

DIFFRACTIVE JET PRODUCTION IN PHOTON-NUCLEUS COLLISIONS

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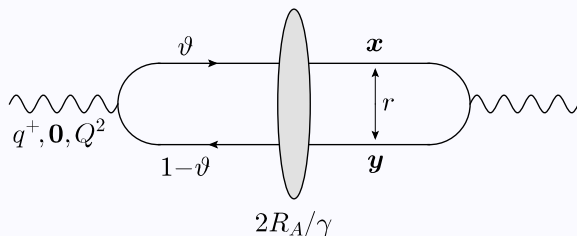
Diffraction and gluon saturation at the LHC and the EIC

Trento, 13 June, 2024

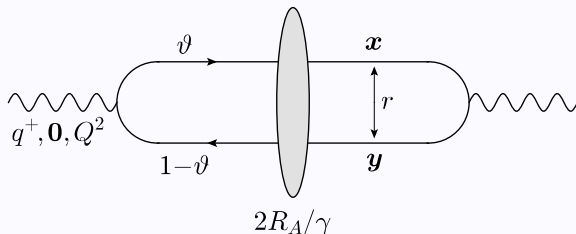
- ⊙ S. Hauksson, E. Iancu, A.H. Mueller, DT, S.Y. Wei: 2402.14748 (JHEP) [and 2304.12401 (EPJC), 2207.06268 (JHEP), 2112.06353 (PRL)]
- ⊙ B. Rodriguez-Aguilar, DT, S.Y. Wei: 2302.01106 (PRD), 240n.nnnnn

- ◉ Deep inelastic scattering in the dipole picture
- ◉ “2 + 1” jets in coherent diffraction as probes of saturation
- ◉ Factorization: Gluon and quark diffractive TMDs
- ◉ DTMDs in SIDIS
- ◉ 2 and “2 + 1” jets in incoherent diffraction

DIS AT SMALL- x IN DIPOLE PICTURE: TIME SCALES

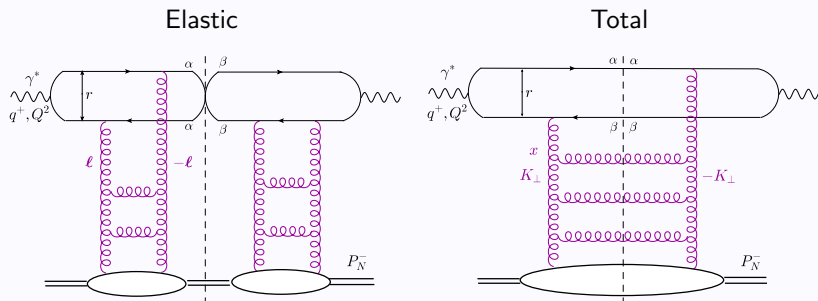


- Right moving off-shell γ^* , $q^\mu = (q^+, \mathbf{0}, -Q^2/2q^+)$
- Left moving nucleus, $p^\mu = (M_N^2/2P_N^-, \mathbf{0}, P_N^-)$ per nucleon
- Projectile lifetime $\tau \sim 2q^+/Q^2$
- Nucleus contracted length $L \sim 2R_A M_N/P_N^- \sim A^{1/3}/P_N^-$
- $L \ll \tau \iff xA^{1/3} \ll 1$



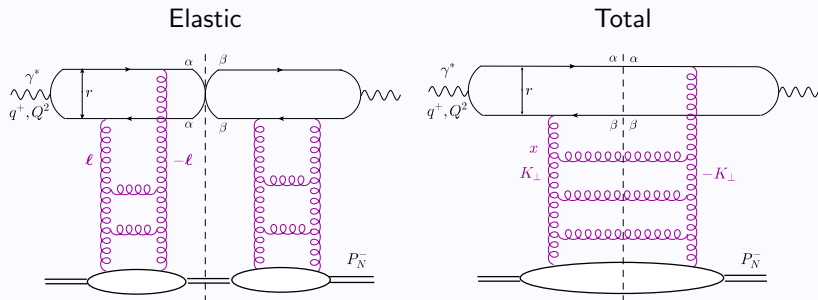
$$\sigma^{\gamma^*A}(x, Q^2) = \int d^2\mathbf{r} \int_0^1 d\vartheta |\Psi_{\gamma^* \rightarrow q\bar{q}}(Q^2; \mathbf{r}, \vartheta)|^2 2\pi R_A^2 T(r, x)$$

- All QCD dynamics in $T(r, x)$
- Virtuality limits large dipoles: $r \lesssim 1/\bar{Q}$, with $\bar{Q}^2 = \vartheta(1 - \vartheta)Q^2$
- Saturation requires $r \gtrsim 1/Q_s$, hence $\bar{Q}^2 \lesssim Q_s^2$
- When $Q^2 \gg Q_s^2$ dominant contribution from **weak scattering**



- Rapidity **gap**: wide angular region **void of particles**
- Elastic for projectile, no nuclear break-up (coherent reaction)
- Close color at amplitude level
- At least **two gluons** exchanged at amplitude

LARGE DIPOLES IN DIFFRACTION

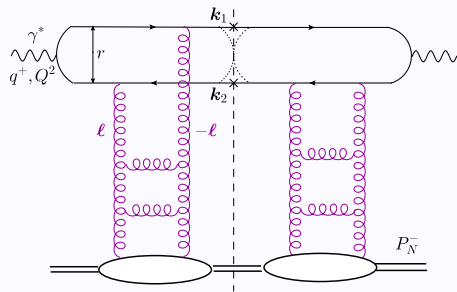


$$\sigma_D^{\gamma^* A}(x, Q^2) = \int d^2\mathbf{r} \int_0^1 d\vartheta |\Psi_{\gamma^* \rightarrow q\bar{q}}(Q^2; \mathbf{r}, \vartheta)|^2 \pi R_A^2 [T(r, x)]^2$$

- T^2 : Diffractive cross section less sensitive to small dipoles
- Even for $Q^2 \gg Q_s^2$ dipoles with $r \gtrsim 1/Q_s$ and $\vartheta \sim Q_s^2/Q^2 \ll 1$ (“aligned jets”) dominate diffractive cross section

HARD DIJET IN DIFFRACTION

- More **exclusive** processes? Measured jets or hadrons?
- Hard scale sets dipole size $r \sim 1/P_{\perp}$, **weak scattering**
- Hard, symmetric, back to back $q\bar{q}$ pair:
 $k_{1\perp} \simeq k_{2\perp} \equiv P_{\perp} \sim Q \gg Q_s, \vartheta_{1,2} \sim 1/2$

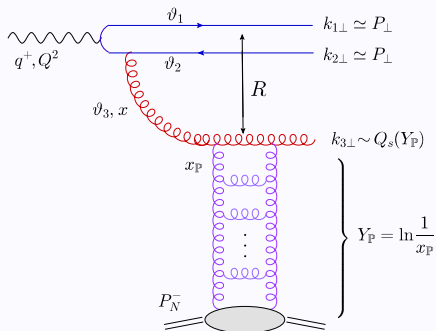


$$\frac{d\sigma_D^{\gamma^*A \rightarrow q\bar{q}A}}{d\vartheta_1 d^2\mathbf{P}} \propto \pi R_A^2 \underbrace{\frac{1}{Q^2}}_{\gamma^*q\bar{q}} \underbrace{\frac{Q_s^4}{P_{\perp}^4}}_{T^2(r)}$$

$$\sim \frac{1}{P_{\perp}^6} \text{ power suppressed}$$

2+1 JETS IN DIFFRACTION

- Diffractive dijet at leading twist $1/P_{\perp}^4$?
- Yes, two hard jets $P_{\perp} \gg Q_s$ and one semi-hard $k_{3\perp} \sim Q_s \ll P_{\perp}$
- Third, semi-hard, jet provides dijet imbalance

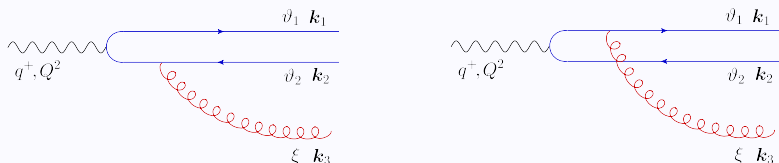


- $\mathcal{O}(\alpha_s)$ suppression
- $R \gg r$: gluon dipole
- $T_g(R, Y_P) \simeq \mathcal{O}(1)$
- Hard factor

$$H \propto \underbrace{\frac{1}{Q^2}}_{\gamma^* q \bar{q}} \times \underbrace{r^2}_{\text{gluon emission}} \sim \frac{1}{P_{\perp}^4}$$

- $x_P P_N^-$ puts trijet on-shell: $x_P \simeq \frac{1}{2q^+ P_N^-} \left(\frac{P_{\perp}^2}{\vartheta_1 \vartheta_2} + \frac{k_{3\perp}^2}{\vartheta_3} + Q^2 \right)$

GLUON DIPOLE WAVEFUNCTION



- Gluon formation time must be small enough:

$$k_3^+ / k_{3\perp}^2 \lesssim q^+ / Q^2 \rightsquigarrow \vartheta_3 \lesssim k_{3\perp}^2 / P_\perp^2 \ll 1, \text{ gluon is soft}$$

- Momentum space LCWF

$$\left[\frac{k_1^l \left(k_3^j + \frac{\xi}{1-\vartheta_1} k_1^j \right)}{k_{1\perp}^2 + \bar{Q}^2} + \frac{k_2^l \left(k_3^j + \frac{\xi}{1-\vartheta_2} k_2^j \right)}{k_{2\perp}^2 + \bar{Q}^2} \right] \frac{1}{k_{3\perp}^2 + \xi \left(\frac{k_{1\perp}^2}{\vartheta_1} + \frac{k_{2\perp}^2}{\vartheta_2} + Q^2 \right)}$$

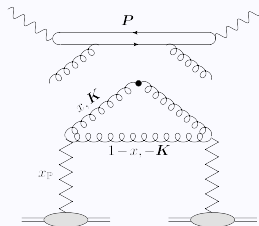
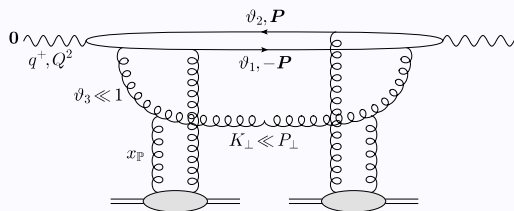
- Expand for $k_{3\perp} \ll P_\perp$ and $\xi \ll k_{3\perp} / P_\perp$ (no recoil)
- Leading terms cancel \rightsquigarrow Non-eikonal emission
- Scattering is eikonal (Wilson lines)
- Add instantaneous quark propagator graph

GLUON FROM THE POMERON

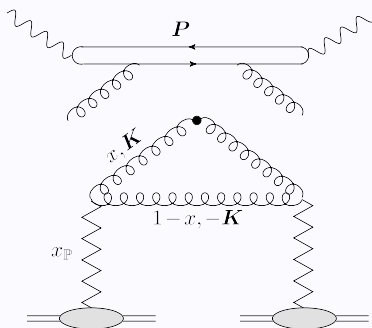
- ⊙ Scales separation \rightsquigarrow Factorization?
- ⊙ View gluon as part of Pomeron. Variable change from ξ to x :

$$x = \frac{x_{q\bar{q}}}{x_{\mathbb{P}}} = \frac{\frac{P_{\perp}^2}{\vartheta_1 \vartheta_2} + Q^2}{\frac{P_{\perp}^2}{\vartheta_1 \vartheta_2} + \frac{k_{3\perp}^2}{\vartheta_3} + Q^2} \quad \text{or} \quad x = \beta \frac{x_{q\bar{q}}}{x_{Bj}} \simeq \beta \frac{\bar{Q}^2 + P_{\perp}^2}{P_{\perp}^2}$$

- ⊙ For given x_{Bj} and hard jets, only one of ξ , $x_{\mathbb{P}}$ and x is independent



TMD FACTORIZATION AND CROSS SECTION



$$\frac{d\sigma_D^{\gamma_{T,L}^* A \rightarrow q\bar{q}gA}}{d\vartheta_1 d\vartheta_2 d^2\mathbf{P} d^2\mathbf{K} dY_{\mathbb{P}}} = H_{T,L}(\vartheta_1, \vartheta_2, Q^2, P_{\perp}^2) \frac{dx G_{\mathbb{P}}(x, x_{\mathbb{P}}, K_{\perp}^2)}{d^2\mathbf{K}}$$

- Hard factor as in inclusive $q\bar{q}$ dijet cross section

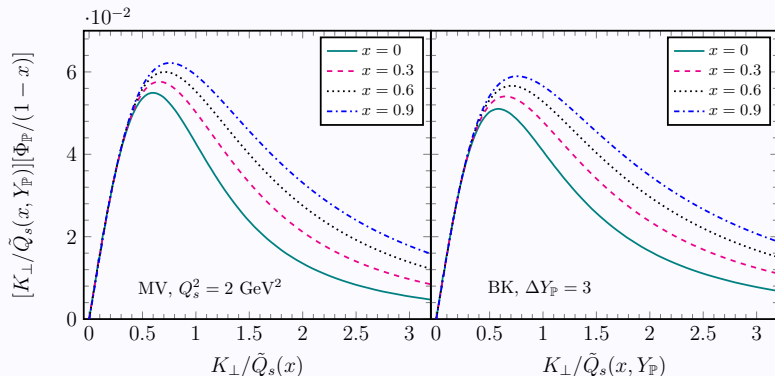
$$H_T(\vartheta_1, \vartheta_2, Q^2, P_{\perp}^2) \equiv \alpha_{em} \alpha_s \left(\sum e_f^2 \right) \delta_{\vartheta} \underbrace{(\vartheta_1^2 + \vartheta_2^2)}_{2P_{q\gamma}(\vartheta_1)} \underbrace{\frac{P_{\perp}^4 + \bar{Q}^4}{(P_{\perp}^2 + \bar{Q}^2)^4}}_{\sim 1/P_{\perp}^4}$$

$$\frac{dxG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_{\perp}^2)}{d^2\mathbf{K}} = \underbrace{\frac{S_{\perp}(N_c^2 - 1)}{4\pi^3}}_{\text{d.o.f.}} \underbrace{\Phi_{\mathbb{P}}(x, x_{\mathbb{P}}, K_{\perp}^2)}_{\text{occupation number}}$$

- Explicit in terms of elastic amplitude $T_g(R, x_{\mathbb{P}})$

$$\Phi_{\mathbb{P}}(x, x_{\mathbb{P}}, K_{\perp}^2) \approx \frac{1-x}{2\pi} \begin{cases} 1 & \text{for } K_{\perp} \ll \tilde{Q}_s(x) \\ \frac{\tilde{Q}_s^4(x, Y_{\mathbb{P}})}{K_{\perp}^4} & \text{for } K_{\perp} \gg \tilde{Q}_s(x) \end{cases}$$

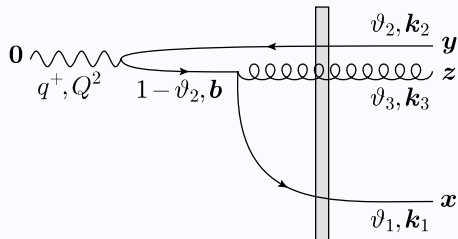
- Valid for large gaps: $x_{\mathbb{P}} \lesssim 10^{-2}$
- Effective saturation momentum $\tilde{Q}_s^2(x) \equiv (1-x)Q_s^2$
- Bulk of distribution at saturation $K_{\perp} \ll \tilde{Q}_s(x)$



- Multiplied by K_{\perp} (cf. measure $d^2\mathbf{K}$)
- Pronounced maximum at $K_{\perp} \sim \tilde{Q}_s(x)$

SOFT QUARK IN 2 + 1 JETS

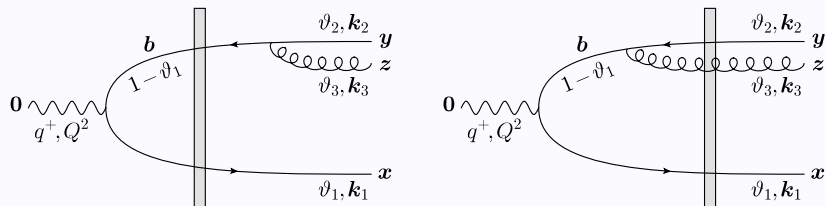
- Hard \bar{q} and g , $P_\perp \gg Q_s$, semi-hard q , $k_{1\perp} \sim Q_s \ll P_\perp$



- $\mathcal{O}(\alpha_s)$ suppression
- Large quark dipole
- $T(R, Y_{\mathbb{P}}) \simeq \mathcal{O}(1)$
- Hard factor $\sim 1/P_\perp^4$

- (NB: Scattering before emission is small, like in soft g case)
- Quark must be soft $v_1 \lesssim k_{1\perp}^2/P_\perp^2$

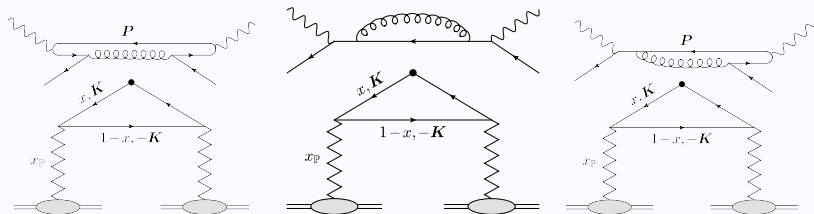
ANOTHER CONFIGURATION WITH A SOFT QUARK



- Large initial $q\bar{q}$ pair, hard QCD vertex
- Same scattering **before or after** gluon emission, fine cancellations
- Also consider **interference** between these and previous diagram

TMD FACTORIZATION AND CROSS SECTION

- Variable change from ϑ_1 to x
- Quark with fraction $1 - x$ in final state
- Antiquark “transfers” fraction x and imbalance \mathbf{K} to dijet



$$\frac{d\sigma_D^{\gamma_{T,L}^* A \rightarrow q\bar{q}gA}}{d\vartheta_1 d\vartheta_2 d^2\mathbf{P} d^2\mathbf{K} dY_{\mathbb{P}}} = \frac{4\pi^2 \alpha_{\text{em}}}{Q^2} \left(\sum e_f^2 \right) H_{T,L}(\vartheta_2, \vartheta_3, Q^2, P_{\perp}^2) \frac{dx q_{\mathbb{P}}(x, x_{\mathbb{P}}, K_{\perp}^2)}{d^2\mathbf{K}}$$

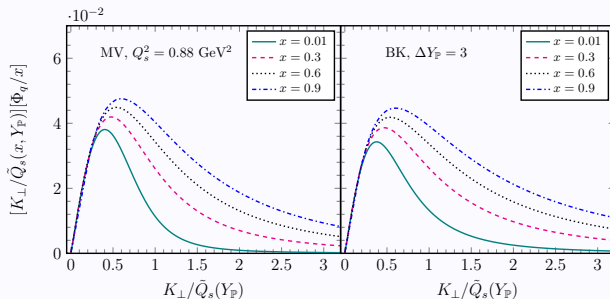
$$H_T(\vartheta_2, \vartheta_3, P_{\perp}^2, \tilde{Q}^2) = \delta_{\vartheta} \frac{\alpha_s C_F}{\pi^2} \frac{1}{2\vartheta_3} \frac{\tilde{Q}^2 [(P_{\perp}^2 + \tilde{Q}^2)^2 + \vartheta_2^2 \tilde{Q}^4 + \vartheta_3^2 P_{\perp}^4]}{P_{\perp}^2 (P_{\perp}^2 + \tilde{Q}^2)^3}.$$

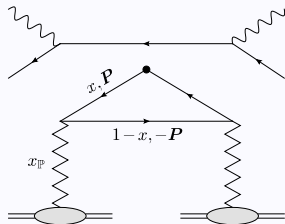
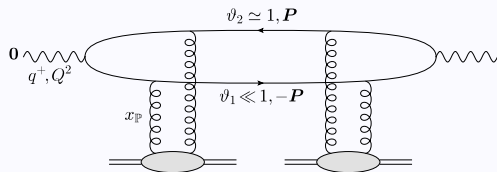
SEMI-HARD FACTOR: QUARK DIFFRACTIVE TMD

$$\frac{dxq_{\mathbb{P}}(x, x_{\mathbb{P}}, K_{\perp}^2)}{d^2\mathbf{K}} = \underbrace{\frac{S_{\perp} N_c}{4\pi^3}}_{\text{d.o.f.}} \underbrace{\Psi_{\mathbb{P}}(x, x_{\mathbb{P}}, K_{\perp}^2)}_{\text{occupation number}}$$

Explicit in terms of elastic amplitude $T(R, x_{\mathbb{P}})$ (fundamental)

$$\Psi_{\mathbb{P}}(x, x_{\mathbb{P}}, K_{\perp}^2) \approx \frac{x}{2\pi} \begin{cases} 1 & \text{for } K_{\perp} \ll \tilde{Q}_s(x) \\ \frac{\tilde{Q}_s^4(x, Y_{\mathbb{P}})}{K_{\perp}^4} & \text{for } K_{\perp} \gg \tilde{Q}_s(x) \end{cases}$$





- Consider dijet cross section obtained in the dipole picture
- Integrate one jet keeping β (gap) fixed \rightsquigarrow change from ϑ_1 to β
- If (and only if) aligned jet configuration ($\vartheta_1 \ll 1$) and $P_{\perp}^2 \ll Q^2$:

$$\frac{d\sigma^{\gamma_T^* A \rightarrow q\bar{q}A}}{d\ln(1/\beta) d^2\mathbf{P}} = \frac{4\pi^2\alpha_{em}}{Q^2} \left(\sum e_f^2 \right) 2 \frac{dxq_{\mathbb{P}}(x, x_{\mathbb{P}}, P_{\perp}^2)}{d^2\mathbf{P}} \Bigg|_{x=\beta}$$

- Leading twist result, same quark TMD encountered in 2+1 jets

- Consider hard antiquark-gluon pair and soft quark configuration
- In SIDIS we measure antiquark, integrate the gluon
- Dominant contribution from gluon such that

$$\vartheta_2 \simeq 1 \gg \vartheta_3 \sim \frac{P_{\perp}^2}{Q^2} \gg \vartheta_1 \sim \frac{k_{1\perp}^2}{Q^2}$$

- Integrate at fixed β

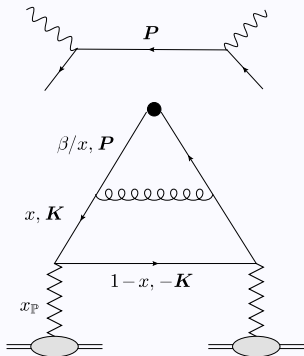
$$\begin{aligned} \frac{d\sigma^{\gamma_T^* A \rightarrow (q)\bar{q}gA}}{d^2\mathbf{P} d\ln(1/\beta)} &= \int d\vartheta_2 d\vartheta_3 \int \frac{dx}{x} \beta \delta\left(\beta - x \frac{\tilde{Q}^2}{\tilde{Q}^2 + P_{\perp}^2}\right) \\ &\times H_T(\vartheta_2, \vartheta_3, Q^2, P_{\perp}^2) \int d^2\mathbf{K} \frac{dx q_{\mathbb{P}}(x, x_{\mathbb{P}}, K_{\perp}^2)}{d^2\mathbf{K}} \end{aligned}$$

- \mathbf{K} -integration gives DPDF

EMERGENCE OF DGLAP

- Hard factor becomes (real part of) DGLAP splitting function

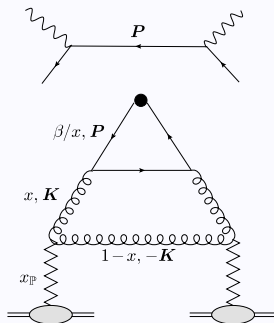
$$\frac{d\sigma^{\gamma_T^* A \rightarrow (q)\bar{q}gA}}{d^2\mathbf{P} d\ln(1/\beta)} = \frac{4\pi^2\alpha_{\text{em}}}{Q^2} \left(\sum e_f^2 \right) \times \frac{\alpha_s}{2\pi^2} \frac{1}{P_{\perp}^2} \int_{x_{\min}}^1 \frac{dx}{x} \frac{\beta}{x} P_{qg} \left(\frac{\beta}{x} \right) x q_{\mathbb{P}}(x, x_{\mathbb{P}}, P_{\perp}^2).$$



- Target: gluon emission before γ^* absorption by struck antiquark
- All projectile diagrams contribute to simple target picture

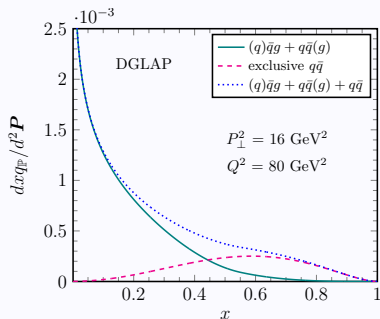
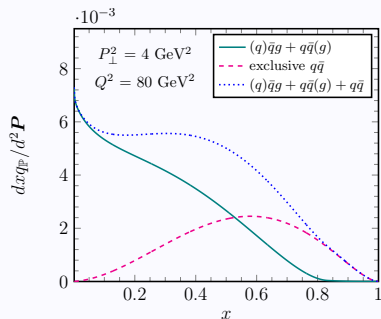
- Consider hard quark-antiquark pair and soft gluon
- Integrate the quark with fixed β to get SIDIS

$$\frac{d\sigma^{\gamma_T^* A \rightarrow q\bar{q}(g)A}}{d^2\mathbf{P} d\ln(1/\beta)} = \frac{4\pi^2\alpha_{\text{em}}}{Q^2} \left(\sum e_f^2 \right) \times \frac{\alpha_s}{2\pi^2} \frac{1}{P_\perp^2} \int_\beta^1 \frac{dx}{x} \frac{\beta}{x} P_{qg} \left(\frac{\beta}{x} \right) x G_{\mathbb{P}}(x, x_\mathbb{P}, P_\perp^2).$$



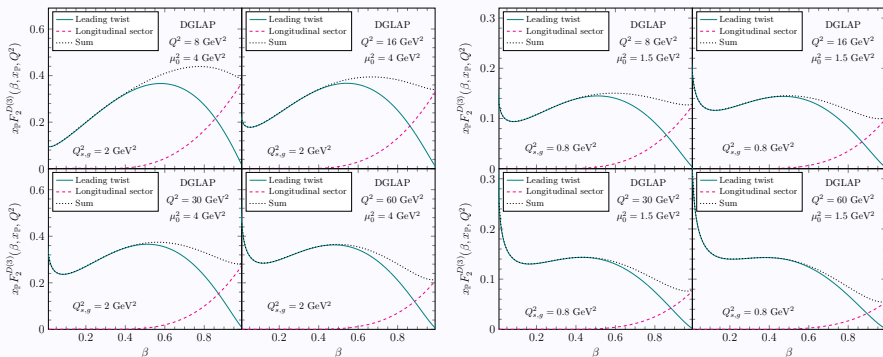
“TOTAL” DTMDs

- Absorb 2+1 jet contributions into the quark DTMD
- 2 jets piece: $\sim 1/P_{\perp}^4$
- 2+1 jets pieces: $\sim \alpha_s/P_{\perp}^2$
- Similarly for the gluon DTMD (would need an extra step since it does not appear in 2 jets)



DIFFRACTIVE STRUCTURE FUNCTION

- Integrate over $d^2P \rightsquigarrow$ Diffractive structure function
- Large nucleus $Q_{sg}^2 = 2 \text{ GeV}^2$, proton $Q_{sg}^2 = 0.8 \text{ GeV}^2$



- Shape difference attributed to starting point μ_0^2 , magnitude $\sim Q_s^2$

Target average to be taken with CGC wave-function

- ◉ $\langle T(\mathbf{x}, \mathbf{y})T(\bar{\mathbf{y}}, \bar{\mathbf{x}}) \rangle \rightarrow$ Total diffraction
- ◉ $\langle T(\mathbf{x}, \mathbf{y}) \rangle \langle T(\bar{\mathbf{y}}, \bar{\mathbf{x}}) \rangle \rightarrow$ Coherent diffraction
- ◉ $\langle T(\mathbf{x}, \mathbf{y})T(\bar{\mathbf{y}}, \bar{\mathbf{x}}) \rangle - \langle T(\mathbf{x}, \mathbf{y}) \rangle \langle T(\bar{\mathbf{y}}, \bar{\mathbf{x}}) \rangle \rightarrow$ Incoherent diffraction

Homogeneous target:

- ◉ Coherent diffraction $\sim \delta^2(\Delta)$ (smeared to $1/R_A$)
Negligible momentum transfer from target to projectile
- ◉ Momentum transfer conjugate to difference B of impact parameters in DA and CCA \rightsquigarrow non-zero momentum transfer

Variance of scattering amplitude determined by target fluctuations

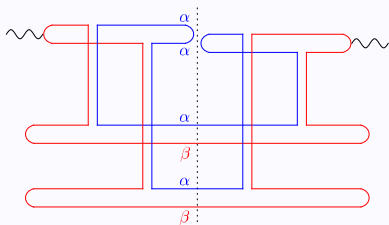
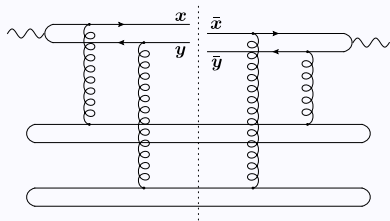
- ◉ Pomeron loops: particle number fluctuations in target
- ◉ Hot spots (shape fluctuations)
- ◉ $1/N_c^2$ color fluctuations (MV model, JIMWLK)
- ◉ ...

Study color fluctuations

- ◉ Q_s sets the scale for color fluctuations
- ◉ Expect power-law tail for $\Delta_{\perp} > Q_s$

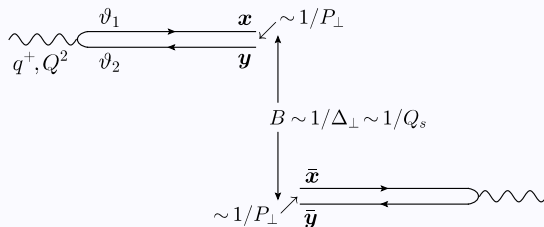
CORRELATOR AT 4-GLUON EXCHANGE

Assume Gaussian CGC WF, only pieces connecting DA with CCA survive
Just for illustration assume 4-gluon exchange



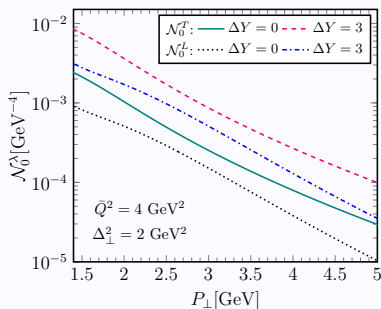
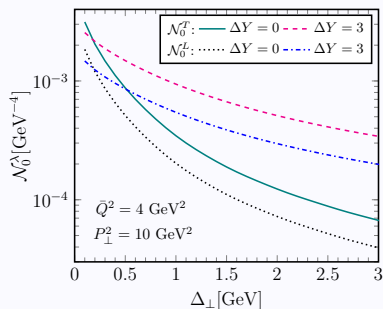
- Projectile can be either quark or gluon dipole
- Elastic on projectile
- Colorless nucleus substructures scatter inelastically
- Exchange of color among them
- Incoherent scattering $\leftrightarrow 1/N_c^2$ suppression

With only two hard jets, pair imbalance is momentum transfer $\mathbf{K} = \Delta$
 Interested in $P_{\perp} \gg \Delta_{\perp}, Q_s \longleftrightarrow r, \bar{r} \ll B, 1/Q_s$



- Correlator known (at finite- N_c)
- Expand for $r, \bar{r} \ll B, 1/Q_s$, other than that B is arbitrary

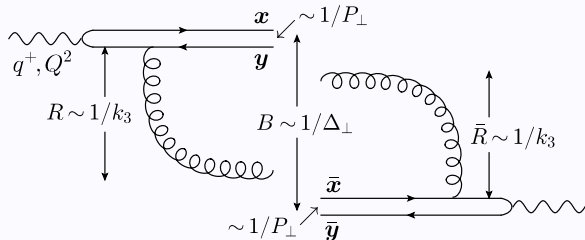
2 HARD JETS, AVERAGED (OVER ANGLE) CROSS SECTION



There is angular $\mathbf{P} \cdot \mathbf{\Delta}$ dependence, focus on averaged cross section

$$\frac{d\sigma_D^{\gamma_\lambda^* A \rightarrow q\bar{q}X}}{d\vartheta_1 d\vartheta_2 d^2\mathbf{P} d^2\mathbf{\Delta}} \propto \frac{\alpha_{\text{em}} S_\perp}{N_c} \frac{Q_s^2}{P_\perp^6} \underbrace{f(\Delta_\perp/Q_s)}_{\text{dim/less}}$$

$$f(\Delta_\perp/Q_s) \sim \begin{cases} \mathcal{O}(1) & \text{for } \Delta_\perp \ll Q_s \\ Q_s^2/\Delta_\perp^2 & \text{for } \Delta_\perp \gg Q_s \end{cases}$$



- Pair imbalance determined by momentum transfer Δ and recoil due to soft jet emission, $\mathbf{K} = \Delta - \mathbf{k}_3$
- Integrate over \mathbf{k}_3 with fixed Δ

Interested again in $P_\perp \gg \Delta_\perp, Q_s$, but now:

- Distribution in $k_{3\perp}$ is peaked at Q_s
- Size of scattering dipole $R \sim 1/k_{3\perp} \sim 1/Q_s$ is large
- No expansion in the dipole-dipole QCD correlator

Tensor structure $H^{ij}(\mathbf{P}) G^{ij}(\mathbf{\Delta})$

- Soft gluon

$$H^{ij}(\mathbf{P}) = H(P_{\perp}) \delta^{ij} + H_1(P_{\perp}) \hat{P}^{ij}$$

$$G^{ij}(\mathbf{\Delta}) = G(\Delta_{\perp}) \delta^{ij} + \mathbf{0} \times \hat{\Delta}^{ij}$$

- Soft quark

$$H^{ij}(\mathbf{P}) = H(P_{\perp}) \delta^{ij} + \mathbf{0} \times \hat{P}^{ij}$$

$$G^{ij}(\mathbf{\Delta}) = G(\Delta_{\perp}) \delta^{ij} + G_1(\Delta_{\perp}) \hat{\Delta}^{ij}$$

- No $2\hat{P}^{ij} \hat{\Delta}^{ij} = \cos 2\phi$ coefficient in both cases, no angle dependence

[NB: $V^{ij} \equiv V^i V^j / V_{\perp}^2 - \delta^{ij} / 2$]

- Integrate numerically over B , R , \bar{R} (and k_3)
- Well behaved quantity

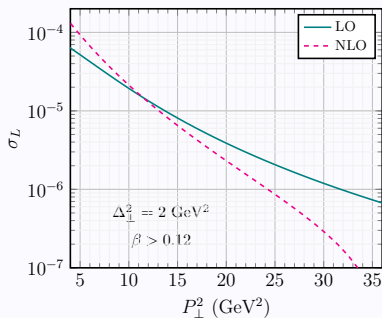
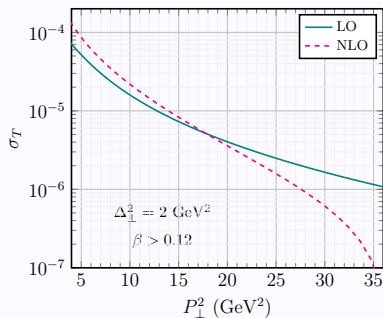
$$\frac{d\sigma_D^{\gamma^* A \rightarrow q\bar{q}gX}}{d\vartheta_1 d\vartheta_2 d^2 P d^2 \Delta} \propto \frac{\alpha_{\text{em}} S_\perp}{N_c} \frac{\alpha_s}{P_\perp^4} \underbrace{f_1(\Delta_\perp/Q_s)}_{\text{dim/less}}$$

$$f_1(\Delta_\perp/Q_s) \sim \begin{cases} \mathcal{O}(1) & \text{for } \Delta_\perp \ll Q_s \\ Q_s^4/\Delta_\perp^4 & \text{for } \Delta_\perp \gg Q_s \end{cases}$$

- Parametrically** larger than 2 jets: $\alpha_s(P_\perp)/P_\perp^4$ vs $1/P_\perp^6$

FORWARD DIJET WITH MINIMUM RAPIDITY GAP

- Fix Q^2 and s , so that $Y_{Bj} = \ln 1/x_{Bj} = 6.1$ is fixed
- Require a minimum rapidity gap $Y_{\text{gap}}^{\text{min}} = 4$ (i.e. $x_{\mathbb{P}} \sim 0.02$)
- This implies a Y_{β}^{max} or $\beta_{\text{min}} = 0.12$
- Fix $\vartheta_1 = \vartheta_2 = 1/2$ and $\Delta_{\perp}^2 = 2\text{GeV}^2$
- Then P_{\perp}^2 cannot exceed a max value



- ◉ Diffraction at **hard** momenta in γA collisions in CGC
- ◉ Diffractive **hard** dijet cross sections dominated by 2+1 jets due to scattering near unitarity limit
- ◉ Seed of semi-hard jet either a gluon or a quark
- ◉ For sufficiently large rapidity gaps and/or large nuclei **gluon and quark** DTMDs and DPDFs calculated from “**first principles**”
- ◉ CGC as initial condition for DGLAP