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Towards a consistent formulation of high energy factorization at NLO

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Outline

1. running coupling corrections & the dipole amplitude
2. Challenges
3. The tool: Lipatov's effective action (and a brief re-derivation)
4. Renormalization and results in the dilute limit
5. First results for the dense limit
6. Conclusions

Running coupling in the Wilson line

dilute-dense collisions e.g. $F_2(x, Q^2) \sim \int d^2r \int d^2b \int_0^1 dz \, |\psi(z, r, Q^2)|^2 N(x, r, b)$

with

$$\hat{N}(x, \mathbf{r}, \mathbf{b}) = \left\langle \frac{1}{N_c} \mathrm{tr} \left(\mathbb{1} - W(\mathbf{b} + \mathbf{r}/2) W^\dagger(\mathbf{b} - \mathbf{r}/2) \right) \right\rangle_x = \mathcal{O}(\alpha_s)$$

and

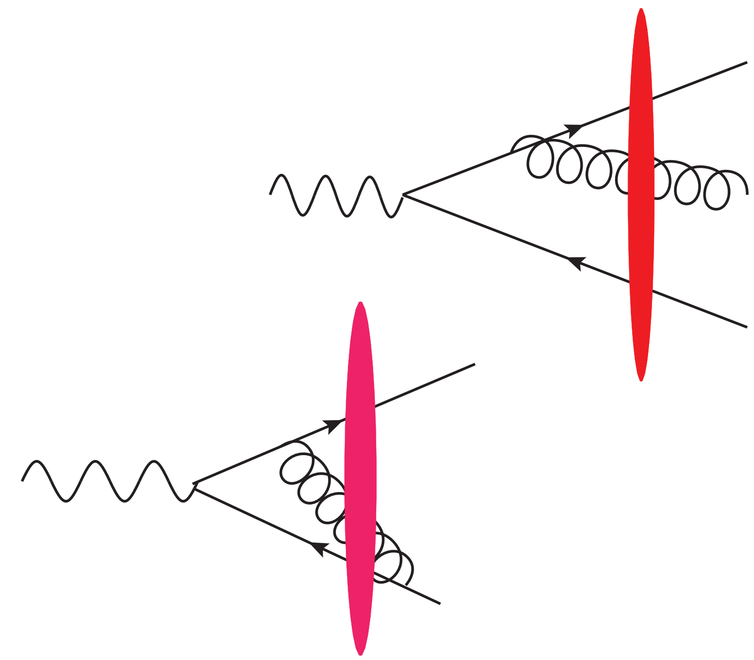
$$W(\boldsymbol{z}) = \mathrm{P} \exp \left(-\frac{g}{2} \int_{-\infty}^{\infty} dz^+ A_+ \right)$$

- Dilute limit: overall $\alpha_s(\mu)$ clearly part of virtual photon impact factor
  collinear limit: splitting function $P_{qg}(z)$
- renormalization scale: must be constrained by NLO (and higher order) corrections
- expect the same for high parton densities, so far not taken into account
- phenomenology: changes normalization (at NLO) of dipole amplitude and generalizations therefore

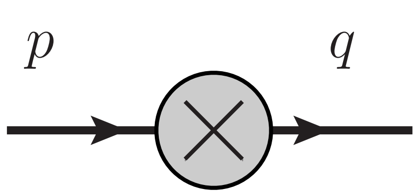
How to take this into account?

NLO corrections: 1st: perturbative corrections to the light-front wave function

real & virtual



but also to the interaction with the background field (=resummed propagator, Wilson lines, …) should receive a NLO corrections:



$$= \tau_F(q, -p) = 2\pi\delta(p^+ - q^+)\not{n}^+ \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})}$$

$$\cdot \Big[\theta(p^+)\,[W(\boldsymbol{z}) - 1] - \theta(-p^+)\,\big[W^\dagger(\boldsymbol{z}) - 1\big]\Big],$$

$$W(\boldsymbol{z}) = \mathrm{P}\exp\left(-\frac{g}{2}\int_{-\infty}^{\infty} dz^+ A_+\right)$$

natural if we start from interaction with single gluon & generalize to many

Challenges

From a technical point of view, this is not completely trivial ….

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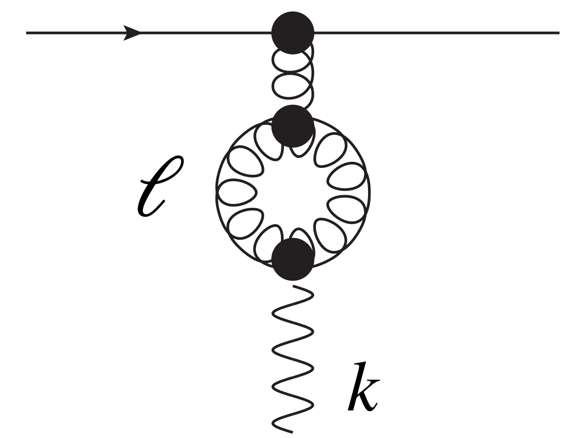

such corrections appear to be zero

$$I = \int \frac{dl^- d^+ d^{2+2\epsilon}\mathbf{l}}{(l^+ l^- - \mathbf{l}^2 + i0^+)(l^+(l^- - k^-) - (\mathbf{l} - \mathbf{k})^2 + i0^+)}$$

poles:

$$l^- = \frac{\mathbf{l}^2 - i0^+}{l^+}$$

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at same side → integral seems to vanish

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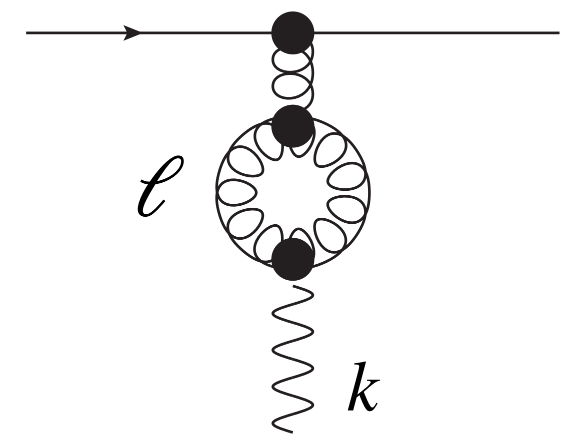

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→ one of the two results is wrong!

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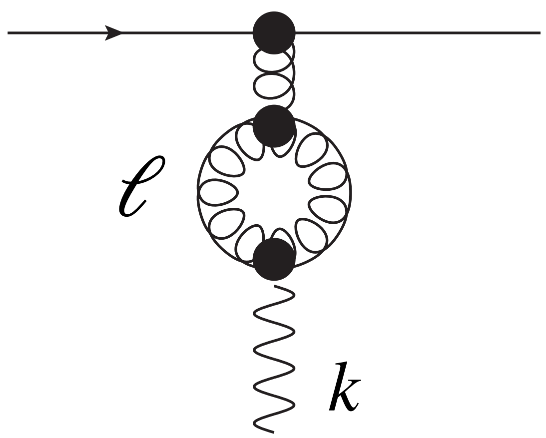

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$$l^- = \frac{(\mathbf{l}-\mathbf{k})^2 - i0^+}{l^+}$$ at same side → integral seems to vanish

BUT: conventional calculation ($d^d l$, Wick rotation etc.): yields finite result

→ one of the two results is wrong!

solution: $I \sim \delta(l^+)$ [Yan (1973)], [Heinzl (2003)] outlined in [Collins (2011)]

integrals **non-zero + UV divergent** (in the case of interest to us)

More challenges …

don't use cut-off regulator

if we regulate rapidity divergencies through tilting light-cone directions $n^+ \to n, n^2 \neq 0$

one finds integrals $\sim 1/n^2$ [Chachamis, MH, Madrigal, Sabio Vera; 1212.4992]

means: one cannot set $n^2 \to 0$ before evaluating integrals

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similar issues with alternative regulators $$\frac{1}{k^+ + i0^+} \to \frac{1}{k^+ + I\Delta}$$

used for TMDs eg. [Echevarria, Idilbi, Scimemi; 1111.4996]

even though $\Delta \to 0$, one needs $$\frac{l^+}{l^+ + i\Delta} = 1 - \frac{i\Delta}{l^+ + i\Delta} \neq 1$$

since $$\int d^d l \frac{\Delta}{l^2(l-k^2)(l^+ + i\Delta)} = \text{ finite}$$

cannot use simplified theory before the integral is evaluated

The tool for our study: Lipatov's effective action

Tool to be used for this study

Lipatov's high energy effective action [Lipatov; hep-ph/9502308]

✗ original derivation slightly opaque
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✓ leading order Balitsky JIMWLK evolution contained [MH, 1802.06755]

✓ NLO corrections well understood in the dilute limit *e.g.*

- forward jets and trajectory [MH, Sabio Vera; 1110.6741][Chachamis, MH, Madrigal, Sabio Vera;1202.0649, 1212.4992,1307.2591], forward jets with rapidity gap [MH, Madrigal, Murdaca, Sabio Vera, 1404.2937, 1406.5625, 1409.6704]

- forward Higgs [MH, Kutak, van Hameren, arXiv:2011.03193]

- TMD factorization [MH, 2107.06203]

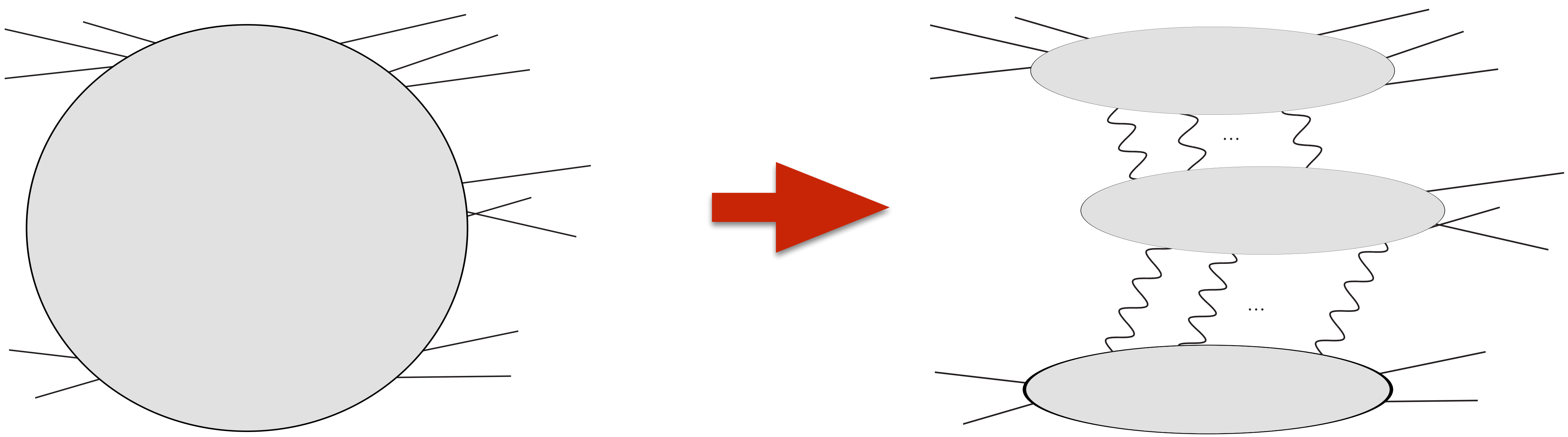
with correct UV properties (agrees with limits of scattering amplitudes etc),

Note: does not agree with results on NLO forward jets/hadrons within shockwave picture e.g.
[Chirilli, Xiao, Yuan; 1203.6139],
[Altinoluk, Armesto, Beuf, Kovner, Lublinsky; 1411.2869]

Brief derivation of Lipatov's action

first presented in [MH, Gomez Bock, Sabio Vera; 2010.03621]
for electroweak theory

basic idea:

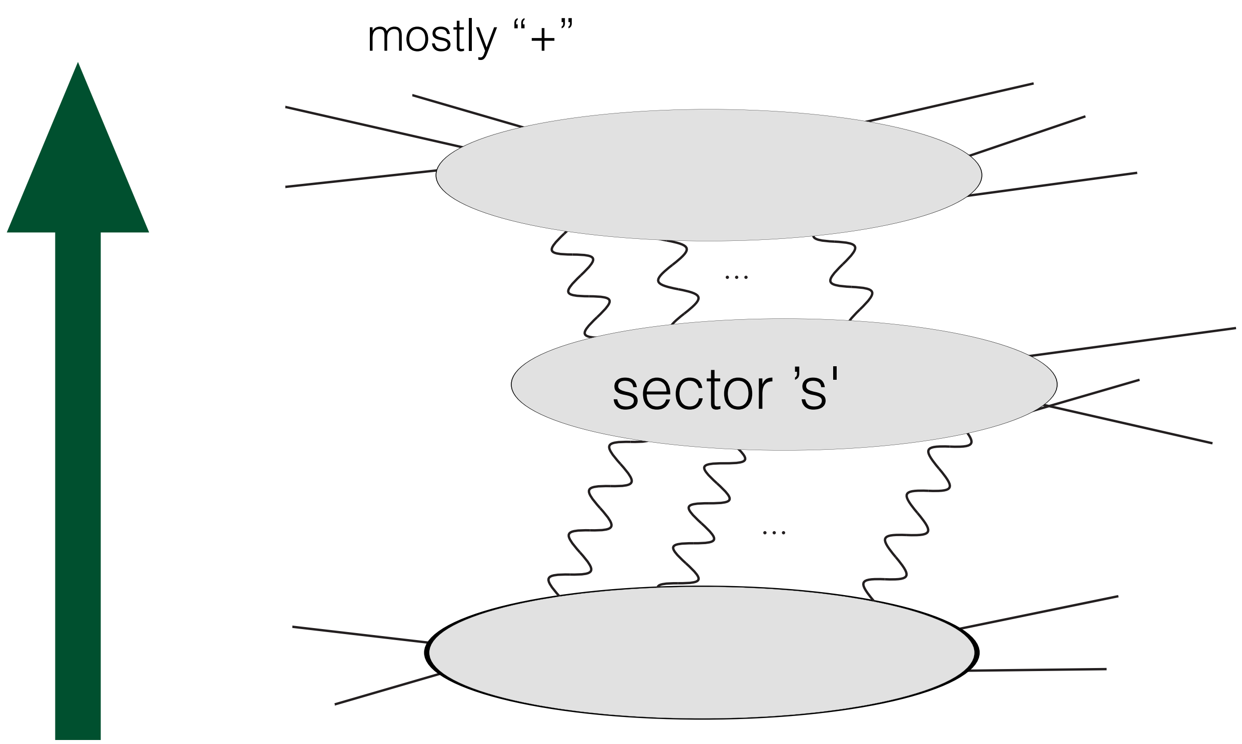


group particles/fields into clusters local in rapidity

& study their interactions with fields with significantly different rapidity
study behavior of vector fields under boosts

Boosting fields

the usual results everyone knows



rapidity $\eta = \frac{1}{2}\ln\frac{k^+}{k^-}$

- can be also done for fermions and scalars (if needed)
- allows in general for a systematic expansion
- Lipatov action: the leading term

pick a sector & boost the sources in other sectors

- fields connecting to sources in same sector are not modified $V_\mu^s \sim 1$
- fields connecting to a source boosted in +/- direction

$$V_+^{\eta>\eta(s)} \sim V_-^{\eta<\eta(s)} \sim 1$$

plus/minus component is leading

$$V_{\mu,\perp}^{\eta>\eta(s)} \sim V_{\mu,\perp}^{\eta<\eta(s)} \sim e^{-|\eta-\eta(s)|}$$

$$V_-^{\eta>\eta(s)} \sim V_+^{\eta<\eta(s)} \sim e^{-2|\eta-\eta(s)|}$$

transverse/conjugate light-cone directions are suppressed

Constructing the action

for fields connecting to boosted sources: dependence on light-cone time is frozen for $\eta \to \pm\infty$

$$V_+^{\eta>\eta_l}(x) = V_+^{\eta>\eta_l}(x_+ e^{\eta_l - \eta}, x_-, x_\perp) \simeq V_+^{\eta>\eta_l}(0, x_-, x_\perp)$$
$$V_-^{\eta<\eta_l}(x) = V_-^{\eta<\eta_l}(x_+, x_- e^{\eta - \eta_l}, x_\perp) \simeq V_-^{\eta<\eta_l}(x_+, 0, x_\perp)$$

for the chosen convention: $\partial_- V_\mu^{\eta>\eta_l}(x) = 0 = \partial_+ V_\mu^{\eta<\eta_l}(x)$

Constructing the action

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what does this imply for the local QCD action?

- the two point function of a field connection to a local and a field connecting to a boosted source vanishes
- to construct the action: remove those terms
- in practice: need to decompose gluonic field into local/non-local components

$$v_\mu(x) = V_\mu^{local(s)} + \frac{n^+}{2} V_+^{\eta>\eta(s)} + \frac{n^-}{2} V_-^{\eta<\eta(s)}$$

and remove the bilinear terms

Constructing the action

In practice: more economic to write down the action for the complete field $v_\mu(x)$ and the non-local field

$$S^{(s)} = \int d^4x \mathcal{L}_{\text{QCD}}[v_\mu, \psi, \bar{\psi}] + \int d^4x \, \text{tr}\left[\left(v_- - V_-^{\eta<\eta(s)}\right)\partial^2 V_+^{\eta>\eta(s)}\right] + \int d^4x \, \text{tr}\left[\left(\underbrace{v_+ - V_+^{\eta>\eta(s)}}_{\text{“local" field}}\right)\partial^2 V_-^{\eta<\eta(s)}\right]$$

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$$+ \int d^4x \,{\rm tr}\left[\underbrace{\left(v_+ - V_+^{\eta>\eta(s)}\right)}_{\text{“local" field}}\partial^2 V_-^{\eta<\eta(s)}\right]$$

last steps:

- promote non-local fields $V_\pm$ to gauge invariant fields $\delta_L A_\pm = 0$ (the reggeized gluon fields)
- finally replace $v_\pm(x) \to v_\pm(x) U[v_\pm(x)] = -\frac{1}{g}\partial_\pm U[v_\pm(x)]$

to arrive at gauge invariant local action

$$S_{eff} = S_{QCD}[v_\mu, \psi, \bar\psi] + \int d^4x \;{\rm tr}\left[(v_- U[v_-] - A_-)\,\partial^2 A_+\right] + ('+' \leftrightarrow '-')$$

= action presented in [Lipatov; hep-ph/9502308]
& used for many calculations

Remarks on the regulator

$$S_{eff} = S_{QCD}[v_\mu, \psi, \bar{\psi}] + \int d^4x \ \mathrm{tr}\left[(v_- U[v_-] - A_-)\, \partial^2 A_+\right] + ('+' \leftrightarrow '-')$$

- action is gauge invariant, without invoking $n_\pm^2 = 0$
- tilting light cone directions is therefore a gauge invariant deformation of the original action
- good regulator

UV renormalization

original publication does not address this problem

QCD fields:

$$\psi_{\text{bare}} = Z_2^{\frac{1}{2}} \psi_R, \qquad v^{\mu}_{\text{bare}} = Z_3^{\frac{1}{2}} v^{\mu}_R$$

couplings: $g_{\text{bare}} = Z_g \mu^{-\epsilon} g_R, \qquad Z_1 = Z_g Z_2 Z_3^{\frac{1}{2}}$

have in mind $\overline{MS}$ scheme; as a start: massless theory

reggeized gluon field:

$$S_{eff} = S_{QCD}[v_\mu, \psi, \bar{\psi}] + \int d^4x \ \text{tr} \left[\underbrace{(v_- U[v_-] - A_-)\, \partial^2 A_+} \right] + ('+' \leftrightarrow '-')$$

removes bilinear term if $v_\pm \to v_\pm + A_\pm$

requires $A_{\pm,\text{bare}} = Z_3^{\frac{1}{2}} A_{\pm,R}$

also coupling in the path ordered exponential $U[v_-]$ should be renormalized

Rescaling fields

Commonly done:

$$\psi_R, \bar{\psi}_R \to Z_2^{-\frac{1}{2}} \psi_R, Z_2^{-\frac{1}{2}} \bar{\psi}_R, v_R^\mu \to Z_3^{-\frac{1}{2}} v_R^\mu$$

similar: $A_{\pm,R} \to Z_3^{-\frac{1}{2}} A_{\pm,R}$

advantage: only need one counter-term related to the coupling constant (and masses for massive theory)

disadvantage: individual correlation functions are not finite (usually not a problem)

here: rescale QCD field as usually

scheme 1: reggeized gluon field NOT rescaled (counter term for 2 point function)

scheme 2: reggeized gluon field rescaled (no counter term for 2 point function)

Testing ground: dilute calculation

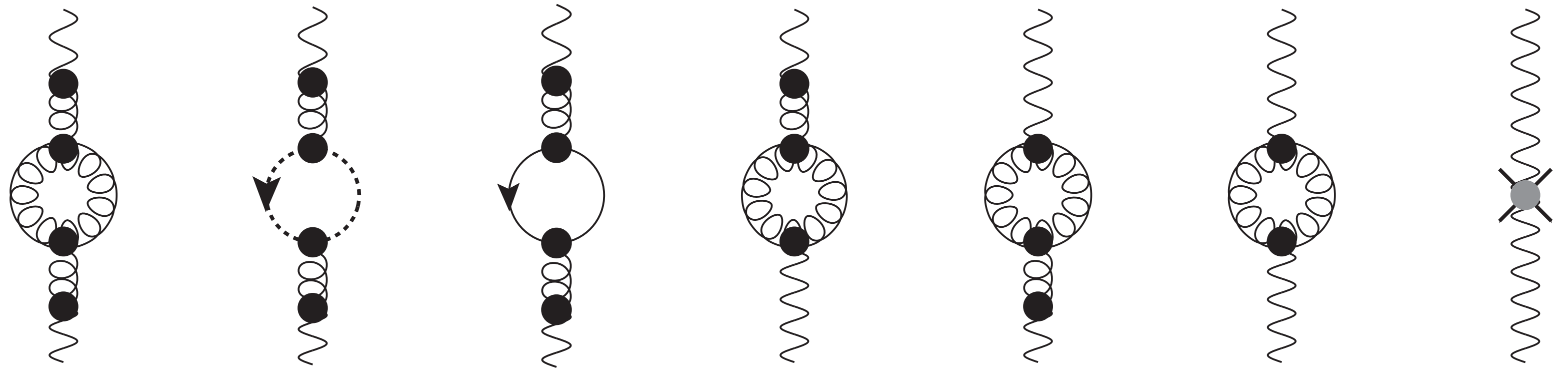
loop integrals etc. already done in

[MH, Sabio Vera; 1110.6741]
[Chachamis, MH, Madrigal, Sabio Vera;1202.0649, 1212.4992,1307.2591]

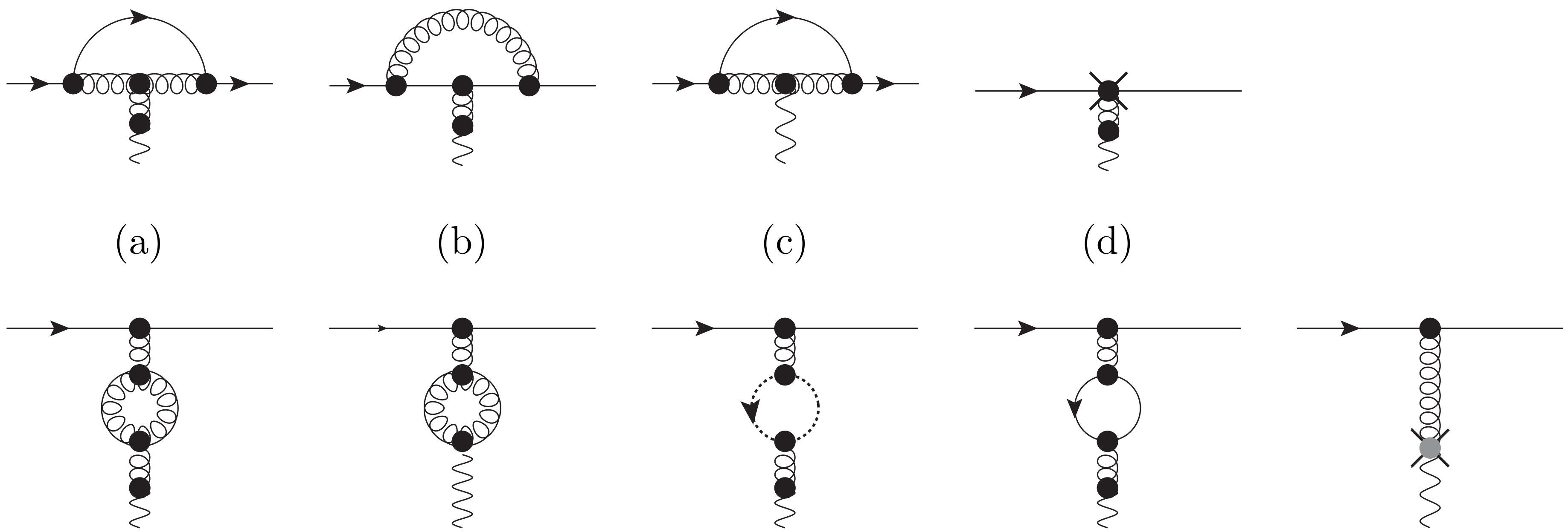
tilt light cone directions $n^{\pm} \to n, \bar{n} = n^{\pm} + e^{-\rho} n^{\mp}$

goal: focus on UV terms

$$\frac{2i}{\boldsymbol{k}^2}\Sigma^{(1)}\left(\rho, \epsilon, \frac{\boldsymbol{k}^2}{\mu^2}\right) = \frac{\alpha_s}{4\pi}\left(\frac{\boldsymbol{k}^2}{\mu^2}\right)^{\epsilon}\left[-\frac{C_A(2\rho - i\pi)}{\epsilon} - \frac{1}{\epsilon}\left(\frac{5C_A}{3} - \frac{2n_f}{3}\right) + \frac{31C_A}{9} - \frac{10n_f}{9} + \mathcal{O}(\epsilon)\right] + \left[\left(\frac{5}{3}C_a - \frac{2}{3}n_f\right)\frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\right]_{\text{scheme 1}}.$$



Testing: quark vertex



$$\frac{h^{(1)}\left(\rho,\epsilon;\boldsymbol{k}^2,\mu^2\right)}{h_q^{(0)}(\boldsymbol{k})} = \frac{\alpha_s}{2\pi}\left(\frac{\boldsymbol{k}^2}{\mu^2}\right)^{\epsilon}\Bigg[2C_F\left(-\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}-4+\frac{\pi^2}{6}\right)+C_A\Bigg(-\frac{8}{3\epsilon} - \frac{2}{\epsilon}\ln\left(e^{\rho/2}\frac{p_a^+}{|\boldsymbol{k}|}\right)+\frac{58}{9}+\frac{\pi^2}{2}\Bigg)+n_f\left(\frac{2}{3\epsilon}-\frac{10}{9}\right)+\frac{\beta_0}{2\epsilon}+\left[\frac{5C_A}{6\epsilon}-\frac{2n_f}{6\epsilon}\right]_{\text{scheme 1}}$$

within scheme 1: both elements are finite
scheme 2: uncanceled UV divergences (as expected)

Combining

a) subtract factorized contribution from NLO quark reggeized gluon vertex

$$C_q^B\left(\rho, \epsilon; \boldsymbol{k}^2, \mu^2\right) = h_q^B\left(\rho, \epsilon; \boldsymbol{k}^2, \mu^2\right) - 2h_q^{(0)}(\boldsymbol{k})\frac{2i}{\boldsymbol{k}^2}\Sigma^{(1)}\left(\rho; \epsilon, \frac{\boldsymbol{k}^2}{\mu^2}\right)$$

b) bare reggeized gluon Green's function

$$\hat{G}^B\left(\rho; \epsilon, \boldsymbol{k}^2, \mu^2\right) = \frac{2i}{\boldsymbol{k}^2}G^B\left(\rho; \epsilon, \frac{\boldsymbol{k}^2}{\mu^2}\right)$$

$$G^B\left(\rho; \epsilon, \frac{\boldsymbol{k}^2}{\mu^2}\right) = \left\{1 + \frac{2i}{\boldsymbol{k}^2}\Sigma\left(\rho, \epsilon, \frac{\boldsymbol{k}^2}{\mu^2}\right) + \left[\frac{2i}{\boldsymbol{k}^2}\Sigma\left(\rho, \epsilon, \frac{\boldsymbol{k}^2}{\mu^2}\right)\right]^2 + \ldots\right\}$$

c) cross-section independent ρ at NLO, individual elements are ρ dependent

$$\left.\frac{d\hat{\sigma}}{d^{2+2\epsilon}\boldsymbol{k}}\right|_{\text{virt.}} = C_q^B\left(\rho, \epsilon; p_a^+, \boldsymbol{k}^2, \mu^2\right) \cdot \left|G^B\left(\rho; \epsilon, \frac{\boldsymbol{k}^2}{\mu^2}\right)\right|^2 C_q^B\left(\rho, \epsilon; p_b^-, \boldsymbol{k}^2, \mu^2\right)$$

Renormalization for rapidity divergencies

transition function $Z^{\pm}$ to obtain finite elements:

$$G^R\left(\eta,\epsilon;\frac{\boldsymbol{k}^2}{\mu^2}\right) = \frac{G^B\left(\rho,\epsilon;\frac{\boldsymbol{k}^2}{\mu^2}\right)}{Z^+\left(\eta,\rho;\epsilon,\frac{\boldsymbol{k}^2}{\mu^2}\right)Z^-\left(\eta,\rho;\epsilon,\frac{\boldsymbol{k}^2}{\mu^2}\right)}$$

$$C_q^R\left(\eta,\epsilon;p_{a,b}^{\pm},\boldsymbol{k}^2,\mu^2\right) = \left[Z^{\pm}\left(\eta,\rho;\epsilon,\frac{\boldsymbol{k}^2}{\mu^2}\right)\right]^2\cdot C_q^B\left(\rho,\epsilon;p_{a,b}^{\pm},\boldsymbol{k}^2,\mu^2\right)$$

NLO cross-section:

$$\left.\frac{d\hat{\sigma}}{d^{2+2\epsilon}\boldsymbol{k}}\right|_{\text{virt.}} = C_q^R\left(\eta,\epsilon;p_a^+,\boldsymbol{k}^2,\mu^2\right)\cdot\left|G^R\left(\eta;\epsilon,\frac{\boldsymbol{k}^2}{\mu^2}\right)\right|^2 C_q^R\left(\eta,\epsilon;p_b^-\boldsymbol{k}^2,\mu^2\right)$$

$$Z^{\pm}\left(\eta,\rho;\epsilon,\frac{\boldsymbol{q}^2}{\mu^2}\right) = \exp\left[\frac{\rho-\eta}{2}\omega\left(\epsilon,\frac{\boldsymbol{q}^2}{\mu^2}\right)+f^{\pm}\left(\epsilon,\frac{\boldsymbol{q}^2}{\mu^2}\right)\right]$$

gluon trajectory:

$\boldsymbol{\rho}$ independent/finite term must fix UV finiteness

'Finite' term in the transition function

fixed by symmetry: [MH, Sabio Vera; 1110.6741]
[Chachamis, MH, Madrigal, Sabio Vera;1202.0649, 1212.4992,1307.2591]

for scheme 2: must be such that UV divergencies are cancelled between Green's function & vertex

$$f^{(1)}\left(\epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right) = \frac{\alpha_s}{4\pi}\left(\frac{2n_f - 5C_A}{6}\ln\frac{\boldsymbol{k}^2}{\mu^2} + \frac{31C_A - 10n_f}{18} + \left[\frac{2n_f - 5C_A}{6\epsilon}\right]_{\text{scheme 2}}\right)$$

finally:

$$\frac{d}{d\eta}G^R\left(\eta;\epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right) = \omega\left(\epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right)G^R\left(\eta;\epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right)$$

reggeized gluon:

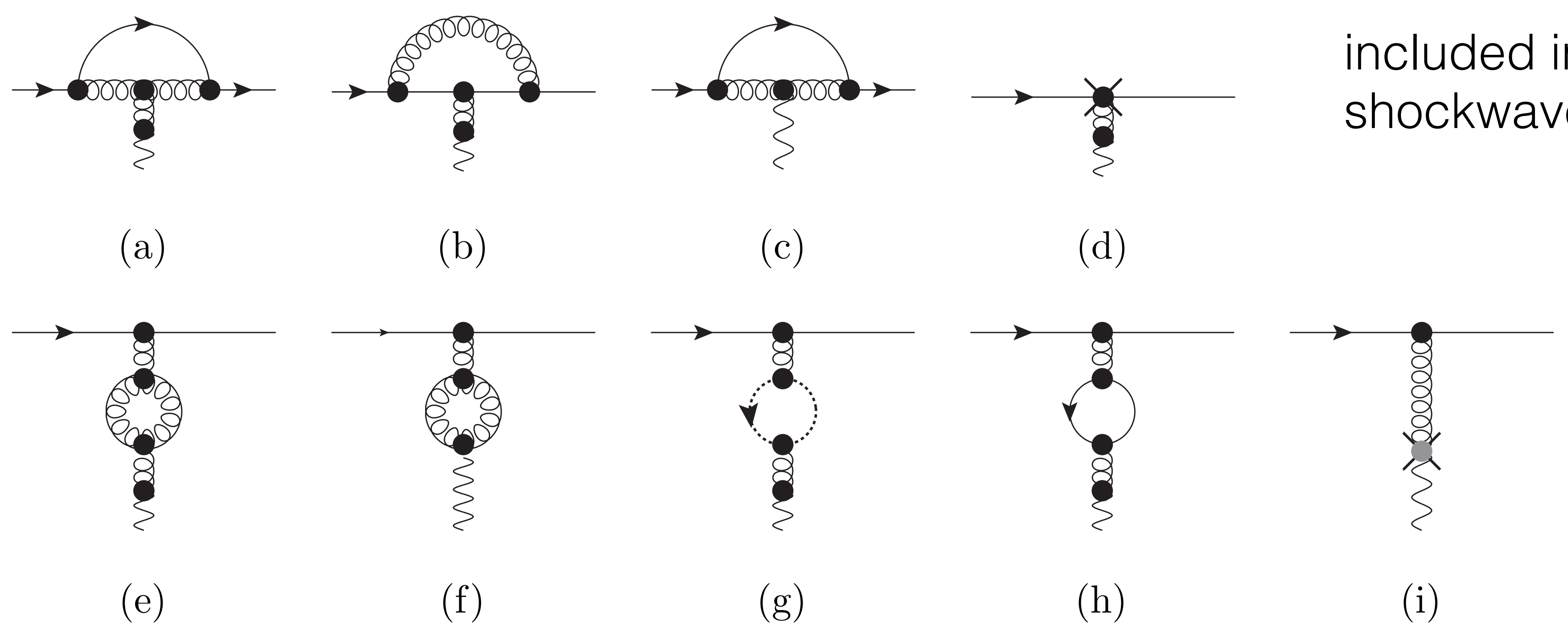
$$G^R\left(\eta;\epsilon, \frac{\boldsymbol{q}^2}{\mu^2}\right) = \exp\left[\eta\cdot\omega^{(1)}\left(\epsilon, \frac{\boldsymbol{k}^2}{\mu^2}\right)\right]$$

(solution to RG equation for $A_\pm$ field)

What about the shock wave picture?

used for strong fields $gA_+ \sim 1$, equally valid for dilute $gA_+ \ll 1$

idea: separate NLO corrections into
- formation of the partonic wave (1 or 2 partons)
- interaction with the shockwave



included in the shockwave picture

belongs somehow to the target

effective action does not support this separation of contributions

A test: use different gauges

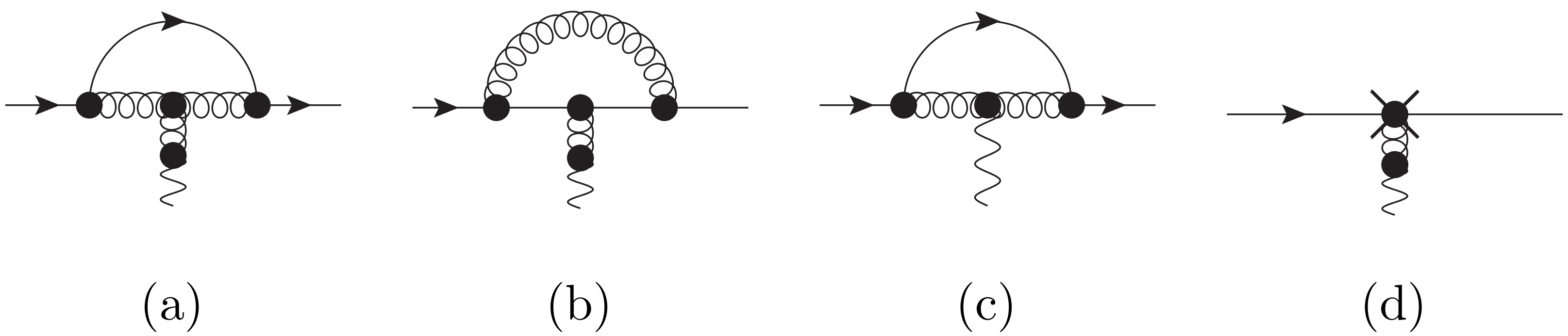
1) covariant (Lorentz-Feynman) gauge
2) axial gauge (not light-cone due to tilted directions)

$$d_{\mu\nu}(l,n) = -g_{\mu\nu} + \frac{l^\mu n^\nu + n^\mu l^\nu}{l \cdot n} - \frac{n^2 l^\mu l^\nu}{(l \cdot n)^2}$$

some diagrams yield different results

$$\text{Fig. 4.a + Fig. 4.c}\Big|_{\text{cov.}} = \frac{\alpha_s C_A}{4\pi}\left(\frac{\boldsymbol{k}^2}{\mu^2}\right)^\epsilon \left[\frac{-1}{\epsilon^2} + \frac{1}{2\epsilon} - \frac{2}{\epsilon}\ln\left(e^{\rho/2}\frac{p_a^+}{|\boldsymbol{k}|}\right) + \frac{2\pi^2}{3} - 1\right] + \mathcal{O}(\epsilon),$$

$$\text{Fig. 4.a + Fig. 4.c}\Big|_{\text{axial}} = \frac{\alpha_s C_A}{4\pi}\left(\frac{\boldsymbol{k}^2}{\mu^2}\right)^\epsilon \left[\frac{-1}{\epsilon^2} + \frac{5}{2\epsilon} - \frac{2}{\epsilon}\ln\left(e^{\rho/2}\frac{p_a^+}{|\boldsymbol{k}|}\right) + \frac{2\pi^2}{3} - 1\right] + \mathcal{O}(\epsilon).$$



- differences cancel if complete set of corrections is considered
- not true for individual “shock-wave” and “self-energy” diagrams
- there is no gauge invariant separation of both contributions

Good news: not everything is lost

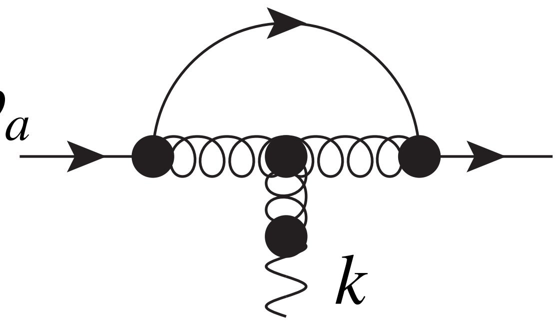
shockwave like/external contributions and remainder correspond to different types of loop integrals

- shockwave like integrals (as obtained within light front perturbation theory): loop integrals which contain a momentum with $n \cdot p \neq 0$
  - yields eikonalization (reggeized field resummed through Wilson line)
  - no $1/n^2$ terms

- bubble like integrals: all external momenta in the loop $n \cdot p = 0$
  - no eikonalization possible (only a single A_+ couples to the loop)
  - can yield $1/n^2, 1/(n^2)^2$ etc.
  - care is needed with the limit $n^2 \to 0$
  - seem to vanish (but they don't)

BUT: separation not possible on a diagrammatic basis & not a consequence of high energy factorization

Reason



contains integrals

$$\int \frac{d^d l\,(l\cdot p_a)^m}{l^2(l+k)^2(l\cdot n)^{a_1}}, \quad k\cdot n = 0$$

- UV divergent
- absent if one ignores (incorrectly) the $\delta(l^+)$ configuration
- but you can’t do that; it’s non-zero

my procedure:
- identify all those contributions
- evaluate remainder using more conventional methods (but using consistent regulator i.e. tilting)

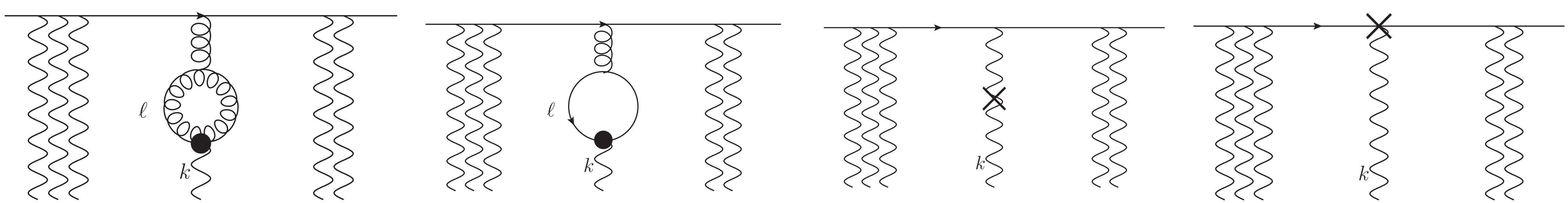
First result for the dilute dense case

Finally $gA_+ \sim 1$

Lipatov's effective action allows for resummation of strong field for arbitrary gauge
[MH, 1802.06755]

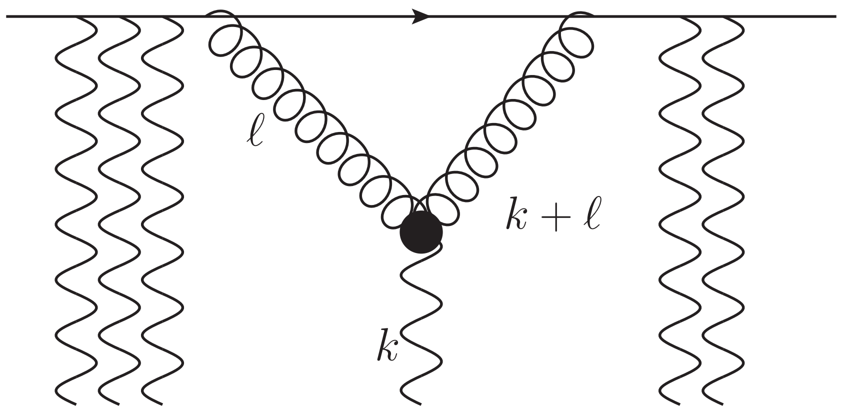
NLO: requires new set of induced vertices (non-local shifted version) or cancellations between individual diagrams

both problems are absent for axial gauge → use that + shift $v_\pm \to v_\pm + A_\pm$
also not free of issues, but can be overcome



- evaluation of self-energy diagrams (and counter-terms) relatively straightforward

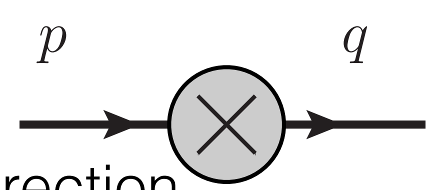
UV terms with quark propagator



- more problematic; naively absent (“emission inside the shockwave”)
- possible to isolate “bubble" configuration in triangle diagram using Ward like identities

Result

for the quasi-elastic correction



$$= \tau_F(q,-p) = 2\pi\delta(p^+ - q^+) n\!\!\!/^+ \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})}$$

$$\cdot \left[\theta(p^+)\left[W(\boldsymbol{z}) - 1\right] - \theta(-p^+)\left[W^\dagger(\boldsymbol{z}) - 1\right]\right],$$

$$W(\boldsymbol{z}) = W^{(0)}(\boldsymbol{z})\Bigg\{1 + \frac{\alpha_s C_A}{4\pi}\left[\left(-\frac{8}{3} + \frac{2n_f}{3C_A}\right)\ln\frac{4e^{-2\gamma_E}}{\mu^2\delta} + \frac{49}{9} - \frac{10n_f}{9C_A}\right]$$
$$+ \frac{\alpha_s C_A}{4\pi}\left(\frac{8}{3} - \frac{2n_f}{3C_A}\right)\sum_{n=0}^{\infty}\frac{1}{n!}\sum_{k=0}^{n-1}\left[ig\alpha^a(\boldsymbol{z})t^a\right]^k$$
$$\int \frac{d^2\boldsymbol{x}}{\pi}\frac{\theta\left[(\boldsymbol{x}-\boldsymbol{z})^2 - \delta\right]}{(\boldsymbol{x}-\boldsymbol{z})^2} ig\alpha^c(\boldsymbol{x})t^c\left[ig\alpha^b(\boldsymbol{z})t^b\right]^{n-k-1}\Bigg\}$$

Fourier transform of transverse log: [Diehl, Ostermeier, Schäfer; 1111.0910]

$$\alpha^a(\boldsymbol{z}) = \frac{1}{2}\int dx^+ A_+^a(x^+,\boldsymbol{z}) = \frac{1}{gN_c} f^{abc}\log U^{bc}(\boldsymbol{z})$$

[Caron-Huot; 1309.6521]
[MH,1802.06755]

using scheme 1 (otherwise uncanceled UV pole)

still lacks “shock wave integrals”; yields essentially result in the literature (+ tilted regulator) [Chirilli, Xiao, Yuan; 1203.6139], [Altinoluk, Armesto, Beuf, Kovner, Lublinsky; 1411.2869]

central corrections (UV)

1st naive attempt: follow[MH, 1802.06755] and determine fluctuations of Wilson line

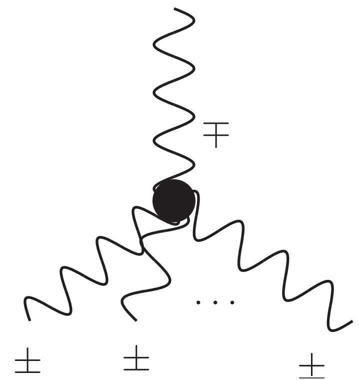
→ not what the action tells you to do; does not work (uncanceled UV divergencies)

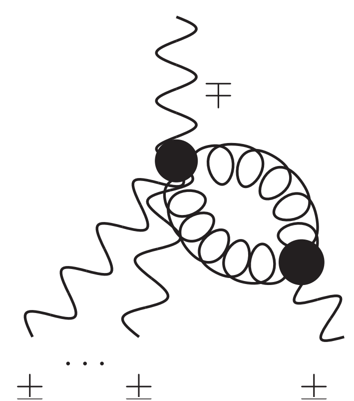
correct procedure:

- calculate corrections to *n* reggeized gluon state
- as far as bubble/UV configurations are concerned, this is straightforward

important observation:

- generalizing pole prescription of eikonal denominators developed in [MH, 1112.4509] to *n* reggeized gluons, the $A_\pm \to (A_\pm)^n$ transition vanishes after integration over longitudinal momenta
- also the counter-term & associated self-energy correction vanish
- same applies to the loop correction to this vertex

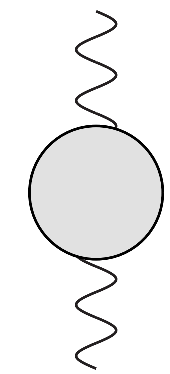




UV configuration: reggeized gluon self energy

only relevant configuration (for UV):

- self energy of the reggeized gluon *without* ρ dependent (rapidity divergent) term
- the latter: not a “bubble" configuration, allows for eikonalization + needs to be combined with other corrections



for scheme 1
in momentum space:

$$\frac{\alpha_s}{4\pi}\left[-\left(\frac{5C_A}{3}-\frac{2n_f}{3}\right)\ln\left(\frac{\mathbf{k}^2}{\mu^2}\right)+\frac{31C_A}{9}-\frac{10n_f}{9}+\mathcal{O}(\epsilon)\right]$$

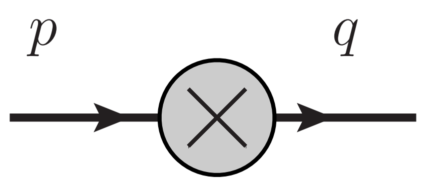
Remainder: need to construct “shockwave like correction” for the *n* reggeized gluon state (can’t use directly the Wilson line as in [MH, 1802.06755])

expect: JIMWLK evolution at 1-loop (to be confirmed, work in progress)

next step: combine central/factorized and quasi-elastic correction using subtraction + transition function

requires complete 1-loop correction to *n* reggeized gluon state; but can anticipate what happens to the UV terms (essentially the same as for the dilute case)

Preliminary final result for UV enhanced terms



$$= \tau_F(q,-p) = 2\pi\delta(p^+ - q^+)\not{n}^+ \int d^2\boldsymbol{z} e^{i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})}$$

$$\cdot \left[\theta(p^+)\,[W(\boldsymbol{z})-1] - \theta(-p^+)\left[W^\dagger(\boldsymbol{z})-1\right]\right],$$

$$\begin{aligned}
W(\boldsymbol{z}) = W^{(0)}(\boldsymbol{z})\Bigg\{ & 1 - \frac{\alpha_s}{4\pi}\left[\frac{\beta_0}{2}\ln\frac{4e^{-2\gamma_E}}{\mu^2\delta} - \frac{67C_A}{18} + \frac{10n_f}{18}\right] \\
& + \frac{\alpha_s C_A}{4\pi}\frac{\beta_0}{2}\sum_{n=0}^{\infty}\frac{1}{n!}\sum_{k=0}^{n-1}[ig\alpha^a(\boldsymbol{z})t^a]^k \\
& \qquad \int\frac{d^2\boldsymbol{x}}{\pi}\frac{\theta\left[(\boldsymbol{x}-\boldsymbol{z})^2-\delta\right]}{(\boldsymbol{x}-\boldsymbol{z})^2} ig\alpha^c(\boldsymbol{x})t^c\left[ig\alpha^b(\boldsymbol{z})t^b\right]^{n-k-1}\Bigg\} \\
= \exp\Bigg[& ig\alpha^a(\boldsymbol{z})t^a\left\{1 - \frac{\alpha_s}{4\pi}\left[\frac{\beta_0}{2}\ln\frac{4e^{-2\gamma_E}}{\mu^2\delta} - \frac{67C_A}{18} + \frac{10n_f}{18}\right]\right\} \\
& + \frac{\alpha_s}{4\pi}\frac{\beta_0}{2}\int\frac{d^2\boldsymbol{x}}{\pi}\frac{\theta\left[(\boldsymbol{x}-\boldsymbol{z})^2-\delta\right]}{(\boldsymbol{x}-\boldsymbol{z})^2} ig\alpha^c(\boldsymbol{x})t^c\Bigg] + \mathcal{O}(\alpha_s^2)
\end{aligned}$$

Final result for UV enhanced terms

note:

$$W^{(0)}(\mathbf{z}) = \exp\left(ig\alpha^a(\mathbf{z})t^a\right), \quad \alpha^a(\mathbf{z}) = \frac{2}{ig}\,\mathrm{tr}\left[t^a \ln W^{(0)}(\mathbf{z})\right]$$

the Wilson line enters somehow the optimal scale of the running coupling

$$\begin{aligned}
W(\boldsymbol{z}) &= W^{(0)}(\boldsymbol{z})\left\{1 - \frac{\alpha_s}{4\pi}\left[\frac{\beta_0}{2}\ln\frac{4e^{-2\gamma_E}}{\mu^2\delta} - \frac{67C_A}{18} + \frac{10n_f}{18}\right]\right. \\
&\quad + \frac{\alpha_s C_A}{4\pi}\frac{\beta_0}{2}\sum_{n=0}^{\infty}\frac{1}{n!}\sum_{k=0}^{n-1}\left[ig\alpha^a(\boldsymbol{z})t^a\right]^k \\
&\qquad \left.\int\frac{d^2\boldsymbol{x}}{\pi}\frac{\theta\left[(\boldsymbol{x}-\boldsymbol{z})^2-\delta\right]}{(\boldsymbol{x}-\boldsymbol{z})^2} ig\alpha^c(\boldsymbol{x})t^c\left[ig\alpha^b(\boldsymbol{z})t^b\right]^{n-k-1}\right\} \\
&= \exp\left[ig\alpha^a(\boldsymbol{z})t^a\left\{1 - \frac{\alpha_s}{4\pi}\left[\frac{\beta_0}{2}\ln\frac{4e^{-2\gamma_E}}{\mu^2\delta} - \frac{67C_A}{18} + \frac{10n_f}{18}\right]\right\}\right. \\
&\qquad \left. + \frac{\alpha_s}{4\pi}\frac{\beta_0}{2}\int\frac{d^2\boldsymbol{x}}{\pi}\frac{\theta\left[(\boldsymbol{x}-\boldsymbol{z})^2-\delta\right]}{(\boldsymbol{x}-\boldsymbol{z})^2} ig\alpha^c(\boldsymbol{x})t^c\right] + \mathcal{O}(\alpha_s^2)
\end{aligned}$$

Discussion

$$\exp\left[ig\alpha^a(\boldsymbol{z})t^a\left\{1-\frac{\alpha_s}{4\pi}\left[\frac{\beta_0}{2}\ln\frac{4e^{-2\gamma_E}}{\mu^2\delta}-\frac{67C_A}{18}+\frac{10n_f}{18}\right]\right\}\right.$$
$$\left.+\frac{\alpha_s}{4\pi}\frac{\beta_0}{2}\int\frac{d^2\boldsymbol{x}}{\pi}\frac{\theta\left[(\boldsymbol{x}-\boldsymbol{z})^2-\delta\right]}{(\boldsymbol{x}-\boldsymbol{z})^2}ig\alpha^c(\boldsymbol{x})t^c\right] \; -1$$

- Can we absorb these corrections into the “target”?
  Technically (I believe) yes: a special scheme (a special choice for the “f" function)

  Argument against: depends on the “projectile" renormalization scale
  Also: needs this for correct anomalous dimension of TMD operator
  Effective action: “projectile" not “target" correction

[MH, 2107.06203]

- In this work we separated UV and conventional shock wave contributions through different characteristics of look integrals
  Question: is there a more general organizing principle?

Discussion & Conclusion

- We have many impressive NLO results, e.g.

Caucal, Salazar, Schenke, Stebel, Venugopalan; 2308.00022
Beuf, Lappi, Mäntysaari, Paatelainen, Penttala, 2401.17251
Boussarie, Grabovsky, Wallon; 1905.07371
Bergabo, Jalilian-Marian; 2301.03117
Balitsky, Chirilli; 1207.3844
….

but to my understanding, high energy factorization at NLO with a dense target is not yet fully worked out;

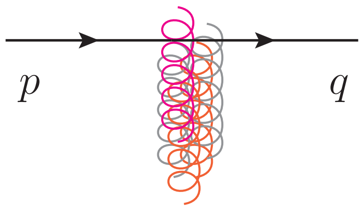
- My take home message: Don't ignore UV divergencies, even if they do not seem to manifest, they are there & should be taken into account

- Still in work in progress;

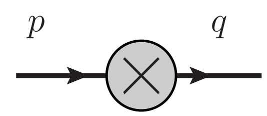
Appendix

Tool: propagators in background field

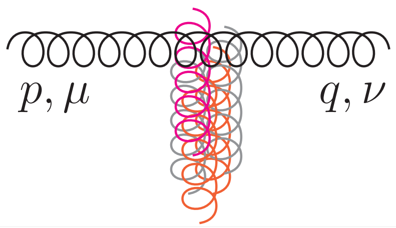
use light-cone gauge, with $k^- = n^- \cdot k$, $(n^-)^2=0$, $n^- \sim$ target momentum



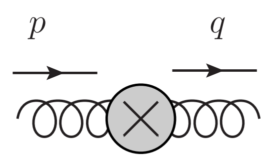
$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{S}_F^{(0)}(p) + \tilde{S}_F^{(0)}(p)$$



$$\tilde{S}_F^{(0)}(q)$$



$$= (2\pi)^d \delta^{(d)}(p-q)\tilde{G}_{\mu\nu}^{(0)}(p) + \tilde{G}_{\mu\alpha}^{(0)}(p)$$



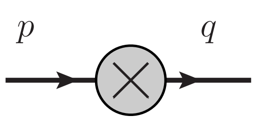
$$\tilde{G}_{\alpha\nu}^{(0)}(q)$$

$$\tilde{S}_F^{(0)}(p) = \frac{i\not{p} + m}{p^2 - m^2 + i0} \qquad \tilde{G}_{\mu\nu}^{(0)}(p) = \frac{i d_{\mu\nu}(p)}{p^2 + i0}$$

$$d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{n_\mu^- p_\nu + p_\mu n_\nu^-}{n^- \cdot p}$$

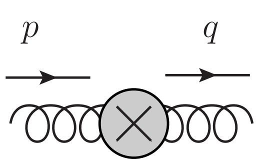
[Balitsky, Belitsky; NPB 629 (2002) 290], [Ayala, Jalilian-Marian, McLerran, Venugopalan, PRD 52 (1995) 2935-2943], …

interaction with the background field:



$$= 2\pi\delta(p^- - q^-)\not{n}^- \int d^{d-2}\boldsymbol{z}\, e^{-i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})}$$

$$\cdot \left\{\theta(p^-)[V(\boldsymbol{z}) - 1] - \theta(-p^-)[V^\dagger(\boldsymbol{z}) - 1]\right\}$$



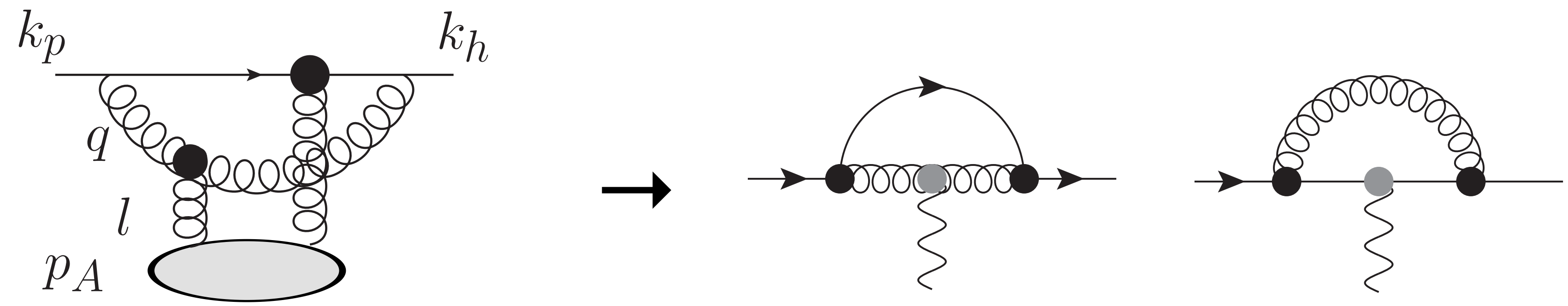
$$= -2\pi\delta(p^- - q^-) 2p^- \int d^{d-2}\boldsymbol{z}\, e^{-i\boldsymbol{z}\cdot(\boldsymbol{p}-\boldsymbol{q})}$$

$$\cdot \left\{\theta(p^-)[U(\boldsymbol{z}) - 1] - \theta(-p^-)[U^\dagger(\boldsymbol{z}) - 1]\right\}$$

$$V(\boldsymbol{z}) \equiv V_{ij}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^- A^{+,c}(x^-, \boldsymbol{z}) t^c$$

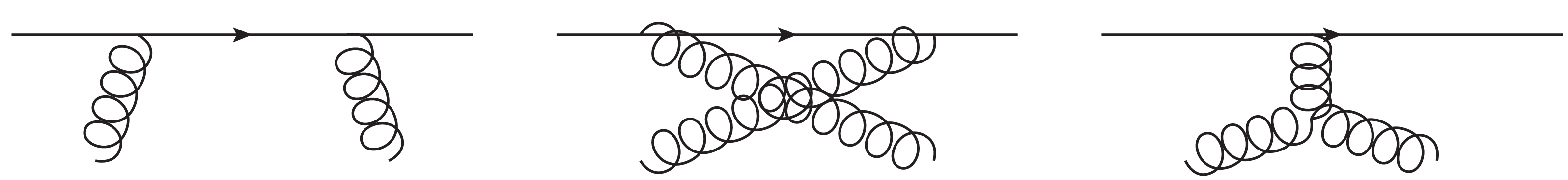
$$U(\boldsymbol{z}) \equiv U^{ab}(\boldsymbol{z}) \equiv \mathrm{P}\exp ig \int_{-\infty}^{\infty} dx^- A^{+,c}(x^-, \boldsymbol{z}) T^c$$

strong background field resummed into path ordered exponentials (Wilson lines) issue: “shockwave approach”, only 2 diagrams

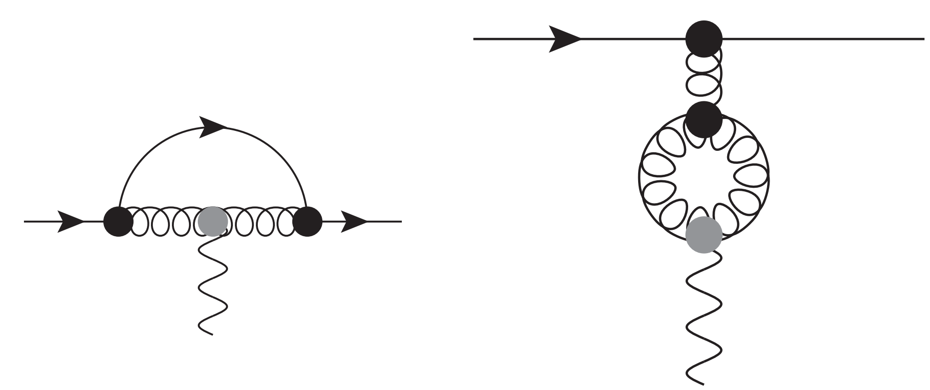


→ difference between different gauges do not cancel → <u>gauge dependent</u>

is this to be expected?



YES: non-abelian gauge theory requires 3 diagrams for current conservation



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