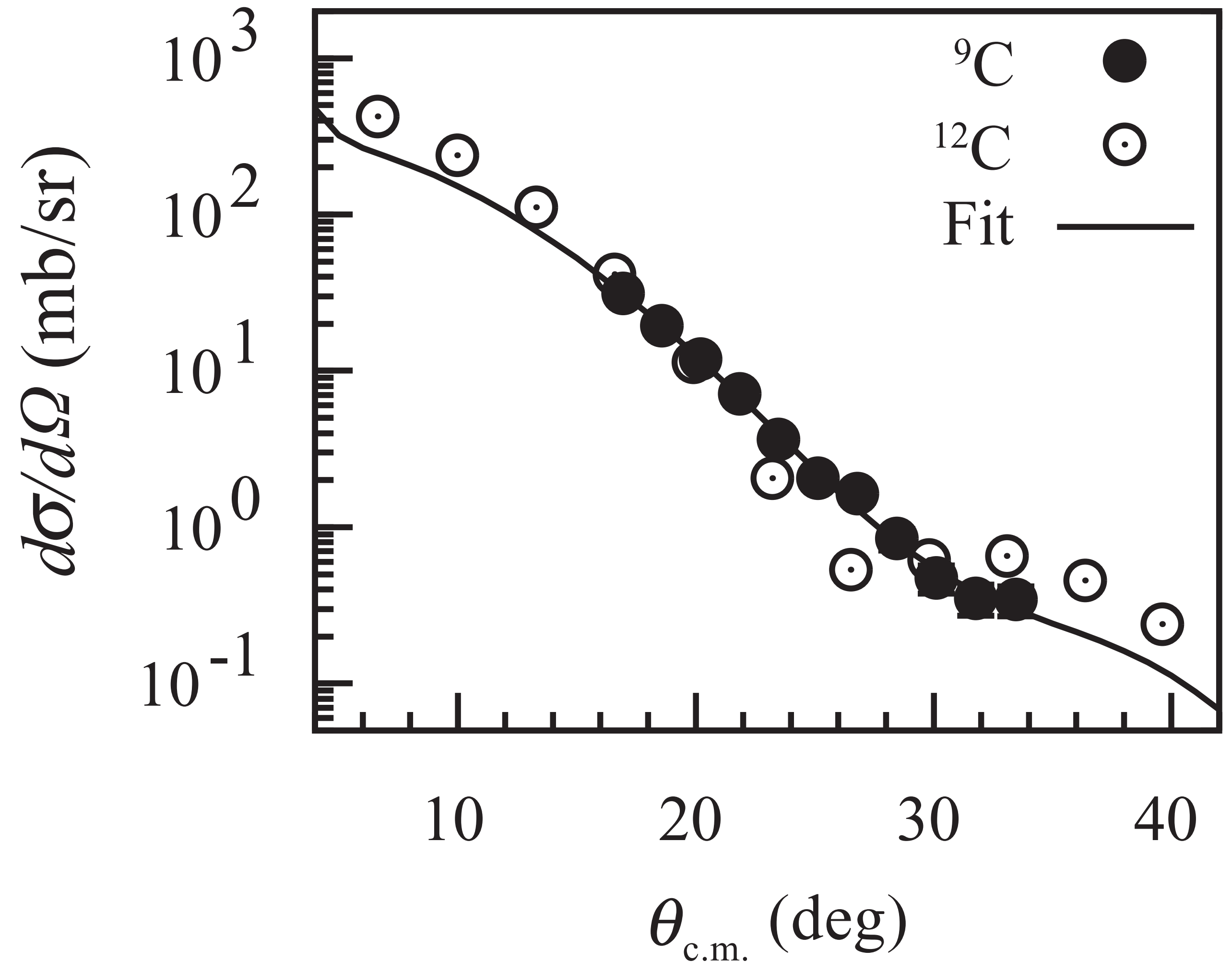
[Sakaguchi, Zenihiro, PPNP 97 (2017) 1–52]

- Measurements are not free from sizeable uncertainties
- The Glauber model is used to analyse the data
- An essential step in the data analysis is the subtraction of contributions from the inelastic scattering

Develop a microscopic approach to make reliable predictions for elastic and inelastic scattering



[Dobrovolsky et al., NPA 1008 (2021) 122154]



[Matsuda et al., PRC **87**, 034614 (2013)]

Optical potential

Phenomenological

Unfortunately, current used optical potentials for low-energy reactions are phenomenological and primarily constrained by elastic scattering data.

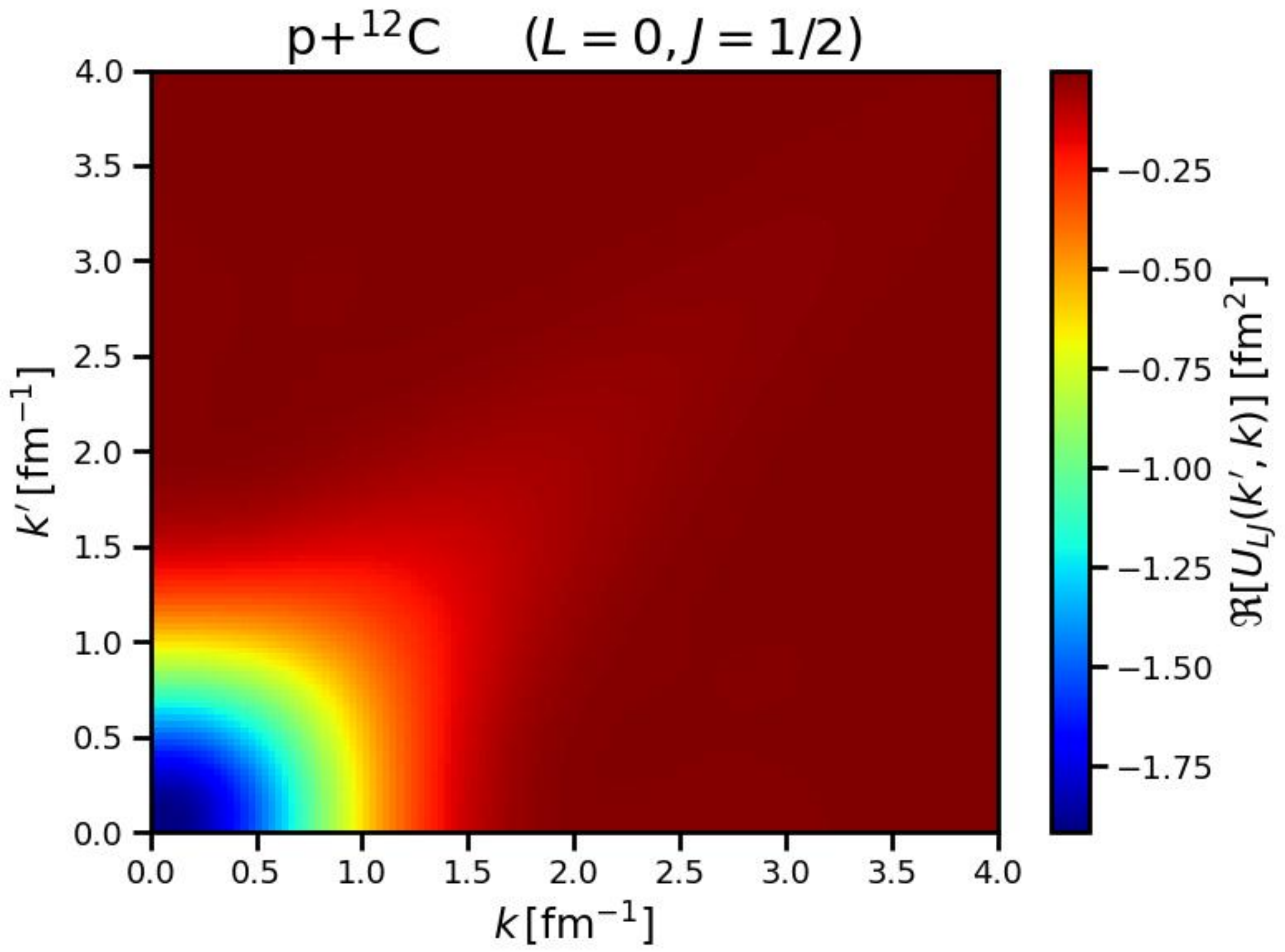
Unreliable when extrapolated beyond their fitted range in energy and nuclei

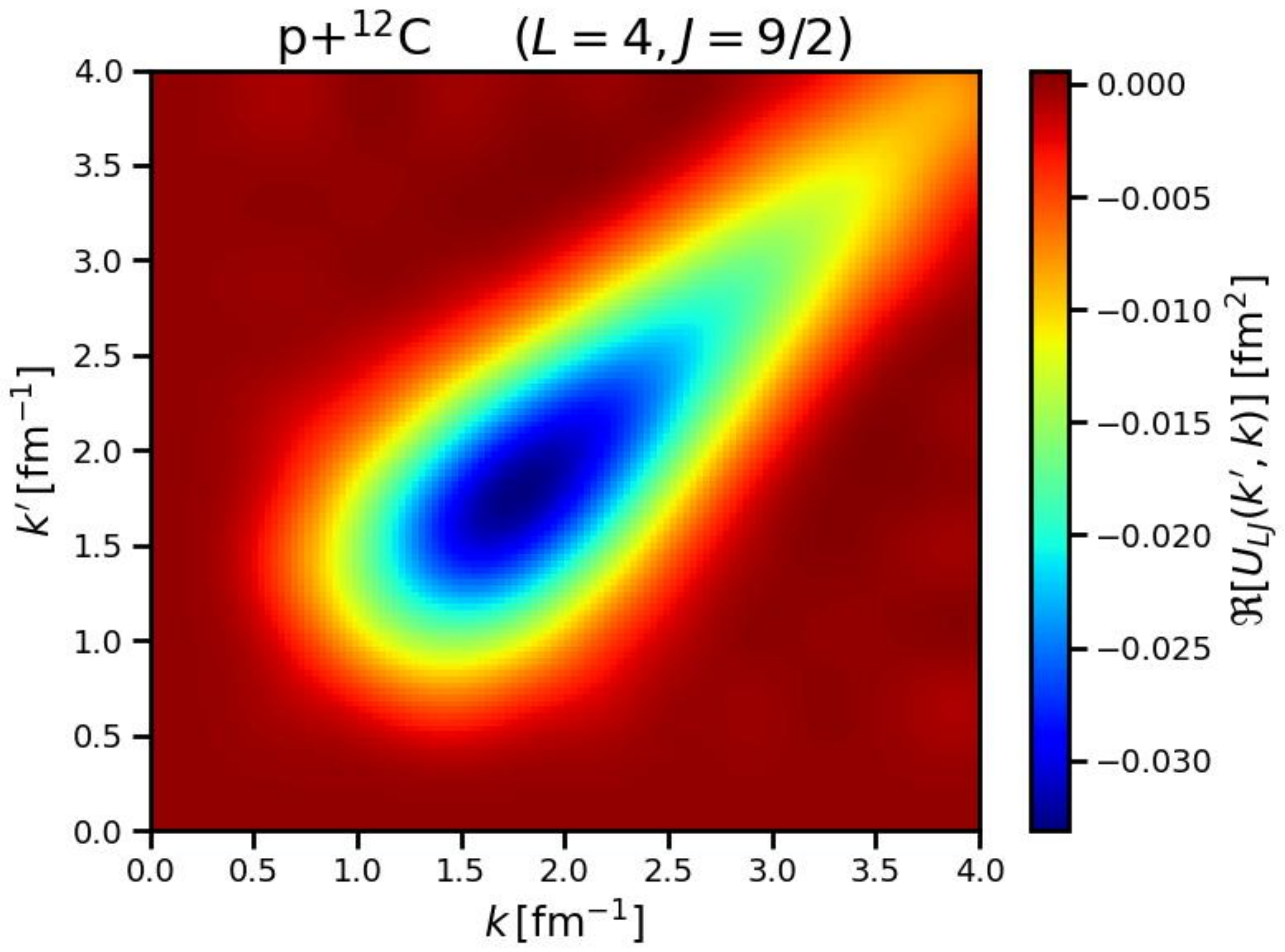
Microscopic

Existing microscopic optical potentials can be developed in a low- (Feshbach theory) or high-energy regime (Watson multiple scattering theory). Calculations are more difficult.

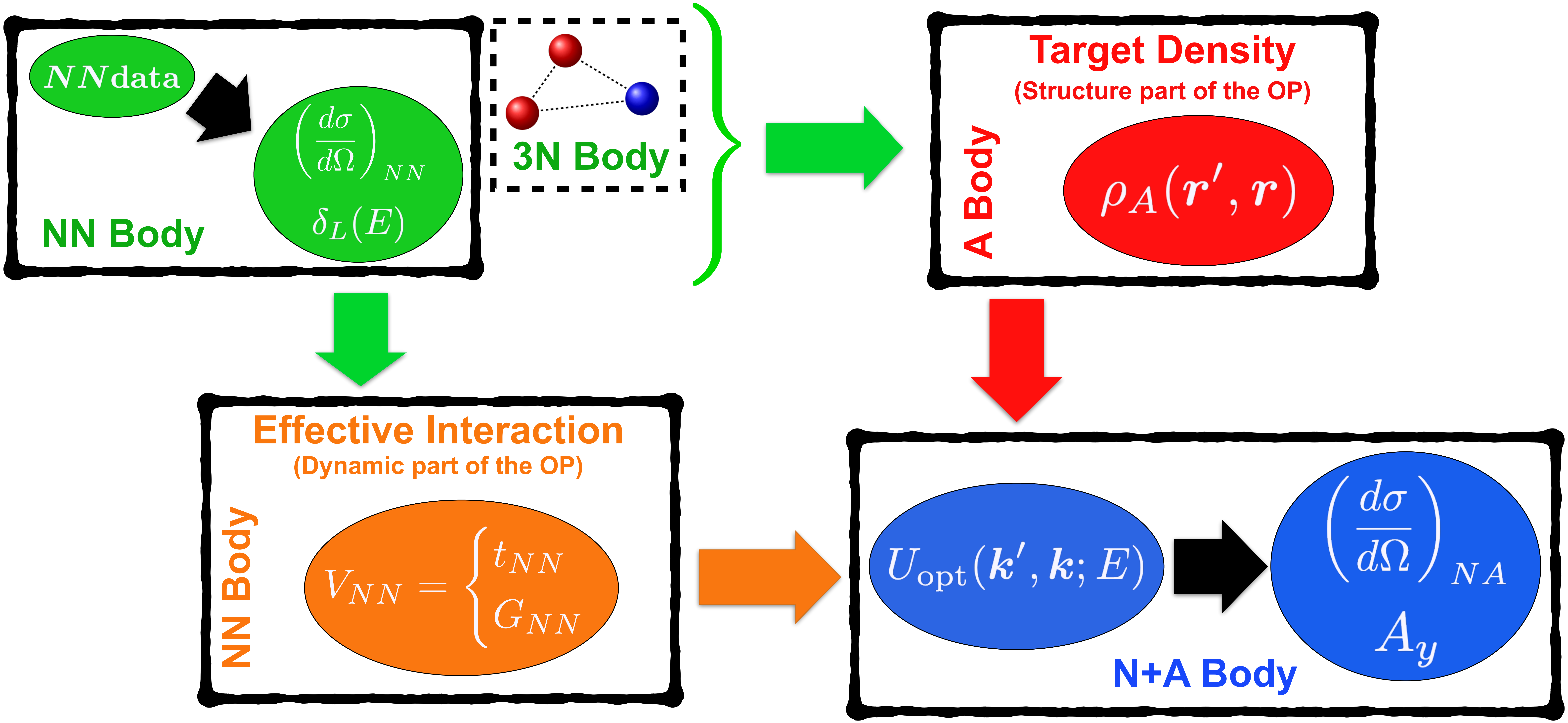
No fit to experimental data

$$
\begin{aligned}
V(r) &= -V_R f_R(r) - iW_V f_V(r) \\
&+ 4a_{VD} V_D \frac{d}{dr} f_{VD}(r) + 4ia_{WD} \frac{d}{dr} f_{WD}(r) \\
&+ \frac{\lambda_\pi^2}{r} \left[V_{SO} \frac{d}{dr} f_{VSO}(r) + iW_{SO} \frac{d}{dr} f_{WSO}(r) \right] \vec{\sigma} \cdot \vec{l}
\end{aligned}
$$





Road map to the optical potential



Theoretical framework

Lippmann-Schwinger equation for the nucleon-nucleus transition amplitude

$$T = V + VG_0(E)T$$

Projectile-target interaction

$$V = \sum_{i=1}^{A} v_{0i} + \sum_{i<j}^{A} w_{0ij}$$

Full 3N interaction
is still missing!

Many-body propagator

$$G_0(E) = (E - H_0 + i\epsilon)^{-1}$$

$$H_0 = h_0 + H_A$$

h_0 = kinetic term of the projectile

Target Hamiltonian

$$H_A|\Psi_0\rangle = E_0|\Psi_0\rangle$$

Theoretical framework

Lippmann-Schwinger equation for the nucleon-nucleus transition amplitude

$$T = V + VG_0(E)T$$

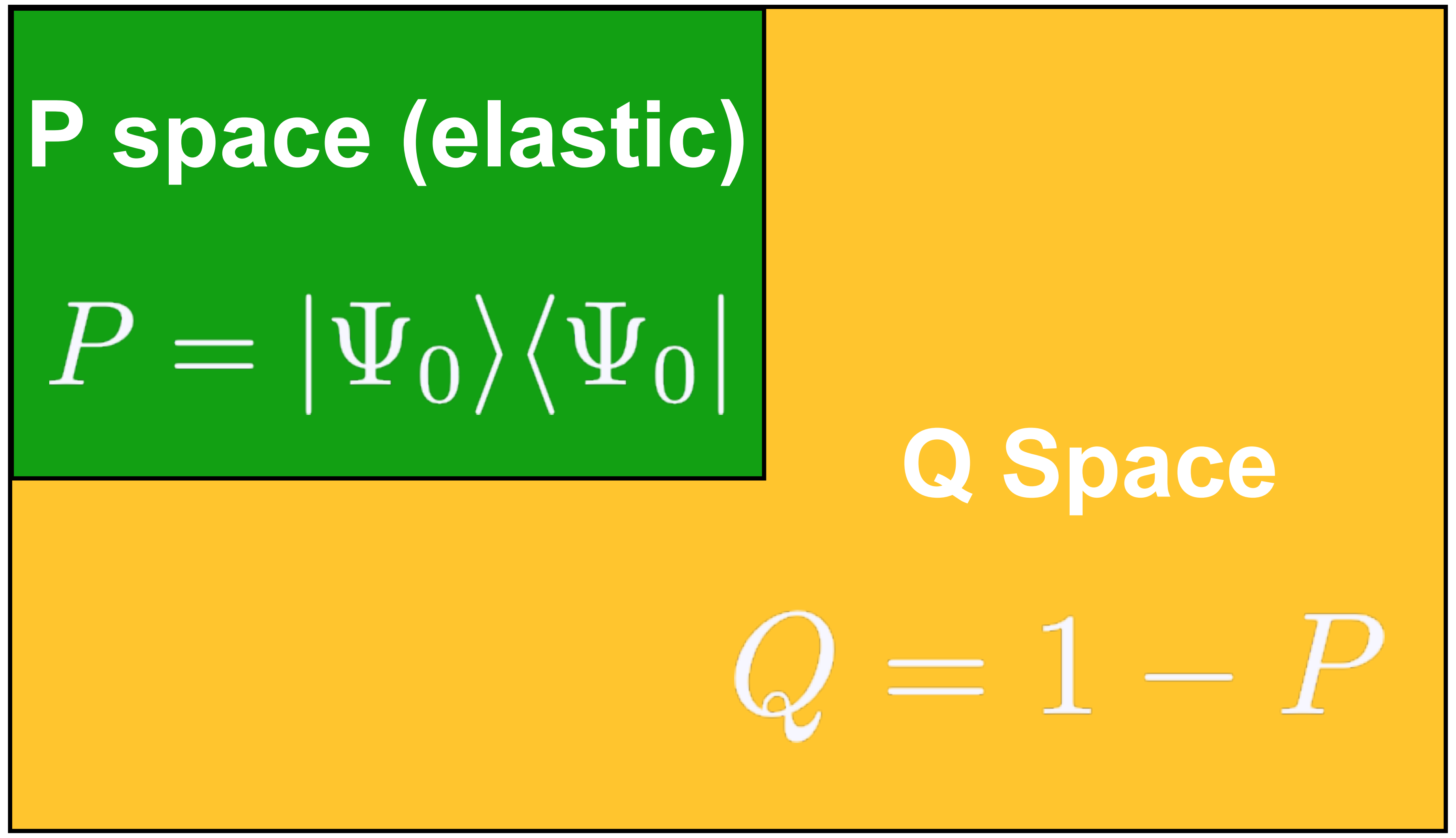
Let's introduce the optical potential *U*

$$T = U + UG_0(E)PT$$

$$U = V + VG_0(E)QU$$

Projection operators

$$P + Q = 1$$



Theoretical framework

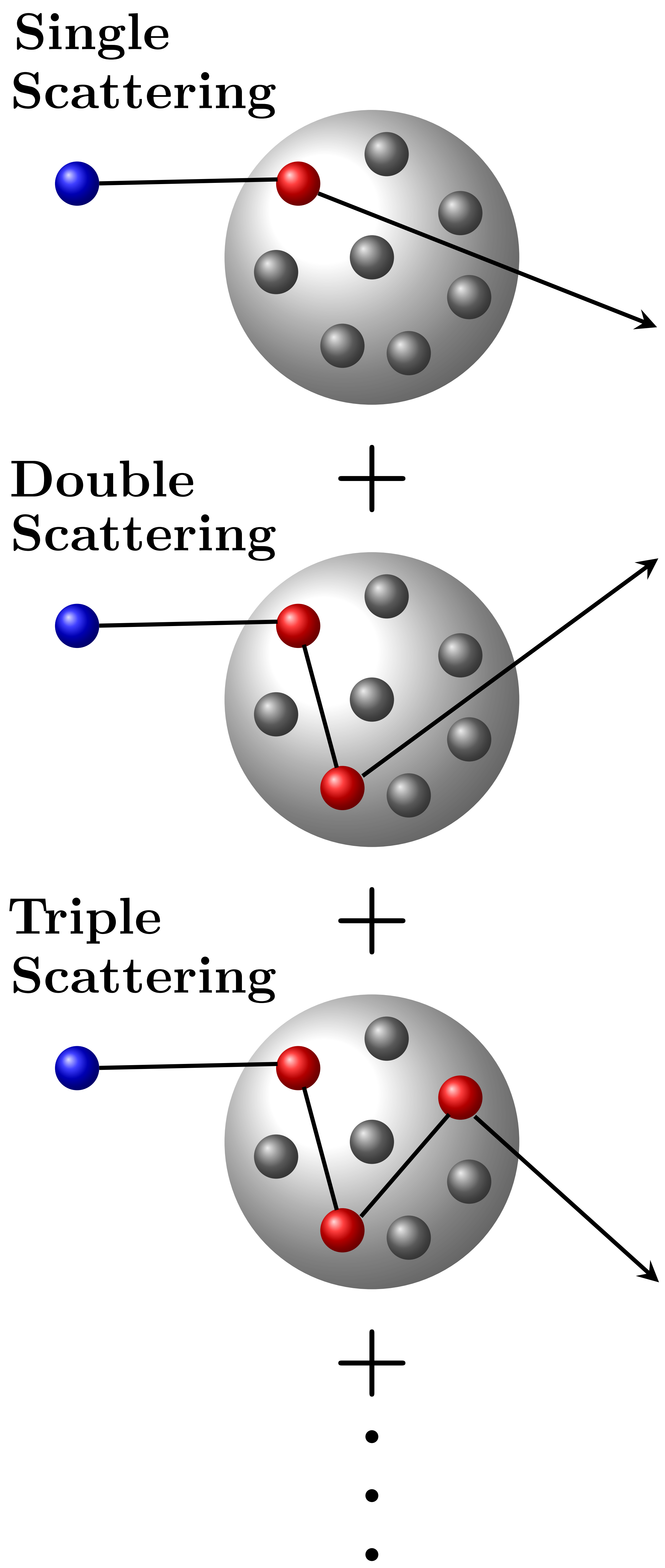
Transition amplitude for elastic scattering

$$T_{\mathrm{el}} \equiv PTP = PUP + PUPG_0(E)T_{\mathrm{el}}$$

The spectator expansion

[Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)]

$$U = \underbrace{\sum_{i=1}^{A} \tau_{0i} + \sum_{i,j\neq i}^{A} \tau_{0ij} + \sum_{i,j\neq i,k\neq i,j}^{A} \tau_{0ijk} + \dots}_{\textbf{A terms}}$$



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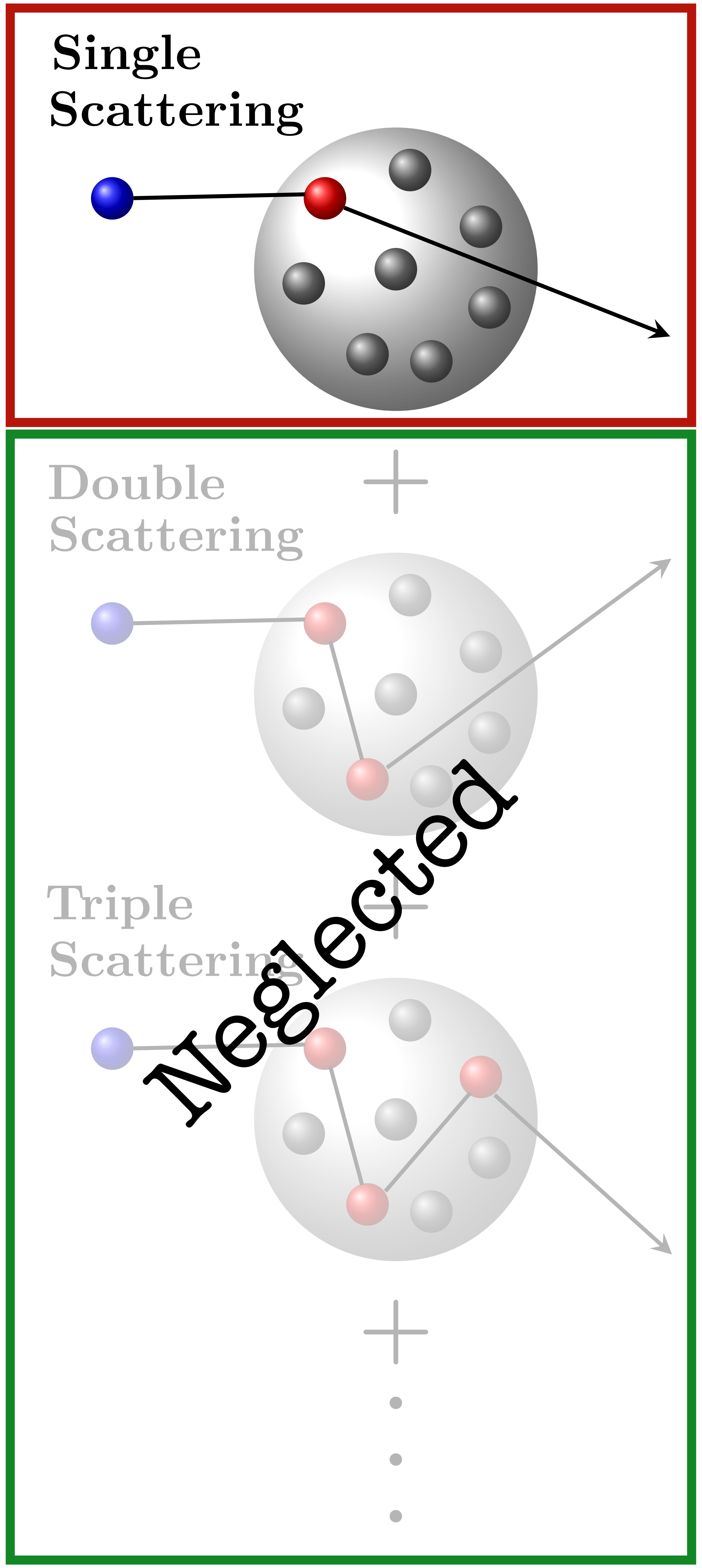
[Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)]

$$U \simeq \sum_{i=1}^{A} \tau_{0i} \qquad \tau_{0i} = v_{0i} + v_{0i}G_0(E)\tau_{0i}$$

(A+1)-body propagator



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$$T_{\rm el} \equiv PTP = PUP + PUPG_0(E)T_{\rm el}$$

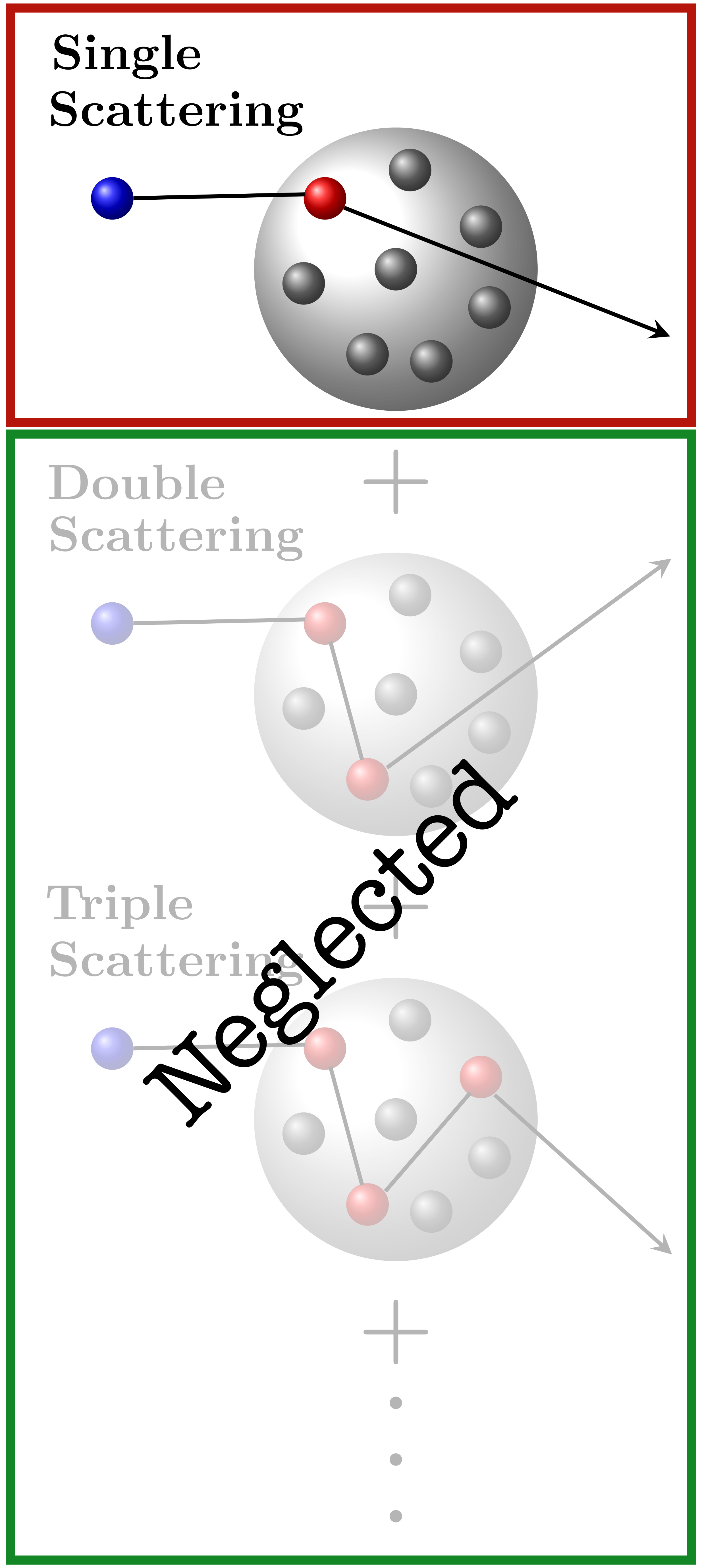
The spectator expansion

[Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)]

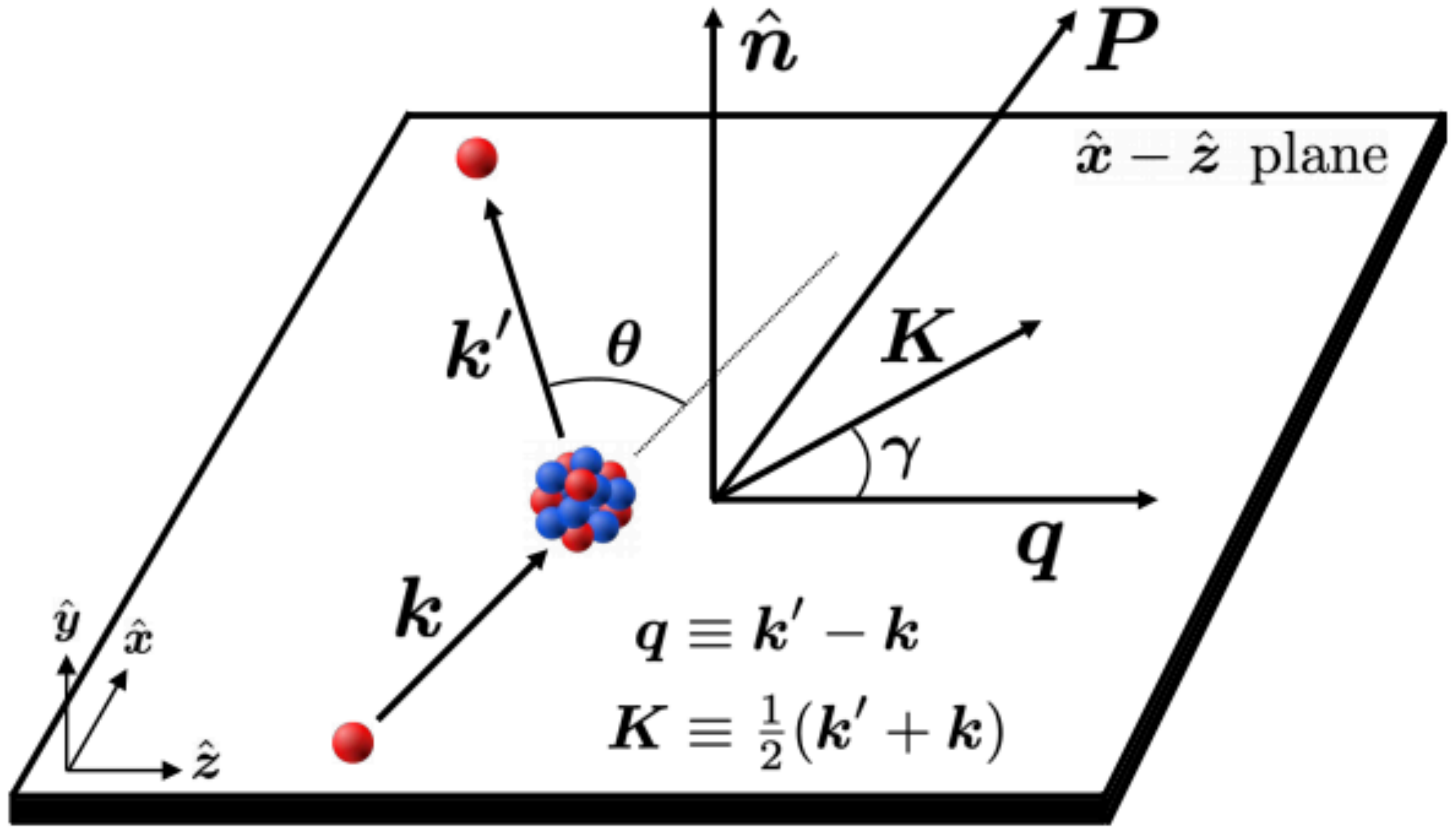
$$U \simeq \sum_{i=1}^{A} \tau_{0i} \quad \rightarrow \quad \tau_{0i} \approx t_{0i} = v_{0i} + v_{0i}\, g_0(E)\, t_{0i}$$

Impulse approximation

Two-body propagator



The first-order optical potential



$$U_{\mathbf{p}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{\mathbf{p}N}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N(\boldsymbol{q}, \boldsymbol{P})$$

$\mathbf{p} = (p, n, \bar{p})$

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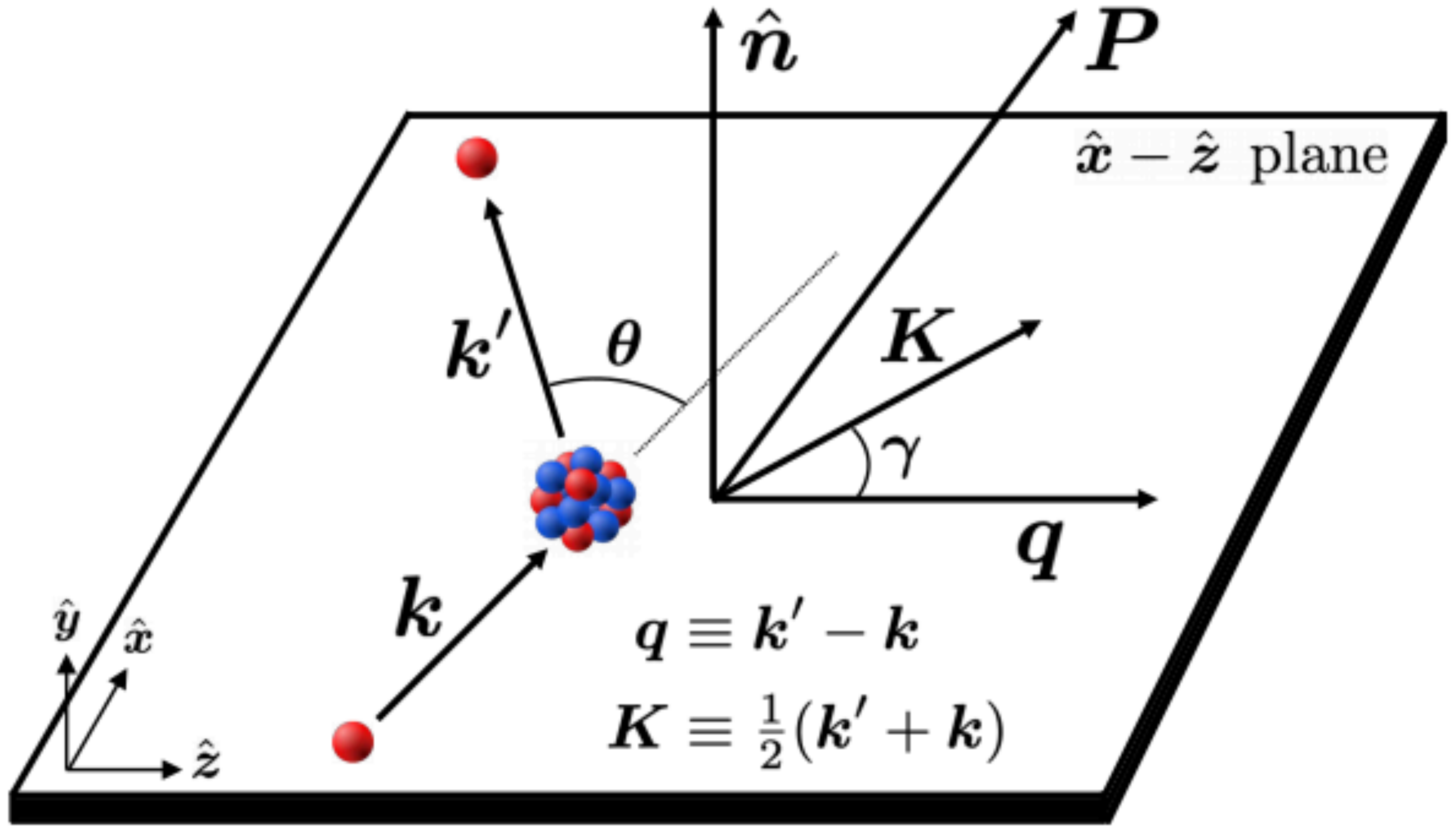

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$\mathbf{p} = (p, n, \bar{p})$

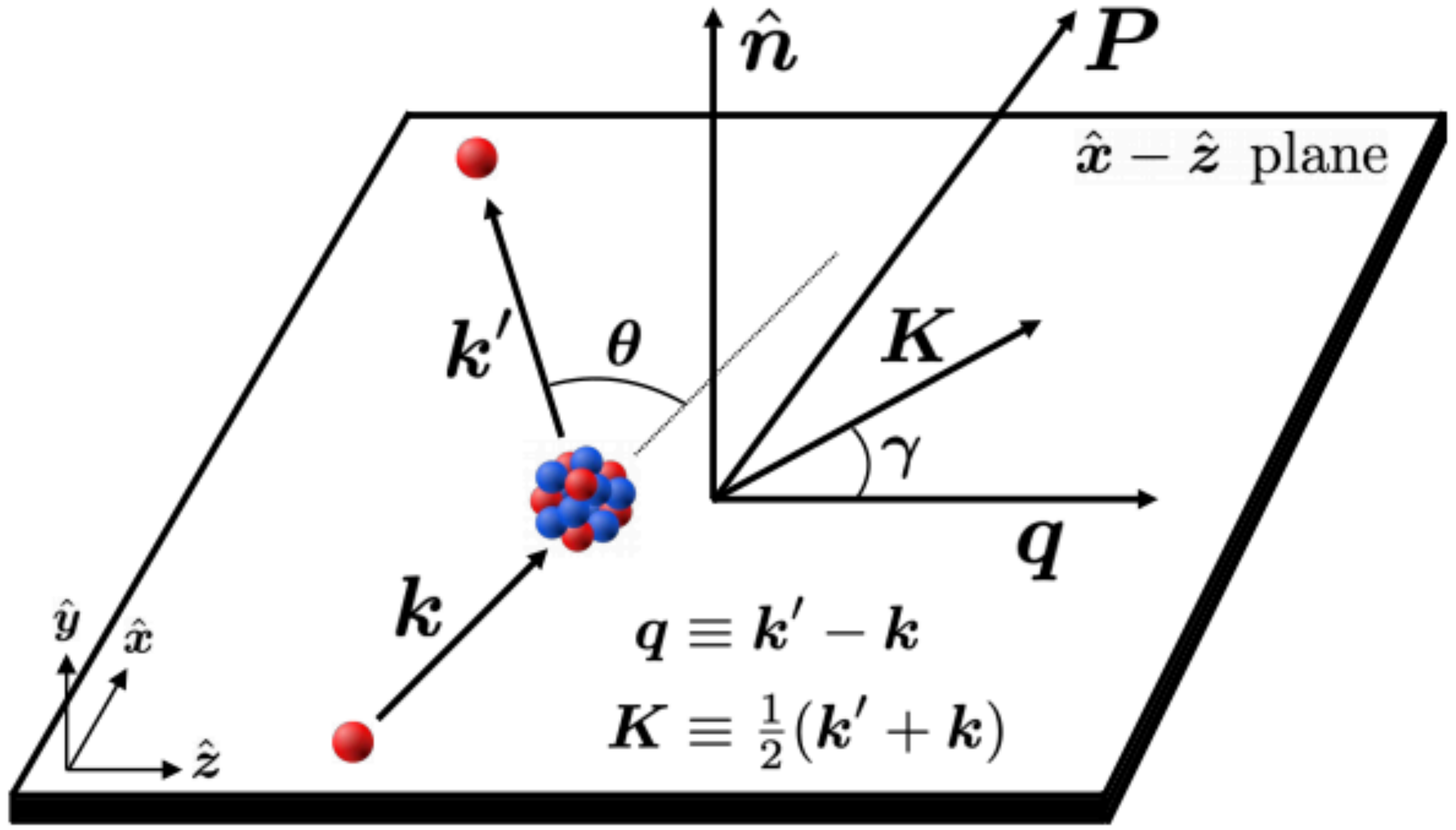
Free two-body scattering matrix

$$t_{0i} = v_{0i} + v_{0i}\, g_{0i}\, t_{0i}$$

$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

- Simple one-body equation
- Can be solved easily
- Only **NN** interaction

The first-order optical potential



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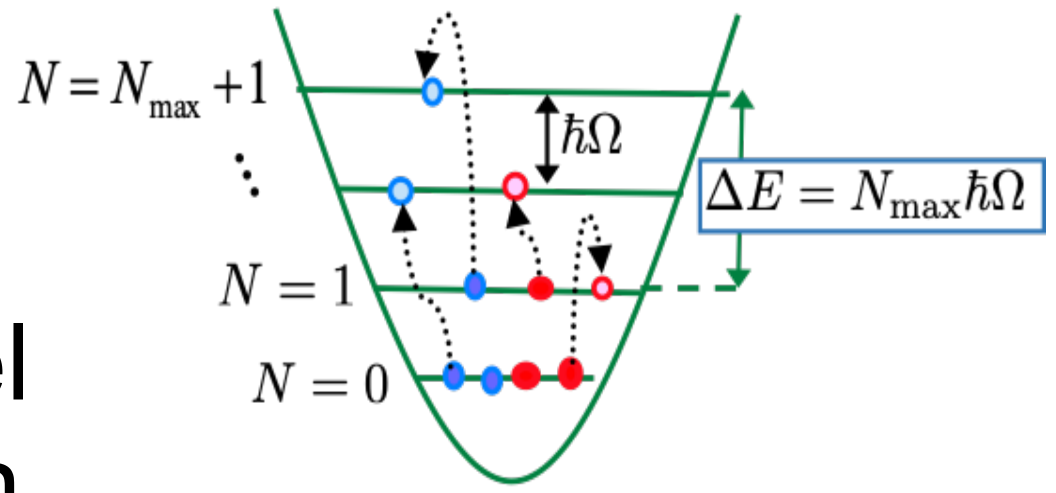
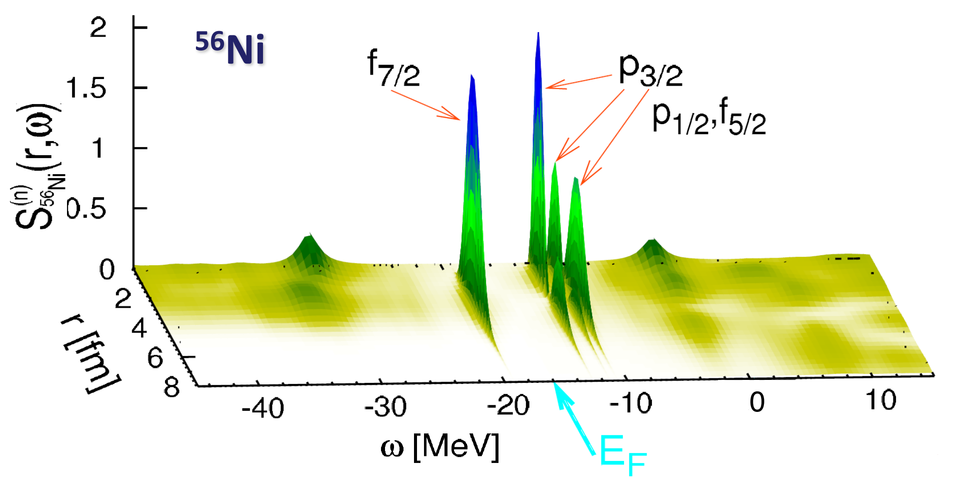
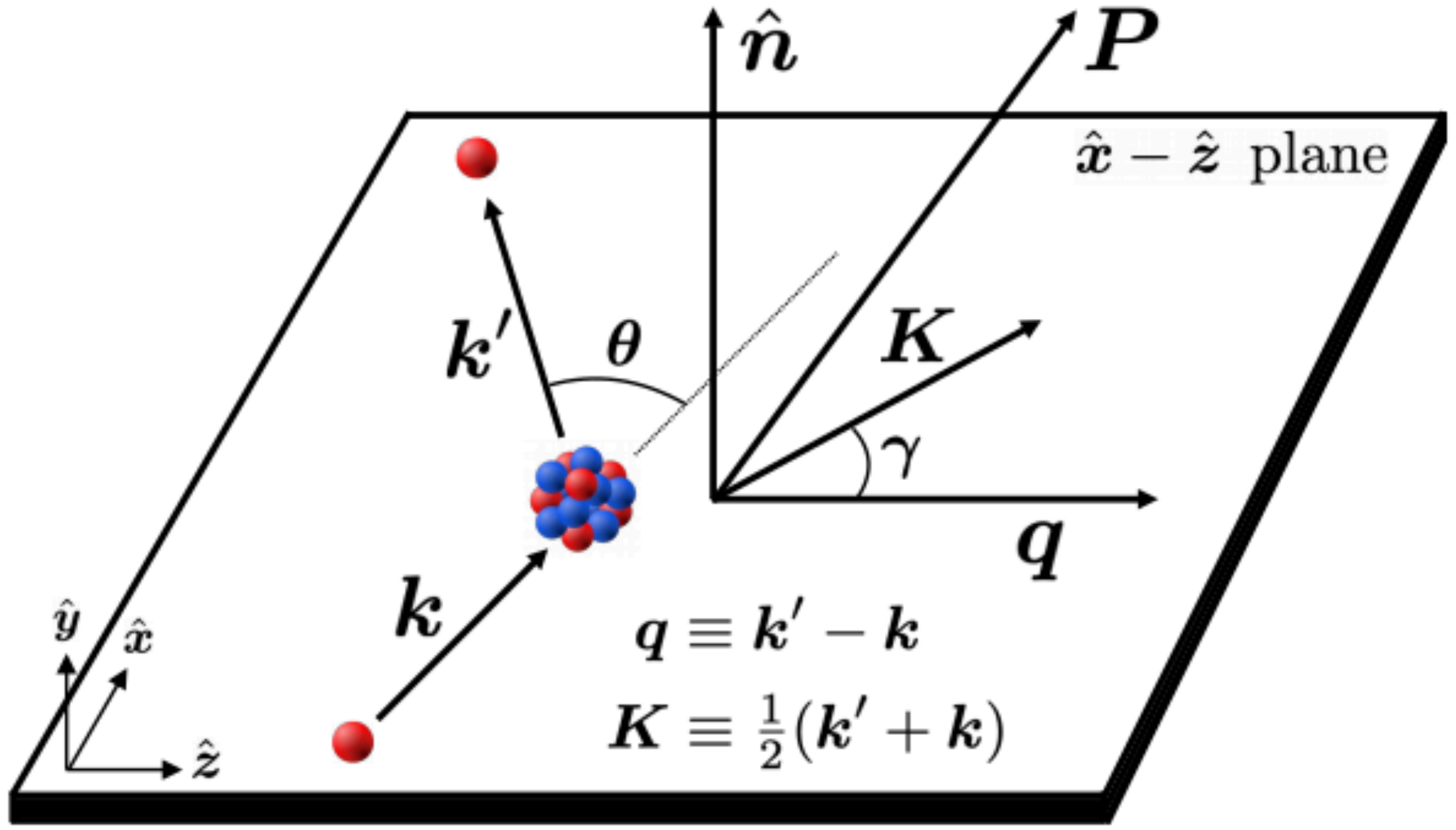
Nonlocal one-body density

- Computationally expensive
- Obtained from the No-Core Shell Model or the Self-Consistent Green's Function
- Calculation performed with **NN** and **3N** interaction





The first-order optical potential



Møller factor

$$t_{\mathbf{p}N}^{(NA)} = \eta\, t_{\mathbf{p}N}^{(NN)}$$

It imposes the Lorentz invariance of flux when we pass from the NA to the NN frame where the t matrices are evaluated

$$U_{\mathbf{p}}(\boldsymbol{q},\boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q},\boldsymbol{K},\boldsymbol{P})\, t_{\mathbf{p}N}(\boldsymbol{q},\boldsymbol{K},\boldsymbol{P})\, \rho_N(\boldsymbol{q},\boldsymbol{P})$$

$\mathbf{p} = (p, n, \bar{p})$

Free two-body scattering matrix

$$t_{0i} = v_{0i} + v_{0i}\, g_{0i}\, t_{0i}$$
$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

- Simple one-body equation
- Can be solved easily
- Only **NN** interaction

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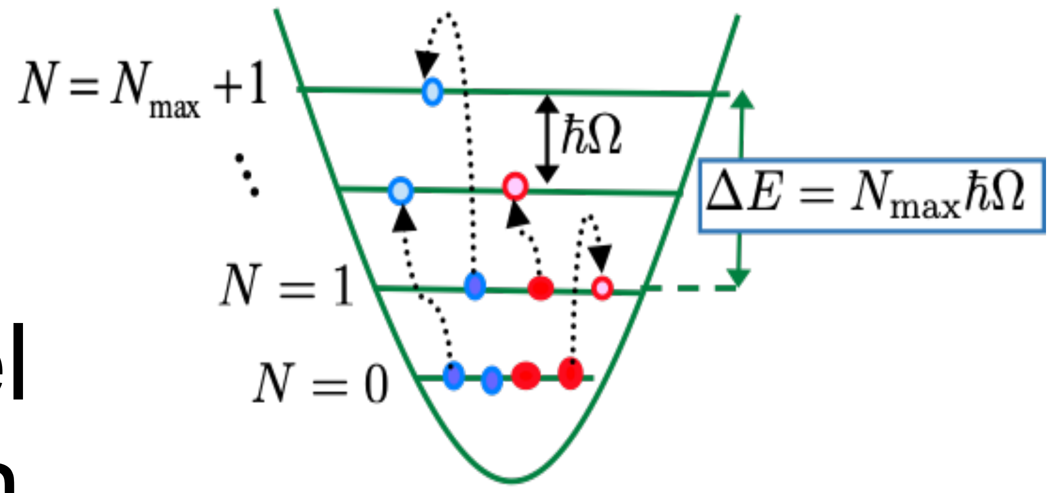
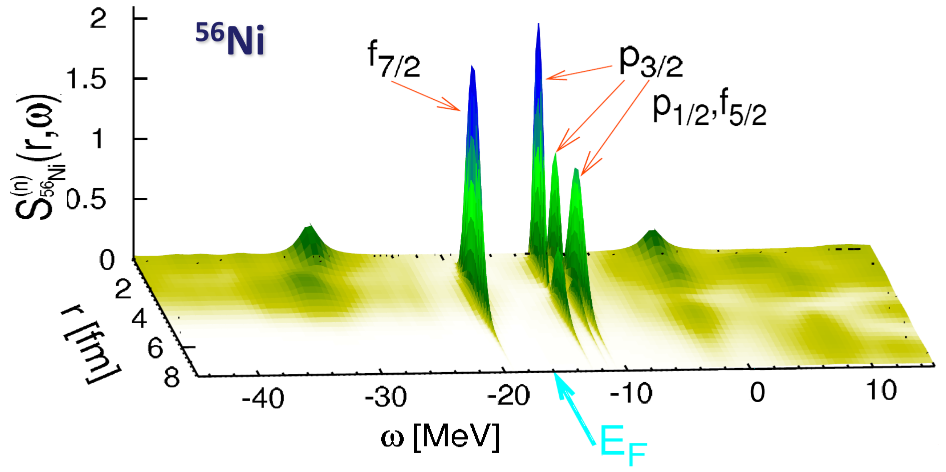


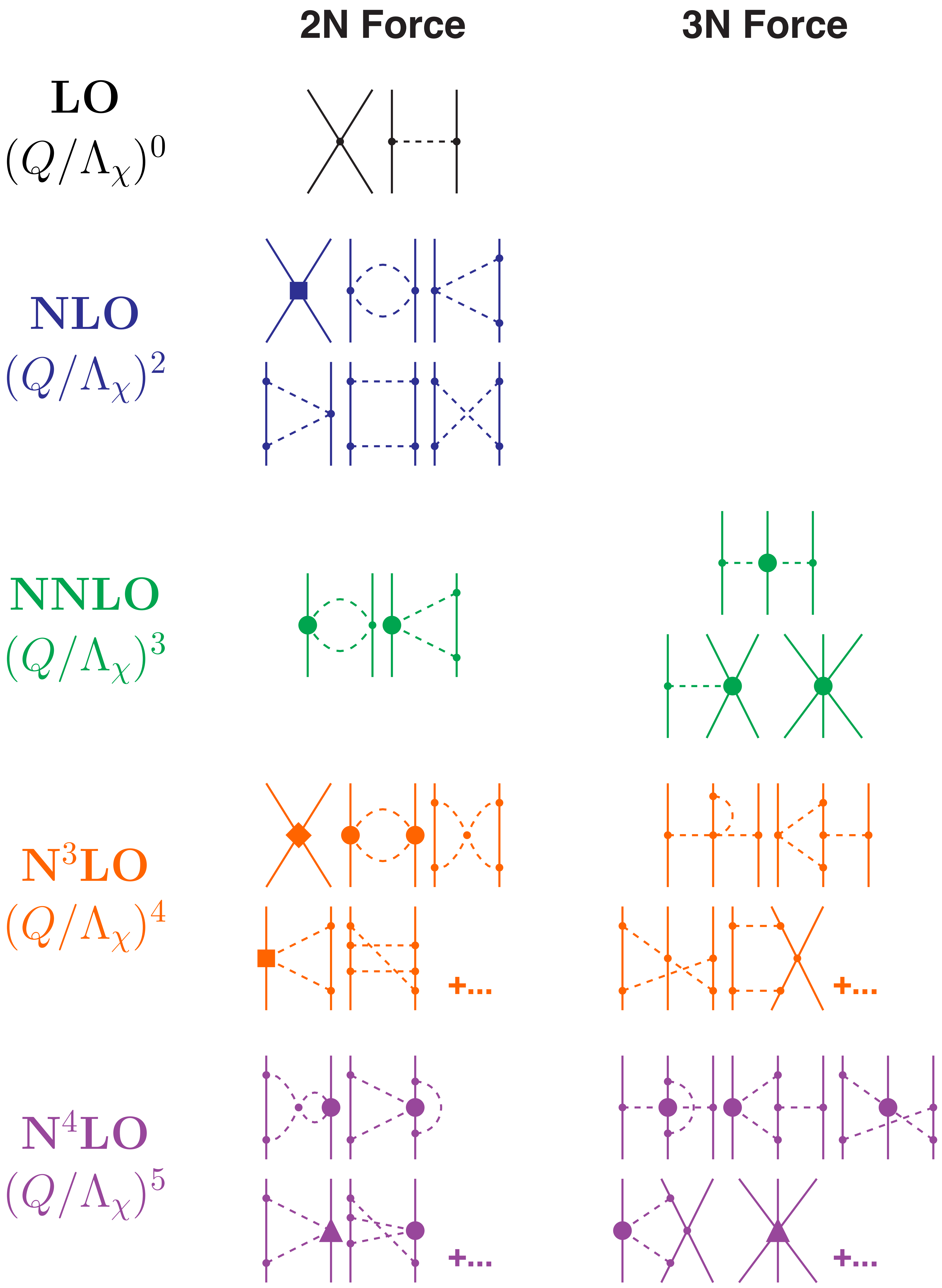


Chiral interactions

Advantages

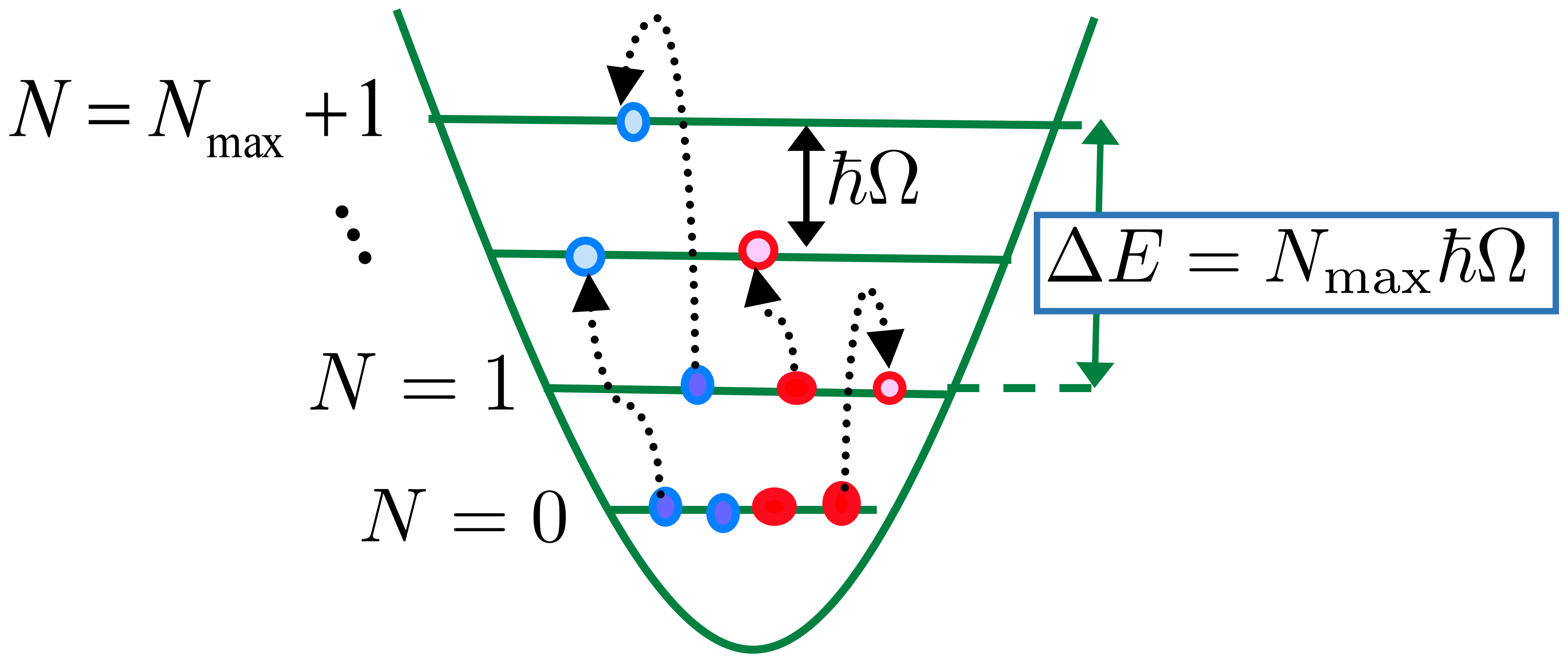
- QCD symmetries are consistently respected
- Systematic expansion (order by order we know exactly the terms to be included)
- Theoretical errors
- Two- and three-nucleon forces belong to the same framework

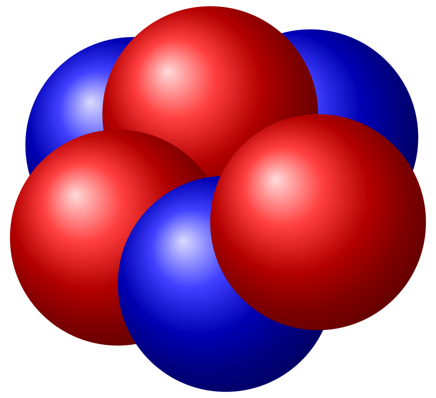
We use these interactions as the **only** input to calculate the **effective interaction** between projectile and target and the **target density**



Target description

No-Core Shell Model



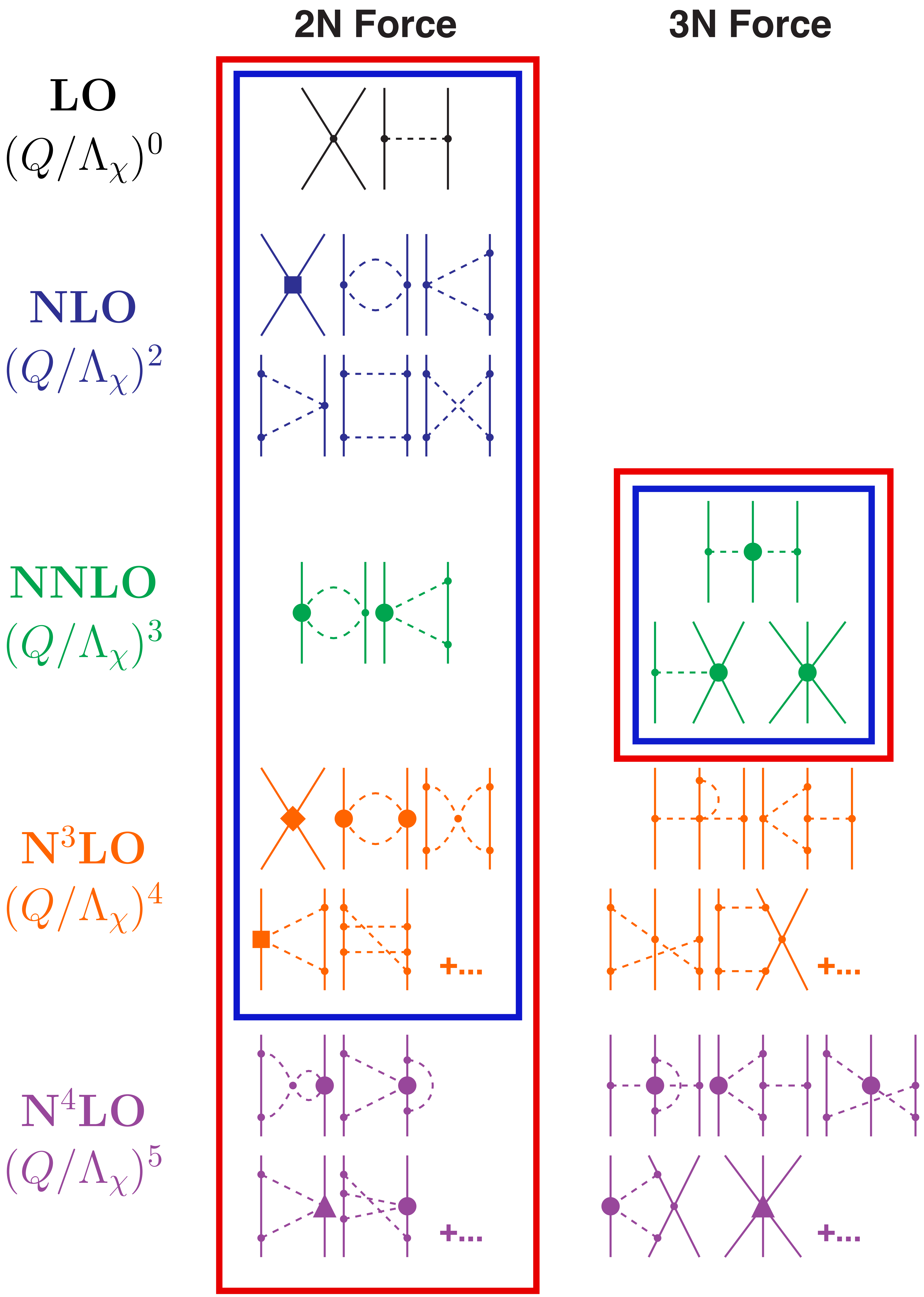


NCSM

In collaboration with P. Navrátil and M. Gennari (TRIUMF)

- **NN-N^{4}LO + 3NlnI (^{12}C, ^{16}O)**
  - N^{4}LO: Entem et al., Phys. Rev. C **96**, 024004 (2017)
  - 3NlnI: Navrátil, Few-Body Syst. **41**, 117 (2007)
  - c_D & c_E: Kravvaris et al., Phys. Rev. C **102**, 024616 (2020)

- **NN-N^{3}LO + 3NlnI (^{9,13}C, ^{6,7}Li, ^{10}B)**
  - N^{3}LO: E&M, Phys. Rev. C **68**, 041001(R) (2003)
  - 3NlnI: Navrátil, Few-Body Syst. **41**, 117 (2007)
  - c_D & c_E: Somà et al., Phys. Rev. C **101**, 014318 (2020)



Assessing the impact of the 3N interaction

General equation for the optical potential

$$U = (V_{NN} + V_{3N}) + (V_{NN} + V_{3N}) G_0(E) Q U$$

Treatment of the 3N force

[Holt et al., Phys. Rev. C **81**, 024002 (2010)]

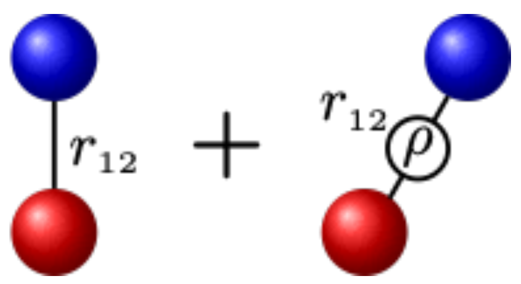
$$V_{3N} = \frac{1}{2} \sum_{i=1}^{A} \sum_{\substack{j=1 \\ j \neq i}}^{A} w_{0ij} \approx \sum_{i=1}^{A} \langle w_{0i} \rangle$$

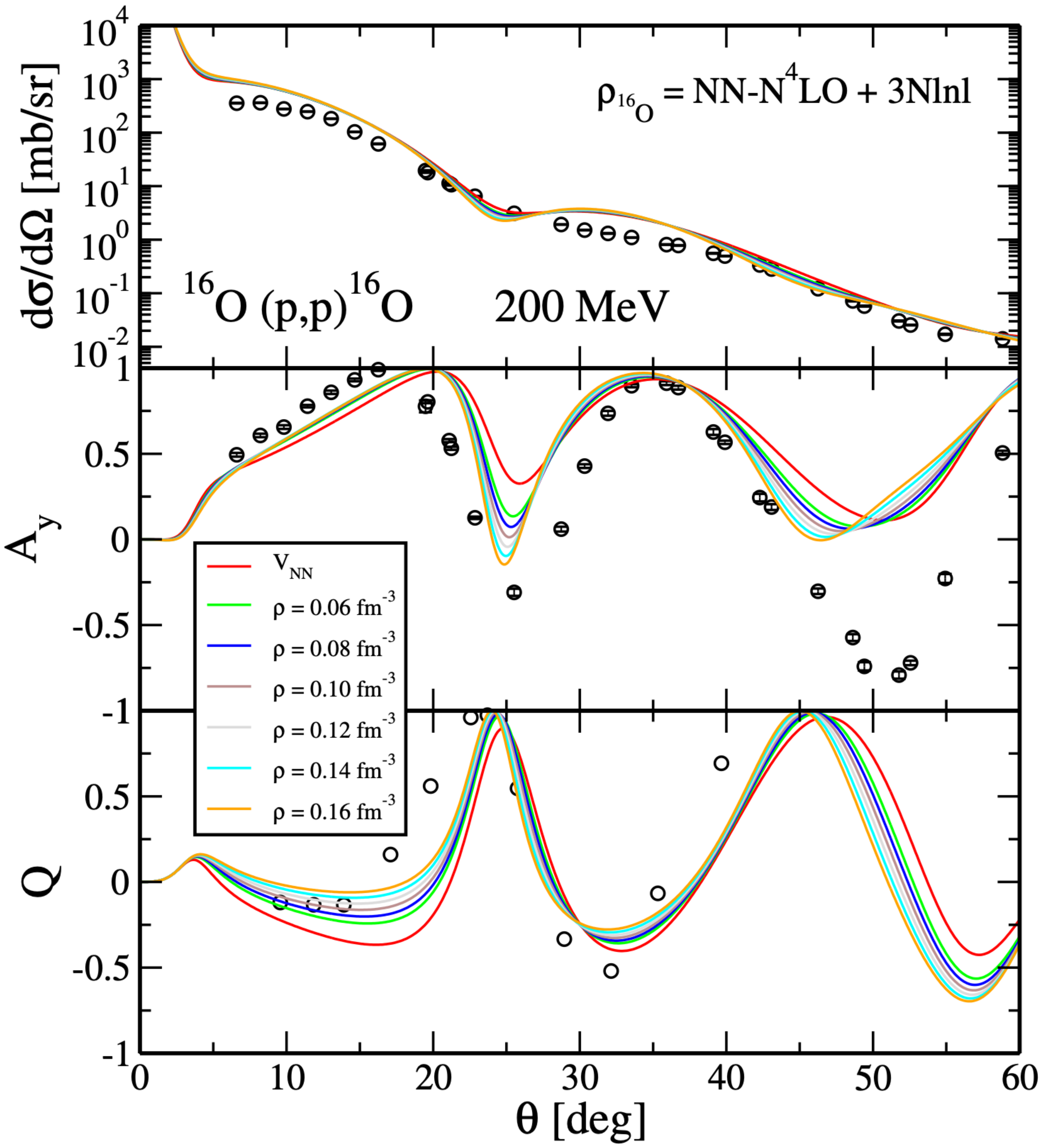
Density dependent

Modification of the *t* matrix

$$t_{0i} = v_{0i}^{(1)} + v_{0i}^{(2)} g_{0i} t_{0i}$$

$$v_{0i}^{(1)} = v_{0i} + \frac{1}{2} \langle w_{0i} \rangle$$
$$v_{0i}^{(2)} = v_{0i} + \langle w_{0i} \rangle$$

$v =$ 



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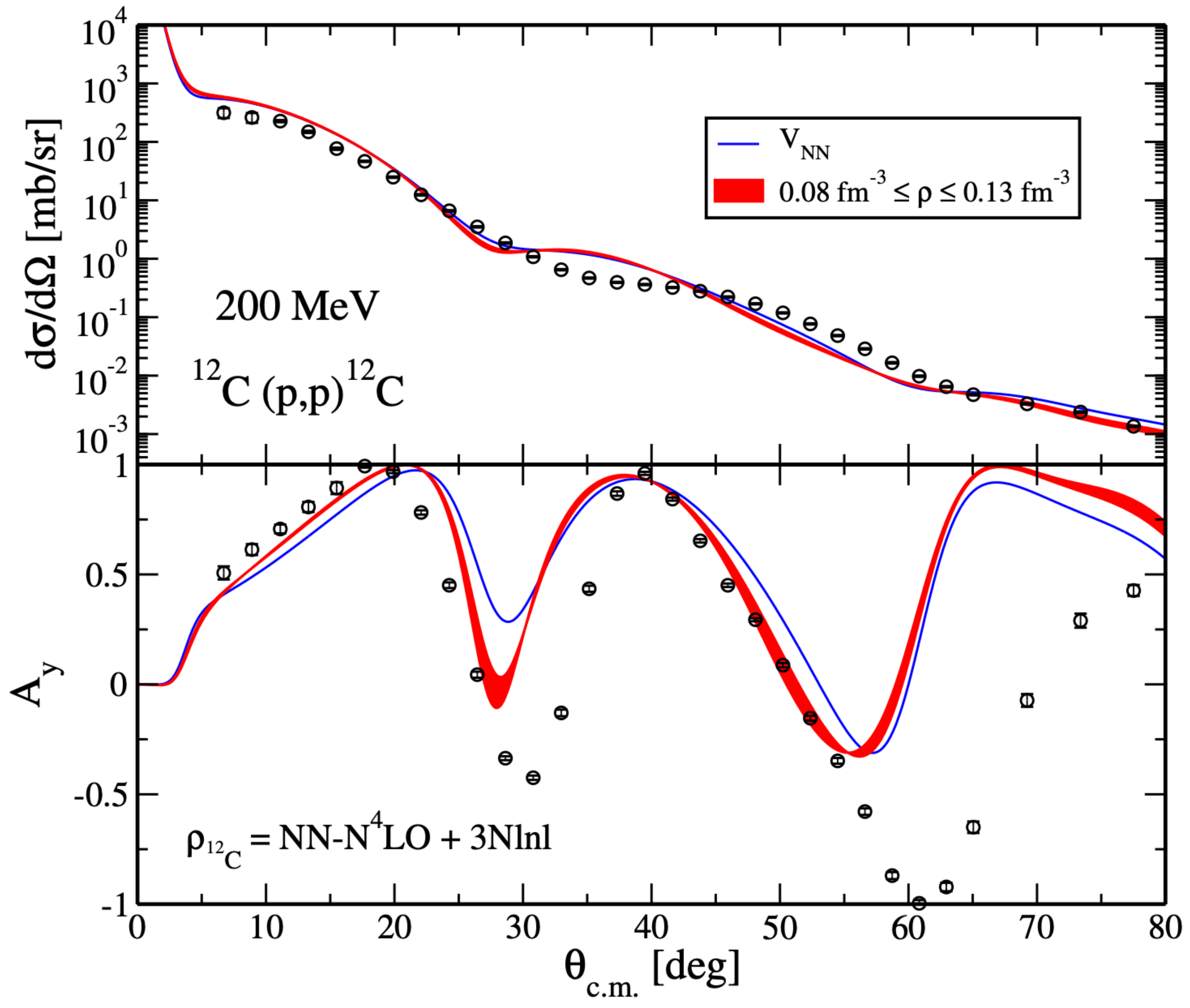
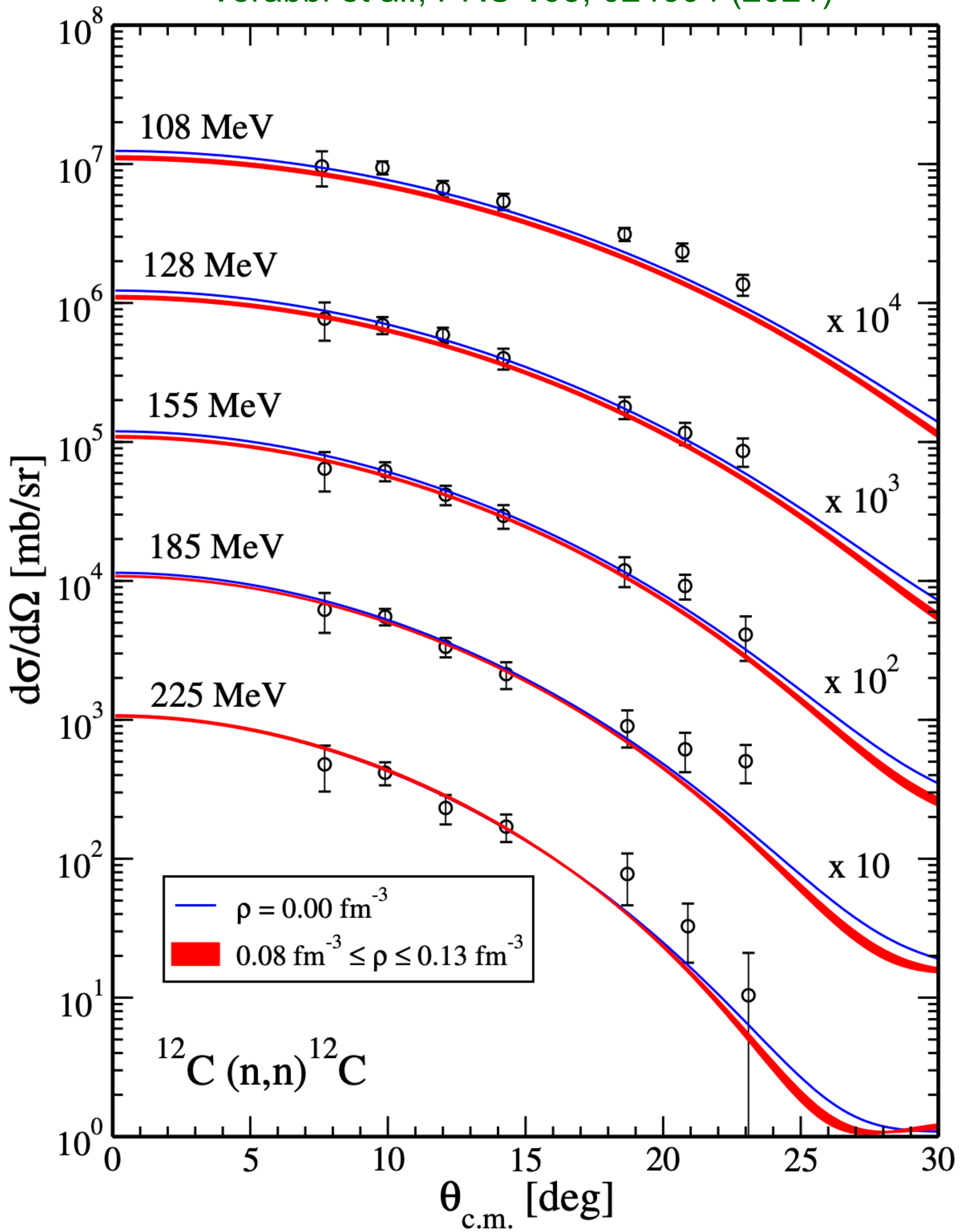


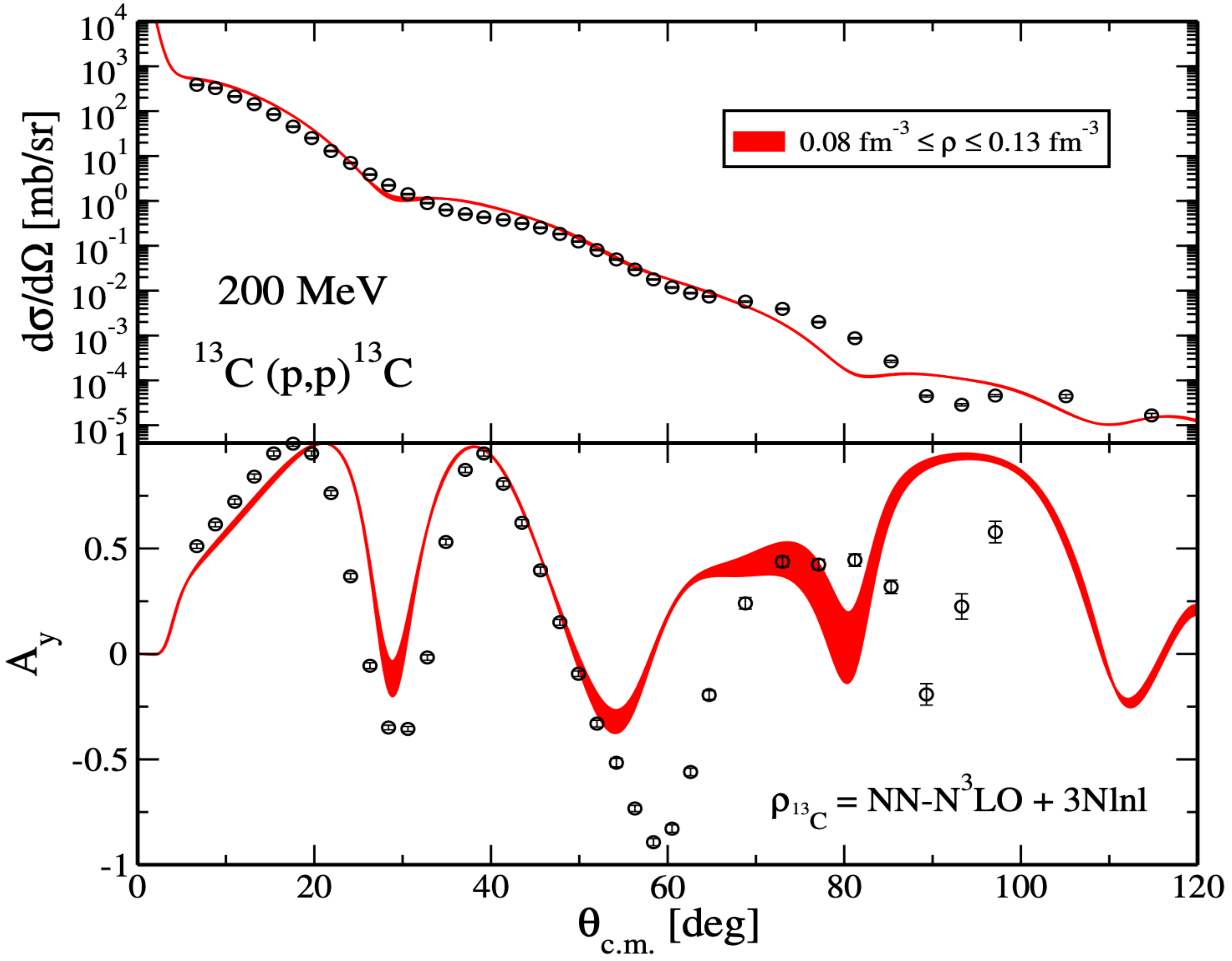
Vorabbi et al., PRC **103**, 024604 (2021)



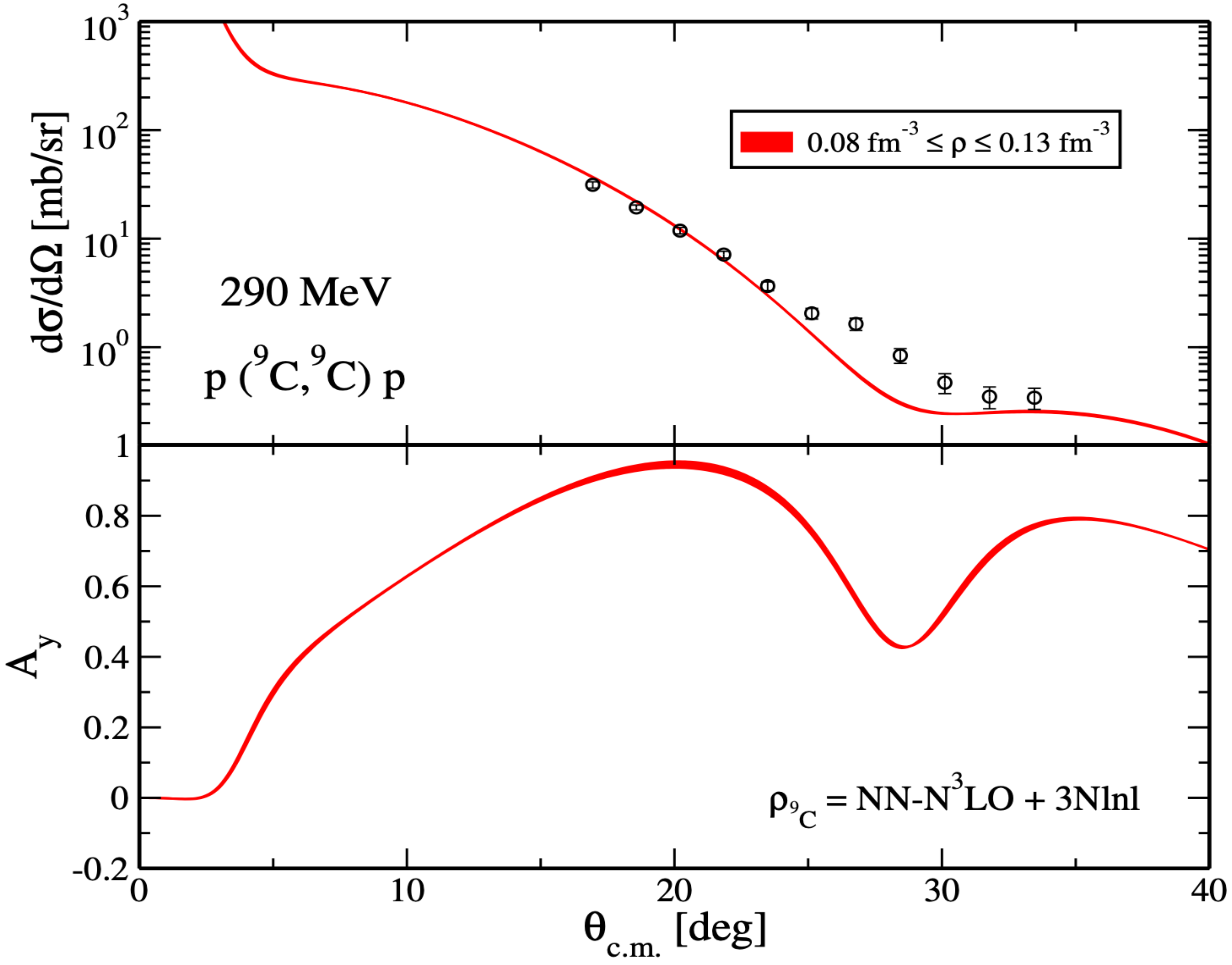
- For all nuclei we found very small contributions to the differential cross section
- The contributions to the spin observable are larger and they seem to improve the agreement with the data

Extension to non-zero spin targets

$J^\pi = 1/2^-$



$J^\pi = 3/2^-$



[Vorabbi et al., Phys. Rev. C **105**, 014621 (2022)]

Convergence in the chiral expansion

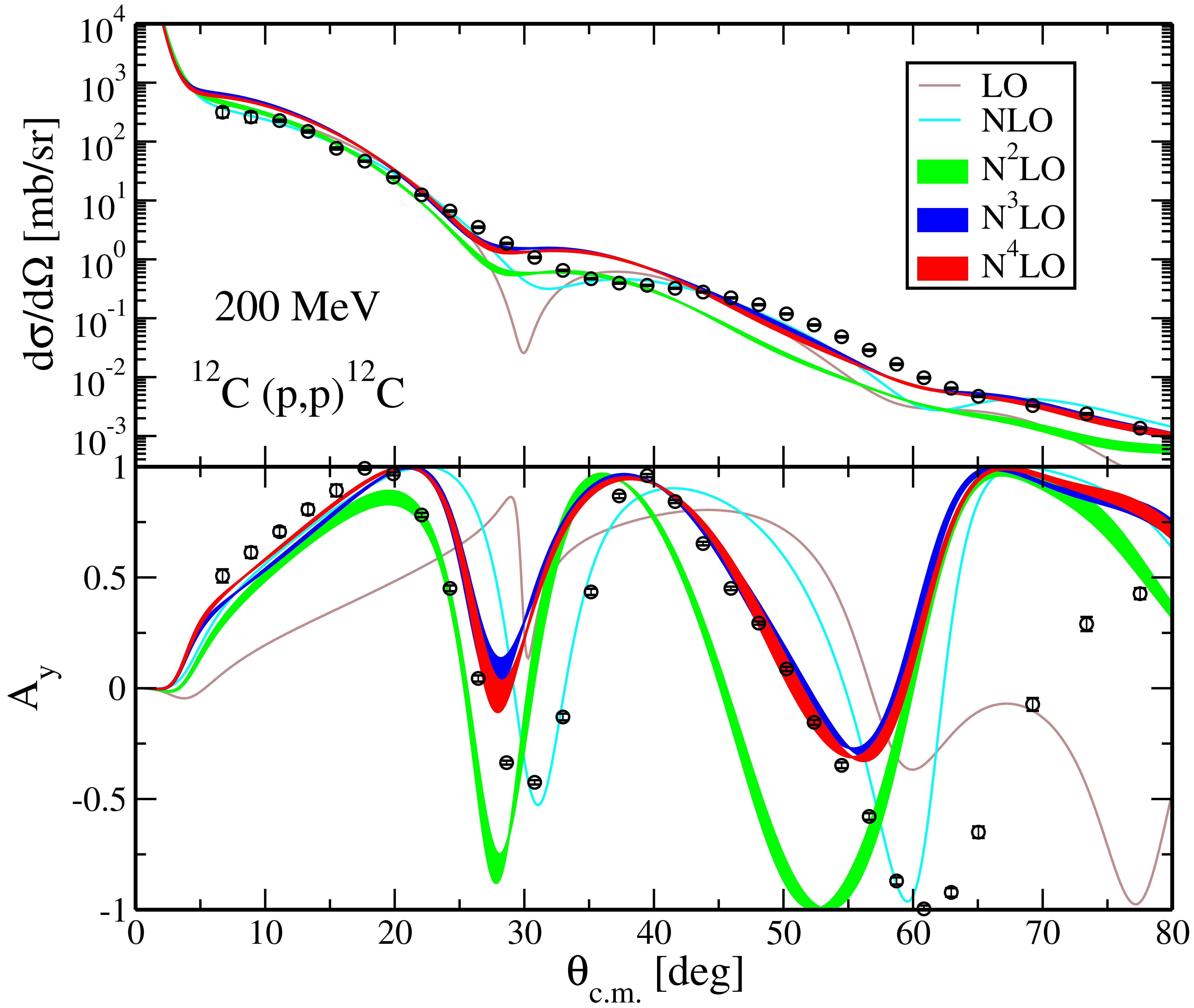
- Density computed with NN+3N interaction

  -NN interactions at all orders taken from
  Entem et al., PRC **96**, 024004 (2017)

  -3N only at N²LO with c_D and c_E refitted at each order
  Kravvaris et al., PRC **102**, 024616 (2020)

- Bands are obtained starting from N²LO when the matter density ρ is allowed to vary between 0.08 and 0.13 fm^{-3}

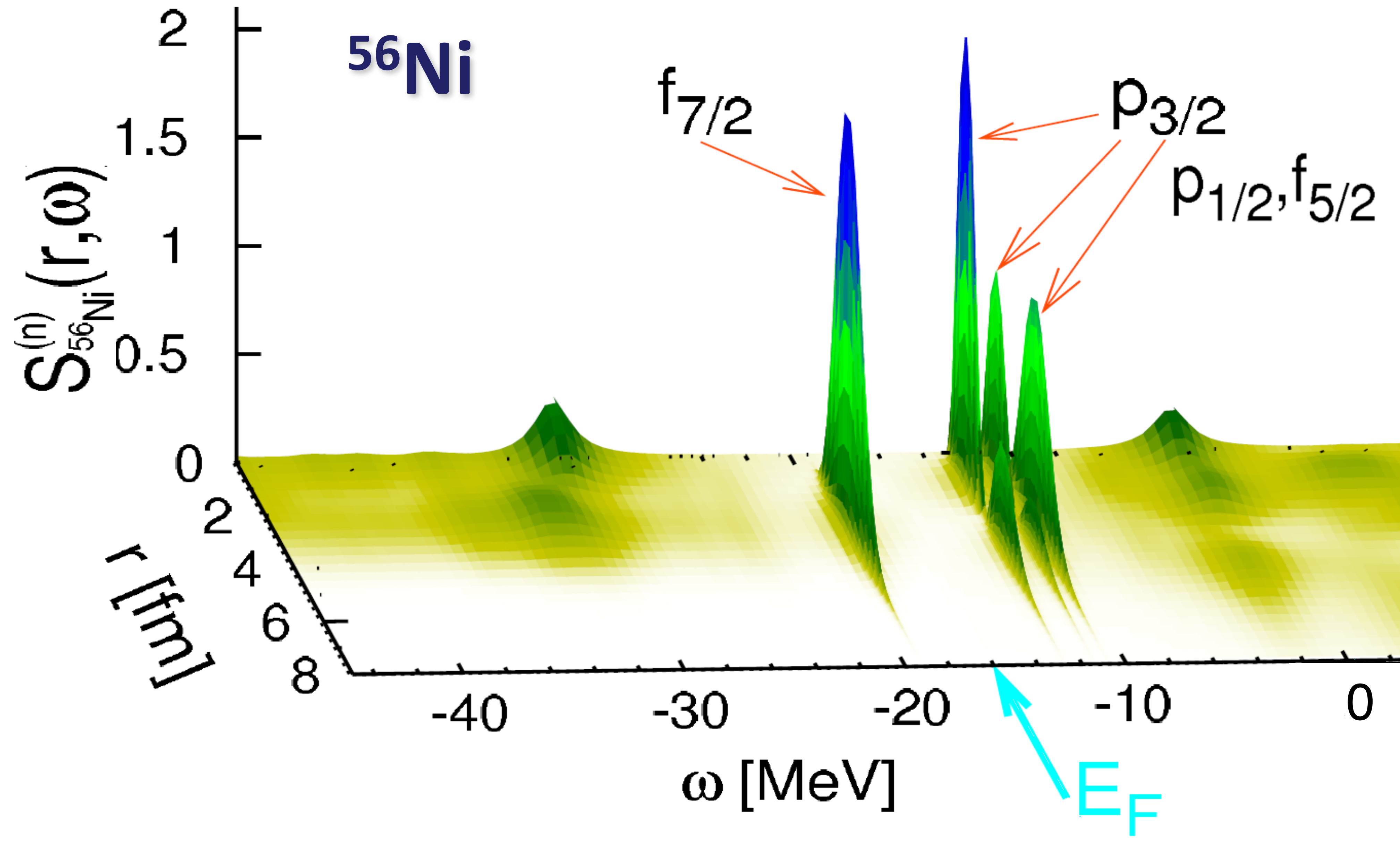
- At N³LO the results seem to achieve a good degree of convergence



Vorabbi et al., PRC **103**, 024604 (2021)

Extension to heavier nuclei

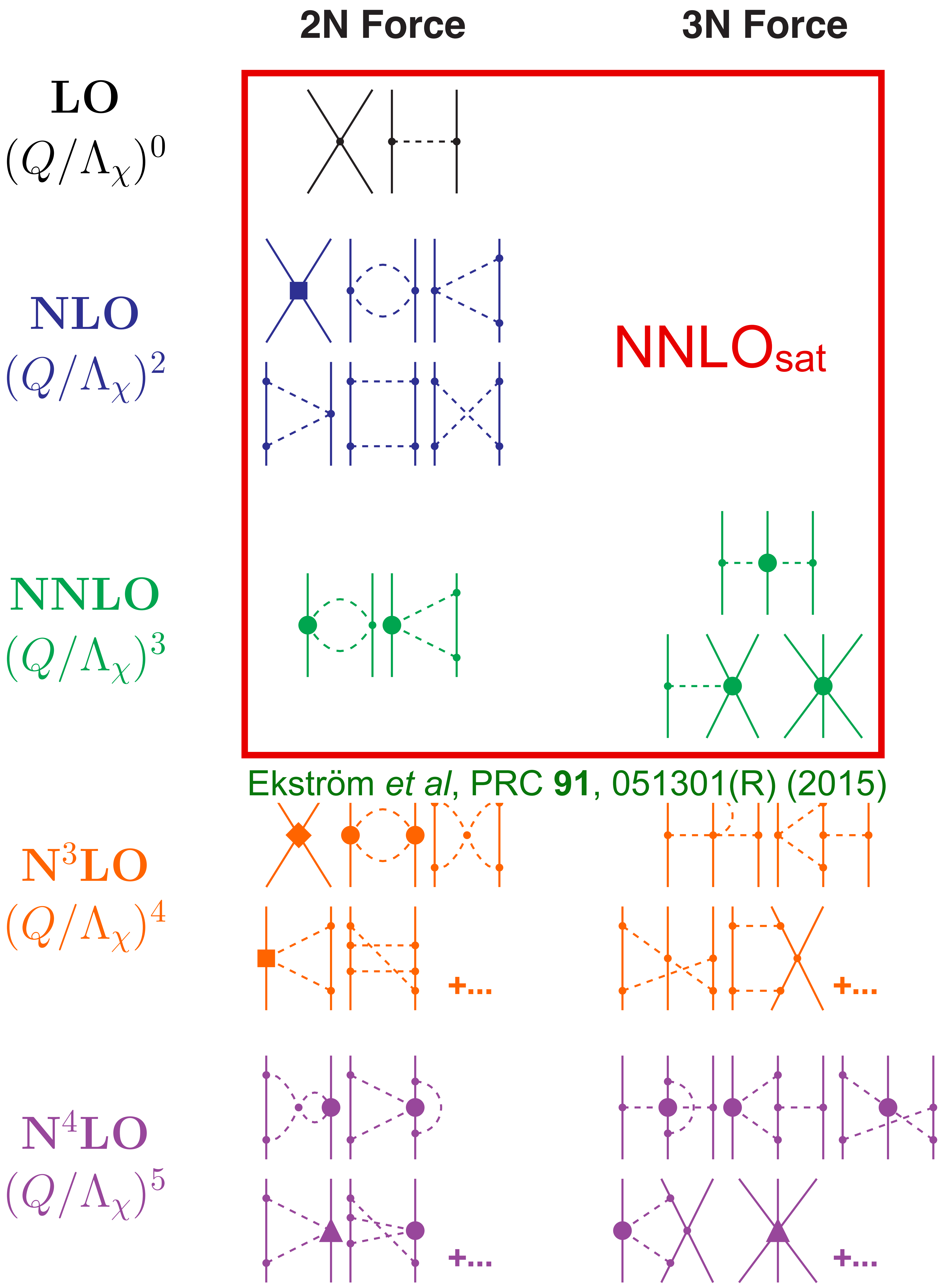
Self Consistent Green's Function (SCGF)



In collaboration with C. Barbieri (Milan) and V. Somà (Paris)

Somà, *SCGF Theory for Atomic Nuclei*, Frontiers **8** (2020) 340

$$U_{\mathbf{p}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{\mathbf{p}N}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N(\boldsymbol{q}, \boldsymbol{P})$$



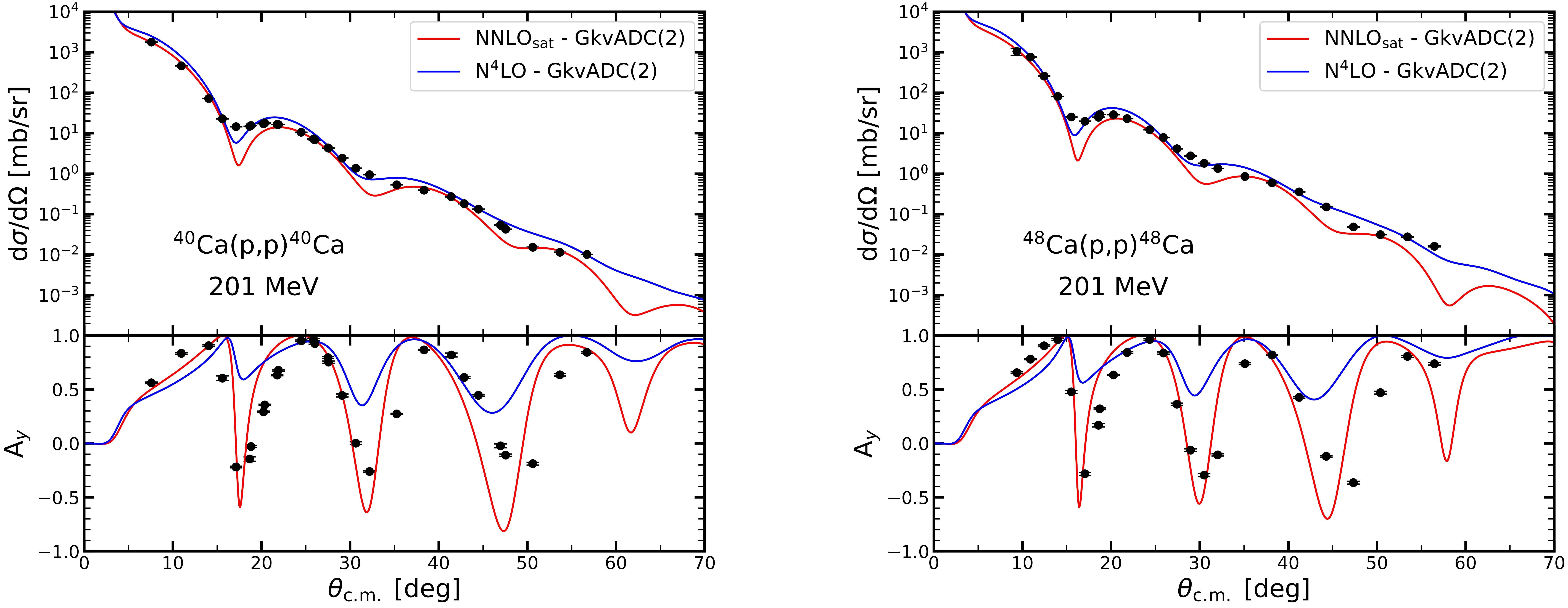
Ekström *et al*, PRC **91**, 051301(R) (2015)

NN amplitudes from $NNLO_{sat}$

The $NNLO_{sat}$ does not reproduce the NN scattering amplitudes at the energies considered, however the disagreement does not seem to get worse with the increasing energy

Results for proton scattering off ^{40,48}Ca

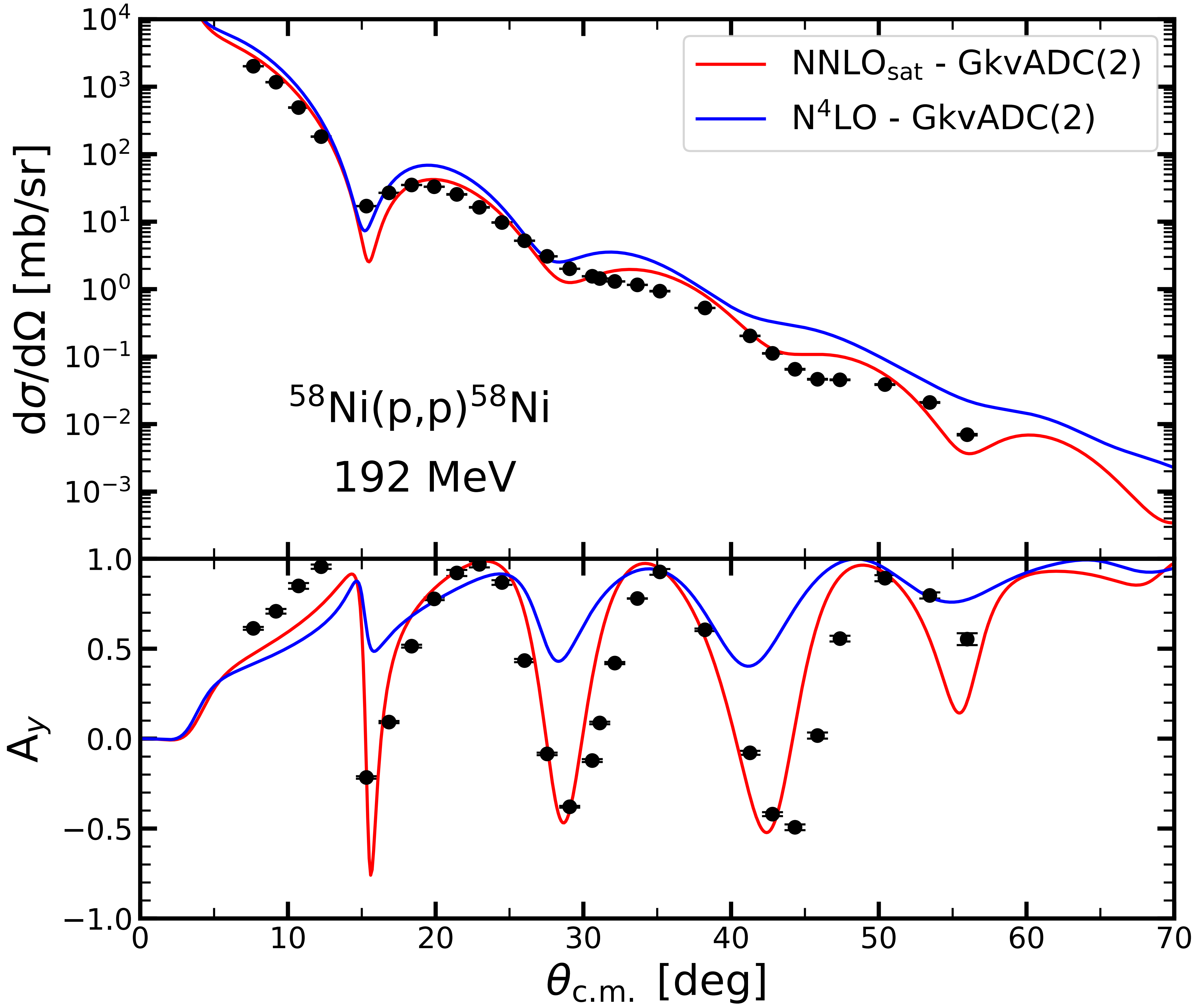
[Vorabbi et al., PRC **109**, 034613 (2024)]

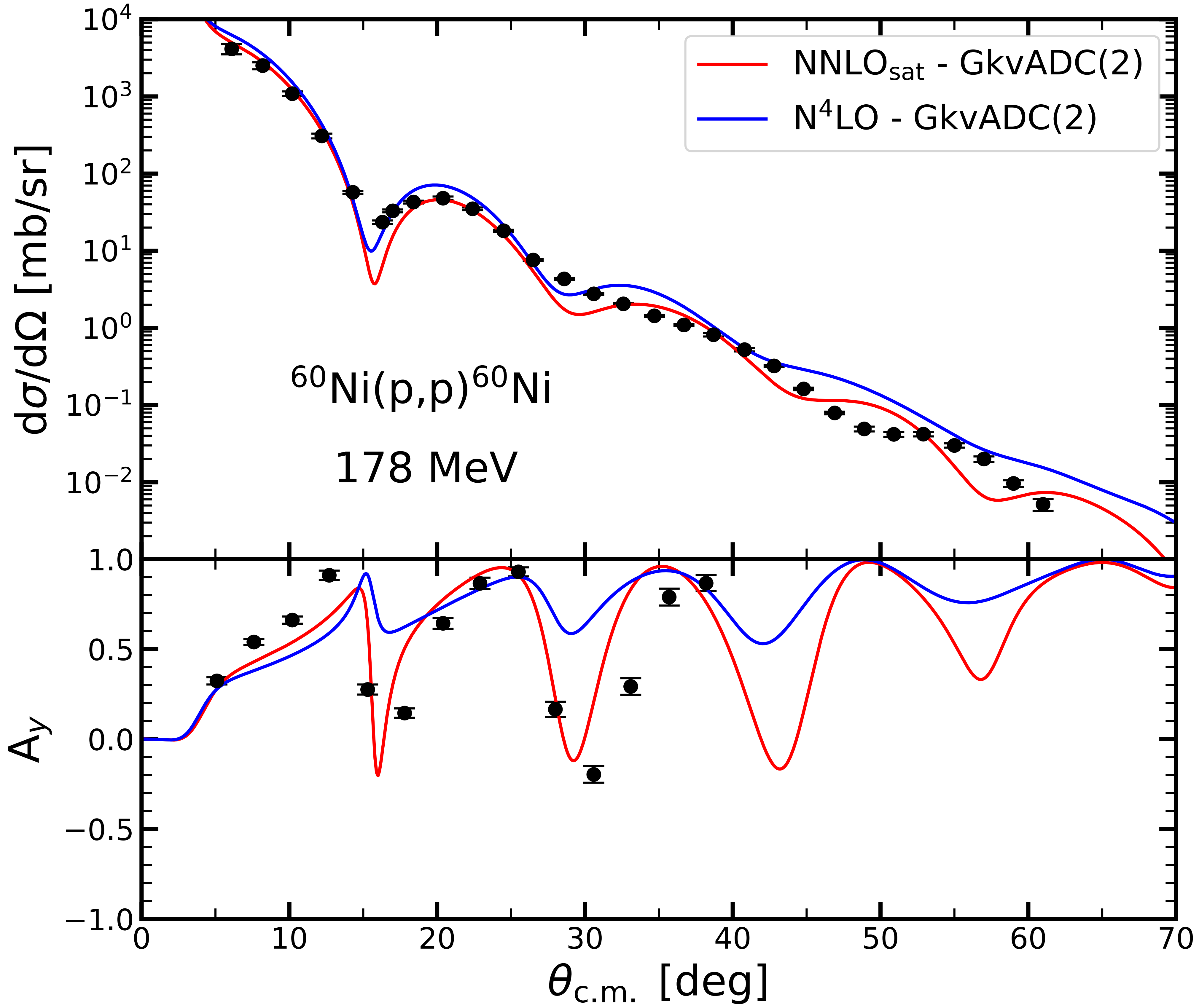


- First microscopic optical potential for calcium and nickel from *ab initio* densities
- For this comparison the densities are always computed with the NNLO$_{sat}$

Results for proton scattering off ^{58,60}Ni

[Vorabbi et al., PRC **109**, 034613 (2024)]

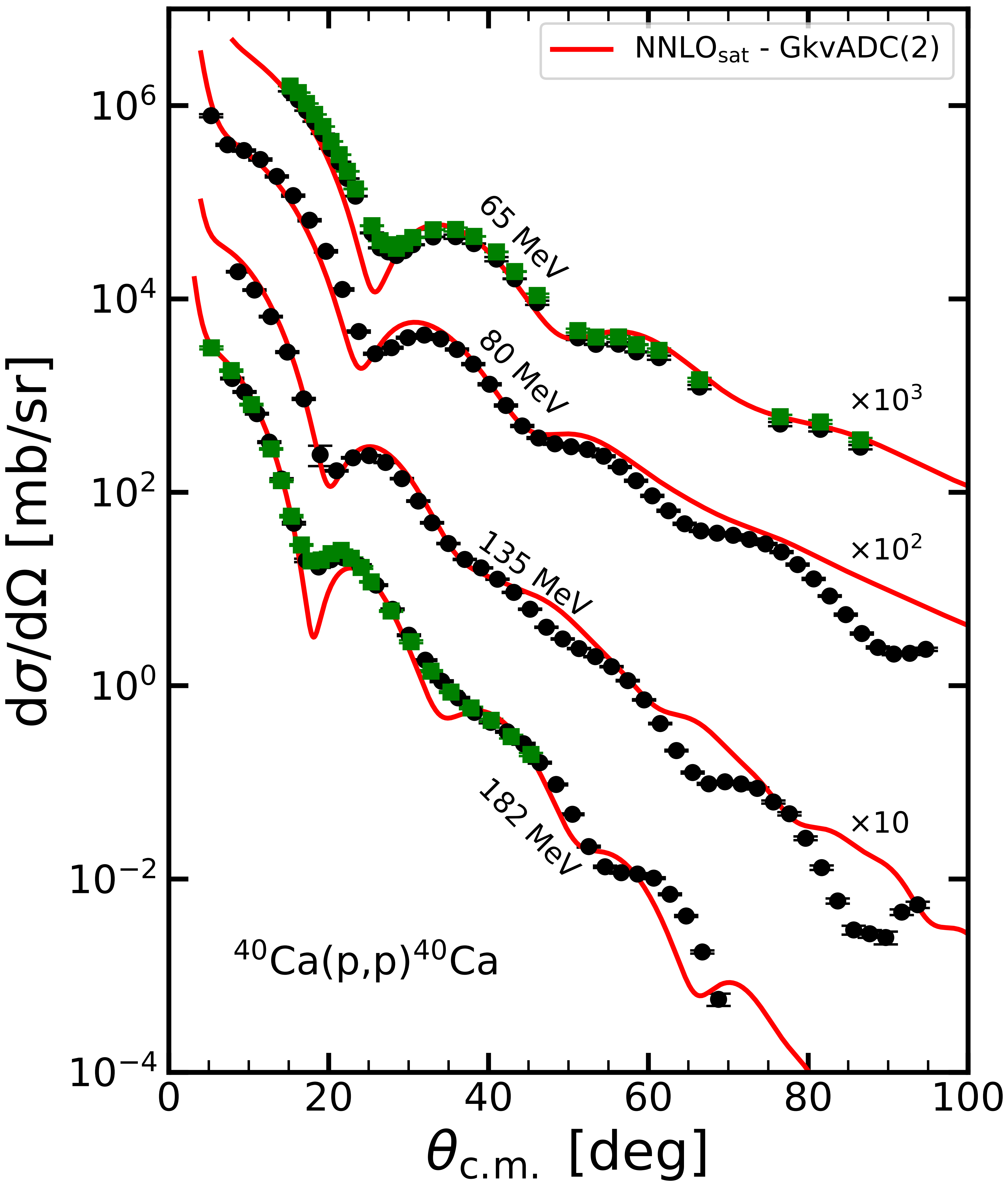




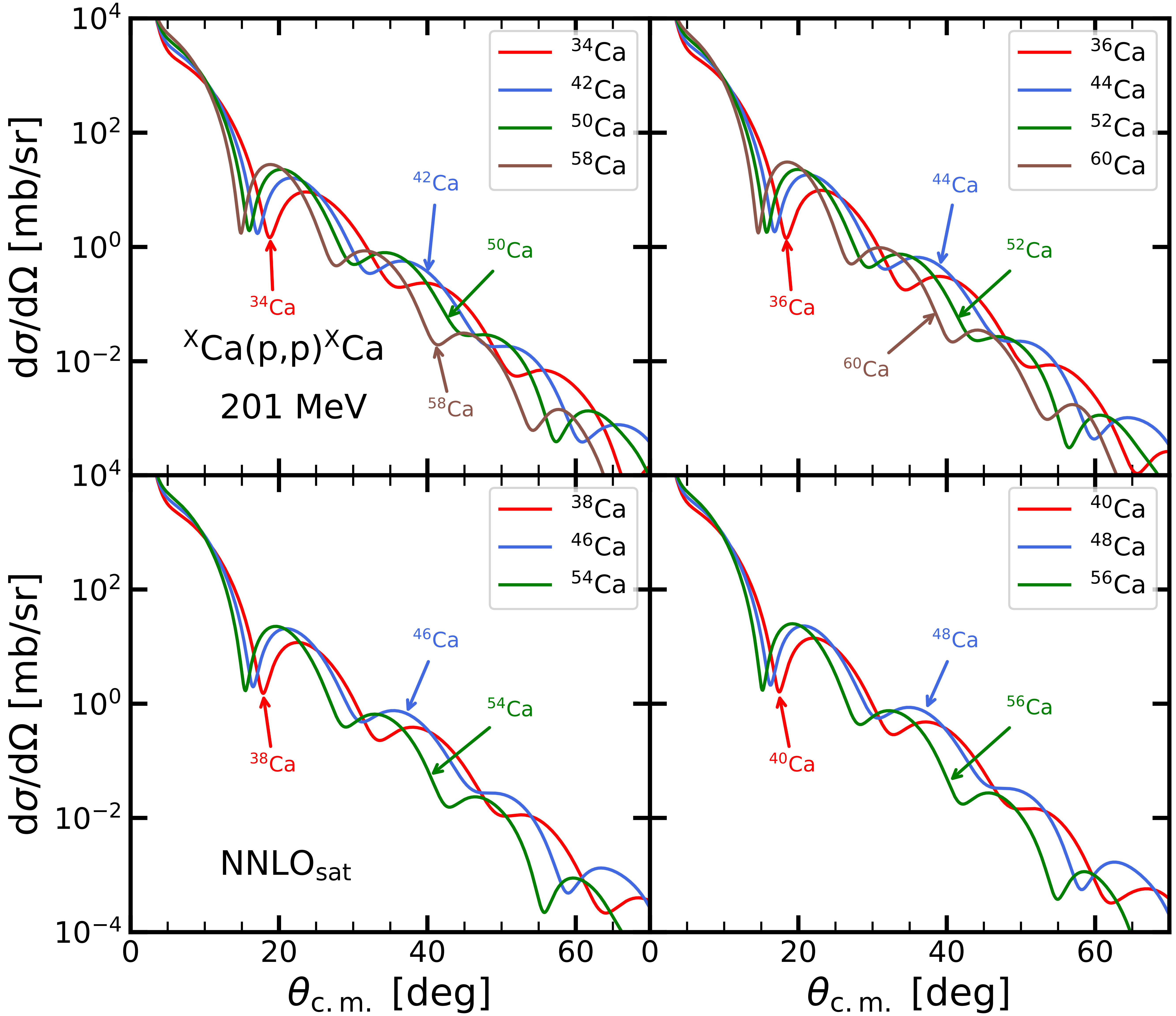
The data for the analysing power is remarkably well described!

(but remember that the NN potential does not reproduce the NN amplitudes)

Results for Calcium isotopic chain



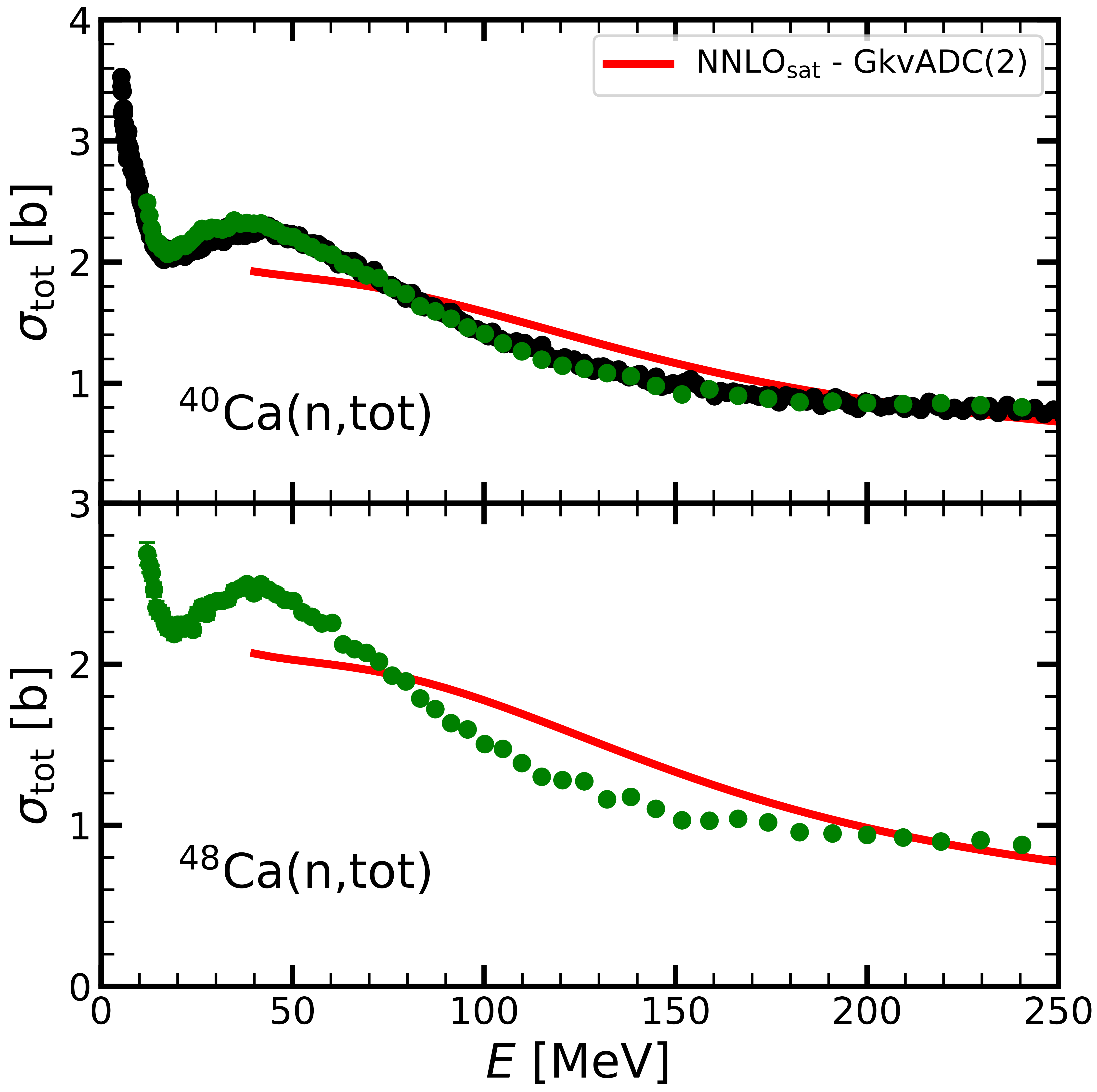
[Vorabbi et al., PRC **109**, 034613 (2024)]



Total cross sections

- At high energies, 180-250 MeV, is able to describe the data
- The adopted impulse approximation gradually worsens as the energy decreases, without any abrupt divergence
- In the range 70-180 MeV, data are overestimated
- Below 70 MeV, the impulse approximation is no longer valid

[Vorabbi et al., PRC **109**, 034613 (2024)]



Distorted wave theory of inelastic scattering

[Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1215 (1982)]
[Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1233 (1982)]

The inelastic transition amplitude

$$T_{\mathrm{inel}}(\boldsymbol{k}_*, \boldsymbol{k}_0) = \int d\boldsymbol{r}' \int d\boldsymbol{r}\, \psi^\dagger(\boldsymbol{k}_*, \boldsymbol{r}')\, U_{\mathrm{tr}}(\boldsymbol{r}', \boldsymbol{r})\, \psi(\boldsymbol{k}_0, \boldsymbol{r})$$

Required potentials

$$U_{\mathrm{ex}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{pN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N^{(\mathrm{ex})}(\boldsymbol{q}, \boldsymbol{P})$$

$$U_{\mathrm{tr}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{pN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N^{(\mathrm{tr})}(\boldsymbol{q}, \boldsymbol{P})$$

$$U_{\mathrm{gs}}(\boldsymbol{q}, \boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, t_{pN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P})\, \rho_N^{(\mathrm{gs})}(\boldsymbol{q}, \boldsymbol{P})$$

Distorted wave theory of inelastic scattering

[Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1215 (1982)]
[Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1233 (1982)]

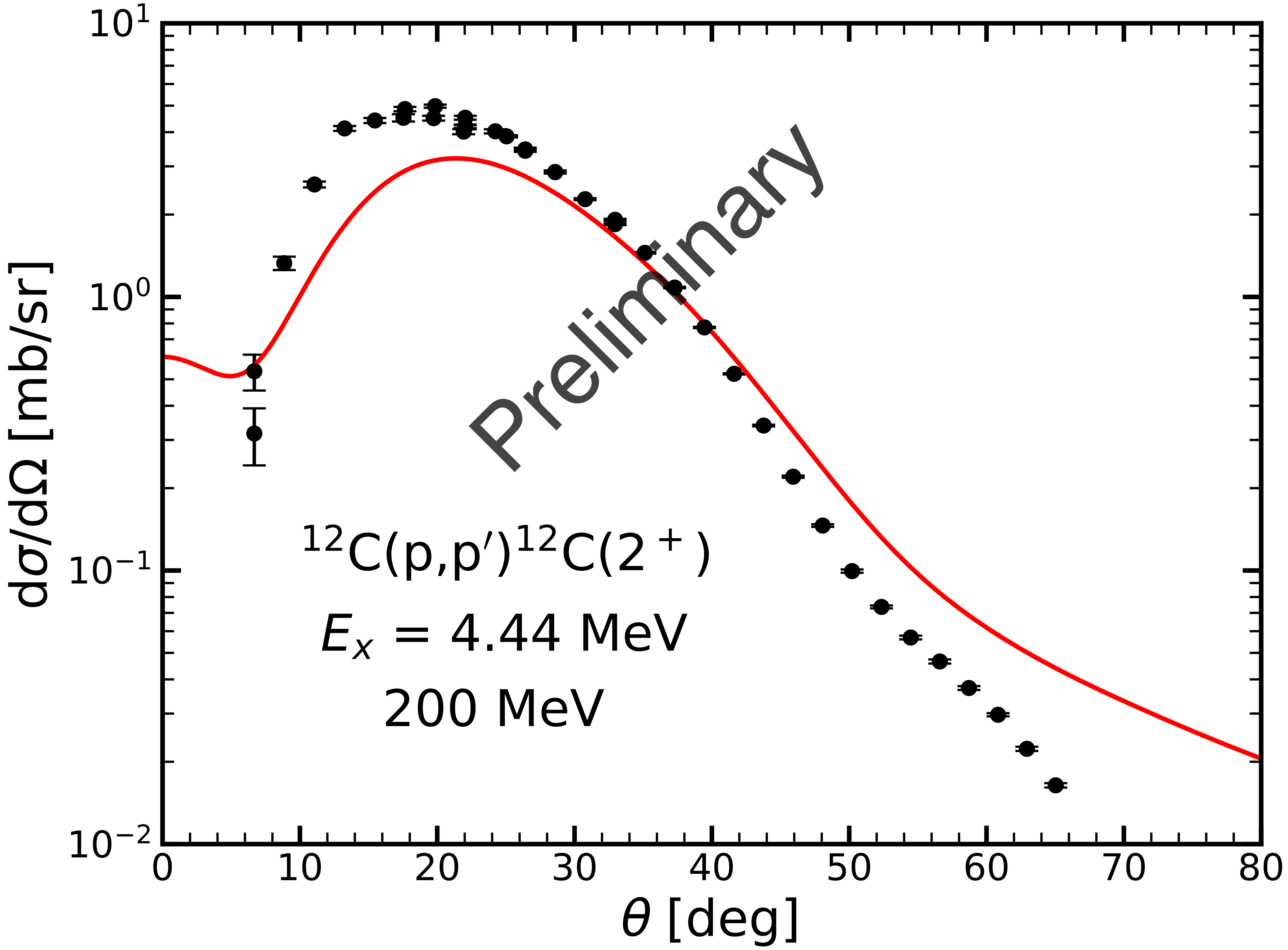
The inelastic transition amplitude

$$T_{\text{inel}}(\boldsymbol{k}_*, \boldsymbol{k}_0) = \int d\boldsymbol{r}' \int d\boldsymbol{r}\, \psi^\dagger(\boldsymbol{k}_*, \boldsymbol{r}')\, U_{\text{tr}}(\boldsymbol{r}', \boldsymbol{r})\, \psi(\boldsymbol{k}_0, \boldsymbol{r})$$

- The NN *t* matrix adopted for the calculation of the 3 potentials only contains two terms

$$A + i(\boldsymbol{\sigma} \cdot \hat{\boldsymbol{n}})C$$

- The peak of the curve is slightly shifted and the data are a bit underestimated but the overall agreement is good



Inclusion of medium effects

First-order term of the spectator expansion

$$\tau_{0i} = v_{0i} + v_{0i} G_0(E) \tau_{0i}$$

(A+1)-body propagator
The simplest approximation is

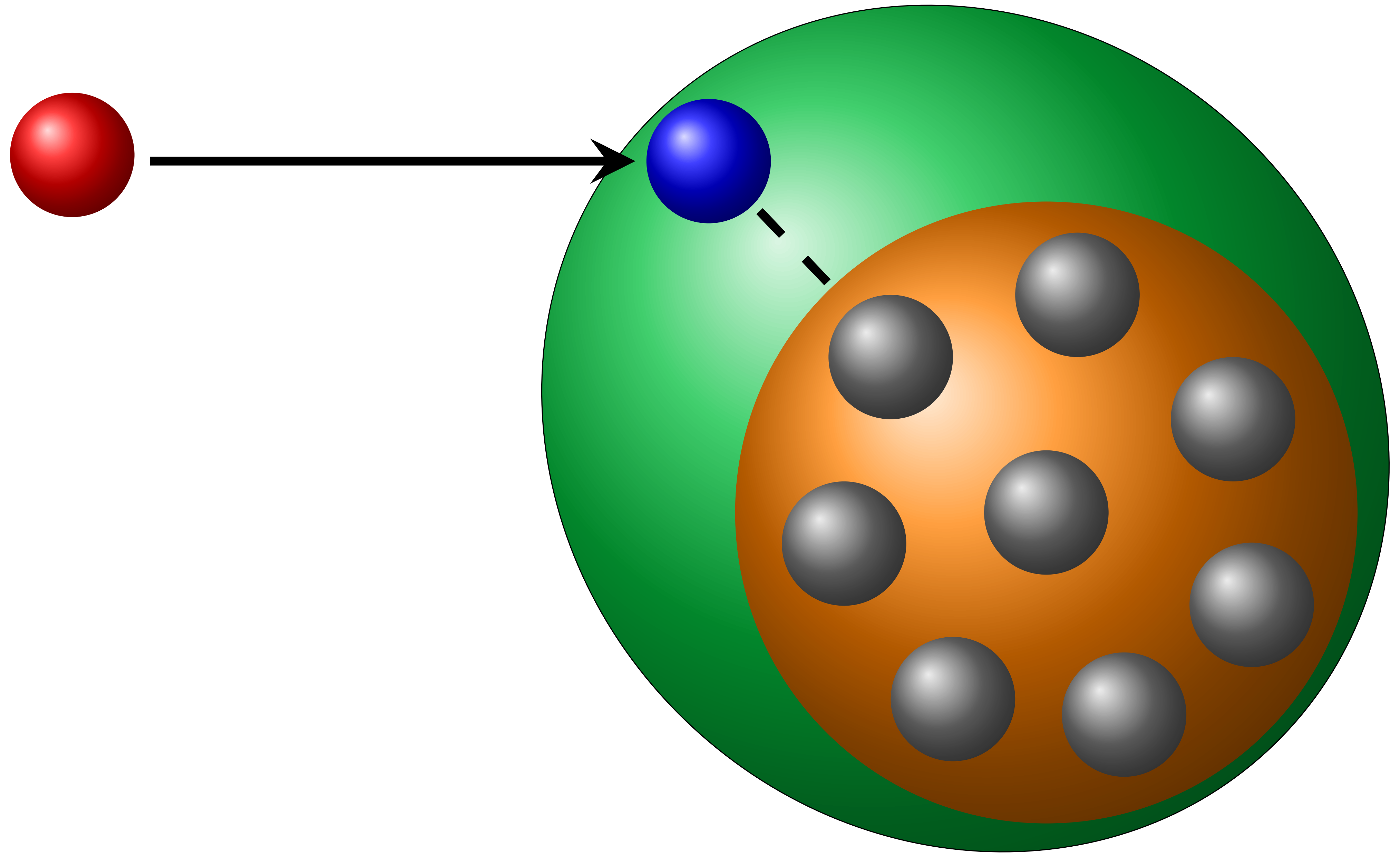
$$G_0(E) \approx g_0(E)$$

but there is not an intermediate one

Inclusion of medium effects

- Work has been done to include these effects at a mean-field level [Chinn *et al.*, PRC **52**, 1992 (1995)]
- We can use the SCGF to calculate the many-body propagator and the excitation spectrum

The first-order term is a 3-body problem



Inclusion of double scattering

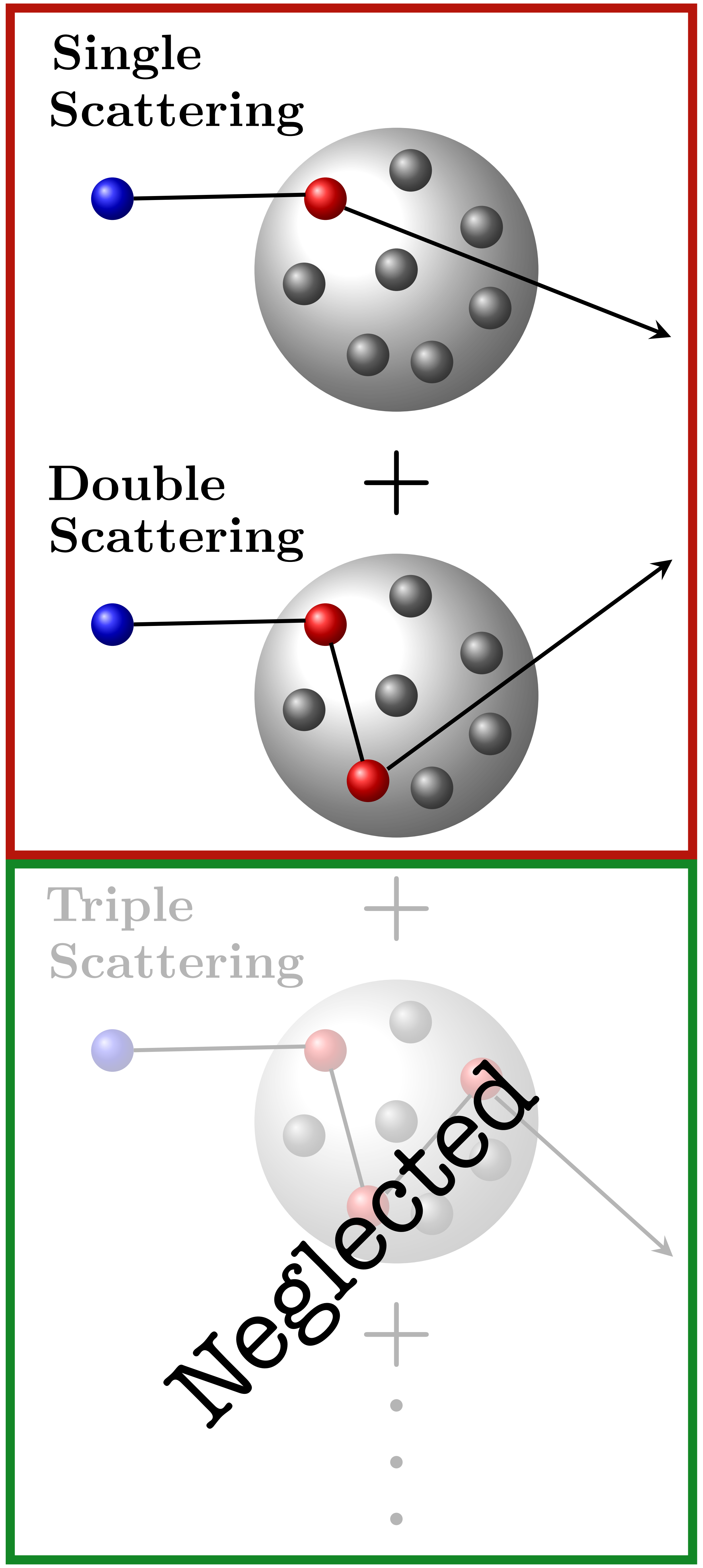
- Inclusion of the second-order term of the spectator expansion [Crespo *et al.*, PRC **46**, 279 (1992)]

$$U^{(2)} = \sum_{i=1}^{A} \tau_{0i} + \sum_{i,j\neq i}^{A} \tau_{0ij}$$

- Requires:
  1. Two-body density matrix (from NCSM)

     $$\rho(\boldsymbol{r}'_1, \boldsymbol{r}'_2, \boldsymbol{r}_1, \boldsymbol{r}_2)$$

  2. Solution of the three-body scattering equation for τ_{0ij}



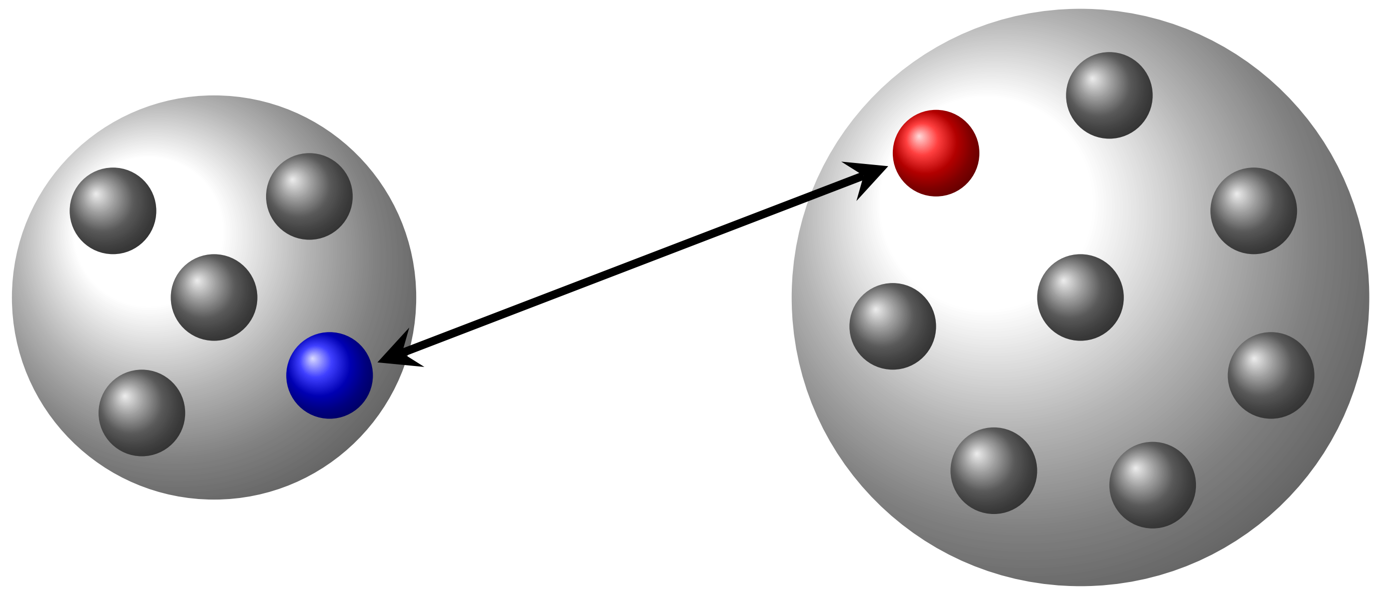
Optical potential for nucleus-nucleus elastic scattering

Transition amplitude for elastic scattering

$$T_{\rm el} \equiv PTP = PUP + PUPG_0(E)T_{\rm el}$$

The spectator expansion

[Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)]



$$U \simeq \sum_{i=1}^{A} \sum_{j=A+1}^{A+B} \tau_{ij} \qquad \tau_{ij} = v_{ij} + v_{ij} G_0(E) \tau_{ij}$$

(A+B)-body propagator
More complicated than
the NA case

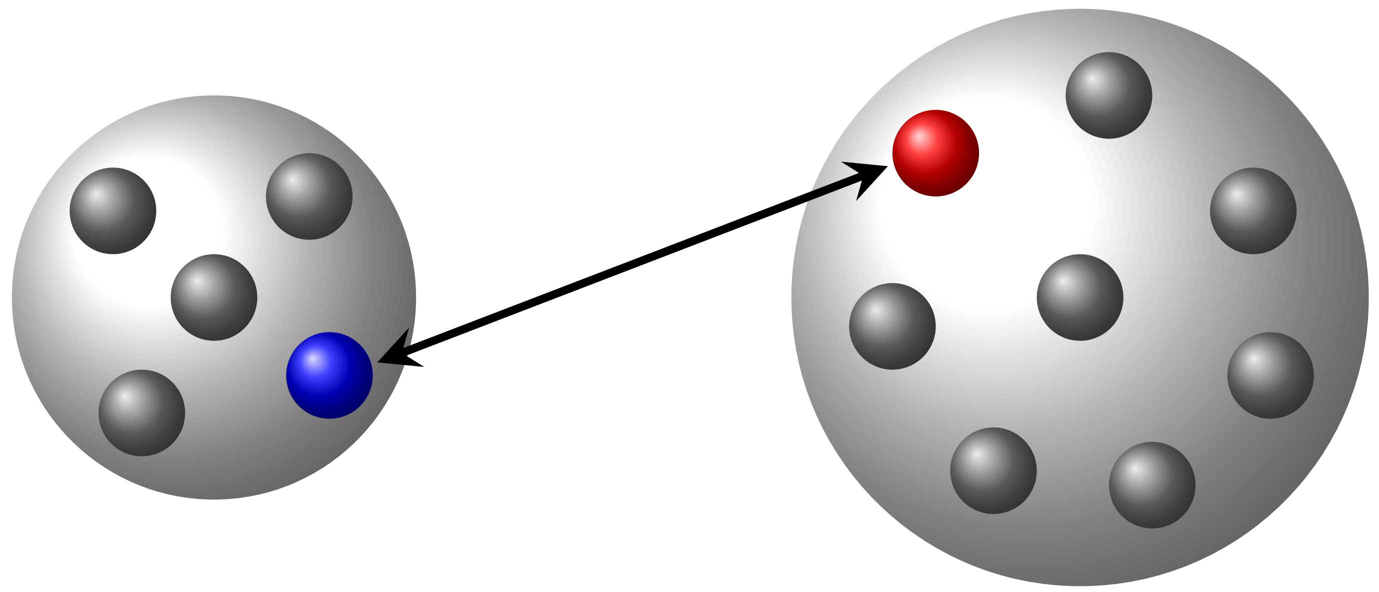
Optical potential for nucleus-nucleus elastic scattering

Transition amplitude for elastic scattering

$$T_{\rm el} \equiv PTP = PUP + PUPG_0(E)T_{\rm el}$$

The spectator expansion

[Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)]



$$U \simeq \sum_{i=1}^{A} \sum_{j=A+1}^{A+B} \tau_{ij} \qquad \tau_{ij} \approx t_{ij} = v_{ij} + v_{ij}\, g_0(E)\, t_{ij}$$

Not the best approximation,
but the simplest!

Two-body propagator

The first-order optical potential

Møller factor

$$t_{NN}^{(AB)} = \eta \, t_{NN}^{(NN)}$$

It imposes the Lorentz invariance of flux when we pass from the AB to the NN frame where the t matrices are evaluated

Projectile density

Target density

$$U(\boldsymbol{q}, \boldsymbol{K}) = \sum_{\alpha=n,p} \sum_{\beta=n,p} \int d\boldsymbol{P} \int d\boldsymbol{Q} \, \eta(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P}, \boldsymbol{Q}) \, t_{\alpha\beta}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P}, \boldsymbol{Q}; \mathcal{E}) \, \rho_{\alpha}^{(\mathbb{P})}(\boldsymbol{q}, \boldsymbol{P}) \, \rho_{\beta}^{(\mathbb{T})}(\boldsymbol{q}, \boldsymbol{Q})$$

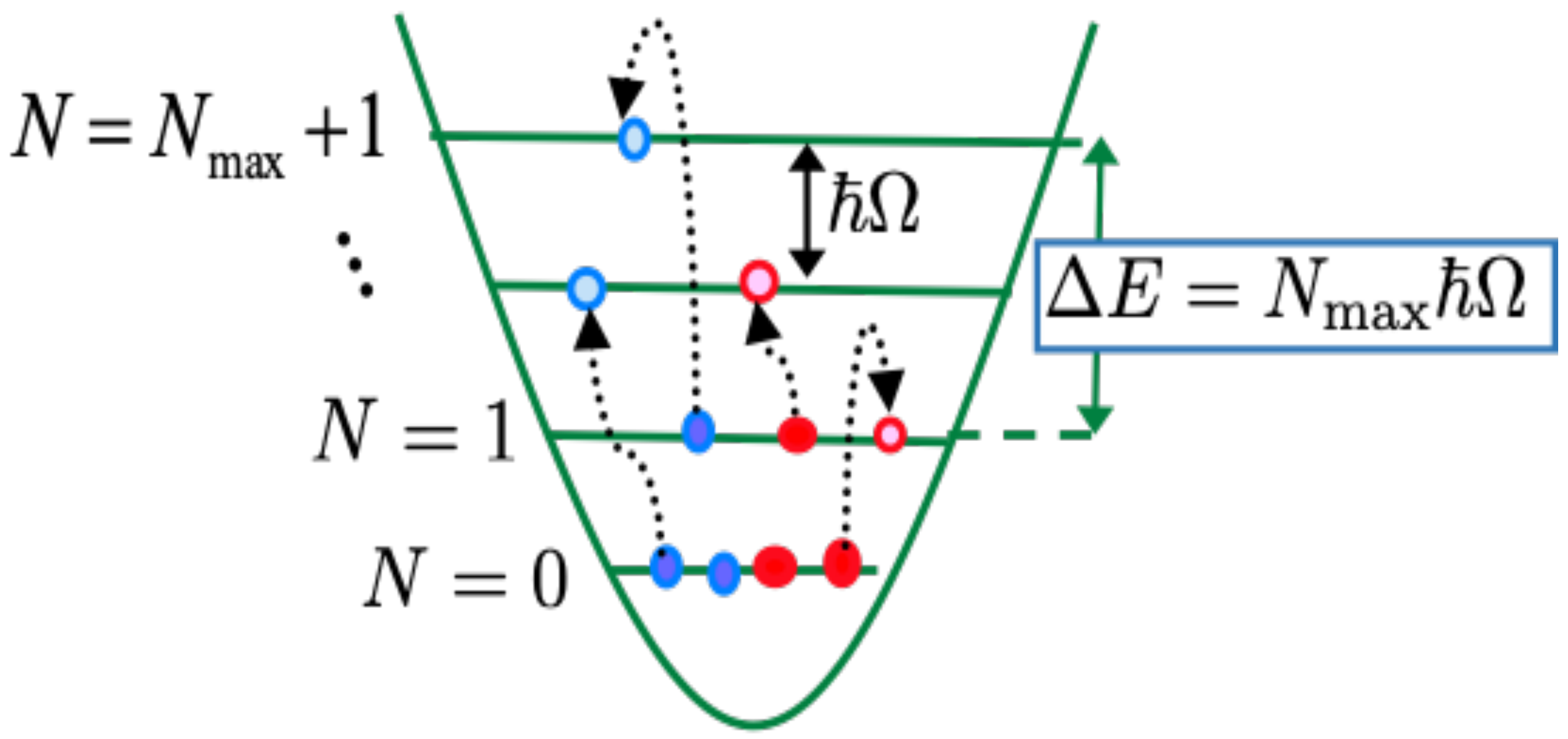
Free two-body scattering matrix

$$t_{0i} = v_{0i} + v_{0i} \, g_{0i} \, t_{0i}$$

$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

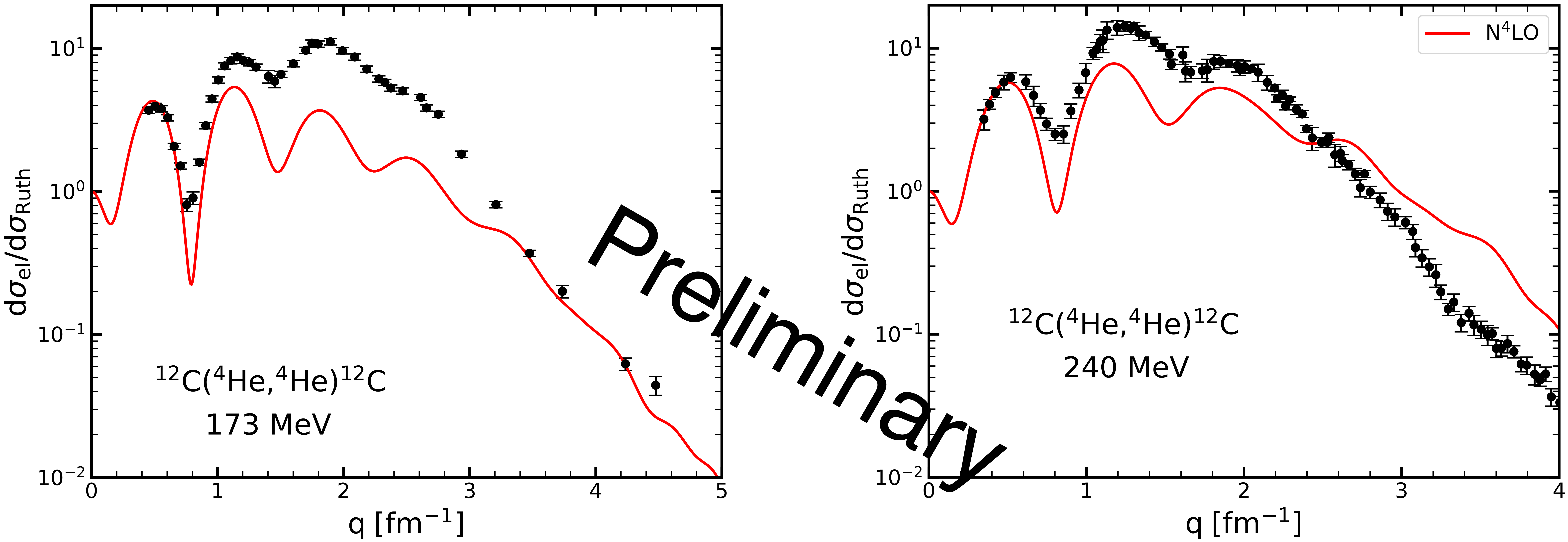
- Simple one-body equation
- Can be solved easily
- Only **NN** interaction

Nonlocal one-body density



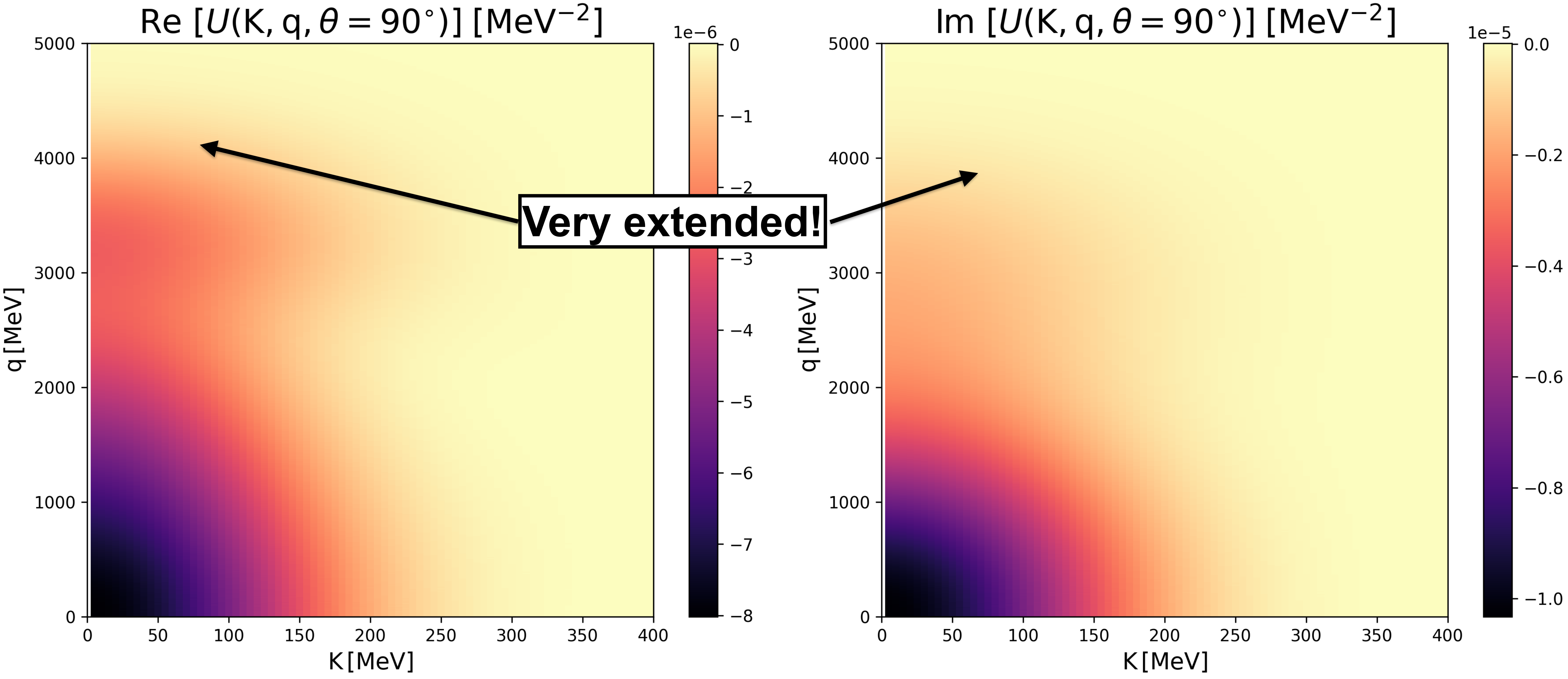
- Computationally expensive
- Obtained from the No-Core Shell Model
- Calculation performed with **NN** and **3N** interaction

Results for elastic α-^{12}C scattering



- Interesting results despite the approximations!
- The potential seems to be too absorptive

Potential for elastic α-^{12}C scattering at 173 MeV



How to reduce the absorption

A simple rescaling of the imaginary part seems to confirm that!

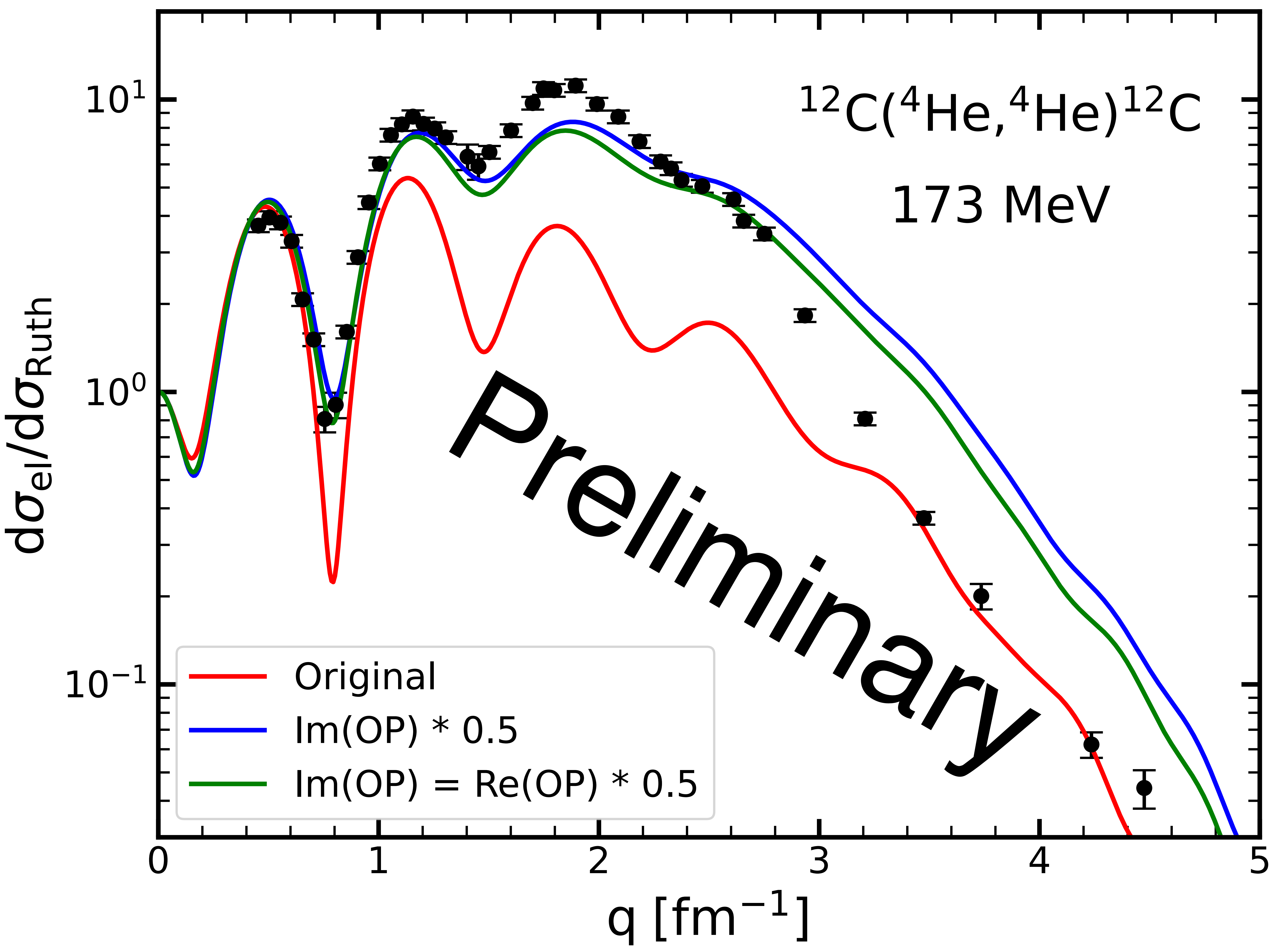
How can we decrease the absorption?

- Inclusion of medium effects
- Introducing the energy dependence of the t matrix in the double-folding integral

$$t_{NN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P}, \boldsymbol{Q}, \mathcal{E})$$

- Adding the double scattering term

[Crespo *et al.*, PRC **46**, 279 (1992)]



Summary & outlook

- ☑ The choice of the NN interaction is crucial to define the energy limits of applicability of the optical potential
- ☑ The combination of MST and SCGF looks promising for future calculations heavy systems
- ☑ Achieved a first step in the derivation of a nucleus-nucleus optical potential

- ☐ Extend the high- and low-energy limits of applicability of the optical potential
- ☐ Inclusion of the second-order term of the spectator expansion
- ☐ Consistent treatment of the full 3N interaction
- ☐ Reducing the absorption in the nucleus-nucleus optical potential