Incoherent diffractive dijet production and gluon Bose enhancement in the nuclear wavefunction

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'BEYOND EIKONAL METHODS IN HIGH ENERGY SCATTERING',

ECT*.

(BASED ON: **TK**, A. Kovner, M. Li, V. V. Skokov arXiv:2312.04493)

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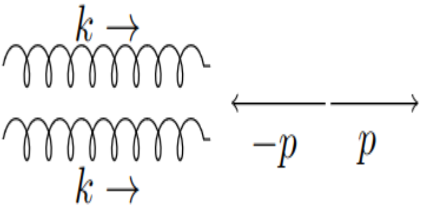
Outline

- Introduction
- Deep inelastic scattering (DIS) and Bose enhancement
- Results
- Conclusion

- Prior to scattering, gluons in the hadronic wavefunction are correlated via Bose enhancement.
- At small x DIS, there is a non-zero probability that the dipole's quark and the antiquark scatter on the correlated gluons.
- The final jets' momenta will carry the information about the correlation in the hadronic wavefunction.



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In CGC, the nuclear wavefunction is split into the valence modes and the soft modes.

$$|\Psi\rangle = |s_v\rangle \otimes |v\rangle$$

- Computing the expectation value of an operator $\mathcal{O}$,

$$\langle\Psi|\,\mathcal{O}\,|\Psi\rangle = \int [\mathcal{D}\rho^a][\mathcal{D}A^{\mu,a}]W[\rho]e^{iS[\rho,A]}\mathcal{O}[\rho,A]$$

  with the normalisation

$$\langle\Psi|\,\Psi\rangle = 1$$

- The weight function is a Gaussian.

$$W[\rho] = e^{-\int \frac{\mathrm{d}^2\underline{k}}{(2\pi)^2}\frac{1}{2\mu^2}\rho_a(\underline{k})\rho_a^*(\underline{k})}$$

  The McLerran-Venugopalan model.

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Density matrix in CGC [H. Duan, C. Akkaya, A. Kovner, V. V. Skokov arXiv:hep-ph/2001.01726]

$$\hat{\rho}_r = \mathcal{N} \int D\rho e^{-\int_{\underline{k}} \frac{1}{2\mu^2} \rho_a(\underline{k}) \rho_a^*(\underline{k})} \mathcal{C}(\rho_b, \phi_b^i) \left|0\right\rangle \left\langle 0\right| \mathcal{C}^\dagger(\rho_c, \phi_c^j)$$

Normalization of states are given by

$$\left\langle n_c(\underline{k}) \middle| n_c(\underline{k}') \right\rangle = \langle 0| \frac{[a_c(\underline{k})]^n}{\sqrt{n!}} \frac{[a_c^\dagger(\underline{k}')]^n}{\sqrt{n!}} |0\rangle$$

Multigluon states:

$$\prod_c \prod_k |n_c(\underline{k}), m_c(-\underline{k})\rangle = \prod_c \prod_k \frac{[a_c(\underline{k})]^n}{\sqrt{n!}} \frac{[a_c^\dagger(-\underline{k}')]^m}{\sqrt{n!}} |0\rangle$$

The action of the coherent operator on the soft gluon vacuum:

$$\mathcal{C}\left|0\right\rangle = e^{\int_{\underline{k}} b_c^i(\underline{k})[a_c^{i\dagger}(\underline{k}) + a_c^i(-\underline{k})]} \left|0\right\rangle$$

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Calculation in the CGC

- The matrix elements of $\hat{\rho}_r$ between states in the momentum space Fock basis:

$$\rho_{n,m,\alpha,\beta} \equiv \langle n_c(\underline{q}), m_c(-\underline{q})|\hat{\rho}_r(\underline{q})|\alpha_c(\underline{q}), \beta_c(-\underline{q})\rangle = (1-R)\frac{(n+\beta)!}{\sqrt{n!m!\alpha!\beta!}}\left(\frac{R}{2}\right)^{n+\beta} \times \delta_{(n+\beta),(m+\alpha)} ,$$

$$R = \left(1 + \frac{\underline{q}^2}{2g^2\mu^2}\right)^{-1}$$

- Correlator of two gluons:

$$D(\underline{k},\underline{p}) = \mathrm{Tr}\left(\hat{\rho}_r a_b^+(\underline{k}) a_c^+(\underline{p}) a_b(\underline{k}) a_c(\underline{p})\right) .$$

$$\langle a_a^+(\underline{k}_1) a_b(\underline{k}_2)\rangle = \mathrm{Tr}\left(\hat{\rho}_r a_a^+(\underline{k}_1) a_b(\underline{k}_2)\right) = (2\pi)^2\delta^{(2)}(\underline{k}_1 - \underline{k}_2)\delta_{ab}\sum_{n,m} n\rho_{n,m,n,m}$$
$$= (2\pi)^2\delta^{(2)}(\underline{k}_1 - \underline{k}_2)\delta_{ab}\frac{g^2\bar{\mu}^2}{k_1^2} .$$

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Bose Enhancement [T. Altinoluk, N. Armesto, G. Beuf, A. Kovner, M. Lublinsky arXiv: hep-ph/1503.07126]

- Probability amplitude for finding a boson at r and the other at r' is

$$\langle\phi|\,\hat{\varphi}^\dagger(r)\hat{\varphi}^\dagger(r')\hat{\varphi}(r')\hat{\varphi}(r)\,|\phi\rangle = \quad n^2 + \left|\int \frac{\mathrm{d}^3p}{(2\pi)^3} e^{ip.(r-r')} n(p)\right|^2$$

- Considering a coherent state,

$$|b(x)\rangle \equiv \exp\left(i\int \mathrm{d}^3x b^i(x)(a^i(x) + a^{i\dagger}(x))\right)|0\rangle$$

Computing the 2-particle correlator in this state gives

$$\begin{aligned}\langle b(x)|\, a^{i\dagger}(x)a^{j\dagger}(y)a^i(x)a^j(y)\,|b(x)\rangle =& b^i(x)b^i(x)b^j(y)b^j(y)\\ =& n(x)n(y)\end{aligned}$$

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What about the CGC then? [T. Altinoluk, N. Armesto, G. Beuf, A. Kovner, M. Lublinsky arXiv: hep-ph/1503.07126]

- In the MV model,

$$\langle a_a^+(\underline{k}_1)a_b^+(\underline{k}_2)\rangle \neq 0$$

Instead

$$\begin{aligned}\langle a_a^+(\underline{k}_1)a_b^+(\underline{k}_2)\rangle =&(2\pi)^2\delta^{(2)}(\underline{k}_1+\underline{k}_2)\delta_{ab}\frac{g^2\bar{\mu}^2}{k_1^2}\\ =&\langle a_a(\underline{k}_1)a_b(\underline{k}_2)\rangle\end{aligned}$$

- Hence, the 2-gluon correlator is

$$\begin{aligned}D(\underline{k},\underline{p}) =&\mathrm{Tr}\left(\hat{\rho}_r a_b^+(\underline{k})a_c^+(\underline{p})a_b(\underline{k})a_c(\underline{p})\right)\\ =&\underbrace{\left(\underline{S}(N_c^2-1)\frac{g^2\bar{\mu}^2}{\underline{k}^2}\right)}_{n(\underline{k})}\underbrace{\left(\underline{S}(N_c^2-1)\frac{g^2\bar{\mu}^2}{\underline{p}^2}\right)}_{n(\underline{p})}\\ &+(2\pi)^2(N_c^2-1)\underline{S}\left(\frac{g^2\bar{\mu}^2}{\underline{k}^2}\right)^2\left[\underbrace{\delta^{(2)}(\underline{k}+\underline{p})}_{\text{back-to-back}}+\underbrace{\delta^{(2)}(\underline{k}-\underline{p})}_{\text{collinear}}\right]\end{aligned}$$

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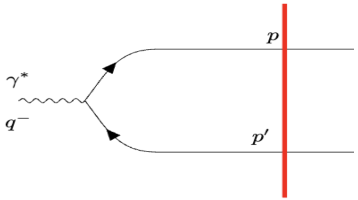
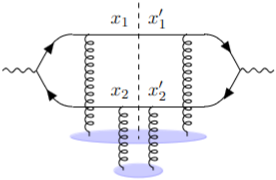
Dipole Picture of DIS [Y. V. Kovchegov, E. Levin Quantum Chromodynamics at High Energy]



- In high energy we use the dipole picture of DIS.
- The photon splits into a $q\bar{q}$ pair and interacts with the target.
- The total cross section is given by

$$\sigma^{\gamma^* A}(x, Q^2) = \int \mathrm{d}^2\underline{x} \int_0^1 \frac{\mathrm{d}z}{z(1-z)} |\Psi^{\gamma^* \to q\bar{q}}(\underline{x}, z)|^2 \sigma^{q\bar{q}A}(\underline{x}, Y)$$

Incoherent Diffractive Dijet Production in DIS [B. Rodriguez-Aguilar, D. N. Triantafyllopoulos, S. Y. Wei arXiv: hep-ph/2302.01106]



- A rapidity gap between the scattered $q\bar{q}$ pair and the target remnants.
- The $q\bar{q}$ is in a color singlet state.

The part of the cross section that describes the interaction of the dipole with the target

$$\mathcal{N}_{\text{incoherent diffractive}} = \frac{1}{N_c^2}\left\langle \mathrm{Tr}\left[V^\dagger(\underline{x}_2)V(\underline{x}_1)\right]\mathrm{Tr}\left[V^\dagger(\underline{x}_1')V(\underline{x}_2')\right]\right\rangle - \frac{1}{N_c^2}\mathrm{Tr}\langle V^\dagger(\underline{x}_2)V(\underline{x}_1)\rangle\mathrm{Tr}\langle V^\dagger(\underline{x}_2')V(\underline{x}_1')\rangle$$

$$\sigma^{\gamma^* A}(x, Q^2) = \int d^2\underline{x}\int_0^1 \frac{dz}{z(1-z)}|\Psi^{\gamma^*\to q\bar{q}}(\underline{x}, z)|^2\sigma^{q\bar{q}A}(\underline{x}, Y)$$

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Averaging in MV model

- MV model defines the correlation between the static color charges

$$\langle \rho^a(x^-,\underline{x})\rho^b(y^-,\underline{y})\rangle = \delta^{ab}\mu^2(x^-)\delta(x^- - y^-)\delta^{(2)}(\underline{x}-\underline{y})$$

- In the covariant gauge,

$$\partial^2 A_a^+(x^-,\underline{x}) = g\rho_a(x^-,\underline{x})$$

$$\Longrightarrow \; A_a^+(x^-,\underline{x}) = -\frac{g}{2\pi}\int d^2\underline{y}\;\ln\Big(|\underline{x}-\underline{y}|\Lambda\Big)\;\rho(x^-,\underline{y})$$

- Hence, the correlation between the soft gluon fields is

$$\langle A^a(x^-,\underline{x})A^b(y^-,\underline{y})\rangle = \delta^{ab}g^2\mu^2(x^-)\delta(x^- - y^-)L(\underline{x}-\underline{y})$$

where

$$L(\underline{x}-\underline{y}) = \frac{g^2}{(2\pi)^2}\int d^2z\,\ln(|\underline{x}-\underline{z}|\Lambda)\ln\Big(|\underline{z}-\underline{y}|\Lambda\Big)\,.$$

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- Expanding the Wilson line,

$$V(\underline{x}) \approx 1 + \frac{(ig)^4}{2}\left(\frac{C_f g^2 \bar{\mu}^2 L(\underline{0})}{2}\right)^2 + igt^a \alpha_a(\underline{x})\left(1 + (ig)^2 \frac{C_f \bar{g}^2 \mu^2 L(\underline{0})}{2}\right)$$
$$+ (ig)^2 \int_{-\infty}^{+\infty} dx_0^- \int_{-\infty}^{x_0^-} dx_1^- t^a t^b A_a^+(x_0^-, \underline{x}) A_b^+(x_1^-, \underline{x})\,.$$

- The dipole factor:

$$\frac{1}{N_c}\mathrm{Tr}\, V^\dagger(\underline{y})V(\underline{x}) = 1 + \frac{(ig)^2}{2}\frac{1}{N_c}\mathrm{Tr}(t_a t_b)(\alpha_a(\underline{y}) - \alpha_a(\underline{x}))(\alpha_b(\underline{y}) - \alpha_b(\underline{x}))$$
$$+ \frac{(ig)^4 (C_f g^2 \bar{\mu}^2)^2}{2}[L(\underline{0}) - L(\underline{x} - \underline{y})]^2$$

- We can drop the first and the last term as the diffractive cross-section contains $\frac{1}{N_c}\mathrm{Tr}\, V^\dagger(\underline{y})V(\underline{x}) - \left\langle \frac{1}{N_c}\mathrm{Tr}\, V^\dagger(\underline{y})V(\underline{x})\right\rangle$.

$$\therefore \mathcal{N}_{\text{diffractive}} \approx \frac{C_f g^8 \bar{\mu}^4}{4N_c}(L(\underline{x}_1 - \underline{x}_1') - L(\underline{x}_1 - \underline{x}_2') - L(\underline{x}_1' - \underline{x}_2) + L(\underline{x}_2 - \underline{x}_2'))^2$$

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Analysis of the cross-section

- The cross-section is

$$E_1 E_2 \frac{d\sigma^{\gamma_L^* A\to q\bar{q}X}}{d^3k_1 d^3k_2}\Bigg|_D$$

$$=\alpha_{em} e_q^2 Q^2 z^2 \bar{z}^2 \frac{C_f g^8 \bar{\mu}^4 \underline{S}}{(2\pi)^4} \int \frac{\mathrm{d}^2 \underline{q}}{(2\pi)^2} L(\underline{q}) L(\underline{k}_1 + \underline{k}_2 - \underline{q})$$

$$\times \left(\frac{1}{\epsilon_f^2 + (\underline{k}_1 - \underline{q})^2} - \frac{1}{\epsilon_f^2 + \underline{k}_1^2} + \frac{1}{\epsilon_f^2 + (\underline{k}_2 - \underline{q})^2} - \frac{1}{\epsilon_f^2 + \underline{k}_2^2} \right)^2 .$$

-

$$\alpha_{em} e_q^2 \frac{C_f g^8 \bar{\mu}^4 \underline{S}}{4(2\pi)^6} \int \frac{\mathrm{d}^2 \underline{q}}{(2\pi)^2} L(\underline{q}) L(\underline{k}_1 + \underline{k}_2 - \underline{q})$$

$$\Bigg[\left(\frac{1}{\epsilon_f^2 + \underline{k}_2^2} + \frac{1}{\epsilon_f^2 + \underline{k}_1^2} \right)^2 + \frac{1}{\epsilon_f^2 + (\underline{k}_1 - \underline{q})^2} \Bigg(\frac{1}{\epsilon_f^2 + (\underline{k}_1 - \underline{q})^2}$$

$$- 2\left(\frac{1}{\epsilon_f^2 + \underline{k}_1^2} + \frac{1}{\epsilon_f^2 + \underline{k}_2^2} \right) \Bigg) + \frac{1}{\epsilon_f^2 + (\underline{k}_2 - \underline{q})^2} \Bigg(\frac{1}{\epsilon_f^2 + (\underline{k}_2 - \underline{q})^2}$$

$$- 2\left(\frac{1}{\epsilon_f^2 + \underline{k}_1^2} + \frac{1}{\epsilon_f^2 + \underline{k}_2^2} \right) \Bigg) + \underbrace{\mathbf{2} \frac{\mathbf{1}}{\epsilon_{\mathbf{f}}^{\mathbf{2}} + (\underline{\mathbf{k}}_{\mathbf{1}} - \underline{\mathbf{q}})^{\mathbf{2}}} \frac{\mathbf{1}}{\epsilon_{\mathbf{f}}^{\mathbf{2}} + (\underline{\mathbf{k}}_{\mathbf{2}} - \underline{\mathbf{q}})^{\mathbf{2}}}}_{\text{Bose-enhanced}} \Bigg]$$

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Finite color neutralisation scale

- The sum of all source is color neutral.

$$\tilde{\rho}(k) = \int \mathrm{d}y e^{-ik.y} \rho(y) \xrightarrow{k=0} \int \mathrm{d}y \rho(y) = 0$$

- We introduce a color neutralisation scale.

$$\langle \rho^a(x^-, \underline{k}) \rho^b(y^-, \underline{k}') \rangle = \frac{\mu^2 k^2}{k^2 + m^2} \delta^{ab} \delta(x^- - y^-) \delta(\underline{k} + \underline{k}')$$

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Result for dilute approximation

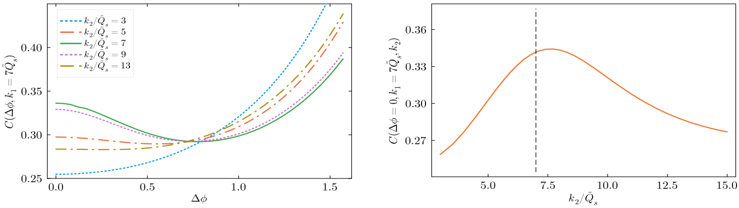


Figure: MV model with no color neutralization, $m \to 0$.

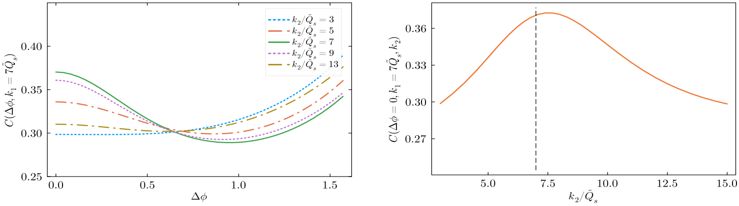
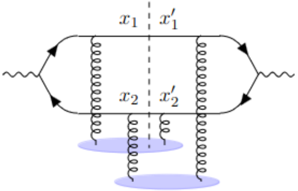


Figure: MV model with the color neutralization scale $m = \tilde{Q}_s$.

Why not Inclusive DIS?



- The term in the cross section describing the interaction of the dipole with nucleus is

$$\mathcal{N}_\mathrm{I} = 1 + \frac{1}{N_c}\mathrm{Tr}\langle V^\dagger(\underline{x}_2)V(\underline{x}_1)\left[V^\dagger(\underline{x}_2')V(\underline{x}_1')\right]^\dagger\rangle - \frac{1}{N_c}\mathrm{Tr}\langle V^\dagger(\underline{x}_2)V(\underline{x}_1)\rangle - \frac{1}{N_c}\mathrm{Tr}\langle V^\dagger(\underline{x}_2')V(\underline{x}_1')\rangle$$

- The term that gave rise to Bose enhancement in diffractive process was proportional to
  $L(\underline{x}_1 - \underline{x}_2')L(\underline{x}_2 - \underline{x}_1') \times \delta_{ab'}\delta_{ba'}\left[\frac{1}{N_c}\,\mathrm{tr}(t^a t^b)\right]\left[\frac{1}{N_c}\,\mathrm{tr}(t^{a'} t^{b'})\right]$.
  And,

$$\delta_{ab'}\delta_{ba'}\left[\frac{1}{N_c}\,\mathrm{tr}(t^a t^b)\right]\left[\frac{1}{N_c}\,\mathrm{tr}(t^{a'} t^{b'})\right] = \frac{C_f}{2N_c}$$

- The same combination of L contributes to the inclusive case as well, but with a different color factor

$$\delta_{ab'}\delta_{ba'}\left[\frac{1}{N_c}\,\mathrm{tr}(t^a t^{a'} t^{b'} t^b)\right] = -\frac{C_f}{2N_c}$$

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- As x decreases, no. of partons as well as the parton density increases.
- Taking the MV model as the initial point of the theory and then evolving it in x using the JIMWLK evolution equation.

$$\partial_Y W_Y = \frac{\alpha_s}{2} \int \mathrm{d}^2\underline{x}\,\mathrm{d}^2\underline{y} \frac{\delta^2}{\delta\alpha^a(x^-,\underline{x})\delta\alpha^b(y^-,\underline{y})}(\eta^{ab} W_Y) - \alpha_s \int \mathrm{d}^2\underline{x} \frac{\delta}{\delta\alpha^a(x^-,\underline{x})}(v^a_{\underline{x}} W_Y)$$

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Result for Beyond Dilute Approximaion and including small-x evolution

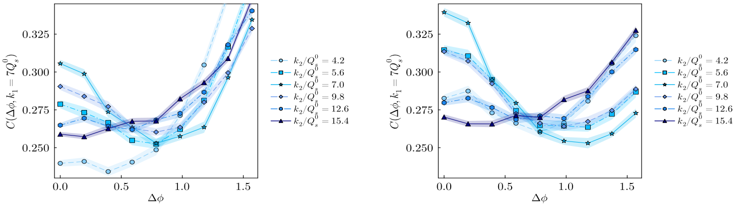

Figure: $\alpha_s Y = 0.0, \alpha_s Y = 0.4$

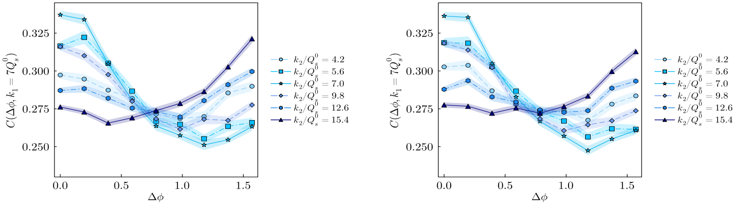

Figure: $\alpha_s Y = 0.8, \alpha_s Y = 1.0$

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Result contd...

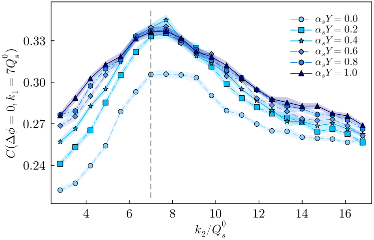

Figure: Incoherent Diffractive production for collinear configuration $\Delta\phi = 0$ as a function o f k_2: $k_1 = 7Q_s^0$, $\epsilon_f = 2Q_s^0$.

Conclusion

- In dilute approximation, for $m = 0$ as well as $m = Q_s$, the correlator has a maximum at zero angle as long as the momenta of the two gluons are close to each other.
- The color neutralization scale makes the maximum more robust.
- For inclusive dijet production there is a 'dip' from the Bose enhancement terms.
- At higher energies we see a similar outcome due to the fact that upon evolving in energy the theory naturally generates a color neutralisation scale.
- Evolving from $\alpha_s Y = 0$ to $\alpha_s Y = .4$ significantly increases the Bose enhancement signal.
- Further evolution to $\alpha_s Y = .8$ doesn't change the correlation.

THANK YOU!

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