Gaussian quantum entanglement in curved spacetime

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Outline

- influence of Hawking radiation on quantum entanglement (E) for bimodal Gaussian states near a Schwarzschild black hole is investigated
- for a thermal squeezed state of a bimodal bosonic system the Hawking radiation reduces and even can destroy E between the mode of a Kruskal observer Alice and the mode of Bob, who is an accelerated observer hovering outside the event horizon of black hole
- by contrary, the Hawking radiation increases and even can generate q. E between Bob and anti-Bob, who is a hypothetical observer inside event horizon
- in both these scenarios the competition between the contrary influences produced by Hawking temperature, squeezing and field frequency, may favour the preservation of q. E

Outline

- we investigate also the influence of the thermal environment on the behaviour in time of E between the considered observers and show that E is destroyed in a finite time for both considered bipartite scenarios of observers Alice and Bob, and respectively Bob and anti-Bob, for non-zero values of the temperature of the thermal environment, i.e. the phenomenon of entanglement sudden death (ESD) takes place
- for a zero temperature of the thermal bath the initial existing E is decreasing over time, but it keeps for all finite times a non-zero value and the logarithmic negativity tends to zero only in the limit of infinite time

Introduction

- realistic q. ss. are essentially non-inertial and manifest relativ. and gravit. characteristics, therefore relativ. q. investigations are of basic importance, both for applications in q. information protocols, and also to better understand the features of the universe
- investigations: behaviour of q. correls in relativ. framework (E manifests a sudden death phenomenon in the case of bosons, and it tends to a nonzero value in the case of fermions, when the acceleration tends to ∞; tasks of q. information processing in relativ. framework by using q. correls; in curved spacetime it is possible to generate q. E, opening a large possibility to be applied for q. information processing and transmission protocols; E is an important ingredient in the physics of black holes; Gaussian q. steering between a Kruskal observer and another one accelerated, near a Schwarzschild black hole; q. coherence of a bimodal Gaussian state in the asymptotically flat region of a black hole)

Introduction

- we describe the q. field dynamics of free massless bosonic modes near a Schwarzschild black hole
- we consider a system consisting of a stationary observer Alice in a region that is asymptotically flat (or who is freely falling in the black hole), with the associated mode A, an observer Bob hovering uniformly accelerated near the black hole event horizon, with the associated mode B, and anti-Bob, a hypothetical observer situated inside of the event horizon, with the associated mode $\bar{B}$
- we suppose that a Gaussian bimodal squeezed thermal state is shared by Alice and Bob, and we intend to investigate the influence of Schwarzschild black hole on q. E, by considering different scenarios
- we extend the study of the E of Schwarzschild modes by immersing them in a thermal reservoir and investigate its influence on the dynamics of q. E for all observers

Unruh-Hawking effect

- the radiation produced by the black hole through the Unruh-Hawking effect can be expressed in terms of an amplification Gaussian bosonic channel
- we take a metric characterizing the background of the spacetime in the region of an asymptotically flat and static Schwarzschild black hole of the form (we set the natural units $\hbar = G = c = k_B = 1$)

$$\begin{aligned} ds^2 = & -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 \\ & + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right), \end{aligned} \tag{1}$$

where r is the radial coordinate, t denotes the time, angles (θ, φ) define the metric on the two-sphere and M denotes the black hole mass

Unruh-Hawking effect

- massless bosonic field ϕ satisfies in the background of the black hole the following Klein-Gordon equation:

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\mu}}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial \phi}{\partial x^{\nu}}\right) = 0, \tag{2}$$

where $x^{\mu} = (t, r, \theta, \varphi)$ denotes the general coordinate and $g = \det g^{\mu\nu}$

Unruh-Hawking effect

- the bosonic field may be split into a region inside and a region outside of the black hole event horizon
- by solving the Klein-Gordon equation near the event horizon of the Schwarzschild black hole, we obtain the outgoing modes of positive frequency Ω in the form of plane waves:

$$\begin{aligned} \Phi^{+}_{\Omega,\text{in}} &\sim e^{\mathrm{i}\Omega u}, \\ \Phi^{+}_{\Omega,\text{out}} &\sim e^{-\mathrm{i}\Omega u}, \end{aligned} \tag{3}$$

where $\Phi^{+}_{\Omega,\text{in}}$ and $\Phi^{+}_{\Omega,\text{out}}$ denote the modes located inside the Schwarzschild black hole, respectively outside the region of the black hole, and we introduced the tortoise coordinate

$$u = t - (r + 2M \ln \frac{r - 2M}{2M}) \tag{4}$$

- using the Schwarzschild modes (3), the expression of the scalar field becomes:

$$\phi = \int d\Omega \left[\hat{a}_{\Omega,\mathrm{out}} \Phi^{+}_{\Omega,\mathrm{out}} + \hat{b}^{\dagger}_{\Omega,\mathrm{out}} \Phi^{-}_{\Omega,\mathrm{out}} + \hat{a}_{\Omega,\mathrm{in}} \Phi^{+}_{\Omega,\mathrm{in}} + \hat{b}^{\dagger}_{\Omega,\mathrm{in}} \Phi^{-}_{\Omega,\mathrm{in}} \right], \tag{5}$$

where by $\hat{a}_{\Omega,\mathrm{out}}$ and $\hat{b}^{\dagger}_{\Omega,\mathrm{out}}$ are denoted the boson annihilation and antiboson creation operators that act on states outside the black hole, respectively, while by $\hat{a}_{\Omega,\mathrm{in}}$ and $\hat{b}^{\dagger}_{\Omega,\mathrm{in}}$ we denote the boson annihilation and antiboson creation operators that act on inside states, respectively

Unruh-Hawking effect

- relation between the bosonic field at the black hole and the bosonic field in the flat spacetime can be obtained by introducing Unruh operators, which are connected with the Schwarzschild operators through the following Bogoliubov transformations:

$$C_{\Omega,\mathrm{R}} = \left(\cosh r_\Omega \; \hat{a}_{\Omega,\mathrm{out}} - \sinh r_\Omega \; \hat{b}^\dagger_{\Omega,\mathrm{in}}\right), \\ C_{\Omega,\mathrm{L}} = \left(\cosh r_\Omega \; \hat{a}_{\Omega,\mathrm{in}} - \sinh r_\Omega \; \hat{b}^\dagger_{\Omega,\mathrm{out}}\right), \tag{6}$$

  therefore the Unruh operators relate the creation and annihilation operators of particles and antiparticles in the regions outside and inside of the Schwarzschild black hole
- $\sinh r_\Omega = \left(e^{\frac{\Omega}{T_H}} - 1\right)^{-\frac{1}{2}}$ and T_H denotes the Hawking temperature
- Hawking temperature parameter r_Ω is a function that monotonically increases with T_H

Unruh-Hawking effect

- we introduce the generic Schwarzschild-Fock state $|nn, mm>_\Omega$, which describes the particles and antiparticles of the event horizon:

$$|nn, mm>_\Omega = |n_\Omega>^+_{\mathrm{out}} |n_{-\Omega}>^-_{\mathrm{in}} |m_{-\Omega}>^-_{\mathrm{out}} |m_\Omega>^+_{\mathrm{in}}, \quad (7)$$

  where the superscripts $\{+, -\}$ are used to indicate the particle and antiparticle modes, respectively

- by employing the Bogoliubov transformations between the Unruh modes and the Schwarzschild modes, the Unruh vacuum can be expressed as

$$|0_\Omega>_U = \frac{1}{\cosh^2 r_\Omega} \sum_{n,m=0}^{\infty} (\tanh r_\Omega)^{n+m} |nn, mm>_\Omega, \quad (8)$$

  each Unruh mode Ω corresponding to a Schwarzschild mode Ω

Unruh-Hawking effect

- we consider a bipartite system, where Alice is a stationary observer in asymptotically flat region, and Bob is a Schwarzschild observer who hovers uniformly accelerated near event horizon of the black hole
- Unruh vacuum state in single-mode approximation, when only bosons exist outside the event horizon (only particles can be detected as Hawking radiation, i.e. we assume that Bob has a detector sensitive only to particle modes) and antibosons are living inside event horizon, reduces to a two-mode squeezing state

$$|0_\Omega >_H = \frac{1}{\cosh r_\Omega} \sum_{n=0}^{\infty} (\tanh r_\Omega)^n \, |n>_{\rm out} \, |n>_{\rm in}, \qquad (9)$$

where by $|n>_{\rm out}$ and $|n>_{\rm in}$ are denoted the bosonic and antibosonic states, outside and inside of the event horizon, that belong to the observer Bob and, respectively, to the imaginary observer anti-Bob

Amplification bosonic channel

- we write the expression in Eq. (9) as the action of the following bimodal squeezing operator $\hat{U}(r)$ on associated states $|n>_{\rm in}$ and $|n>_{\rm out}$, inside and outside regions of event horizon (we abreviate $r_\Omega \equiv r$ for simplicity):

$$\hat{U}(r) = e^{r\left(\hat{b}^\dagger_{\Omega,{\rm out}}\hat{b}^\dagger_{\Omega,{\rm in}} - \hat{a}_{\Omega,\rm out}\,\hat{a}_{\Omega,\rm in}\right)}. \qquad (10)$$

- therefore, from Eq. (9) it follows that the Unruh-Hawking radiation can be expressed as an amplification bosonic channel with the squeezing operator $\hat{U}(r)$

Squeezing transformation

- the two-mode squeezing transformation $\hat{U}(r)$ is a Gaussian operation which preserves the Gaussianity of the input states, and it has the following symplectic phase space representation:

$$S_{B,\bar{B}}(r) = \begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\sinh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix} \tag{11}$$

- indeed, a unitary Gaussian operation $\hat{U}$ amounts, in phase space, to a symplectic transformations S (which preserves the symplectic form $\Omega = S^{\mathrm{T}}\Omega S$) acting by congruence on a covariance matrix σ, i.e., so that $\sigma \to S\sigma S^{\mathrm{T}}$
- then for instance, the two-mode squeezing operator $\hat{U}$ (10) corresponds to the symplectic transformation (11)

Bimodal Gaussian states

- we consider that the scalar massless field ϕ has two Unruh modes A and B, that share a bimodal Gaussian state with the density matrix ρ_{AB}
- we denote by $\mathbf{R} = \{x, p_x, y, p_y\}^{\mathrm{T}}$ the operators of canonical quadratures of two bosonic modes and by σ_{AB} the two-mode covariance matrix, whose elements are given by the statistical moments of second order of quadrature operators, that completely characterise the two-mode Gaussian states:

$$\sigma_{ij} = \mathrm{Tr}[(R_i R_j + R_j R_i)\rho_{AB}], i, j = 1, \dots, 4. \qquad (12)$$

- we neglect the moments of the first order, since they can be made zero by performing local displacements in the phase space
- the phase space operators R_i satisfy the canonical commutation relations $[R_i, R_j] = \mathrm{i}\Omega_{ij}$ and the covariance matrix fulfils the uncertainty relation $\sigma_{AB} + \mathrm{i}\Omega_{AB} \geq 0$, where $\Omega_{AB} = \oplus_1^2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is the symplectic form

Amplification channel

- the change between Unruh modes and Schwarzschild modes, given by the previously introduced amplification channel is expressed by the bimodal squeezing operation associated with symplectic transformation (11)
- the amplification maps the mode B into the modes B and $\bar{B}$ situated outside and inside of event horizon, respectively; therefore, from the point of view of a Schwarzschild observer, becomes relevant an additional mode $\bar{B}$
- consequently, the initial bipartite state is transformed into a three-partite state, with the Alice mode A, mode B of the Schwarzschild observer Bob, and mode $\bar{B}$ of a hypothetical observer anti-Bob, situated inside event horizon of Schwarzschild black hole

Covariance matrix

- therefore, the three-mode system can be described by the following covariance matrix:

$$\sigma_{AB\bar{B}}(s,r) = \left[I_A \oplus S_{B,\bar{B}}(r)\right] \left[\sigma^0_{AB}(s) \oplus I_{\bar{B}}\right] \left[I_A \oplus S_{B,\bar{B}}(r)\right]^{\mathrm{T}}, \tag{13}$$

where by $\sigma^0_{AB}(s)$ we denote the covariance matrix of the bimodal Gaussian state of bosonic fields, shared by Alice and Bob (s is the squeezing parameter)

Bimodal squeezed thermal state

- we suppose that Alice and Bob share a bimodal squeezed thermal state, which has the following covariance matrix in the asymptotically flat region:

$$\sigma_{AB}^{0} = \begin{pmatrix} a_0 & 0 & c_0 & 0 \\ 0 & a_0 & 0 & -c_0 \\ c_0 & 0 & b_0 & 0 \\ 0 & -c_0 & 0 & b_0 \end{pmatrix}, \qquad (14)$$

$$\begin{aligned} a_0 &= 2n_1 \cosh^2 s + 2n_2 \sinh^2 s + \cosh 2s, \\ b_0 &= 2n_2 \cosh^2 s + 2n_1 \sinh^2 s + \cosh 2s, \\ c_0 &= (n_1 + n_2 + 1) \sinh 2s \end{aligned} \qquad (15)$$

- s denotes the parameter of squeezing parameter of the state and n_1, n_2 denote the associated average thermal photon numbers; a two-mode thermal squeezed state is entangled if it is satisfied the relation

$$s > s_e, \quad \cosh^2 s_e = \frac{(n_1+1)(n_2+1)}{n_1+n_2+1} \qquad (16)$$

Case I.

- an observer situated outside black hole is causally disconnected from the inside region $\rightarrow$ Alice, who lives in asymptotically flat region, and Bob, who hovers uniformly accelerated near event horizon cannot access mode $\bar{B}$
- covariance matrix for the outside region can be obtained by performing trace over mode $\bar{B}$ situated inside black hole, associated with the hypothetical observer anti-Bob
- we obtain from Eq. (13) the following covariance matrix of Alice and Bob:

$$\sigma_{AB}(s, r) = \begin{pmatrix} \mathcal{A} & \mathcal{C} \\ \mathcal{C}^{\mathrm{T}} & \mathcal{B} \end{pmatrix}, \tag{17}$$

where:

$$\begin{aligned} \mathcal{A} &= a_0 \mathcal{I}_2, \\ \mathcal{B} &= \left[b_0 \cosh^2 r + \sinh^2 r \right] \mathcal{I}_2, \\ \mathcal{C}_{AB} &= c_0 \cosh r \mathcal{Z}_2, \end{aligned} \tag{18}$$

$\mathcal{I}_2$ is identity matrix and $\mathcal{Z}_2$ is Z-Pauli matrix

Logarithmic negativity

- to quantify quantum entanglement we use as a measure the logarithmic negativity, which can be expressed through the symplectic invariants of covariance matrix σ_{AB}:

$$E_N = -\log_2 g(\sigma), \tag{19}$$

where

$$g(\sigma) = \frac{1}{\sqrt{2}}\sqrt{\det\mathcal{A} + \det\mathcal{B} - 2\det\mathcal{C} - \sqrt{(\det\mathcal{A} + \det\mathcal{B} - 2\det\mathcal{C})^2 - 4\det\sigma_{AB}}} \tag{20}$$

- for $E_N \leq 0$ the state is separable and $E_N > 0$ determines the strength of the entanglement

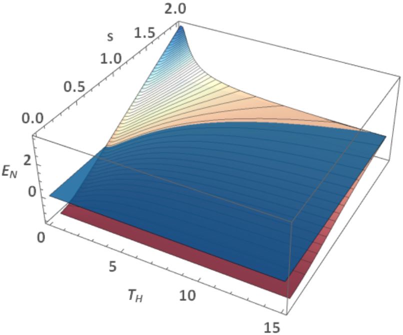

Figure 1: Alice-Bob logarithmic negativity E_N versus Hawking temperature T_H and squeezing parameter s: STS (left), SVS (right).

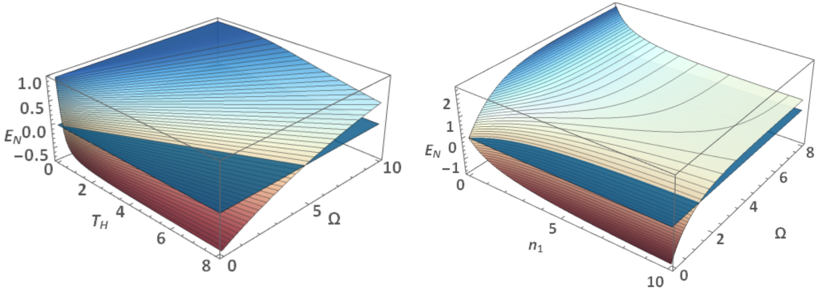

Figure 2: Alice-Bob logarithmic negativity E_N versus frequency Ω and: Hawking temperature T_H (left), and average thermal photon number n_1 (right).

Case II.

- recently there have been obtained interesting results by studying quantum correlations between causally disconnected regions of spacetime
- investigation of quantum entanglement between physically inaccessible regions would contribute to better understand the connection between black hole physics and quantum information
- covariance matrix of Bob and anti-Bob is obtained by taking in Eq. (13) the partial trace over the Alice mode:

$$\sigma_{B\bar{B}}(s,r) = \begin{pmatrix} \mathcal{A} & \mathcal{C} \\ \mathcal{C}^{\mathrm{T}} & \mathcal{B} \end{pmatrix}, \tag{21}$$

where:

$$\begin{aligned} \mathcal{A} &= \left[b_0 \cosh^2 r + \sinh^2 r\right] \mathcal{I}_2, \\ \mathcal{B} &= \left[\cosh^2 r + b_0 \sinh^2 r\right] \mathcal{I}_2, \\ \mathcal{C}_{B\bar{B}} &= \left[(1+b_0)\sinh r \cosh r\right] \mathcal{Z}_2 \end{aligned} \tag{22}$$

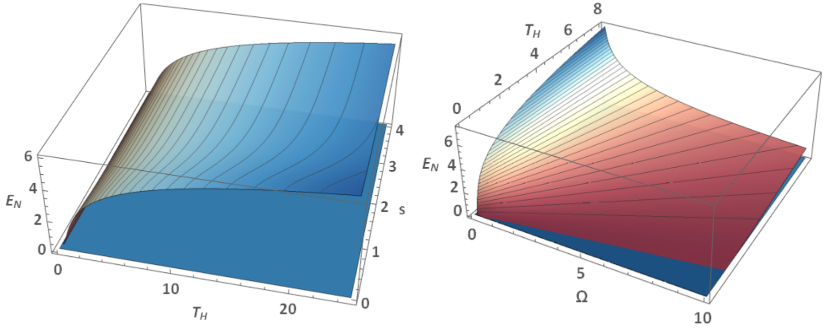

Figure 3: Bob-anti-Bob logarithmic negativity E_N versus Hawking temperature T_H and: squeezing parameter s (left), and frequency Ω (right).

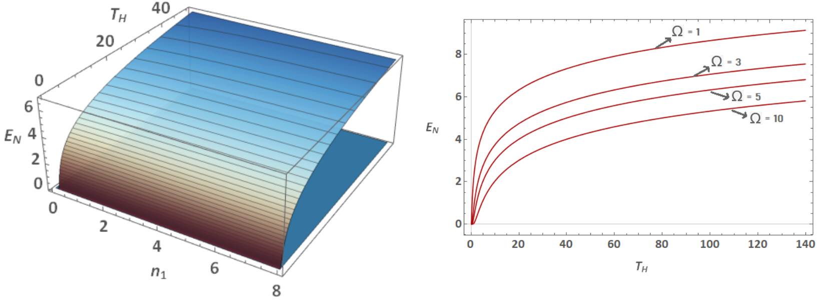

Figure 4: Bob-anti-Bob logarithmic negativity E_N versus Hawking temperature T_H and: thermal photon number n_1 (left), and for different values of frequency Ω (right) for SVS.

Dependence on s

- in general, the physically inaccessible quantum correlations and coherence increase by increasing the parameter s
- at the same time, in our case the logarithmic negativity between Bob and Anti-Bob decreases as the parameter s increases; however, the decreasing of the logarithmic negativity by increasing the initial parameter s is very weak, and it takes place for relatively small values of s, while the main feature of the behaviour of the logarithmic negativity is that it tends to reach a plateau in the limit of large values of s

Case III.

- finally, we analyse the existence of quantum correlations of Alice and anti-Bob modes
- their covariance matrix can be obtained by performing in Eq. (13) the partial trace over the Bob mode:

$$\begin{aligned} \mathcal{A} &= a_0 \mathcal{I}_2, \\ \mathcal{B} &= \left[\cosh^2 r + b_0 \sinh^2 r\right] \mathcal{I}_2, \\ \mathcal{C}_{A\bar{B}} &= c_0 \sinh r \mathcal{Z}_2, \end{aligned} \tag{23}$$

and the calculations show that $E_N < 0$, so that the state of Alice and anti-Bob is always separable

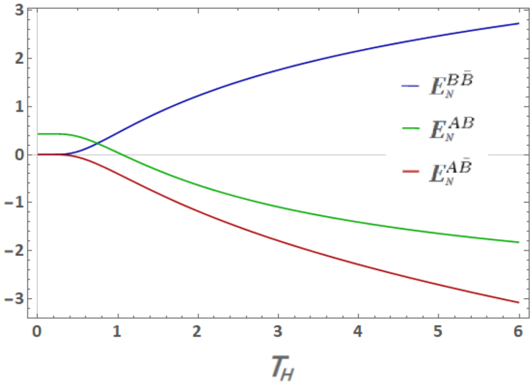

Figure 5: Logarithmic negativity E_N for different scenarios: Alice and Bob (green), Alice and anti-Bob (red), Bob and anti-Bob (blue) for $s = 0.4, \Omega = 1, n_1 = 25, n_2 = 0$.

Open quantum systems

- we will investigate the time evolution of the entanglement between the observers by considering that the two bosonic modes are immersed in a common thermal bath, and they evolve also under the influence of the Hawking radiation, manifested in the curved space-time associated with the Schwarzschild black hole
- to study the dynamics of the considered system, we use the axiomatic formalism based on completely positive quantum dynamical semigroups
- in this framework the Markovian irreversible time evolution of an open system is described by the following Gorini-Kossakowski-Sudarshan-Lindblad master equation for the density operator:

$$\frac{d\rho(t)}{dt} = -i[H, \rho(t)] + \frac{1}{2}\sum_j (2B_j\rho(t)B_j^\dagger - \{\rho(t), B_j^\dagger B_j\}_+) \quad (24)$$

Open quantum systems

- for identical frequencies $\omega = 1$ for the two modes

$$H = \frac{1}{2}(x^2 + p_x^2 + y^2 + p_y^2) \tag{25}$$

  (Hamiltonian of the open system)
- operators $B_j, B_j^\dagger$, defined on the Hilbert space of H, describe the interaction of the system with a general environment
- if the operators B_j are taken polynomials of first degree in the canonically conjugated quadrature operators x, p_x, y, p_y of the two bosonic modes and if we choose initial Gaussian states, then Gaussianity is preserved in time due to the linear character of the dynamics

- from the master equation (24) we obtain that the time evolution of the corresponding bimodal covariance matrix $\sigma(t)$ is given by the following equation of motion, in the form of the Lyapunov equation:

$$\frac{d\sigma(t)}{dt} = Y\sigma(t) + \sigma(t)Y^{\mathrm{T}} + D, \tag{26}$$

where

$$Y = \begin{pmatrix} -\lambda & 1 & 0 & 0 \\ -1 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 1 \\ 0 & 0 & -1 & -\lambda \end{pmatrix} \tag{27}$$

is the drift matrix, λ is the dissipation rate

- D is the diffusion matrix:

$$D = 2\,\mathrm{diag}\{\lambda \coth \frac{1}{2k_B T}, \lambda \coth \frac{1}{2k_B T}, \lambda \coth \frac{1}{2k_B T}, \lambda \coth \frac{1}{2k_B T}\}, \tag{28}$$

  where k_B is the Boltzmann constant and T is the temperature of the thermal reservoir
- the time-dependent solution of Eq. (26) is given by

$$\sigma(t) = \gamma(t)[\sigma_{XY}(s,r) - \sigma_T]\gamma^{\mathrm{T}}(t) + \sigma_T, \tag{29}$$

  where $\sigma_{XY}(s,r)$ is the initial covariance matrix given by Eq. (17) for the observers Alice and Bob and, respectively, by Eq. (21) for Bob and anti-Bob, and $\gamma(t) \equiv \exp(Yt)$, with $\gamma(t) \to 0$ when $t \to \infty$
- the described evolution generated by a Gaussian completely positive map is determined by the two 4 x 4 real matrices γ and $A = \sigma_T - \gamma\sigma_T\gamma^{\mathrm{T}}$, which satisfy $A + i\Omega_{AB} \geq i\gamma\Omega_{AB}\gamma^{\mathrm{T}}$

Open quantum systems

- the covariance matrix corresponding to the asymptotic Gibbs state of the system of two bosonic modes, interacting with the thermal bath of temperature T, is given by (we set Boltzmann constant $k_B = 1$):

$$\sigma_T = \begin{pmatrix} \coth\frac{1}{2T} & 0 & 0 & 0 \\ 0 & \coth\frac{1}{2T} & 0 & 0 \\ 0 & 0 & \coth\frac{1}{2T} & 0 \\ 0 & 0 & 0 & \coth\frac{1}{2T} \end{pmatrix}$$

- to describe the time evolution of the logarithmic negativity as a measure of the Gaussian quantum entanglement between the considered observers, we suppose that Alice and Bob share initially a bimodal symmetric thermal squeezed state, with the covariance matrix given by Eq. (14) in which we denote $n \equiv n_1 = n_2 = 1$

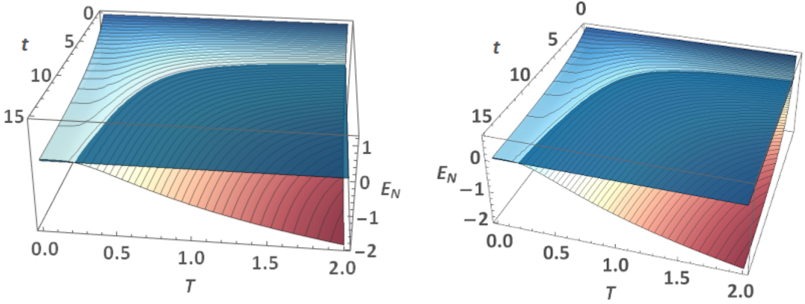

Figure 6: Logarithmic negativity E_N versus bath temperature T and time t for Alice-Bob (left), and for Bob-anti-Bob (right).

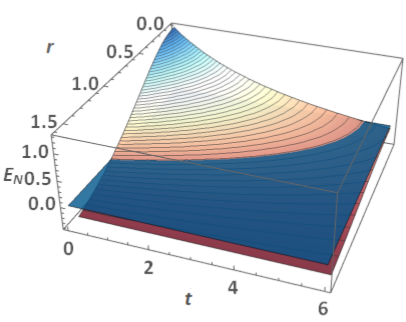

Figure 7: Logarithmic negativity E_N versus Hawking parameter r and time t for Alice-Bob (left), and for Bob-anti-Bob (right).

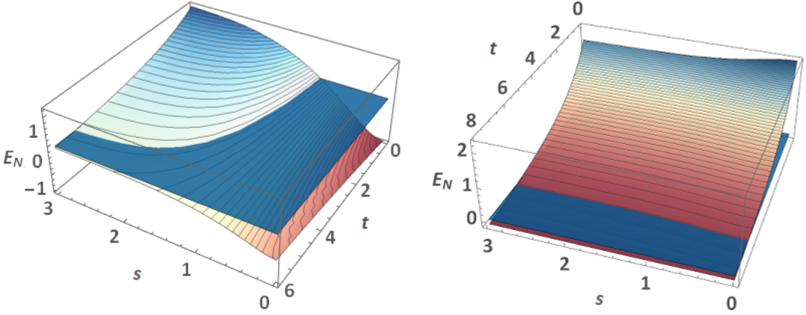

Figure 8: Logarithmic negativity E_N versus squeezing parameter s and time t for Alice-Bob (left), and for Bob-anti-Bob (right)

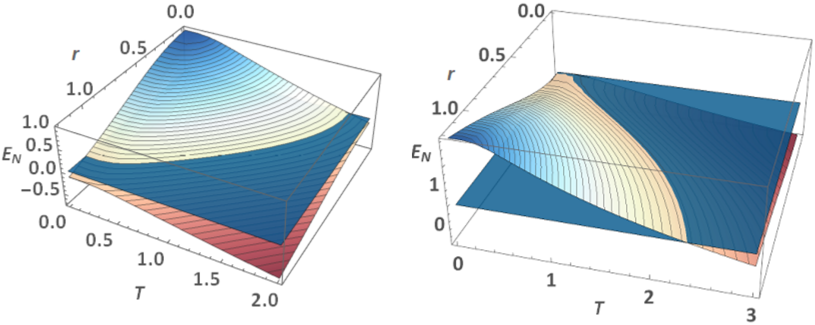

Figure 9: Logarithmic negativity E_N versus bath temperature T and Hawking parameter r for Alice-Bob (left), and for Bob-anti-Bob (right).

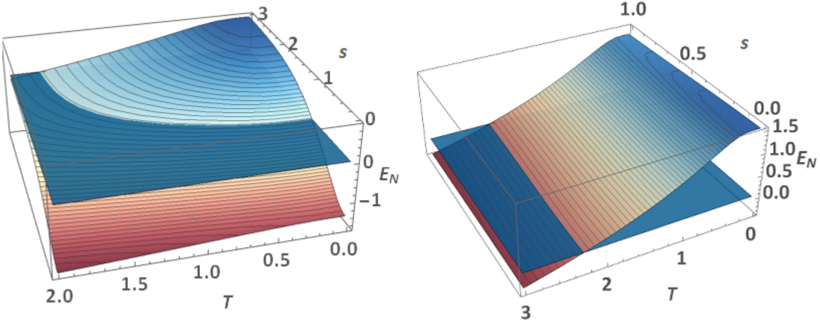

Figure 10: Logarithmic negativity E_N versus bath temperature T and squeezing s for Alice-Bob (left), and for Bob-anti-Bob (right).

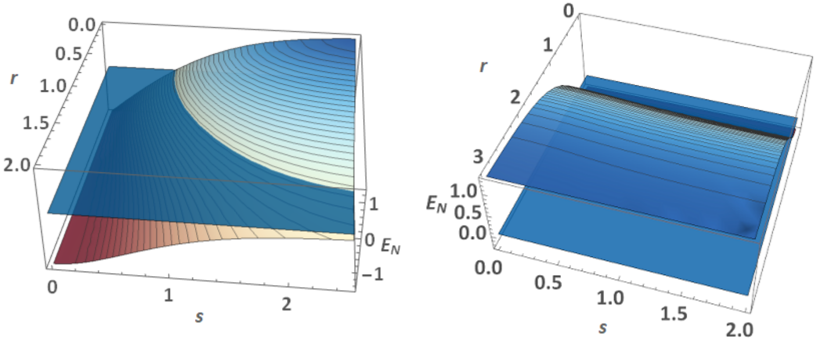

Figure 11: Logarithmic negativity E_N versus Hawking parameter r and squeezing parameter s for Alice-Bob (left), and for Bob-anti-Bob (right).

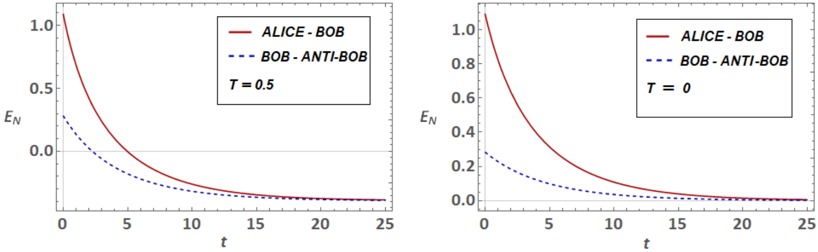

Figure 12: Evolution of logarithmic negativity E_N over time for both Alice-Bob and Bob-anti-Bob for a non-zero temperature of the bath (left) and zero temperature (right).

Conclusions

- we investigated the influence of the Hawking effect on Gaussian q. E for the modes of a massless scalar field in the presence of a Schwarzschild black hole in various scenarios
- for a system consisting of a Kruskal observer Alice and an accelerated observer Bob hovering outside the event horizon of black hole, E depends on the squeezing of modes, the average numbers of thermal photons of the squeezed thermal state, and on the Hawking parameter, through the Hawking temperature and frequency
- E of Alice and Bob is decreasing by increasing Hawking temperature.
- Hawking radiation induces a thermal noise that can cause the decay of q. E in the system
- q. E increases with the squeezing parameter of modes, tending to a definite value for large values of squeezing, and decreases by increasing the numbers of thermal photons

Conclusions

- in the limit of large frequency of the bosonic field, the Hawking effect tends to vanish, and therefore the loss of entanglement, caused by Hawking effect, can be reduced by increasing the frequency of the field, and, moreover, it is also possible even to generate entanglement, for definite values of the frequency
- consequently, the competition between these contrary influences, produced by the Hawking temperature, squeezing parameter and frequency of the field, may facilitate the preservation of the accessible entanglement, that can be useful for the quantum information processing in relativistic setting, and possibly for applications in quantum technologies

Conclusions

- while there exists no entanglement between the causally disconnected mode of a Kruskal observer Alice and a mode associated with an imaginary observer anti-Bob, in the case of modes that are causally disconnected, associated with observers Bob and anti-Bob, by increasing the Hawking temperature strengthens the entanglement of modes, in comparison with the first scenario of modes associated with Alice and Bob, where logarithmic negativity decreases by increasing the Hawking temperature
- it is also possible even the generation of quantum entanglement for the observers Bob and anti-Bob, due to the effect of Hawking radiation, in other words the Gaussian amplification operation can create quantum entanglement

Conclusions

- this behaviour of quantum entanglement is also proved by the fact that the increase of squeezing of the modes leads to a slight degradation of quantum entanglement, while, for a given value of the squeezing parameter, the logarithmic negativity increases with Hawking temperature
- in this scenario the competition between the influences produced by Hawking temperature, squeezing and frequency of the field, may lead to the survival of the entanglement, that could present interest in the investigation of black holes

Conclusions

- we have also described the influence of the thermal environment on the behaviour in time of the entanglement between the considered observers, evolving also under the influence of the Hawking radiation
- due to decoherence and dissipation induced by the environment, the entanglement is destroyed in a finite time for both considered bipartite scenarios of observers Alice and Bob, and respectively Bob and anti-Bob, for non-zero values of the temperature of the thermal environment, i.e. the phenomenon of entanglement sudden death takes place
- the only exception is that for a zero temperature of the thermal bath, the initial existing entanglement is decreasing over time, but it keeps for all finite times a non-zero value and the logarithmic negativity tends to zero only in the limit of infinite time

Conclusions

- we have also illustrated that the logarithmic negativity is decreasing by increasing the temperature of the environment and that the survival time of the entanglement is decreasing by increasing the temperature in both scenarios, while the survival time decreases by increasing the Hawking parameter for the observers Alice and Bob, and it increases with the Hawking parameter for the observers Bob and anti-Bob
- conversely, the survival time of the entanglement increases with the squeezing parameter for the observers Alice and Bob and slightly decreases by increasing the squeezing parameter for the observers Bob and anti-Bob
- in addition, for the modes associated with Bob and anti-Bob the logarithmic negativity is increasing with the Hawking parameter, so that for sufficiently large values of it, the entanglement can be generated at a given moment of time

Conclusions

- since the realistic systems evolve under the influence of the environment, manifested through the quantum decoherence, it may present interest to describe, besides quantum entanglement and quantum steering, the time evolution in curved spacetime of other quantum correlations (for instance quantum discord), mutual information and quantum coherence, for a system composed of bimodal bosonic fields interacting with a thermal environment or a squeezed thermal environment

Thank you!