Diffractive jet production in photon-nucleus interactions

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Beyond-Eikonal Methods in High-Energy Scattering

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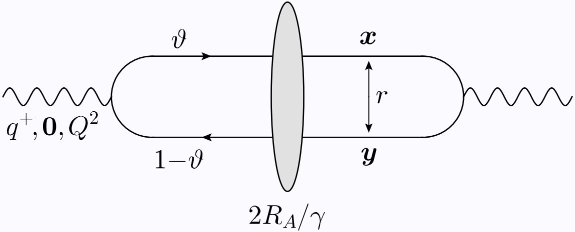
- S. Hauksson, E. Iancu, A.H. Mueller, DT, S.Y. Wei: 2402.14748 (JHEP) [and 2304.12401 (EPJC), 2207.06268 (JHEP), 2112.06353 (PRL)]
- B. Rodriguez-Aguilar, DT, S.Y. Wei: 2302.01106 (PRD), 240n.nnnnn

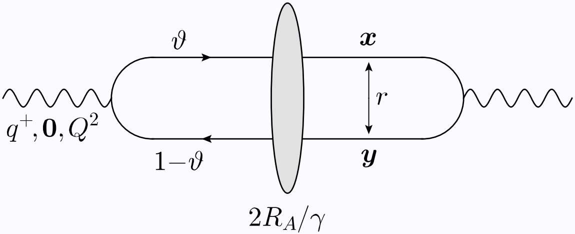
Content

- Deep inelastic scattering in the dipole picture
- “$2+1$” jets in coherent diffraction as probes of saturation
- Factorization: Gluon and quark diffractive TMDs
- DTMDs in SIDIS
- 2 and “$2+1$” jets in incoherent diffraction



- Right moving off-shell γ^*, $q^\mu = (q^+, \mathbf{0}, -Q^2/2q^+)$
- Left moving nucleus, $p^\mu = (M_N^2/2P_N^-, \mathbf{0}, P_N^-)$ per nucleon
- Projectile lifetime $\tau \sim 2q^+/Q^2$
- Nucleus contracted length $L \sim 2R_A M_N/P_N^- \sim A^{1/3}/P_N^-$
- $L \ll \tau \Longleftrightarrow xA^{1/3} \ll 1$

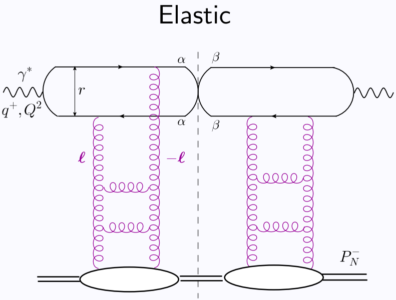
DIS at Small-x in Dipole Picture: Factorization



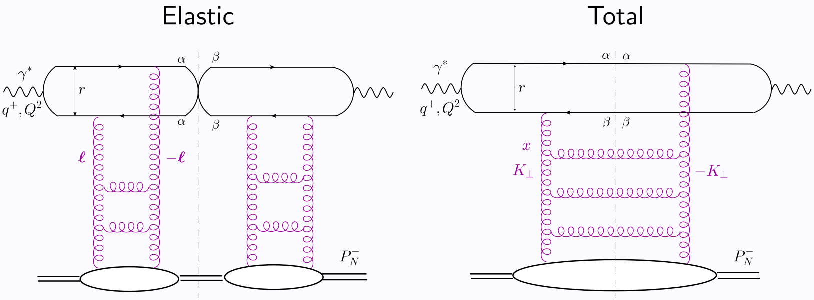
$$\sigma^{\gamma^* A}(x, Q^2) = \int \mathrm{d}^2\boldsymbol{r} \int_0^1 \mathrm{d}\vartheta \left|\Psi_{\gamma^*\to q\bar{q}}(Q^2; \boldsymbol{r}, \vartheta)\right|^2 2\pi R_A^2 T(r, x)$$

- All QCD dynamics in $T(r, x)$
- Virtuality limits large dipoles: $r \lesssim 1/\bar{Q}$, with $\bar{Q}^2 = \vartheta(1-\vartheta)Q^2$
- Saturation requires $r \gtrsim 1/Q_s$, hence $\bar{Q}^2 \lesssim Q_s^2$
- When $Q^2 \gg Q_s^2$ dominant contribution from weak scattering

Diffraction/Elastic Scattering



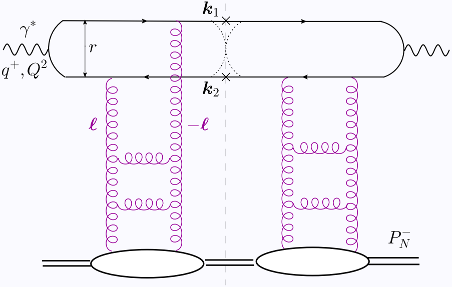
- Rapidity gap: wide angular region void of particles
- Elastic for projectile, no nuclear break-up (coherent reaction)
- Close color at amplitude level
- At least two gluons exchanged at amplitude



$$\sigma_{\rm D}^{\gamma^* A}(x, Q^2) = \int {\rm d}^2\boldsymbol{r} \int_0^1 {\rm d}\vartheta \left|\Psi_{\gamma^*\to q\bar{q}}(Q^2; \boldsymbol{r}, \vartheta)\right|^2 \pi R_A^2 [T(r,x)]^2$$

- T^2 : Diffractive cross section less sensitive to small dipoles
- Even for $Q^2 \gg Q_s^2$ dipoles with $r \gtrsim 1/Q_s$ and $\vartheta \sim Q_s^2/Q^2 \ll 1$ ("aligned jets") dominate diffractive cross section

- More exclusive processes? Measured jets or hadrons?
- Hard scale sets dipole size $r \sim 1/P_\perp$, weak scattering
- Hard, symmetric, back to back $q\bar{q}$ pair:
  $k_{1\perp} \simeq k_{2\perp} \equiv P_\perp \sim Q \gg Q_s$, $\vartheta_{1,2} \sim 1/2$

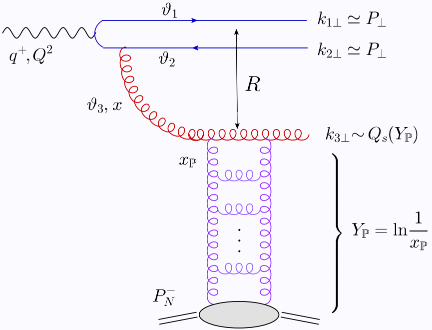


$$\frac{\mathrm{d}\sigma_{\mathrm{D}}^{\gamma^* A \to q\bar{q}A}}{\mathrm{d}\vartheta_1 \mathrm{d}^2\boldsymbol{P}} \propto \pi R_A^2 \underbrace{\frac{1}{Q^2}}_{\gamma^* q\bar{q}} \underbrace{\frac{Q_s^4}{P_\perp^4}}_{T^2(r)}$$

$$\sim \frac{1}{P_\perp^6} \text{ higher twist}$$

2+1 Jets in Diffraction

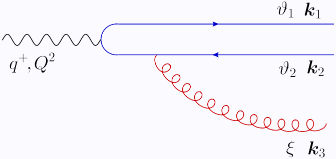
- Diffractive dijet at leading twist $1/P_\perp^4$?
- Yes, two hard jets $P_\perp \gg Q_s$ and one semi-hard $k_{3\perp} \sim Q_s \ll P_\perp$
- Third, semi-hard, jet provides dijet imbalance

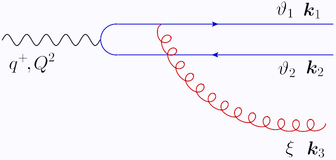


- $\mathcal{O}(\alpha_s)$ suppression
- $R \gg r$: gluon dipole
- $T_g(R, Y_{\mathbb{P}}) \simeq \mathcal{O}(1)$
- Hard factor

$$H \propto \underbrace{\frac{1}{Q^2}}_{\gamma^* q\bar{q}} \times \underbrace{r^2}_{\text{gluon emission}} \sim \frac{1}{P_\perp^4}$$

- $x_{\mathbb{P}} P_N^-$ puts trijet on-shell: $x_{\mathbb{P}} \simeq \frac{1}{2q^+ P_N^-}\left(\frac{P_\perp^2}{\vartheta_1 \vartheta_2} + \frac{k_{3\perp}^2}{\vartheta_3} + Q^2\right)$





- Gluon formation time must be small enough:
  $k_3^+/k_{3\perp}^2 \lesssim q^+/Q^2 \rightsquigarrow \vartheta_3 \lesssim k_{3\perp}^2/P_\perp^2 \ll 1$, gluon is soft
- Momentum space LCWF

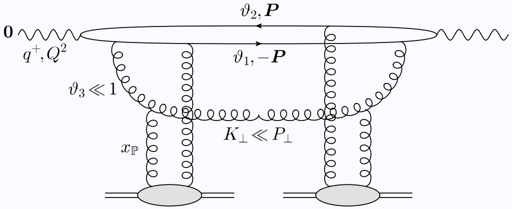
$$\left[\frac{k_1^l\left(k_3^j + \frac{\xi}{1-\vartheta_1}\,k_1^j\right)}{k_{1\perp}^2+\bar{Q}^2} + \frac{k_2^l\left(k_3^j + \frac{\xi}{1-\vartheta_2}\,k_2^j\right)}{k_{2\perp}^2+\bar{Q}^2}\right]\frac{1}{k_{3\perp}^2+\xi\left(\frac{k_{1\perp}^2}{\vartheta_1}+\frac{k_{2\perp}^2}{\vartheta_2}+Q^2\right)}$$

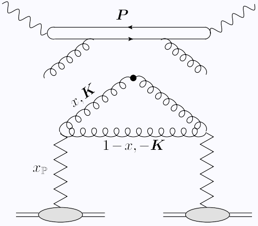
- Expand for $k_{3\perp} \ll P_\perp$ and $\xi \ll k_{3\perp}/P_\perp$ (no recoil)
- Leading terms cancel $\rightsquigarrow$ Non-eikonal emission
- Scattering is eikonal (Wilson lines)
- Add instantaneous quark propagator graph

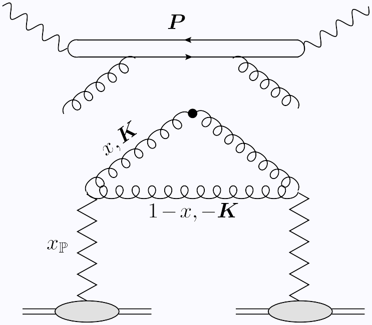
- Scales separation ⇝ Factorization?
- View gluon as part of Pomeron. Variable change from ξ to x:

$$x = \frac{x_{q\bar{q}}}{x_{\mathbb{P}}} = \frac{\frac{P_\perp^2}{\vartheta_1\vartheta_2} + Q^2}{\frac{P_\perp^2}{\vartheta_1\vartheta_2} + \frac{k_{3\perp}^2}{\vartheta_3} + Q^2} \quad \text{or} \quad x = \beta\,\frac{x_{q\bar{q}}}{x_{\rm Bj}} \simeq \beta\,\frac{\bar{Q}^2 + P_\perp^2}{P_\perp^2}$$

- For given $x_{\rm Bj}$ and hard jets, only one of ξ, $x_{\mathbb{P}}$ and x is independnent







$$\frac{\mathrm{d}\sigma_{\mathrm{D}}^{\gamma^*_{T,L}A\to q\bar{q}gA}}{\mathrm{d}\vartheta_1\mathrm{d}\vartheta_2\mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{K}\mathrm{d}Y_{\mathbb{P}}} = H_{T,L}(\vartheta_1,\vartheta_2,Q^2,P_\perp^2)\,\frac{\mathrm{d}xG_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}}$$

- Hard factor as in inclusive $q\bar{q}$ dijet cross section

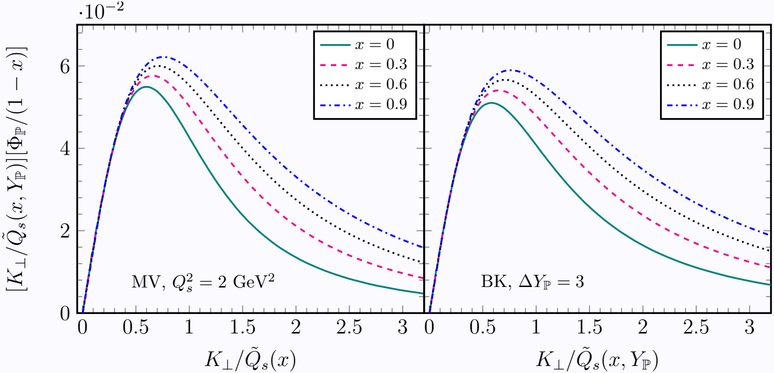
$$H_T(\vartheta_1,\vartheta_2,Q^2,P_\perp^2) \equiv \alpha_{em}\alpha_s\Big(\sum e_f^2\Big)\,\delta_\vartheta\,\underbrace{(\vartheta_1^2+\vartheta_2^2)}_{2P_{q\gamma}(\vartheta_1)}\,\underbrace{\frac{P_\perp^4+\bar{Q}^4}{(P_\perp^2+\bar{Q}^2)^4}}_{\sim 1/P_\perp^4}$$

$$\frac{\mathrm{d}xG_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} = \underbrace{\frac{S_\perp(N_c^2-1)}{4\pi^3}}_{\text{d.o.f.}}\ \underbrace{\Phi_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}_{\text{occupation number}}$$

- Explicit in terms of elastic amplitude $T_g(R,x_{\mathbb{P}})$

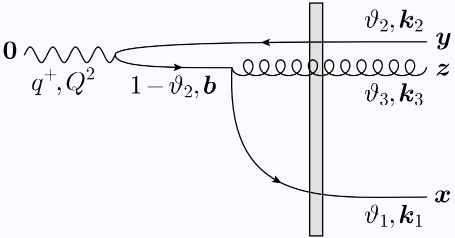
$$\Phi_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2) \approx \frac{1-x}{2\pi}\begin{cases} 1 & \text{for} \quad K_\perp \ll \tilde{Q}_s(x) \\ \dfrac{\tilde{Q}_s^4(x,Y_{\mathbb{P}})}{K_\perp^4} & \text{for} \quad K_\perp \gg \tilde{Q}_s(x) \end{cases}$$

- Valid for large gaps: $x_{\mathbb{P}} \lesssim 10^{-2}$
- Effective saturation momentum $\tilde{Q}_s^2(x) \equiv (1-x)Q_s^2$
- Bulk of distribution at saturation $K_\perp \ll \tilde{Q}_s(x)$



- Multiplied by $K_\perp$ (cf. measure $\mathrm{d}^2\boldsymbol{K}$)
- Pronounced maximum at $K_\perp \sim \tilde{Q}_s(x)$

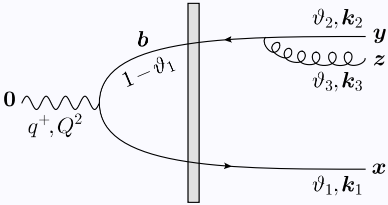
- Hard $\bar{q}$ and g, $P_\perp \gg Q_s$, semi-hard q, $k_{1\perp} \sim Q_s \ll P_\perp$

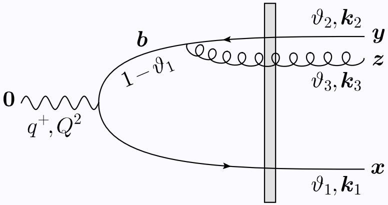


- $\mathcal{O}(\alpha_s)$ suppression
- Large quark dipole
- $T(R, Y_{\mathbb{P}}) \simeq \mathcal{O}(1)$
- Hard factor $\sim 1/P_\perp^4$

- (NB: Scattering before emission is small, like in soft g case)
- Quark must be soft $\vartheta_1 \lesssim k_{1\perp}^2/P_\perp^2$

Another Configuration with a Soft Quark

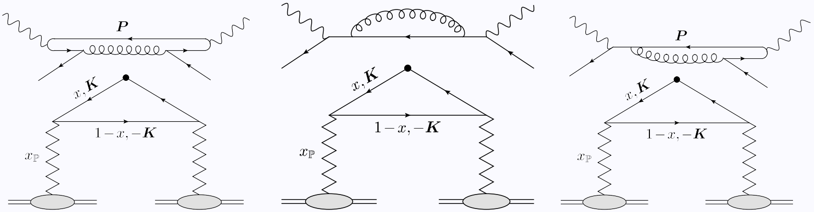




- Large initial $q\bar{q}$ pair, hard QCD vertex
- Same scattering before or after gluon emission, fine cancellations
- Also consider interference between these and previous diagram

TMD Factorization and Cross Section

- Variable change from ϑ_1 to x
- Quark with fraction $1-x$ in final state
- Antiquark "transfers" fraction x and imbalance $\boldsymbol{K}$ to dijet



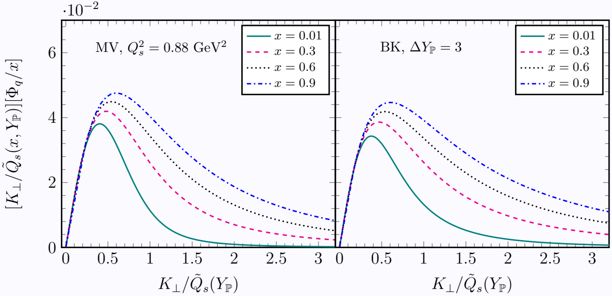
$$\frac{\mathrm{d}\sigma_{\mathrm{D}}^{\gamma^*_{T,L}A\to q\bar{q}gA}}{\mathrm{d}\vartheta_1\mathrm{d}\vartheta_2\mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{K}\mathrm{d}Y_{\mathbb{P}}} = \frac{4\pi^2\alpha_{\mathrm{em}}}{Q^2}\Big(\sum e_f^2\Big)\,H_{T,L}(\vartheta_2,\vartheta_3,Q^2,P_\perp^2)\,\frac{\mathrm{d}xq_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}}$$

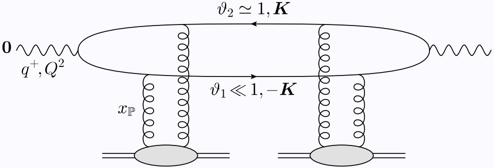
$$H_T(\vartheta_2,\vartheta_3,P_\perp^2,\tilde{Q}^2) = \delta_\vartheta\,\frac{\alpha_s C_F}{\pi^2}\,\frac{1}{2\vartheta_3}\,\frac{\tilde{Q}^2\big[(P_\perp^2+\tilde{Q}^2)^2+\vartheta_2^2\tilde{Q}^4+\vartheta_3^2P_\perp^4\big]}{P_\perp^2(P_\perp^2+\tilde{Q}^2)^3}.$$

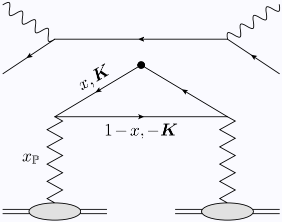
$$\frac{\mathrm{d}xq_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}} = \underbrace{\frac{S_\perp N_c}{4\pi^3}}_{\text{d.o.f.}} \underbrace{\Psi_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2)}_{\text{occupation number}}$$

Explicit in terms of elastic amplitude $T(R,x_{\mathbb{P}})$ (fundamental)

$$\Psi_{\mathbb{P}}(x,x_{\mathbb{P}},K_\perp^2) \approx \frac{x}{2\pi} \begin{cases} 1 & \text{for} \quad K_\perp \ll \tilde{Q}_s(x) \\ \dfrac{\tilde{Q}_s^4(x,Y_{\mathbb{P}})}{K_\perp^4} & \text{for} \quad K_\perp \gg \tilde{Q}_s(x) \end{cases}$$







- Consider dijet cross section obtained in the dipole picture
- Integrate one jet keeping β (gap) fixed $\rightsquigarrow$ change from ϑ_1 to β
- If (and only if) aligned jet configuration ($\vartheta_1 \ll 1$) and $P_\perp^2 \ll Q^2$:

$$\frac{\mathrm{d}\sigma^{\gamma_T^* A \to q\bar{q}A}}{\mathrm{d}\ln(1/\beta)\,\mathrm{d}^2\boldsymbol{P}} = \frac{4\pi^2\alpha_{em}}{Q^2}\Big(\sum e_f^2\Big)\, 2\, \frac{\mathrm{d}x q_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2)}{\mathrm{d}^2\boldsymbol{P}}\bigg|_{x=\beta}$$

- Leading twist result, same quark TMD encountered in 2+1 jets

- Consider hard antiquark-gluon pair and soft quark configuration
- In SIDIS we measure antiquark, integrate the gluon
- Dominant contribution from gluon such that

$$\vartheta_2 \simeq 1 \gg \vartheta_3 \sim \frac{P_\perp^2}{Q^2} \gg \vartheta_1 \sim \frac{k_{1\perp}^2}{Q^2}$$

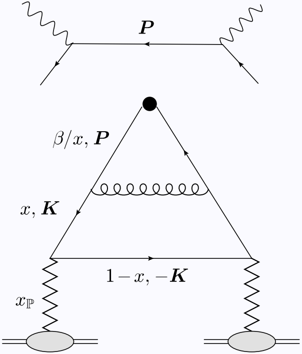
- Integrate at fixed β

$$\frac{\mathrm{d}\sigma^{\gamma_T^* A \to (q)\bar{q}gA}}{\mathrm{d}^2\boldsymbol{P}\,\mathrm{d}\ln(1/\beta)} = \int \mathrm{d}\vartheta_2 \mathrm{d}\vartheta_3 \int \frac{\mathrm{d}x}{x}\,\beta\delta\left(\beta - x\frac{\tilde{Q}^2}{\tilde{Q}^2 + P_\perp^2}\right)$$
$$\times\, H_T(\vartheta_2, \vartheta_3, Q^2, P_\perp^2) \int \mathrm{d}^2\boldsymbol{K}\, \frac{\mathrm{d}x q_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}}$$

- $\boldsymbol{K}$-integration gives DPDF

- Hard factor becomes (real part of) DGLAP splitting function

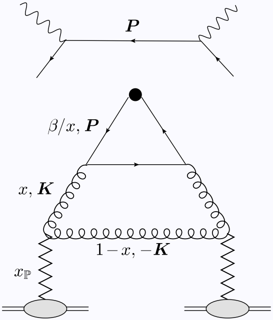
$$\frac{\mathrm{d}\sigma^{\gamma_T^* A\to(q)\bar{q}gA}}{\mathrm{d}^2\boldsymbol{P}\,\mathrm{d}\ln(1/\beta)} = \frac{4\pi^2\alpha_{\rm em}}{Q^2}\left(\sum e_f^2\right) \times \frac{\alpha_s}{2\pi^2}\,\frac{1}{P_\perp^2}\int_{x_{\min}}^{1}\frac{\mathrm{d}x}{x}\,\frac{\beta}{x}\,P_{qq}\left(\frac{\beta}{x}\right) x q_{\mathbb{P}}\left(x, x_{\mathbb{P}}, P_\perp^2\right).$$



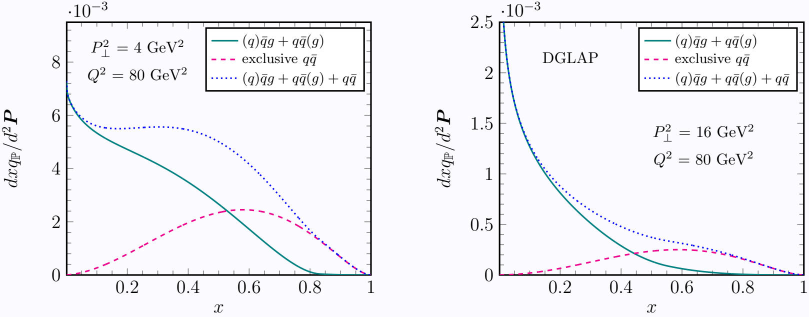
- Target: gluon emission before γ^* absorption by struck antiquark
- All projectile diagrams contribute to simple target picture

- Consider hard quark-antiquark pair and soft gluon
- Integrate the quark with fixed β to get SIDIS

$$\frac{\mathrm{d}\sigma^{\gamma_T^* A \to q\bar{q}(g) A}}{\mathrm{d}^2\boldsymbol{P}\,\mathrm{d}\ln(1/\beta)} = \frac{4\pi^2\alpha_{\rm em}}{Q^2}\left(\sum e_f^2\right) \times \frac{\alpha_s}{2\pi^2}\,\frac{1}{P_\perp^2}\int_\beta^1 \frac{\mathrm{d}x}{x}\,\frac{\beta}{x}\,P_{qg}\left(\frac{\beta}{x}\right) xG_{\mathbb{P}}\left(x, x_{\mathbb{P}}, P_\perp^2\right).$$



- Absorb 2+1 jet contributions into the quark DTMD
- 2 jets piece: $\sim 1/P_\perp^4$
- 2+1 jets pieces: $\sim \alpha_s/P_\perp^2$
- Similarly for the gluon DTMD (would need an extra step since it does not appear in 2 jets)



Incoherent Diffraction

Target average to be taken with CGC wave-function

- $\langle T(\boldsymbol{x},\boldsymbol{y})T(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}})\rangle \rightarrow$ Total diffraction
- $\langle T(\boldsymbol{x},\boldsymbol{y})\rangle\langle T(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}})\rangle \rightarrow$ Coherent diffraction
- $\langle T(\boldsymbol{x},\boldsymbol{y})T(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}})\rangle - \langle T(\boldsymbol{x},\boldsymbol{y})\rangle\langle T(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}})\rangle \rightarrow$ Incoherent diffraction

Homogeneous target:

- Coherent diffraction $\sim \delta^2(\boldsymbol{\Delta})$ (smeared to $1/R_A$)
  Negligible momentum transfer from target to projectile
- Momentum transfer conjugate to difference $\boldsymbol{B}$ of impact parameters in DA and CCA $\rightsquigarrow$ non-zero momentum transfer

Variance of scattering amplitude determined by target fluctuations

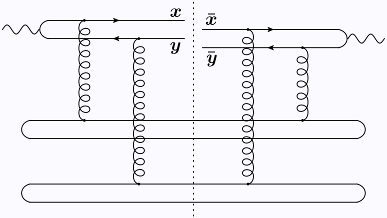
- Pomeron loops: particle number fluctuations in target
- Hot spots (shape fluctuations)
- $1/N_c^2$ color fluctuations (MV model, JIMWLK)
- $\cdots$

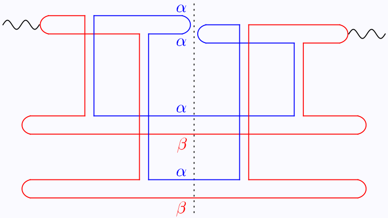
Study color fluctuations

- Q_s sets the scale for color fluctuations
- Expect power-law tail for $\Delta_\perp > Q_s$

Assume Gaussian CGC WF, only pieces connecting DA with CCA survive

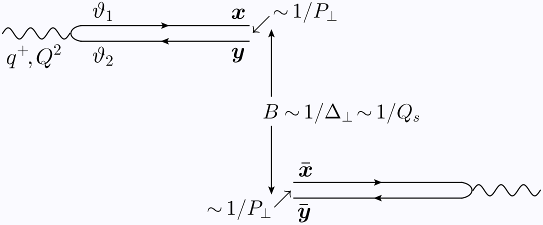
Just for illustration assume 4-gluon exchange





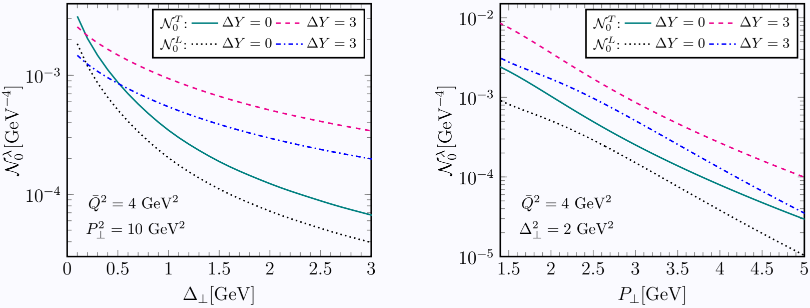
- Projectile can be either quark or gluon dipole
- Elastic on projectile
- Colorless nucleus substructures scatter inelastically
- Exchange of color among them
- Incoherent scattering $\leftrightarrow 1/N_c^2$ suppression

With only two hard jets, pair imbalance is momentum transfer $\boldsymbol{K} = \boldsymbol{\Delta}$
Interested in $P_\perp \gg \Delta_\perp, Q_s \longleftrightarrow r, \bar{r} \ll B, 1/Q_s$



- Correlator known (at finite-N_c)
- Expand for $r, \bar{r} \ll B, 1/Q_s$, other than that B is arbitrary

2 Hard Jets, Averaged (over angle) Cross Section



$$\frac{\mathrm{d}\sigma_{\mathrm{D}}^{\gamma_\lambda^* A \to q\bar{q}X}}{\mathrm{d}\vartheta_1 \mathrm{d}\vartheta_2 \mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{\Delta}} \propto \frac{\alpha_{\mathrm{em}} S_\perp}{N_c} \frac{Q_s^2}{P_\perp^6} \underbrace{f(\Delta_\perp/Q_s)}_{\text{dim/less}}$$

$$f(\Delta_\perp/Q_s) \sim \begin{cases} \mathcal{O}(1) & \text{for} \quad \Delta_\perp \ll Q_s \\ Q_s^2/\Delta_\perp^2 & \text{for} \quad \Delta_\perp \gg Q_s \end{cases}$$

2 + 1 JETS

- Pair imbalance determined by momentum transfer $\boldsymbol{\Delta}$ and recoil due to soft jet emission, $\boldsymbol{K} = \boldsymbol{\Delta} - \boldsymbol{k}_3$
- Integrate over $\boldsymbol{k}_3$ with fixed $\boldsymbol{\Delta}$

Interested again in $P_\perp \gg \Delta_\perp, Q_s$, but now:

- Distribution in $k_{3\perp}$ is peaked at Q_s
- Size of scattering dipole $R \sim 1/k_{3\perp} \sim 1/Q_s$ is large
- No expansion in the QCD correlator

2 +1 Jets Cross Section

- Integrate numerically over $\boldsymbol{B}$, $\boldsymbol{R}$, $\bar{\boldsymbol{R}}$ (and $\boldsymbol{k}_3$)
- Well behaved quantity

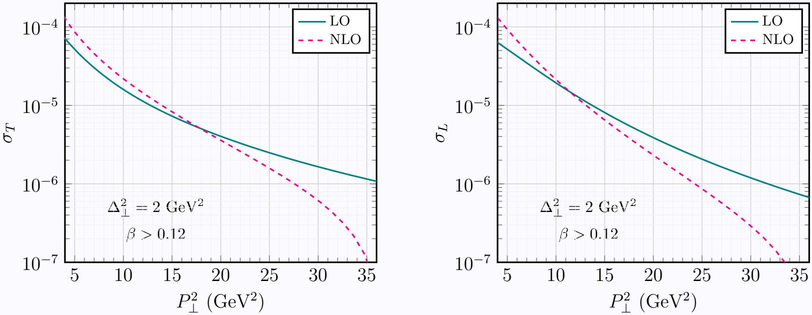
$$\frac{\mathrm{d}\sigma_{\scriptscriptstyle\mathrm{D}}^{\gamma_\lambda^* A\to q\bar{q}gX}}{\mathrm{d}\vartheta_1\mathrm{d}\vartheta_2\mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{\Delta}} \propto \frac{\alpha_{\mathrm{em}}S_\perp}{N_c}\frac{\alpha_s}{P_\perp^4}\underbrace{f_1(\Delta_\perp/Q_s)}_{\mathrm{dim/less}}$$

$$f_1(\Delta_\perp/Q_s) \sim \begin{cases}\mathcal{O}(1) & \text{for} \quad \Delta_\perp \ll Q_s \\ Q_s^4/\Delta_\perp^4 & \text{for} \quad \Delta_\perp \gg Q_s\end{cases}$$

- Larger than 2 jets cross sections since $1/P_\perp^4$ vs $1/P_\perp^6$

Forward dijets with minimum rapidity gap

- Fix Q^2, s and thus $Y_{\rm Bj} = \ln 1/x_{\rm Bj} = 6.1$
- Require a minimum rapidity gap $Y_{\rm gap}^{\rm min} = 4$
- This implies a $Y_{\beta}^{\rm max}$ or $\beta_{\rm min} = 0.12$
- Fix $\vartheta_1 = \vartheta_2 = 1/2$ and $\Delta_\perp^2 = 2 {\rm GeV}^2$
- Then $P_\perp^2$ cannot exceed a max value



Conclusion

- Diffraction at hard momenta in γA collisions in CGC
- Diffractive hard dijet cross sections dominated by 2+1 jets due to scattering near unitarity limit
- Seed of semi-hard jet either a gluon or a quark
- For sufficiently large rapidity gaps and/or large nuclei gluon and quark DTMDs and DPDFs calculated from "first principles"
- CGC as initial condition for DGLAP