NLO calculations in the dipole picture

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Diffraction and gluon saturation at the LHC and the EIC
ECT*, Trento, June 2024

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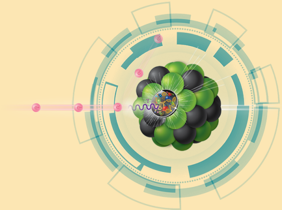
Outline

Outline of this talk

- Dipole + eikonal scattering picture of DIS, power counting
- Factorization and "negativity problem" $\Longrightarrow$ not new, discussion?
- Diffractive structure function $F_{2,L}^{D}$ at NLO
  - Based on G. Beuf, T. L., H. Mäntysaari, R. Paatelainen, J. Penttala 2401.17251
  - Partial results G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari, arXiv:2206.13161
  - Types of contributions
  - Technical remarks about energy denominators

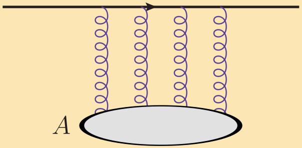
Process of interest

DIS in the high energy saturation regime



High energy collisions as eikonal scattering

Eikonal scattering off target of glue

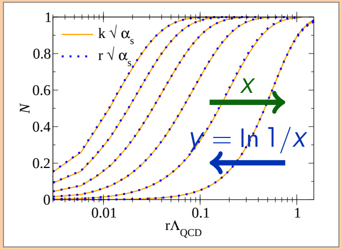


How to measure small-x glue?

- Dilute probe through target color field
- At high energy interaction is **eikonal**, $\perp$ coordinate conserved in scattering

- Amplitude for quark: **Wilson line**

$$\mathbb{P}\exp\left\{-ig\int^{x^+}dy^+A^-(y^+,x^-,\mathbf{x})\right\}\underset{x^+\to\infty}{\approx}V(\mathbf{x})\in SU(N_c)$$



- Amplitude for color dipole

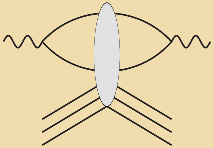
$$\mathcal{N}(r=|\mathbf{x}-\mathbf{y}|)=1-\left\langle\frac{1}{N_c}\operatorname{tr}V^\dagger(\mathbf{x})V(\mathbf{y})\right\rangle$$

- $r=0$: color transparency, $r\gg 1/Q_s$: saturation , nonperturbative!

Dipole picture of DIS

Limit of small x, i.e. high γ^*-target energy. γ^* rightmoving

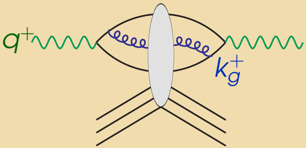
Leading order



- $\gamma^* \to q\bar{q}$ in vacuum
- $q\bar{q}$ interacts eikonally with target

"Dipole model": Nikolaev, Zakharov 1991 ; Mueller

Leading Log: add **soft** gluon



$\Longrightarrow$ Large log $\int_{x_{Bj}} dk_g^+/k_g^+ \sim \ln \frac{1}{x_{Bj}}$

BK-evolution of target

Balitsky 1995, Kovchegov 1999

- Need to **subtract** leading log from cross section:

$$\sigma_{NLO} = \int_{\sim x_{Bj}}^{\sim 1} \frac{dz}{z} \Big[\overbrace{\sigma(z) - \sigma(z=0)}^{\sigma_{\text{sub}}} + \overbrace{\sigma(z=0)}^{\text{absorb in BK}} \Big] \quad z = \frac{k_g^+}{q^+}$$

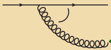
- NLO: The same gluon with full kinematics

Factorization of high energy divergence

Factorization and negativity problem

NLO single inclusive $p + A \to h + X$: first calculation Chirilli, Xiao, Yuan 2011 = "CXY"

$$\sigma_{NLO} = \int_{\sim x} \frac{\mathrm{d}z}{z} \left[\overbrace{\sigma(z) - \sigma(z \approx 0)}^{\sigma_{\mathrm{sub}}} + \overbrace{\sigma(z \approx 0)}^{\text{BK evolution up to} \sim \ln 1/x} \right]$$



$\leftarrow k_g^+ \sim z$

Factorize as CXY $\Longrightarrow$ "negative cross section problem" $\sigma_{\mathrm{sub}} < 0$ at high p_T

Many papers written, lots of terminological confusion! Some personal comments:

- Important: calculation is in k^+-fraction, BK should be in target k^-
- High p_T or high Q^2 have $\ln k^- \neq -\ln k^+$ $\Longrightarrow$ large transverse log
- Cannot mumble "rapidity" and wave hands: define rapidity! Is it $\ln k^+$? $\ln k^-$?
- For pA it is very useful to do finite N_c to understand structure
  - Terms in C_F (associated with DGLAP of pdf & FF in pA) are separate, unproblematic
  - Terms in N_c (associated with BK) give the large negative contribution

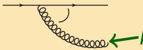
    I believe first pointed out by Z. Kang et al 1403.5221

Solutions to the negativity problem

I'm aware of two classes of solutions that are implemeted in numerics

- For pA: Ioffe time/kinematical constraint
  $\Longrightarrow$ additional correction terms $\ln k^- - (-\ln k^+) \approx \ln {k_T}^2$
  $\Longrightarrow$ at least push problem to higher p_T Xiao et al, starting from 1505.05183
- All-order solution Iancu et al 1608.05293 , successfully used in Mäntysaari, Tawabutr 2310.06640 : write in explicitly positive form (Y=BK evolution rapidity)

$$0 < \sigma_{LO+NLO} = \overbrace{\sigma_{LO}}^{\text{bare}} + \alpha_s \int_{\sim x} \frac{dz}{z} K(z) \overbrace{\mathcal{N}(Y(z))}^{\text{Wilson line op.}}$$



$k_g^+ \sim z$

$$= \overbrace{\sigma_{LO} + \alpha_s \int_{\sim x} \frac{dz}{z} K(z=0)\mathcal{N}(Y(z))}^{\text{bare LO + } Y\text{-evolution = LL}} + \overbrace{\alpha_s \int_{\sim x} \frac{dz}{z} [K(z) - K(z=0)] \underbrace{\mathcal{N}(Y(z))}_{\text{CXY}\to\mathcal{N}(Y(z\to x_{Bj}))}}^{\text{pure NLO}}$$

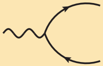
+ Add & subtract exactly same term.
- Not in spirit of k_T-factorization: NLO term integrates over BK evolution history
- Subtracted form sensitive to running α_s etc.

- Threshold resummation??? (Color factor? Inclusive DIS?)

DIS at NLO: subtraction of BK

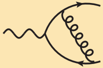
Beuf 2016, 2017, H. Hänninen, T.L., Paatelainen 2017

Evaluate cross section as $\sigma^{\text{NLO}}_{L,T} = \sigma^{\text{LO}}_{L,T} + \sigma^{\text{dip}}_{L,T} + \sigma^{qg}_{L,T,\text{sub.}}$



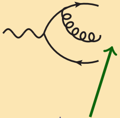
$\Rightarrow$

$$\sigma^{\text{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} |\psi^{\text{LO}}_{\gamma^*\to q\bar{q}}(z_1,\mathbf{x}_0,\mathbf{x}_1)|^2 \mathcal{N}_{01}(x_{Bj})$$



$-\,* \Rightarrow$

$$\sigma^{\text{dip}} \sim \alpha_s C_F \int_{\mathbf{x}_0,\mathbf{x}_1,z_1} \left|\psi^{\text{LO}}_{\gamma^*\to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



$+\,* \Rightarrow$

- LL

$k_g^+ \sim z_2$

$$\sigma^{qg}_{\text{sub.}} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(Y(z_2)) - \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(Y(z_2)) \Big].$$

* UV-divergence

DIS at NLO: subtraction of BK

Beuf 2016, 2017, H. Hänninen, T.L., Paatelainen 2017

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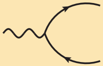

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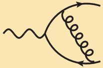
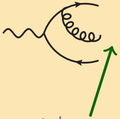


$- * \Rightarrow$

$$\sigma^{\mathrm{dip}} \sim \alpha_s C_F \int_{\mathbf{x}_0,\mathbf{x}_1,z_1} \left|\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



$+ * \Rightarrow$

- LL

$k_g^+ \sim z_2$

$$\sigma^{qg}_{\mathrm{sub.}} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(Y(z_2)) - \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(Y(z_2)) \Big].$$

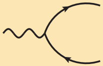
* UV-divergence

LL: subtract leading log, already in BK-evolved $\mathcal{N}$

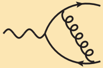
DIS at NLO: subtraction of BK

Beuf 2016, 2017, H. Hänninen, T.L., Paatelainen 2017

Evaluate cross section as $\sigma^{\text{NLO}}_{L,T} = \sigma^{\text{LO}}_{L,T} + \sigma^{\text{dip}}_{L,T} + \sigma^{qg}_{L,T,\text{sub.}}$



$\Rightarrow \quad \sigma^{\text{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} |\psi^{\text{LO}}_{\gamma^*\to q\bar{q}}(z_1,\mathbf{x}_0,\mathbf{x}_1)|^2 \mathcal{N}_{01}(x_{Bj})$



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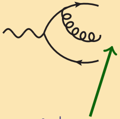


$+ * \Rightarrow$

- LL

$k_g^+ \sim z_2$

$$\sigma^{qg}_{\text{sub.}} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(Y(z_2)) - \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0,\{\mathbf{x}_i\})\right|^2 \mathcal{N}_{012}(Y(z_2)) \Big].$$

* UV-divergence   LL: subtract leading log, already in BK-evolved $\mathcal{N}$

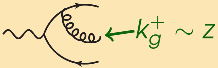
- Parametrically $Y(z_2) \sim x_{Bj}$, but $Y(z_2) \sim \ln(1/z_2)$ essential!

  ("k_T-factorization" with fixed rapidity scale is unstable @ NLO. Analogous to $p + A \to h + X$)

Inclusive DIS: same negativity problem

Ducloué, Hänninen, T.L., Zhu 2017

$$\sigma^{qg,\mathrm{sub.}}_{L,T} \sim \alpha_s C_\mathrm{F} \int_{z_q,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \int_{x_{Bj}/x_0}^{1} \frac{\mathrm{d}z}{z} \left[\mathcal{K}^{\mathrm{NLO}}_{L,T}(z, Y(z)) - \mathcal{K}^{\mathrm{NLO}}_{L,T}(0, Y(z)) \right].$$

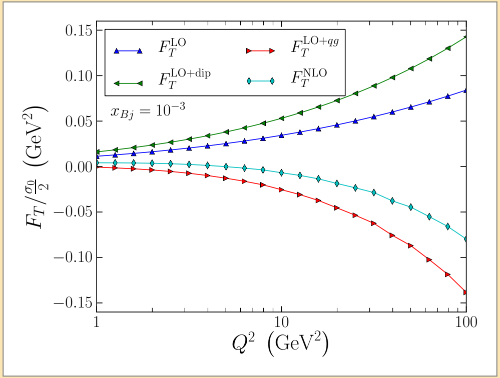


- Target fields evolved to $Y(z)$:
  - $Y(z) = \ln(1/x_{Bj})$: unstable

Like single inclusive!

Question

Your favorite solution to pA negativity, does it solve the DIS negativity problem?

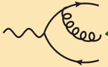


$Y(z) = \ln(1/x_{Bj})$ ("CXY")

Inclusive DIS: same negativity problem

Ducloué, Hänninen, T.L., Zhu 2017

$$\sigma_{L,T}^{qg,\text{sub.}} \sim \alpha_s C_F \int_{z_q,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \int_{x_{Bj}/x_0}^{1} \frac{dz}{z}\left[\mathcal{K}_{L,T}^{\text{NLO}}(z, Y(z)) - \mathcal{K}_{L,T}^{\text{NLO}}(0, Y(z))\right].$$

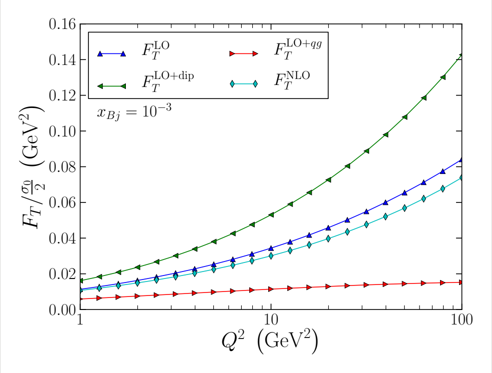


- Target fields evolved to $Y(z)$:
  - $Y(z) = \ln(1/x_{Bj})$: unstable
  - $Y(z) = \ln(x_{Bj}/z)$ OK

  Like single inclusive!

Question

Your favorite solution to pA negativity, does it solve the DIS negativity problem?

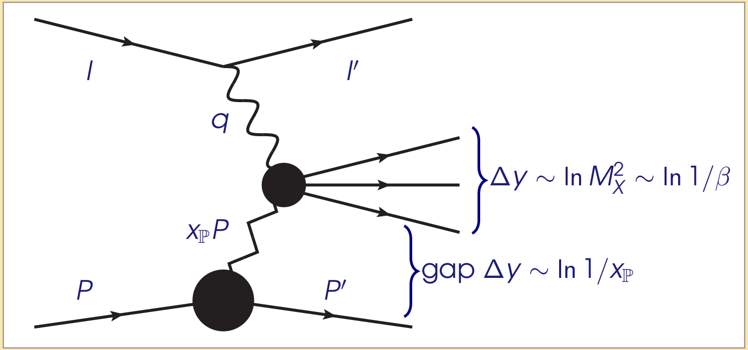


$Y(z) = \ln(z/x_{Bj})$

Diffractive DIS

Inclusive diffraction, kinematics

$\gamma^* + A \to X + A$, differential in M_X



- Momentum transfer $t = (P - P')^2$
- Gap size $x_{\mathbb{P}}$, target evolution rapidity $\sim \ln 1/x_{\mathbb{P}}$
- Diffractive system mass M_X^2, $\beta = Q^2/(Q^2 + M_X^2)$
- Virtuality Q^2
- Lower $x_{\mathbb{P}}$ than dijets (e.g. at EIC)

$$x_{Bj} = x_{\mathbb{P}} \beta$$

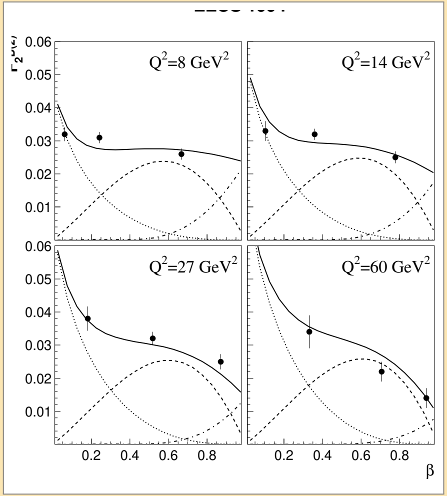
This talk: $x_{\mathbb{P}}$ small, β not.

Dependence on β, i.e. M_X

${M_X}^2$ = photon remnants.

Essential regimes:

- Large $\beta \to 1$ — small M_X:
  longitudinal $\gamma^* \to q\bar{q}$
- Medium $\beta \sim 0.5$ — ${M_X}^2 \sim Q^2$:
  transverse $\gamma^* \to q\bar{q}$
- Small $\beta \ll 1$ — large ${M_X}^2$:
  higher Fock states ($q\bar{q}g$ etc.)



LO $q\bar{q}$ and leading ln Q^2 $q\bar{q}g$

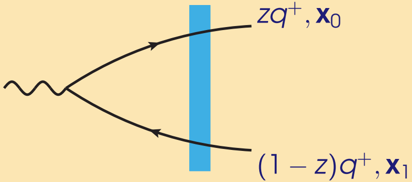
Golec-Biernat& Wüsthoff `hep-ph/9903358`

NLO amplitude for diffractive DIS

Diffractive DIS at leading order

- Earlier: factorized impact parameter dependence
- Full kinematics G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari, arXiv:2206.13161

$$\frac{d\sigma^{D}_{\lambda,q\bar{q}}}{dM_X^2\, d|t|} = \frac{N_c}{4\pi}\int_0^1 dz \int_{\mathbf{x}_0\mathbf{x}_1\bar{\mathbf{x}}_0\bar{\mathbf{x}}_1} \mathcal{I}^{(2)}_{\mathbf{\Delta}}\mathcal{I}^{(2)}_{M_X}$$
$$\times \sum_{f,h_0,h_1} \left(\widetilde{\psi}_{\gamma^*_\lambda\to q_{\bar{0}}\bar{q}_{\bar{1}}}\right)^\dagger \left(\widetilde{\psi}_{\gamma^*_\lambda\to q_0\bar{q}_1}\right) \boxed{\left[S^\dagger_{\bar{0}\bar{1}}-1\right]\left[S_{01}-1\right]}$$



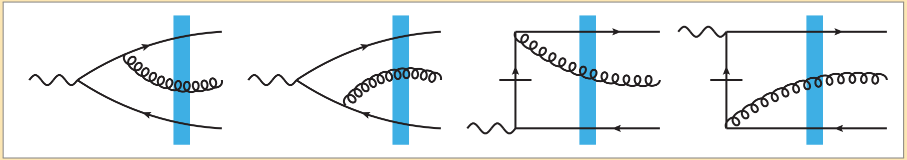
(Blue: shockwave target)

- $q\bar{q}$ crossing shockwave: dipole S_{01}
- "Transfer functions:" relate coordinates at shockwave to:
  - Momentum transfer $t = -\mathbf{\Delta}^2$ $\mathcal{I}^{(2)}_{\mathbf{\Delta}} = \frac{1}{4\pi}J_0\left(\sqrt{|t|}\,\|z\mathbf{x}_{\bar{0}0} - (1-z)\mathbf{x}_{\bar{1}1}\|\right)$
  - Invariant mass $\mathcal{I}^{(2)}_{M_X} = \frac{1}{4\pi}J_0\left(\sqrt{z(1-z)}M_X\|\bar{\mathbf{r}}-\mathbf{r}\|\right)$

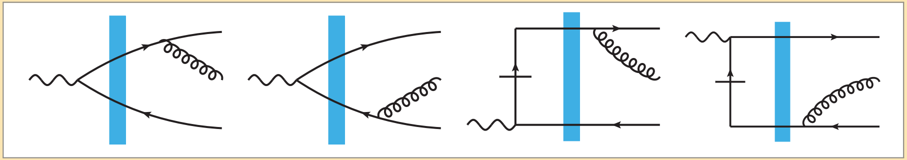
Now to NLO

NLO radiative corrections

- Emission before target

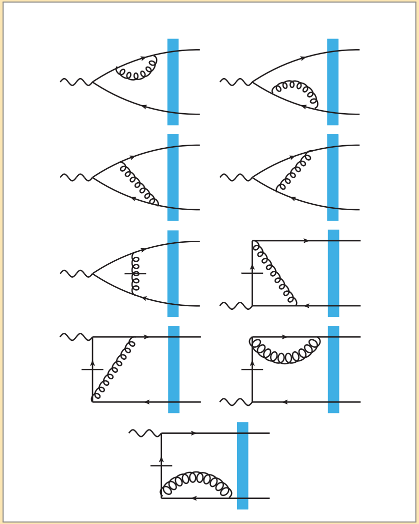


  - Squares already in G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari arXiv:2206.13161
  - Contain leading $\ln Q^2$ contribution
- Emission after target



- Interferences $\Longrightarrow$ simplify with some of the virtual corrections

NLO virtual

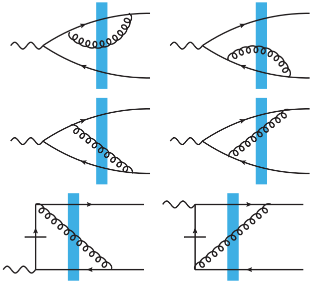


- Vertex corrections:
  known 1-loop $\gamma \to q\bar{q}$ wavefunction

See also Boussarie et al 2014: diffractive jets,

also Caucal et al 2021 inclusive

NLO virtual



- Vertex corrections:
  known 1-loop $\gamma \to q\bar{q}$ wavefunction
- Gluon crosses shockwave, but not the cut:
  - Loop corrections to amplitude,
    tree level wavefunctions
  - 3-point operator of Wilson lines
  - BK/JIMWLK evolution of LO amplitude

See also Boussarie et al 2014: diffractive jets,

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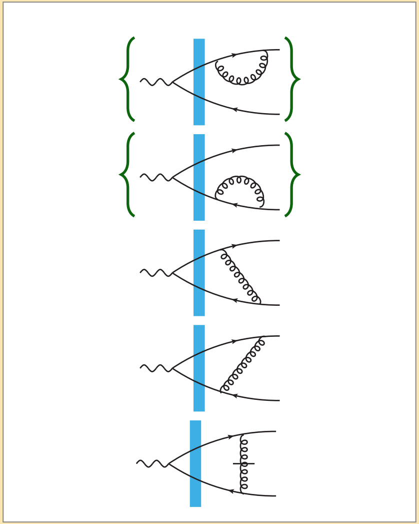
NLO virtual



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    tree level wavefunctions
  - 3-point operator of Wilson lines
  - BK/JIMWLK evolution of LO amplitude
- Final state interactions
  (Propagator corrections $\{\}$ $\to$
  State renormalization, in fact = 0 in dim. reg.)

See also Boussarie et al 2014: diffractive jets,

also Caucal et al 2021 inclusive

NLO cross section

Calculation in 2401.17251 [hep-ph]

We have calculated all these contributions

- Diffractive structure function:
  clean, [perturbative = experimental] final state definition M_X!
  (No fragmentation function, jet definition)
  $\Longrightarrow$ Divergences must cancel
- Explicit expressions will not fit in the slides, but there in 2401.17251 [hep-ph]

Rest of talk: two features of the calculation:

- Divergence structure
- Treatment of energy denominators

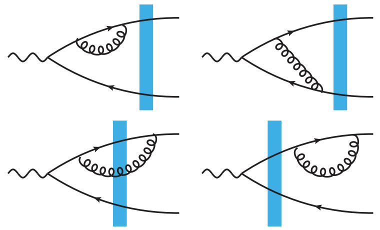
Regularization and divergences

Regularization procedure

- Transverse momentum in $2-2\varepsilon$ dimensions $\Longrightarrow$ $\frac{1}{\varepsilon}$ divergences, collinear or UV
- Longitudinal k^+: cutoff $k^+ > \alpha$, $\alpha \to 0$ $\Longrightarrow$ $1/\alpha$, $\ln^2 \alpha$, $\ln \alpha$ divergences

1. UV $\frac{1}{\varepsilon}$ and $\frac{1}{\varepsilon} \ln \alpha$ divergences:
   $\gamma^* \to q\bar{q}$ vertex, gluon crossing shock, wavefunction renormalization

1. UV $\frac{1}{\varepsilon}$ e.g. in



Regularization and divergences

Regularization procedure

- Transverse momentum in $2 - 2\varepsilon$ dimensions $\Longrightarrow$ $\frac{1}{\varepsilon}$ divergences, collinear or UV
- Longitudinal k^+: cutoff $k^+ > \alpha$, $\alpha \to 0$ $\Longrightarrow$ $1/\alpha$, $\ln^2 \alpha$, $\ln \alpha$ divergences

1. UV $\frac{1}{\varepsilon}$ and $\frac{1}{\varepsilon} \ln \alpha$ divergences:
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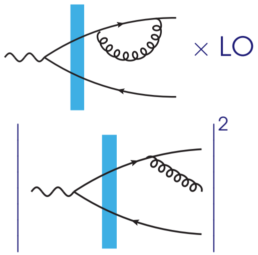
   wavefunction renormalization
2. Collinear $\frac{1}{\varepsilon}$:
   wavef. renormalization, final state emission

2. Collinear $\frac{1}{\varepsilon}$ e.g. in



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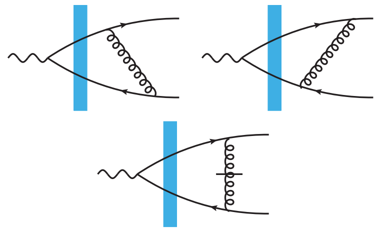

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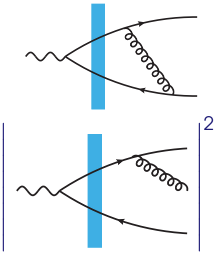
4. $\ln^2 \alpha$ from final state exchange and emission
   (M_X restriction matters here!)

4. $\ln^2 \alpha$ e.g. in



Regularization and divergences

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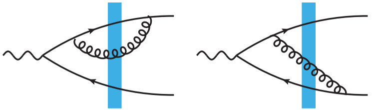
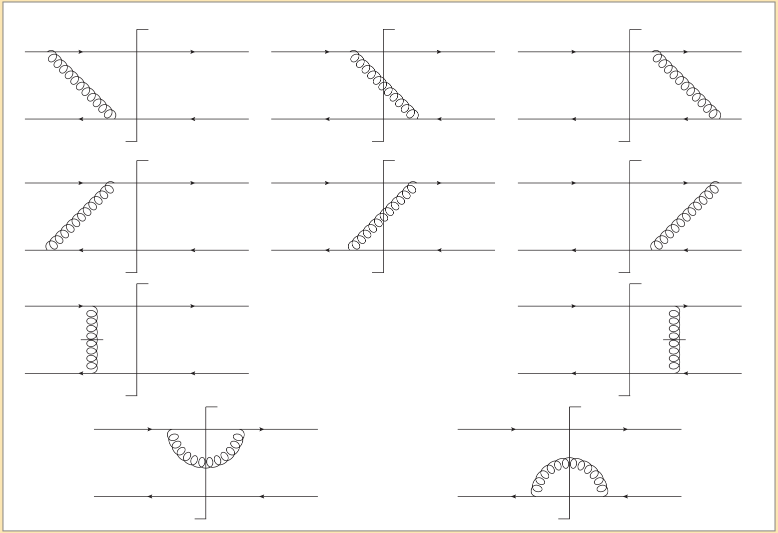
   (M_X restriction matters here!)
5. Remaining $\ln \alpha$ absorbed into BK/JIMWLK

5. $\ln \alpha$ e.g. in



Final state corrections

How to dig out different types of divergences?



Final state corrections

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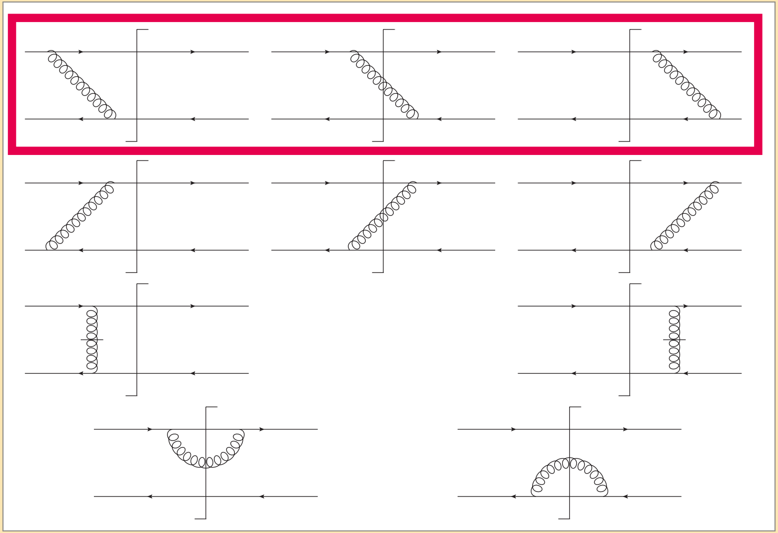
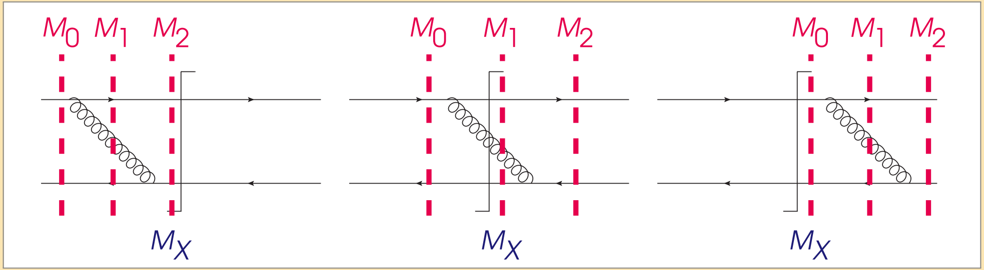


As an example: consider 1st row

Combine energy denominators

"Beuf trick": write M_X delta function as imaginary part of "propagator"



$$\frac{\delta({M_X}^2 - M_2^2)}{(M_2^2 - M_1^2 + i\delta)(M_2^2 - M_0^2 + i\delta)} + \frac{\delta({M_X}^2 - M_1^2)}{(M_1^2 - M_0^2 + i\delta)(M_1^2 - M_2^2 - i\delta)} + \frac{\delta({M_X}^2 - M_0^2)}{(M_0^2 - M_1^2 - i\delta)(M_0^2 - M_2^2 - i\delta)}$$

(Note: sign of $i\delta$ essential)

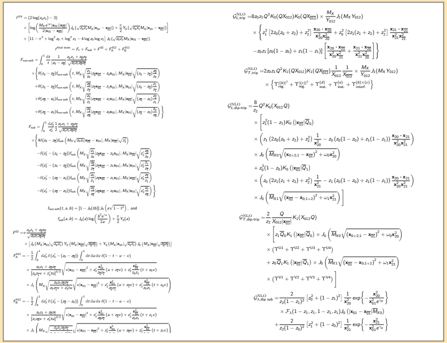
$$= \frac{1}{2\pi i}\left[\frac{1}{({M_X}^2 - M_0^2 - i\delta)({M_X}^2 - M_1^2 - i\delta)({M_X}^2 - M_2^2 - i\delta)} - \text{c.c.}\right]$$

- Then express numerator (⊥ momentum dot products) in terms of M_0^2, M_1^2, M_2^2
- Combine before integration—then separate different divergence types

Conclusions

First: comments on power counting, factorization and negative cross sections . . .

- Diffractive DIS total cross section
  = diffractive structure function F_2^D
  calculated in dipole picture fully at NLO
- General impact parameter dependence
- Reproduces earlier large M_X, ln Q^2 limits,
  used so far in phenomenology
- Future:
  - Numerical implementation
  - Inclusion in global fits



(The result: logs, Bessel functions, polynomials in z_i's)