IGFAE Instituto Galego de Física de Altas Enerxías

~~Beyond~~-Eikonal Methods in High-Energy Scattering

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Discussion on Single inclusive particle production in pA collisions at forward rapidities: beyond the hybrid model

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FONDO EUROPEO DE DESENVOLVEMENTO REXIONAL "Unha maneira de facer Europa"

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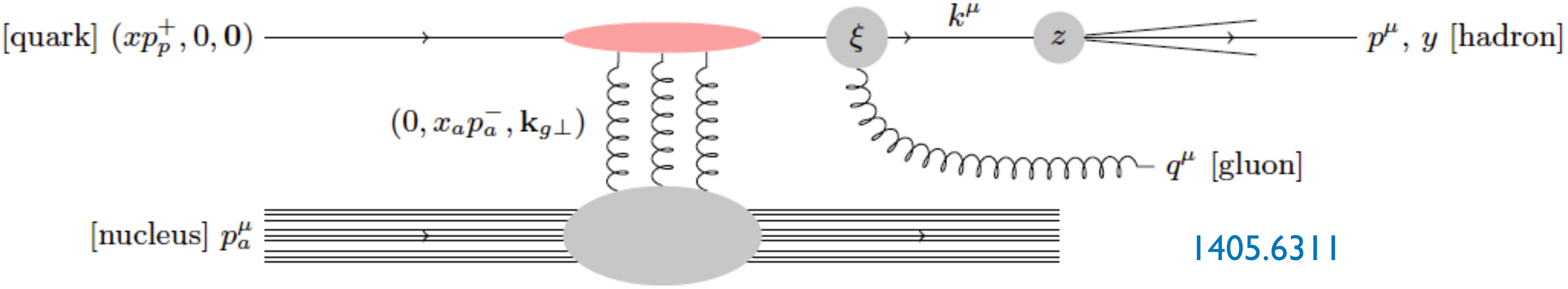
Phys. Rev. D 108 (2023) 7, 074003, 2307.14922 [hep-ph] with Tolga Altinoluk (NCBJ), Alex Kovner (UConn) and Michael Lublinsky (BGU), plus ongoing work with Tolga, Alex, Misha, Guillaume Beuf (NCBJ) and Alina Czajka (NCBJ).

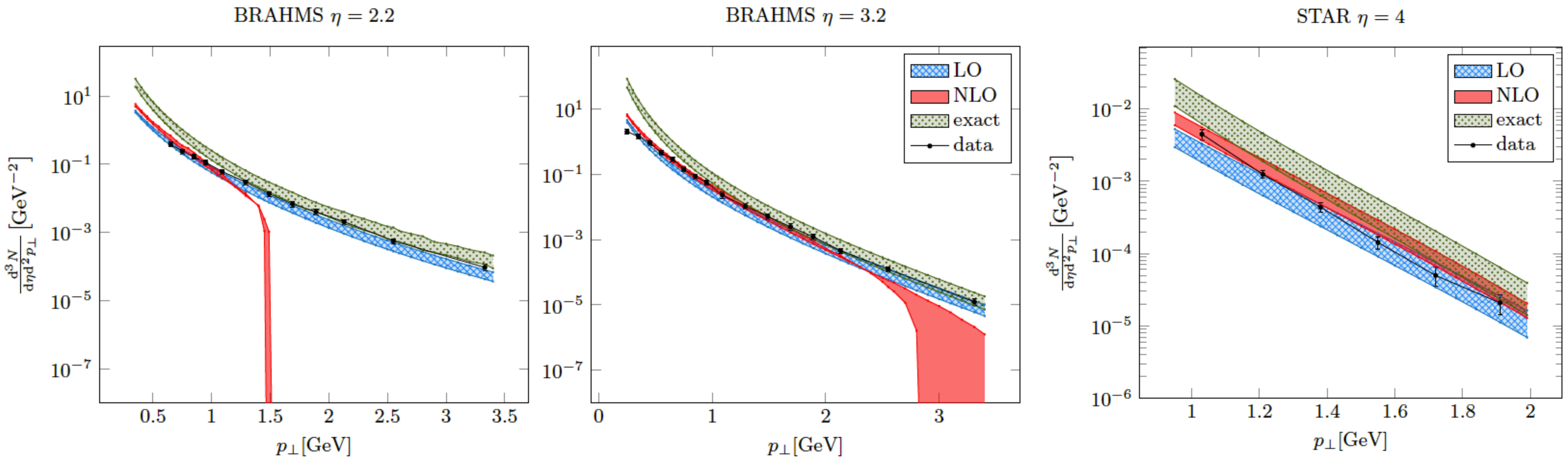
The hybrid model:

• Hybrid model proposed at LO in 2005 (hep-ph/0506308), NLO in 2011 (1112.1061): large x collinear parton which splits, rescatters with the target (eikonally) and fragments onto a hadron.



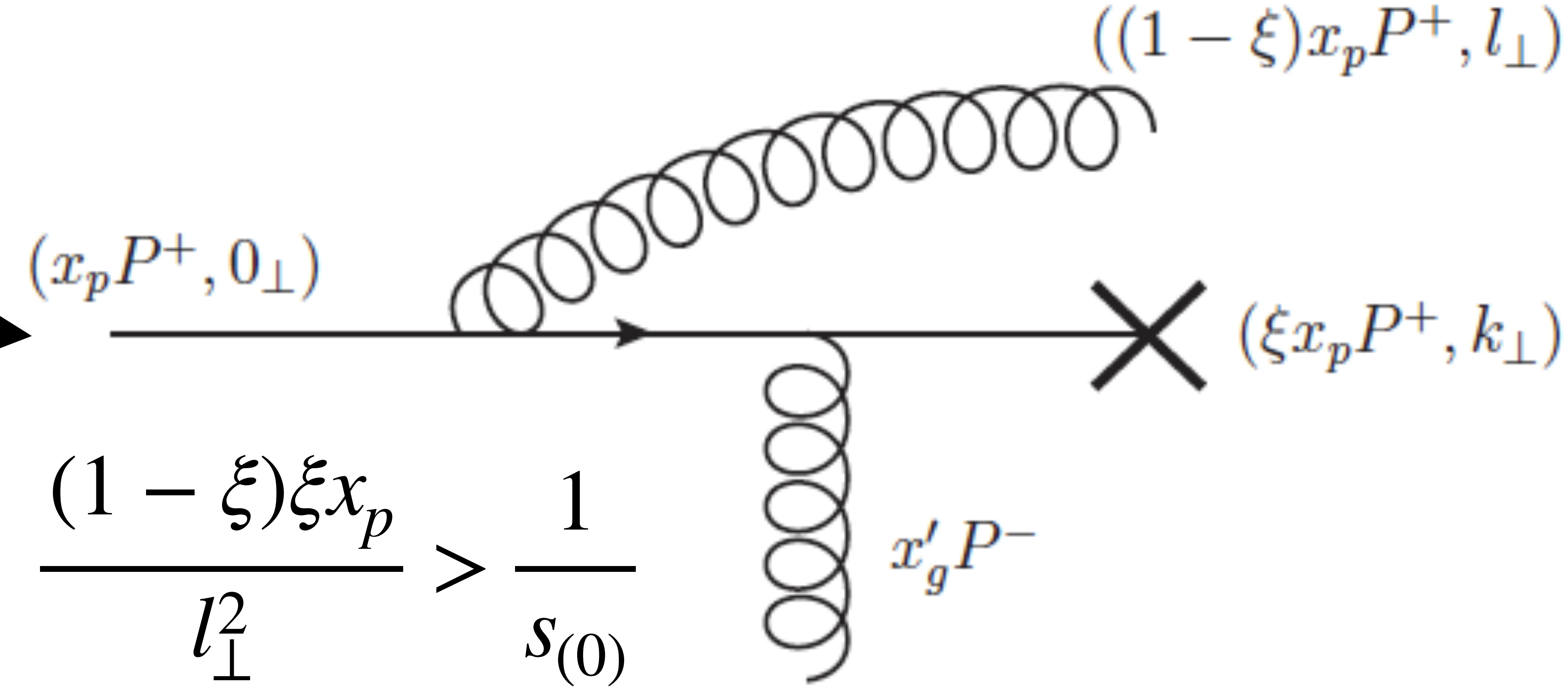
1405.6311

• Cross sections turned out to be negative at large transverse momentum, a problem alleviated at larger rapidities or energies.



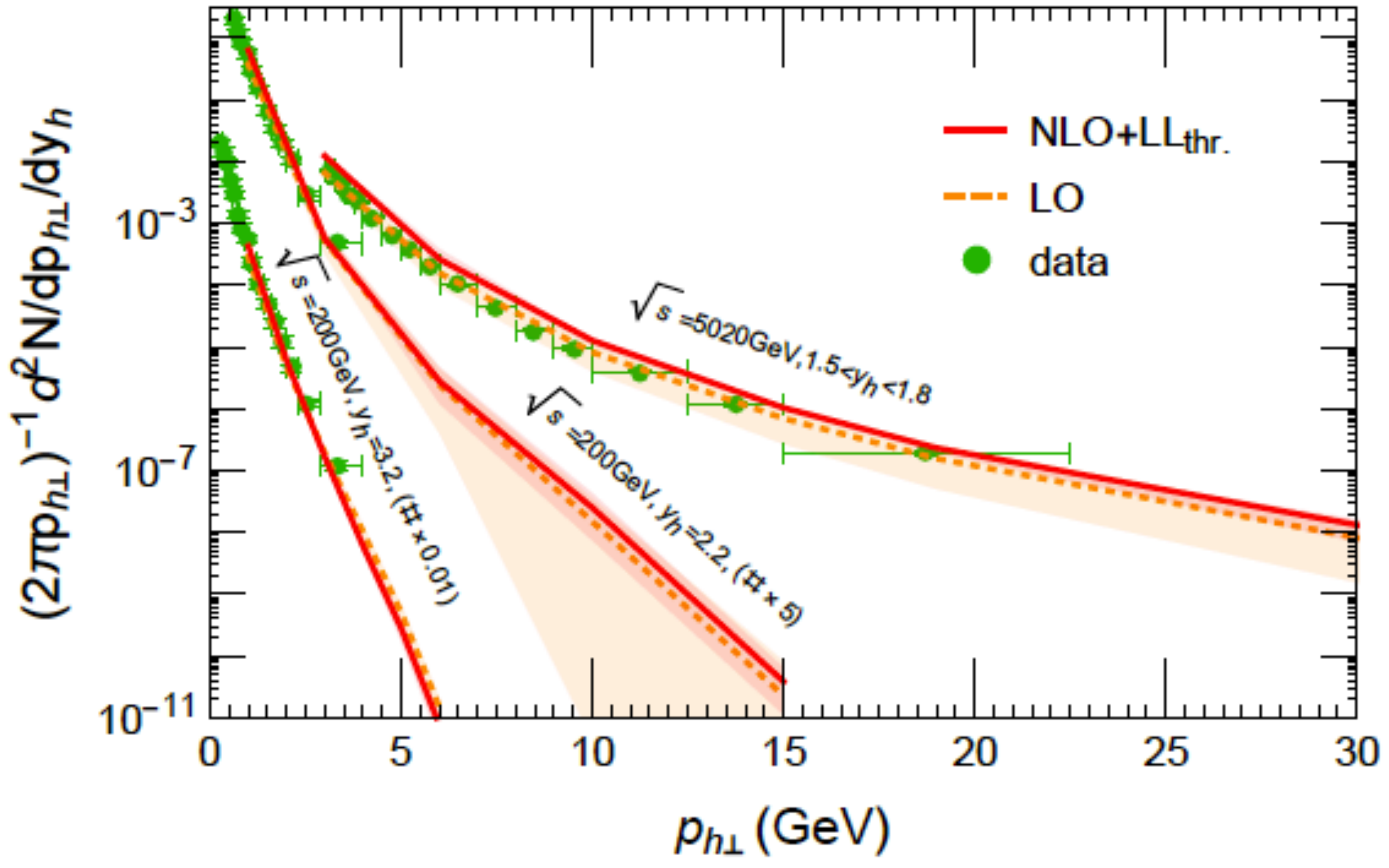
The problem (I):

- Several solutions proposed along the years:
  ➜ Kinematic constraints (1505.05183)/Ioffe time restriction (1411.2869) leading to new, BK-like terms.
  ➜ Choice of rapidity scales (1403.5221,1407.6314,1608.05293).
  ➜ Threshold (2004.11990) and Sudakov (2112.06975) resummation.

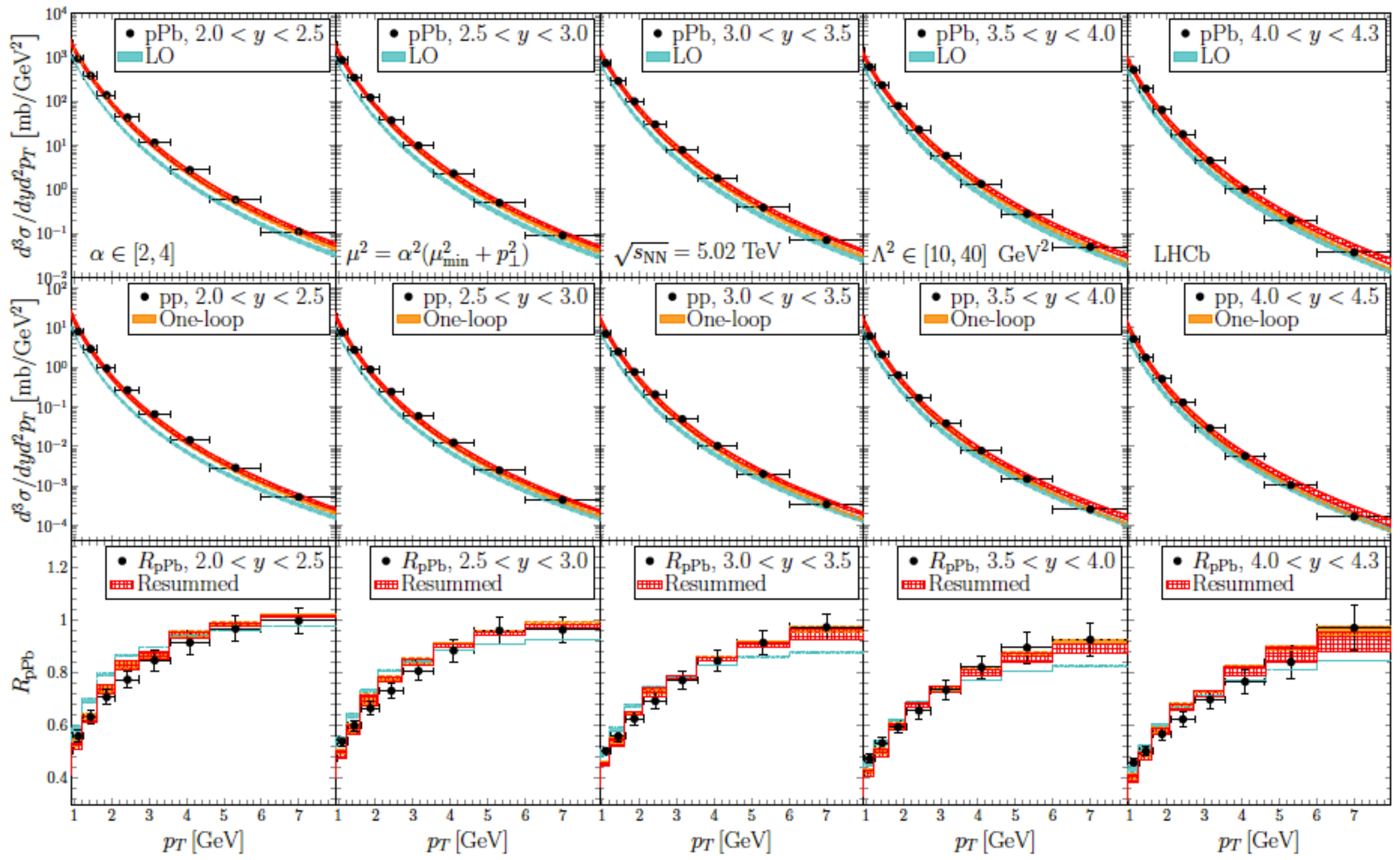


$$\frac{(1-\xi)\xi x_p}{l_\perp^2} > \frac{1}{s_{(0)}}$$

- They lead to a successful description of data but lack of understanding of what was or still is wrong, or of guidance on how to rectify it.



2004.11990



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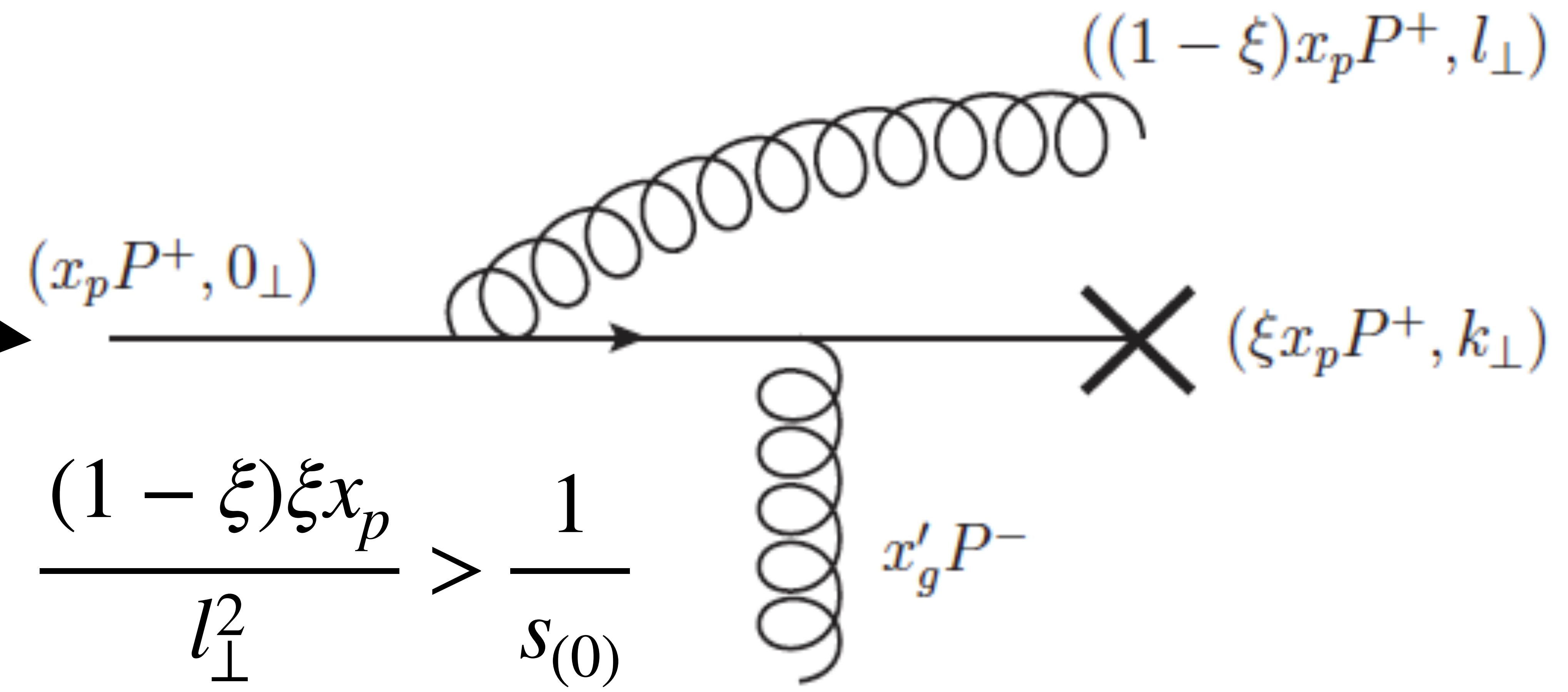

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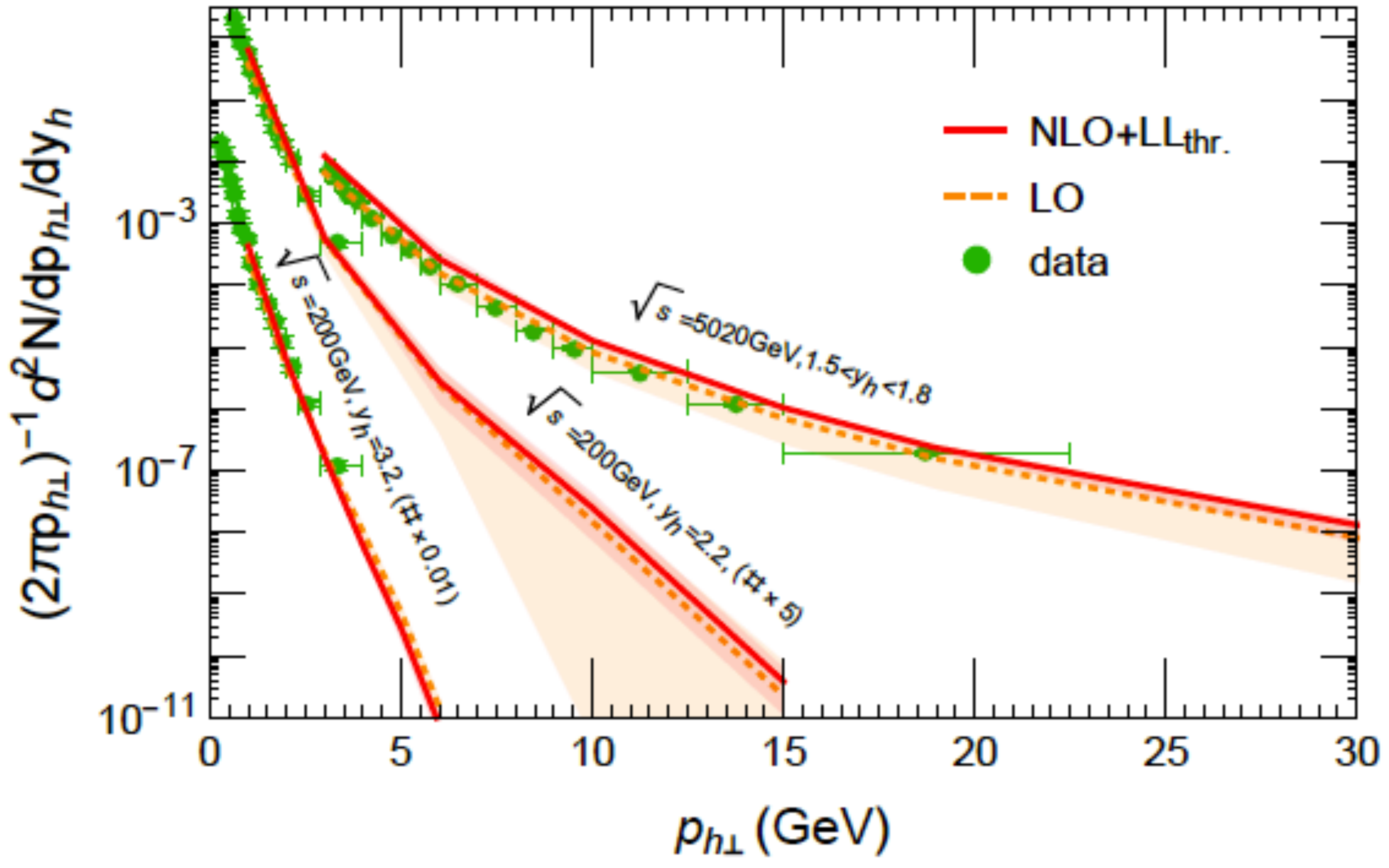
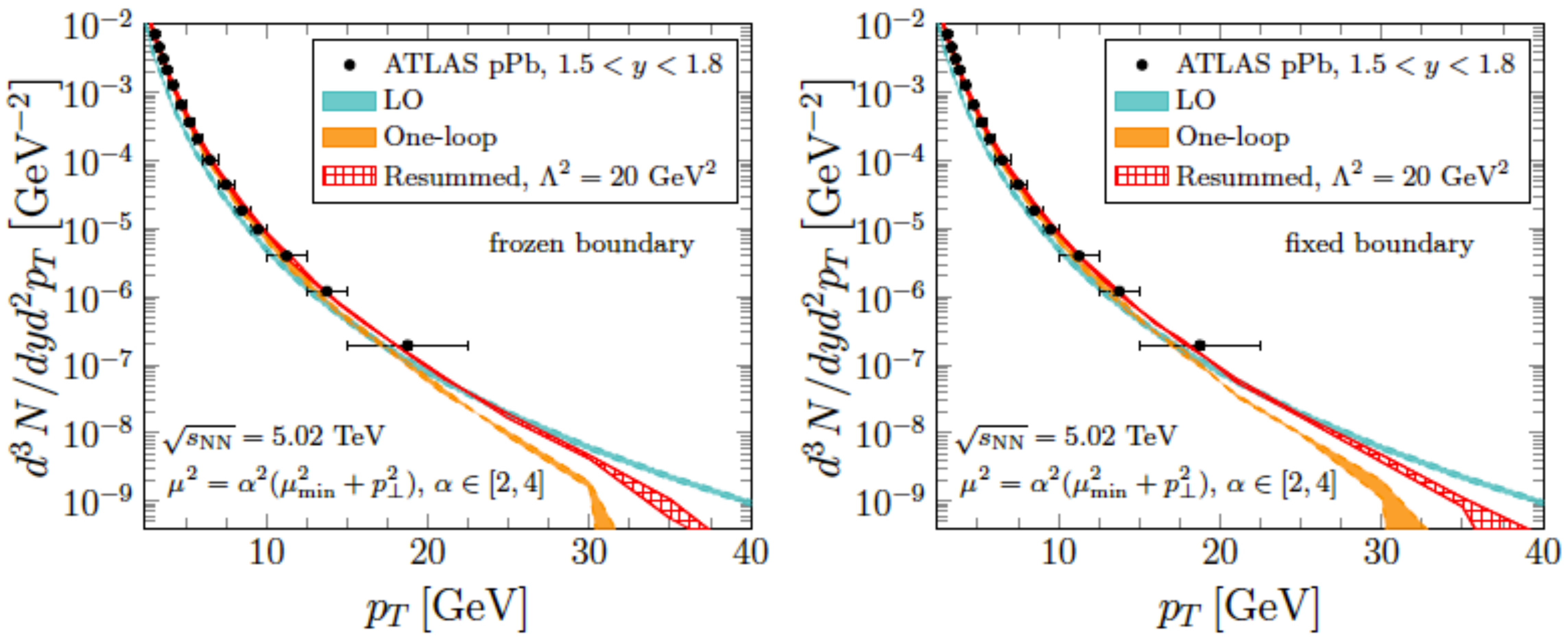


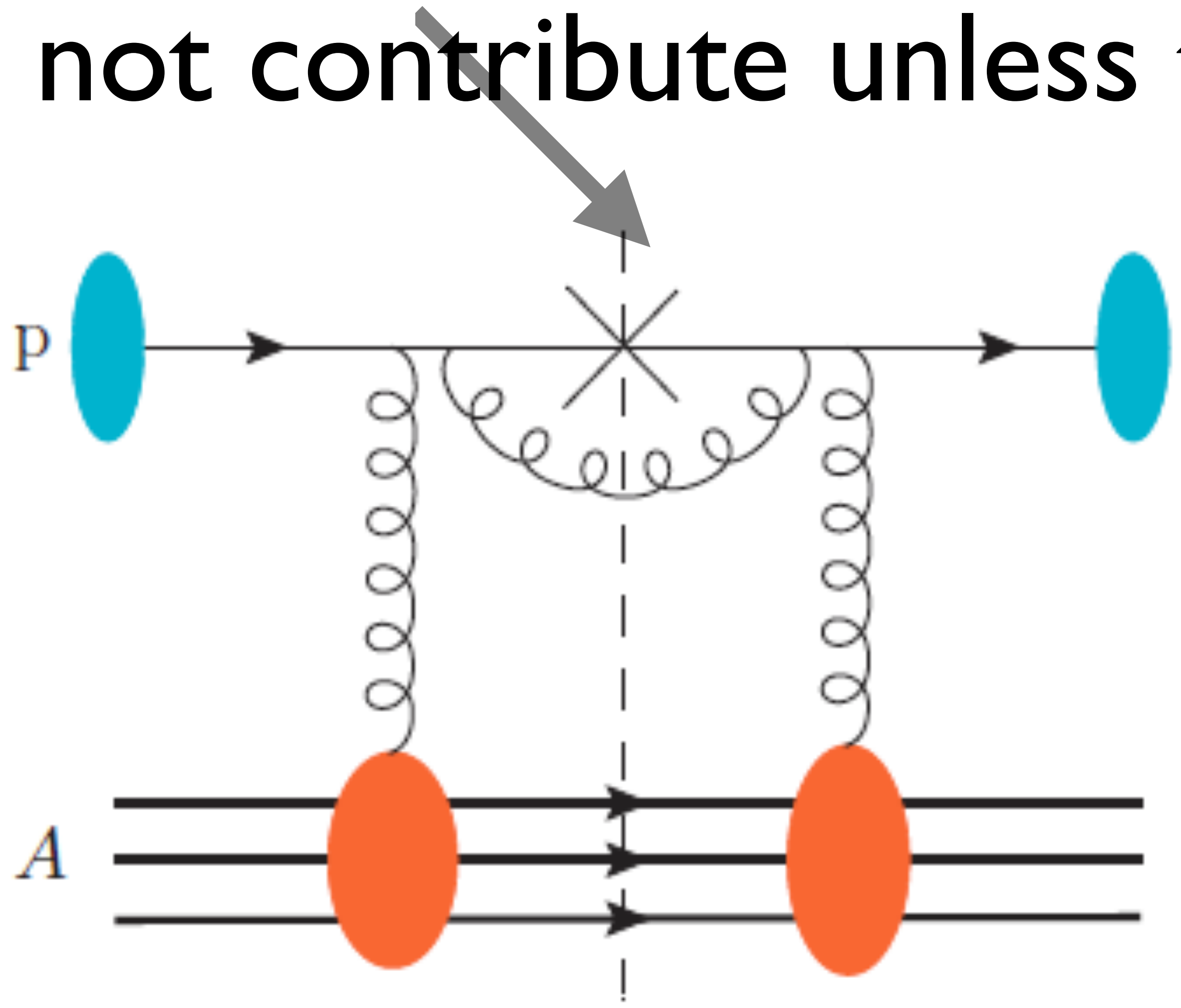
2004.11990

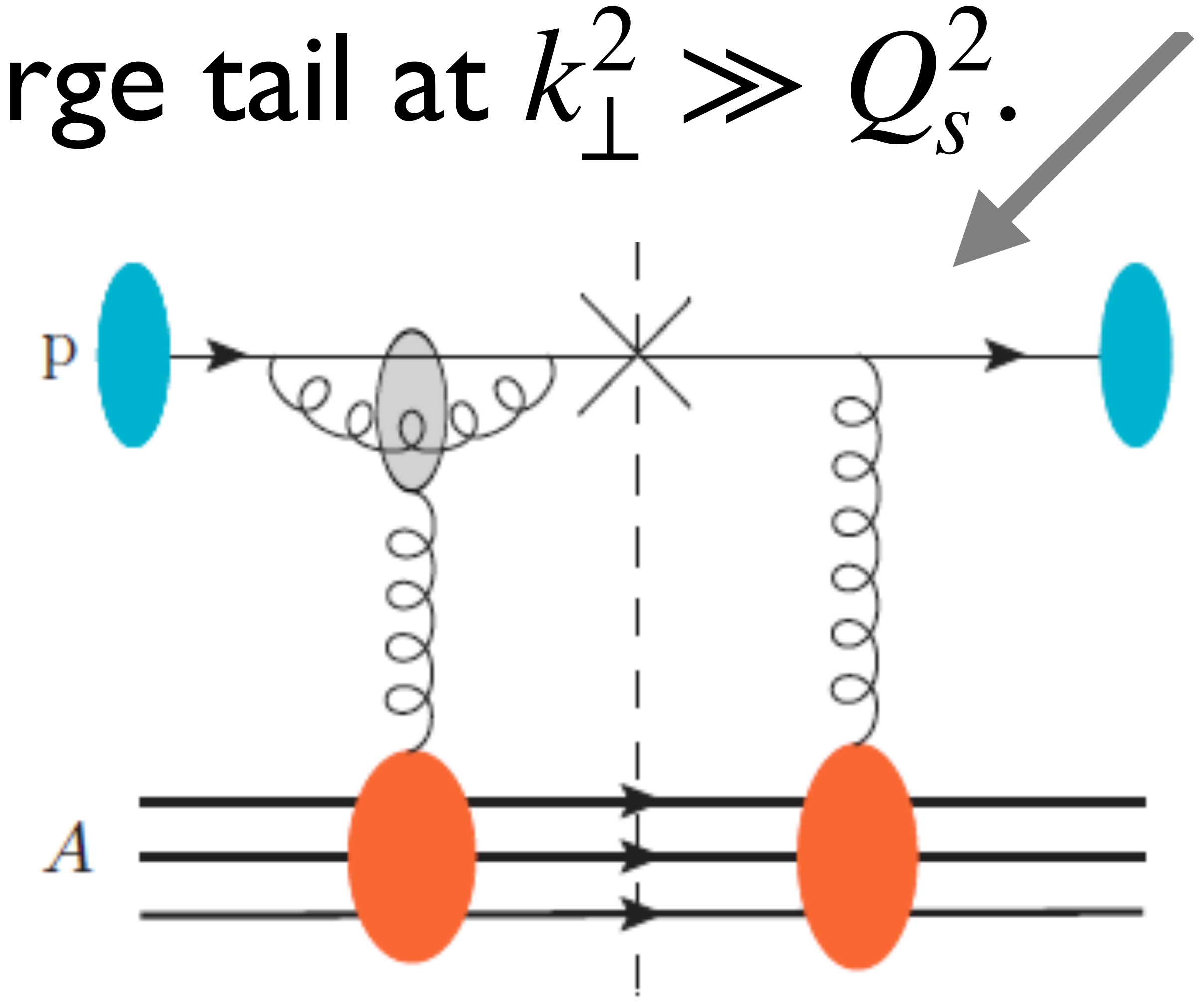


2112.06975

The problem (II):

• Any eventual problem of negativity at NLO should not come from large transverse momentum: inelastic (real NLO) contribution squared (1102.5327), the elastic one (LO+virtual NLO) does not contribute unless the dipole has a large tail at $k_\perp^2 \gg Q_s^2$.





2112.06975

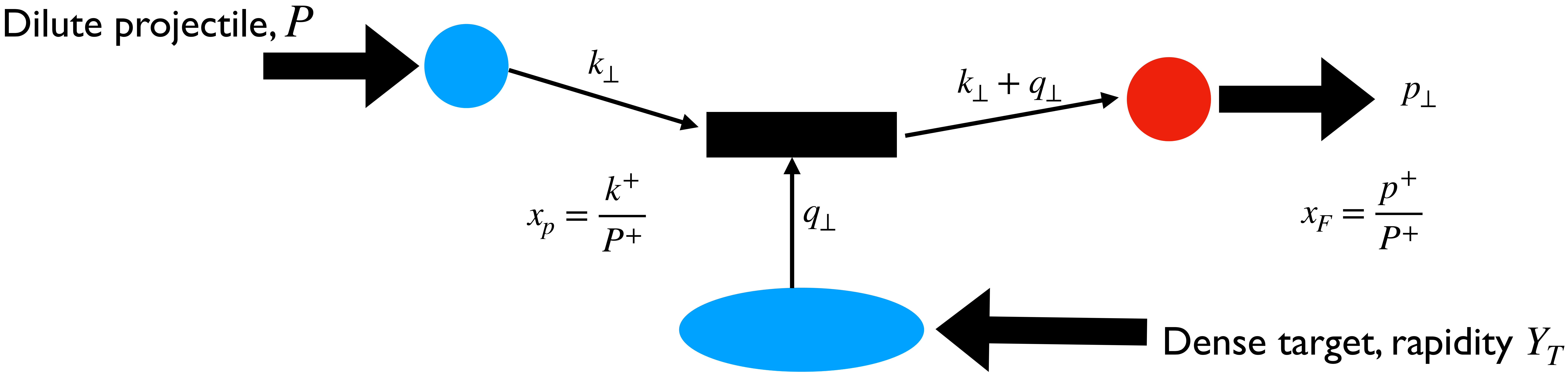
• The reason for the negativity is seemingly an over subtraction: the NLO is extracted collinear pieces that go to the DGLAP evolution of the collinear PDFs and FFs, and a soft piece (through the plus prescription) that goes into the BK evolution of the dipole scattering matrix. The remainder turns out to become negative at large transverse momentum.

• Here we conclude that the correct framework to resum all large logarithms is not collinear factorization for the projectile (the hybrid model) but TMD factorization.

Our setup:

- We work in a frame in which the target nucleus moves fast. We find a TMD-factorized parton model expression:

$$\int \frac{d\zeta}{\zeta^2} \int d^2k_\perp d^2q_\perp \, T\left(\frac{x_F}{\zeta}, k_\perp; \mu_T^2\right) P(k_\perp, q_\perp) \, F\left(\zeta; p_\perp, (k_\perp + q_\perp); \mu_F^2\right) + \mathcal{O}\left(\frac{p_\perp^2, k_\perp^2, Q_s^2, \mu^2}{s_0}\right)$$



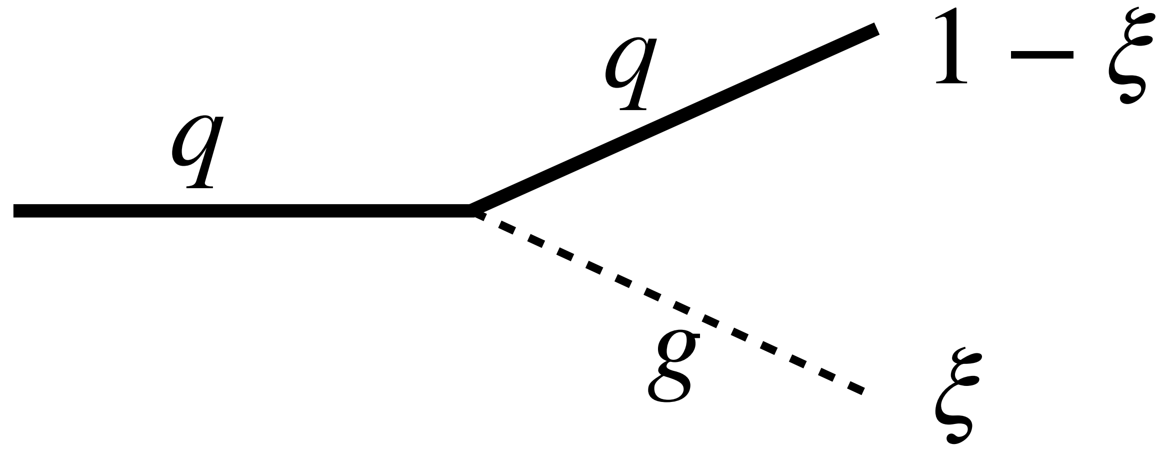
- Our scales are

$$\mu_T^2 = \max\left\{k_\perp^2, q_\perp^2, Q_s^2\right\} \approx \max\left\{(k_\perp + q_\perp)^2, Q_s^2\right\}, \; \mu_F^2 = \left((q_\perp + k_\perp) - p_\perp/\zeta)^2\right) \approx \max\left\{(q_\perp + k_\perp)^2, (p_\perp/\zeta)^2\right\}$$

TMD distributions: one flavor PDFs

- TMD PDFs (single parton species to start with) are generated from collinear ones (**large** k):

$$x\mathcal{T}_q(x,k^2;k^2;\xi_0)=\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^1 d\xi\frac{1+(1-\xi)^2}{\xi}\frac{x}{1-\xi}f^q_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}$$



- Evolution (diagonal in parton species and momentum fraction; the second term corresponds to a loss due to the increase in resolution):

$$x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)=\theta(\mu^2-k^2)\left[x\mathcal{T}_q(x,k^2;k^2;\xi_0)-\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^1 d\xi\frac{1+(1-\xi)^2}{\xi}x\,\mathcal{T}_q\left(x,k^2;l^2;\xi_0\right)\right]$$

- At $\mathcal{O}(\alpha_s)$: $x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)=\theta(\mu^2-k^2)\,x\mathcal{T}_q(x,k^2;k^2;\xi_0)\left[1-\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^1 d\xi\frac{1+(1-\xi)^2}{\xi}\right]$

TMD distributions: one flavor PDFs

- With $xf^q_{\mu^2}(x) = \int_0^{\mu^2} \pi dk^2\, x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)$, we recover DGLAP for the collinear PDFs and their definition is independent of $\xi_0 \ll 1$.

- TMD FFs are defined analogously.

- These definitions can be generalised to n_f massless quarks and antiquarks, and gluons.

- It will turn out in the calculation that $\xi_0 \propto \mu^2/s_0$, with s_0 an energy scale that comes from the Ioffe time restriction (1411.2869).

- Relation of these definitions and the corresponding evolution equations to more standard implementations (see e.g. 2304.03302) of the rapidity cut-off (s_0 in our case that acts as a longitudinal resolution).

TMD distributions: evolution equations

$$\frac{\partial x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)}{\partial \ln \mu^2} = -\frac{\alpha_s}{2\pi}\frac{N_c}{2}\int_{\xi_0}^1 d\xi \frac{1+(1-\xi)^2}{\xi}\, x\,\mathcal{T}_q\left(x,k^2;\mu^2;\xi_0\right) + \mu^2\delta\left(\mu^2-k^2\right) x\mathcal{T}_q(x,k^2;k^2;\xi_0)$$

$$\frac{\partial x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)}{\partial \ln \frac{1}{\xi_0}} = -\frac{\alpha_s}{2\pi}\frac{N_c}{2}\left(1+(1-\xi_0)^2\right)\theta(\mu^2-k^2)$$

$$\left[\int_{k^2}^{\mu^2}\frac{dp^2}{p^2}\, x\,\mathcal{T}_q\left(x,k^2;p^2;\xi_0\right) - \int_0^{k^2} dq^2\, x\,\mathcal{T}_q\left(x,q^2;k^2;\xi_0\right)\frac{1}{k^2}\right]$$

$$\xi_0 = \frac{\mu^2}{\bar{s}_0}, \quad \bar{s}_0 = \frac{x s_0}{\zeta}$$

- Both equations are compatible one each other (à la CSS).

TMD distributions: evolution equations

$$\frac{\partial x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)}{\partial\ln\mu^2} = -\frac{\alpha_s}{2\pi}\frac{N_c}{2}\int_{\xi_0}^1 d\xi\frac{1+(1-\xi)^2}{\xi}\,x\,\mathcal{T}_q\left(x,k^2;\mu^2;\xi_0\right) + \mu^2\delta\left(\mu^2-k^2\right)x\mathcal{T}_q(x,k^2;k^2;\xi_0)$$

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- Both equations are compatible one each other (à la CSS).

- For $\mu^2 \gg k^2 \gg \Lambda^2_{\rm QCD}$, **CSS** for $\zeta_{\rm CSS}=\bar{s}_0^2/\mu^2$.

$$\frac{\partial\ln\mathcal{T}_q(x,k^2;\mu^2;\xi_0)}{\partial\ln\mu^2} = -\frac{\alpha_s}{2\pi}\frac{N_c}{2}\int_{\xi_0}^1 d\xi\frac{1+(1-\xi)^2}{\xi} \approx -\frac{\alpha_s}{2\pi}N_c\left[\ln\frac{1}{\xi_0}-\frac{3}{4}\right],$$

$$\frac{\partial\ln\mathcal{T}_q(x,k^2;\mu^2;\xi_0)}{\partial\ln\frac{1}{\xi_0}} = -\frac{\alpha_s}{2\pi}\frac{N_c}{2}\left(1+(1-\xi_0)^2\right)\ln\frac{\mu^2}{k^2} \approx -\frac{\alpha_s}{2\pi}N_c\ln\frac{\mu^2}{k^2}\,.$$

TMD distributions: evolution equations

$$\frac{\partial x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)}{\partial \ln \mu^2} = -\frac{\alpha_s}{2\pi}\frac{N_c}{2}\int_{\xi_0}^{1} d\xi \frac{1+(1-\xi)^2}{\xi}\, x\,\mathcal{T}_q\left(x,k^2;\mu^2;\xi_0\right) + \mu^2\delta\left(\mu^2-k^2\right) x\mathcal{T}_q(x,k^2;k^2;\xi_0)$$

$$\frac{\partial x\mathcal{T}_q(x,k^2;\mu^2;\xi_0)}{\partial \ln \frac{1}{\xi_0}} = -\frac{\alpha_s}{2\pi}\frac{N_c}{2}\left(1+(1-\xi_0)^2\right)\theta(\mu^2-k^2)$$

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$$\xi_0 = \frac{\mu^2}{\bar{s}_0}, \quad \bar{s}_0 = \frac{xs_0}{\zeta}$$

- Both equations are compatible one each other (à la CSS).

$$\mathcal{T}_q(x,k^2;\mu^2;\xi_0) = e^{-\frac{\alpha_s N_c}{2\pi}\left[\frac{1}{2}\left(\ln^2\frac{\bar{s}_0}{k^2}-\ln^2\frac{\bar{s}_0}{\mu^2}\right)-\frac{3}{4}\ln\frac{\mu^2}{k^2}\right]}\mathcal{T}_q(x,k^2;k^2;\xi_0)$$

$$= e^{-\frac{\alpha_s N_c}{2\pi}\left[\frac{1}{2}\left(2\ln\frac{\bar{s}_0}{\mu^2}\ln\frac{\mu^2}{k^2}+\ln^2\frac{\mu^2}{k^2}\right)-\frac{3}{4}\ln\frac{\mu^2}{k^2}\right]}\mathcal{T}_q(x,k^2;k^2;\xi_0)$$

TMD distributions: one flavor FFs

- TMD FFs (single parton species):

$$\mathscr{F}^q_H(\zeta, k^2; k^2, \xi_0) = \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^1 d\xi\, \frac{1+(1-\xi)^2}{\xi}\,\frac{1}{1-\xi}\, D^q_{H,k^2}\left(\frac{\zeta}{1-\xi}\right)\frac{1}{k^2}$$

$$\mathscr{F}^q_H(x, k^2; \mu^2; \xi_0) = \theta(\mu^2 - k^2)\left[\mathscr{F}^q_H(x, k^2; k^2; \xi_0) - \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^1 d\xi \frac{1+(1-\xi)^2}{\xi}\,\mathscr{F}^q_H\left(x, k^2; l^2; \xi_0\right)\right]$$

$$D^q_{H,\mu^2}(x) = \int_0^{\mu^2}\pi dk^2\, \mathscr{F}^q_H(x, k^2; \mu^2; \xi_0)$$

TMD distributions: all flavor PDFs

- For n_f massless quarks and antiquarks, and gluons:

$$x\mathcal{T}_q(x,k^2;k^2;\xi_0) = \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^1 d\xi \frac{1+(1-\xi)^2}{\xi}\,\frac{x}{1-\xi}\,f^q_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2} + \frac{g^2}{(2\pi)^3}\frac{1}{2}\int_{\xi_0}^1 d\xi\,[\xi^2+(1-\xi)^2]\,\frac{x}{1-\xi}\,f^g_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}$$

$$x\mathcal{T}_g(x,k^2;k^2;\xi_0) = \frac{g^2}{(2\pi)^3}2N_c\int_{\xi_0}^1 d\xi\left[\frac{1-\xi}{\xi}+\frac{\xi}{1-\xi}+\xi(1-\xi)\right]\frac{x}{1-\xi}\,f^g_{k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2} + \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\sum_q\int_{\xi_0}^1 d\xi\,\frac{1+\xi^2}{1-\xi}\,\frac{x}{1-\xi}\left[f^q_{k^2}\left(\frac{x}{1-\xi}\right)+f^{\bar q}_{k^2}\left(\frac{x}{1-\xi}\right)\right]\frac{1}{k^2}$$

$$\begin{aligned} x\mathcal{T}_q(x,k^2;\mu^2;\xi_0) &= \theta(\mu^2-k^2)\Big[x\mathcal{T}_q(x,k^2;k^2;\xi_0) \\ &\quad -\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^1 d\xi\frac{1+(1-\xi)^2}{\xi}x\,\mathcal{T}_q\left(x,k^2;l^2;\xi_0\right)\Big] \end{aligned}$$

$$\begin{aligned} x\mathcal{T}_g(x,k^2;\mu^2;\xi_0) &= \theta(\mu^2-k^2)\Big[x\mathcal{T}_g(x,k^2;k^2;\xi_0) \\ &\quad -\frac{g^2}{(2\pi)^3}N_c\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^1 d\xi\left[\frac{1-\xi}{\xi}+\frac{\xi}{1-\xi}+\xi(1-\xi)\right]x\,\mathcal{T}_g\left(x,k^2;l^2;\xi_0\right) \\ &\quad -\frac{g^2}{(2\pi)^3}\frac{n_f}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^1 d\xi\,[\xi^2+(1-\xi)^2]\,x\,\mathcal{T}_g\left(x,k^2;l^2;\xi_0\right)\Big] \end{aligned}$$

$$xf^g_{\mu^2}(x) = \int_0^{\mu^2}\pi dk^2\, x\mathcal{T}_g(x,k^2;\mu^2;\xi_0)$$

- The collinear PDFs satisfy DGLAP.

TMD distributions: all flavor FFs

- For n_f massless quarks and antiquarks, and gluons:

$$\mathcal{F}^q_H(\zeta,k^2;k^2,\xi_0)=\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^1 d\xi\,\frac{1+(1-\xi)^2}{\xi}\,\frac{1}{1-\xi}\,D^g_{H,k^2}\left(\frac{\zeta}{1-\xi}\right)\frac{1}{k^2}+\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^1 d\xi\,\frac{1+\xi^2}{1-\xi}\,\frac{1}{1-\xi}\,D^q_{H,k^2}\left(\frac{\zeta}{1-\xi}\right)\frac{1}{k^2}\,,$$

$$\mathcal{F}^g_H(x,k^2;k^2;\xi_0)=\frac{g^2}{(2\pi)^3}2N_c\int_{\xi_0}^1 d\xi\left[\frac{1-\xi}{\xi}+\frac{\xi}{1-\xi}+\xi(1-\xi)\right]\frac{1}{1-\xi}D^g_{H,k^2}\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}+\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\sum_q\int_{\xi_0}^1 d\xi\,\left[\xi^2+(1-\xi)^2\right]\frac{1}{1-\xi}\left[D^q_{H,k^2}\left(\frac{x}{1-\xi}\right)+D^{\bar q}_{H,k^2}\left(\frac{x}{1-\xi}\right)\right]\frac{1}{k^2}$$

$$\mathcal{F}^q_H(x,k^2;\mu^2;\xi_0)=\theta(\mu^2-k^2)\left[\mathcal{F}^q_H(x,k^2;k^2;\xi_0)-\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^1 d\xi\frac{1+(1-\xi)^2}{\xi}\mathcal{F}^q_H\left(x,k^2;l^2;\xi_0\right)\right]$$

$$\mathcal{F}^g_H(x,k^2;\mu^2;\xi_0)=\theta(\mu^2-k^2)\Bigg[\mathcal{F}^g_H(x,k^2;k^2;\xi_0)-\frac{g^2}{(2\pi)^3}N_c\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^1 d\xi\left[\frac{1-\xi}{\xi}+\frac{\xi}{1-\xi}+\xi(1-\xi)\right]\mathcal{F}^g_H\left(x,k^2;l^2;\xi_0\right)-\frac{g^2}{(2\pi)^3}\frac{n_f}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^1 d\xi\left[\xi^2+(1-\xi)^2\right]\mathcal{F}^g_H\left(x,k^2;l^2;\xi_0\right)\Bigg].$$

$$D^g_{H,\mu^2}(x)=\int_0^{\mu^2}\pi dk^2\,\mathcal{F}^g_H(x,k^2;\mu^2;\xi_0)$$

- The collinear FFs satisfy DGLAP.

$q \to q \to H$: initial expressions

- We start from the expressions obtained in LCPT in 1411.2869, before collinear subtraction, and we do not implement the plus prescription.
- We work at large N_c, and assume factorisation and translational invariance for the dipoles $s(r) \equiv s_F(r)$.

$$\boxed{\frac{d\bar{\sigma}^{q\to q}}{d^2k d\eta}(k,x_p) = \frac{d\bar{\sigma}_0^{q\to q}}{d^2k d\eta}(k,x_p) + \frac{d\bar{\sigma}_{1,r}^{q\to q}}{d^2k d\eta}(k,x_p) + \frac{d\bar{\sigma}_{1,v}^{q\to q}}{d^2k d\eta}(k,x_p)}$$

$$\frac{d\bar{\sigma}_0^{q\to q}}{d^2k d\eta}(k,x_p) = \frac{1}{(2\pi)^2} x_p f^q_{\mu_0^2}(x_p) \int_{y,\bar{y}} e^{ik\cdot(y-\bar{y})} s[y-\bar{y}]$$

- The WW factors with the Ioffe time restriction read

$$A^i_{\xi,x_p}(y-z) \equiv -i \int_{l^2<\xi(1-\xi)x_p s_0} \frac{d^2l}{(2\pi)^2}\frac{l^i}{l^2} e^{-il\cdot(y-z)}, \quad A^i_\xi(y-z) \equiv A^i_{\xi,x_p/(1-\xi)}(y-z) \simeq A^i_{\xi,x_p}(y-z) \text{ for } k^2/(x_F s_0) \ll 1.$$

$$\frac{d\bar{\sigma}_{1,r}^{q\to q}}{d^2k d\eta}(k,x_p) = \frac{g^2}{(2\pi)^3}\int_0^1 d\xi \int_{y,\bar{y},z} e^{ik\cdot(y-\bar{y})} \frac{1+(1-\xi)^2}{\xi}$$
$$\times\left[\frac{x_p}{1-\xi} f^q_{\mu_0^2}\left(\frac{x_p}{1-\xi}\right) A^i_\xi(y-z)A^i_\xi(\bar{y}-z)\left\{C_F\left[s[y-\bar{y}]+s[(1-\xi)(y-\bar{y})]\right]\right.\right.$$
$$\left.\left.-\frac{N_c}{2}\left[s[(1-\xi)(y-z)]\,s[\bar{y}-z]+s[(1-\xi)(\bar{y}-z)]\,s[y-z]\right]\right\}\right]$$

$$\frac{d\bar{\sigma}_{1,v}^{q\to q}}{d^2k d\eta}(k,x_p) = -\frac{g^2}{(2\pi)^3}\int_0^1 d\xi \int_{y,\bar{y},z} e^{ik\cdot(y-\bar{y})} \frac{1+(1-\xi)^2}{\xi}$$
$$\times\left[x_p f^q_{\mu_0^2}(x_p)\left\{C_F\left[A^i_{\xi,x_p}(y-z)A^i_{\xi,x_p}(y-z)+A^i_{\xi,x_p}(\bar{y}-z)A^i_{\xi,x_p}(\bar{y}-z)\right]s[y-\bar{y}]\right.\right.$$
$$-\left\{A^i_{\xi,x_p}(y-z)A^i_{\xi,x_p}(y-z)\left[\frac{N_c}{2}s\left[y-z\right]s\left[(z-\bar{y})+\xi(y-z)\right]\right]\right.$$
$$\left.\left.\left.+A^i_{\xi,x_p}(\bar{y}-z)A^i_{\xi,x_p}(\bar{y}-z)\frac{N_c}{2}s\left[z-\bar{y}\right]s\left[(y-z)-\xi(\bar{y}-z)\right]\right\}\right\}\right]$$

$q \to q \to H$: initial expressions

- Our dilute projectile contains quarks with transverse momentum smaller than $\mu_0 \sim \Lambda_{QCD}$.

- The dense target sits at some rapidity with no need of further evolution (no large rapidity logarithms found).

- We add fragmentation and Fourier transform to transverse momentum space:

$$\frac{d\bar{\sigma}^{q\to q}}{d^2k d\eta}(k,x_p) = \frac{d\bar{\sigma}_0^{q\to q}}{d^2k d\eta}(k,x_p) + \frac{d\bar{\sigma}_{1,r}^{q\to q}}{d^2k d\eta}(k,x_p) + \frac{d\bar{\sigma}_{1,v}^{q\to q}}{d^2k d\eta}(k,x_p) \qquad \frac{d\sigma^{q\to q\to H}}{d^2p d\eta} = \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D_{H,\mu_0^2}^q(\zeta) \frac{d\bar{\sigma}^{q\to q}}{d^2k d\eta}\left(\frac{p}{\zeta}, \frac{x_F}{\zeta}\right)$$

$$s(k) = \int_r \frac{1}{(2\pi)^2} e^{-ik\cdot r} s(r) \Longrightarrow s(r) = \int_l e^{il\cdot r} s(l) \Longrightarrow s(r=0) = 1 = \int_l s(l)$$

$$\frac{d\sigma_0^{q\to q\to H}}{d^2p d\eta} = S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D_{H,\mu_0^2}^q(\zeta) \frac{x_F}{\zeta} f_{\mu_0^2}^q\left(\frac{x_F}{\zeta}\right) s\left(\frac{p}{\zeta}\right)$$

$q \to q \to H$: real terms

$$\frac{d\sigma_{1,r}^{q\to q\to H}}{d^2p d\eta} = S_\perp \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{H,\mu_0^2}(\zeta) \int_{k,q}\int_{\zeta k^2/(x_F s_0)}^1 d\xi\, \frac{1+(1-\xi)^2}{\xi}\, \frac{x_F}{(1-\xi)\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{(1-\xi)\zeta}\right) s(k)s(q) \frac{[k-(1-\xi)q]^2}{\left(\frac{p}{\zeta}-k\right)^2\left(\frac{p}{\zeta}-(1-\xi)q\right)^2}$$

- It can be written in terms of TMD distributions plus an NLO remainder:

$$\begin{aligned}\frac{d\sigma_{1,r}^{q\to q\to H}}{d^2p d\eta} = {} & S_\perp \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{H,\mu_0^2}(\zeta) \int_{k,q}\int_{\zeta k^2/(x_F s_0)}^1 d\xi\, \frac{1+(1-\xi)^2}{\xi}\, \frac{x_F}{(1-\xi)\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{(1-\xi)\zeta}\right) \\ & \times s(k)s(q)\left\{\left[\frac{\frac{p}{\zeta}-k}{\left(\frac{p}{\zeta}-k\right)^2} - \frac{\frac{p}{\zeta}-(1-\xi)k}{\left(\frac{p}{\zeta}-(1-\xi)k\right)^2}\right]\left[\frac{\frac{p}{\zeta}-q}{\left(\frac{p}{\zeta}-q\right)^2} - \frac{\frac{p}{\zeta}-(1-\xi)q}{\left(\frac{p}{\zeta}-(1-\xi)q\right)^2}\right]\right. \\ & \left. +\frac{1}{2}\left[\frac{(k-q)^2}{\left(\frac{p}{\zeta}-k\right)^2\left(\frac{p}{\zeta}-q\right)^2} + \frac{(1-\xi)^2(k-q)^2}{\left(\frac{p}{\zeta}-(1-\xi)k\right)^2\left(\frac{p}{\zeta}-(1-\xi)q\right)^2}\right]\right\}.\end{aligned}$$

$q \to q \to H$: real terms

$$\frac{d\sigma_{1,r}^{q\to q\to H}}{d^2p d\eta} = S_\perp \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{H,\mu_0^2}(\zeta) \int_{k,q}\int_{\zeta k^2/(x_F s_0)}^1 d\xi \, \frac{1+(1-\xi)^2}{\xi} \, \frac{x_F}{(1-\xi)\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{(1-\xi)\zeta}\right) \, s(k)s(q) \frac{[k-(1-\xi)q]^2}{\left(\frac{p}{\zeta}-k\right)^2\left(\frac{p}{\zeta}-(1-\xi)q\right)^2}$$

- It can be written in terms of TMD distributions plus an NLO remainder:

$$\frac{d\sigma_{1,r}^{q\to q\to H}}{d^2p d\eta} = S_\perp \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{H,\mu_0^2}(\zeta) \int_{k,q}\int_{\zeta k^2/(x_F s_0)}^1 d\xi \, \frac{1+(1-\xi)^2}{\xi} \, \frac{x_F}{(1-\xi)\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{(1-\xi)\zeta}\right)$$
$$\times\, s(k)s(q)\left\{\left[\frac{\frac{p}{\zeta}-k}{\left(\frac{p}{\zeta}-k\right)^2}-\frac{\frac{p}{\zeta}-(1-\xi)k}{\left(\frac{p}{\zeta}-(1-\xi)k\right)^2}\right]\left[\frac{\frac{p}{\zeta}-q}{\left(\frac{p}{\zeta}-q\right)^2}-\frac{\frac{p}{\zeta}-(1-\xi)q}{\left(\frac{p}{\zeta}-(1-\xi)q\right)^2}\right]\right.$$
$$\left. +\frac{1}{2}\left[\frac{(k-q)^2}{\left(\frac{p}{\zeta}-k\right)^2\left(\frac{p}{\zeta}-q\right)^2}+\frac{(1-\xi)^2(k-q)^2}{\left(\frac{p}{\zeta}-(1-\xi)k\right)^2\left(\frac{p}{\zeta}-(1-\xi)q\right)^2}\right]\right\}.$$

$q \to q \to H$: real terms

$$\frac{d\sigma_{1,r}^{q\to q\to H}}{d^2p d\eta} = S_\perp \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{x_F}^1 \frac{d\zeta}{\zeta^2} D_{H,\mu_0^2}^q(\zeta) \int_{k,q}\int_{\zeta k^2/(x_F s_0)}^1 d\xi\, \frac{1+(1-\xi)^2}{\xi} \frac{x_F}{(1-\xi)\zeta} f_{\mu_0^2}^q\left(\frac{x_F}{(1-\xi)\zeta}\right) s(k)s(q) \frac{[k-(1-\xi)q]^2}{\left(\frac{p}{\zeta}-k\right)^2\left(\frac{p}{\zeta}-(1-\xi)q\right)^2}$$

- It can be written in terms of TMD distributions plus an NLO remainder:

$$x\mathcal{T}_q(x,k^2;k^2;\xi_0) = \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{\xi_0}^1 d\xi \frac{1+(1-\xi)^2}{\xi}\frac{x}{1-\xi} f_{k^2}^q\left(\frac{x}{1-\xi}\right)\frac{1}{k^2}$$

$$\frac{d\sigma_{1,r}^{q\to q\to H}}{d^2p d\eta} = S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2}\int_{k^2>\mu_0^2}\frac{x_F}{\zeta}\left\{D_{H,\mu_0^2}^q(\zeta)\mathcal{T}_q\left(\frac{x_F}{\zeta},k^2;k^2,\xi_0=k^2\zeta/(x_F s_0)\right) + f_{\mu_0^2}^q\left(\frac{x_F}{\zeta}\right)\mathcal{F}_H^q\left(\zeta,k^2;k^2,\xi_0=k^2\zeta/(x_F s_0)\right)\right\}$$
$$\times s(-k+p/\zeta)\left[1-\int_q \frac{k\cdot q}{q^2}s(-q+p/\zeta)\right]$$
$$+\frac{g^2}{(2\pi)^3}S_\perp\frac{N_c}{2}\int_{x_F}^1\frac{d\zeta}{\zeta^2}D_{H,\mu_0^2}^q(\zeta)\int_k\int_{k^2\zeta/(x_F s_0)}^1 d\xi\,\frac{x_F}{\zeta(1-\xi)}f_{\mu_0^2}^q\left(\frac{x_F}{\zeta(1-\xi)}\right)\frac{1+(1-\xi)^2}{\xi}$$
$$\times\int_q s(k)s(q)\left[\frac{p/\zeta-k}{(p/\zeta-k)^2}-\frac{p/\zeta-(1-\xi)k}{(p/\zeta-(1-\xi)k)^2}\right]\left[\frac{p/\zeta-q}{(p/\zeta-q)^2}-\frac{p/\zeta-(1-\xi)q}{(p/\zeta-(1-\xi)q)^2}\right].$$

$q \to q \to H$: virtual terms

$$\frac{d\sigma_{1,v}^{q\to q\to H}}{d^2p d\eta} = -2\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2} \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D_{H,\mu_0^2}^q(\zeta) \int_{k^2>\mu_0^2} \int_{k^2\zeta/(x_F s_0)}^1 d\xi \, \frac{x_F}{\zeta} f_{\mu_0^2}^q\left(\frac{x_F}{\zeta}\right) \frac{1+(1-\xi)^2}{\xi} \int_q s\left(\frac{p}{\zeta}\right) s(q) \left[\frac{\frac{p}{\zeta}(1-\xi)-q-k}{(\frac{p}{\zeta}(1-\xi)-q-k)^2}\frac{k}{k^2} + \frac{1}{k^2}\right]$$

- It contains logarithmic divergencies that can be added and subtracted, to be combined with LO providing the evolution of TMDs from μ_0^2 to μ^2:

$$\frac{d\sigma_{1,v}^{q\to q\to H}}{d^2p d\eta} = -2\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2} \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D_{H,\mu_0^2}^q(\zeta) \int_{\mu_0^2}^\infty d^2k \int_{k^2\zeta/(x_F s_0)}^1 d\xi \, \frac{x_F}{\zeta} f_{\mu_0^2}^q\left(\frac{x_F}{\zeta}\right) \frac{1+(1-\xi)^2}{\xi}$$
$$\times \int_q s\left(\frac{p}{\zeta}\right) s(q) \left[\frac{\frac{p}{\zeta}(1-\xi)-q-k}{(\frac{p}{\zeta}(1-\xi)-q-k)^2}\frac{k}{k^2} + \frac{1}{k^2}\right]$$
$$\pm 2\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2} \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D_{H,\mu_0^2}^q(\zeta) \frac{x_F}{\zeta} f_{\mu_0^2}^q\left(\frac{x_F}{\zeta}\right) \int_{\mu_0^2}^{\mu^2} \frac{d^2k}{k^2} \int_{k^2\zeta/(x_F s_0)}^1 d\xi \, \frac{1+(1-\xi)^2}{\xi}$$
$$\times \int_m s\left(\frac{p}{\zeta}\right) s\left(-m+\frac{p}{\zeta}\right).$$

$q \to q \to H$: virtual terms

$$\frac{d\sigma_{1,v}^{q\to q\to H}}{d^2p d\eta} = -2\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2} \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D_{H,\mu_0^2}^q(\zeta) \int_{k^2>\mu_0^2} \int_{k^2\zeta/(x_F s_0)}^1 d\xi \, \frac{x_F}{\zeta} f_{\mu_0^2}^q\left(\frac{x_F}{\zeta}\right) \frac{1+(1-\xi)^2}{\xi} \int_q s\left(\frac{p}{\zeta}\right) s(q) \left[\frac{\frac{p}{\zeta}(1-\xi)-q-k}{(\frac{p}{\zeta}(1-\xi)-q-k)^2}\frac{k}{k^2}+\frac{1}{k^2}\right]$$

- It contains logarithmic divergencies that can be added and subtracted, to be combined with LO providing the evolution of TMDs from μ_0^2 to μ^2:

$$x\mathcal{T}_q(x,k^2;\mu^2;\xi_0) = \theta(\mu^2-k^2)\, x\mathcal{T}_q(x,k^2;k^2;\xi_0)\left[1-\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^1 d\xi \frac{1+(1-\xi)^2}{\xi}\right]$$

$$\frac{d\sigma_0^{q\to q\to H}}{d^2p d\eta} + \frac{d\sigma_{1,v}^{q\to q\to H}}{d^2p d\eta} = S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} \int_m \int_0^{\mu_0^2} d^2l \int_0^{\mu_0^2} d^2k$$
$$\times \mathcal{F}_H^q\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) \frac{x_F}{\zeta}\mathcal{T}_q\left(\frac{x_F}{\zeta}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) s\left(-(k+l)+\frac{p}{\zeta}\right)\left[1-\frac{(k+l)\cdot m}{m^2}\right] s\left(-m+\frac{p}{\zeta}\right)$$
$$-2\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2}\int_{x_F}^1 \frac{d\zeta}{\zeta^2} D_{H,\mu_0^2}^q(\zeta)\int_{\mu_0^2}^\infty d^2k \int_{k^2\zeta/(x_F s_0)}^1 d\xi \, \frac{x_F}{\zeta} f_{\mu_0^2}^q\left(\frac{x_F}{\zeta}\right)\frac{1+(1-\xi)^2}{\xi}$$
$$\times \int_q s\left(\frac{p}{\zeta}\right) s(q)\left[\frac{\frac{p}{\zeta}(1-\xi)-q-k}{(\frac{p}{\zeta}(1-\xi)-q-k)^2}\frac{k}{k^2}+\frac{1}{k^2}\right]$$
$$+2\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2}\int_{x_F}^1 \frac{d\zeta}{\zeta^2} D_{H,\mu_0^2}^q(\zeta)\frac{x_F}{\zeta} f_{\mu_0^2}^q\left(\frac{x_F}{\zeta}\right)\int_{\mu_0^2}^{\mu^2}\frac{d^2k}{k^2}\int_{k^2\zeta/(x_F s_0)}^1 d\xi \, \frac{1+(1-\xi)^2}{\xi}$$
$$\times \int_m s\left(\frac{p}{\zeta}\right) s\left(-m+\frac{p}{\zeta}\right).$$

$q \to q \to H$: virtual terms

$$\frac{d\sigma_{1,v}^{q\to q\to H}}{d^2p d\eta} = -2\frac{g^2}{(2\pi)^3}S_\perp \frac{N_c}{2}\int_{x_F}^1 \frac{d\zeta}{\zeta^2} D_{H,\mu_0^2}^q(\zeta)\int_{k^2>\mu_0^2}\int_{k^2\zeta/(x_F s_0)}^1 d\xi \,\frac{x_F}{\zeta} f_{\mu_0^2}^q\left(\frac{x_F}{\zeta}\right) \frac{1+(1-\xi)^2}{\xi}\int_q s\left(\frac{p}{\zeta}\right) s(q)\left[\frac{\frac{p}{\zeta}(1-\xi)-q-k}{(\frac{p}{\zeta}(1-\xi)-q-k)^2}\frac{k}{k^2}+\frac{1}{k^2}\right]$$

- It contains logarithmic divergencies that can be added and subtracted, to be combined with LO providing the evolution of TMDs from μ_0^2 to μ^2:

$$x\mathcal{T}_q(x,k^2;\mu^2;\xi_0) = \theta(\mu^2-k^2)\,x\mathcal{T}_q(x,k^2;k^2;\xi_0)\left[1-\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^1 d\xi\frac{1+(1-\xi)^2}{\xi}\right]$$

$$\frac{d\sigma_0^{q\to q\to H}}{d^2pd\eta}+\frac{d\sigma_{1,v}^{q\to q\to H}}{d^2pd\eta} = S_\perp\int_{x_F}^1\frac{d\zeta}{\zeta^2}\int_m\int_0^{\mu_0^2}d^2l\int_0^{\mu_0^2}d^2k$$
$$\times\mathcal{F}_H^q\left(\zeta,l^2;\mu^2;\xi_0=\frac{\zeta\mu^2}{x_F s_0}\right)\frac{x_F}{\zeta}\mathcal{T}_q\left(\frac{x_F}{\zeta},k^2;\mu^2;\xi_0=\frac{\zeta\mu^2}{x_F s_0}\right)s\left(-(k+l)+\frac{p}{\zeta}\right)\left[1-\frac{(k+l)\cdot m}{m^2}\right]s\left(-m+\frac{p}{\zeta}\right)$$
$$-2\frac{g^2}{(2\pi)^3}S_\perp\frac{N_c}{2}\int_{x_F}^1\frac{d\zeta}{\zeta^2}D_{H,\mu_0^2}^q(\zeta)\int_{\mu_0^2}^\infty d^2k\int_{k^2\zeta/(x_F s_0)}^1 d\xi\,\frac{x_F}{\zeta}f_{\mu_0^2}^q\left(\frac{x_F}{\zeta}\right)\frac{1+(1-\xi)^2}{\xi}$$
$$\times\int_q s\left(\frac{p}{\zeta}\right)s(q)\left[\frac{\frac{p}{\zeta}(1-\xi)-q-k}{(\frac{p}{\zeta}(1-\xi)-q-k)^2}\frac{k}{k^2}+\frac{1}{k^2}\right]$$
$$+2\frac{g^2}{(2\pi)^3}S_\perp\frac{N_c}{2}\int_{x_F}^1\frac{d\zeta}{\zeta^2}D_{H,\mu_0^2}^q(\zeta)\frac{x_F}{\zeta}f_{\mu_0^2}^q\left(\frac{x_F}{\zeta}\right)\int_{\mu_0^2}^{\mu^2}\frac{d^2k}{k^2}\int_{k^2\zeta/(x_F s_0)}^1 d\xi\,\frac{1+(1-\xi)^2}{\xi}$$
$$\times\int_m s\left(\frac{p}{\zeta}\right)s\left(-m+\frac{p}{\zeta}\right).$$

they can be combined

$q \to q \to H$: virtual terms

$$\frac{d\sigma_{1,v}^{q\to q\to H}}{d^2p d\eta} = -2\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2}\int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{H,\mu_0^2}(\zeta)\int_{k^2>\mu_0^2}\int_{k^2\zeta/(x_F s_0)}^1 d\xi \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\frac{1+(1-\xi)^2}{\xi}\int_q s\left(\frac{p}{\zeta}\right) s(q)\left[\frac{\frac{p}{\zeta}(1-\xi)-q-k}{(\frac{p}{\zeta}(1-\xi)-q-k)^2}\frac{k}{k^2}+\frac{1}{k^2}\right]$$

- It contains logarithmic divergencies that can be added and subtracted, to be combined with LO providing the evolution of TMDs from μ_0^2 to μ^2:

$$x\mathcal{T}_q(x,k^2;\mu^2;\xi_0) = \theta(\mu^2-k^2)\, x\mathcal{T}_q(x,k^2;k^2;\xi_0)\left[1-\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\int_{k^2}^{\mu^2}\frac{\pi dl^2}{l^2}\int_{\xi_0}^1 d\xi\frac{1+(1-\xi)^2}{\xi}\right]$$

$$\frac{d\sigma_0^{q\to q\to H}}{d^2p d\eta}+\frac{d\sigma_{1,v}^{q\to q\to H}}{d^2p d\eta} = S_\perp\int_{x_F}^1\frac{d\zeta}{\zeta^2}\int_m\int_0^{\mu_0^2} d^2l\int_0^{\mu_0^2} d^2k$$
$$\times\mathcal{F}_H^q\left(\zeta,l^2;\mu^2;\xi_0=\frac{\zeta\mu^2}{x_F s_0}\right)\frac{x_F}{\zeta}\mathcal{T}_q\left(\frac{x_F}{\zeta},k^2;\mu^2;\xi_0=\frac{\zeta\mu^2}{x_F s_0}\right) s\left(-(k+l)+\frac{p}{\zeta}\right)\left[1-\frac{(k+l)\cdot m}{m^2}\right] s\left(-m+\frac{p}{\zeta}\right)$$

$$-2\frac{g^2}{(2\pi)^3}S_\perp\frac{N_c}{2}\int^1\frac{d\zeta}{\zeta^2}D^q_{H,\mu_0^2}(\zeta)\int^\infty d^2k\int^1_{\ldots\zeta/(x_F s_0)} d\xi\,\frac{x_F}{\zeta}f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\frac{1+(1-\xi)^2}{\xi}$$

$$\ldots -q-k \ldots \frac{k}{k^2}+\frac{1}{k^2}\Big]$$

$$\ldots\Big)\int_{\mu_0^2}^{\mu^2}\frac{d^2k}{k^2}\int_{k^2\zeta/(x_F s_0)}^1 d\xi\,\frac{1+(1-\xi)^2}{\xi}$$

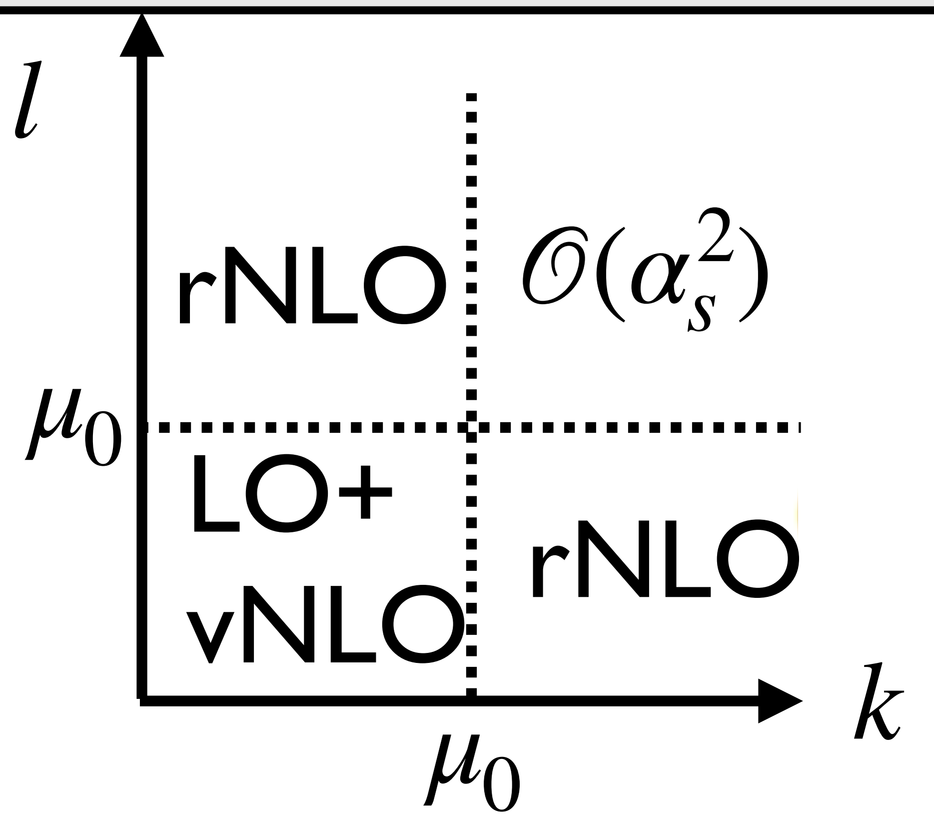
$$\times\int_m s\left(\frac{p}{\zeta}\right) s\left(-m+\frac{p}{\zeta}\right).$$

Note: at this point it can be shown that independence of the choice of s_0 results in LO BK evolution for the dipoles.

they can be combined

$q \to q \to H$: all terms

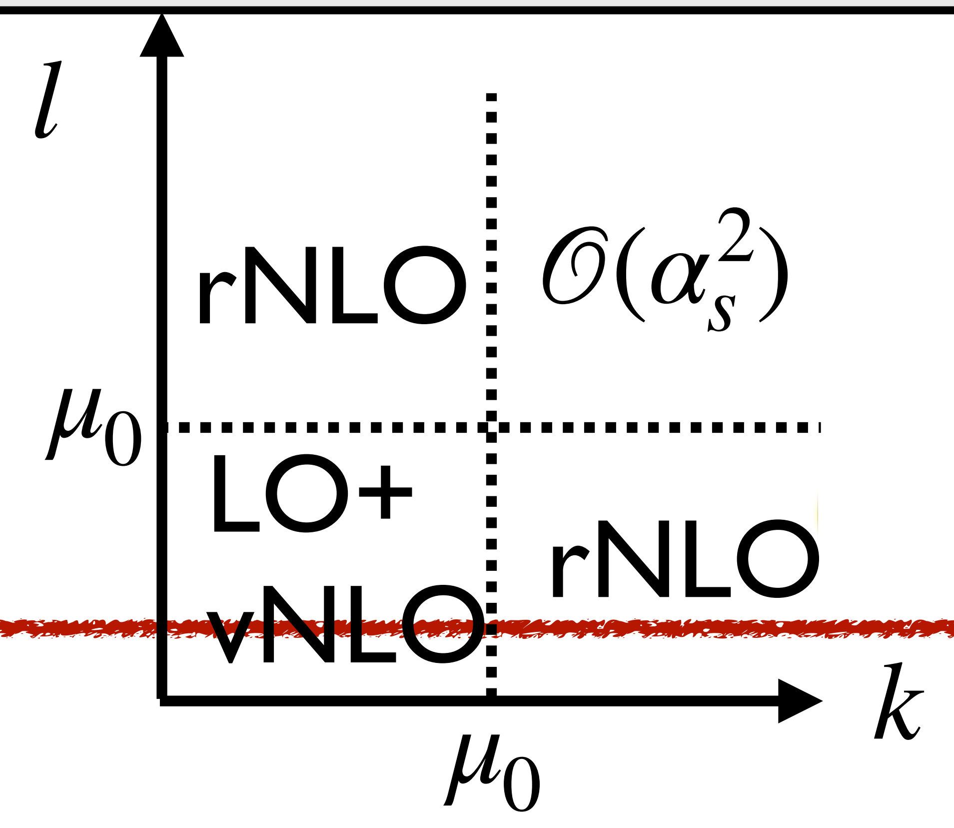
- Adding all terms we get, neglecting terms $\mathcal{O}\left(\frac{p^2, k^2, Q_s^2, \mu^2}{s_0}\right)$, $\mathcal{O}(\alpha_s^2)$:



$$\frac{d\sigma^{q\to q\to H}}{d^2p\,d\eta}$$
$$= S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} \int_m \int d^2l \int d^2k$$
$$\times \mathcal{F}_H^q\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) \frac{x_F}{\zeta} \mathcal{T}_q\left(\frac{x_F}{\zeta}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) s\left(-(k+l) + \frac{p}{\zeta}\right) \left[1 - \frac{(k+l)\cdot m}{m^2}\right] s\left(-m + \frac{p}{\zeta}\right)$$
$$-2\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2} \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{H,\mu_0^2}(\zeta) \frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right) \int_m \int_{\mu^2}^{(-m+\xi p/\zeta)^2} \frac{d^2k}{k^2} \int_{k^2\zeta/(x_F s_0)}^1 d\xi\, \frac{1+(1-\xi)^2}{\xi}$$
$$\times s\left(\frac{p}{\zeta}\right) s\left(-m + \frac{p}{\zeta}\right)$$
$$+\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2} \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{H,\mu_0^2}(\zeta) \int_k \int_{k^2\zeta/(x_F s_0)}^1 d\xi\, \frac{x_F}{\zeta(1-\xi)} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta(1-\xi)}\right) \frac{1+(1-\xi)^2}{\xi}$$
$$\times \int_q s(k)s(q) \left[\frac{p/\zeta - k}{(p/\zeta - k)^2} - \frac{p/\zeta - (1-\xi)k}{(p/\zeta - (1-\xi)k)^2}\right] \left[\frac{p/\zeta - q}{(p/\zeta - q)^2} - \frac{p/\zeta - (1-\xi)q}{(p/\zeta - (1-\xi)q)^2}\right].$$

$q \to q \to H$: all terms

- Adding all terms we get, neglecting terms $\mathcal{O}\left(\frac{p^2, k^2, Q_s^2, \mu^2}{s_0}\right)$, $\mathcal{O}(\alpha_s^2)$:

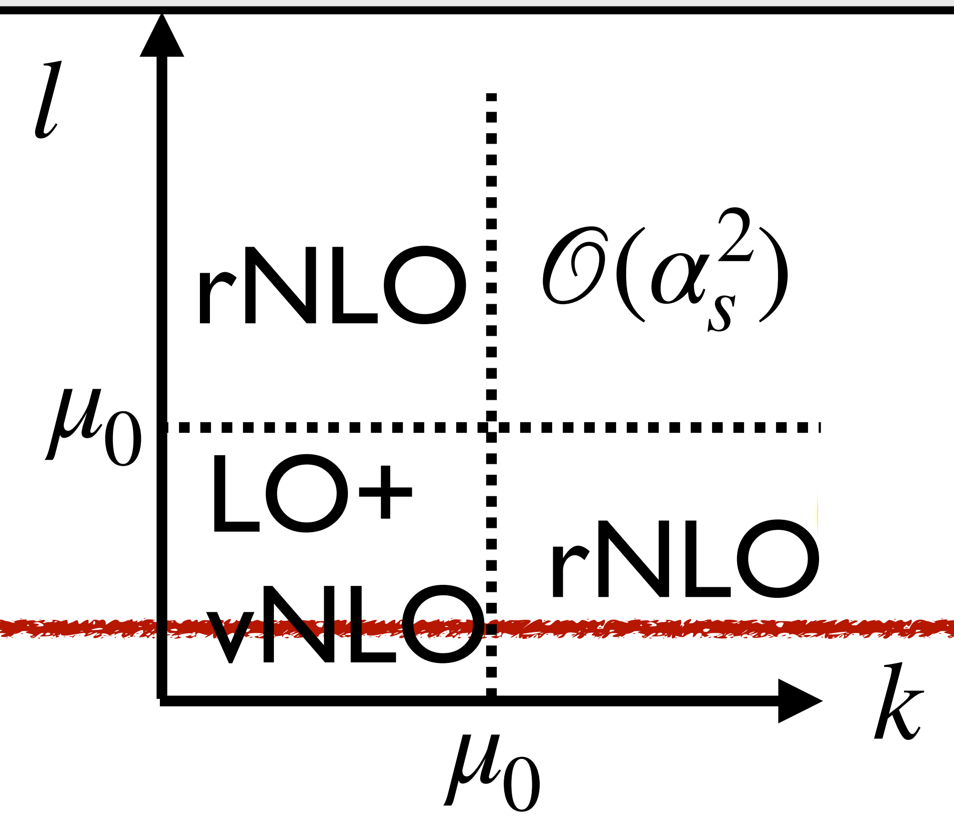


TMDs for PDFs and FFs

$$
\begin{aligned}
\frac{d\sigma^{q\to q\to H}}{d^2p\,d\eta}
&= S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} \int_m \int d^2l \int d^2k \\
&\quad \times \mathcal{F}_H^q\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) \frac{x_F}{\zeta} \mathcal{T}_q\left(\frac{x_F}{\zeta}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) s\left(-(k+l)+\frac{p}{\zeta}\right)\left[1-\frac{(k+l)\cdot m}{m^2}\right] s\left(-m+\frac{p}{\zeta}\right) \\
&\quad -2\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2} \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D_{H,\mu_0^2}^q(\zeta) \frac{x_F}{\zeta} f_{\mu_0^2}^q\left(\frac{x_F}{\zeta}\right) \int_m \int_{\mu^2}^{(-m+\xi p/\zeta)^2} \frac{d^2k}{k^2} \int_{k^2\zeta/(x_F s_0)}^1 d\xi\, \frac{1+(1-\xi)^2}{\xi} \\
&\qquad \times s\left(\frac{p}{\zeta}\right) s\left(-m+\frac{p}{\zeta}\right) \\
&\quad +\frac{g^2}{(2\pi)^3} S_\perp \frac{N_c}{2} \int_{x_F}^1 \frac{d\zeta}{\zeta^2} D_{H,\mu_0^2}^q(\zeta) \int_k \int_{k^2\zeta/(x_F s_0)}^1 d\xi\, \frac{x_F}{\zeta(1-\xi)} f_{\mu_0^2}^q\left(\frac{x_F}{\zeta(1-\xi)}\right) \frac{1+(1-\xi)^2}{\xi} \\
&\qquad \times \int_q s(k)s(q)\left[\frac{p/\zeta-k}{(p/\zeta-k)^2} - \frac{p/\zeta-(1-\xi)k}{(p/\zeta-(1-\xi)k)^2}\right]\left[\frac{p/\zeta-q}{(p/\zeta-q)^2} - \frac{p/\zeta-(1-\xi)q}{(p/\zeta-(1-\xi)q)^2}\right].
\end{aligned}
$$

$q \to q \to H$: all terms

- Adding all terms we get, neglecting terms $\mathcal{O}\left(\frac{p^2,k^2,Q_s^2,\mu^2}{s_0}\right)$, $\mathcal{O}(\alpha_s^2)$:



$$\frac{d\sigma^{q\to q\to H}}{d^2p\,d\eta}$$

$$= S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2}\int_m \int d^2l \int d^2k$$

$$\times \mathcal{F}_H^q\left(\zeta,l^2;\mu^2;\xi_0=\frac{\zeta\mu^2}{x_F s_0}\right)\frac{x_F}{\zeta}\mathcal{T}_q\left(\frac{x_F}{\zeta},k^2;\mu^2;\xi_0=\frac{\zeta\mu^2}{x_F s_0}\right) s\left(-(k+l)+\frac{p}{\zeta}\right)\left[1-\frac{(k+l)\cdot m}{m^2}\right] s\left(-m+\frac{p}{\zeta}\right)$$

TMDs for PDFs and FFs

$$-2\frac{g^2}{(2\pi)^3}S_\perp \frac{N_c}{2}\int_{x_F}^1 \frac{d\zeta}{\zeta^2} D^q_{H,\mu_0^2}(\zeta)\frac{x_F}{\zeta} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta}\right)\int_m \int_{\mu^2}^{(-m+\xi p/\zeta)^2}\frac{d^2k}{k^2}\int_{k^2\zeta/(x_F s_0)}^1 d\xi\, \frac{1+(1-\xi)^2}{\xi}$$

$$\times s\left(\frac{p}{\zeta}\right) s\left(-m+\frac{p}{\zeta}\right)$$

NLO remainders, no large logs for our choice of scales

$$+\frac{g^2}{(2\pi)^3}S_\perp\frac{N_c}{2}\int_{x_F}^1\frac{d\zeta}{\zeta^2}D^q_{H,\mu_0^2}(\zeta)\int_k\int_{k^2\zeta/(x_F s_0)}^1 d\xi\, \frac{x_F}{\zeta(1-\xi)} f^q_{\mu_0^2}\left(\frac{x_F}{\zeta(1-\xi)}\right)\frac{1+(1-\xi)^2}{\xi}$$

$$\times \int_q s(k)s(q)\left[\frac{p/\zeta-k}{(p/\zeta-k)^2}-\frac{p/\zeta-(1-\xi)k}{(p/\zeta-(1-\xi)k)^2}\right]\left[\frac{p/\zeta-q}{(p/\zeta-q)^2}-\frac{p/\zeta-(1-\xi)q}{(p/\zeta-(1-\xi)q)^2}\right].$$

$q \to q \to H$: final expression

- Neglecting terms $\mathcal{O}\left(\frac{p^2, k^2, Q_s^2, \mu^2}{s_0}\right)$, $\mathcal{O}(\alpha_s^2)$, we get a parton model-like expression:

$$\frac{d\sigma^{q\to q\to H}}{d^2p d\eta} = S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} \int d\xi \int d^2l \int d^2k\, \mathcal{F}_H^q\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right)$$
$$\times \frac{x_F}{\zeta(1-\xi)} \mathcal{T}_q\left(\frac{x_F}{\zeta(1-\xi)}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) \mathcal{P}(\xi, \zeta; k+l; p, s_0, \mu^2, \mu_0^2),$$

$$\mathcal{P}(\xi, \zeta; k+l; p, s_0, \mu^2, \mu_0^2) = \int d\lambda \int_m \left\{\delta(\lambda)\delta(\xi-\lambda) s\left(-(k+l)+\frac{p}{\zeta}\right)\left[1-\frac{(k+l)\cdot m}{m^2}\right] s\left(-m+\frac{p}{\zeta}\right)\right.$$
$$+\frac{g^2}{(2\pi)^3}\frac{N_c}{2}\frac{1+(1-\lambda)^2}{\lambda}\theta(1-\lambda)$$
$$\left[\delta(\lambda-\xi)\theta\left(\xi-\frac{m^2\zeta}{x_F s_0}\right)\int_q s(m)s(q)\left[\frac{p/\zeta-m}{(p/\zeta-m)^2}-\frac{p/\zeta-(1-\xi)m}{(p/\zeta-(1-\xi)m)^2}\right]\left[\frac{p/\zeta-q}{(p/\zeta-q)^2}-\frac{p/\zeta-(1-\xi)q}{(p/\zeta-(1-\xi)q)^2}\right]\right.$$
$$\left.\left.-2\delta(\xi)\theta\left(\lambda-\frac{\mu^2\zeta}{x_F s_0}\right)\theta(m^2-\mu_0^2)\, s\left(\frac{p}{\zeta}\right) s\left(m+(1-\lambda)\frac{p}{\zeta}\right)\int_{\mu^2}^{\min[m^2,\lambda\bar{s}_0]}\frac{d^2q}{q^2}\right]\right\}.$$

$q \to q \to H$: final expression

- Neglecting terms $\mathcal{O}\left(\frac{p^2, k^2, Q_s^2, \mu^2}{s_0}\right)$, $\mathcal{O}(\alpha_s^2)$, we get a parton model-like expression:

$$\frac{d\sigma^{q\to q\to H}}{d^2p d\eta} = S_\perp \int_{x_F}^1 \frac{d\zeta}{\zeta^2} \int d\xi \int d^2l \int d^2k\, \mathcal{F}_H^q\left(\zeta, l^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) \times \frac{x_F}{\zeta(1-\xi)} \mathcal{T}_q\left(\frac{x_F}{\zeta(1-\xi)}, k^2; \mu^2; \xi_0 = \frac{\zeta\mu^2}{x_F s_0}\right) \mathcal{P}(\xi, \zeta; k+l; p, s_0, \mu^2, \mu_0^2),$$

$$\mathcal{P}(\xi,\zeta;k+l;p,s_0,\mu^2,\mu_0^2) = \int d\lambda \int_m \left\{ \delta(\lambda)\delta(\xi-\lambda) s\left(-(k+l)+\frac{p}{\zeta}\right)\left[1-\frac{(k+l)\cdot m}{m^2}\right] s\left(-m+\frac{p}{\zeta}\right) \right.$$
$$+ \frac{g^2}{(2\pi)^3}\frac{N_c}{2}\frac{1+(1-\lambda)^2}{\lambda}\theta(1-\lambda)$$
$$\left[\delta(\lambda-\xi)\theta\left(\xi-\frac{m^2\zeta}{x_F s_0}\right)\int_q s(m)s(q)\left[\frac{p/\zeta-m}{(p/\zeta-m)^2}-\frac{p/\zeta-(1-\xi)m}{(p/\zeta-(1-\xi)m)^2}\right]\left[\frac{p/\zeta-q}{(p/\zeta-q)^2}-\frac{p/\zeta-(1-\xi)q}{(p/\zeta-(1-\xi)q)^2}\right]\right.$$
$$\left.\left.-2\delta(\xi)\theta\left(\lambda-\frac{\mu^2\zeta}{x_F s_0}\right)\theta(m^2-\mu_0^2)\, s\left(\frac{p}{\zeta}\right)\, s\left(m+(1-\lambda)\frac{p}{\zeta}\right)\int_{\mu^2}^{\min[m^2,\lambda\bar{s}_0]}\frac{d^2q}{q^2}\right]\right\}.$$

quark scattering

qg scattering due to $q \to qg$

The other channels:

- TMD PDFs:
  - ➜ For quark: it gets contributions from $q \to q$ and $g \to q$.
  - ➜ For antiquark: it gets contributions from $\bar{q} \to \bar{q}$ and $g \to \bar{q}$.
  - ➜ For gluon: it gets contributions from $g \to g, q \to g$ and $\bar{q} \to g$.

- TMD FFs:
  - ➜ For quark: it gets contributions from $q \to q \to H$ and $q \to g \to H$.
  - ➜ For antiquark: it gets contributions from $\bar{q} \to \bar{q} \to H$ and $\bar{q} \to g \to H$.
  - ➜ For gluon: it gets contributions from $g \to g \to H, g \to q \to H$ and $g \to \bar{q} \to H$.

- The complete quark piece of the parton-like formula keeps the form with additional NLO remainders.

- The gluon piece of the parton-like formula contains 3 dipoles in the fundamental representation, and additional NLO remainders.

Summary:

• In view of the negativity problem and the recent LHCb data, we revisit the calculation of single inclusive particle production at NLO in the hybrid model, avoiding collinear and soft (BK-like) subtractions. We assume large N_c, and factorisation/translational invariance for dipoles.

• We see that divergencies are absorbed into TMD PDFs and FFs defined from collinear ones.
• We get a parton model-like formula with a probability interpretation plus NLO, not log-enhanced remainders.

$$\int \frac{d\zeta}{\zeta^2} \int d^2k_\perp d^2q_\perp \, T\left(\frac{x_F}{\zeta}, k_\perp; \mu_T^2\right) P(k_\perp, q_\perp) \, F\left(\zeta; p_\perp, (k_\perp + q_\perp); \mu_F^2\right) + \text{NLO remainders} + \mathcal{O}\left(\frac{p_\perp^2, k_\perp^2, Q_s^2, \mu^2}{s_0}\right)$$

• We conclude that the correct framework to resum all large logarithms is not collinear factorization for the projectile (the hybrid model) but TMD factorization.

• Outlook:

➜ Small-x evolution equations in this setup.
➜ Implement numerically our results.

Summary:

● In view of the negativity problem and the recent LHCb data, we revisit the calculation of single inclusive p … r and soft (BK-like) subtractio … or dipoles.

Thanks a lot to you for your attention, and to the organisers to the ECT people!!!*

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Backup: