TMD evolution at small x

Edmond Iancu

IPhT, Université Paris-Saclay

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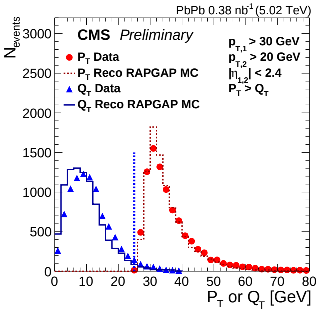
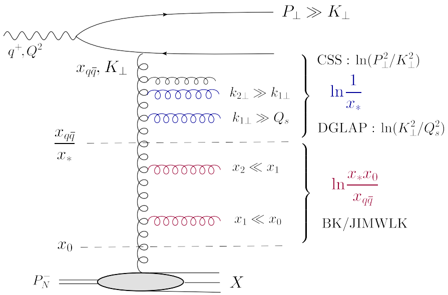
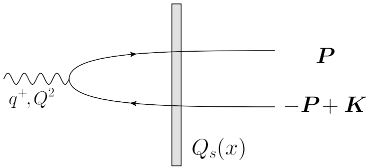
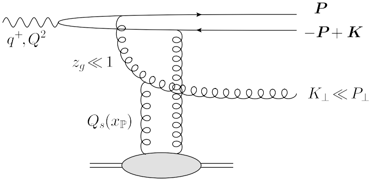




Introduction

- Particle production at high energy/small x and in the presence of widely separate transverse scales (momenta, masses, virtuality)
- Hard dijets in DIS: relative momentum $P_\perp \sim Q \gg$ imbalance $K_\perp \gtrsim Q_s(x)$
- Diffractive 2+1 jets in DIS *(cf. talk by Dionysis T.)*





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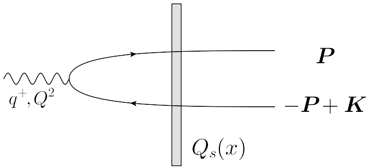

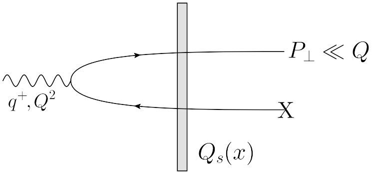

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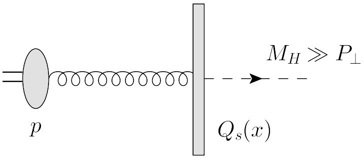
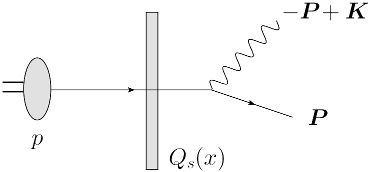




- Semi-inclusive SIDIS: $Q^2 \gg P_\perp^2 \gtrsim Q_s^2(x)$
- Higgs production ($gg \to H$) in pA collisions: $M_H \gg P_\perp \gtrsim Q_s(x)$
- Hard γ-jet pair production in in pA collisions: $P_\perp \gg K_\perp \gtrsim Q_s(x)$

Introduction (2)

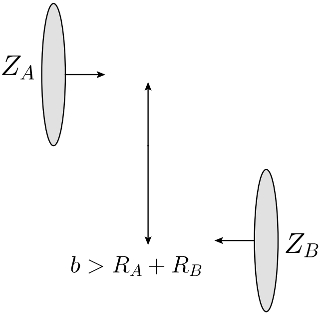
- Radiative corrections enhanced by large logarithms: rapidity and transverse

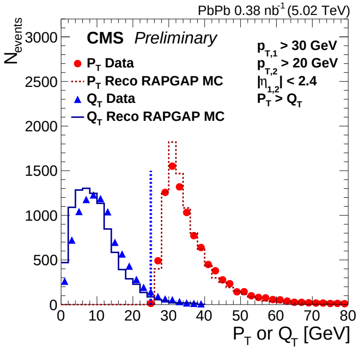
$$\alpha_s \ln\frac{1}{x}\ (\text{small-}x)\,,\quad \alpha_s \ln^2\frac{P_\perp^2}{K_\perp^2}\,,\ \alpha_s \ln\frac{P_\perp^2}{K_\perp^2}\ (\text{Sudakov})\,,\quad \alpha_s \ln\frac{K_\perp^2}{Q_s^2}\ (\text{DGLAP})$$

- ... that we would like to resum to all orders ⇒ evolution equations
- Small-x logs are resummed by the BK/JIMWLK equations (1997-2000)
- Sudakov logs are resummed by the CSS equation (1981-85)
  - within the high-energy factorisation (CGC): Mueller, Xiao, Yuan, 2013
- DGLAP well understood in the collinear factorisation (target parton picture)
- ... but overlooked so far in the CGC theory/colour dipole picture
  - see however the collinear improvement of the BK equation (Beuf 2014, E.I. et al. 2015, Ducloué et al. 2019)

Motivation: Pb+Pb UPCs at the LHC

- All 3 evolutions relevant for dijet production in Pb+Pb UPCs at the LHC
  - x values of order 10^{-3}
  - very hard jets with $P_\perp \geq 30\,\text{GeV}$
  - equally hard dijet imbalance $K_\perp \sim 10\,\text{GeV}$





- Similar results by ATLAS (ATLAS-CONF-2022-02)

Outline

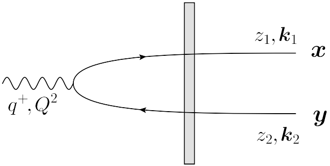
- This work: A unified picture of the 3 types of evolution
  - a succession of 3 evolutions: BK/JIMWLK, DGLAP and CSS
- The framework: TMD factorisation emerging from the CGC effective theory
- The context: the production of hard dijets in DIS at NLO
  - identify all the large transverse logarithms which occur at NLO
  - separate the DGLAP logs and the Sudakov logs from the transverse logs included in the collinearly improved BK equation
- A partonic picture for the target (proton, nucleus) emerging from the evolution of the projectile (dipole)
- A step towards unifying collinear (TMD) and CGC factorisations at small x

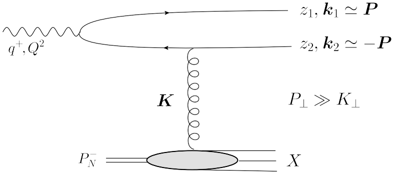
Hard dijet production in DIS

- Two back-to-back jets in the transverse plane: $P_\perp \sim Q \gg K_\perp \gtrsim Q_s$

$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k}_{1\perp} - z_1 \boldsymbol{k}_{2\perp}\,, \qquad \boldsymbol{K}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$$

- Small $q\bar{q}$ dipole: $r = |\boldsymbol{x} - \boldsymbol{y}| \sim 1/P_\perp \ll 1/Q_s \implies$ single scattering





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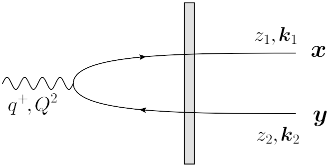
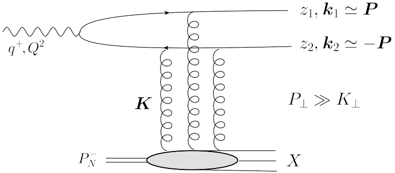




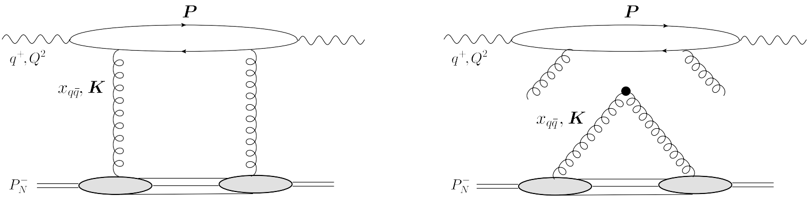
- Multiple scattering still important for the momentum imbalance: $K_\perp \sim Q_s$

scattering amplitude: $$V_{\boldsymbol{x}}V_{\boldsymbol{y}}^\dagger - 1 \simeq r^j\big(V_{\boldsymbol{b}}\partial^j V_{\boldsymbol{b}}^\dagger\big), \quad \boldsymbol{b} = z_1\boldsymbol{x} + z_2\boldsymbol{y}$$

- $r \sim 1/P_\perp$ dependence factorises from the $b \sim 1/K_\perp$ dependence

TMD factorisation for inclusive dijets

$$\frac{\mathrm{d}\sigma^{\gamma^*_{T,L}A\to q\bar{q}A}}{\mathrm{d}z_1\mathrm{d}z_2\mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{K}} = H_{T,L}(z_1, z_2, Q^2, P_\perp^2)\,\mathcal{F}_g(x_{q\bar{q}}, K_\perp^2)$$



- Hard factor encoding the kinematics of the $q\bar{q}$ pair

$$H_T = \alpha_{em}\alpha_s e_f^2\delta(1 - z_1 - z_2)\big(z_1^2 + z_2^2\big)\frac{P_\perp^4 + \bar{Q}^4}{(P_\perp^2 + \bar{Q}^2)^4} \qquad (\bar{Q}^2 = z_1 z_2 Q^2)$$

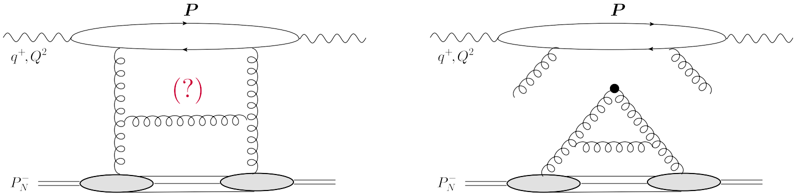
- Weizsäcker-Williams gluon TMD: unintegrated gluon distribution

$$\mathcal{F}_g(x, K_\perp^2) = \int_{\boldsymbol{b},\bar{\boldsymbol{b}}} \frac{\mathrm{e}^{-i\boldsymbol{K}\cdot(\boldsymbol{b}-\bar{\boldsymbol{b}})}}{(2\pi)^4}\,\frac{-2}{\alpha_s}\left\langle \mathrm{Tr}\Big[(\partial^i V_{\boldsymbol{b}})V_{\boldsymbol{b}}^\dagger(\partial^i V_{\bar{\boldsymbol{b}}})V_{\bar{\boldsymbol{b}}}^\dagger\Big]\right\rangle_x$$

Collinear factorisation

- If the dijet imbalance it not measured $\Rightarrow$ integrate over $\boldsymbol{K}$ $\Rightarrow$ gluon PDF

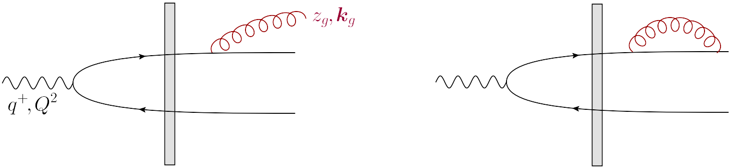
$$\frac{\mathrm{d}\sigma^{\gamma^*_{T,L}A\to q\bar{q}A}}{\mathrm{d}z_1\mathrm{d}z_2\mathrm{d}^2\boldsymbol{P}\mathrm{d}^2} = H_{T,L}(z_1, z_2, Q^2, P_\perp^2)\, xG(x, P_\perp^2)$$



- $xG(x, P_\perp^2)$ is well known to obey DGLAP evolution with increasing $P_\perp^2$
- Standard one-loop calculation in the target picture
  - Bjorken frame $P_N^- \to \infty$, target light-cone gauge $A^- = 0$
- Is this also encoded in the NLO corrections in the dipole picture ?

The Sudakov effect

- $P_\perp \gg K_\perp \Rightarrow$ large phase-space for final state emissions



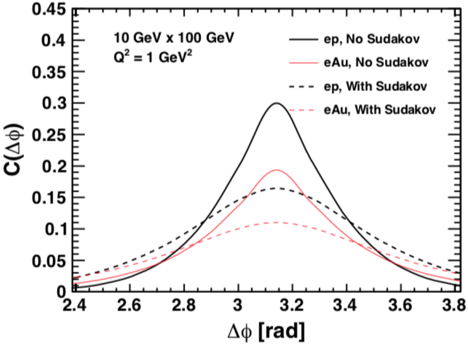
- Double-logarithmic integration: $K_\perp \ll k_{g\perp} \ll P_\perp$ and $z_g \ll 1$
- Virtual corrections dominate: suppression of the cross-section

$$\Delta\mathcal{F}_{\rm Sud}(x, K_\perp^2, P_\perp^2) = -\frac{\alpha_s N_c}{4\pi}\,\ln^2\frac{P_\perp^2}{K_\perp^2}\,\mathcal{F}_g(x, K_\perp^2)\,.$$

- The one-loop result exponentiates: $\mathrm{e}^{-\Delta\mathcal{F}_{\rm Sud}}$
- Strong effect: it washes out the back-to-back correlation

Azimuthal correlations in inclusive dijets

- $P_\perp \gg K_\perp$: the final jets are nearly back to back in the transverse plane
  - azimuthal distribution shows a pronounced peak at $\Delta\phi = \pi$
- Additional broadening due to final-state radiation: Sudakov effect

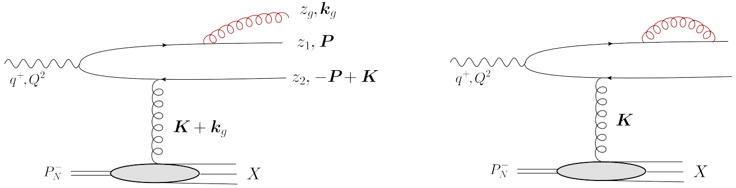


(Zheng, Aschenauer, Lee, and Xiao, arXiv:1403.2413)

- The effects of saturation are essentially washed out ☹

Final state emissions

- The final state emissions of soft gluons factorise $\Rightarrow \Delta\mathcal{F}_{\rm Sud}$



- Direct emissions by the quark: real & virtual

$$\Delta\mathcal{F}_{\rm Sud}^{qq} = \frac{\alpha_s C_F}{\pi^2}\int \mathrm{d}^2\boldsymbol{k}_g \int_{k_{g\perp}^2/P_\perp^2}^{1} \frac{\mathrm{d}z_g}{z_g}\,\frac{\mathcal{F}_g(x,\boldsymbol{K}+\boldsymbol{k}_g)-\mathcal{F}_g(x,\boldsymbol{K})}{\left(\boldsymbol{k}_g-\frac{z_g}{z_1}\boldsymbol{P}\right)^2}$$

- Lower limit on z_g: the boundary with the high energy evolution
- BK/JIMWLK: very soft gluon emissions which occur close the collision:

$$\tau_g \simeq \frac{2z_g q^+}{k_{g\perp}^2} \ll \tau_\gamma \simeq \frac{2q^+}{Q^2} \Longrightarrow z_g \ll \frac{k_{g\perp}^2}{Q^2}$$

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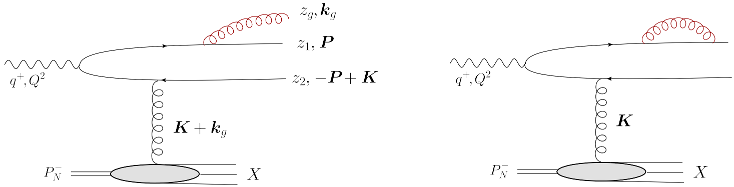

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- Collinear singularity when $\boldsymbol{k}_g/z_g = \boldsymbol{P}/z_1$ or $\theta_g = \theta_1$
- Removed via the renormalisation of the quark fragmentation function
- DGLAP evolution of quark fragmentation into hadrons

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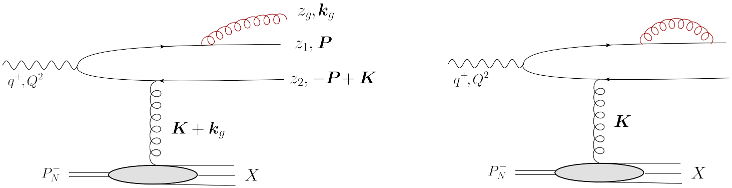

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- However, we are interested in the production of jets $\Rightarrow$ large angles

$$\theta_g \sim \frac{k_{g\perp}}{z_g q^+} > \theta_1 \sim \frac{P_\perp}{z_1 q^+} \;\Rightarrow\; z_g < \frac{k_{g\perp}}{P_\perp} \;\Rightarrow\; \text{logarithmic phase-space} \;\int \frac{{\rm d}^2\boldsymbol{k}_g}{\boldsymbol{k}_g^2}$$

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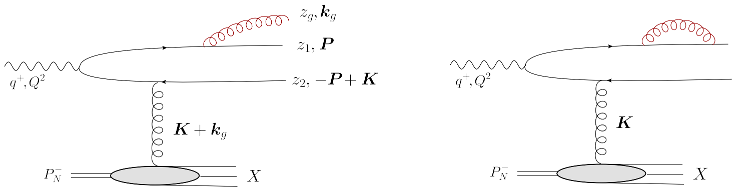

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- Emissions with low $k_{g\perp}\ll K_\perp$ cancel between real and virtual
  - unobservable: cannot affect the final state
- Real gluons with $k_{g\perp}\gg K_\perp$ are suppressed: $\mathcal{F}_g(\boldsymbol{K}+\boldsymbol{k}_g)\simeq\mathcal{F}_g(\boldsymbol{k}_g)\sim 1/k^2_{g\perp}$

The “virtual” Sudakov

- The “standard” Sudakov, as coming from virtual corrections:

$$\Delta\mathcal{F}^{qq}_{\rm Sud} = -\frac{\alpha_s C_F}{\pi}\,\mathcal{F}_g(x, K_\perp^2)\int_{K_\perp^2}^{P_\perp^2}\frac{{\rm d}k_{g\perp}^2}{k_{g\perp}^2}\int_{k_{g\perp}^2/P_\perp^2}^{k_{g\perp}/P_\perp}\frac{{\rm d}z_g}{z_g}$$

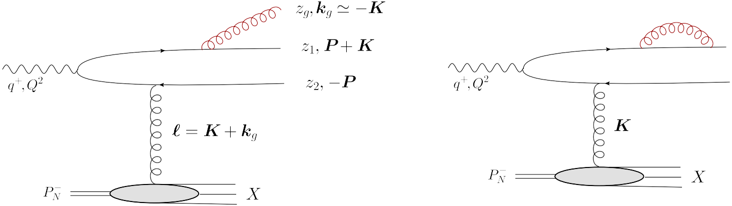
- Similar result for direct emissions by the antiquark
- Large angle $\Rightarrow$ interference between emissions by the q and by the $\bar{q}$
- Leading-twist interference effects are suppressed at large N_c
- Overall color factor: $C_F + C_F + 1/N_c = N_c$

$$\Delta\mathcal{F}^{\rm V}_{\rm Sud}(x, K_\perp^2, P_\perp^2) = -\frac{\alpha_s N_c}{4\pi}\,\ln^2\frac{P_\perp^2}{K_\perp^2}\,\mathcal{F}_g(x, K_\perp^2)\,.$$

  - a large angle emission sees the overall colour charge: a gluon

The "real" Sudakov

- To DLA, there is also a "real" Sudakov effect (real gluon emission)
- The dijet imbalance can also be caused by the gluon recoil: $\boldsymbol{k}_g \simeq -\boldsymbol{K}$



$$\Delta\mathcal{F}_{\rm Sud}^{\rm R} = \frac{\alpha_s N_c}{\pi^2}\int \frac{{\rm d}^2\boldsymbol{k}_g}{\boldsymbol{k}_g^2}\int_{k_{g\perp}^2/P_\perp^2}^{k_{g\perp}/P_\perp} \frac{{\rm d}z_g}{z_g}\,\mathcal{F}_g(x, \boldsymbol{K}+\boldsymbol{k}_g)$$

- Replace $\boldsymbol{k}_g = \boldsymbol{\ell} - \boldsymbol{K}$ and use $\ell_\perp \equiv |\boldsymbol{K}+\boldsymbol{k}_g| \ll K_\perp$.
- Implicit assumption: $Q_s \ll K_\perp \ll P_\perp$

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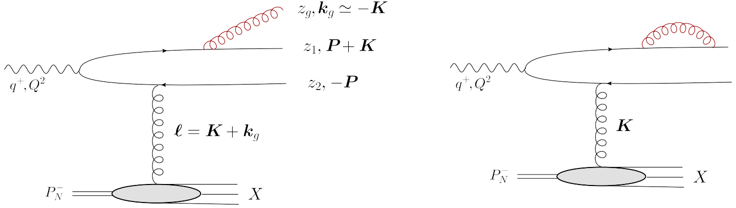


$$\Delta\mathcal{F}^{\rm R}_{\rm Sud}(x, K_\perp^2, P_\perp^2) = \frac{\alpha_s N_c}{\pi^2}\frac{1}{K_\perp^2}\frac{1}{2}\ln\frac{P_\perp^2}{K_\perp^2}\int_{\Lambda^2}^{K_\perp^2} {\rm d}^2\boldsymbol{\ell}\,\mathcal{F}_g(x, \ell_\perp^2)$$

- The integral over $\boldsymbol{\ell}$ yields the gluon PDF at the scale $K_\perp^2$

$$\int_{\Lambda^2}^{K_\perp^2} {\rm d}^2\boldsymbol{\ell}\,\mathcal{F}_g(x, \ell_\perp^2) = xG(x, K_\perp^2) \sim \ln\frac{K_\perp^2}{Q_s^2}$$

More on the Sudakov dynamics

$$\mathcal{F}_g(x, K_\perp^2, P_\perp^2) = \mathcal{F}_g^{(0)}(x, K_\perp^2) + \Delta\mathcal{F}_{\rm Sud}^{\rm R} + \Delta\mathcal{F}_{\rm Sud}^{\rm V}$$

- $P_\perp$ dependence ("resolution scale") introduced by the loop corrections
- The gluon PDF in the presence of the resolution scale:

$$xG(x, P_\perp^2) = \int_{\Lambda^2}^{P_\perp^2} {\rm d}^2\boldsymbol{K}\, \mathcal{F}_g(x, K_\perp^2, P_\perp^2)$$

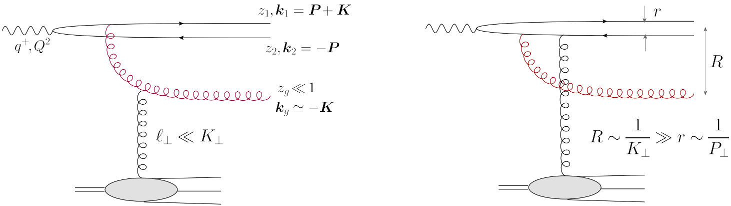
- After integrating over $K_\perp$, "real" and "virtual" Sudakov mutually cancel:

$$\int_{\Lambda^2}^{P_\perp^2} {\rm d}^2\boldsymbol{K} \left(\Delta\mathcal{F}_{\rm Sud}^{\rm R} + \Delta\mathcal{F}_{\rm Sud}^{\rm V}\right) = 0$$

  - final-state emissions irrelevant if the imbalance $K_\perp$ not measured
- Sudakov dynamics does not change the total number of gluons
- In order to uncover DGLAP dynamics, we need to go beyond DLA

Real gluon emissions in the initial state

- No collinear singularity $\Rightarrow$ just one logarithm: $xG(x, K_\perp^2)$
- Special NLO: leading order in 2 power expansions: $K_\perp/P_\perp$ and $\ell_\perp/K_\perp$
- Soft gluon $\Rightarrow$ effective gluon-gluon dipole *(cf. talk by Dionysis T.)*



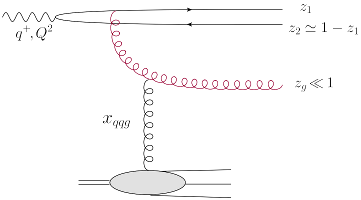
- TMD factorisation: same hard factor $H(z_1, z_2, Q^2, P_\perp^2)$ as at leading order
- Same WW colour operator, but evaluated at the lower scale $\ell_\perp$:

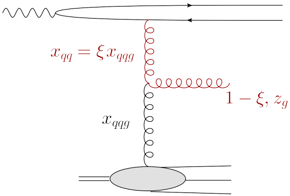
$$U_{\boldsymbol{z}}^{ac}\left(V_{\boldsymbol{x}} t^c V_{\boldsymbol{y}}^\dagger\right) - t^a \simeq -R^j \left(U_{\boldsymbol{z}} \partial^j U_{\boldsymbol{z}}^\dagger\right)^{ac} t^c.$$

- Renormalisation group... as expected for one step in DGLAP

From projectile to target rapidity

- DGLAP must refer to the gluon TMD: a parton distribution in the target
  - the gluon emission should occur in the target wavefunction
  - splitting function $P_{gg}(\xi)$ with ξ a fraction of k^-

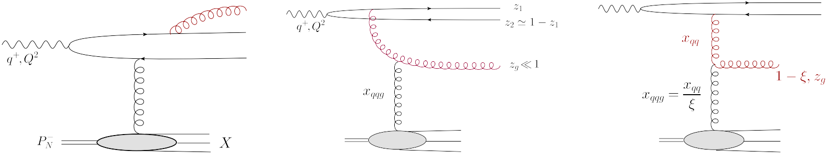




- Exchange z_g for ξ: transfer the gluon from the dipole to the target
  - always possible for a soft gluon: $z_g \ll 1$
  - s-channel gluon must be on-shell: $2(z_g q^+)(1-\xi)P_N^- = K_\perp^2$
- Integration limits on z_g transmit to corresponding limits on ξ

The DGLAP splitting function

$$\Delta\mathcal{F}_{\rm R} = \frac{\alpha_s}{2\pi^2 K_\perp^2}\int_{x_\star}^{1-K_\perp/P_\perp} {\rm d}\xi\, P_{gg}\left(\xi\right)\, \frac{x}{\xi} G\left(\frac{x}{\xi}, K_\perp^2\right)$$



$$P_{gg}(\xi) = 2N_c\,\frac{1+(1-\xi)^2(1+\xi^2)-(1-\xi^2)}{\xi(1-\xi)}$$

- Final-state (singular at $\xi \to 1$) + initial-state + interference
- Non-local in x: $x \equiv x_{q\bar q}$ and $\frac{x}{\xi} = x_{q\bar q g}$
- Upper limit $1 - K_\perp/P_\perp$ on ξ comes from $z_g \lesssim K_\perp/P_\perp$
- Lower limit $x_* < 1$ on ξ comes from $z_g \gtrsim x_*(K_\perp/P_\perp)^2$: $\alpha_s \ln\frac{1}{x_*} \ll 1$

Emergent DGLAP & CSS evolutions

$$\Delta\mathcal{F}_{\rm R} = \frac{\alpha_s}{2\pi^2 K_\perp^2}\int_{x_\star}^{1} {\rm d}\xi\, P_{gg}^{(+)}(\xi)\, \frac{x}{\xi} G\left(\frac{x}{\xi}, K_\perp^2\right) + \Delta\mathcal{F}_{\rm Sud}^{\rm R}$$

- "Plus" prescription for the pole at $\xi \to 1$:

$$P_{gg}(\xi) = 2N_c\,\frac{1+(1-\xi)^2(1+\xi^2)-(1-\xi^2)}{\xi(1-\xi)_+}$$

- $P_\perp$–dependence only in the Sudakov piece (final-state emission)

$$\Delta\mathcal{F}_{\rm Sud}^{\rm R}(x, K_\perp^2, P_\perp^2) = \frac{\alpha_s N_c}{\pi^2}\,\frac{1}{K_\perp^2}\,\frac{1}{2}\ln\frac{P_\perp^2}{K_\perp^2}\, xG(x, K_\perp^2)$$

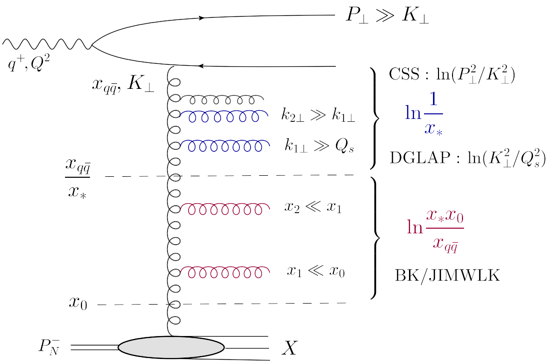
- The corresponding virtual correction: Sudakov + running coupling

$$\Delta\mathcal{F}_{\rm V} = -\frac{\alpha_s N_c}{\pi}\left(\frac{1}{4}\ln^2\frac{P_\perp^2}{K_\perp^2} - \beta_0 \ln\frac{P_\perp^2}{K_\perp^2}\right)\mathcal{F}_g^{(0)}(x, K_\perp^2)$$

- $\Delta\mathcal{F}_{\rm Sud}^{\rm R} + \Delta\mathcal{F}_{\rm V}$: one step in CSS evolution (with increasing $P_\perp^2$)

Three successive evolutions

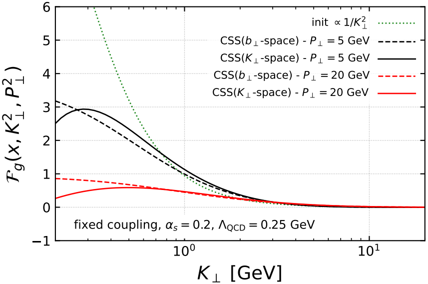
- JIMWLK for the WW TMD from $x_0 \sim 10^{-2}$ down to $x_{q\bar{q}}$: $\mathcal{F}_g^{(0)}(x_{q\bar{q}}, K_\perp^2)$
- DGLAP evolution from Q_s^2 up to $K_\perp^2$ using $\mathcal{F}_g^{(0)}$ as a source term
- CSS evolution from $K_\perp^2$ up to $P_\perp^2$ with initial condition $\mathcal{F}_g(x, K_\perp^2)$



CSS evolution: a rate equation

$$\frac{\partial \mathcal{F}_g(x,K_\perp^2,P_\perp^2)}{\partial \ln P_\perp^2} = \frac{\alpha_s N_c}{2\pi}\left\{\frac{1}{K_\perp^2}\int_{\Lambda^2}^{K_\perp^2} \mathrm{d}\ell_\perp^2\, \mathcal{F}_g(x,\ell_\perp^2,P_\perp^2) - \int_{K_\perp^2}^{P_\perp^2} \frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\mathcal{F}_g(x,K_\perp^2,P_\perp^2)\right\}$$

- With increasing $P_\perp$, one increases the phase-space for soft gluon emissions
- The number of gluons in the bin $K_\perp$ can
  - increase via the splitting of gluons with $\ell_\perp \ll K_\perp$
  - decrease via the splitting into gluons with $K_\perp \ll \ell_\perp \ll P_\perp$
  - the total number of gluons with $K_\perp < P_\perp$ remains unchanged



- Solution with initial condition

$$\mathcal{F}_g^{(0)}(x,K_\perp^2) = \frac{1}{K_\perp^2}$$

- 2 values of $P_\perp$: 5 and 20 GeV

CSS evolution: a rate equation

$$\frac{\partial \mathcal{F}_g(x, K_\perp^2, P_\perp^2)}{\partial \ln P_\perp^2} = \frac{\alpha_s N_c}{2\pi} \left\{ \frac{1}{K_\perp^2} \int_{\Lambda^2}^{K_\perp^2} \mathrm{d}\ell_\perp^2 \, \mathcal{F}_g(x, \ell_\perp^2, P_\perp^2) - \int_{K_\perp^2}^{P_\perp^2} \frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2} \mathcal{F}_g(x, K_\perp^2, P_\perp^2) \right\}$$

- With increasing $P_\perp$, one increases the phase-space for soft gluon emissions
- The number of gluons in the bin $K_\perp$ can
  - increase via the splitting of gluons with $\ell_\perp \ll K_\perp$
  - decrease via the splitting into gluons with $K_\perp \ll \ell_\perp \ll P_\perp$
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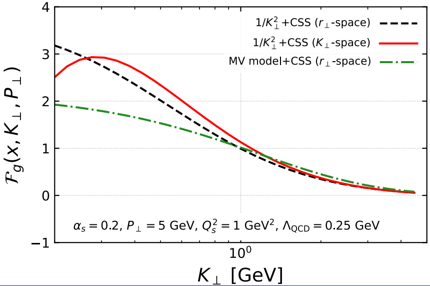

- Adding saturation (MV model)

$$\tilde{\mathcal{F}}_g^{(0)}(x, R_\perp^2) = \frac{T_{q\bar{q}}(R)}{R^2}$$

- $P_\perp = 5$ GeV (green curve)

Conclusions & Open questions

- Particle production at small x $(x < 10^{-2})$ and large transverse momenta $(P_\perp \gg Q_s(x))$ requires several types of evolutions
- TMD factorisation within the CGC theory seems to be the right approach
- A strategy to combine BK/JIMWLK, DGLAP and CSS evolutions in a unified framework
- Not a unique, super-equation, but rather a succession of evolutions
- Numerical implementation of the whole scheme is under the way
- Demonstrated on the exemple of hard dijet production in γA
- Similar results for many other processes in γA and pA collisions
- E.g. SIDIS involves the quark TMD and its evolutions

Back-up: More about DGLAP & CSS

- $\mathcal{F}_g^{(0)}(x, K_\perp^2)$: the gluon WW TMD without DGLAP and CSS
- DGLAP evolution from Q_s^2 up to $K_\perp^2$ using $\mathcal{F}_g^{(0)}$ as a source term:

$$\frac{\partial xG(x,Q^2)}{\partial \ln Q^2} = \pi Q^2 \mathcal{F}_g^{(0)}(x,Q^2) + \frac{\alpha_s}{2\pi}\int_{x_*}^{1} \mathrm{d}\xi\, \mathcal{P}_{gg}(\xi)\, \frac{x}{\xi} G\left(\frac{x}{\xi}, Q^2\right),$$

- The gluon TMD including JIMWLK and DGLAP:

$$\mathcal{F}_g(x, K_\perp^2) = \frac{1}{\pi\alpha_s(K_\perp^2)} \frac{\partial}{\partial K_\perp^2}\left[\alpha_s(K_\perp^2) xG(x, K_\perp^2)\right]$$

- CSS evolution from $K_\perp^2$ up to $P_\perp^2$ with initial condition $\mathcal{F}_g(x, K_\perp^2)$:

$$\frac{\partial \mathcal{F}_g(x,K_\perp^2,P_\perp^2)}{\partial \ln P_\perp^2} = \frac{\alpha_s N_c}{2\pi}\left\{\frac{1}{K_\perp^2}\int_{\Lambda^2}^{K_\perp^2} \mathrm{d}\ell_\perp^2\, \mathcal{F}_g(x,\ell_\perp^2,P_\perp^2) - \int_{K_\perp^2}^{P_\perp^2} \frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\mathcal{F}_g(x,K_\perp^2,P_\perp^2)\right\}$$

$$+\beta_0 \frac{\alpha_s N_c}{\pi}\mathcal{F}_g(x,K_\perp^2,P_\perp^2)$$

Back-up: CSS equation in coordinate space

- The CSS equation is usually written/solved in transverse coordinate space
- DGLAP evolution from Q_s^2 up to $K_\perp^2$ using $\mathcal{F}_g^{(0)}$ as a source term:

$$\tilde{\mathcal{F}}_g(x, R^2, Q^2) \equiv \int \frac{\mathrm{d}^2\boldsymbol{K}}{(2\pi)^2}\,\mathrm{e}^{-i\boldsymbol{K}\cdot\boldsymbol{R}}\,\mathcal{F}_g(x, K_\perp^2, Q^2)$$

- The respective equation has no real piece ! Smearing of the $K_\perp$–distribution:

$$\frac{\partial\tilde{\mathcal{F}}_g(x, R^2, Q^2)}{\partial \ln Q^2} = \frac{N_c}{\pi}\left\{-\frac{1}{2}\int_{1/R^2}^{Q^2}\frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\,\alpha_s(\ell_\perp^2) + \beta_0\alpha_s(Q^2)\right\}\tilde{\mathcal{F}}_g$$

- Local in both Q^2 and R^2 $\Rightarrow$ trivial to solve

$$\tilde{\mathcal{F}}_g(x, R^2, Q^2) = \tilde{\mathcal{F}}_0(x, R^2)\left\{-\frac{N_c}{\pi}\int_{1/R^2}^{Q^2}\frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\,\alpha_s(\ell_\perp^2)\left[\frac{1}{2}\ln\frac{Q^2}{\ell_\perp^2} - \beta_0\right]\right\}$$

- ... but the Fourier transform back to momentum space can be tricky !