

# RG improved JIMWLK Hamiltonian: running coupling and DGLAP resummation

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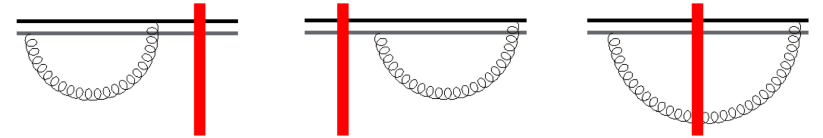
A. Kovner, M. Lublinsky, V. V. Skokov and Z. Zhao, arXiv:2308.15545 [hep-ph].

T. Altinoluk, G. Beuf, M. Lublinsky and V. V. Skokov, arXiv:2310.10738 [hep-ph]

## LO JIMWLK Hamiltonian

$$\mathcal{H}_{\text{LO}}^{\text{JIMWLK}} = \int_{\mathbf{x}, \mathbf{y}, \mathbf{z}} \mathbf{K}_{\text{LO}} \left\{ \mathbf{J}_{\text{L}}^{\text{a}}(\mathbf{x}) \mathbf{J}_{\text{L}}^{\text{a}}(\mathbf{y}) + \mathbf{J}_{\text{R}}^{\text{a}}(\mathbf{x}) \mathbf{J}_{\text{R}}^{\text{a}}(\mathbf{y}) - 2 \mathbf{J}_{\text{L}}^{\text{a}}(\mathbf{x}) \mathbf{S}_{\text{A}}^{\text{ab}}(\mathbf{z}) \mathbf{J}_{\text{R}}^{\text{b}}(\mathbf{y}) \right\}$$

$$\mathbf{K}_{\text{LO}}(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \frac{\alpha_s}{2\pi^2} \frac{(\mathbf{x} - \mathbf{z})_i (\mathbf{y} - \mathbf{z})_i}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{y} - \mathbf{z})^2} \equiv \frac{\alpha_s}{2\pi^2} \frac{\mathbf{X}_i \mathbf{Y}_i}{\mathbf{X}^2 \mathbf{Y}^2}$$



Here  $\mathbf{J}_{\text{L}}$  and  $\mathbf{J}_{\text{R}}$  are left and right  $\text{SU}(N)$  generators:

$$\mathbf{J}_{\text{L}}^{\text{a}}(\mathbf{x}) \mathbf{S}_{\text{A}}^{\text{ij}}(\mathbf{z}) = (\mathbf{T}^{\text{a}} \mathbf{S}_{\text{A}}(\mathbf{z}))^{\text{ij}} \delta^2(\mathbf{x} - \mathbf{z})$$

$$\mathbf{J}_{\text{R}}^{\text{a}}(\mathbf{x}) \mathbf{S}_{\text{A}}^{\text{ij}}(\mathbf{z}) = (\mathbf{S}_{\text{A}}(\mathbf{z}) \mathbf{T}^{\text{a}})^{\text{ij}} \delta^2(\mathbf{x} - \mathbf{z})$$

$$\mathcal{H}_{\text{LO}} = \frac{\alpha_s}{2} \int_{\mathbf{z}} \mathbf{Q}_i^{\text{a}}(\mathbf{z}) \mathbf{Q}_i^{\text{a}}(\mathbf{z}), \quad \mathbf{Q}_i^{\text{a}}(\mathbf{z}) = \frac{1}{\pi} \int_{\mathbf{x}} \frac{\mathbf{X}_i}{\mathbf{X}^2} [\mathbf{S}_{\text{A}}(\mathbf{x}) - \mathbf{S}_{\text{A}}(\mathbf{z})]^{\text{ab}} \mathbf{J}_{\text{R}}^{\text{b}}(\mathbf{x}).$$

**JIMWLK is valid for dilute-on-dense collisions only ( $Q_s^P \ll Q_s^T$ )**

# Stochastic formulation/Langevin

The rapidity evolution operator from  $\eta_0$  to  $\eta_1$  is

$$\mathcal{U}(\eta_0, \eta_1) = \mathcal{P} e^{-\int_{\eta_0}^{\eta_1} d\eta \mathcal{H}_{LO}} = \int \mathbf{D}\xi \mathcal{U}_\xi(\eta_0, \eta_1) e^{-\int_{\eta_0}^{\eta_1} d\eta \int_{\mathbf{z}} \frac{1}{2} \vec{\xi}^2(\eta, \mathbf{z})}$$

$$\mathcal{U}_\xi(\eta_0, \eta_1) = \mathcal{P}_\eta \exp \left\{ -i \int_{\eta_0}^{\eta_1} d\eta \int_{\mathbf{z}} \sqrt{\alpha_s} \mathbf{Q}_i^a(\mathbf{z}) \xi_i^a(\eta, \mathbf{z}) \right\}$$

for an infinitesimally small rapidity interval  $\Delta$

$$\mathcal{U}_\xi \equiv \mathcal{U}_\xi(\eta, \eta + \Delta) = \exp \left\{ -i\Delta \int_{\mathbf{z}} \sqrt{\alpha_s} \mathbf{Q}_i^a(\mathbf{z}) \xi_i^a(\eta, \mathbf{z}) \right\}$$

Rapidity evolution of any operator that is built out of Wilson lines, such as a color dipole, is performed in two steps: first one computes evolution of the Wilson lines on a fixed configuration of the noise and then averages the operator over the noise.

The success of the Langevin reformulation lies in the observation that  $\mathcal{H}_{LO}$  is quadratic in the operator  $\mathbf{Q}$ . The Hamiltonian  $\mathcal{H}_{LO}$  has all its eigenvalues non-negative.

## LO JIMWLK kernel beyond LO

$$\mathcal{H} = \int_{\mathbf{x}, \mathbf{y}, \mathbf{z}} \mathbf{K}(\mathbf{x}, \mathbf{y}; \mathbf{z}) \left[ \mathbf{J}_L^{\mathbf{a}}(\mathbf{x}) \mathbf{J}_L^{\mathbf{a}}(\mathbf{y}) + \mathbf{J}_R^{\mathbf{a}}(\mathbf{x}) \mathbf{J}_R^{\mathbf{a}}(\mathbf{y}) - 2\mathbf{J}_L^{\mathbf{a}}(\mathbf{x}) \mathbf{S}_A^{\mathbf{ab}}(\mathbf{z}) \mathbf{J}_R^{\mathbf{b}}(\mathbf{y}) \right]$$

**An effective kernel**  $\mathbf{K} = \mathbf{K}_{\text{LO}} + \mathbf{K}_{\text{NLO}} + \dots \sim \alpha_s(\# + \alpha_s(\# + \text{Logs}) + \dots)$

**Large transverse logarithms emerge at NLO. There are various types of large Logs - all have to be identified, clearly separated, and independently resummed.**

**Proper resummation requires understanding of physics beyond NLO!**

- **Running coupling effects (UV divergent) – rcJIMWLK:**

$$\mathbf{K}_{\text{LO}} = \frac{\alpha_s}{2\pi^2} \frac{\mathbf{XY}}{\mathbf{X}^2 \mathbf{Y}^2} \rightarrow \mathbf{K}_{\text{rc}} = \frac{\alpha_s[\text{running}]}{2\pi^2} \frac{\mathbf{XY}}{\mathbf{X}^2 \mathbf{Y}^2}$$

- **DGLAP logs: Large transverse logs of the  $\log(Q_s^{\text{T}}/Q_s^{\text{P}})$  type (dilute-on-dense).**

## rcJIMWLK kernel?

Action on a dipole  $S(\mathbf{u}, \mathbf{v})$  (BK equation):

$$\mathcal{H} S(\mathbf{u}, \mathbf{v}) = N_c \int_{\mathbf{z}} \mathbf{K}_{\text{dipole}}(\mathbf{u}, \mathbf{v}, \mathbf{z}) [S(\mathbf{u}, \mathbf{z}) S(\mathbf{z}, \mathbf{v}) - S(\mathbf{u}, \mathbf{v})]$$

$$\mathbf{K}_{\text{dipole}}(\mathbf{u}, \mathbf{v}, \mathbf{z}) = \mathbf{K}(\mathbf{u}, \mathbf{u}, \mathbf{z}) + \mathbf{K}(\mathbf{v}, \mathbf{v}, \mathbf{z}) - \mathbf{K}(\mathbf{u}, \mathbf{v}, \mathbf{z}) - \mathbf{K}(\mathbf{v}, \mathbf{u}, \mathbf{z})$$

$\mathbf{K}_{\text{rc}} \leftrightarrow$  **rcBK** (parent dipole or minimal daughter dipole, etc, not consistent with NLO)

$$\mathbf{K}^{\text{Bal}} = \frac{\alpha_s(\mathbf{X} - \mathbf{Y}) \mathbf{X} \cdot \mathbf{Y}}{2\pi^2 \mathbf{X}^2 \mathbf{Y}^2} + \frac{\alpha_s(\mathbf{X})}{4\pi^2} \frac{1}{\mathbf{X}^2} \left( 1 - \frac{\alpha_s(\mathbf{X} - \mathbf{Y})}{\alpha_s(\mathbf{Y})} \right) + \frac{\alpha_s(\mathbf{Y})}{4\pi^2} \frac{1}{\mathbf{Y}^2} \left( 1 - \frac{\alpha_s(\mathbf{X} - \mathbf{Y})}{\alpha_s(\mathbf{X})} \right)$$

$$\mathbf{K}^{\text{KW}} = \frac{\alpha_s(\mathbf{X}) \alpha_s(\mathbf{Y})}{\alpha_s(\mathbf{R}(\mathbf{X}, \mathbf{Y}))} \frac{1}{2\pi^2} \frac{\mathbf{X} \cdot \mathbf{Y}}{\mathbf{X}^2 \mathbf{Y}^2}, \quad \mathbf{R} = \mathbf{R}(\mathbf{X}, \mathbf{Y})$$

These results are problematic: what  $\alpha_s$  is doing in the denominators? why is the charge renormalization of the emitter at position  $x$  sensitive to position of another emitter at  $y$ ?

Any constraints on  $\mathbf{K}$ ?

## Positive semi-definiteness of the JIMWLK kernel

The Hamiltonian  $\mathcal{H}$  must have all its eigenvalues non-negative.

This is equivalent to the positive semi-definiteness of the kernel  $K$ .

$$\int d^2\mathbf{X} d^2\mathbf{Y} f(\mathbf{X}) K(\mathbf{X}, \mathbf{Y}) f(\mathbf{Y}) \geq 0 \quad \text{for any } f$$

$$A_1^2 K(\mathbf{X}_1, \mathbf{X}_1) + A_2^2 K(\mathbf{X}_2, \mathbf{X}_2) + 2A_1 A_2 K(\mathbf{X}_1, \mathbf{X}_2) \geq 0 \quad \text{for any } A_1 \text{ and } A_2.$$

$$K(\mathbf{X}, \mathbf{X}) + K(\mathbf{Y}, \mathbf{Y}) - 2K(\mathbf{X}, \mathbf{Y}) \geq 0 \quad \rightarrow \quad K_{\text{dipole}}(\mathbf{X}, \mathbf{Y}) \geq 0$$

$$K(\mathbf{X}, \mathbf{X}) + K(\mathbf{Y}, \mathbf{Y}) - K^2(\mathbf{X}, \mathbf{Y}) \geq 0$$

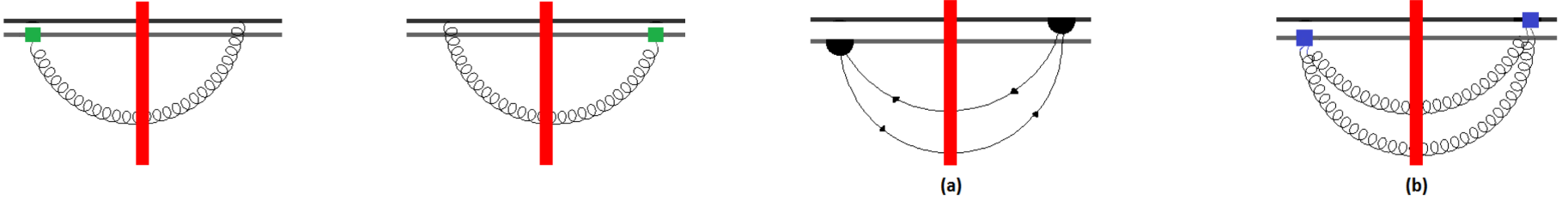
For any positive semidefinite kernel, it is possible to define a “square root”  $\Psi$

$$\int_{\mathbf{V}} \Psi_i(\mathbf{X}, \mathbf{V}) \Psi_i(\mathbf{Y}, \mathbf{V}) = K(\mathbf{X}, \mathbf{Y}).$$

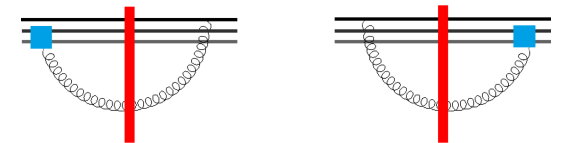
None of the rcJIMWLK prescriptions satisfy positive semi-definiteness!

# JIMWLK Hamiltonian @ NLO

Kovner, ML & Mulian (2013) based on Balitsky & Chirilli (2007), Grabovsky (2013); ML & Mulian (2016)



$$\begin{aligned}
 \mathcal{H}^{NLO \text{ JIMWLK}} = & \int_{x,y,z} K_{JSJ}(x,y;z) \left[ J_L^a(x) J_L^a(y) + J_R^a(x) J_R^a(y) - 2 J_L^a(x) S_A^{ab}(z) J_R^b(y) \right] \\
 & + \int_{x,y,z,z'} K_{JSSJ}(x,y;z,z') \left[ f^{abc} f^{def} J_L^a(x) S_A^{be}(z) S_A^{cf}(z') J_R^d(y) - N_c J_L^a(x) S_A^{ab}(z) J_R^b(y) \right] \\
 & + \int_{x,y,z,z'} K_{q\bar{q}}(x,y;z,z') \left[ 2 J_L^a(x) \text{tr}[S_F^\dagger(z) t^a S_F(z') t^b] J_R^b(y) - J_L^a(x) S_A^{ab}(z) J_R^b(y) \right] \\
 & + \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z') f^{acb} \left[ J_L^d(x) J_L^e(y) S_A^{dc}(z) S_A^{eb}(z') J_R^a(w) - J_L^a(w) S_A^{cd}(z) S_A^{be}(z') J_R^d(x) J_R^e(y) \right] \\
 & + \int_{w,x,y,z} K_{JJSJ}(w;x,y;z) f^{bde} \left[ J_L^d(x) J_L^e(y) S_A^{ba}(z) J_R^a(w) - J_L^a(w) S_A^{ab}(z) J_R^d(x) J_R^e(y) \right] \\
 & + \int_{w,x,y} K_{JJJ}(w;x,y) f^{deb} \left[ J_L^d(x) J_L^e(y) J_L^b(w) - J_R^d(x) J_R^e(y) J_R^b(w) \right].
 \end{aligned}$$



# NLO Kernels (Large UV Logs only)

$$\begin{aligned} \mathbf{X} &= \mathbf{x} - \mathbf{z} \\ \mathbf{Y} &= \mathbf{y} - \mathbf{z} \end{aligned}$$

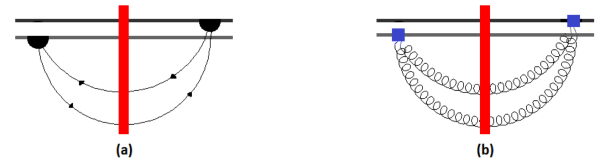
$$\mathbf{K}_{JSJ}(\text{b terms}) = \frac{\alpha_s^2}{16\pi^3} \left\{ -\mathbf{b} \frac{(\mathbf{x} - \mathbf{y})^2}{\mathbf{X}^2 \mathbf{Y}^2} \ln(\mathbf{x} - \mathbf{y})^2 \mu^2 + \frac{\mathbf{b}}{\mathbf{X}^2} \ln \mathbf{Y}^2 \mu^2 + \frac{\mathbf{b}}{\mathbf{Y}^2} \ln \mathbf{X}^2 \mu^2 \right\} + \dots$$

Here  $\mu$  is the normalization point,  $\mathbf{b} = \frac{11}{3}\mathbf{N}_c - \frac{2}{3}\mathbf{n}_f$ ,  $\mathbf{b} \ln Q^2/\mu^2 \rightarrow \alpha_s(Q^2)$   
 Huge ambiguity in identifying  $Q$ .  $K_{JSJ}$  is not positive semi-definite

Resumm large Logs into an effective kernel  $\mathbf{K} = \mathbf{K}_{LO} + \mathbf{K}_{JSJ} + \dots$

$$\int_{\mathbf{x} \mathbf{y} \mathbf{z}, \mathbf{z}'} \mathbf{K}_{JSSJ}(\mathbf{x}, \mathbf{y}; \mathbf{z}, \mathbf{z}') \mathbf{J}_L^a(\mathbf{x}) \mathbf{J}_R^b(\mathbf{y}) \left[ \mathbf{D}^{ab}(\mathbf{z}, \mathbf{z}') \right] \sim \mathbf{b} \times (\text{UV divergent Log})$$

$$\mathbf{D}^{ab}(\mathbf{z}, \mathbf{z}') \equiv \text{Tr}[\mathbf{T}^a \mathbf{S}_A(\mathbf{z}) \mathbf{T}^b \mathbf{S}_A^+(\mathbf{z}')]$$



The UV divergence in  $JSSJ$  is trivial: when the two gluons are too close to each other ( $z \sim z'$ ), they cannot be resolved by the target and hence should be counted as a single gluon scattering. We are thus prompted to introduce a "resolution scale"  $Q$



## Dressed Wilson line

Within the finite resolution  $Q$  bare gluons  $\rightarrow$  dressed gluons,  
bare Wilson lines  $\rightarrow$  *dressed Wilson lines*,  $S \rightarrow S_Q$

$$S_Q^{\text{ab}}(\mathbf{z}) = S_A^{\text{ab}}(\mathbf{z}) + \frac{\alpha_s}{2\pi^2} \int_0^1 d\xi \sigma(\xi) \int^{\mathbf{Q}^{-1}} \frac{d^2\mathbf{Z}}{Z^2} \left( D^{\text{ab}}(\mathbf{z} + (1-\xi)\mathbf{Z}, \mathbf{z} - \xi\mathbf{Z}) - N_c S_A^{\text{ab}}(\mathbf{z}) \right)$$

$\xi$  is the fraction of longitudinal momentum carried by one of the gluons.

$$\sigma(\xi) = \left[ \frac{1}{\xi(1-\xi)} \left( \xi^2 + (1-\xi)^2 + \xi^2(1-\xi)^2 \right) \right]_+ ; \quad 2N_c \int_0^1 d\xi \sigma(\xi) = -\frac{11N_c}{3} \rightarrow -b$$

This is a  $P_{gg}$  splitting function except that we introduce the "+" prescription both for  $\xi = 1$  and  $\xi = 0$  poles. The "+" prescription emerges from the  $1/\xi$  subtraction absorbed into  $(\text{LO})^2$  part of the evolution.

The sign is negative – correcting for the over-subtraction in the LO.

We go beyond the usual DGLAP: we allow simultaneous scattering of all gluons.

For  $Q > Q_s^T$ ,  $S_Q \simeq S_A$  - the target does not resolve gluon splitting at distances smaller than  $1/Q_s^T$ .

## Resolution scale and the running coupling

Express  $S$  in terms of  $S_Q$  and substitute it into the LO+NLO JIMWLK Hamiltonian.  $\mathcal{H}[S] \rightarrow \mathcal{H}[S_Q]$ . The Hamiltonian will feature  $\ln Q^2$  terms such as  $\ln(Q^2 X^2)$ .

$$\mathbf{K} = \mathbf{K}_{\text{LO}} \left( 1 + \frac{\alpha_s}{4\pi} \mathbf{b} (\ln X^2 \mu^2 + \ln Y^2 \mu^2 - \ln Q^{-2} \mu^2) \right) + \text{other } O(\alpha_s^2) \text{ terms}$$

We assume existence of a typical scale  $Q_s^P \ll Q_s^T$  associated with the projectile, such that  $\ln(Q_s^P X^2)$  are small. The UV finite parts of the Hamiltonian proportional to  $\mathbf{b}$  do not have any large Logs

$$\mathbf{K}_{\text{in}} = \mathbf{K}(Q = Q_s^P) = \frac{\sqrt{\alpha_s(\mathbf{X}) \alpha_s(\mathbf{Y})}}{2\pi^2} \frac{\mathbf{X}\mathbf{Y}}{\mathbf{X}^2 \mathbf{Y}^2} \left[ 1 + \frac{\alpha_s}{8\pi} \mathbf{b} (\text{small logs}) \right]$$

$\sqrt{\alpha_s(\mathbf{X}) \alpha_s(\mathbf{Y})}$  is a positive semi-definite prescription for rcJIMWLK.

Has been already used in Langevin

However, at  $Q = Q_s^P$ ,  $S_Q$  is very different from  $S_A$ ,  $S_Q \sim S_A [1 + \alpha_s \# \text{Log}(Q^2/Q_s^T)]$ . This large Log has to be resummed via inclusion of multiple consecutive DGLAP splittings:

$$\frac{\partial S_Q(\mathbf{z})}{\partial \ln Q} = -\frac{\alpha_s}{2\pi^2} \int_{\xi} \sigma(\xi) \int_{\phi_Q} [\mathbf{D}_Q(\mathbf{z}) - \mathbf{N}_c S_Q(\mathbf{z})]$$

$$\mathbf{D}_Q(\mathbf{z}_1, \mathbf{z}_2) \equiv \text{Tr}[\mathbf{T}^a S_Q(\mathbf{z}_1) \mathbf{T}^b S_Q^+(\mathbf{z}_2)]$$

If we were to take  $Q = Q_s^T$  then  $S_Q \simeq S_A$  but the  $\ln Q^2$  terms in the Hamiltonian would be large and have to be resummed.

Either way, we have to resum large logs of the order  $\log Q_s^T/Q_s^P$ .

## Functional RG

The resummed Hamiltonian should be  $Q$ -independent:

$$\frac{d\mathcal{H}}{d \ln Q} = \frac{\partial \mathcal{H}}{\partial \ln Q} + \int_{\mathbf{u}} \left[ \frac{\delta \mathcal{H}}{\delta \mathbf{S}_Q(\mathbf{u})} \frac{\partial \mathbf{S}_Q(\mathbf{u})}{\partial \ln Q} \right] = 0$$

DGLAP-like evolution for the Hamiltonian (evolution in the space of Hamiltonians):

$$\mathcal{H}[\mathbf{Q}_s^P] = \text{Exp} \left[ \int_{\mathbf{Q}_s^P}^{\mathbf{Q}_s^T} \frac{d\mathbf{Q}}{Q} \mathbf{H}_{\text{DGLAP}} \right] \mathcal{H}_{\text{in}}$$

$$\mathbf{H}_{\text{DGLAP}} = \frac{\alpha_s}{2\pi^2} \int_{\mathbf{u}} \int_{\xi} \sigma(\xi) \int_{\phi_Q} \text{Tr} \left( [\mathbf{D}_Q(\mathbf{u}) - \mathbf{N}_c \mathbf{S}_Q(\mathbf{u})] \frac{\delta}{\delta \mathbf{S}_Q(\mathbf{u})} \right)$$

$\mathbf{Q}_s^P = \mathbf{Q}_s^P(\eta)$  –  $Q_s^P$  is dynamical (rapidity dependent);  
hence the resummed Hamiltonian is too.

## Weak target field approximation – linearization

$$S_Q^{ab} = \delta^{ab} + f^{abc} \alpha_Q^c; \quad D_Q^{ab}(z_1, z_2) = N_c \left( \delta^{ab} + \frac{1}{2} f^{abc} \left[ \alpha_Q^c(z_1) + (\alpha_Q^c(z_2))^* \right] \right)$$

Expand the Hamiltonian (BFKL-like)

$$\mathcal{H} = \int_{\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{z}'} \mathbf{K}_Q(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{z}') (\alpha_Q(\mathbf{x}) - \alpha_Q(\mathbf{z}))^a (\alpha_Q(\mathbf{y}) - \alpha_Q(\mathbf{z}'))^b \frac{\delta}{\delta \alpha_Q^a(\mathbf{x})} \frac{\delta}{\delta \alpha_Q^b(\mathbf{y})}$$

$$\mathcal{H} = \int_{\mathbf{q}_1, \mathbf{q}_2, \mathbf{p}_1, \mathbf{p}_2} \mathbf{K}_Q(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q}_1, \mathbf{q}_2) \alpha_Q^a(\mathbf{q}_1) \alpha_Q^b(\mathbf{q}_2) \frac{\delta}{\delta \alpha_Q^a(\mathbf{p}_1)} \frac{\delta}{\delta \alpha_Q^b(\mathbf{p}_2)}$$

$$\mathbf{H}_{\text{DGLAP}} \sim \alpha_Q \frac{\delta}{\delta \alpha_Q}$$

$\mathbf{H}_{\text{DGLAP}}$  is homogeneous

The DGLAP-like RG evolution becomes a closed equation for the kernel  $K_Q$ .

$$\mathbf{K}_Q(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q}_1, \mathbf{q}_2) = \mathbf{Exp} \left( \int_{Q_S^P}^{Q_S^T} \frac{dQ}{Q} (\mathbf{R}(\mathbf{q}_1, Q) + \mathbf{R}(\mathbf{q}_2, Q)) \right) \mathbf{K}_{in}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{q}_1, \mathbf{q}_2)$$

Here  $\mathbf{K}_{in}$  is the non-forward BFKL kernel, which is multiplicatively "renormalized"

$$\mathbf{R}(\mathbf{q}, Q) = \frac{\alpha_s \mathbf{N}_c}{2\pi^2} \int d\xi \sigma(\xi) \int d\phi \left( e^{i(1-\xi)Q^{-1}\vec{q}\cdot\vec{e}_\phi} - 1 \right)$$

Alternatively,

$$\alpha_Q(\mathbf{z}) \approx \left( \frac{Q_T}{Q} \right)^{\frac{\alpha_s b}{2\pi}} \alpha(\mathbf{z}) + \int_{1/Q_T < |\mathbf{x}-\mathbf{z}| < 1/Q} d^2\mathbf{x} \frac{\alpha_s b Q^2}{4\pi \pi} \left[ 1 - \left( \frac{1}{Q^2(\mathbf{x}-\mathbf{z})^2} \right)^{1+\frac{\alpha_s b}{4\pi}} \right] \alpha(\mathbf{x})$$

For small distances, the WW fields get smeared

## Saturation region

$$\mathbf{H}_{\text{DGLAP}} = \frac{\alpha_s}{2\pi^2} \int_{\mathbf{u}} \int_{\xi} \sigma(\xi) \int_{\phi_Q} \text{Tr} \left( [\mathbf{D}_Q(\mathbf{u}) - \mathbf{N}_c \mathbf{S}_Q(\mathbf{u})] \frac{\delta}{\delta \mathbf{S}_Q(\mathbf{u})} \right)$$

Since  $|z_1 - z_2| = 1/Q > 1/Q_s^T$ , the two gluons are well separated and outside the correlation region in the target (in the sense of averaging over the target).

$\mathbf{D}_Q \ll \mathbf{N}_c \mathbf{S}_Q \rightarrow \mathbf{H}_{\text{DGLAP}}$  is again homogeneous

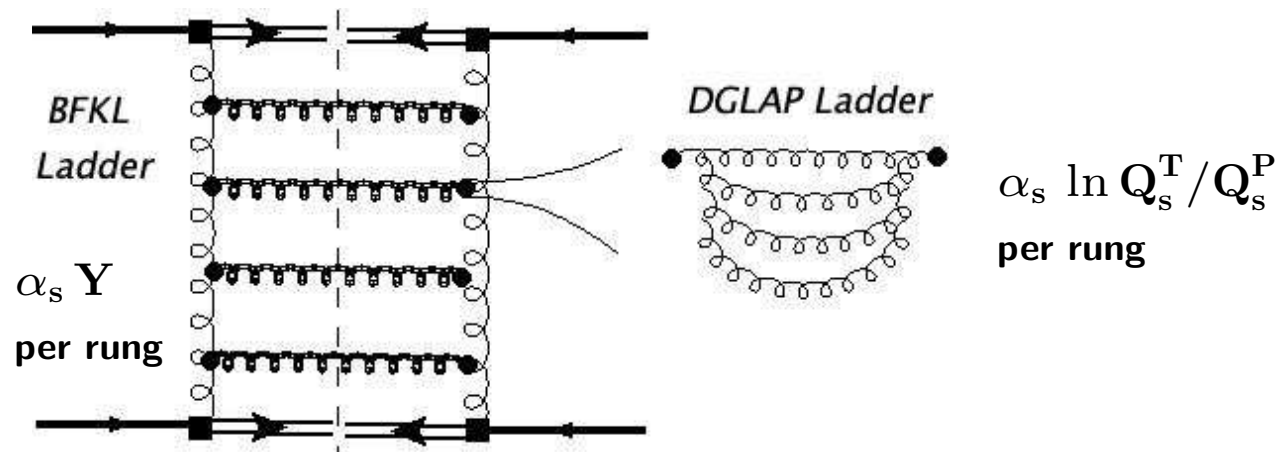
$$\frac{\partial S_Q(z)}{\partial \ln Q^2} = \frac{\alpha_s N_c}{\pi} \int_{\xi} \sigma(\xi) S_Q(z)$$

$$\mathbf{S}_Q = \left[ \frac{Q^T}{Q_s} \right]^{\frac{\alpha_s b}{2\pi}} \mathbf{S}$$

## Summary/Outlook

- DGLAP-like resummation inside the JIMWLK Hamiltonian has been performed. These DGLAP corrections are large whenever there is a large disparity between the correlation lengths (or saturation momenta) in the projectile and the target. This is precisely JIMWLK's regime of validity.

The result is a smearing of the WW fields within the  $1/Q_s^T$  distance



- rcJIMWLK emerges with the scale choice for the running coupling:

$$\mathbf{K} \sim \sqrt{\alpha_s(\mathbf{X})\alpha_s(\mathbf{Y})}$$



# "High energy QCD: from the LHC to the EIC"

**When:** 2025, Aug 4 - Aug 15

**Where:** Centro de Ciencias de Benasque Pedro Pascual



**Organizers:**

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N. Armesto (Universidade de Santiago de Compostela, Spain),  
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