The Color-Glass Condensate and the Impact Parameter

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SURGE collaboration

Diffraction and gluon saturation at the LHC and the EIC

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Motivation for impact-parameter dependence

Dipole amplitude

$$D(\mathbf{r},\mathbf{b}) = 1 - \frac{1}{N_c} \mathrm{Tr} \left\langle V(\mathbf{x}) V^\dagger(\mathbf{y}) \right\rangle$$

$\mathbf{r} = \mathbf{x} - \mathbf{y} = \text{dipole size}$ $\qquad$ $\mathbf{b} = \frac{1}{2}(\mathbf{x} + \mathbf{y}) = \text{impact parameter}$

Infinite-target approximation: dependence on $\mathbf{b}$ very slow $\Rightarrow D(\mathbf{r},\mathbf{b}) \approx D(\mathbf{r})$

However, problems from neglecting the impact parameter:

1. Proton is not very large! Is this approximation justified?
2. Diffractive observables: $\mathbf{b}$-dependence needed for t-spectrum

$$i\mathcal{M}(t = -\boldsymbol{\Delta}^2) \sim \int \mathrm{d}^2\mathbf{b}\, e^{-i\mathbf{b}\cdot\boldsymbol{\Delta}} D(\mathbf{r},\mathbf{b})$$

Outline

1. Impact parameter in the initial condition
2. Impact parameter in the evolution
3. Saturation effects in exclusive vector meson production

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With Farid Salazar, work-in-progress

With Christophe Royon, work-in-progress

Impact parameter in the initial condition

McLerran–Venugopalan (MV) model

MV model: no dependence on impact parameter

- Starting from the correlator:

$$\left\langle \rho^a(\mathbf{x}, x^+)\rho^b(\mathbf{y}, y^+)\right\rangle = \delta^{ab}\delta^2(\mathbf{x}-\mathbf{y})\delta(x^+ - y^+)\mu^2$$

we get

$$D(r) = 1 - \exp\left[-\frac{Q_s^2 r^2}{4}\log\left(\frac{1}{mr}\right)\right]$$

where m is an infrared regulator and $Q_s^2 \sim \mu^2$

Impact parameter McLerran–Venugopalan model (IP-MV)

We can introduce dependence on the impact parameter by changing $\mu^2 \to \mu^2(\mathbf{x}, x^+)$

- Starting from the correlator:

$$\left\langle \rho^a(\mathbf{x}, x^+)\rho^b(\mathbf{y}, y^+)\right\rangle = \delta^{ab}\delta^2(\mathbf{x}-\mathbf{y})\delta(x^+ - y^+)\mu^2(\mathbf{x}, x^+)$$

we get

$$D(\mathbf{r}, \mathbf{b}) = 1 - \exp\left[-\frac{\alpha_s C_F}{2\pi}\int \mathrm{d}^2\mathbf{z}\,\mathrm{d}z^+\,\mu^2(\mathbf{z}, z^+)\Gamma_{\mathbf{z}}(\mathbf{x}, \mathbf{y})\right]$$

where

$$\Gamma_{\mathbf{z}}(\mathbf{x}, \mathbf{y}) = \left[K_0(m|\mathbf{x}-\mathbf{z}|) - K_0(m|\mathbf{y}-\mathbf{z}|)\right]^2$$

Considered before in Iancu, Rezaeian; 1702.03943

Also similar to the JIMWLK initial condition used in Mäntysaari, Salazar, Schenke; 2207.03712

IP-MV at small and large dipoles

We will consider a simple model for protons:

$$\frac{\alpha_s C_F}{2\pi} \int \mathrm{d}z^+ \, \mu^2(\mathbf{z}, z^+) \equiv \mu_0^2 T(\mathbf{z})$$

with a Gaussian thickness function $T(\mathbf{b}) = \frac{1}{2\pi B_p} e^{-\mathbf{b}^2/(2B_p)}$

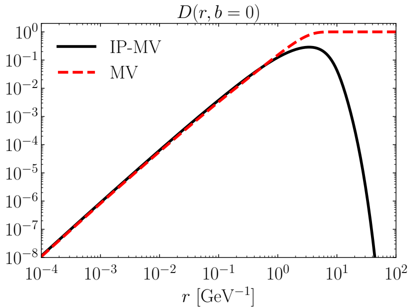
1. Small dipoles $\mathbf{r}^2 \ll B_p$: reduces to the MV model

$$D(\mathbf{r}, \mathbf{b} = 0) \approx 1 - \exp\left[-\frac{\mu_0^2 r^2}{2B_p} \log\left(\frac{\sqrt{B_p}}{r} \right) \right]$$

2. Large dipoles $\mathbf{r}^2 \gg B_p$: $D \to 0$

   Very different from the MV model: $D \to 1$!

   Effectively $V(\mathbf{x}) \to 1$ when $|\mathbf{x}| \to \infty$: no scattering



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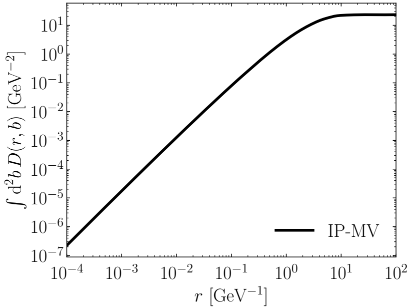
Integrated dipole amplitude in the IP-MV model

IP-integrated dipole amplitude: $\int \mathrm{d}^2\mathbf{b}\, D(\mathbf{r},\mathbf{b})$

- Saturates to a constant for large dipoles
- Behavior much closer to the MV model!
- Large r: contribution from configurations where

  $|\mathbf{x}| \lesssim$ target radius, $|\mathbf{y}| \to \infty$

  $\Rightarrow D(\mathbf{x},\mathbf{y}) \approx 1 - \frac{1}{N_c}\mathrm{Tr}\,\langle V(\mathbf{x})\rangle \equiv M(\mathbf{x}) =$ "*monopole*"



- Total quark-target scattering cross section σ_q by the optical theorem:

$$\sigma_q = 2\int \mathrm{d}^2\mathbf{b}\,\mathrm{Re}\,M(\mathbf{b})$$

$\Rightarrow$ We can write $\int \mathrm{d}^2\mathbf{b}\, D(\mathbf{r},\mathbf{b}) = \sigma_q \widetilde{D}(r)$ where $\widetilde{D}(r) \to 1$ for large r

Dipole gluon TMD in the IP-MV model

Gluon dipole TMD in the small-x limit

$$G_{\rm dip}(k) = \frac{-2}{\alpha_s} \int \frac{\mathrm{d}^2\mathbf{x}\,\mathrm{d}^2\mathbf{y}}{(2\pi)^2} e^{-i(\mathbf{x}-\mathbf{y})\cdot\mathbf{k}}\, \mathrm{Tr}\left\langle \partial^i_{\mathbf{x}} V(\mathbf{x}) \partial^i_{\mathbf{y}} V^\dagger(\mathbf{y}) \right\rangle$$

- Can be simplified by partial integration
- **However**, one needs to take into account that $V(\mathbf{x}) \to 1$ when $|\mathbf{x}| \to \infty$!

$$G_{\rm dip}(k) = \frac{2\mathbf{k}^2}{\alpha_s} \int \frac{\mathrm{d}^2\mathbf{x}\,\mathrm{d}^2\mathbf{y}}{(2\pi)^2} e^{-i(\mathbf{x}-\mathbf{y})\cdot\mathbf{k}}\, \mathrm{Tr}\left\langle \left[V(\mathbf{x}) - 1\right]\left[V^\dagger(\mathbf{y}) - 1\right] \right\rangle$$

$$= \frac{2N_c\mathbf{k}^2}{\alpha_s} \int \frac{\mathrm{d}^2\mathbf{x}\,\mathrm{d}^2\mathbf{y}}{(2\pi)^2} e^{-i(\mathbf{x}-\mathbf{y})\cdot\mathbf{k}} [M(\mathbf{x}) + M^*(\mathbf{y}) - D(\mathbf{x},\mathbf{y})] = \frac{2N_c}{\alpha_s}\mathbf{k}^2 \sigma_q \int \frac{\mathrm{d}^2\mathbf{r}}{(2\pi)^4} e^{-i\mathbf{k}\cdot\mathbf{r}} \left[1 - \widetilde{D}(r)\right]$$

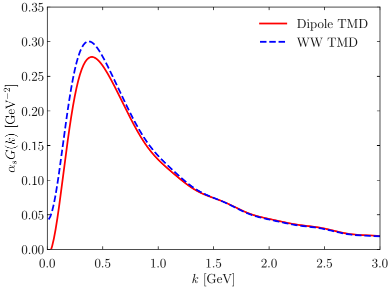
Similar forms common in the literature – note that here $\widetilde{D}(r)$ already integrated over IP!

Weizsäcker–Williams (WW) TMD in the IP-MV model

Gluon WW TMD in the small-x limit

$$G_{\mathrm{WW}}(k) = \frac{-2}{\alpha_s} \int \frac{\mathrm{d}^2\mathbf{x}\,\mathrm{d}^2\mathbf{y}}{(2\pi)^2} e^{-i(\mathbf{x}-\mathbf{y})\cdot\mathbf{k}} \,\mathrm{Tr}\left\langle \left[V(\mathbf{x}) \left(\partial^i_{\mathbf{x}} V^\dagger(\mathbf{x})\right) V(\mathbf{y}) \left(\partial^i_{\mathbf{y}} V^\dagger(\mathbf{y})\right)\right]\right\rangle$$

- Large k: similar to the dipole TMD as expected
- Small k: approaches a constant (c.f. $G_{\mathrm{dip}} \to 0$)
- Note: using $D(r) = 1 - \exp\left[-\frac{Q_s^2 r^2}{4} \log\left(\frac{1}{mr} + e\right)\right]$ we have $G_{\mathrm{WW}} \to \log Q_s^2/k^2$ at $k = 0$
- With IP: can also calculate GTMDs and Wigner distributions



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Impact parameter in the evolution

High-energy evolution

Balitsky–Kovchegov equation

$$\partial_Y D(\mathbf{x}_0, \mathbf{x}_1, Y) = \int \mathrm{d}^2\mathbf{x}_2\, K_{\mathrm{BK}}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)\Big[D(\mathbf{x}_0, \mathbf{x}_2, Y) + D(\mathbf{x}_2, \mathbf{x}_1, Y) - D(\mathbf{x}_0, \mathbf{x}_1, Y) - D(\mathbf{x}_0, \mathbf{x}_2, Y)D(\mathbf{x}_2, \mathbf{x}_1, Y)\Big]$$

where the kernel is given by ($\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$):

$$K_{\mathrm{BK}}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2) = \frac{\alpha_s N_c}{2\pi^2} \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{20}^2 \mathbf{x}_{21}^2}$$

We also introduce an IR cutoff m': Replace $\frac{\mathbf{x}^i}{\mathbf{x}^2} \to \frac{\mathbf{x}^i}{\mathbf{x}^2} \times |\mathbf{x}| m' K_1(m'|\mathbf{x}|)$ in K_{BK}

- $m' \equiv m = 0.4\,\mathrm{GeV}$ IR regulator, needed to suppress contribution from large dipoles (see e.g. Kovner, Wiedemann; hep-ph/0112140)
- Similar prescription as in Mäntysaari, Salazar, Schenke; 2207.03712 for JIMWLK evolution

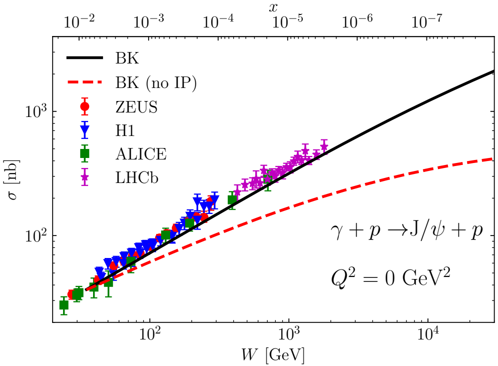
Comparison: BK evolution with and without impact parameter

- BK evolution with the same initial condition
  - Without IP: integrate over IP first, then evolve the normalized result $\widetilde{D}(r)$
- Calculate exclusive J/ψ production
  - t-integration for the IP-independent case:

$$\sigma_{\text{no IP}} \equiv \left.\frac{\mathrm{d}\sigma_{\text{no IP}}}{\mathrm{d}t}\right|_{t=0} \times \left(\sigma_{\text{IP}} \Big/ \left.\frac{\mathrm{d}\sigma_{\text{IP}}}{\mathrm{d}t}\right|_{t=0}\right)$$

- Huge difference with the two prescriptions

  $\Rightarrow$ IP-integration and evolution do not commute!
- Difference due to the nonlinear term



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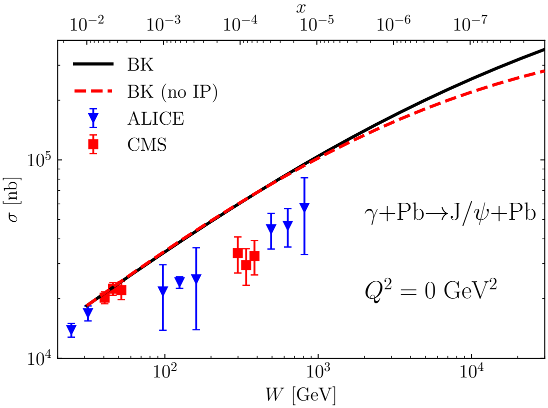
Comparison: BK evolution with and without impact parameter

- We can also compare for nuclear targets
  $\Rightarrow$ Switch the thickness function $T(\mathbf{z})$ for Woods–Saxon
- Nucleus much larger than proton
  $\Rightarrow$ Neglecting impact parameter more justified



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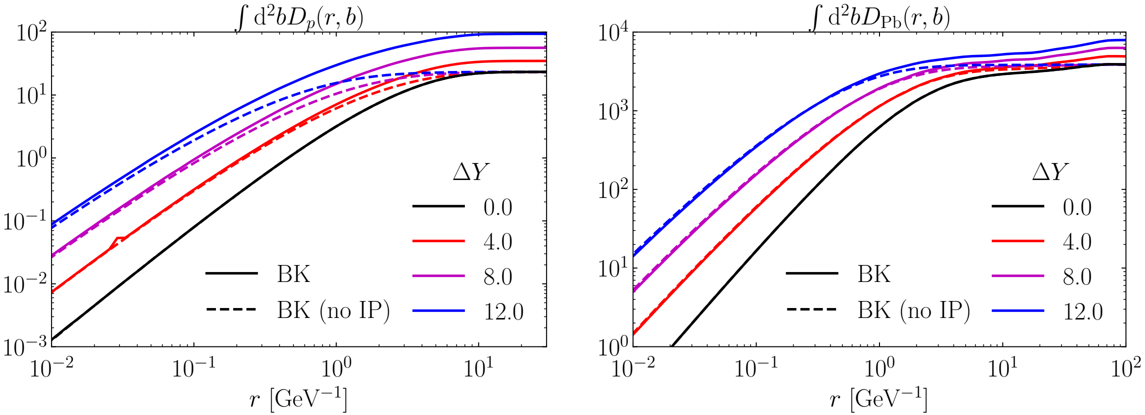
Comparison: BK evolution with and without impact parameter

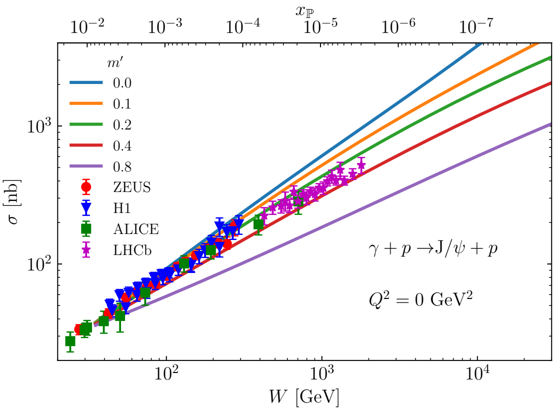


Differences can be seen in the integrated dipole amplitude:

The quark-target scattering cross section σ_q grows with energy

Sensitivity to the IR regulator

- Problem with the IP BK: need to regulate the IR region (large dipoles)
- Compare exclusive J/ψ production with different m' in the evolution:
  Sensitivity to the IR regulator can be large
- Note: we can compensate for m' by changing α_s
- Dependence on m' might be ameliorated by NLL corrections to the evolution
  - Can be done with BK! (WIP)



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Saturation effects in exclusive vector meson production

Estimating saturation effects

BK equation

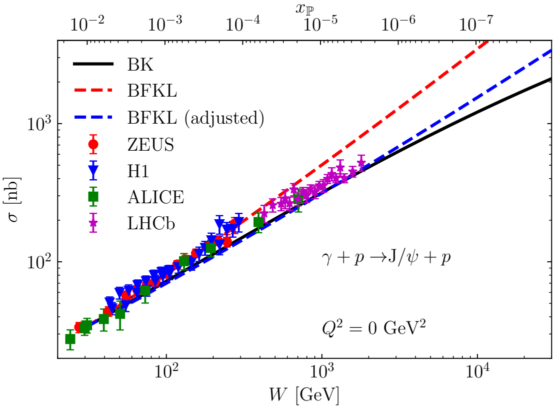
$$\partial_Y D(\mathbf{x}_0, \mathbf{x}_1, Y) = \int \mathrm{d}^2\mathbf{x}_2\, K_{\mathrm{BK}}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)\Big[D(\mathbf{x}_0, \mathbf{x}_2, Y) + D(\mathbf{x}_2, \mathbf{x}_1, Y) - D(\mathbf{x}_0, \mathbf{x}_1, Y) - D(\mathbf{x}_0, \mathbf{x}_2, Y)D(\mathbf{x}_2, \mathbf{x}_1, Y)\Big]$$

- Saturation effects introduced by the nonlinear term in the BK equation
- Without nonlinear term: BFKL evolution

  $\Rightarrow$ Compare BK and BFKL evolutions to estimate saturation effects
- Use the same setup with the IR regulator m' for both evolution equations
  - Consistent comparison of the equations
  - IP-dependence important for protons to avoid overestimating saturation

Exclusive J/ψ production: proton targets

- Slight difference between BFKL and BK in the slope
  - Can be compensated by adjusting α_s for BFKL
- Fit the evolution to the proton data
  $\Rightarrow$ Predictions for heavy nuclei not sensitive to the IR regulator
- Proton data described well by both BK and BFKL evolution



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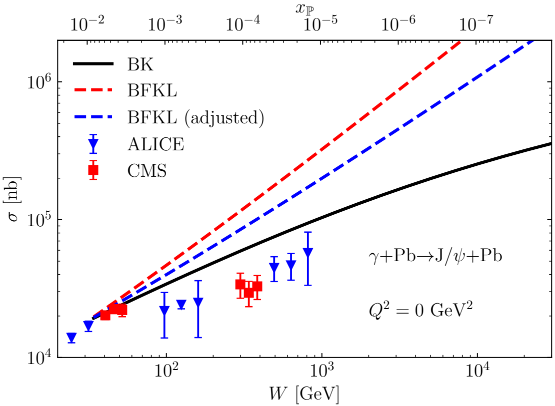
Exclusive J/ψ production: nuclear targets

- Differences between BK and BFKL: a factor of 2 for $W \sim 1000\,\mathrm{GeV}$
- BFKL results linear as predicted
- BK describes the data much better
- Still not exact agreement: A well-known problem, see e.g. Mäntysaari, Salazar, Schenke; 2312.04194



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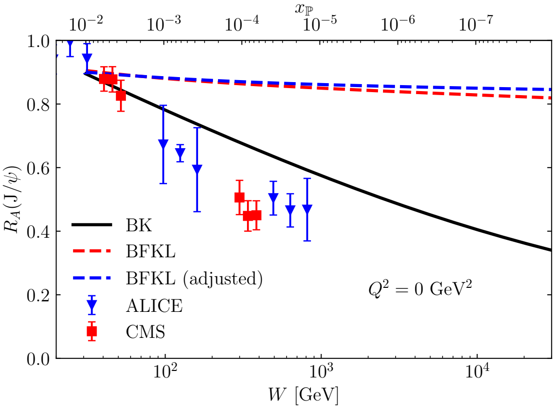
Exclusive J/ψ production: nuclear suppression

- Plot the nuclear suppression defined as

$$R_A = \sqrt{\sigma_A/\sigma_{\text{IA}}}$$

  where σ_{IA} is the impulse approximation

- BFKL essentially constant
  - Linear evolution: energy dependence for protons and nuclei expected to be similar
  - Clear disagreement with the data
  - Saturation provides a natural explanation



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Summary

- Initial condition modified with the impact parameter
- Evolution modified with the impact parameter (for protons)
- With IP: sensitivity to the large dipoles (nonperturbative region) – needs to be regulated
  - NLL evolution might decrease contribution form large dipoles
- Impact parameter makes it possible to compare BFKL and BK consistently
  - Saturation effects already visible in $\gamma + \text{Pb} \to \text{J}/\psi + \text{Pb}$?
- Many features of the JIMWLK can reproduced with BK
- Articles out soon...

The dependence on the impact parameter should not be neglected!