Forward Higgs production in the infinite-top-mass limit

Alessandro Papa

Università della Calabria & INFN - Cosenza

based on M. Fucilla, M.A. Nefedov, A. Papa, JHEP 04 (2024) 078

Diffraction and gluon saturation at the LHC and the EIC

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DIPARTIMENTO DI FISICA

INFN
Istituto Nazionale di Fisica Nucleare
GRUPPO COLLEGATO DI COSENZA

Outline

Introduction and Motivation

- The importance of exploring the Higgs sector of the Standard Model can hardly be overestimated; opening new channels of theoretical investigation or improving the reliability of known ones can only be beneficial.

- An interesting new option in this respect is the inclusive production at the LHC (and future colliders) of a forward Higgs, possibly in association with a backward jet or hadron.

- It belongs to the class of semihard processes, where the scale hierarchy

$$s \gg Q^2 \gg \Lambda^2_{\rm QCD} \;, \quad Q \text{ a hard scale } (Q = m_{H\perp} \text{ in the Higgs case}) \;,$$

  holds, making fixed-order perturbative calculations insufficient, due to $\alpha_s(Q) \log s \sim 1$, and calling for an all-order resummation of energy logarithms.

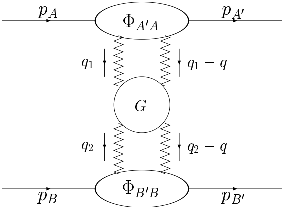
- The Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach provides a general framework for the large-s / high-energy resummation: I will briefly review the BFKL approach and present its (challenging) application to the forward Higgs production case.

Outline

BFKL factorization

Scattering $A+B \longrightarrow A'+B'$ in the Regge kinematical region $s\to\infty$, t fixed

$\Rightarrow$ BFKL factorization for $\mathrm{Im}_s\mathcal{A}$:
convolution of a Green's function with the impact factors of the colliding particles.



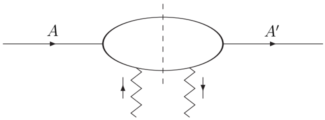
Valid both in
LLA (resummation of all terms $(\alpha_s \ln s)^n$)
NLA (resummation of all terms $\alpha_s(\alpha_s \ln s)^n$).

$$\mathrm{Im}_s\mathcal{A} = \frac{s}{(2\pi)^{D-2}}\int\frac{d^{D-2}\vec{q}_1}{\vec{q}_1^{\,2}}\Phi_{AA'}(\vec{q}_1,\vec{q};s_0)\int\frac{d^{D-2}\vec{q}_2}{\vec{q}_2^{\,2}}\Phi_{BB'}(-\vec{q}_2,-\vec{q};s_0)$$
$$\times\int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega}G_\omega(\vec{q}_1,\vec{q}_2)$$

The Green's function is process-independent and is determined through the BFKL equation. [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1,\vec{q}_2) = \delta^{D-2}(\vec{q}_1-\vec{q}_2) + \int d^{D-2}\vec{q}\, K(\vec{q}_1,\vec{q})\, G_\omega(\vec{q},\vec{q}_1)$$

Impact factors are process-dependent;
only very few of them known in the NLA ...



- $A = A'$=quark, $A = A'$=gluon [V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)] [M. Ciafaloni and G. Rodrigo (2000)]
- $A = \gamma^*$, $A' = V$, with $V = \rho^0, \omega, \phi$ (forward)
  twist-2 (long. polarization) [D.Yu. Ivanov, M.I. Kotsky, A.P. (2004)]
  twist-3 (transv. polarization) [I.V. Anikin et al. (2009)]
- $A = A' = \gamma^*$ (forward) [J. Bartels, S. Gieseke, C.F. Qiao (2001)] [J. Bartels, S. Gieseke, A. Kyrieleis (2002)] [J. Bartels, D. Colferai, S. Gieseke, A. Kyrieleis (2002)] [V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2003)] [J. Bartels, A. Kyrieleis (2004)] [I. Balitsky, G.A. Chirilli (2013)] [G.A. Chirilli, Yu.V. Kovchegov (2014)]

... so that only a very limited number of predictions can be built for exclusive processes or total cross sections (among them the 'gold plated' $\gamma^*\gamma^* \to$ all), even hardly testable in present colliders.

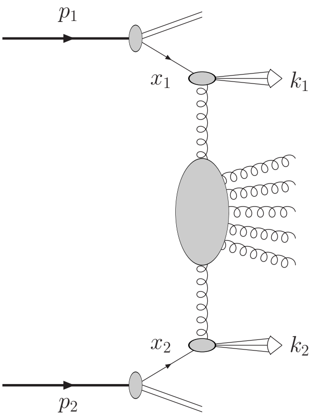
Outline

1. Introduction and Motivation
2. Theoretical background / BFKL approach
   - Exclusive processes and total cross sections
   - Inclusive processes
3. The Higgs impact factor
   - Leading-order case
   - NLO real corrections
   - NLO virtual corrections
4. Conclusions and outlook

Inclusive processes

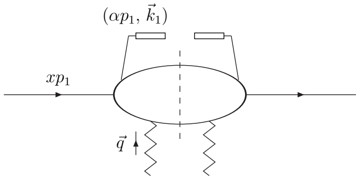
A lot more possibilities open for inclusive processes, with jets or identified particles in the final state, produced in the fragmentation regions...

... and if the fragmentation subprocess is hybridized with collinear factorization.

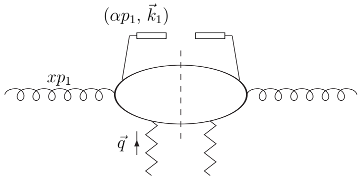


Identified 'object' (jet, hadron) with momentum k_1 (k_2) in the forward (backward) region; all the rest undetected.

Straightforward adaptation of the BFKL factorization: just restrict the summation over final states entering the definition of impact factors.



quark vertex



gluon vertex

- "open" one of the integrations over the phase space of the intermediate state to allow one (or more) parton(s) to generate a jet or a Higgs or one parton to fragment into a given hadron
- use QCD collinear factorization

$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark vertex}) \otimes (D_a^h \ / \ S_a^J) \ / \ H) \quad + \quad f_g \otimes (\text{gluon vertex}) \otimes (D_g^h \ / \ S_g^J \ / \ H)$$

$f_{a,g}$: unpolarized collinear PDFs, $D^h_{a,g}$: unpolarized collinear FFs,

$S^J_{a,g}$: jet selection functions, H: Higgs vertex

A few more impact factors became available in the hybrid collinear/high-energy factorization in the NLA ...

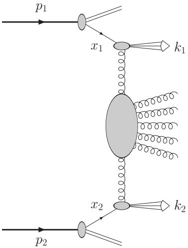
- jet vertex [J. Bartels, D. Colferai, G.P. Vacca (2003)]
  [F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P., A. Perri (2011)]
  [D.Yu. Ivanov, A.P. (2012)] (small-cone approximation)
  [D. Colferai, A. Niccoli (2015)]

- hadron vertex [D.Yu. Ivanov, A.P. (2012)]

- Higgs vertex
  [M.A. Nefedov (2019)] [M. Hentschinski, K. Kutak, A. van Hameren (2020)]
  [F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2022)]

... to be added to several LA ones:

- J/Ψ vertex [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]
- Drell-Yan pair vertex [K. Golec-Biernat, L. Motyka, T. Stebel (2018)]
- heavy-quark production vertex
  [A.D. Bolognino, F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, A.P. (2019)]



$$\text{proton}(p_1) + \text{proton}(p_2) \longrightarrow \text{object}_1(k_1) + X + \text{object}_2(k_2)$$

Taking also into account the large variety of available FFs, a plethora of predictions were recently produced for the inclusive production at the LHC of a forward and a backward identified 'objects'.

Full-NLA analyses (for LHC and possibly FCC):

- jet + jet (Mueller-Navelet) [D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)] [B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)] [F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P. (2014)] [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (2015,2016)] [F.G. Celiberto, A.P. (2022)]
- light hadron + light hadron [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (2016,2017)]
- light hadron + jet [A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2018)]
- $\Lambda + \Lambda$, Λ + jet [F.G. Celiberto, D.Yu. Ivanov, A.P. (2020)]
- $\Lambda_c + \Lambda_c$, Λ_c + jet [F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, A.P. (2020)]
- b-hadron + b-hadron, b-hadron + jet [F.G. Celiberto, M. Fucilla, M.M.A. Mohammed, A.P. (2022)]
- $B_c(B_c^*)$ + b-hadron, $B_c(B_c^*)$ + jet [F.G. Celiberto (2022)]
- J/ψ or Υ + jet [F.G. Celiberto, M. Fucilla (2022)]
- heavy-light tetraquark + b, c-hadron or jet [F.G. Celiberto, A. Papa (2023)]

Partial-NLA analyses:

- J/Ψ + jet [R. Boussarie, B. Ducloué, L. Szymanowski, S. Wallon (2018)]
- Drell-Yan pair + jet [K. Golec-Biernat, L. Motyka, T. Stebel (2018)]
- Higgs + jet [F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2021)]
- light jet + heavy jet [A.D. Bolognino, F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, A.P. (2021)]
- Higgs + c-hadron [F.G. Celiberto, M. Fucilla, M.M.A. Mohammed, A.P. (2022)]

Observables

BFKL-factorized form, with "differential" impact factors:

$$\frac{d\sigma}{dy_1 dy_2\, d|\vec{k}_1|\, d|\vec{k}_2| d\phi_1 d\phi_2} = \frac{d\Phi_1}{dy_1\, d|\vec{k}_1| d\phi_1} \otimes G \otimes \frac{d\Phi_2}{dy_2\, d|\vec{k}_2| d\phi_2}$$

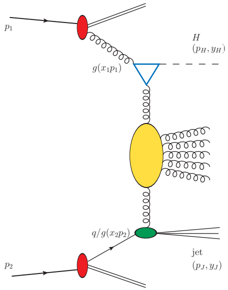
$$= \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\,\mathcal{C}_n\right] , \quad \phi = \phi_1 - \phi_2 - \pi$$

- azimuthal correlations, $\langle\cos(n\phi)\rangle = \mathcal{C}_n/\mathcal{C}_0$, and ratios between them [A. Sabio Vera, F. Schwennsen (2007)]
- cross sections differential in $Y \equiv y_1 - y_2$
- ...

Higgs plus jet as a paradigm

Inclusive **Higgs plus jet** production in p-p collisions [V. Del Duca, C.R. Schmidt (1994)]
(+ Sudakov logs) [B. Xiao, F. Yuan (2018)]

- Full NLA Green function + partially NLO impact factor (full m_t-dependence) [F.G. Celiberto, D. Yu. Ivanov, M.M.A. Mohammed, A. Papa (2021)]
- Same process in HEJ framework (full m_t,m_b-dep.) [J. Andersen *et al.* (2022)]



$$\frac{d\sigma_{\rm pp}}{dy_H dy_J d|\vec{p}_H| d|\vec{p}_J| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2}$$
$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} \left(\mathcal{V}_H^{(g)}(\vec{q}_1, s_0, x_1, \vec{p}_H) \otimes f_g(x_1) \right)$$
$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_H x_J s}{s_0} \right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$
$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} \left(\sum_p \mathcal{V}_J^{(p)}(\vec{q}_2, s_0, x_2, \vec{p}_J) \otimes f_p(x_2) \right)$$

Hadronic cross section expanded in azimuthal coefficients

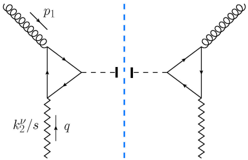
$$\frac{d\sigma_{\rm pp}}{dy_H dy_J d|\vec{p}_H| d|\vec{p}_J| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2} \left[\mathcal{C}_0 + 2 \sum_{n=1}^{\infty} \cos(n\phi) \mathcal{C}_n \right] \qquad \phi = \phi_1 - \phi_2 - \pi$$

Outline

LO Higgs impact factor

Gluon-Reggeon → Higgs
(through the top quark loop)

Off-shell t-channel gluon with effective $\frac{k_2^\nu}{s}$ polarization



- LO impact factor [V. Del Duca, C.R. Schmidt (1994)]

$$\frac{d\Phi_{PP}^{\{H\}(0)}}{dx_H d^2\vec{p}_H} = \frac{\alpha_s^2}{v^2}\frac{\vec{q}^{\,2}|\mathcal{F}(m_t, m_H, \vec{q}^{\,2})|^2}{128\pi^2\sqrt{N^2-1}} f_g(x_H)\delta^{(2)}(\vec{p}_H - \vec{q})$$

$$\xrightarrow{m_t\to\infty} \frac{\alpha_s^2}{v^2}\frac{\vec{q}^{\,2}}{72\pi^2\sqrt{N^2-1}} f_g(x_H)\delta^{(2)}(\vec{p}_H - \vec{q})$$

- NLO impact factor: for $m_t \to \infty$, effective Lagrangian

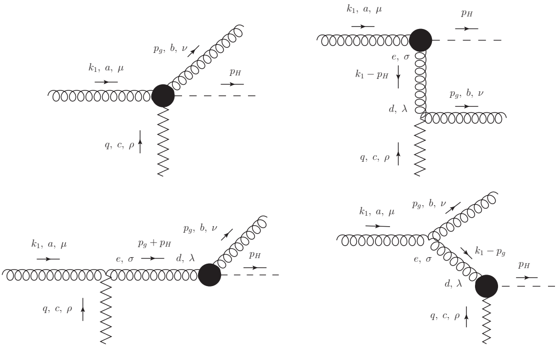
$$\mathcal{L}_{ggH} = -\frac{1}{4} g_H F_{\mu\nu}^a F^{\mu\nu,a} H \,, \qquad g_H = \frac{\alpha_s}{3\pi v} + \mathcal{O}(\alpha_s^2)$$

[M. Nefedov (2019)] [M. Hentschinski, K. Kutak, A. van Hameren (2020)]
[F.G. Celiberto, M. Fucilla, D.Yu. Ivanov, M.M.A. Mohammed, A.P. (2022)]

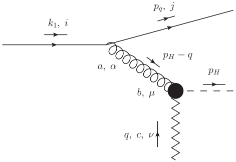
Outline

NLO Higgs impact factor: Real corrections

- Gluon-initiated subprocess

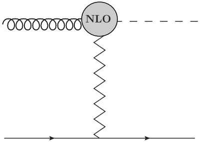


- Quark-initiated subprocess



Outline

NLO Higgs impact factor: Virtual corrections



Target: 1-loop ggH effective vertex

$$\Gamma^{ac(1)}_{\{H\}g}(q) = \underbrace{\Gamma^{ac(0)}_{\{H\}g}(q)}_{\frac{g_H \delta^{ac}(\varepsilon_\perp \cdot q_\perp)}{2}} \; [1 + \delta_{\rm NLO}]$$

(reference process: g+q $\to$ H + q)

Strategy: compare a suitable high-energy amplitude with the Regge form

$$\mathcal{A}^{(8,-)}_{gq\to Hq} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{c}_{qq} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right)\right]\Gamma^{c(0)}_{qq} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(1)}_{qq} + \Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

Virtual corrections to the impact factor:

$$\frac{d\Phi^{\{H\}(1)}_{gg}}{dz_H d^2\vec{p}_H} = \frac{d\Phi^{\{H\}(0)}_{gg}}{dz_H d^2\vec{p}_H}\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\frac{C_A}{\epsilon^2} + \frac{11C_A - 2n_f}{6\epsilon}\right.$$

$$\left.-\frac{C_A}{\epsilon}\ln\left(\frac{\vec{q}^{\,2}}{s_0}\right) - \frac{5n_f}{9} + C_A\left(2\,\Re\left(\mathrm{Li}_2\left(1+\frac{m_H^2}{\vec{q}^{\,2}}\right)\right) + \frac{\pi^2}{3} + \frac{67}{18}\right) + 11\right]$$

Agreement with [M. Nefedov (2019)] (tool: Lipatov high-energy effective theory)

NLO Higgs impact factor: Virtual corrections

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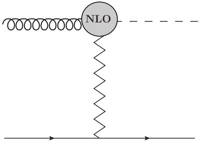

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$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right)\right]\Gamma^{c(0)}_{qq} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(1)}_{qq} + \Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{c(0)}_{qq}$$

Virtual corrections to the impact factor:

$$\frac{d\Phi^{\{H\}(1)}_{gg}}{dz_H d^2\vec{p}_H} = \frac{d\Phi^{\{H\}(0)}_{gg}}{dz_H d^2\vec{p}_H}\,\frac{\bar{\alpha}_s}{2\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\frac{C_A}{\epsilon^2} + \frac{11C_A - 2n_f}{6\epsilon}\right.$$

$$\left. -\frac{C_A}{\epsilon}\ln\left(\frac{\vec{q}^{\,2}}{s_0}\right) - \frac{5n_f}{9} + C_A\left(2\,\Re\left(\mathrm{Li}_2\left(1+\frac{m_H^2}{\vec{q}^{\,2}}\right)\right) + \frac{\pi^2}{3} + \frac{67}{18}\right) + 11\right]$$

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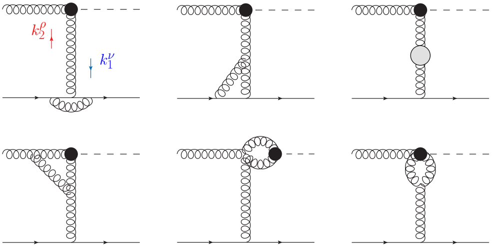
Anatomy of the calculation: “non-Gribov” terms

- Single-gluon in the *t*-channel



Gribov’s prescription (eikonal approximation): $g^{\rho\nu} = g^{\rho\nu}_{\perp\perp} + 2\frac{k_1^\rho k_2^\nu + k_1^\nu k_2^\rho}{s} \to 2s\frac{k_1^\nu}{s}\frac{k_2^\rho}{s}$

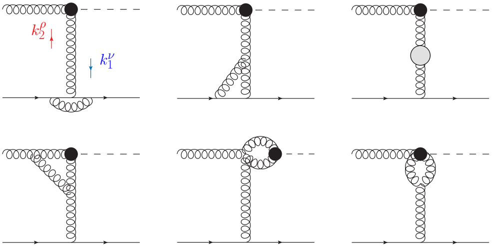
- Two gluons in the *t*-channel

Dimension-5 operator in $\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{ggH}$ $\to$ Gribov’s trick modification

$$g^{\rho\nu} = g^{\rho\nu}_{\perp\perp} + 2\frac{k_1^\rho k_2^\nu + k_1^\nu k_2^\rho}{s} \to 2s\frac{k_1^\nu}{s}\frac{k_2^\rho}{s} + g^{\rho\nu}_{\perp\perp}$$

Anatomy of the calculation: “non-Gribov” terms

- Single-gluon in the t-channel



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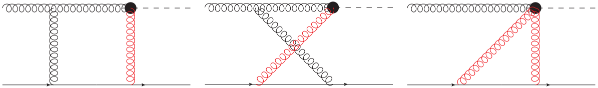
- Two gluons in the t-channel



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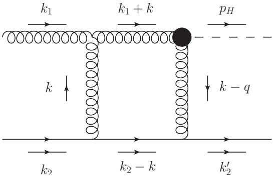
Non-Gribov terms: comparison with pure QCD



- Non-Gribov term in $\mathcal{A}_{gq\to Hq}$

$$-s\,\bar{u}(k_2')\gamma_{\perp,\sigma}u(k_2)\,\varepsilon_\perp^\nu(k_1)\,H_\nu{}^\sigma(-k_1-k,k-q)$$

$$H^{\nu\sigma}(p_1,p_2) = g^{\nu\sigma}(p_1\cdot p_2) - p_1^\nu p_2^\sigma$$

$\longrightarrow$ order s after integration over loop momentum k

- Non-Gribov term in $\mathcal{A}_{gq\to gq}$

$$-s\,\bar{u}(k_2')\gamma_{\perp,\sigma}u(k_2)\,\varepsilon_\perp^\nu(k_1)\left(\varepsilon^*_{\perp,\beta}(k_1') - \frac{\varepsilon^*_\perp(k_1')\cdot k'_{1,\perp}}{k_{1'}\cdot k_2}k_{2,\beta}\right)A_\nu{}^{\sigma\beta}(k-q,k_1+k)$$

$$A^{\nu\sigma\beta}(k-q,-k_1-k) = g^{\sigma\beta}(q-k_1-2k)^\nu + g^{\nu\sigma}(k-2q-k_1)^\beta + g^{\nu\beta}(2k_1+k+q)$$

$\longrightarrow$ order s^0 after integration over loop momentum k

Non-Gribov terms: impact on Regge form of amplitude

- Born helicity structure:

$$\begin{aligned}\mathcal{H}_{\rm Born} &\equiv (\varepsilon_\perp(k_1)\cdot q_\perp)\,\bar{u}(k_2-q)\,\frac{\hat{k}_1}{s}\,u(k_2)\\ &= (\varepsilon_\perp(k_1)\cdot q_\perp)\,\bar{u}(k_2-q)\,\frac{\hat{q}_\perp}{|q_\perp|^2}\,u(k_2) \quad \text{by Sudakov decomposition of } q\end{aligned}$$

Basis $n_q^\mu = \frac{q_\perp^\mu}{|q_\perp|}$, $\quad n_{\tilde{q}}^\mu = \epsilon^{\mu\nu+-}\frac{q_\perp^\nu}{|q_\perp|}$

$$\mathcal{H}_{\rm Born} = (\varepsilon_\perp(k_1)\cdot n_q)\,\bar{u}(k_2-q)\hat{n}_q u(k_2)$$

- Helicity structure of a non-Gribov term

$$\bar{u}(k_2-q)\hat{\varepsilon}_\perp(k_1)u(k_2) = -\bar{u}(k_2-q)\gamma_\mu u(k_2)\left(n_q^\mu n_q^\nu + n_{\tilde{q}}^\mu n_{\tilde{q}}^\nu\right)\varepsilon_{\perp,\nu} = -\mathcal{H}_{\rm Born} - \mathcal{H}_{\rm anomalous}$$

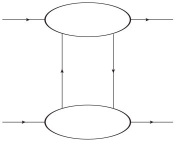
Interference between $\mathcal{H}_{\rm Born}$ and $\mathcal{H}_{\text{anomalous}}$ + spin sum gives zero

The anomalous helicity structure vanishes at amplitude level

- Nonetheless, non-Gribov terms give a total contribution

$$\delta^{\rm n.G.}_{gq\to Hq} = g^2(-2C_A)B_0(q^2) = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left[-\frac{2C_A}{\epsilon} - 4C_A\right]$$

Gribov part: strategy of rapidity regions, QCD case



Strategy for the calculation of two-gluon t-channel diagrams
[V.S. Fadin, A.D. Martin (1999)] [V.S. Fadin, R. Fiore (2001)]

- Feynman gauge and Gribov's trick

$$g^{\mu\nu} = g_{\perp\perp}^{\mu\nu} + 2\frac{k_2^\mu k_1^\nu + k_2^\mu k_1^\nu}{s} \longrightarrow \frac{2k_2^\mu k_1^\nu}{s}$$

- Loop momentum k decomposed à la Sudakov: $k = \beta k_1 + \alpha k_2 + k_\perp$

$$\begin{cases} \textbf{Central region} & |\alpha| \lesssim \alpha_0 \ , \ |\beta| \lesssim \beta_0 \ , \\ \textbf{Region A} & |\alpha| \lesssim \alpha_0 \ , \ |\beta| > \beta_0 \ , \\ \textbf{Region B} & |\alpha| > \alpha_0 \ , \ |\beta| \lesssim \beta_0 \ , \\ \textbf{Region C} & |\alpha| > \alpha_0 \ , \ |\beta| > \beta_0 \ , \end{cases}$$

$$\alpha_0 \ll 1 \ , \qquad \beta_0 \ll 1 \ , \qquad s\alpha_0\beta_0 \gg |t|$$

- Factorization of vertices:
  in the region $|\alpha| \ll 1 \longrightarrow \Gamma_{B'B}^{(0)}$ in the region $|\beta| \ll 1 \longrightarrow \Gamma_{A'A}^{(0)}$

Gribov part: strategy of rapidity regions, QCD case

- Region C is suppressed by a factor $|t|/\alpha_0\beta_0 s \ll 1$
- Central region: box + crossed diagram

$$\mathcal{A}^{(8,-)}_{\mathrm{Central}} = \Gamma^{(0)}_{A'A}\frac{2s}{t}\Gamma^{(0)}_{B'B}\,\omega^{(1)}(t)\left[\frac{1}{2}\ln\left(\frac{-s}{t}\right)+\frac{1}{2}\ln\left(\frac{-s}{-t}\right)+\phi(\alpha_0)+\phi(\beta_0)\right]$$

$$\phi(z) = \ln z + \frac{1}{2}\left(-\frac{1}{\epsilon} - \psi(1) + \psi(1+\epsilon) - 2\psi(1-\epsilon) + 2\psi(1-2\epsilon)\right)$$

Correction to the upper and lower effective vertex from the central region

$$\Gamma^{(\mathrm{Central})}_{A'A} = \Gamma^{(0)}_{A'A}\,\omega^{(1)}(t)\phi(\beta_0) \qquad \Gamma^{(\mathrm{Central})}_{B'B} = \Gamma^{(0)}_{B'B}\,\omega^{(1)}(t)\phi(\alpha_0)$$

- Region A

$$\Gamma^{(\mathrm{A})}_{A'A} = \Gamma^{(0)}_{A'A}\delta^{(\mathrm{A})}_{\mathrm{NLO}} = \Gamma^{(0)}_{A'A}\left[-\omega(t)\ln\beta_0 + \tilde{\delta}^{(\mathrm{A})}_{\mathrm{NLO}}\right]$$

- Region B

$$\Gamma^{(\mathrm{B})}_{B'B} = \Gamma^{(0)}_{B'B}\,\delta^{(\mathrm{B})}_{\mathrm{NLO}} = \Gamma^{(0)}_{B'B}\left[-\omega(t)\ln\alpha_0 + \tilde{\delta}^{(\mathrm{B})}_{\mathrm{NLO}}\right]$$

Gribov part: strategy of rapidity regions, with Higgs

Reference amplitude: gluon + quark $\to$ Higgs + quark

- Region C is suppressed by a factor $|t|/\alpha_0\beta_0 s \ll 1$
- Central region

$$\mathcal{A}_{\text{Box,Central}} = \Gamma^{c(0)}_{q'q}\left(\frac{2s}{t}\right) g_H \epsilon_\mu(k_1)\delta^{ac}\left(-\frac{g^2 C_A s t}{2}\right)\frac{s}{2}\int_{-\alpha_0}^{\alpha_0} d\alpha \int_{-\beta_0}^{\beta_0} d\beta$$

$$\times \int \frac{d^{D-2}k_\perp}{(2\pi)^D i}\frac{q_\perp^\mu - k_\perp^\mu}{(\alpha\beta s + k_\perp^2 + i0)(\alpha\beta s + (q-k)_\perp^2 + i0)(-\beta s + i0)(\alpha s + i0)}$$

Apparently, no factorization of the $\Gamma^{ac(0)}_{gH}$ vertex.

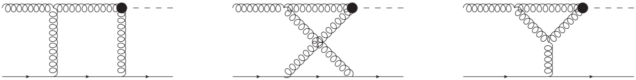
But, the change of variables $k_\perp \to q_\perp - k_\perp$ implies $q_\perp \to \frac{1}{2}q_\perp$ in the numerator. Then

$$\mathcal{A}^{(8,-)}_{\text{Central}} = \Gamma^{ac(0)}_{gH}\frac{2s}{t}\Gamma^{c(0)}_{q'q}\,\omega^{(1)}(t)\left[\frac{1}{2}\ln\left(\frac{-s}{t}\right) + \frac{1}{2}\ln\left(\frac{-s}{-t}\right) + \phi(\alpha_0) + \phi(\beta_0)\right]$$

Correct factorization!

Gribov part: strategy of rapidity regions, with Higgs

- Region A + triangular diagram



Proper factorization of the upper and lower vertex and

$$\delta_{\rm NLO}^{\rm (Tri+A)} = -\omega^{(1)}(t)\ln\beta_0 + \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}$$

$$\times\left\{\frac{7}{3}\frac{C_A}{\epsilon} + \frac{85}{18}C_A + \frac{1}{6}C_A\ln\left(-\frac{m_H^2}{\vec{q}^{\,2}}\right) + 2C_A\left(\frac{\pi^2}{6} + {\rm Li}_2\left(1+\frac{m_H^2}{\vec{q}^{\,2}}\right)\right)\right\} + \mathcal{O}(\epsilon)$$

Gribov part: strategy of rapidity regions, with Higgs

- Region B

  Expectation: factorization of the upper and lower vertices and extraction of the NLO correction to the lower (quark) one.

  Instead,

$$\mathcal{A}_{\rm B} = \Gamma^{c(0)}_{qq'} \left(\frac{2s}{t}\right) \frac{\epsilon_\mu(k_1)\delta^{ac} g_H}{2} g^2 C_A t \int_{\alpha_0}^1 \frac{d\alpha}{\alpha}(1-\alpha)^2 \int \frac{d^{D-2}k}{(2\pi)^{D-1}} \frac{(q-k)_\perp^\mu}{k_\perp^2 (k_\perp - (1-\alpha)q_\perp)^2}$$

$$\frac{\mathcal{A}_B}{\Gamma^{ac(0)}_{gH} \left(\frac{2s}{t}\right) \Gamma^{c(0)}_{qq'}} = \frac{g^2 C_A t}{2} \int_{\alpha_0}^1 \frac{d\alpha}{\alpha}(1-\alpha)^2 (1+\alpha) \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-1}} \frac{1}{k_\perp^2 (k_\perp - (1-\alpha)q_\perp)^2}$$

The "anomalous" term can only be assigned to the Higgs vertex:

$$\delta^{\rm (B)}_{\rm NLO} = \frac{\bar{\alpha}_s}{4\pi} \left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon} \left[\frac{2C_A}{\epsilon} + 4C_A\right]$$

Compare with non-Gribov contribution ...

$$\delta^{\rm n.G.}_{gq\to Hq} = g^2(-2C_A)B_0(q^2) = \frac{\bar{\alpha}_s}{4\pi} \left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon} \left[-\frac{2C_A}{\epsilon} - 4C_A\right]$$

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$$\frac{\mathcal{A}_B}{\Gamma^{ac(0)}_{gH}\left(\frac{2s}{t}\right)\Gamma^{c(0)}_{qq'}} = \frac{g^2 C_A t}{2} \int_{\alpha_0}^1 \frac{d\alpha}{\alpha}(1-\alpha)^2 (1+\alpha) \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-1}} \frac{1}{k_\perp^2 (k_\perp - (1-\alpha)q_\perp)^2}$$

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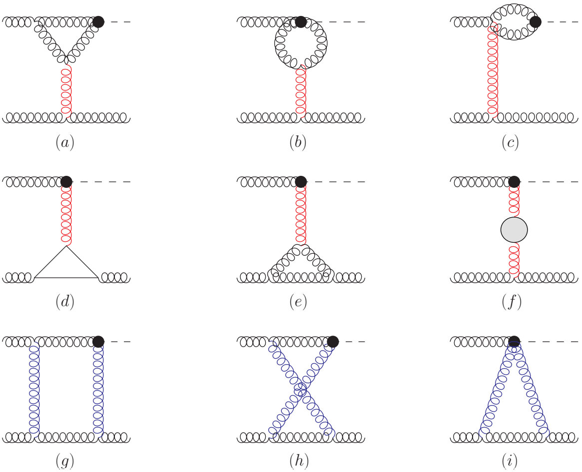
Gribov *vs* non-Gribov: $\mathcal{A}_{gg\to Hg}$ amplitude

Diffusion of a gluon off a gluon to produce a Higgs plus a gluon



Gribov *vs* non-Gribov: $\mathcal{A}_{gg\to Hg}$ amplitude

- Compare with the Regge form

$$\mathcal{A}^{(8,-)}_{gg\to Hg} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{bcd(0)}_{gg} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right)\right]\Gamma^{bcd(0)}_{gg} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(1)}_{gg} + \Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

- Extracted effective vertex same as from $\mathcal{A}_{gq\to Hq}$

$$\delta_{\rm NLO} \simeq \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left\{-\frac{C_A}{\epsilon^2} + \frac{11C_A - 2n_f}{6\epsilon} - \frac{5n_f}{9} + C_A\left(2\text{Li}_2\left(1+\frac{m_H^2}{\vec{q}^{\,2}}\right) + \frac{\pi^2}{3} + \frac{67}{18}\right)\right\}$$

- Non-Gribov contributions ...

$$\delta^{\rm n.G.}_{gg\to Hg} = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\frac{C_A}{4}\left[\frac{1}{\epsilon^2} - \frac{5}{\epsilon} - 9 - \zeta(2)\right] + \mathcal{O}(\epsilon)$$

... exactly compensate the “anomalous” term from the region B

$$\delta^{\rm (B)}_{\rm NLO} = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\frac{C_A}{4}\left[-\frac{1}{\epsilon^2} + \frac{5}{\epsilon} + 9 + \zeta(2)\right] + \mathcal{O}(\epsilon)$$

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$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right)+\ln\left(\frac{-s}{-t}\right)\right]\Gamma^{bcd(0)}_{gg}+\Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(1)}_{gg}+\Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

- Extracted effective vertex same as from $\mathcal{A}_{gq\to Hq}$

$$\delta_{\rm NLO} \simeq \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left\{-\frac{C_A}{\epsilon^2}+\frac{11C_A-2n_f}{6\epsilon}-\frac{5n_f}{9}+C_A\left(2{\rm Li}_2\left(1+\frac{m_H^2}{\vec{q}^{\,2}}\right)+\frac{\pi^2}{3}+\frac{67}{18}\right)\right\}$$

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$$\delta^{\rm n.G.}_{gg\to Hg} = \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\frac{C_A}{4}\left[\frac{1}{\epsilon^2}-\frac{5}{\epsilon}-9-\zeta(2)\right]+\mathcal{O}(\epsilon)$$

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$$\mathcal{A}^{(8,-)}_{gg\to Hg} = \Gamma^{ac}_{\{H\}g}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma^{bcd(0)}_{gg} \approx \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

$$+\Gamma^{ac(0)}_{\{H\}g}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right)\right]\Gamma^{bcd(0)}_{gg} + \Gamma^{ac(0)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(1)}_{gg} + \Gamma^{ac(1)}_{\{H\}g}\frac{2s}{t}\Gamma^{bcd(0)}_{gg}$$

- Extracted effective vertex same as from $\mathcal{A}_{gq\to Hq}$

$$\delta_{\rm NLO} \simeq \frac{\bar{\alpha}_s}{4\pi}\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^{-\epsilon}\left\{-\frac{C_A}{\epsilon^2} + \frac{11C_A - 2n_f}{6\epsilon} - \frac{5n_f}{9} + C_A\left(2{\rm Li}_2\left(1 + \frac{m_H^2}{\vec{q}^{\,2}}\right) + \frac{\pi^2}{3} + \frac{67}{18}\right)\right\}$$

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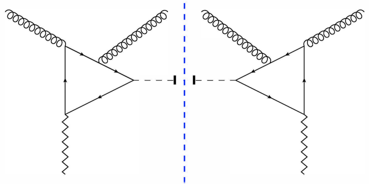
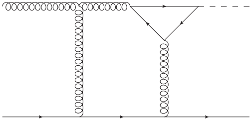
Next step: restoring physical top-mass dependence

- Real corrections with full top-mass dependence
  [F.G. Celiberto, L. Delle Rose, M. Fucilla, G. Gatto, A. Papa (in preparation)]



- Limit $m_t \to \infty$ checked

- Virtual corrections involve two-loop amplitudes with several scales



Conclusions and outlook

- The calculation of the impact factor for forward Higgs production in the infinite-top-mass limit turned out to be very challenging, due to the presence of a dimension-5 non-renormalizable effective interaction
- Unexpected terms beyond eikonal approximation (“non-Gribov” terms) appear, potentially jeopardizing the Regge form of one-loop amplitudes

  In fact, terms not in accordance with the Regge form in different diagrams finally cancel out

  Reggeization outweighs renormalizability?
- Unexpected terms arise also within eikonal approximation (“Gribov” part), in rapidity regions where all next-to-leading order corrections would naively be attributed to the impact factor of the backward produced particle
- Relieving cancellation among the two unexpected effects.
- Prospects:

  - restore physical top mass

  - applications to phenomenology

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