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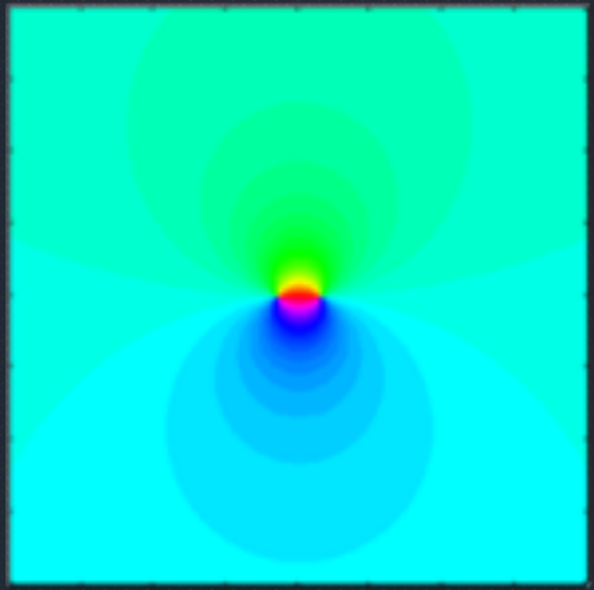
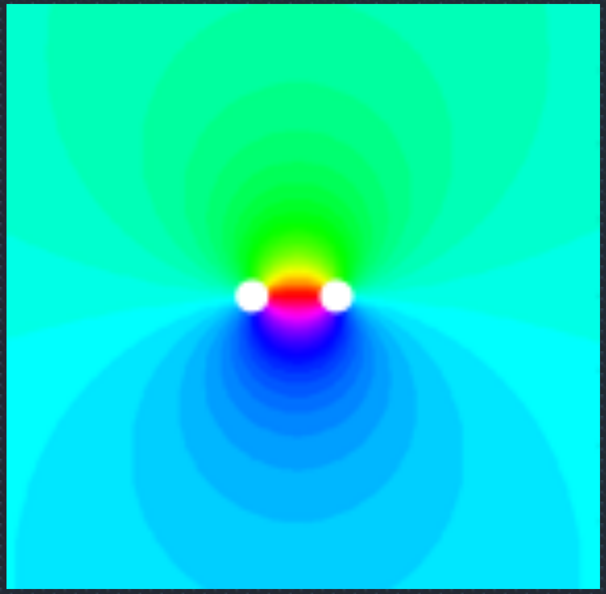
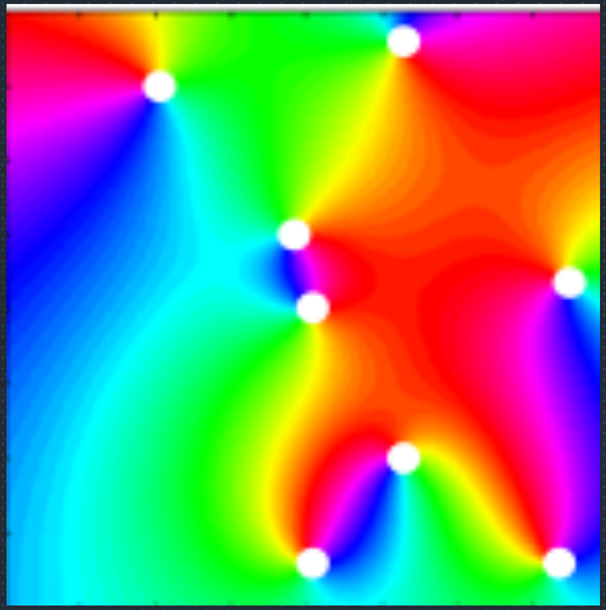






Nicolò Defenu

XY Model

$$\beta H_{XY} = -K \sum_{\langle ij \rangle} \left[\cos\left(\theta_i - \theta_j\right) - 1\right]$$

Low temperature

$$\beta H_{\mathrm{sw}} = \frac{K}{2} \int \left(\nabla \theta\right)^2 d^2 x.$$

Non-periodic

Critical scaling at all temperatures

Spin-spin correlation: $$G_{ij} = \langle \cos(\theta_i - \theta_j) \rangle.$$

Power law behavior at all temperatures:

$$M(x) = e^{-\frac{1}{K}\mathcal{G}(0)} \to M_L \propto \left(\frac{a}{L}\right)^{\frac{1}{2\pi K}}$$

$$G(x) = e^{\frac{1}{K}[\mathcal{G}(x)-\mathcal{G}(0)]} \propto \left(\frac{a\pi}{x}\right)^{\frac{1}{2\pi K}}$$

Critical scaling at all temperatures

Spin-wave

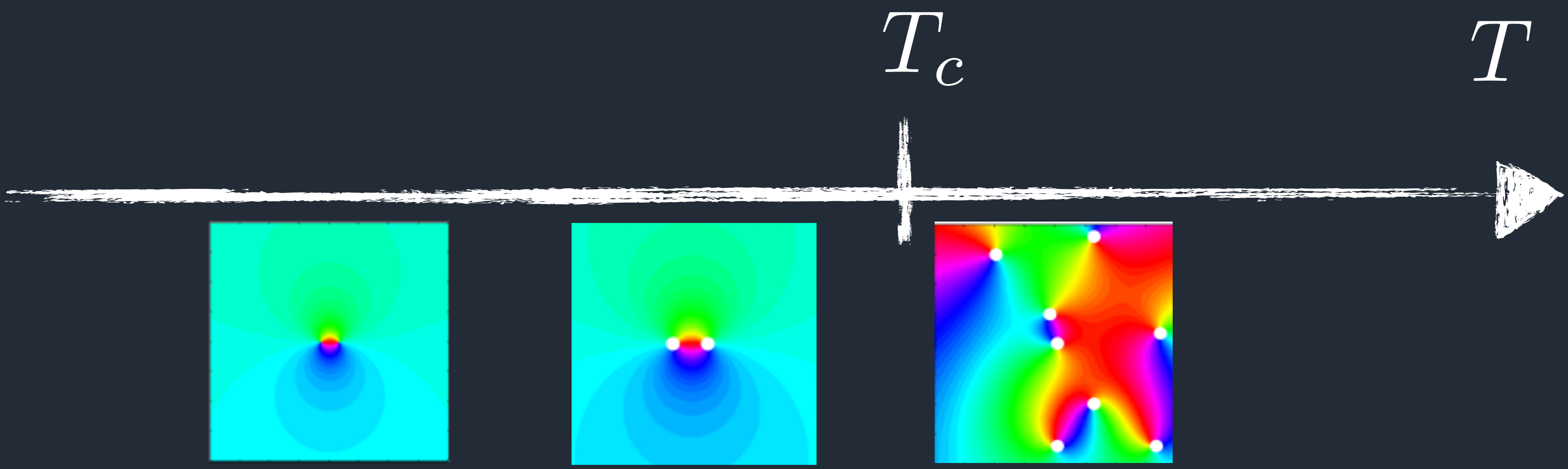
$$\nabla \times \mathbf{j}_{||} = 0$$

$$\int \mathbf{j}_{\perp} \cdot \mathbf{j}_{||} d\mathbf{r} = 0$$

Vortexes

$$\nabla \cdot \mathbf{j}_{\perp} = 0$$

$$\oint \mathbf{j}_{\perp} dl = 2\pi \sum_i q_i$$



Bilayer effect in the XY model

$$\mathscr{H} = -J\sum_{\langle ij\rangle}\cos\left(\phi_i - \phi_j\right) - J\sum_{\langle ij\rangle}\cos\left(\psi_i - \psi_j\right) - K\sum_{i}\cos\left(\phi_i - \psi_i\right),$$

Intra-plane correlations

$$c_{\uparrow}(k) = \sum_{|i-j|=k} \exp(\mathrm{i}\phi_i - \mathrm{i}\phi_j)$$

$$c_{\downarrow}(k) = \sum_{|i-j|=k} \exp(\mathrm{i}\psi_i - \mathrm{i}\psi_j)$$

Inter-plane correlations

$$z(k) = \sum_{|i-j|=k} \exp(\mathrm{i}\phi_i + \mathrm{i}\psi_i - \mathrm{i}\phi_j - \mathrm{i}\psi_j)$$

Field theoretic representation of the bilayer XY

$$S[\phi] = \frac{1}{2}\sum_{\sigma,q}\frac{\varphi_\sigma(q)\varphi_\sigma(q)}{K^\sigma(q)} + \sum_{l}\int U\left(\left|\varphi_l\right|\right) d^2x$$

New variables

$$\varphi_\sigma = \sqrt{\rho_\sigma} e^{\mathrm{i}\theta_\sigma(x)}$$

+

$$\varphi_\pm(q) = \left(\varphi_1(q) \pm \varphi_2(q)\right)/\sqrt{2}$$

Mean-field solution

$$S_{\text{kin}}[\phi] = \frac{1}{2}\sum_{\sigma,q} \varphi_q^\sigma \left(\frac{1}{K^\sigma(q)} - \frac{1}{K^\sigma(0)} \right) \varphi_{-q}^\sigma$$

The inverse mass explicitly depends on K

$$m_\pm^{-1} = \frac{J}{K^\pm(0)^2} \quad \text{with} \quad K^\sigma(q) = 2J\varepsilon_0(q) + 2\mu + \sigma 2K$$

New phase: $1 - K - \mu < 2J < 1 + K - \mu$ $\langle \varphi_+ \rangle > 0$ but $\langle \varphi_- \rangle = 0$

Mean-field phase action

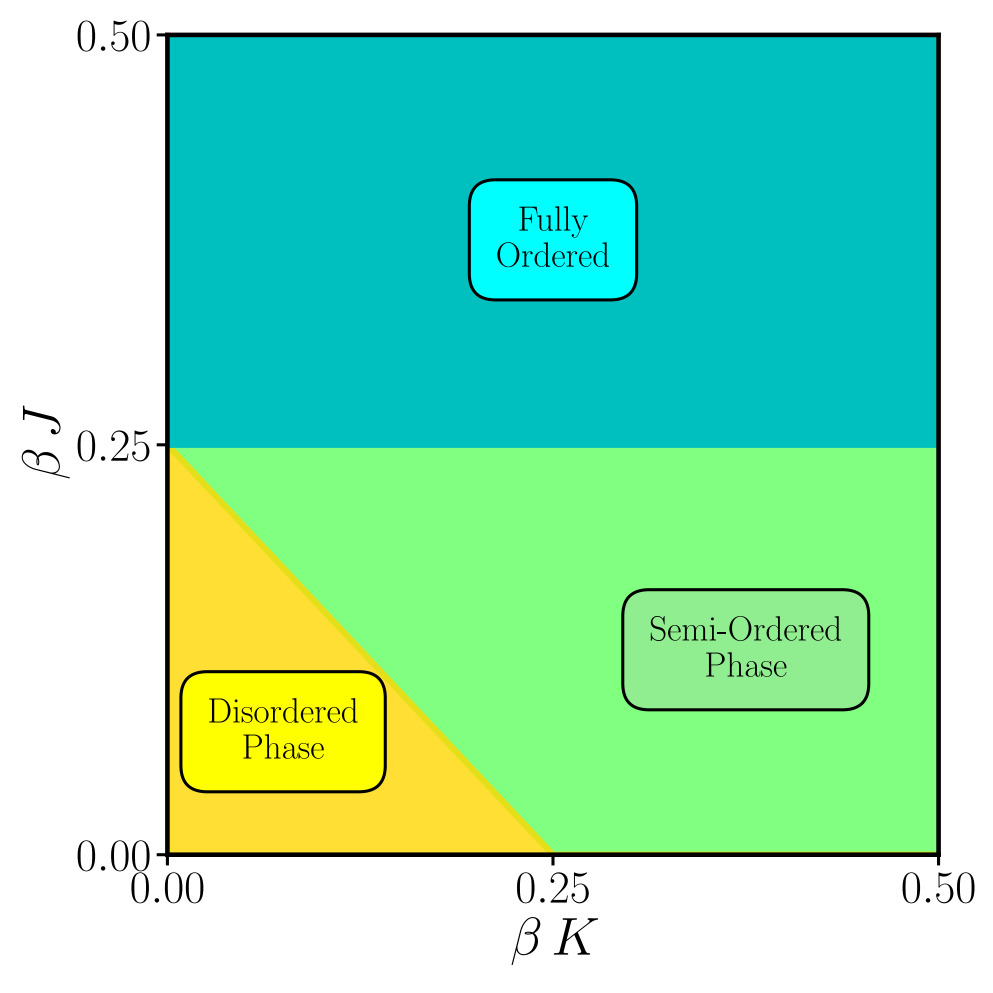
$$S[\theta] = \sum_{\sigma} \int d^2x \frac{\rho_\sigma}{2m_\sigma} \partial_\mu \theta_\sigma \partial_\mu \theta_\sigma$$

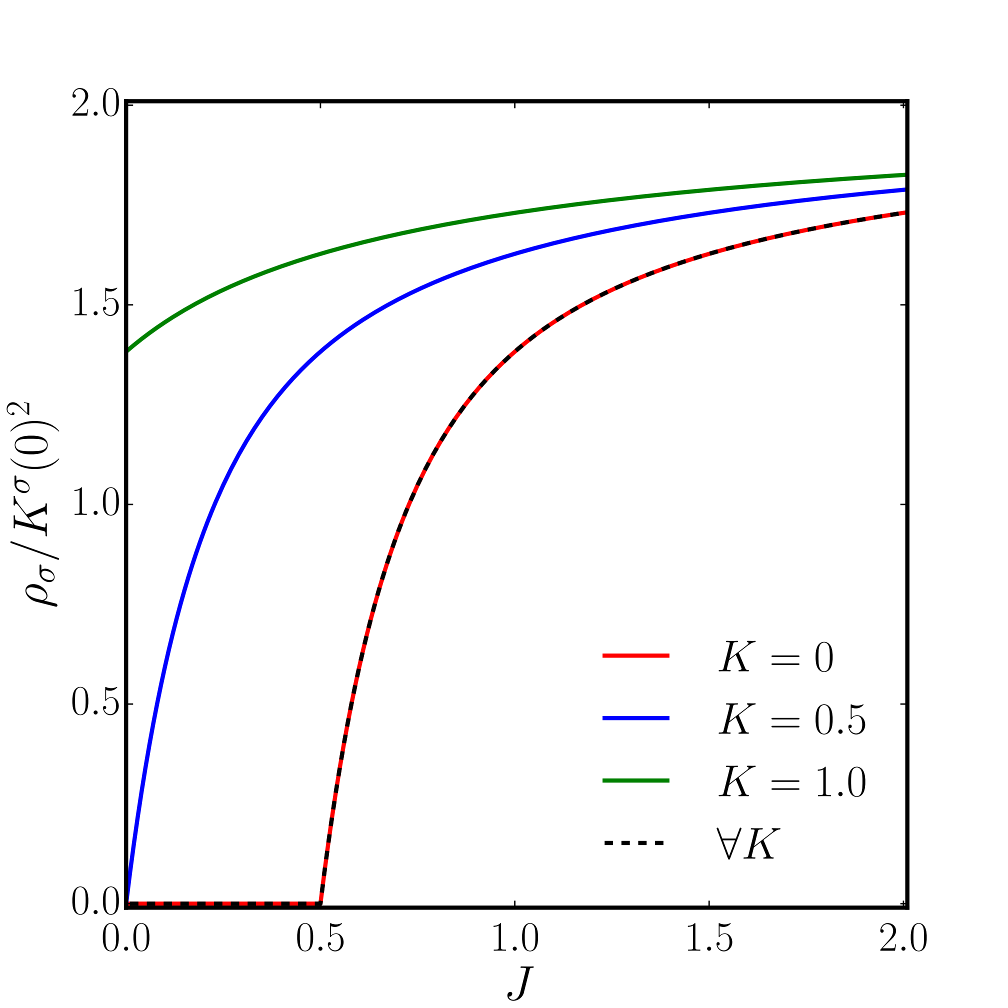
The inverse mass explicitly depends on the mode

$$m_{\pm}^{-1} = \frac{J}{K^{\pm}(0)^2} \quad \text{with} \quad K^{\sigma}(q) = 2J\varepsilon_0(q) + 2\mu + \sigma 2K$$

- "Berezinskii-Kosterlitz-Thouless Paired Phase in Coupled XY Models." G. Bighin, ND, et al., Phys. Rev. Lett. 123, 100601 (2019).

Mean-field effective stiffness





Mean-field + Renormalization Group

Kosterlitz-Thouless flow

$$\partial_t K_k = -\pi g_k^2 K_k^2$$

$$\partial_t g_k = \pi \left(\frac{2}{\pi} - K_k \right) g_k$$

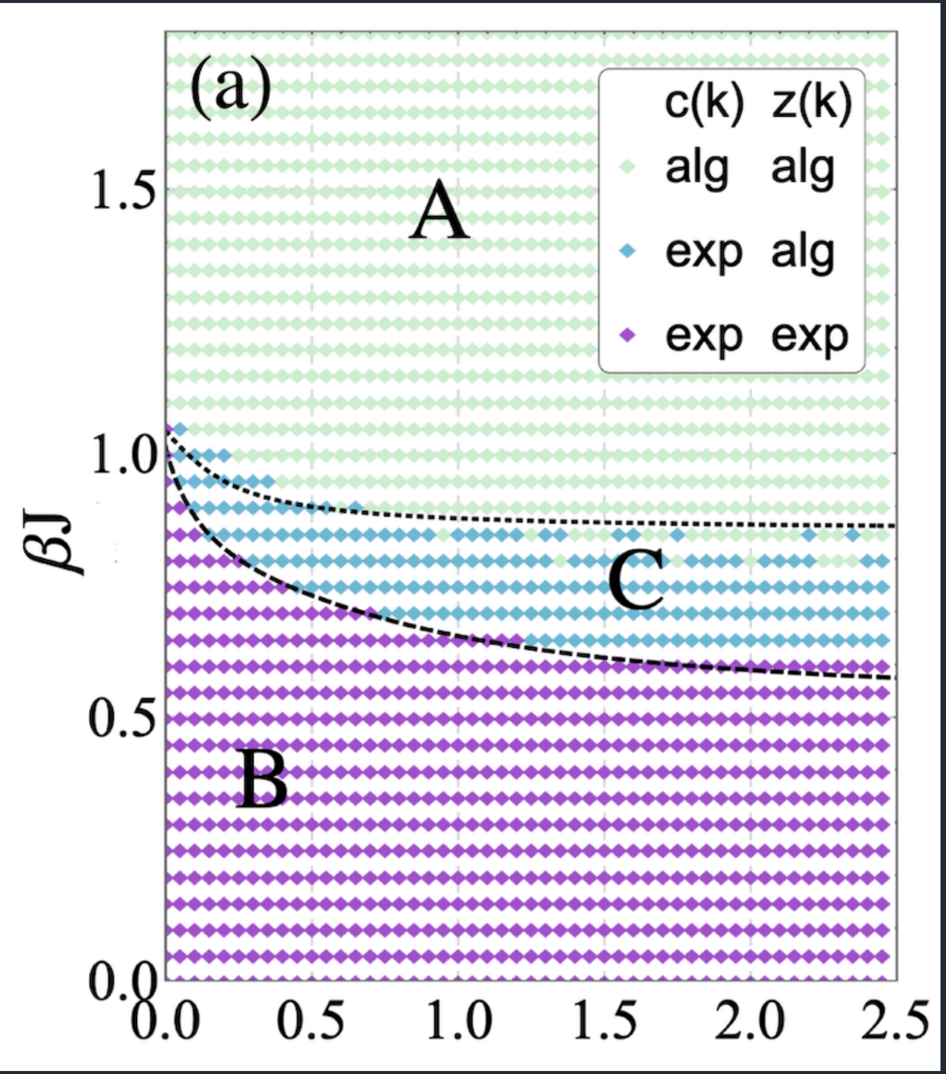
Initial conditions

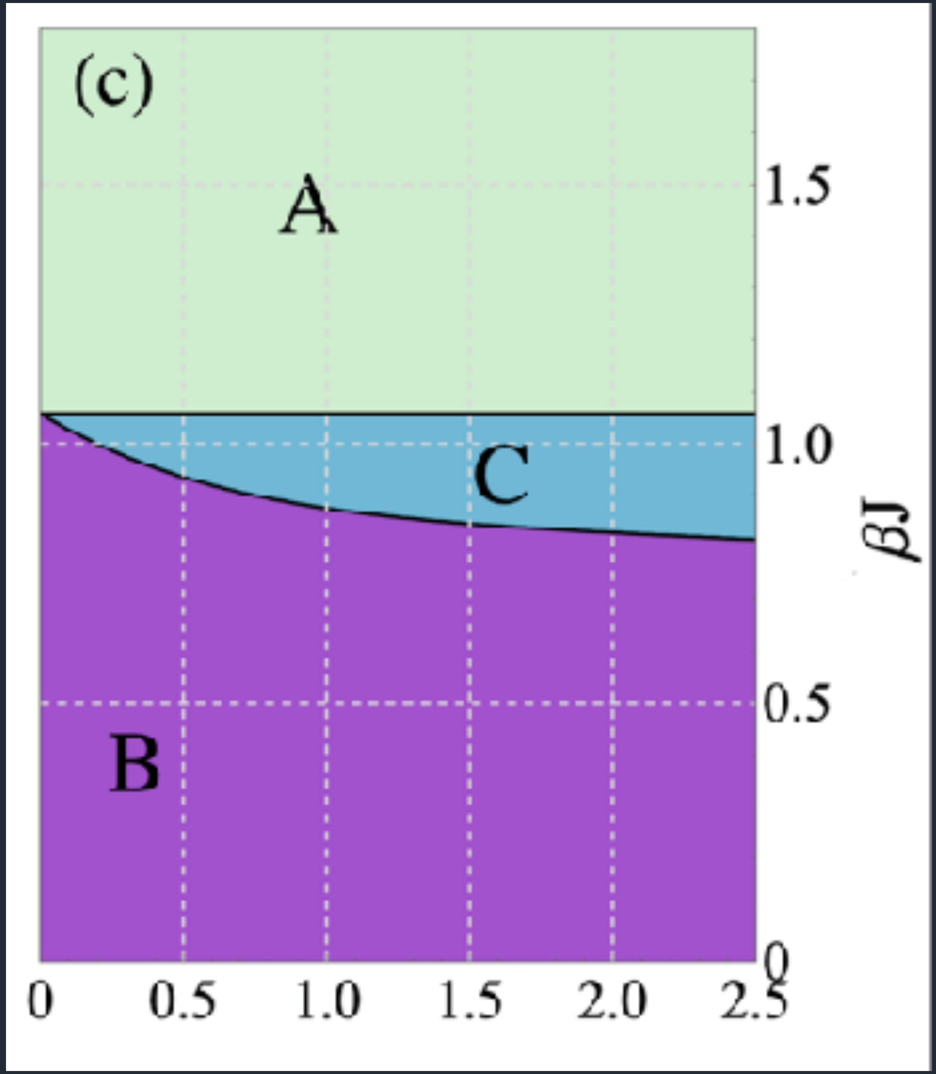
$$K_\Lambda = J_{\text{eff}}^\sigma,$$

$$g_\Lambda = 2\pi e^{-\pi^2 K_\Lambda / 2}$$

$$J_{\text{eff}}^\sigma = \frac{\rho_\sigma}{2m_\sigma}.$$

BKT flow made quantitative

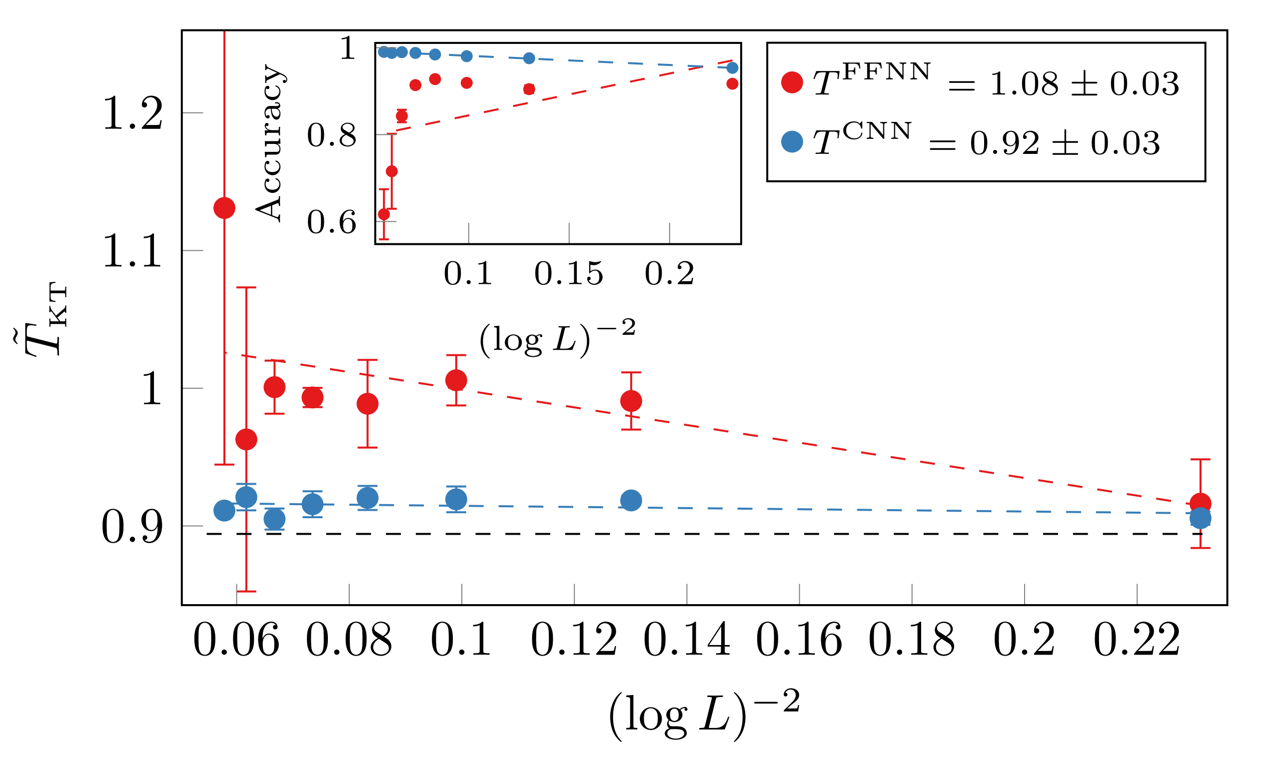


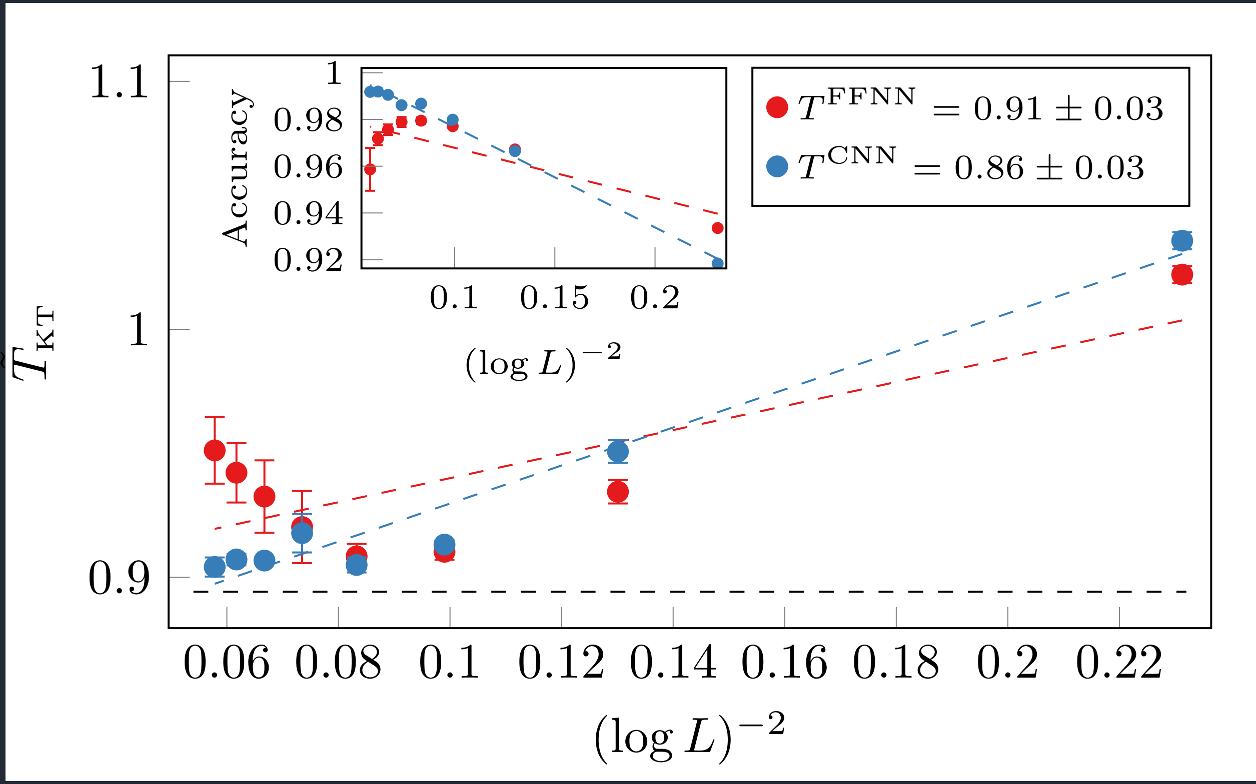


- “Berezinskii-Kosterlitz-Thouless Paired Phase in Coupled XY Models.” G. Bighin, ND, et al., Phys. Rev. Lett. 123, 100601 (2019).

Machine learning BKT Phase

Pre-processing is needed to learn the BKT phase





- "Machine learning vortices at the Kosterlitz-Thouless transition." M.J.S. Beach, et al., Phys. Rev. B 97, 045207 (2018).

Machine learning bilayer models

Bilayer Hamiltonians are characterised by two parameters K-J

The training of the CNN learns to distinguish MC snapshots belonging to two different points (J1, K1) and (J2, K2), in the phase diagram.

Classification accuracy: $$\varphi = \frac{N_s}{N}$$

- "Detecting composite orders in layered models via machine learning." W. Rzadkowski, et al., New J. Phys. **22** (2020) 093026.

Pseudo-distance and confusion

If the two points belong to the same phase the algorithm will be confused $\varphi \approx 0.5$

We introduce the notion of pseudo-distance in the phase diagram

$$d\Big((J_1, K_1), (J_2, K_2)\Big) = 2(\varphi - 0.5)\Theta(\varphi - 0.5)$$

Identical phases

$$d \approx 0$$

Different phases

$$d \approx 1$$

- "Detecting composite orders in layered models via machine learning." W. Rzadkowski, et al., New J. Phys. **22** (2020) 093026.

Similarity measure in phase space

We introduce a similarity measure $u(J, K)$ for adjacent points in the phase diagram

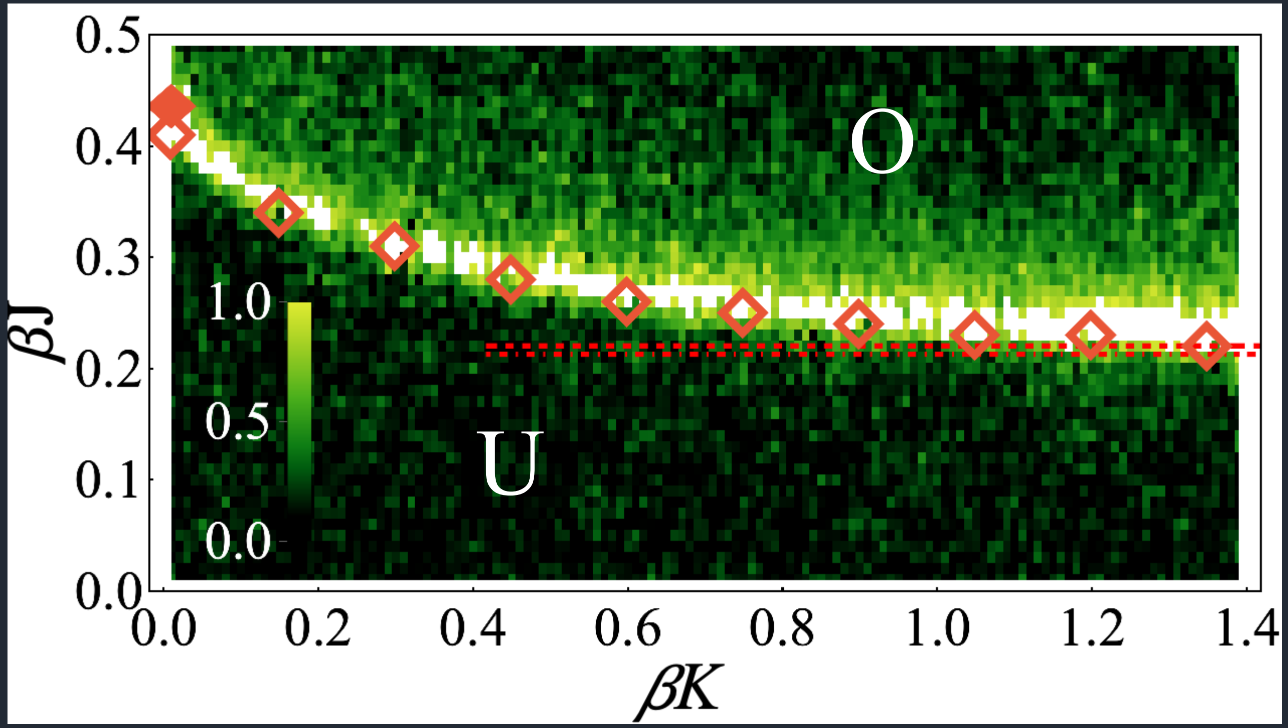
$$\nabla u(J, K) \equiv \begin{pmatrix} d((J + \Delta J, K), (J, K))/\Delta J \\ d((J, K + \Delta K), (J, K))/\Delta K \end{pmatrix}.$$

A peak of $\nabla^2 u(j, J)$ signals a phase transition

- "Detecting composite orders in layered models via machine learning." W. Rzadkowski, et al., New J. Phys. **22** (2020) 093026.

Toy model (1): bilayer Ising model

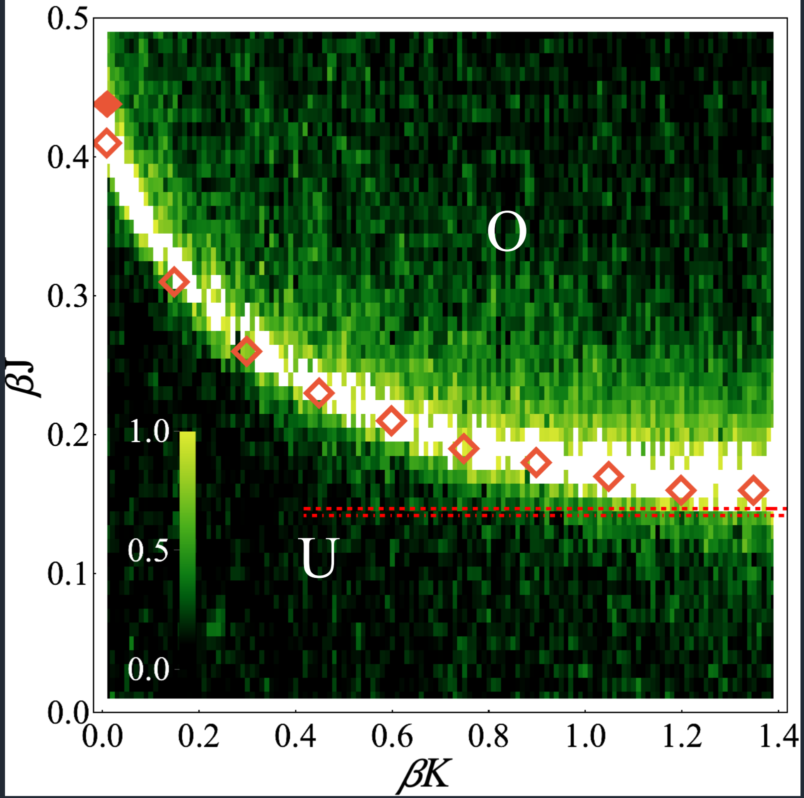
$$H_b = -J\sum_{\langle ij\rangle}\sigma_i\sigma_j - J\sum_{\langle ij\rangle}\tau_i\tau_j - K\sum_i \sigma_i\tau_i,$$

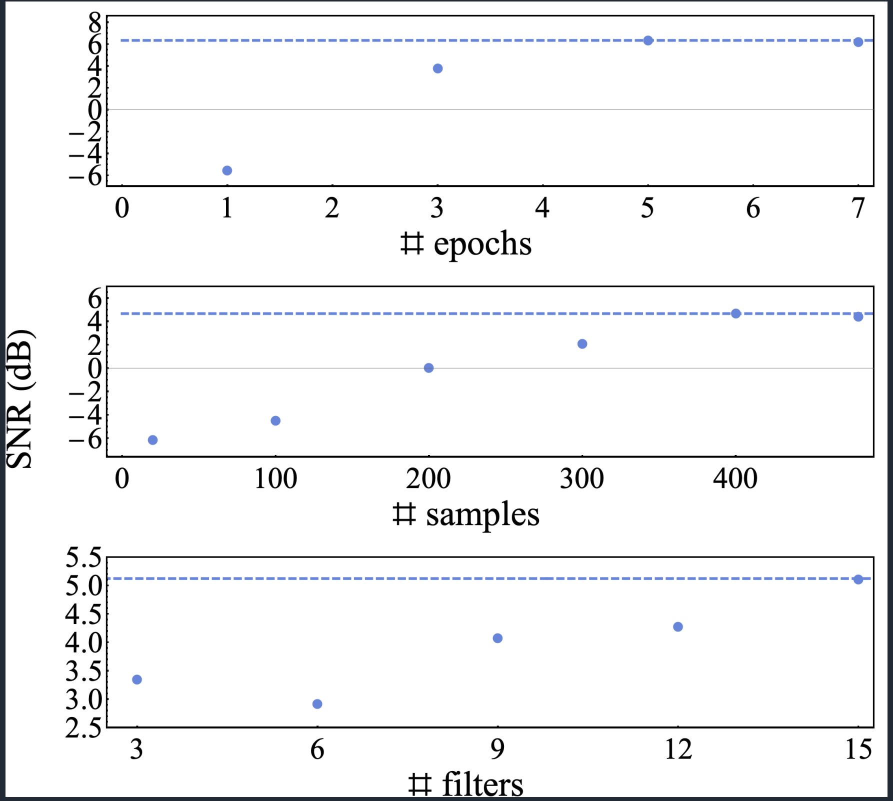


- "Detecting composite orders in layered models via machine learning." W. Rzadkowski, et al., New J. Phys. **22** (2020) 093026.

Toy model (2): bilayer Ising model

$$H_t = -J\sum_{\langle ij\rangle}\sigma_i\sigma_j - J\sum_{\langle ij\rangle}\tau_i\tau_j - J\sum_{\langle ij\rangle}v_iv_j - K\sum_i\sigma_i\tau_i - K\sum_i\tau_iv_i,$$





- "Detecting composite orders in layered models via machine learning." W. Rzadkowski, et al., New J. Phys. **22** (2020) 093026.

Ashkin-Teller model

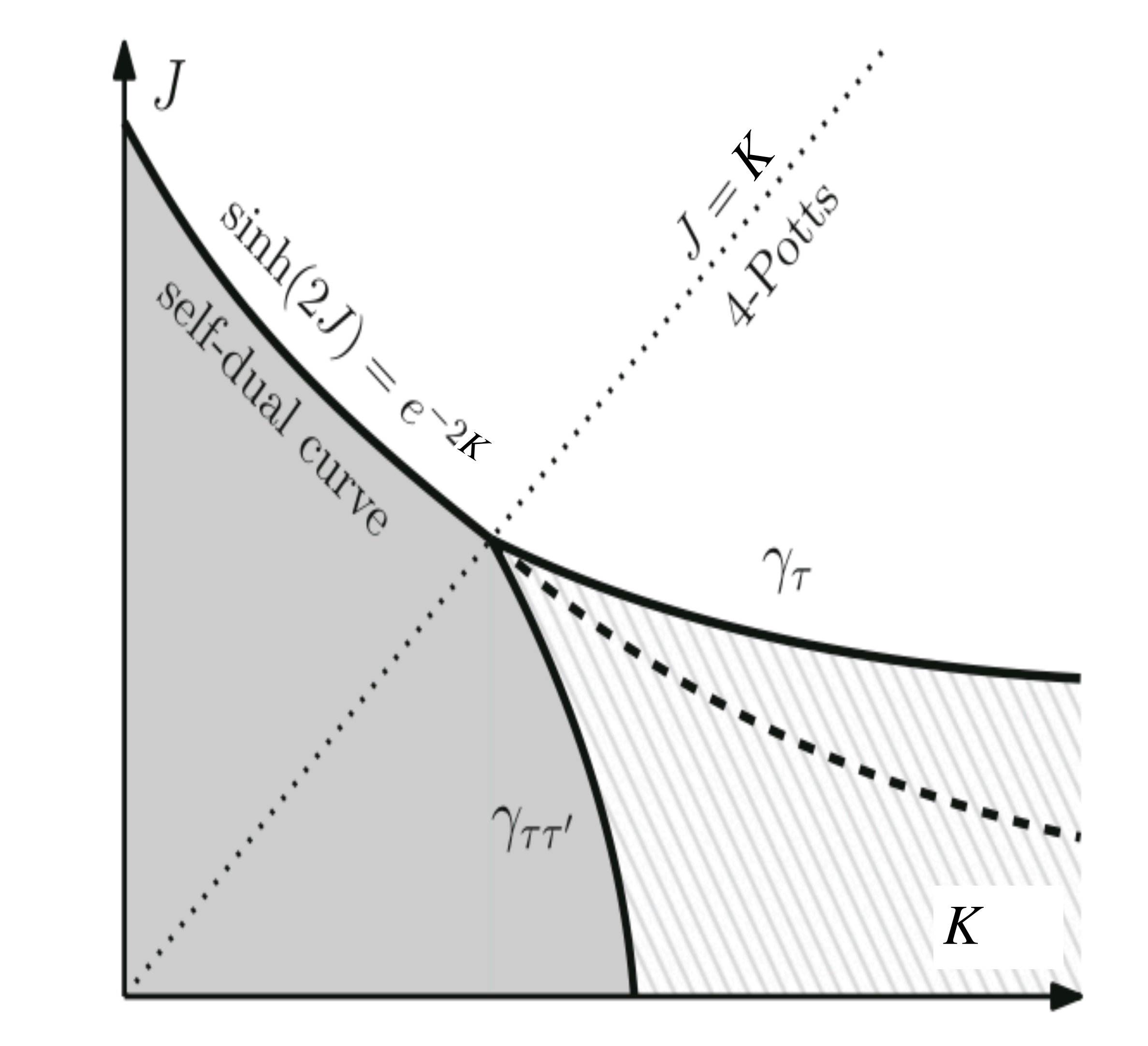
$$H_{\mathrm{AT}} = -J\sum_{\langle ij\rangle}\sigma_i\sigma_j - J\sum_{\langle ij\rangle}\tau_i\tau_j - K\sum_{\langle ij\rangle}\sigma_i\sigma_j\tau_i\tau_j$$

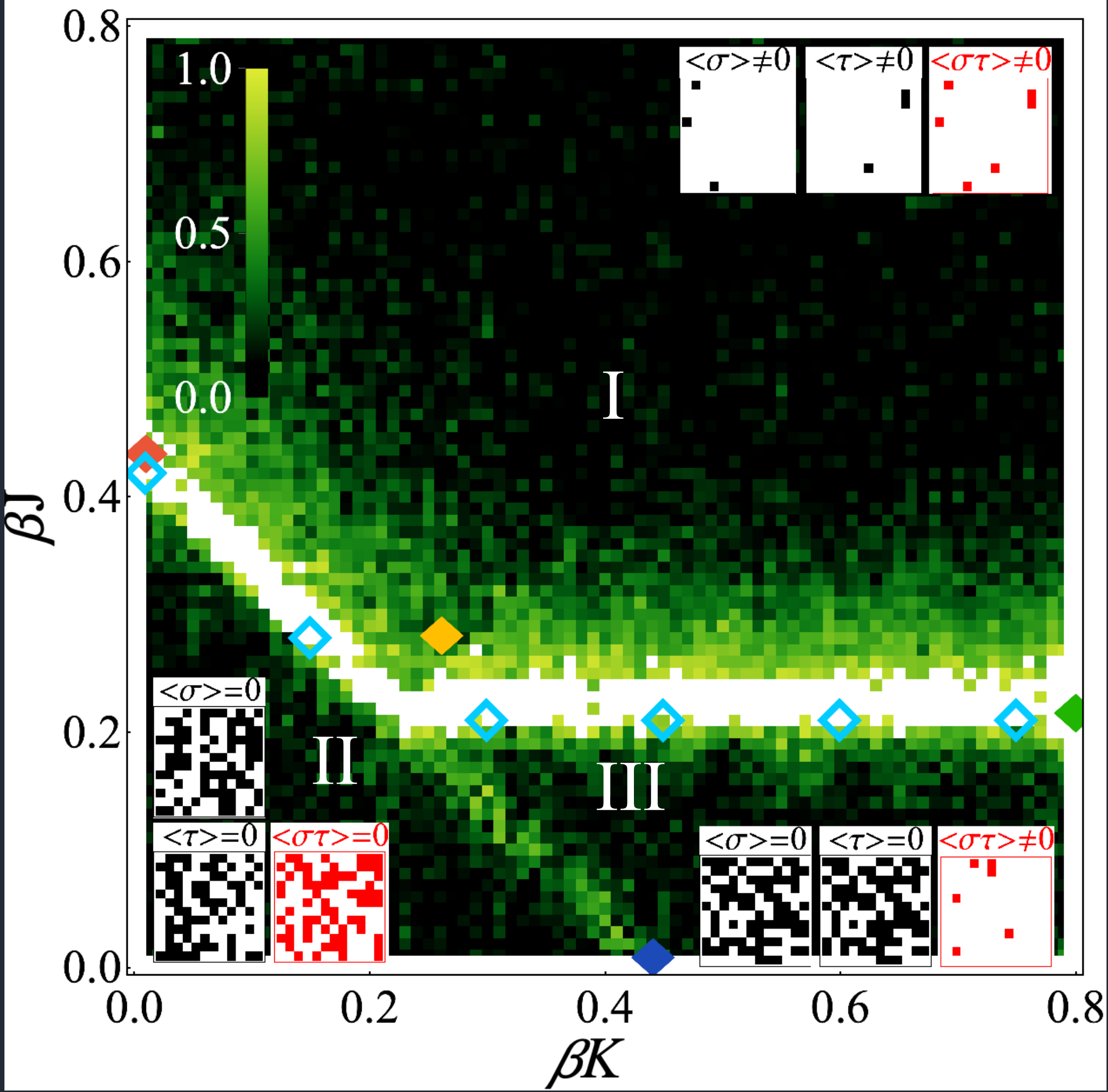
$$\langle\tau_0\tau_x\rangle_{\beta J,\beta K} \approx \begin{cases} e^{-c_\beta\cdot|x|} & \text{if } \beta < \beta_c^{\tau} \\ c_\beta & \text{if } \beta > \beta_c^{\tau} \end{cases}$$

$$\left\langle\tau_0\sigma_0\tau_x\sigma_x\right\rangle_{\beta J,\beta K} \approx \begin{cases} e^{-c_\beta\cdot|x|} & \text{if } \beta < \beta_c^{\tau\tau'} \\ c_\beta & \text{if } \beta > \beta_c^{\tau\sigma} \end{cases}$$

- “Phase Diagram of the Ashkin–Teller Model” Y. Aoun, et al., Commun. Math. Phys. (2024) 405:37.

Toy model: bilayer Ising model





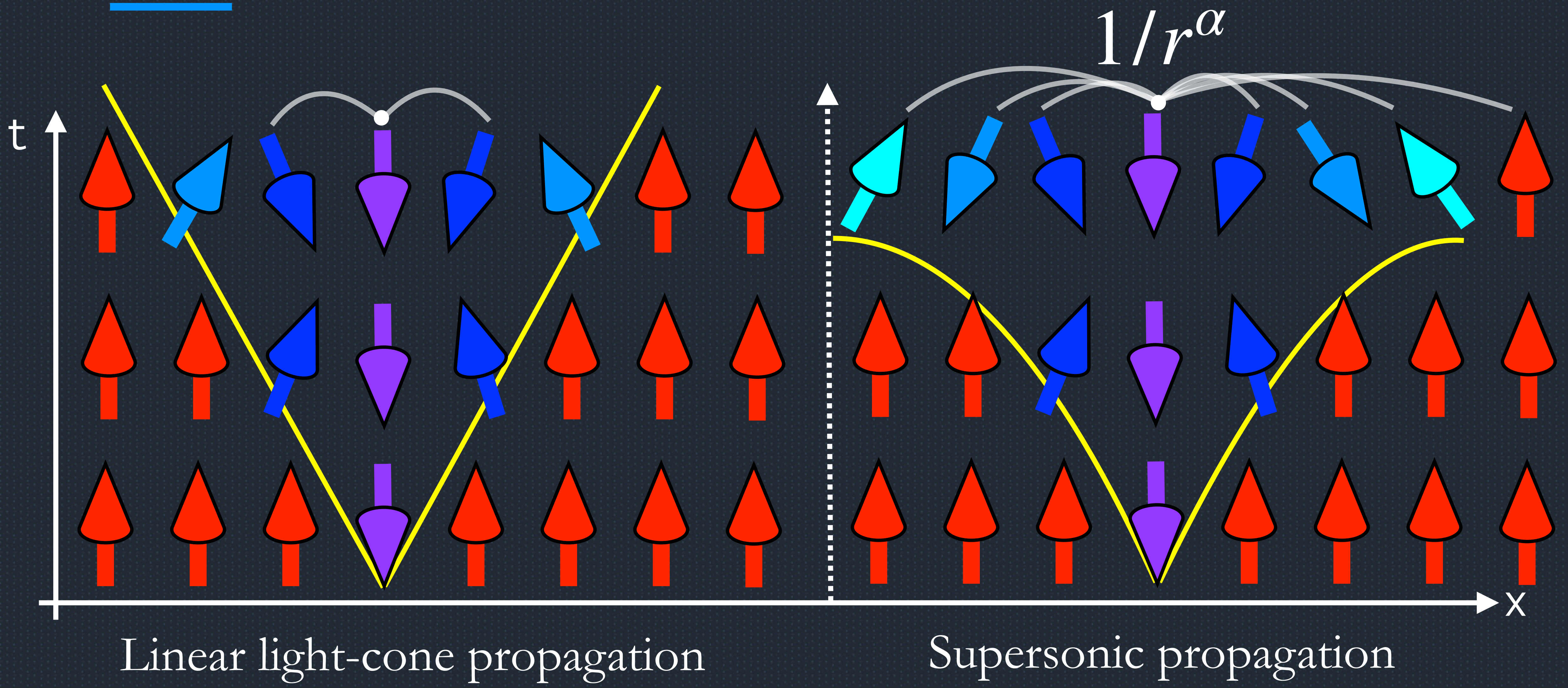
- "Detecting composite orders in layered models via machine learning." W. Rzadkowski, et al., New J. Phys. **22** (2020) 093026.

Understanding of BKT with ML?

Thank you

Back up slides

Fast entanglement spreading



P. Richerme, et al., Nature 511, 198 (2014).

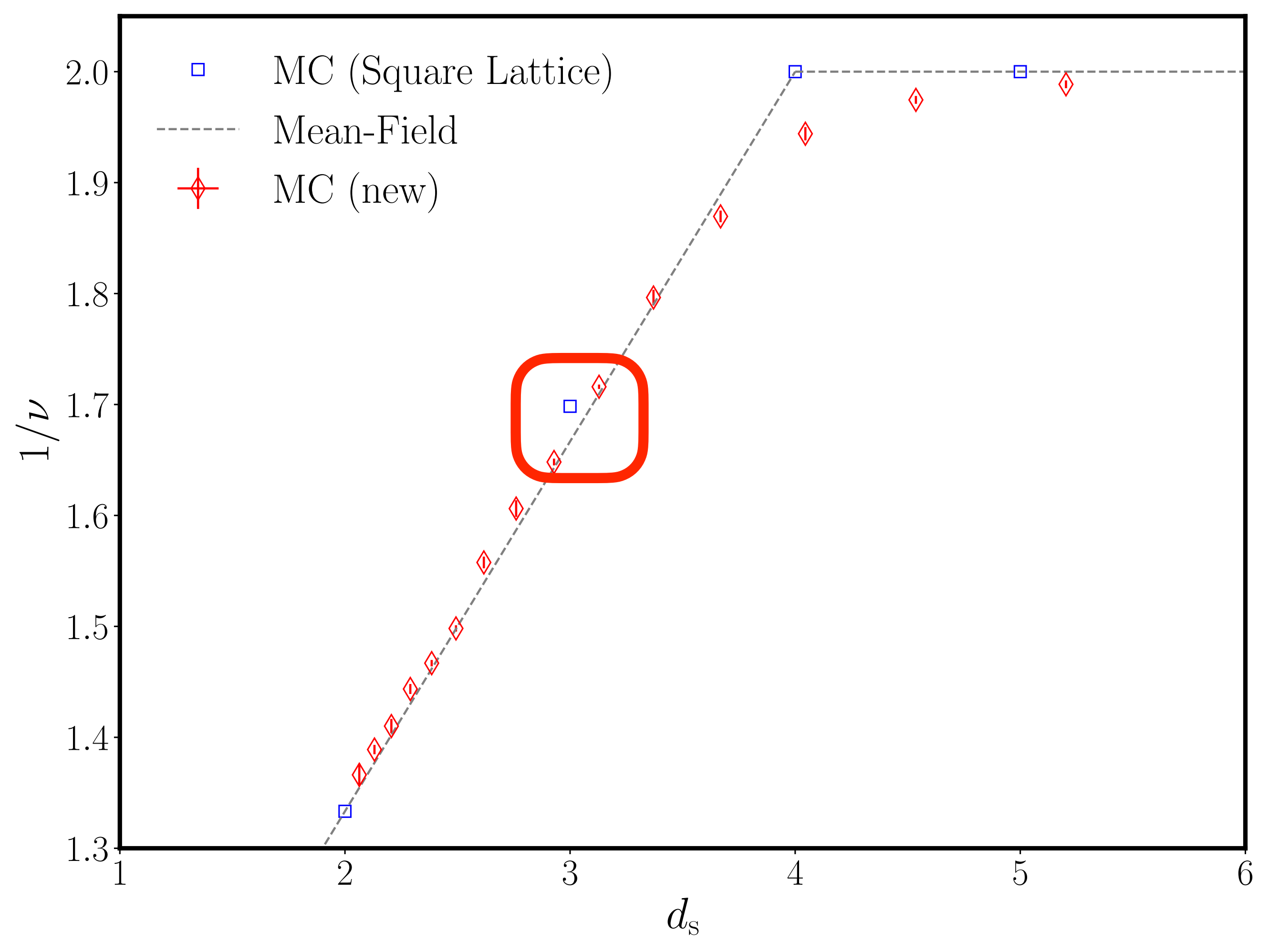
Preliminar Results: Classical SAW

MC Simulations on Regular Lattice

Slade Gordon, 2019, Self-avoiding walk, spin systems and renormalization Proc. R. Soc. A.47520180549 20180549

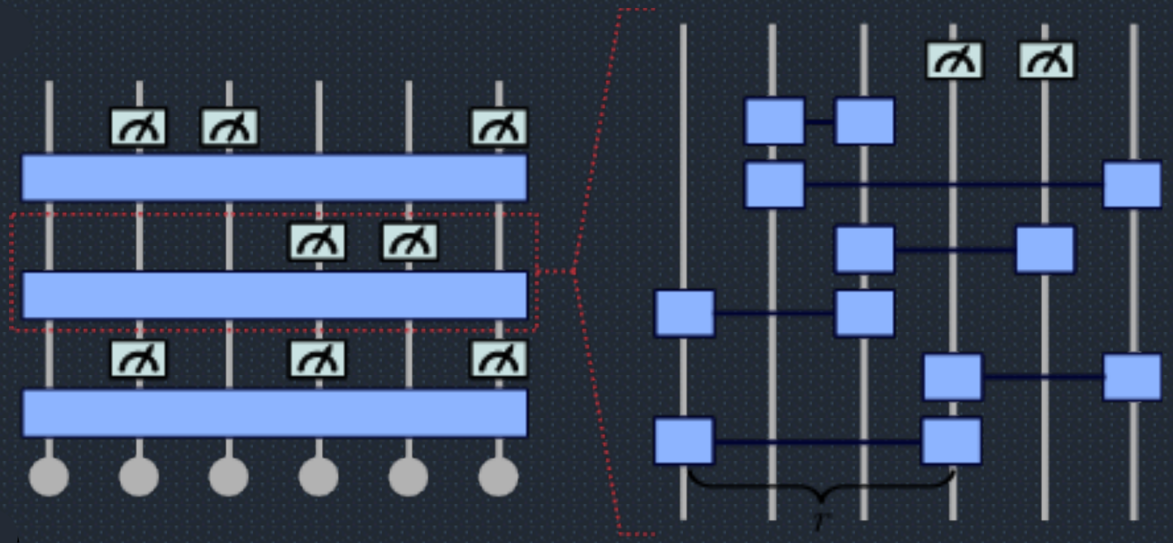
MC Simulations on LR diluted graph

The critical exponent is extracted by the finite size scaling of the gyration ratio of the walk length.



Competitors: Quantum Circuits at Berkley

M. Block, Y. Bao, S. Choi, E. Altman, N. Yao, *Phys. Rev. Lett.* 128, 010604 (2022).



Long-range gate occurrence probability

$$p \sim \frac{1}{r_{ij}^{\alpha}}$$

Equivalent to the Ising model studied in

N. Defenu, et al. *Phys. Rev.* B 96, 104432 (2017).

It is a purely geometric property!

A. P. Millán, G. Gori, F. Battiston, T. Enss, N. Defenu, *Phys. Rev. Res.* 3, 023015 (2021).

Project Team

Team leader

Direct supervision

Scientific guidance

Many-body theory team

Thesis: Functional RG study of dynamical universality in the regime $\alpha > d$

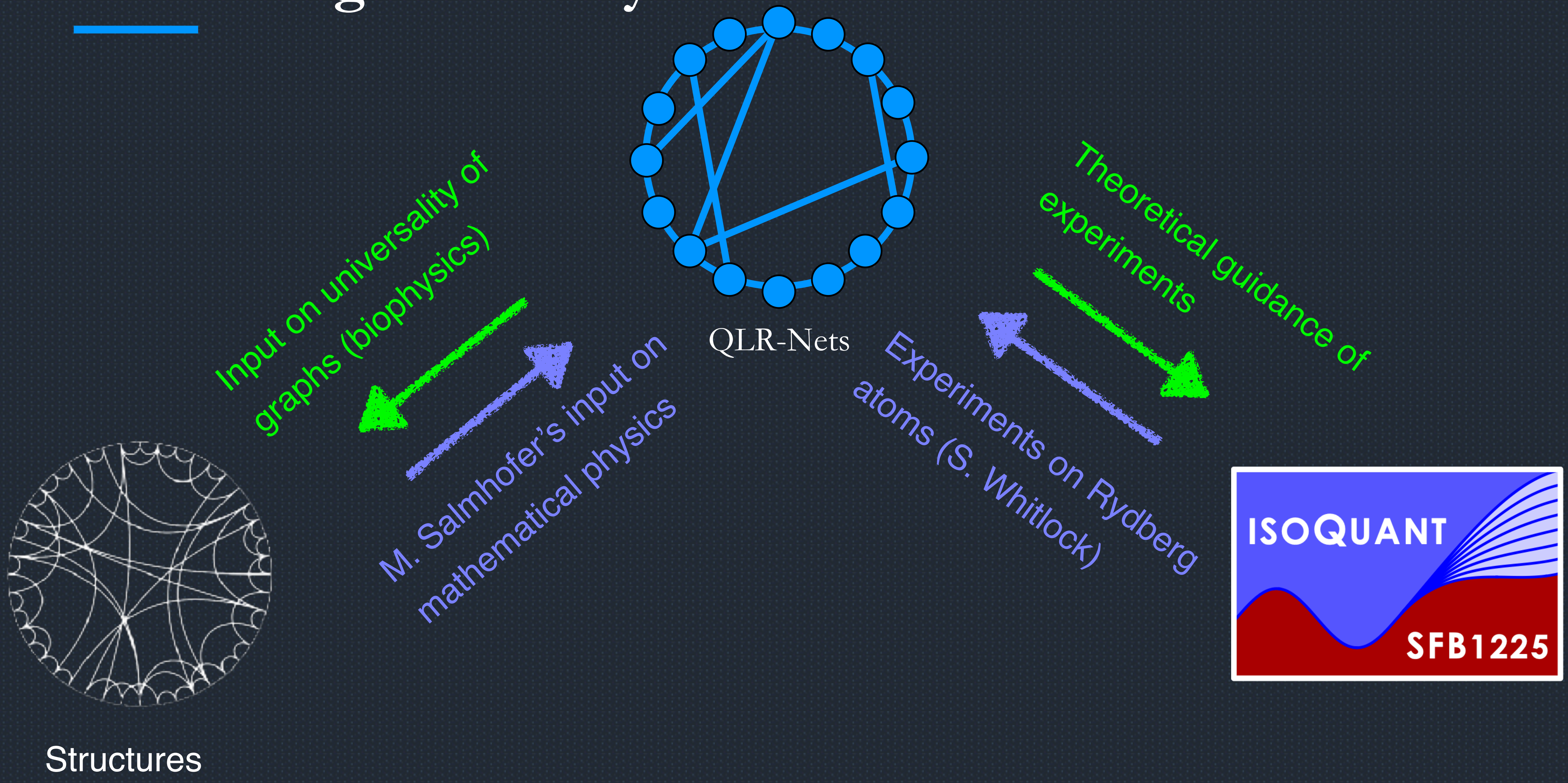
1/N expansion of quantum spin Hamiltonians at $\alpha < d$

Numerical simulations team

Out-of-equilibrium QMC study of universality at $\alpha > d$

Variational QMC analysis of driven-dissipative QLR-Nets using neural network ansatz

Heidelberg University



Funded projects and research supervision

PI of 2 Exploratory Projects (150’000€)

1. Critical behavior of epidemic models on distinct network topologies and applications to the study of brain disease
2. Universality on Network Structures from Quantum Dynamics to Big Data

PI of SNSF Project Funding Scheme (500’000€)

1. Out-of-equilibrium criticality of long-range interacting quantum systems

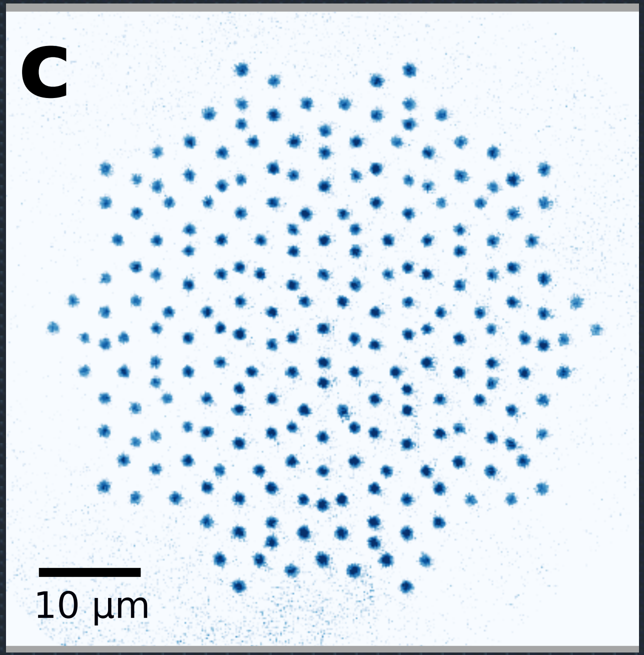
M. Sc. & PhD Students Supervision

1. Marvin Syed (M. Sc.)
2. Guido Giachetti (PhD)
3. Andrea Solfanelli (PhD)
4. Benjamin Liegeois (PhD)
5. Ka Rin Sim (PhD)
6. Cristiano Muzzi (PhD)

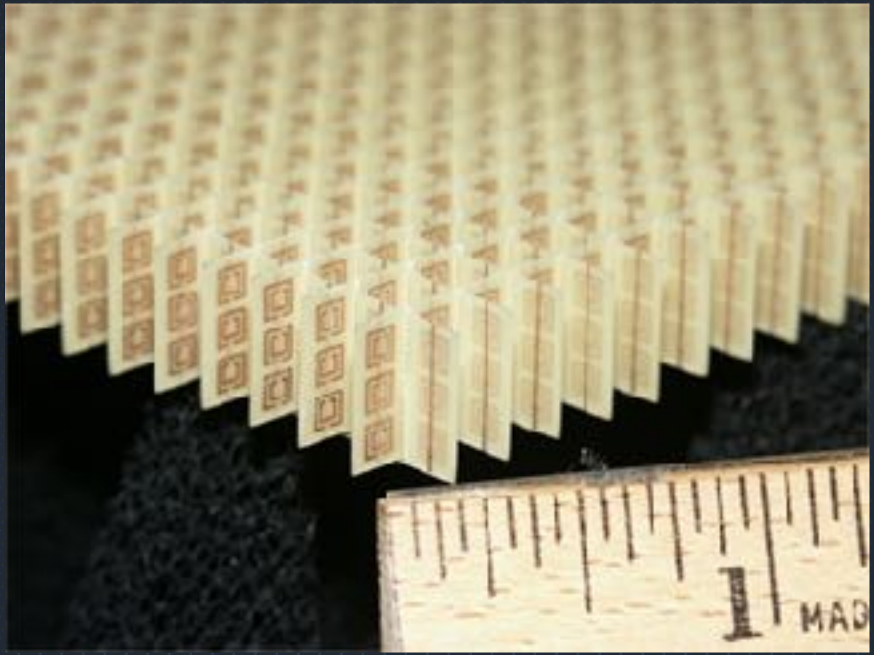
1. M. Syed, T. Enss, ND, Phys. Rev. B 103, 064306 (2021).
2. M. Syed, T. Enss, ND, Phys. Rev. B 105, 224302 (2022).
3. G. Giachetti, ND, arXiv:2112.11488.
4. G. Giachetti, A. Solfanelli, ND, arXiv:2203.16562 (2022).
5. A. Solfanelli et al., arXiv:2208.09492 (2022).
6. *B. Liegeois, R. Chitra, ND, In preparation (2022).*
7. *K. R. Sim, ND, P. Molignini, R. Chitra, In preparation (2022).*

Broad Impact

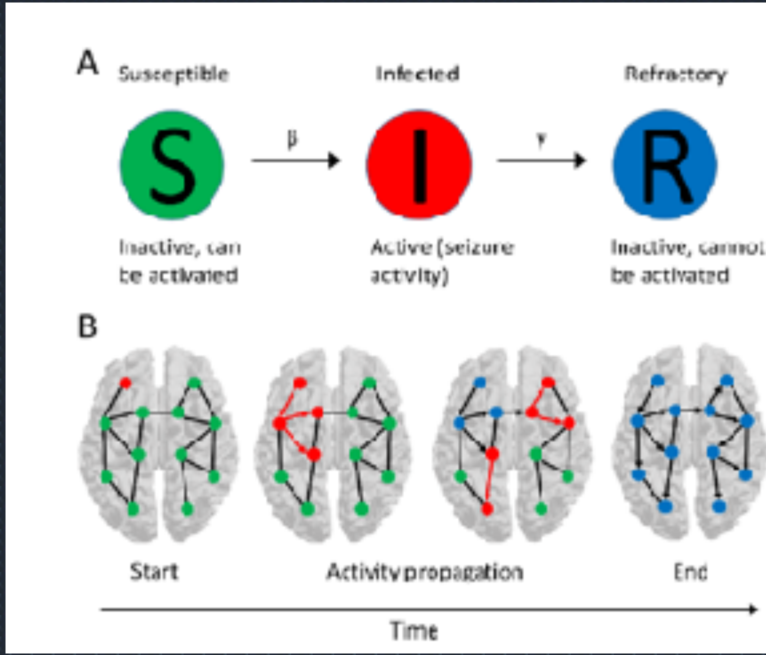
Rydberg atoms in optical tweezers



Engineered materials



Brain seizure modelling (SIR universality)



$$V(r) \sim r^{-\alpha}$$