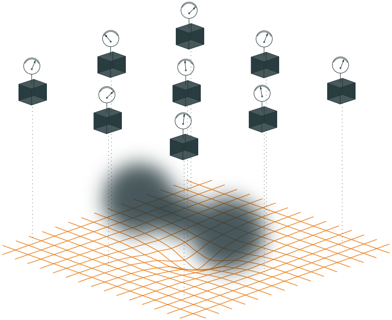
The general structure of quantum-classical dynamics

for gravity and more...



Antoine Tilloy

June 4th, 2024

ECT, Trento – **A Modern Odyssey:** Quantum Gravity meets Quantum Collapse at Atomic and Nuclear physics energy scales in the Cosmic Silence

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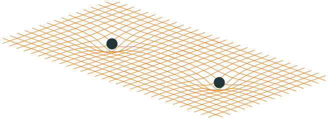
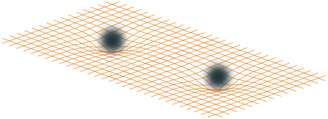
Prolegomena

Classical gravity



- **Matter** is classical
- Spacetime is classical

Semiclassical gravity

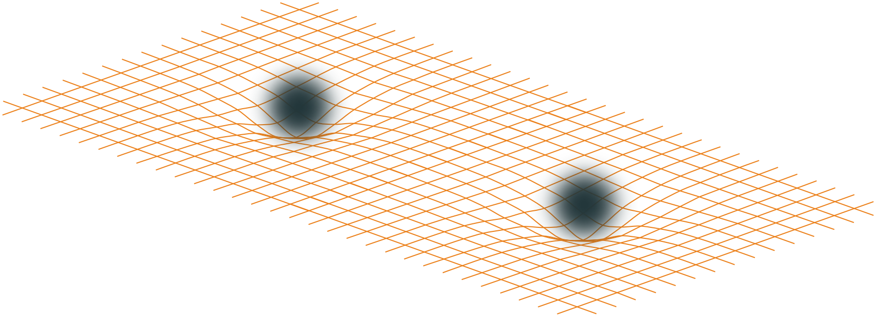


- **Matter** is quantum
- Spacetime is classical

Fully quantum gravity

- **Matter** is quantum
- Spacetime is quantum

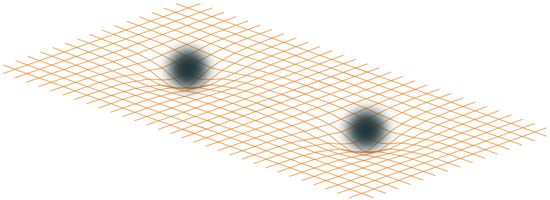
Standard semiclassical gravity



“Standard” semi-classical gravity

A semi-classical theory of gravity tells 2 stories:

1. Quantum matter moves in a curved classical space-time
2. The classical space time is curved by quantum matter



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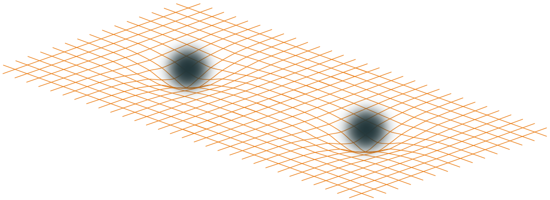


1 is known (QFTCST), **2** is not

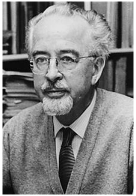
The crucial question of semi-classical gravity is to know how quantum matter should source curvature.

Møller-Rosenfeld semi-classical gravity

The **CHOICE** of Møller and Rosenfeld it to take:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G \,\langle \hat{T}_{\mu\nu} \rangle$$

$\rightarrow$ source gravity via expectation values



Christian Møller

Leon Rosenfeld

There are:

- **technical relativistic** difficulties [renormalization of $\langle T_{\mu\nu} \rangle$]
- **conceptual non-relativistic** difficulties [Born rule, $\cdots$].

Schrödinger-Newton

1. Non-relativistic limit of the "sourcing" equation:

$$\nabla^2 \Phi(x,t) = 4\pi G \, \langle \psi_t | \hat{M}(x) | \psi_t \rangle$$

2. Non-relativistic limit of QFTCST (just external field)

$$\frac{\mathrm{d}}{\mathrm{d}t} |\psi\rangle = -i \left(H_0 + \int \mathrm{d}x \, \Phi(x,t) \hat{M}(x) \right) |\psi_t\rangle,$$

Putting the two together:

$$\frac{\mathrm{d}}{\mathrm{d}t} |\psi_t\rangle = -i H_0 |\psi_t\rangle + i\, G \int \mathrm{d}x \, \mathrm{d}y \, \frac{\langle \psi_t | \hat{M}(x) | \psi_t \rangle \, \hat{M}(y)}{|x-y|} |\psi_t\rangle.$$

The big question

What mathematical object can one construct to source the gravitational field while keeping things consistent?

The big question, generalized

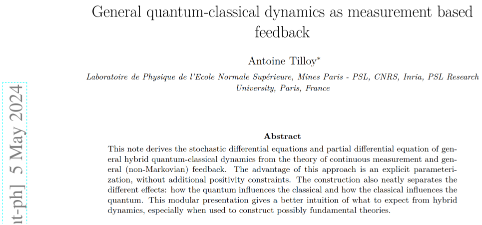
How can one consistently couple quantum and classical variables?

$$\rho_t \longleftrightarrow z_t$$

3 ways to do this

1. Continuous measurement and feedback
2. Spontaneous collapse models
3. General continuous dynamics with a classical subspace [used by Oppenheim]

Main result from 2403.19748: the 3 are **mathematically equivalent**



General quantum-classical dynamics as measurement based feedback

Antoine Tilloy*

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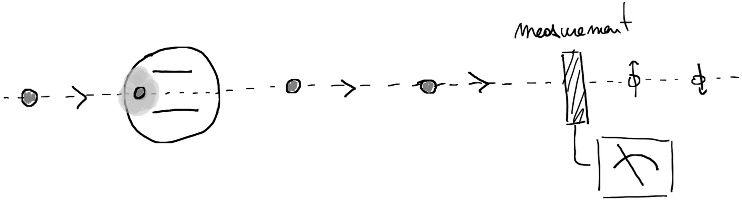
Abstract

This note derives the stochastic differential equations and partial differential equation of general hybrid quantum-classical dynamics from the theory of continuous measurement and general (non-Markovian) feedback. The advantage of this approach is an explicit parameterization, without additional positivity constraints. The construction also neatly separates the different effects: how the quantum influences the classical and how the classical influences the quantum. This modular presentation gives a better intuition of what to expect from hybrid dynamics, especially when used to construct possibly fundamental theories.

Continuous measurement and feedback

Continuous quantum measurement – derivation

Continuous measurement – without Zeno effect



- time between ancillas $\Delta t \propto \varepsilon$
- interaction strength $\omega \propto \sqrt{\epsilon}$

Continuous quantum measurement

Stochastic Master Equation **(~ 1987)**

Density matrix:

$$d\rho_t = \underbrace{-i[H, \rho_t]\, dt}_{\text{standard quantum dynamics}} + \underbrace{\mathcal{D}[\hat{c}](\rho_t)\, dt}_{\text{decoherence}} + \underbrace{\mathcal{H}[\hat{c}](\rho_t)\, dW_t}_{\text{measurement backaction}}$$

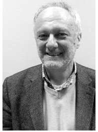
Signal:

$$dr_t = \mathrm{tr}\left[(\hat{c} + \hat{c}^\dagger)\, \rho_t\right] dt + dW_t$$

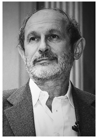
with:

- $\mathcal{D}[\mathcal{O}](\rho) = \mathcal{O}\rho\mathcal{O}^\dagger - \frac{1}{2}\left(\mathcal{O}^\dagger\mathcal{O}\rho + \rho\mathcal{O}^\dagger\mathcal{O}\right)$
- $\mathcal{H}[\mathcal{O}](\rho) = \mathcal{O}\rho + \rho\mathcal{O}^\dagger - \mathrm{tr}\left[(\mathcal{O} + \mathcal{O}^\dagger)\, \rho\right] \rho$
- $\frac{dW_t}{dt}$ "white noise"

V. Belavkin



A. Barchielli



L. Diósi

The measurement signal

The signal – or continuous result

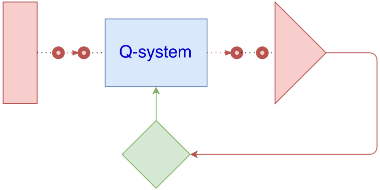
$$\mathrm{d}r_t = \mathrm{tr}\left[\left(\hat{c} + \hat{c}^\dagger\right)\rho_t\right]\,\mathrm{d}t + \mathrm{d}W_t$$

$\rightarrow$ A noisy version of the quantum expectation value

Experimental aside

The signal y_t is routinely measured, for various measured operators, in superconducting circuits using via homodyne / heterodyne detection

Measurement based feedback



Step 1: Have the classical z_t depend on r_t

$$\mathrm{d}z_t = F(z_t)\,\mathrm{d}t + G(z_t)\,\mathrm{d}r_t$$

Step 2: Have the quantum depend on the classical

$$H \longrightarrow H + V(z_t)$$

Consistent by construction since derivable as *effective* from Copenhaguen QM

Most general stochastic equations

Quantum stochastic master equation

$$\mathrm{d}\rho = -i[H_0 + V(\mathbf{z}), \rho]\,\mathrm{d}t + \sum_{k=1}^{n} \mathcal{D}[\hat{c}_k](\rho)\,\mathrm{d}t + \sqrt{\eta_k}\,\mathcal{M}[\hat{c}_k](\rho)\,\mathrm{d}W_k,$$

Signal stochastic differential equation (the quantum classical glue)

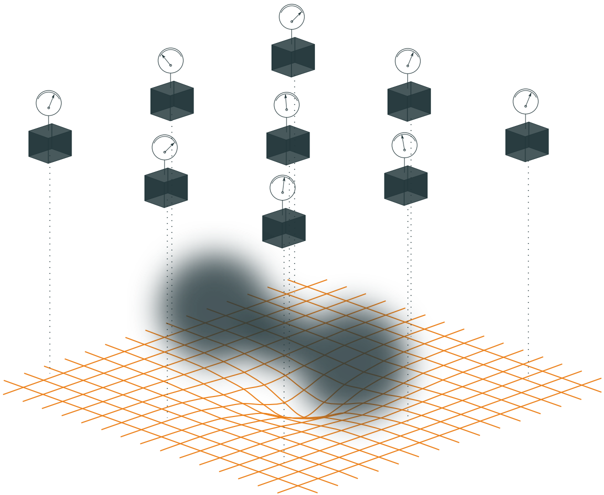
$$\mathrm{d}r_k = \frac{1}{2}\mathrm{tr}[(\hat{c}_k + \hat{c}_k^\dagger)\rho]\,\mathrm{d}t + \frac{1}{2\sqrt{\eta_k}}\mathrm{d}W_k.$$

Classical stochastic differential equation

$$\mathrm{d}z_a = F_a(\mathbf{z})\,\mathrm{d}t + G_{ak}(\mathbf{z})\,\sqrt{\eta_k}\,\mathrm{d}r_k$$

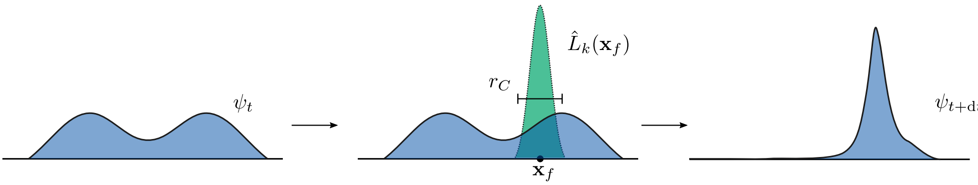
F, G are functions – $V(\mathbf{z})$, $\hat{c}_k$ are operators

“Intuition pump” picture for gravity



AS IF – “There are detectors in space-time measuring the mass density continuously and curving space-time accordingly.” $\longrightarrow$ explains consistency

Spontaneous collapse models



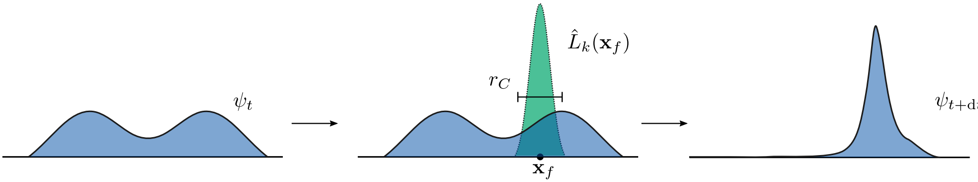
The idea of collapse models

Other names: [Objective / spontaneous / dynamical] [reduction / collapse] [model / program]

Schödinger equation + tiny non-linear bit

$$\frac{\mathrm{d}}{\mathrm{d}t}\psi_t = -\frac{i}{\hbar} H \psi_t + \varepsilon(\psi) \,,$$

H is the Standard Model Hamiltonian (or non-relativistic approx)



Spontaneous collapse models

Mathematically, (continuous) Markovian spontaneous collapse models are equivalent to continuous measurement of appropriate observables

$$\mathcal{O} \longrightarrow \hat{M}_\sigma(x)$$

Metaphysics – Ontology – beables

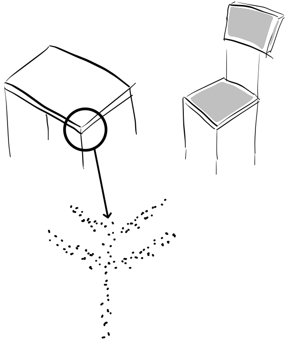
What is real ? What is the world made of ?

1. `GRW0` The wave-function ψ_t itself (but infinite literature of subtleties)

2. `GRWm` The mass density $\langle \hat{M}(x) \rangle$

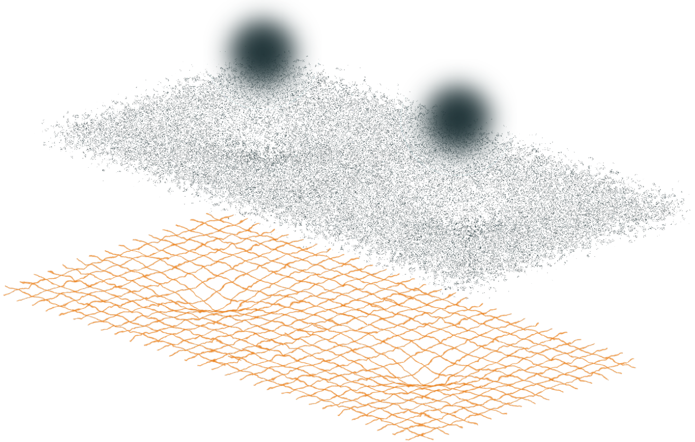
$$\langle \hat{M}(x) \rangle = \sum_k \int \mathrm{d}x_1 \cdots \mathrm{d}x_n \, |\psi(x_1, \cdots, \underbrace{x}_{x \text{ in } k^{\text{th}} \text{ position}}, \cdots, x_n)|^2$$

3. `GRWf` The events (t_f, x_f) where the wave-function collapse (the flashes) – [Bell's choice!]



Fact: (continuous) flashes = signal

Collapse model picture of hybrid dynamics



"The gravitational interaction is mediated by a stochastic field, which is the **local beable** of the theory"

Embedding a classical sector in quantum dynamics

$$\rho(z, t)$$

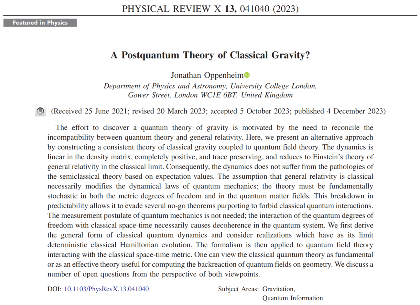
Formulation of the problem

Quantum-classical state

A state diagonal in the classical variables z

$$\rho_{QC} = \int dz\, \rho_Q(z)\, |z\rangle\langle z|$$

- used early on (Diosi, Halliwell, Gisin)
- starting point of Oppenheim



PHYSICAL REVIEW X **13,** 041040 (2023)

Featured in Physics

A Postquantum Theory of Classical Gravity?

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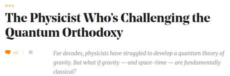
(Received 25 June 2021; revised 20 March 2023; accepted 5 October 2023; published 4 December 2023)

The effort to discover a quantum theory of gravity is motivated by the need to reconcile the incompatibility between quantum theory and general relativity. Here, we present an alternative approach by constructing a consistent theory of classical gravity coupled to quantum field theory. The dynamics is linear in the density matrix, completely positive, and trace preserving, and reduces to Einstein's theory of general relativity in the classical limit. Consequently, the dynamics does not suffer from the pathologies of the semiclassical theory based on expectation values. The assumption that general relativity is classical necessarily modifies the dynamical laws of quantum mechanics; the theory must be fundamentally stochastic in both the metric degrees of freedom and in the quantum matter fields. This breakdown in predictability allows it to evade several no-go theorems purporting to forbid classical quantum interactions. The measurement postulate of quantum mechanics is not needed; the interaction of the quantum degrees of freedom with classical space-time necessarily causes decoherence in the quantum system. We first derive the general form of classical quantum dynamics and consider realizations which have as its limit deterministic classical Hamiltonian evolution. The formalism is then applied to quantum field theory interacting with the classical space-time metric. One can view the classical quantum theory as fundamental or as an effective theory useful for computing the backreaction of quantum fields on geometry. We discuss a number of open questions from the perspective of both viewpoints.

DOI: 10.1103/PhysRevX.13.041040 Subject Areas: Gravitation, Quantum Information


Q & A

The Physicist Who's Challenging the Quantum Orthodoxy

For decades, physicists have struggled to develop a quantum theory of gravity. But what if gravity — and space-time — are fundamentally classical?

Most general second order PDE

Constraints:

- Assuming z evolves continuously $\rightarrow$ at most second order derivatives
- $\rho(z)$ physical $\rightarrow$ positivity conditions

$$
\begin{aligned}
\frac{\partial \rho_t(\mathbf{z})}{\partial t} =& - i\,[H, \rho_t(\mathbf{z})] + \sum_{k=1}^{n} \mathcal{D}[\hat{c}_k](\rho_t(\mathbf{z})) \\
& - \frac{\partial}{\partial z_a}\left[F_a(\mathbf{z})\rho_t(\mathbf{z}) + \frac{\sqrt{\eta_k}\, G_{ak}(\mathbf{z})}{2}\left(\hat{c}_k\rho_t(\mathbf{z}) + \rho_t(\mathbf{z})\hat{c}_k^\dagger\right)\right] \\
& + \frac{1}{2}\frac{\partial^2}{\partial z_a \partial z_b}\left[\frac{G_{ak}(\mathbf{z})\, G_{bk}(\mathbf{z})}{4}\rho_t(\mathbf{z})\right]
\end{aligned}
$$

Equivalence via Ito's lemma

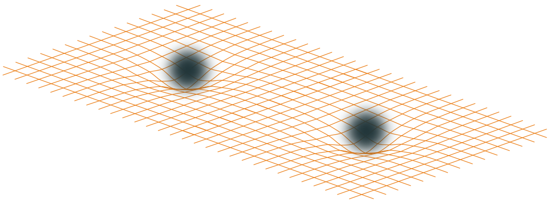
This PDE is the "**Fokker-Planck**" version of the "**Langevin**" dynamics of measurement $+$ feedback:

Equivalence

Using Itô's lemma, one has:

$$\forall f \qquad \underbrace{\mathbb{E}[\rho_t\, f(\mathbf{z}_t)]}_{\text{measurement and feedback}} = \underbrace{\int f(\mathbf{z})\, \rho_t(\mathbf{z})\, \mathrm{d}\mathbf{z}}_{\text{hybrid PDE}}\,.$$

Back to gravity



History

Newtonian early work

Source gravity by measuring the mass density:

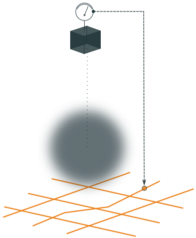
$$\nabla^2\Phi(x) = 4\pi G \mathscr{S}_{\hat{M}}(x)$$

toy model – [Kafri, Taylor, Milburn 2014]
full Newtonian potential – [Diósi & T 2015]

General relativistic extensions

Construct a PDE for $\rho(z)$ for z the gravitational degrees of freedom in ADM general relativity

[Oppenheim, Weller-Davies, Layton, Soda, Russo, ... 2018 $\rightarrow$ today]



Markovian/Newtonian limit

Technically

Newtonian limit $=$ Markovian feedback limit

$$z \propto \mathrm{d}r$$

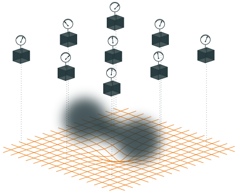
$\implies$ technically infinitely easier $\implies$ one can say something

Experimentally

Hard to probe anything else in the near future

Model

1. **Step 1: continuous mass density measurement**
   We **imagine** that space-time is filled with detectors weakly measuring the mass density:



The equation for matter is now as before with

$\mathcal{O} \to \hat{M}(x), \ \forall x \in \mathbb{R}^3$

$\gamma \to \gamma(x, y)$ coding detector strength and correlation

and there is a "mass density signal" $\mathsf{S}(x)$ in every point.

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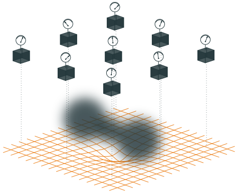
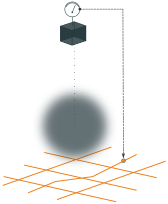


2. **Step 2: Feedback**

   We take the mass density signal $S(x)$ to source the gravitational field φ:

   $$\nabla^2 \varphi(x) = 4\pi\, G\, S(x)$$

   which is **formally** equivalent to quantum feedback.



Result

Standard quantum feedback like computations give for $\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$:

$$\partial_t \rho = -i\left[H_0 + \frac{1}{2}\iint \mathrm{d}x\mathrm{d}y\,\mathscr{V}(x,y)\hat{M}(x)\hat{M}(y), \rho_t\right]$$
$$-\frac{1}{8}\iint \mathrm{d}x\mathrm{d}y\,\mathscr{D}(x,y)\left[\hat{M}(x), \left[\hat{M}(y), \rho_t\right]\right],$$

with the **gravitational pair-potential**

$$\mathscr{V} = \left[\frac{4\pi G}{\nabla^2}\right](x,y) = -\frac{G}{|x-y|},$$

and the **positional decoherence**

$$\mathscr{D}(x,y) = \left[\frac{\gamma}{4} + \mathscr{V}\circ\gamma^{-1}\circ\mathscr{V}^\top\right](x,y)$$

Hence the expected pair potential has been generated consistently at the price of more decoherence.

Principle of least decoherence

$$\mathscr{D}(x, y) = \left[\frac{\gamma}{4} + \mathscr{V} \circ \gamma^{-1} \circ \mathscr{V}^\top\right](x, y)$$

There is still a (functional) degree of freedom $\gamma(x, y)$:

- Large $\|\gamma\| \implies$ strong "measurement" induced decoherence
- Small $\|\gamma\| \implies$ strong "feedback" decoherence

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There is an optimal kernel that minimizes decoherence.

Diagonalizing in Fourier, one gets a global minimum for

$$\gamma = 2\sqrt{\mathscr{V} \circ \mathscr{V}^{\top}} = -2\mathscr{V}$$

Hence:

$$\mathscr{D}(x,y) = -\mathscr{V}(x,y) = \frac{G}{|x-y|}$$

This is just the decoherence kernel of the Diósi-Penrose model (erstwhile heuristically derived)!

Regularization

Even for the minimal decoherence prescription, the decoherence is **infinite**.

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Adding a regulator at a length scale σ has 2 effects:

- It tames decoherence, making it finite
- It regularizes the pair potential $\propto \frac{1}{r}$ for $r \lesssim \sigma$

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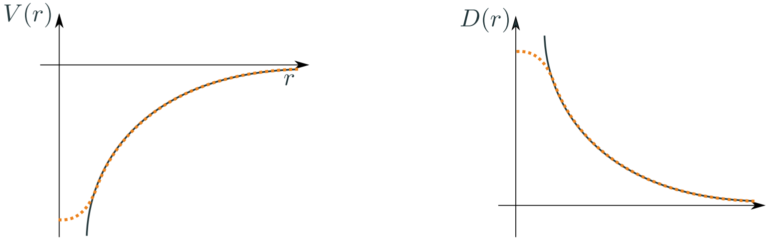
$\implies$ there is a **trade-off**.



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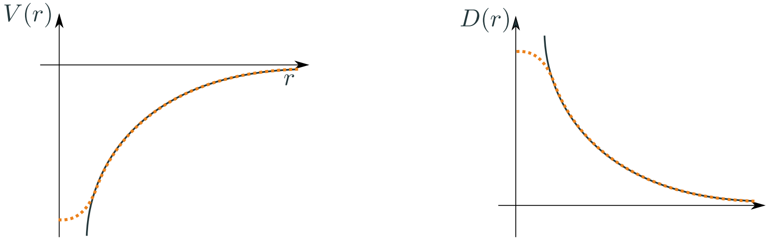


Experimentally:

$$\underbrace{10^{-15} \to 10^{-10} m}_{\text{decoherence constraint}} \ll \sigma \leqslant \underbrace{10^{-4} m}_{\text{gravitational constraint}}$$

Importantly $\sigma > \ell_{\text{Compton}} \gg \ell_{\text{Planck}}$.

Summary

Conceptually: 3 equivalent ways to construct hybrid quantum-classical dynamics

1. **Measurement and feedback** which shows **consistency**
2. **Spontaneous collapse** which shows **empirical effects** + **measurement problem** solution
3. **Quantum Classical PDE** which shows **generality**

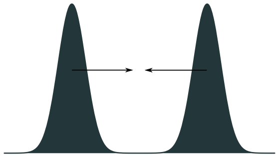
Quantum classical dynamics are *possible* and *well understood*

For gravity

- Newtonian limit: well defined model – minimizing decoherence gives DP + Newtonian potential
- General case: being explored by Oppenheim et al. – big progress, but not clear all constraints can be met

BONUS SLIDES

The problems with Schrödinger-Newton

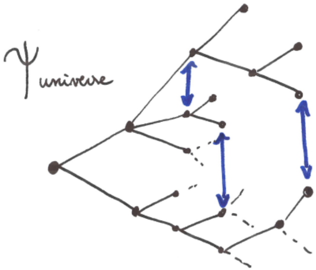


The SN equation is problematic for a fundamental theory because of its **deterministic non-linearity** (Gisin, Diósi, Polchinski)

- If there is **no fundamental collapse** [Many Worlds, Bohm,$\cdots$], super weird world unlike our own
- If there is **fundamental collapse** [Copenhaguen, Collapse models]: break down of the statistical interpretation of states & instantaneous signaling

The problems with Schrödinger-Newton

Without collapse upon measurement (Bohm, Many Worlds, $\cdots$)



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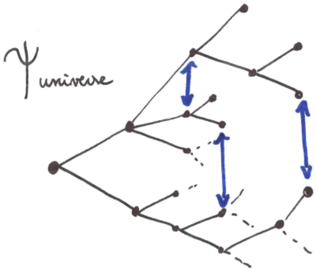
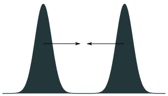
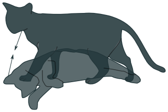


Decohered branches interact with each other $\rightarrow$ **empirically inadequate**



$\Longrightarrow$



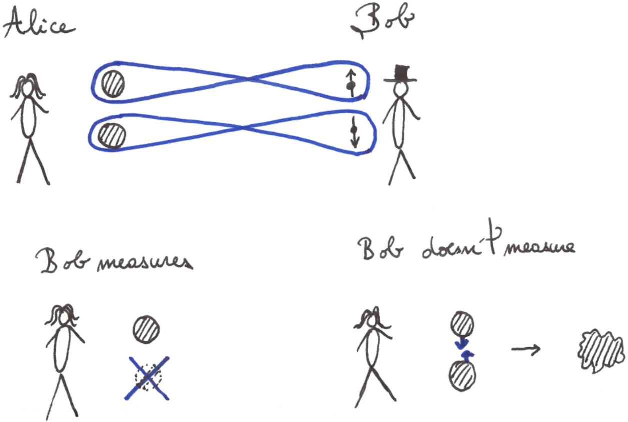
The problems with Schrödinger-Newton

With collapse upon measurement (either from pure Copenhaguen or collapse models).

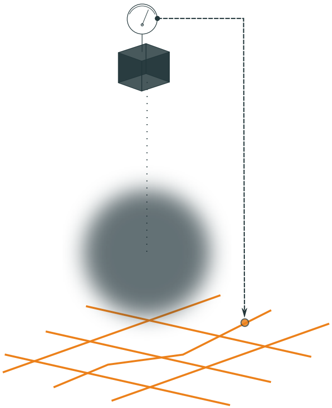
Consider a mass entangled with a spin far away:

$$|\Psi\rangle \propto |\mathsf{left}\rangle^{\text{Alice}} \otimes |\uparrow\rangle^{\text{Bob}} + |\mathsf{right}\rangle^{\text{Alice}} \otimes |\downarrow\rangle^{\text{Bob}}.$$

Bob can decide to whether or not he measures his spin:

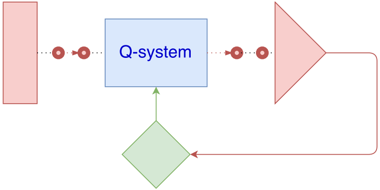


Feedback approach



Measurement + feedback

In orthodox quantum theory, trivial way to do quantum-classical coupling: **measurement** & **feedback** [Diósi & Halliwell]



The state of the controler is the classical variable

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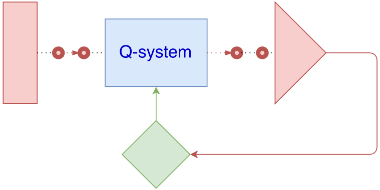


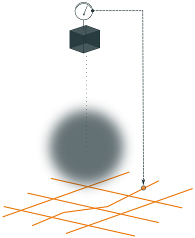
The state of the controler is the classical variable

Idea:

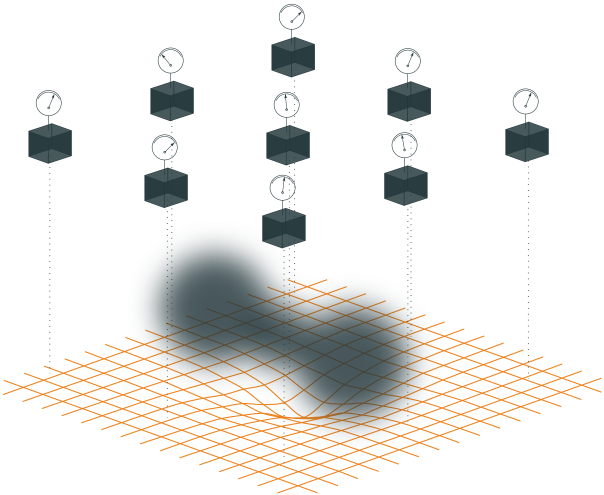
Source gravity by measuring the mass density:

$$\nabla^2 \Phi(x) = 4\pi G \mathscr{S}_{\hat{M}}(x)$$

[Kafri, Taylor & Milburn 2014]
[Diósi & T 2015]

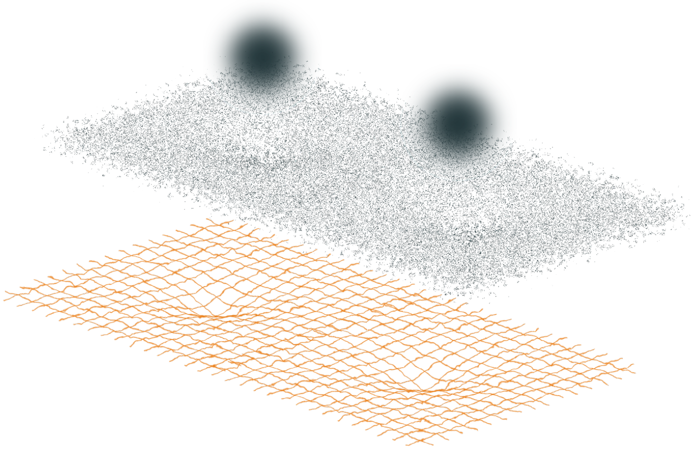


Formal / “intuition pump” picture



“There are detectors in space-time measuring the mass density continuously and curving space-time accordingly.” $\longrightarrow$ this is why it works

Ontological picture



"The gravitational interaction is mediated by a stochastic field, which is the **primitive ontology** of the theory" $\rightarrow$ this is how it should be understood physically

Model

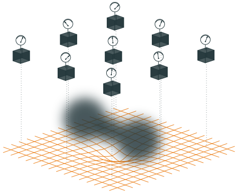
1. **Step 1: continuous mass density measurement**
   We **imagine** that space-time is filled with detectors weakly measuring the mass density:

The equation for matter is now as before with

$$\mathcal{O} \to \hat{M}(x), \ \ \forall x \in \mathbb{R}^3$$

$\gamma \to \gamma(x, y)$ coding detector strength and correlation

and there is a "mass density signal" $\mathsf{S}(x)$ in every point.



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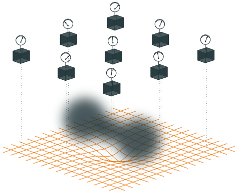
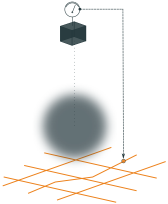


2. **Step 2: Feedback**

   We take the mass density signal $S(x)$ to source the gravitational field φ:

   $$\nabla^2 \varphi(x) = 4\pi\, G\, S(x)$$

   which is **formally** equivalent to quantum feedback.



Result

Standard quantum feedback like computations give for $\rho_t = \mathbb{E}[|\psi_t\rangle\langle\psi_t|]$:

$$\partial_t \rho = -i\left[H_0 + \frac{1}{2}\iint \mathrm{d}x\mathrm{d}y\,\mathscr{V}(x,y)\hat{M}(x)\hat{M}(y), \rho_t\right] - \frac{1}{8}\iint \mathrm{d}x\mathrm{d}y\,\mathscr{D}(x,y)\left[\hat{M}(x),\left[\hat{M}(y),\rho_t\right]\right],$$

with the **gravitational pair-potential**

$$\mathscr{V} = \left[\frac{4\pi G}{\nabla^2}\right](x,y) = -\frac{G}{|x-y|},$$

and the **positional decoherence**

$$\mathscr{D}(x,y) = \left[\frac{\gamma}{4} + \mathscr{V}\circ\gamma^{-1}\circ\mathscr{V}^{\top}\right](x,y)$$

Hence the expected pair potential has been generated consistently at the price of more decoherence.

Principle of least decoherence

$$\mathscr{D}(x, y) = \left[\frac{\gamma}{4} + \mathscr{V} \circ \gamma^{-1} \circ \mathscr{V}^{\top}\right](x, y)$$

There is still a (functional) degree of freedom $\gamma(x, y)$:

- Large $\|\gamma\| \implies$ strong "measurement" induced decoherence
- Small $\|\gamma\| \implies$ strong "feedback" decoherence

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There is an optimal kernel that minimizes decoherence.

Diagonalizing in Fourier, one gets a global minimum for

$$\gamma = 2\sqrt{\mathscr{V} \circ \mathscr{V}^\top} = -2\mathscr{V}$$

Hence:

$$\mathscr{D}(x,y) = -\mathscr{V}(x,y) = \frac{G}{|x-y|}$$

This is just the decoherence kernel of the Diósi-Penrose model (erstwhile heuristically derived)!

Regularization

Even for the minimal decoherence prescription, the decoherence is **infinite**.

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Adding a regulator at a length scale σ has 2 effects:

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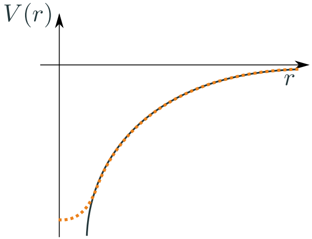

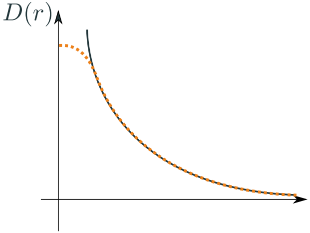

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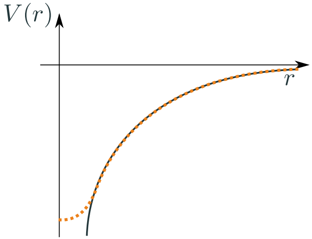
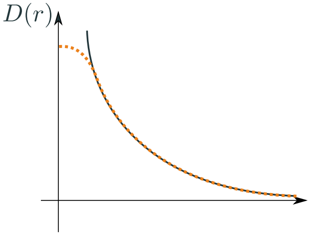




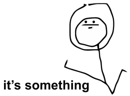
Experimentally:

$$\underset{\text{decoherence constraint}}{10^{-15}m} \ll \sigma \leqslant \underset{\text{gravitational constraint}}{10^{-4}m}$$

Importantly $\sigma > \ell_{\text{Compton}} \gg \ell_{\text{Planck}}$.

Summary of the approach

1. **Most importantly**: Constructing consistent models of semiclassical gravity is possible... in the Newtonian limit



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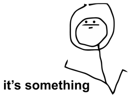

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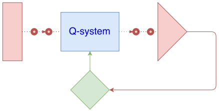

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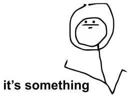

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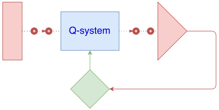

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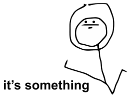

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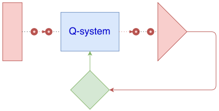

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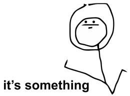

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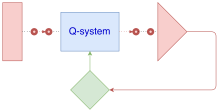

3. The price to pay for semiclassical coupling is intrinsic and gravitational decoherence
4. Minimizing total decoherence gives a parameter free model
5. $\cdots$ up to regularization σ, which is upper bounded and lower bounded experimentally:

$$\underset{\text{decoherence constraint}}{10^{-15}m} \ll \sigma \leqslant \underset{\text{gravitational constraint}}{10^{-4}m}$$

Experimental final word

PRL 119, 240401 (2017) PHYSICAL REVIEW LETTERS week ending 15 DECEMBER 2017

Spin Entanglement Witness for Quantum Gravity

Sougato Bose,[1] Anupam Mazumdar,[2] Gavin W. Morley,[3] Hendrik Ulbricht,[4] Marko Toroš,[4] Mauro Paternostro,[5] Andrew A. Geraci,[6] Peter F. Barker,[1] M. S. Kim,[7] and Gerard Milburn[7,8]

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[6]*Department of Physics, University of Nevada, Reno, 89557 Nevada, USA*
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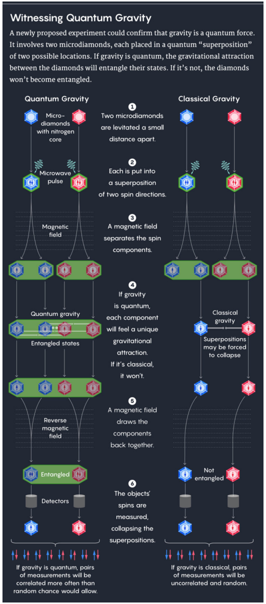
(Received 6 September 2017; revised manuscript received 6 November 2017; published 13 December 2017)

Understanding gravity in the framework of quantum mechanics is one of the great challenges in modern physics. However, the lack of empirical evidence has lead to a debate on whether gravity is a quantum entity. Despite varied proposed probes for quantum gravity, it is fair to say that there are no feasible ideas yet to test its quantum coherent behavior directly in a laboratory experiment. Here, we introduce an idea for such a test based on the principle that two objects cannot be entangled without a quantum mediator. We show that despite the weakness of gravity, the phase evolution induced by the gravitational interaction of two micron size test masses in adjacent matter-wave interferometers can detectably entangle them even when they are placed far apart enough to keep Casimir-Polder forces at bay. We provide a prescription for witnessing this entanglement, which certifies gravity as a quantum coherent mediator, through simple spin correlation measurements.

DOI: 10.1103/PhysRevLett.119.240401



Lucy Reading-Ikkanda/Quanta Magazine

How seriously should we take it?

Antoine, do you seriously believe the world is like in your theory?

Sheldon Goldstein

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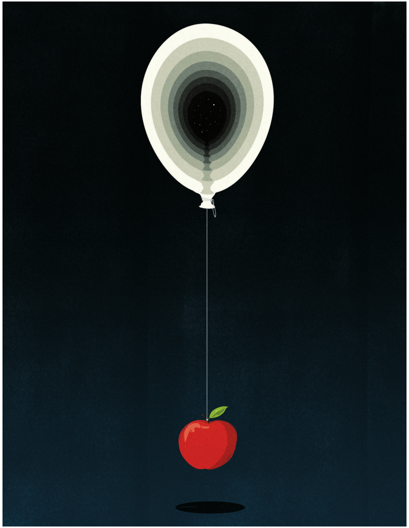
I bet 99 to one that the outcome will be consistent with gravity having quantum properties.

Carlo Rovelli



NewScientist — 14 April 2018

Conclusion

Does gravity need to be quantized? No

- Weak arguments grounded on **hope** and **aesthetics**
- Strong argument: standard approach to semiclassical gravity **empirically inadequate**

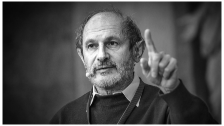
Counter example

- Semiclassical coupling $\equiv$ Measurement based feedback
- Parameter free model up to regularization

Experimentally

- Quantitatively: additional decoherence with a very specific form
- Qualitatively: cannot entangle

Acknowledgments

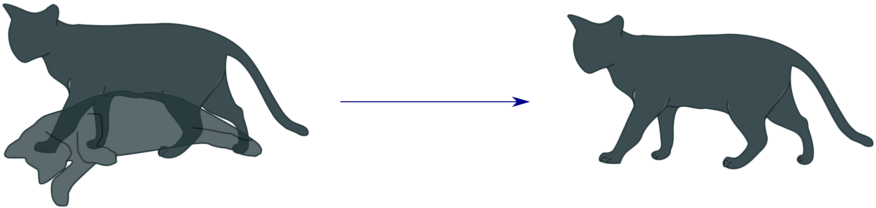


Lajos Diósi

References

1. **Kafri, Taylor, Milburn** *A classical channel model for gravitational decoherence*, NJPhys 2014 and *Bounds on quantum communication via Newtonian gravity* NJPhys 2015
2. **T, Diósi** *Sourcing semiclassical gravity from spontaneously localized quantum matter* PRD 2016 and *Principle of least decoherence for Newtonian semiclassical gravity* PRD 2017
3. **T**, *Ghirardi Rimini Weber model with massive flashes* PRD 2018 and *Binding quantum matter and space time without romanticism*

IV – Link with collapse models



Collapse models

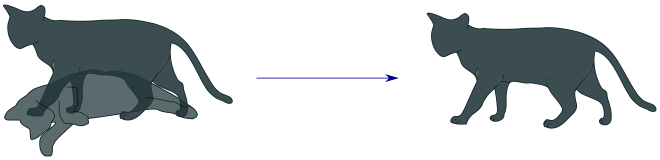
Naive definition

Collapse models are an attempt to solve the measurement problem of quantum mechanics through an *ad hoc*, non-linear, and stochastic modification of the Schrödinger equation.

$$\partial_t|\psi_t\rangle = -iH|\psi_t\rangle + \varepsilon\, f_\xi(|\psi_t\rangle)$$

A few names:

Pearle, Ghirardi, Rimini, Weber, Diósi, Adler, Gisin, Tumulka, Bedingham, Penrose, Percival, Bassi, Ferialdi, Weinberg ...



Collapse models

The modification is such that:

Weak collapse

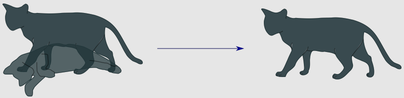
A single particle *extremely rarely* collapses in the position basis

- Microscopic dynamics unchanged

Amplification

The effective collapse rate is renormalized for macroscopic superpositions:

- Macroscopic superpositions almost instantly collapse



We have a collapse model!

Actually, the continuous measurement of the regularized mass density gives:

- The Continuous Spontaneous Localization (CSL) model for $\gamma(x,y) \propto \delta(x,y)$ i.e. maximally local (up to regularization)
- The Diósi-Penrose (DP) model for $\gamma(x,y)$ minimizing decoherence

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Consequences

1. Our model **solves the measurement problem**. There are no macroscopic superpositions

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Consequences

1. Our model **solves the measurement problem**. There are no macroscopic superpositions
2. It is tempting make an analog construction for GRW

The GRW model

GRW model for N spinless particles

- Standard linear evolution between jumps

$$\partial_t |\psi_t\rangle = -iH|\psi_t\rangle$$

- Jump hitting particle k in x_f at a rate λ

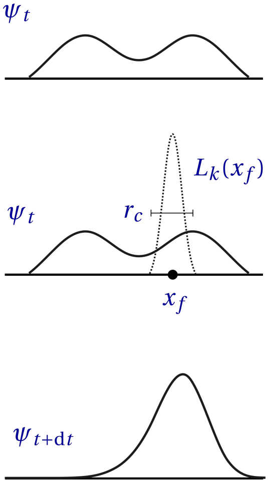
$$|\psi_t\rangle \to \frac{\hat{L}_k(x_f)|\psi_t\rangle}{\|\hat{L}_k(x_f)|\psi_t\rangle\|}$$

with

$$\mathbb{P}(x_f) = \|\hat{L}_k(x_f)|\psi_t\rangle\|^2$$

and

$$\hat{L}_k(x_f) = \frac{1}{(\pi r_c^2)^{3/2}} \mathrm{e}^{(\hat{x}_k - x_f)^2/(2r_c^2)}$$



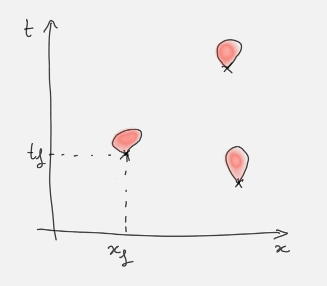
Ghirardi, G. C., Rimini, A., & Weber, T. (1986) Phys. Rev. D, **34**(2), 470.

GRW with massive flashes

Sourcing equation –general case–

Gravitational Φ field created by a single flash (x_f, t_f):

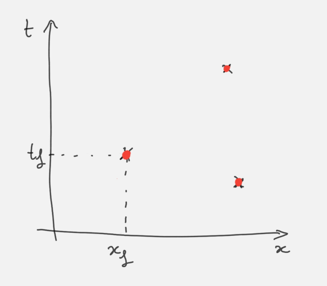
$$\nabla^2\Phi(x,t) = 4\pi G m_k \lambda^{-1} f(t - t_f, x - x_f)$$



Sourcing equation –sharp limit–

Gravitational Φ field created by a single flash (x_f, t_f):

$$\nabla^2\Phi(x,t) = 4\pi G m_k \lambda^{-1} \delta(t - t_f, x - x_f)$$



GRW with massive flashes

Add the gravitational field in the Schrödinger equation

$$
\begin{aligned}
\widehat{V}_G &= \int \mathrm{d}x \; \Phi(x) \widehat{M}(x) \\
&= -G\lambda^{-1} \sum_{\ell=1}^{N} m_k m_l \int \mathrm{d}x \frac{f(t-t_f, x-x_f)}{|x-\hat{x}_\ell|}
\end{aligned}
$$

with $\widehat{M}(x) = \sum_{\ell=1}^{N} m_\ell \delta(x - \hat{x}_\ell)$.

In the limit of sharp sources, $\widehat{V}_G$ is ill-defined but the corresponding unitary is fine:

$$
\begin{aligned}
\widehat{U}_k(x_f) &= \exp\left(-\frac{i}{\hbar} \int_{t_f}^{+\infty} \mathrm{d}t \, \widehat{V}_G(t)\right) \\
&= \exp\left(i \frac{G}{\lambda\hbar} \sum_{\ell=1}^{N} \frac{m_k m_\ell}{|x_f - \hat{x}_\ell|}\right)
\end{aligned}
$$

GRW with massive flashes

Just after a jump, a **jump dependent** unitary is applied to the N-particle system:

$$|\psi_t\rangle \to \hat{U}_k(x_f)\frac{\hat{L}_k(x_f)|\psi_t\rangle}{\|\hat{L}_k(x_f)|\psi_t\rangle\|} = \frac{\hat{U}_k(x_f)\hat{L}_k(x_f)|\psi_t\rangle}{\|\hat{U}_k(x_f)\hat{L}_k(x_f)|\psi_t\rangle\|} := \frac{\hat{B}_k(x_f)|\psi_t\rangle}{\|\hat{B}_k(x_f)|\psi_t\rangle\|}$$

It is just like changing the collapse operators to non self-adjoint ones!

In the end, all the empirical content lies in the master equation:

$$\partial_t\rho_t = -\frac{i}{\hbar}[H,\rho_t] + \lambda\sum_{k=1}^{n}\int \mathrm{d}x_f\ \hat{B}_k(x_f)\rho_t\hat{B}_k(x_f) - \rho_t$$

GRW with massive flashes: phenomenology

Single particle master equation

Consider the density matrix

$$\begin{aligned} \rho : \mathbb{R}^3 \times \mathbb{R}^3 &\longrightarrow \mathbb{C} \\ (x, y) &\longmapsto \rho(x, y) \end{aligned}$$

It obeys:

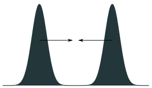
$$\partial_t \rho_t(x, y) = \lambda \left(\Gamma(x, y) - 1\right) \rho(x, y)$$

with

$$\Gamma(x, y) = \int \frac{\mathrm{d}x_f}{(\pi r_C^2)^{3/2}} \exp\left(i \frac{Gm^2}{\lambda \hbar} \left[\frac{1}{|x - x_f|} - \frac{1}{|y - x_f|} \right] \right) \times \exp\left(- \frac{(x - x_f)^2 + (y - x_f)^2}{2 r_C^2} \right)$$

Lemma 1:

- $\Gamma(x, y)$ is **real** $\rightarrow$ pure decoherence
- No self-attraction



Lemma 2:

- The model is falsifiable for "all" values of λ

GRW with massive flashes: recovering Newtonian gravity

Two lengths scales in the problem:

- r_c the collapse regularization radius
- $r_G = Gm^2/(\hbar\lambda)$ a new gravitational length scale

For distances d larger than these two length scales:

- One can neglect the Gaussian smearing of the collapse
- The fact that gravity "kicks" instead of being continuous can be neglected on the average evolution:

$$U_k(x_f) \simeq 1 + i\frac{G}{\lambda\hbar}\sum_{\ell=1}^{N}\frac{m_k m_\ell}{|x_f - \hat{x}_\ell|}$$

We then recover Newton's potential! (+ decoherence)

Bonus: Survival bias