TBA

Beyond-Eikonal Methods in High-Energy Scattering - Trento - May 24, 2024

~~TBA~~ $\longrightarrow$ TBC

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Looking for DGLAP in small-x Helicity-evolution

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Part 1: Setup of the discussion

A picture of the proton spin.

Spin sum rule (Jaffe Manohar decomposition [Nucl. Phys. B337, 509 (1990)])

Quark OAM
Gluon OAM

$$\frac{1}{2} = \frac{1}{2}\Sigma_q + \Sigma_g + L_q + L_g$$

Gluon spin
Quark spin
Proton spin

Possibilities:

- Gluon spin
- Quark/Gluon angular orbital momentum

Large and low x region. Experiments only access a finite range of x...

$$\Sigma_q = \int_0^1 \mathrm{d}x \left(q^{\uparrow}(x) - q^{\downarrow}(x)\right) \qquad (1)$$

Possibilities

- Large-x?
- Small-x?

At sub-eikonal order:

$$
\begin{aligned}
V_{\underline{x},\underline{y};\sigma',\sigma} &= V_{\underline{x}}\,\delta^2(\underline{x}-\underline{y})\,\delta_{\sigma,\sigma'} \\
&+ \frac{i\,P^+}{s}\int_{-\infty}^{\infty} dz^-\,d^2z\; V_{\underline{x}}[\infty,z^-]\,\delta^2(\underline{x}-\underline{z})\,\Big[-\delta_{\sigma,\sigma'}\overleftarrow{D}^i\,D^i + g\,\sigma\,\delta_{\sigma,\sigma'}\,F^{12}\Big](z^-,\underline{z})\,V_{\underline{y}}[z^-,-\infty]\,\delta^2(\underline{y}-\underline{z}) \\
&- \frac{g^2P^+}{2\,s}\delta^2(\underline{x}-\underline{y})\int_{-\infty}^{\infty} dz_1^-\int_{z_1^-}^{\infty} dz_2^-\,V_{\underline{x}}[\infty,z_2^-]\,t^b\,\psi_\beta(z_2^-,\underline{x})\,U_{\underline{x}}^{ba}[z_2^-,z_1^-]\,\Big[\delta_{\sigma,\sigma'}\,\gamma^+ - \sigma\,\delta_{\sigma,\sigma'}\,\gamma^+\gamma^5\Big]_{\alpha\beta} \\
&\qquad\times\bar{\psi}_\alpha(z_1^-,\underline{x})\,t^a\,V_{\underline{x}}[z_1^-,-\infty],
\end{aligned}
\tag{2}
$$

Remarks

- Blue $\longrightarrow$ Already used in previous V^{pol}. Label *of the first kind*; notation $V^{pol\,[1]}$. Proportional to $\sigma\delta_{\sigma\sigma'}$.
- Red $\longrightarrow$ "NEW" (in our framework). Label *of the second kind*; notation $V^{pol\,[2]}$. Proportional to $\delta_{\sigma\sigma'}$.

Picture?

For the quark S-matrix at sub eikonal order, see also:

- Balitsky and Tarasov, e.g. [1505.02151]
- Chirilli, e.g. [1807.11435]
- Altinoluk et al., e.g. [2012.03886]
- Kovchegov et al., e.g. [1808.09010] [2108.03667]

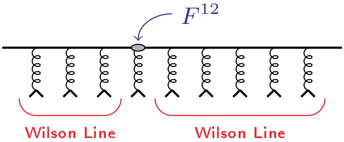
Wilson lines and eikonal expansion - Pictures!

Polarized WL,

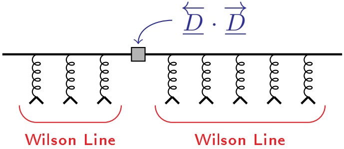
$$V_{\underline{x}}^{\mathrm{pol}[1]} = \underbrace{V_{\underline{x}}^{\mathrm{G}[1]} + V_{\underline{x}}^{\mathrm{q}[1]}}_{\sigma\,\delta_{\sigma\sigma'}}, \quad V_{\underline{x},\underline{y}}^{\mathrm{pol}[2]} = \underbrace{V_{\underline{x},\underline{y}}^{\mathrm{G}[2]} + V_{\underline{x}}^{\mathrm{q}[2]}\,\delta^2(\underline{x}-\underline{y})}_{\delta_{\sigma\sigma'}}\,.$$

can be represented as

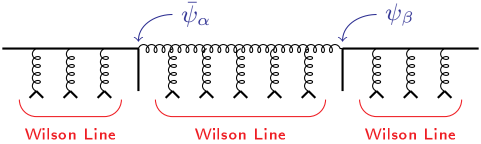
$V_{\underline{x}}^{G[1]} =$



$V_{\underline{x}}^{G[2]} =$



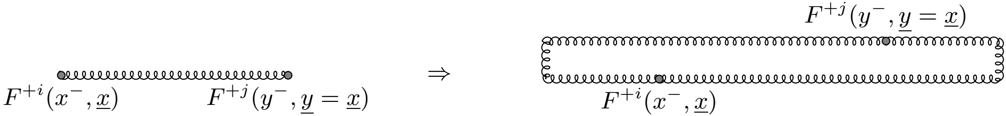
$V_{\underline{x}}^{q[1/2]} =$



Contraction with $(\gamma^+\gamma^5)_{\alpha\beta} \times \sigma\delta_{\sigma\sigma'}$ or $\gamma^+_{\alpha\beta} \times \delta_{\sigma\sigma'}$

From The Jaffe-Manohar (JM) gluon helicity PDF

$$\Delta G(x, Q^2) = \int^{Q^2} d^2k \, g_{1L}^{G\,dip}(x, k_T^2) = \frac{-2i}{x\,P^+} \frac{1}{4\pi} \frac{1}{2} \sum_{S_L} S_L \int_{-\infty}^{\infty} d\xi^- \, e^{ixP^+ \xi^-}$$
$$\times \langle P, S_L | \, \epsilon^{ij} \, F^{a+i}(0^+, 0^-, \underline{0}) \, U_{\underline{0}}^{ab}[0, \xi^-] \, F^{b+j}(0^+, \xi^-, \underline{0}) \, |P, S_L\rangle \, , \quad (3)$$



Gluon helicity

Identify after some algebra the dipole gluon helicity TMD

$$g_{1L}^{G\,dip}(x,k_T^2) = \frac{-2i}{x\,P^+\,V^-}\,\frac{1}{(2\pi)^3}\,\frac{1}{2}\sum_{S_L} S_L\ \langle P, S_L|\,\epsilon^{ij}\,\mathrm{tr}\Big[L^{i\,\dagger}(x,\underline{k})\,L^j(x,\underline{k})\Big]\,|P,S_L\rangle \tag{4}$$

where we define the Lipatov vertex:

$$L^j(x,\underline{k}) \equiv \int\limits_{-\infty}^{\infty} d\xi^-\,d^2\xi\,e^{ixP^+\xi^- - i\underline{k}\cdot\underline{\xi}}\ V_{\underline{\xi}}[\infty,\xi^-]\,(\partial^j A^+ + ixP^+ A^j)\,V_{\underline{\xi}}[\xi^-,-\infty] \tag{5}$$

Expanding the Lipatov vertex in Bjorken x

$$L^j(x,\underline{k}) = \int\limits_{-\infty}^{\infty} d\xi^-\,d^2\xi\,e^{-i\underline{k}\cdot\underline{\xi}}\ V_{\underline{\xi}}[\infty,\xi^-]\ \Big[\partial^j A^+ + ixP^+\Big(\xi^-\,\partial^j A^+ + A^j\Big) + \mathcal{O}(x^2)\Big]\ V_{\underline{\xi}}[\xi^-,-\infty], \tag{6}$$

which we can write

$$L^j(x,\underline{k}) = -\frac{k^j}{g}\int d^2\xi\,e^{-i\underline{k}\cdot\underline{\xi}}\,V_{\underline{\xi}} - \frac{xP^+}{2g}\int d^2\xi\,e^{-i\underline{k}\cdot\underline{\xi}}\int\limits_{-\infty}^{\infty} dz^-\,V_{\underline{\xi}}[\infty,z^-]\,\Big[D^j - \overleftarrow{D}^j\Big]\,V_{\underline{\xi}}[z^-,-\infty] \tag{7}$$

Gluon helicity

Performing the helicity dependent "CGC average"

$$g_{1L}^{G\,dip}(x,k_T^2) = \frac{-4i}{g^2\,(2\pi)^3}\,\epsilon^{ij}\,k^i \int d^2\zeta\,d^2\xi\,e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})}\,\underbrace{\left\langle\!\!\left\langle \text{tr}\left[V_{\underline{\zeta}}^{\dagger}\,V_{\underline{\xi}}^{j\;\mathrm{G}[2]} + \left(V_{\underline{\xi}}^{j\;\mathrm{G}[2]}\right)^{\dagger} V_{\underline{\zeta}}\right]\right\rangle\!\!\right\rangle}_{=2N_c G^j_{\underline{\xi},\underline{\zeta}}(zs)}, \quad (8)$$

with a polarized Wilson line of the second kind

$$V_{\underline{z}}^{i\;\mathrm{G}[2]} \equiv \frac{P^+}{2s}\int_{-\infty}^{\infty} dz^-\,V_{\underline{z}}[\infty,z^-]\left[D^i(z^-,\underline{z}) - \overleftarrow{D}^i(z^-,\underline{z})\right]V_{\underline{z}}[z^-,-\infty]. \quad (9)$$

$\Longrightarrow$ We call $G^j_{\underline{\xi},\underline{\zeta}}(zs)$ a Polarized dipole amplitude of the second kind.

I will also call it $\overleftrightarrow{D}$.

$\rightarrow$ Notice the link with the $\overleftarrow{\underline{D}}\cdot\overrightarrow{\underline{D}}$ operator!

DLA Small-x evolution at large N_c

$$G(x_{10}^2, zs) = G^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[\Gamma(x_{10}^2, x_{21}^2, z's) + 3\, G(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's) + 2\, \Gamma_2(x_{10}^2, x_{21}^2, z's) \Big], \tag{132a}$$

$$\Gamma(x_{10}^2, x_{21}^2, z's) = G^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z''s}}^{\min\left[x_{10}^2, x_{21}^2 \frac{z'}{z''}\right]} \frac{dx_{32}^2}{x_{32}^2} \Big[\Gamma(x_{10}^2, x_{32}^2, z''s) + 3\, G(x_{32}^2, z''s) + 2\, G_2(x_{32}^2, z''s) + 2\, \Gamma_2(x_{10}^2, x_{32}^2, z''s) \Big], \tag{132b}$$

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max\left[x_{10}^2, \frac{1}{z's}\right]}^{\min\left[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{21}^2}{x_{21}^2} \left[G(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's) \right], \tag{132c}$$

$$\Gamma_2(x_{10}^2, x_{21}^2, z's) = G_2^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int\limits_{\max\left[x_{10}^2, \frac{1}{z''s}\right]}^{\min\left[\frac{z'}{z''} x_{21}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{32}^2}{x_{32}^2} \left[G(x_{32}^2, z''s) + 2\, G_2(x_{32}^2, z''s) \right]. \tag{132d}$$

Recovering small-x pol DGLAP

Pol DGLAP splitting function at small-x is

$$\Delta P_{gg}(z) \to \frac{\alpha_s}{2\pi} 4N_c + \left(\frac{\alpha_s}{2\pi}\right)^2 4N_c^2 \ln^2 z + \left(\frac{\alpha_s}{2\pi}\right)^3 \frac{7}{3} N_c^3 \ln^4 z \tag{10}$$

From the large N_C equations, start evolution with

$$G^{(0)}(x_1 0^2, zs) = 0, \qquad G_2^{(0)}(x_1 0^2, zs) = 1 \tag{11}$$

iterate three times, one finds

$$\Delta G^{(3)}(x, Q^2) = \left(\frac{\alpha_s}{\pi}\right)^3 \left[\underbrace{\frac{7}{120} \ln^5\left(\frac{1}{x}\right) \ln\left(\frac{Q^2}{\Lambda^2}\right)}_{NNLO\ \ DGLAP_{gg}} + \frac{1}{6} \ln^4\left(\frac{1}{x}\right) \ln^2\left(\frac{Q^2}{\Lambda^2}\right) + \underbrace{\frac{2}{9} \ln^3\left(\frac{1}{x}\right) \ln^3\left(\frac{Q^2}{\Lambda^2}\right)}_{(LO)^3\ \ DGLAP_{gg}} \right]$$

using

$$1/x_{10}^2 \to Q^2, \qquad zs z_{10}^2 \to 1/x \tag{12}$$

Topic of the discussion

Recap

- Helicity evolution, mixing between helicity-dependent and helicity independent operators.
- Link: JM-operator $\epsilon_{ij} F^{+i} F^{+j} e^{ixP^+ z^-} \longrightarrow \overleftrightarrow{D}$ operator $\longleftarrow \underline{\overleftarrow{D}} \cdot \underline{\overrightarrow{D}}$ operator
- Small-x helicity evolution, contains small-x polarized DGLAP evolution: $\Delta P_{gg}(z)$ @ small-x

Discussion's topic Can we recover the full polarized DGLAP evolution with minimal modification to small-x evolution framework?

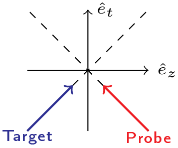
- Full $\Delta P_{gg}(z)$ instead of $\Delta P_{gg}(z \ll 1)$ at one loop?
- How do we iterate? (phase)

Part 2: Exploration

Ordering

Small-x evolution

- Strong ordering in k^-
- All $\underline{k}^2$ of the same order
- Strong lifetime (Ioffe-time) ordering



(IR)-**DGLAP evolution**

- Strong $\underline{k}^2$ ordering. Decrease from the projectile to the target!
- k^+ ordering
- Emission far from the initial probe

Back to the JM-operator

$$\Delta G(x,Q^2) = \int^{Q^2} d^2k\, g_{1L}^{G\,dip}(x,k_T^2) = \frac{-2i}{x\,P^+}\frac{1}{4\pi}\frac{1}{2}\sum_{S_L} S_L \int d\xi^-\, e^{ixP^+\xi^-}$$
$$\times \langle P, S_L|\, \epsilon^{ij}\, F^{a+i}(0^+,0^-,\underline{0})\, U_{\underline{0}}^{ab}[0,\xi^-]\, F^{b+j}(0^+,\xi^-,\underline{0})\,|P,S_L\rangle \tag{13}$$
$$\propto \int^{Q^2} \frac{\mathrm{d}^2\underline{k}}{(2\pi)^2}\int d\xi^- d\zeta^- \mathrm{d}^2\underline{\xi}\mathrm{d}^2\underline{\zeta}\; e^{ixP^+(\xi^--\zeta^-)}e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})}$$
$$\times \Big\langle\!\Big\langle \epsilon^{ij}\mathrm{tr}\,[\infty,\xi^-]F^{+i}(\xi)[\xi^-,-\infty][-\infty,\zeta^-]F^{+j}(\zeta)[\zeta^-,\infty]\Big\rangle\!\Big\rangle \tag{14}$$

Introduce the following L:

$$L^m(x,\underline{x}) \equiv \int dx^-\; e^{ixP^+x^-}\; [-\infty,x^-]_{\underline{x}}F^{+m}(x^-,\underline{x})[x^-,-\infty]_{\underline{x}} \tag{15}$$

such that

$$\Delta G(x,Q^2) = \int^{Q^2} đ^2\underline{k}\int \mathrm{d}^2\underline{\xi}\mathrm{d}^2\underline{\zeta}\; e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})}\epsilon_{ij}\langle\langle L^i(x,\underline{\xi})L^{\dagger j}(x,\underline{\zeta})\rangle\rangle \tag{16}$$

Splitting the fields

Consider the background field A_0^μ and split it in the following way

$$A_0^\mu \to A^\mu + a^\mu \tag{17}$$

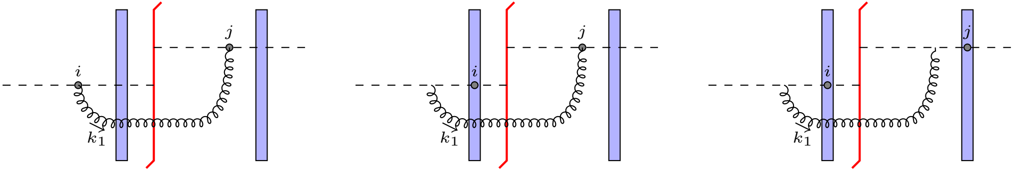
Plug into the expression for $L^i(x^-, \underline{x})$:

$$\begin{aligned} -L^i(x^-, \underline{x}) \supset\ & e^{ixP^+x^-} \left\{ \int dw^- [\infty, w^-]_{\underline{x}} a^+_{\underline{x}} [w^-, x^-]_{\underline{x}} (\check{\partial}^+_x A^i + \partial^i_x A^+)_{\underline{x}} [x^-, -\infty]_{\underline{x}} \theta(w^- - x^-) \right\} \\ & + e^{ixP^+x^-} \left\{ \int dw^- [\infty, x^-]_{\underline{x}} (\check{\partial}^+_x A^i + \partial^i_x A^+)_{\underline{x}} [x^-, w^-]_{\underline{x}} a^+_{\underline{x}} [w^-, -\infty]_{\underline{x}} \theta(x^- - w^-) \right\} \\ & + e^{ixP^+x^-} \left\{ [\infty, x^-]_{\underline{x}} (\check{\partial}^+_x a^i + \partial^i_x a^+)_{\underline{x}} [x^-, -\infty]_{\underline{x}} \right\} \end{aligned} \tag{18}$$

Emission from the WL + Emission from $F^{+\perp}$

One step of evolution

Topologies of interest



δ_{FF} δ_{FA} δ_{AA}

- Interaction of the new gluon at the blue rectangle is undefined at this point. (eikonal, sub-eikonal, next-to-eikonal, sub-to-next-to-eikonal, [...]-eikonal)
- Assuming a width $L^{-} \to 0^{-}$

Looking for polarized DGLAP ΔP_{gg}.

On the lookout for ΔP_{gg}

Consider only the topology δ_{FF}

$$\delta_{FF} = \epsilon_{ij} \int^{q^2} \text{đ}^2\underline{k} \int \mathrm{d}^2\underline{\xi}\mathrm{d}^2\underline{\zeta}\; e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} \int d\xi^- d\zeta^-\; e^{ixP^+(\xi^- - \zeta^-)}$$
$$\times\, \mathrm{tr}\,\{[\infty,\xi^-]_{\underline{\xi}}(\check{\partial}^+_\xi a^i + \partial^i_\xi a^+)[\xi^-,-\infty]_{\underline{\xi}}\}\,\{[\infty,\zeta^-]_{\underline{\zeta}}(\check{\partial}^+_\zeta a^j + \partial^j_\zeta a^+)[\zeta^-,-\infty]_{\underline{\zeta}}\}^\dagger \quad (19)$$

- Assume the interaction with the new gluon at the s.w. to be eikonal. $\longrightarrow$ Regular WL.
- Do not expand the phase $e^{ixP^+(\xi-\zeta)}$!
- Pick the $k^+_{1/2}$ propagator poles.
- Integrate longitudinal emission points to the 0^- of the s.w. (Energy denominators).

$$\delta_{FF} = -\epsilon_{ij} \int \frac{\text{đ}k^-}{2k^-} \int^{q^2} \text{đ}^2\underline{k} \int \mathrm{d}^2\underline{\xi}\mathrm{d}^2\underline{\zeta}\; e^{-i\underline{k}\cdot(\underline{\xi}-\underline{\zeta})} \int \mathrm{d}^2\underline{w}\; U^{ba}_{\underline{w}}\; \mathrm{tr}\,\{[\infty,-\infty]_{\underline{\xi}} t^a\}\,\{t^b[\infty,-\infty]_{\underline{\zeta}}\}^\dagger$$
$$\times \int \text{đ}^2\underline{k}_1 \text{đ}^2\underline{k}_2\; e^{+i\underline{k}_1(\underline{w}-\underline{\xi})+i\underline{k}_2(\underline{\zeta}-\underline{w})} \left\{ \frac{(xP^+\delta^{i\ell} + \frac{k_1^i k_1^\ell}{k^-})}{(xP^+ + \frac{k_1^2}{2k^-})} \frac{(xP^+\delta^{jk} + \frac{k_2^j k_2^k}{k^-})}{(xP^+ + \frac{k_2^2}{2k^-})} \right\} \delta^{\ell k} \quad (20)$$

On the lookout for ΔP_{gg}

To identify the contribution to ΔP_{gg}, act with $\epsilon_{ab}\nabla^a_{\underline{k}_1}\nabla^b_{\underline{k}_2}$ on the curly bracket, then set $\underline{k}_{1/2} \to k$

$$\epsilon_{ab}\nabla^a_{\underline{k}_1}\nabla^b_{\underline{k}_2}\{\cdots\} \longrightarrow \frac{-2\underline{k}^2}{(2xP^+ + \underline{k}^2)^2} + \frac{4\underline{k}^4}{(2xP^+ + \underline{k}^2)^3} \tag{21}$$

which is part of the kernel generating polarized DGLAP.

$\to$ Adding δ_{FA} and δ_{AA} produces the real piece of polarized DGLAP kernel.

Remarks

- To get $\epsilon_{ab}\nabla^a_{\underline{k}_1}\nabla^b_{\underline{k}_2}$ acting on the kernel, expand separations between WLs.
- The operators at this step will read $\epsilon_{nm}\underline{\partial}^n\underline{\partial}^m U \mathrm{tr}\{[\infty,-\infty]t^a\}\ \{t^b[\infty,-\infty]\}^\dagger$ which can be interpreted as the leading part of $\epsilon_{nm}F^{+n}F^{+m}$
- There is no associated phase depending on x_B anymore! (recall that we start with the JM-operator $\epsilon_{ij}F^{+i}F^{+j}e^{ixP^+z^-}$)
- Cannot further iterate this process...

Can we fix the lack of phase after one step of evolution?

On the lookout for the phase $e^{i\frac{x_B}{z_A}P^+z^-}$

Finding solace in the subeikonal correction...

A/ Promote the WL at the new position by adding the $\overleftarrow{\underline{D}} \cdot \overrightarrow{\underline{D}}$

$$V_{\underline{x},\underline{y};\sigma',\sigma} = V_{\underline{x}}\,\delta^2(\underline{x}-\underline{y})\,\delta_{\sigma,\sigma'} + \frac{i\,P^+}{s}\int_{-\infty}^{\infty} dz^-\,d^2z\;V_{\underline{x}}[\infty,z^-]\,\delta^2(\underline{x}-\underline{z})\,\left[-\delta_{\sigma,\sigma'}\overleftarrow{\underline{D}}\cdot\overrightarrow{\underline{D}}\right](z^-,\underline{z})\,V_{\underline{y}}[z^-,-\infty]\,\delta^2(\underline{y}-\underline{z}) + \cdots \tag{22}$$

- By parts and act with the derivatives outside of the WL to recover a factor $\underline{k}^2/2k^- \times L^-$ factoring the previous kernel in δ_{FF}

B/ Introduce a width of the s.w. L^-:

- After integration over $d\xi^-d\zeta^-$ recover an extra factor $e^{ixP^+L^-}$ which can be expanded into $1+ixP^+L^-$

On the lookout for the phase $e^{i\frac{x_B}{z_A}P^+z^-}$

Schematic form obtained for δ_{FF}:

$$\delta_{FF} \propto \left[1 + i\left(xP^+ + \frac{\underline{k}^2}{2k^-}\right)L^-\right] \times \epsilon_{ab}\nabla^a_{\underline{k}_1}\nabla^b_{\underline{k}_2}\{\cdots\} \otimes \underline{\partial}^n\underline{\partial}^m U \mathrm{tr}\{[\infty,-\infty]t^a\}\,\{t^b[\infty,-\infty]\}^\dagger$$

- Green part builds DGLAP kernel beyond the small-x limit.
- Blue part builds a phase $e^{i\frac{x_B}{z_A}P^+L^-}$ where $z_A \equiv \frac{xP^+}{xP^+ + \underline{k}^2/2k^-}$
- Red part is the eikonal part of the JM operator (without the phase)

Postulate Parts of the Sub^n-eikonal corrections resums into the JM operator.

Remains to understand how L^- becomes the separation between the two $F^{+\perp}$ insertions.
(@ sub-sub-eikonal?)

Summary and Outlooks

Summary

- Corrected small-x helicity-evolution involved mixing between unpolarized and polarized -operators
- Also contains small-x polarized DGLAP evolution.
  It involves the unpolarized operator $\underline{x} \times \overleftrightarrow{\underline{D}}$.
- Subeikonal effect allows the extraction of the action of the pol-DGLAP kernel exactly (i.e. the full dependence on x of ΔP_{gg}) acting on the eikonal part of the JM-operators
- Use subeikonal corrections to recover the phase of the JM operator.

Outlook

- Issue remains with the explicit dependence in x^- of the phase. At which order is it fixed?
- Check that the JM operators can be reconstructed beyond the eikonal part:
  $(\partial^i A^+)(\partial^j A^+) \to F^{+i} F^{+j}$
- Interface with the rest of the small-x evolution at SL_T and SL_L.

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Many thanks for the discussions during the week!