Incorporating a spontaneous collapse mechanism in a Wheeler-DeWitt equation

A Modern Odyssey: Quantum Gravity meets Quantum Collapse at Atomic and Nuclear physics energy scales in the Cosmic Silence

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Collapse models and Cosmology

Dark Energy from Violation of Energy Conservation

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Cosmic Microwave Background Constraints Cast a Shadow On Continuous Spontaneous Localization Models

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Discussions about the landscape of possibilities for treatments of cosmic inflation involving continuous spontaneous localization models

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Impact of Dynamical Collapse Models on Inflationary Cosmology

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3+1 Decomposition of General Relativity

- Requiring that the spacetime $(\mathcal{M}, g)$ is globally hyperbolic, one can construct a foliation of spacetime in Cauchy surfaces Σ_t with the corresponding vectorial field given by

$$t^\mu \mathrm{d}t = N n^\mu \mathrm{d}t + N^\mu \mathrm{d}t$$

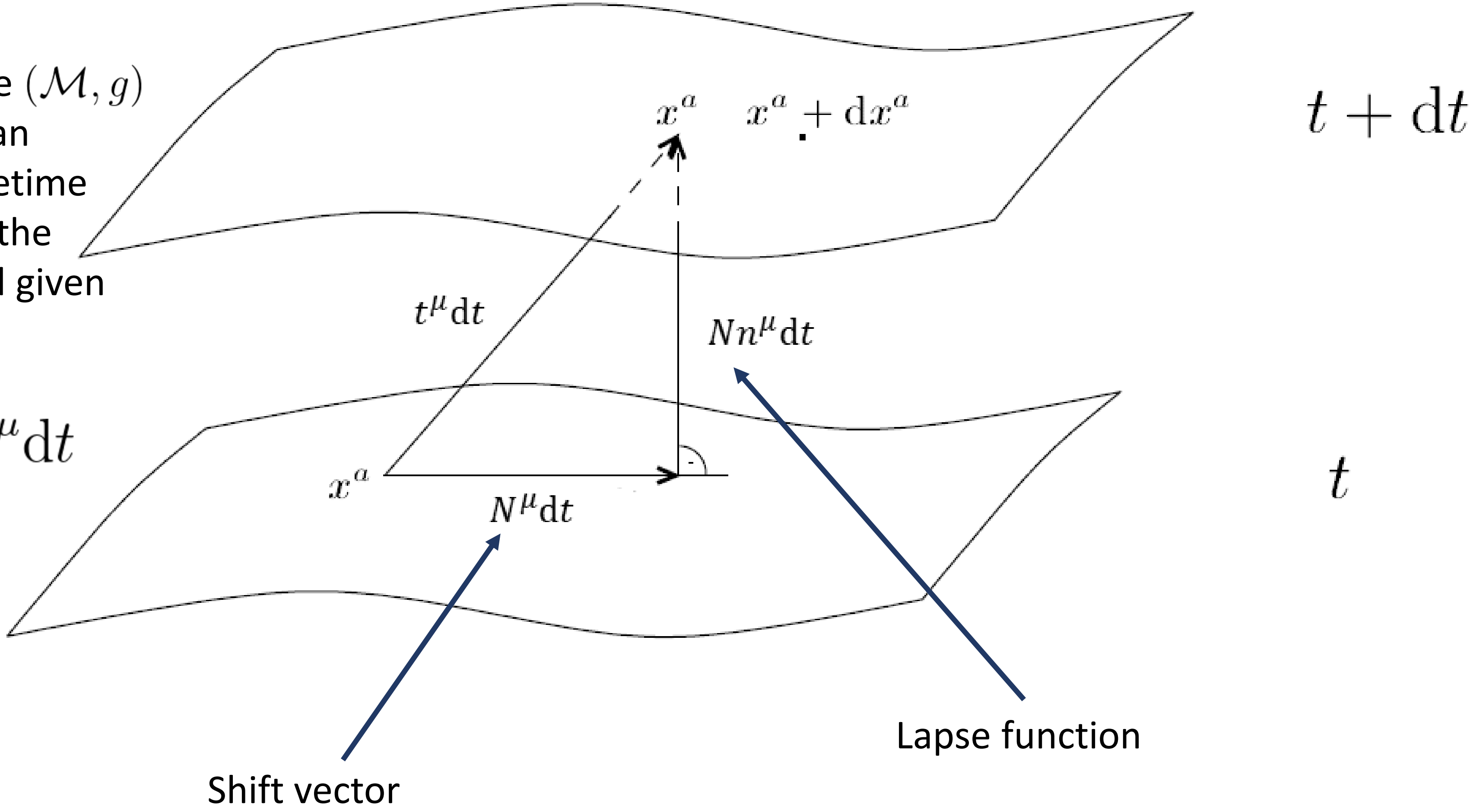

Image: Adapted from C. Kiefer, *Quantum Gravity,* Oxford University Press (2012).

Arnowitt-Deser-Misner (ADM) Action

- Our starting point is the Einstein-Hilbert action

$$S_{\mathrm{EH}} = \frac{c^4}{16\pi G}\int_{\mathcal{M}} \mathrm{d}^4x\sqrt{-g}(R-2\Lambda) - \frac{c^4}{8\pi G}\int_{\partial\mathcal{M}} \mathrm{d}^3x\sqrt{h}K.$$

- Rewriting it in terms of the three-dimensional variables h_{ab} and K_{ab} , one obtains the so-called Arnowitt-Deser-Misner action:

$$S_{\mathrm{EH}} = \frac{1}{16\pi G}\int_{\mathcal{M}} \mathrm{d}t\mathrm{d}^3x N\left(G^{abcd}K_{ab}K_{cd} + \sqrt{h}\left({}^{(3)}R - 2\Lambda\right)\right) \equiv \int_{\mathcal{M}} \mathrm{d}t\mathrm{d}^3x\mathcal{L}^g,$$

with the Wheeler-DeWitt metric given by

$$G^{abcd} = \frac{\sqrt{h}}{2}\left(h^{ab}h^{bd} + h^{ad}h^{bc} - 2h^{ab}h^{cd}\right)$$

Hamiltonian and momentum restrictions

- In terms of the Hamiltonian density $\mathcal{H}_g$, the Einstein Hilbert action reads:

$$S_{\mathrm{EH}} = \frac{1}{16\pi G}\int_{\mathcal{M}} \mathrm{d}t\mathrm{d}^3x \left(p^{ab}\dot{h}_{ab} - N\mathcal{H}^g_\perp - N^a\mathcal{H}^g_a\right).$$

- The variation with respect to N and N^a leads

$$\mathcal{H}^g_\perp = 16\pi G G_{abcd}p^{ab}p^{cd} - \frac{\sqrt{\hbar}}{16\pi G}\left({}^{(3)}R - 2\Lambda\right) \approx 0$$

Hamiltonian restrictions

$$\mathcal{H}^g_a = -2D_b p^b_a \approx 0$$

Diffeomorphism (momentum) restrictions

Quantization of the restrictions

- The quantization of the Poisson parentheses leads to:

$$\{h_{ab}(x), p_{cd}(y)\} = \delta^c_{(a}\delta^d_{b)}\delta(x,y) \Rightarrow [\hat{h}_{ab}(\mathbf{x}), \hat{p}_{cd}(\mathbf{y})] = i\hbar\delta^c_{(a}\delta^d_{b)}\delta(\mathbf{x},\mathbf{y}).$$

- Implementing the operators on wave functionals

$$\hat{h}_{ab}(\mathbf{x})\Psi[h_{ab}(\mathbf{x})] = h_{ab}(\mathbf{x})\Psi[h_{ab}(\mathbf{x})],$$
$$\hat{p}_{cd}(\mathbf{x})\Psi[h_{ab}(\mathbf{x})] = -\frac{i}{\hbar}\frac{\delta}{\delta h_{cd}(\mathbf{x})}\Psi[h_{ab}(\mathbf{x})],$$

we obtain

$$\hat{\mathcal{H}}^g_\perp\Psi \equiv \left(-16\pi G\hbar^2 G_{abcd}\frac{\delta}{\delta h_{ab}\delta h_{cd}} - \frac{\sqrt{\hbar}}{16\pi G}\left({}^{(3)}R - 2\Lambda\right)\right)\Psi = 0.$$

← Wheeler-DeWitt equation

$$\hat{\mathcal{H}}^g_a\Psi \equiv \frac{i}{\hbar}2D_b h_{ac}\frac{\delta\Psi}{\delta h_{bc}} = 0.$$

← Quantum diffeomorphism restrictions

Incorporation of Collapse models into the Wheeler-DeWitt formalism

Found Phys (2014) 44:114–143
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Benefits of Objective Collapse Models for Cosmology and Quantum Gravity

Elias Okon · Daniel Sudarsky

International Journal of Modern Physics D | VOL. 24, NO. 12

Cosmological constant, quantum measurement and the problem of time

Shreya Banerjee, Sayantani Bera, and Tejinder P. Singh

Toy model: Gravity + perfect fluid

- For a plane FLRW Universe, the line element can be written as (τ is the temporal coordinate)

$$\mathrm{d}s^2 = -N^2(\tau)\mathrm{d}\tau^2 + a^2(\tau)h_{ij}\mathrm{d}x^i\mathrm{d}x^j.$$

- Let us consider the dynamical evolution of a perfect fluid. The action reads

$$S_{\mathrm{GR}} = V_0 \int_{\mathbb{R}} \mathrm{d}\tau \left(-\frac{3\dot{a}^2 a}{N} - N\frac{m}{a^{3w}} + m\dot{\chi}\right), \qquad (w \neq -1).$$

- In terms of the canonical conjugated variables $\{v, \pi_v\} = 1, \quad \{t, \lambda\} = 1,$ defined by

$$v = 4\sqrt{V_0/3}a^{3(1-w)/2}/(1-w), \qquad t = \chi/V_0,$$

$$\pi_v = (-6V_0\dot{a}a/N)a^{(3w-1)/2}/\sqrt{12V_0}, \qquad \lambda = mV_0,$$

the Hamiltonian reads

$$\mathcal{H} = Na^{-3w}[-\pi_v^2 + \lambda] \longrightarrow -\pi_v^2 + \lambda = 0.$$

L. Menéndez-Pidal, *The Problem of Time in Quantum Cosmology,* Ph.D. thesis, University of Nottingham (2022).

Toy model: Gravity + perfect fluid (w=-1)

- For this case, let us consider the following action within the context of Parametrised Unimodular Gravity

$$S_{\mathrm{PUM}} = V_0 \int \mathrm{d}\tau \left(-\frac{3\dot{a}^2 a}{N} + \Lambda \dot{T} - N a^3 \Lambda \right)$$

- In terms of the canonical conjugated variables $\{v, \pi_v\} = 1, \quad \{t, \lambda\} = 1,$ defined by

$$v = 2\sqrt{V_0/3}a^3 \qquad t = T/V_0$$

$$\pi_v = (-6V_0 \dot{a} a/N)/(a^2\sqrt{12V_0}) \qquad \lambda = \Lambda V_0$$

the Hamiltonian density is the same as before, namely

$$\mathcal{H} = a^3 N[-\pi_v^2 + \lambda] \longrightarrow -\pi_v^2 + \lambda = 0.$$

Wheeler-DeWitt equation

- Direct substitution $\pi_v \to -i\hbar\frac{\partial}{\partial v}$ and $\lambda \to -i\hbar\frac{\partial}{\partial t}$ yields the following Wheeler-DeWitt equation for the wave function $\Psi(v,t)$

$$\mathcal{H}\Psi(v,t) = \left(\hbar\frac{\partial^2}{\partial v^2} - i\hbar\frac{\partial}{\partial t}\right)\Psi(v,t) = 0.$$

- *NOTE*: The inner product is defined as

$$\langle\Phi|\Psi\rangle = \int_0^\infty \mathrm{d}v\, \Phi^*(v,t)\Psi(v,t)$$

- Under the condition of a self-adjoint Hamiltonian $\gamma\Psi(0,t) - \frac{\partial}{\partial v}\Psi(0,t) = 0$ we get the solutions $(\hbar = 1)$

$$\Psi(v,t) = \int_0^\infty \frac{\mathrm{d}\lambda}{\sqrt{2\pi}} Q(\lambda) e^{i\lambda t} \frac{1}{\sqrt{2\sqrt{\lambda}}} \left(e^{-i\sqrt{\lambda}v} + \frac{i\sqrt{\lambda}+\gamma}{i\sqrt{\lambda}-\gamma} e^{i\sqrt{\lambda}v}\right).$$

Structure of the dynamical collapse equation

- At the level of the wave function, the structure of the dynamical collapse equation reads:

$$\mathrm{d}|\psi_t\rangle = \left(-\frac{i}{\hbar}\hat{H} + \int \mathrm{d}\mathbf{x}\hat{C}_N(\mathbf{x})\xi_t(\mathbf{x})\mathrm{d}t - \frac{1}{2}\int \mathrm{d}\mathbf{x}\int \mathrm{d}\mathbf{y}G(\mathbf{x},\mathbf{y})\hat{C}_N(\mathbf{x})\hat{C}_N(\mathbf{y})\right)|\psi_t\rangle$$

where $\hat{C}_N(\mathbf{x}) = \hat{C}(\mathbf{x}) - \langle\psi_t|\,\hat{C}(\mathbf{x})\,|\psi_t\rangle$

$$\mathbb{E}[\xi_t(\mathbf{x})] = 0 \qquad \mathbb{E}[\xi_t(\mathbf{x})\xi_{t'}(\mathbf{y})] = G(\mathbf{x},\mathbf{y})\delta(t-t')$$

Model	$\hat{C}(\mathbf{x})$	$G(\mathbf{x},\mathbf{y})$
Continuous Spontaneous Localization (CSL)	$\sqrt{\gamma}\hat{M}(\mathbf{x})/m_0$	$\exp[-(\mathbf{x}-\mathbf{y})^2/4r_c^2]/(4\pi r_c^2)^{3/2}$
Diósi-Penrose (DP)	$\hat{M}(\mathbf{x})$	$G/(\hbar\lvert\mathbf{x}-\mathbf{y}\rvert)$

M. Carlesso, S. Donadi, L. Ferialdi, M. Paternostro, H. Ulbricht, A. Bassi, Nat. Phys. **18**, 243-250 (2022).

Wheeler-DeWitt equation and dynamical collapse

- For the collapse equation:

$$\mathrm{d}\,|\Psi_t\rangle = \left(-i\hat{H}\mathrm{d}t + \epsilon(\hat{H} - \langle\hat{H}\rangle_t)\mathrm{d}W_t - \frac{\epsilon^2}{2}(\hat{H} - \langle\hat{H}\rangle_t)\mathrm{d}t\right)|\Psi_t\rangle,$$

we have:

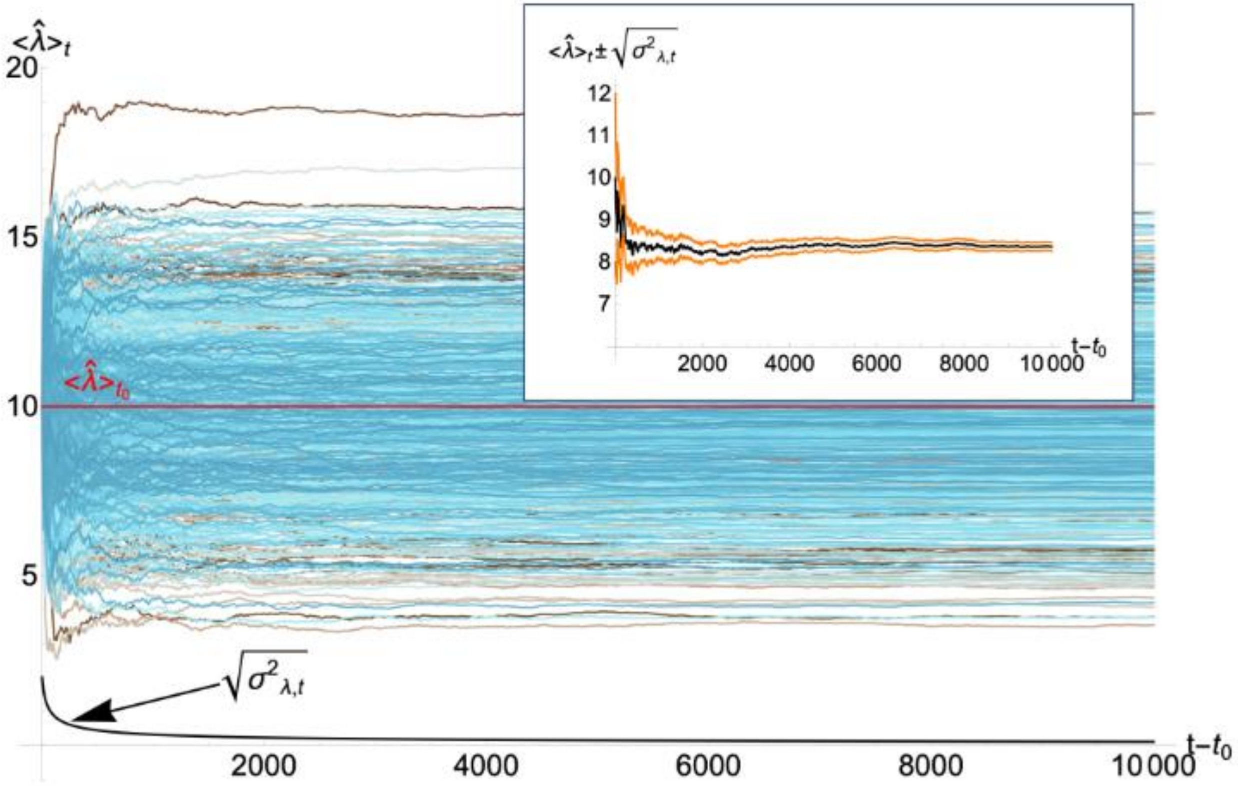
$$\mathrm{d}\langle\hat{\lambda}\rangle_t = -2\epsilon\sigma^2_{\lambda,t}\mathrm{d}W_t,$$

$$\mathrm{d}\sigma^2_{\lambda,t} = -4\epsilon^2(\sigma^2_{\lambda,t})^2\mathrm{d}t - 2\epsilon\Sigma^{(3)}_t\mathrm{d}W_t.$$

- We consider an initial state given by:

$$|\Phi_0\rangle = \int_0^\infty \mathrm{d}\lambda A(\lambda)\,|\lambda\rangle,$$

with $A(\lambda) \propto e^{-(\lambda-\lambda_0)^2/4\sigma_0^2}$.



JLGR, L. Menéndez-Pidal, M. Faizal, M. Carlesso, JHEP **2024**, 193 (2024).

Wheeler-DeWitt equation and dynamical collapse

- Given the wave function and the initial state, one can find an approximate solution using a perturbative approach. To second order in ϵ , we have

$$|\Psi_t\rangle = |\Psi_0\rangle + \epsilon\,|\Psi_1\rangle + \epsilon^2\,|\Psi_2\rangle$$

with

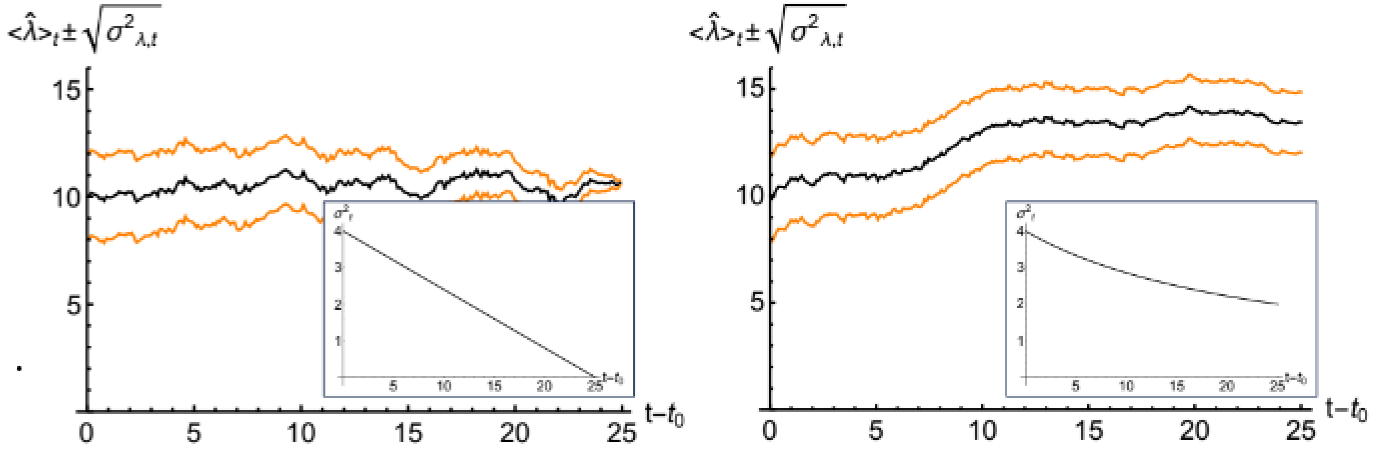
$$|\Psi_j\rangle = \int_0^{\infty} \mathrm{d}\lambda A(\lambda)\mathcal{C}_j(\lambda,t)\exp(i(t-t_0)\lambda)\,|\lambda\rangle\,.$$

Explicitly, we have

$$\mathcal{C}_0(\lambda,t) = 1,$$

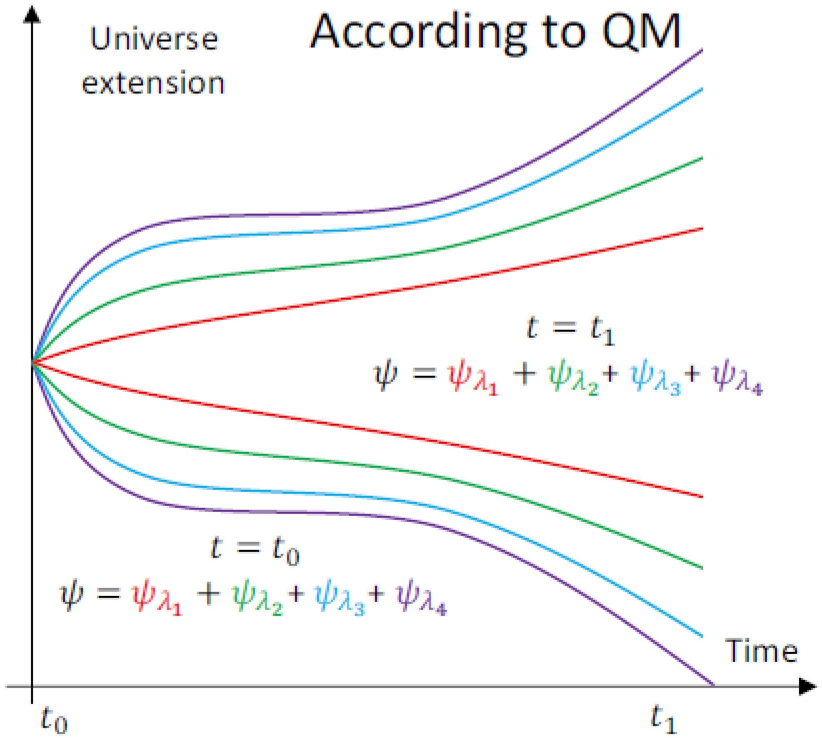
$$\mathcal{C}_1(\lambda,t) = (W(t)-W(t_0))(-\lambda+\langle\lambda\rangle_0),$$

$$\mathcal{C}_2(\lambda,t) = \frac{1}{2}(W(t)-W(t_0))^2(\lambda^2-2\lambda\langle\lambda\rangle_0+3\langle\lambda\rangle_0^2-2\langle\lambda^2\rangle_0)$$
$$-(t-t_0)(\lambda^2-2\lambda\langle\lambda\rangle_0+2\langle\lambda\rangle_0^2-\langle\lambda^2\rangle_0)$$

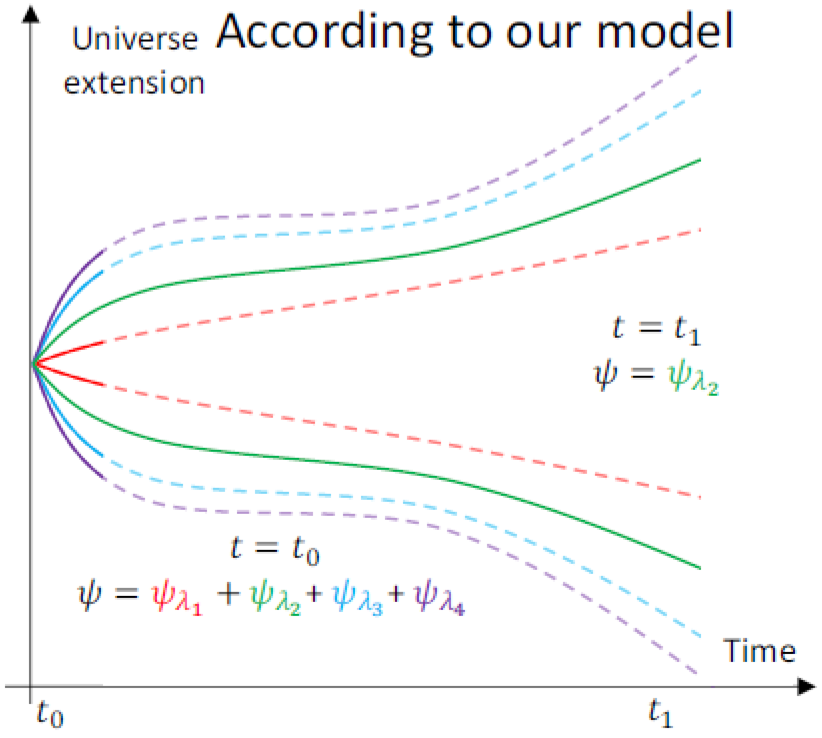


Behaviour of the mean value for the exact and approximate solutions of the variance under the Gaussian assumption.

Recap: Emergence of classicality in the Universe



Schrödinger evolution



Incorporation of dynamical collapse terms in the Wheeler-DeWitt equation

Max Born
(1882-1970)

Robert Oppenheimer
(1904-1967)