Brookhaven National Laboratory

Systematic resummation of large collinear logs in small x evolution

Yacine Mehtar-Tani (BNL)
in collaboration with R. Boussarie (in preparation)

Beyond Eikonal Methods in High Energy Scattering Workshop

ECT*, Trento, May 20 - 24, 2024

Outline

- The ambiguity of small x evolution variable
- Kinematic constraint from first principles
- Collinearly improved BK equation

Ambiguity of small x evolution

- Two "indistinguishable" rapidity variables Y and η

$$\eta = \log \frac{s}{Q^2} = \log \frac{s}{Q_0^2} + \log \frac{Q^2}{Q_0^2} = Y + \rho$$

- Small x rational problematic when collinear log is large $\rho \gg 1$

Inclusive DIS:

$$\rho = \log \frac{Q^2}{Q_s^2}$$

Inclusive TMD:

$$\rho = \log \frac{Q^2}{q_\perp^2}$$

Ambiguity of small x evolution

- Step1: Factorization of fast $\log k^+ > Y$ and slow $\log k^+ < Y$ modes

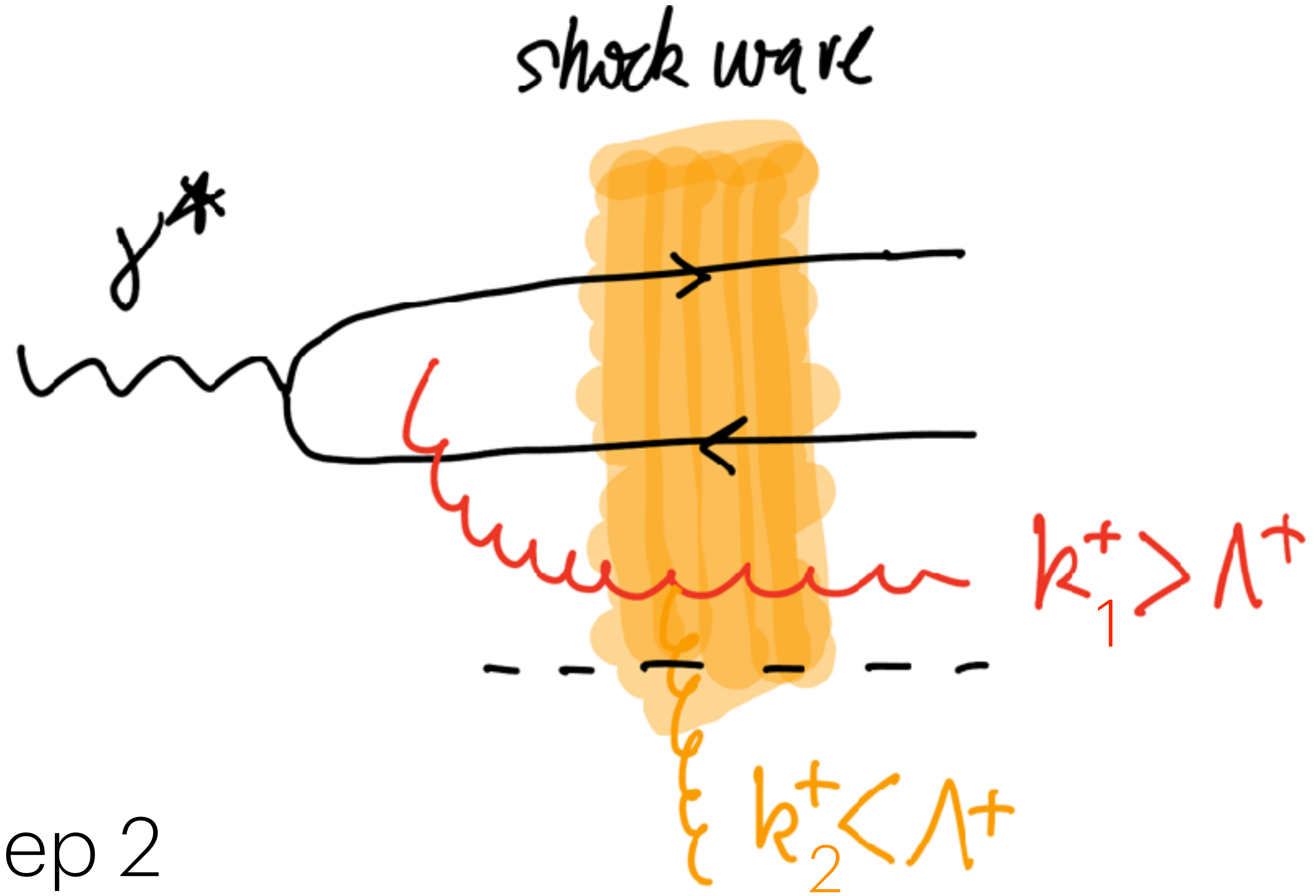
- Step2: Shock wave limit, separate large and short lifetimes

$$k_1^- < k_2^-$$

- If no large collinear log Step 1 implies Step 2

$$k_1^+ > k_2^+ \qquad \Longrightarrow \qquad k_1^- = \frac{k_{\perp 1}^2}{k_1^+} \quad < \quad k_2^- = \frac{k_{\perp 2}^2}{k_1^+}$$

- If large collinear logs: shock wave must be dynamically enforced!



Solutions

- Order-by-order in perturbation theory impose a kinematic constraint on phase space integration

[Beuf (2014) Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos (2015-2019), Salam (1998), Shi, Wang, Wei, Xiao (2021) Liu, Xie, Kang, Liu, (2022) Caucal, Salazar, Schenke, Venugopalan, (2022) Taels, Altinoluk, Beuf, Marquet (2022) ,...]

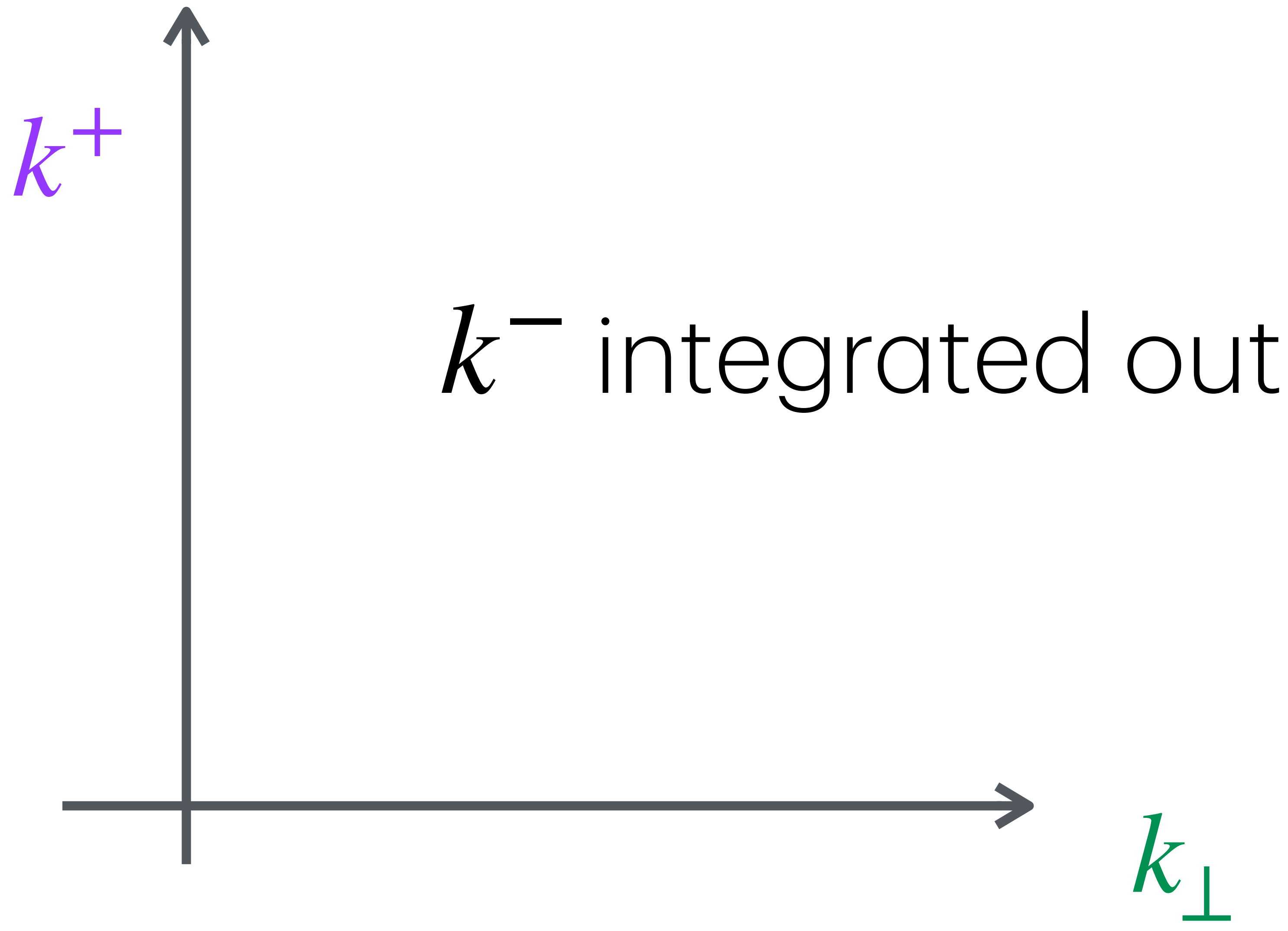
- Implication: Non-locality in rapidity!

Beuf (2014) Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos (2015-2019)

What else can be done?

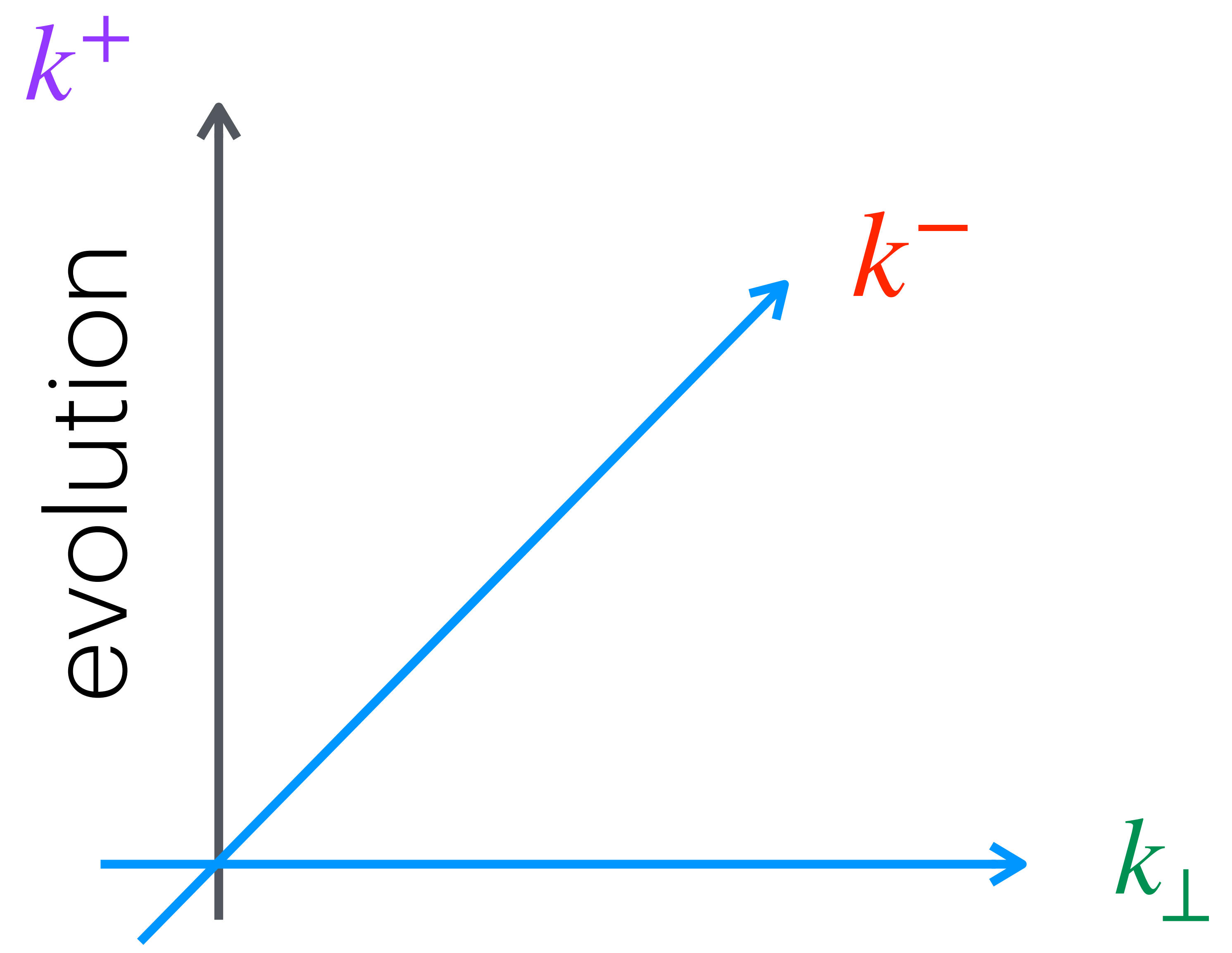
Diagnosing small-x

- Issue in small x: dipole operator is function of 2 variables: $x_\perp \sim 1/k_\perp$ and rapidity $Y = \log k^+$ (or η)

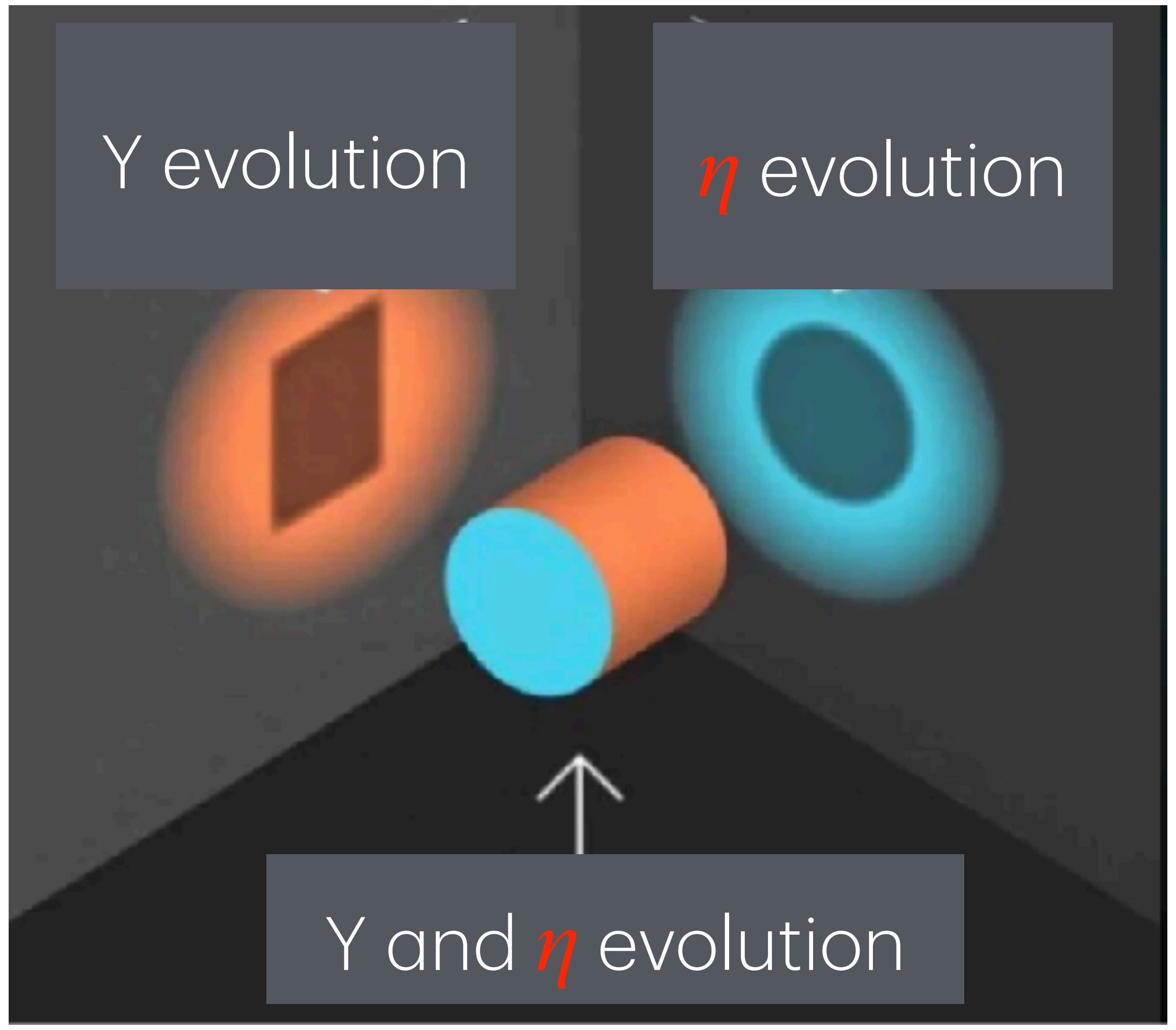


$$S(x_\perp, Y) \sim \langle \mathrm{tr} U^\dagger(0) U^\dagger(x_\perp) \rangle_Y$$

Dimensionally enhanced evolution



- Treat Y and η as independent variables
- The kinematic constraint built in the evolution kernel



$$S(x_\perp, Y) \quad \rightarrow \quad S(x_\perp, \eta, Y)$$

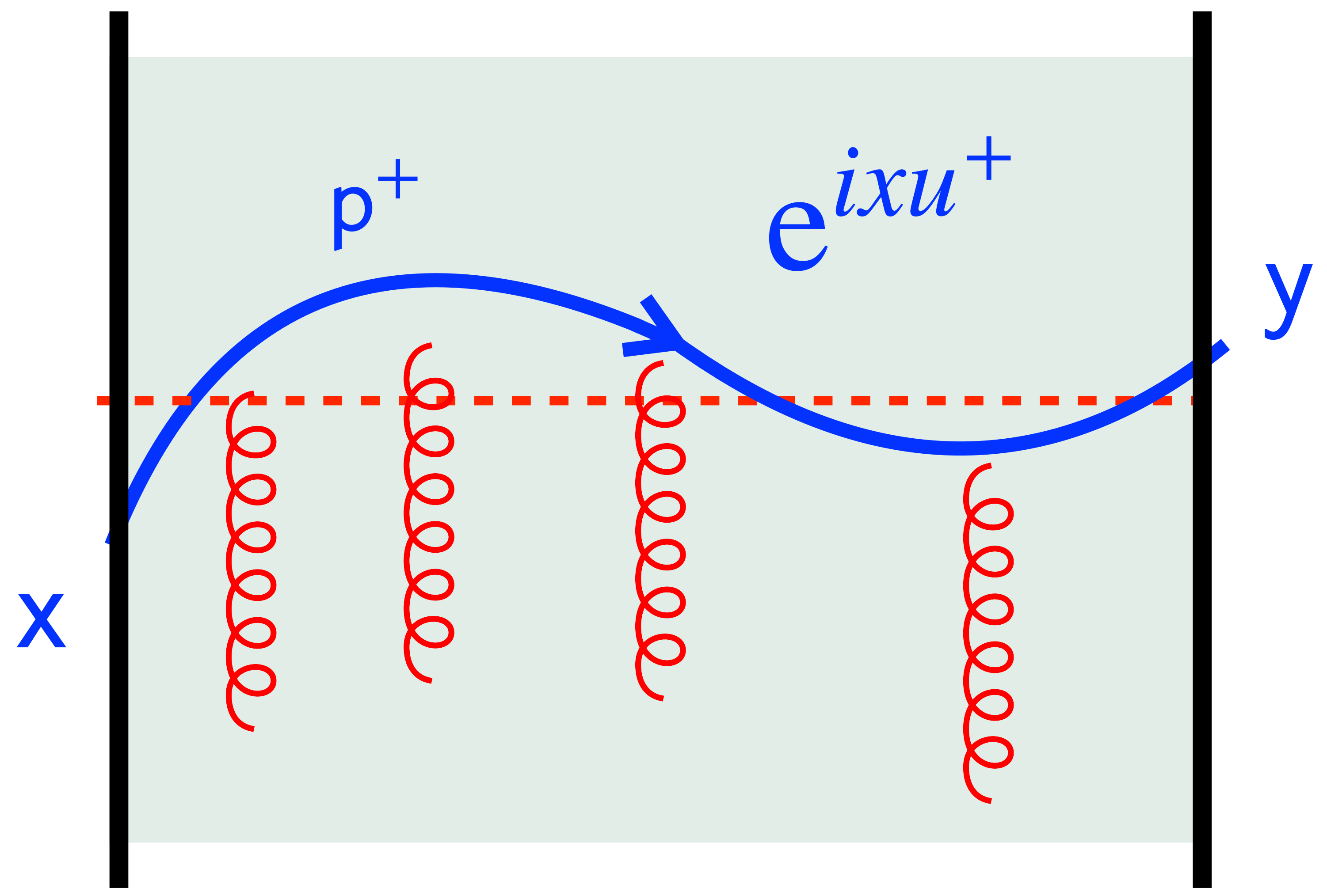
Beyond shockwave approximation

- Sub-eikonal expansion around the shock wave $\delta(x^+)$ [Agostini, Altinoluk, Armesto, Beuf, Martinez, Moscoso, Salgado]
- Expansion in the boost parameter [Chirilli] ; [Altinoluk, Beuf, Czajka, Tymowska]
- Addition of a single hard scattering [Jalilian-Marian]

Our approach:

- Revisit the shock wave factorization scheme to restore the x dependence of the gluon distribution - consistent with factorization in k^+ [Balitsky-Tarasov]
- Perform a partial twist expansion to connect Regge and Bjorken limits

$$f(k_\perp, x) + O\left(\frac{x_{\mathrm{Bj}}}{Q^2}\right)$$



$$u^+ = (y - x)^+$$

Quantum phase

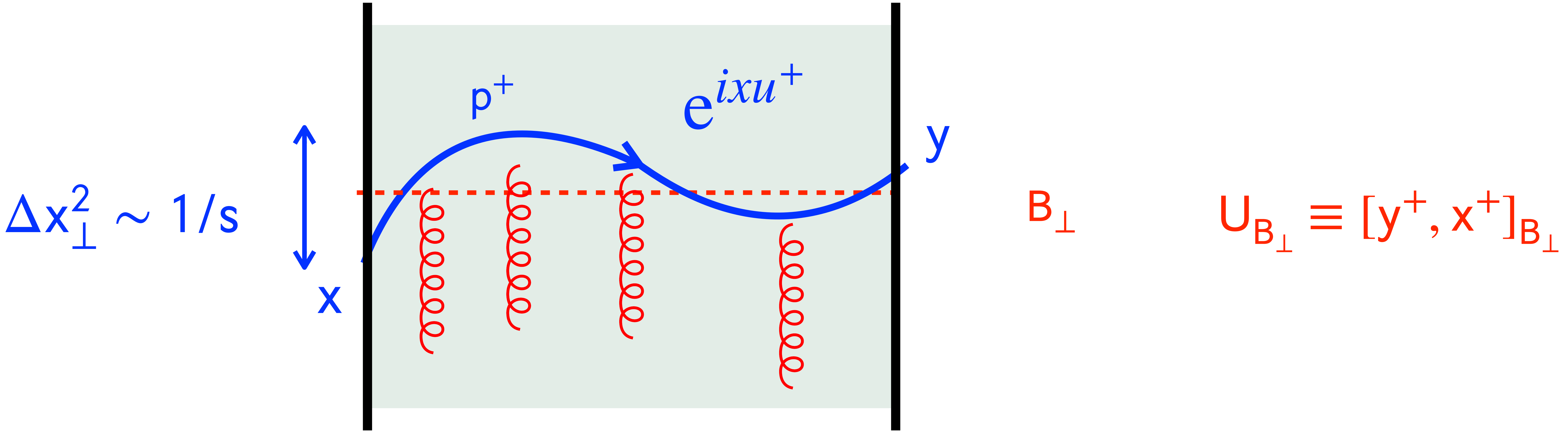
$$e^{ixu^+} \approx$$

$1 + ix\,u^+ + O(x^2)$ Eikonal expansion

$\theta(x < 1/u^+)$ Kinematic constraint

- Observation: kinematic constraint involves all powers of s !

Partial Twist Expansion



- Expand around the classical trajectory: $\Delta x_\perp = x_\perp - y_\perp \ll B_\perp = (x_\perp + y_\perp)/2$

$$D(x-y) \sim \frac{p^+}{2i\pi\Delta x^+} e^{i\frac{(x-y)_\perp^2}{\Delta x^+}p^+} U_B(x^+, y^+) + O(|\Delta x_\perp|/|B_\perp|)$$

Quantum phase Wilson line

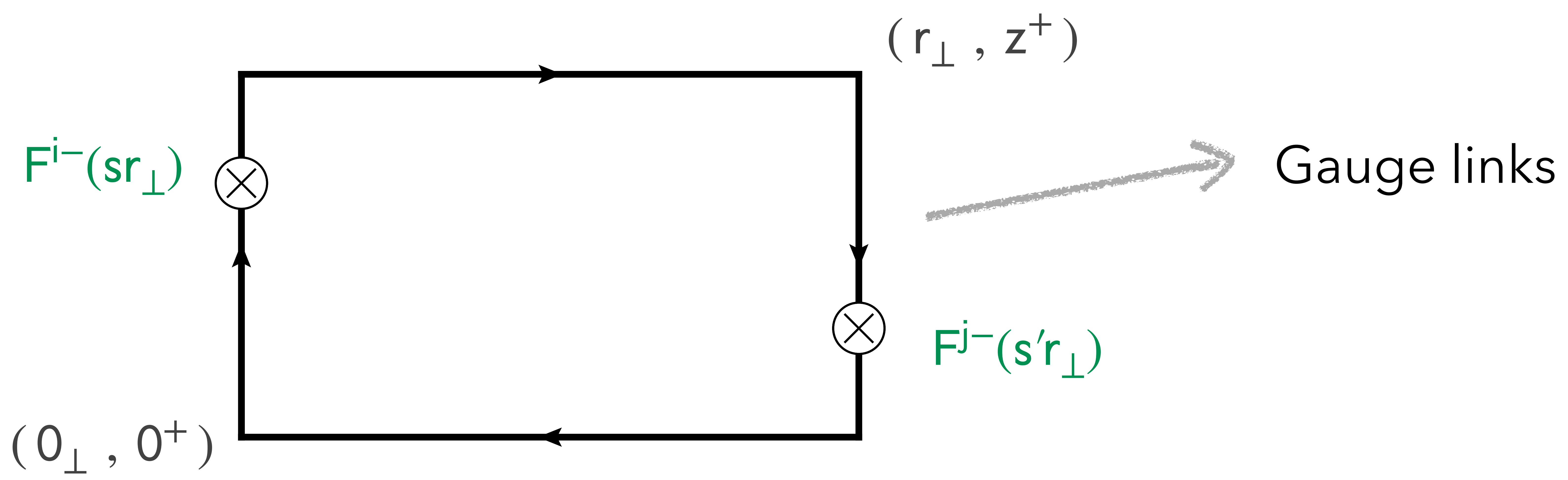
[Altinoluk, Armesto, Beuf, Martinez, Salgado]

- Standard approximation: $P^+ \to +\infty$: $D_F(x-y) \sim \delta(x_\perp - y_\perp)\ U_x(x^+, y^+)$

x-dependent unintegrated gluon GPD

$$G^{ij}(x, k_\perp) \equiv \frac{1}{P^-} \int \frac{dz^+}{2\pi} e^{ixP^- z^+} \int \frac{d^d r_\perp}{(2\pi)^d} e^{-ik_\perp \cdot r_\perp} \int_0^1 ds ds'$$
$$\times \langle p' \big| \mathrm{Tr}\left[z^+, 0^-\right]_0 F^{i-}\left(0^+, s r_\perp\right) \left[0^+, z^+\right]_{r_\perp} F^{j-}\left(z^+, s' r_\perp\right) \big| p \rangle$$

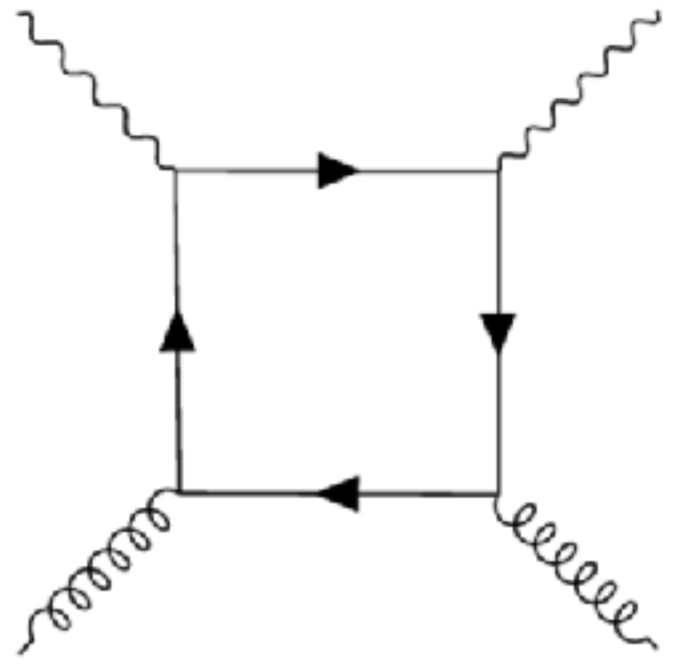
[R. Boussarie, Y. M. T. (2020-2022)
2309.16576 [hep-ph]
2112.01412 [hep-ph]
2006.14569 [hep-ph]



Nonlocal quange-invariant gluon operator in longitudinal and transverse directions

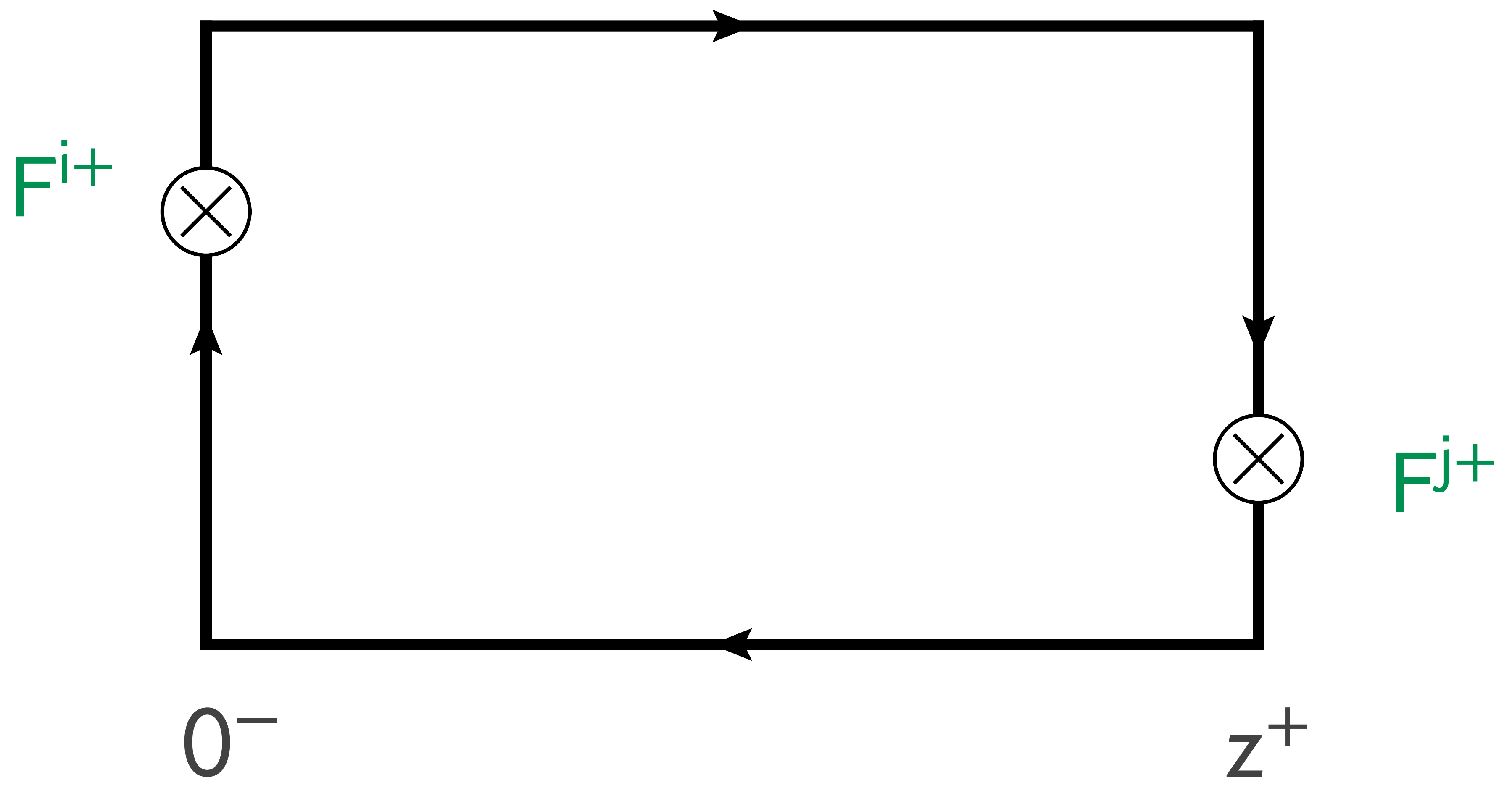
Bjorken limit

- Neglecting transverse momentum transfer from the target



$$k_\perp \ll l_\perp \sim Q$$

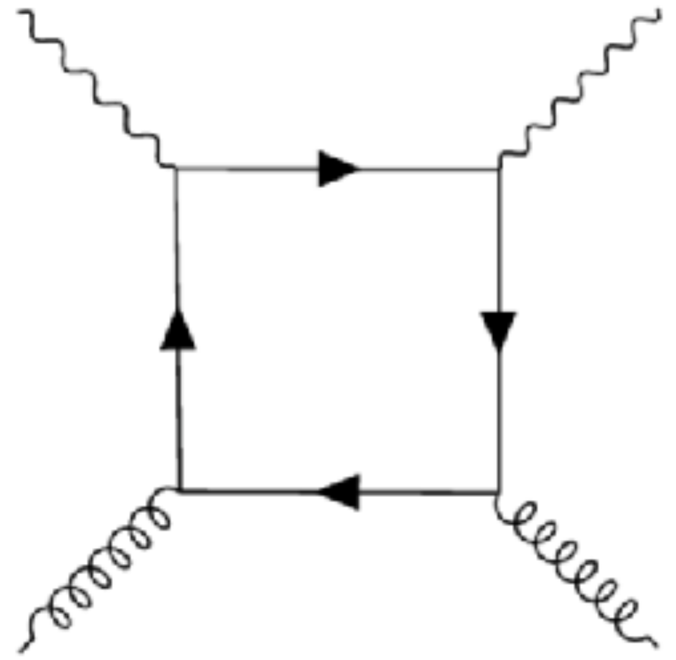
- uGPD integrates into gluon GPD $\longrightarrow$ $\int d^d k_\perp\, G^{ij}(x, k_\perp) = G^{ij}(x, k_\perp)$



$r_\perp \to 0$ $\longrightarrow$

Bjorken limit

- In the collinear limit $Q^2 \to \infty$, we reproduce the 1-loop contribution to the DIS structure function

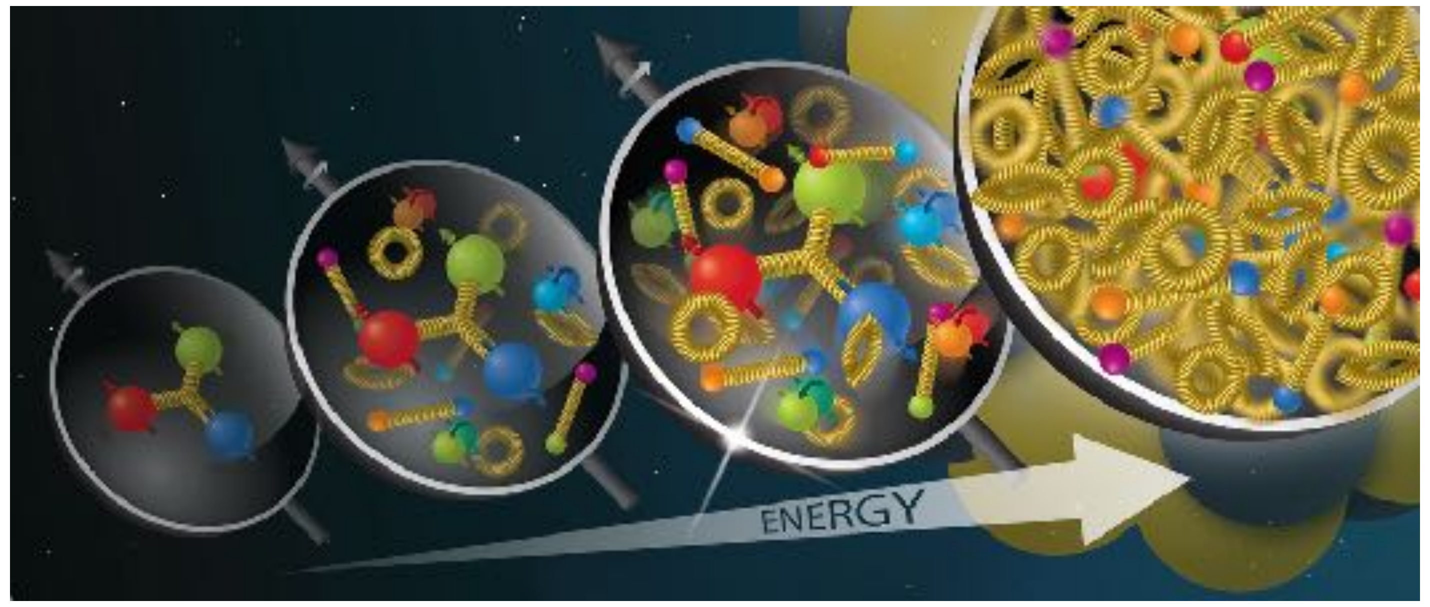


$$F_T(x_{Bj}, Q^2) = \frac{\alpha_s}{\pi} \sum_f q_f^2 \int_{x_{Bj}}^1 dy \; xg(x_{Bj}/y, \mu^2)$$

$$\times \left[\frac{1}{\epsilon} \left(\frac{e^{\gamma_E}}{4\pi} \right)^{\epsilon} P_{qg}(y) + \left[(1-y)^2 + y^2 \right] \log \left[\frac{Q^2(1-y)}{\mu^2 y} \right] - 1 + 4y(1-y) \right]$$

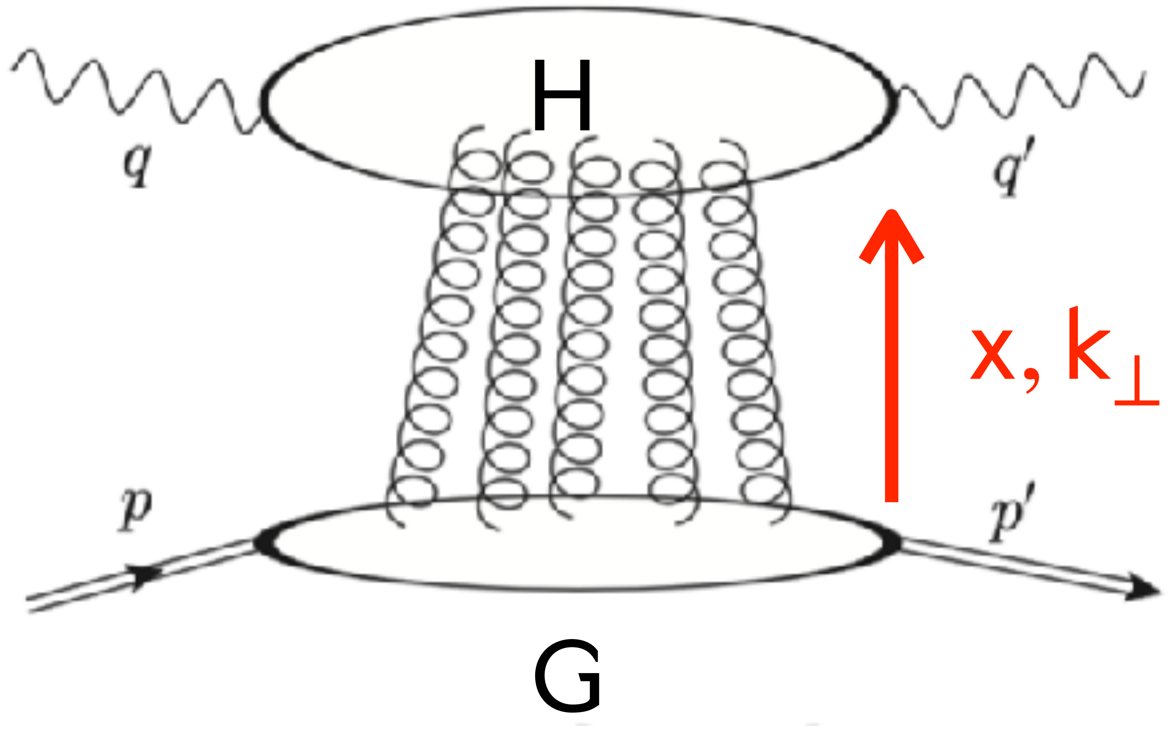
J. Collins, Foundations of pQCD 2011

Interpolating scheme for DIS



Overarching scheme

$$\int dx \int dk_\perp \; H^{ij}(x, k_\perp)\; G^{ij}(x, k_\perp)$$



Bjorken limit: $Q^2 \to +\infty$

$$\int dx \; H^{ij}(x, k_\perp = 0) \left(\int dk_\perp G^{ij}(x, k_\perp) \right)$$

PDF

Regge limit: $s \to +\infty$

$$\int dk_\perp \; G^{ij}(x = 0, k_\perp) \left(\int dx \; H^{ij}(x, k_\perp) \right)$$

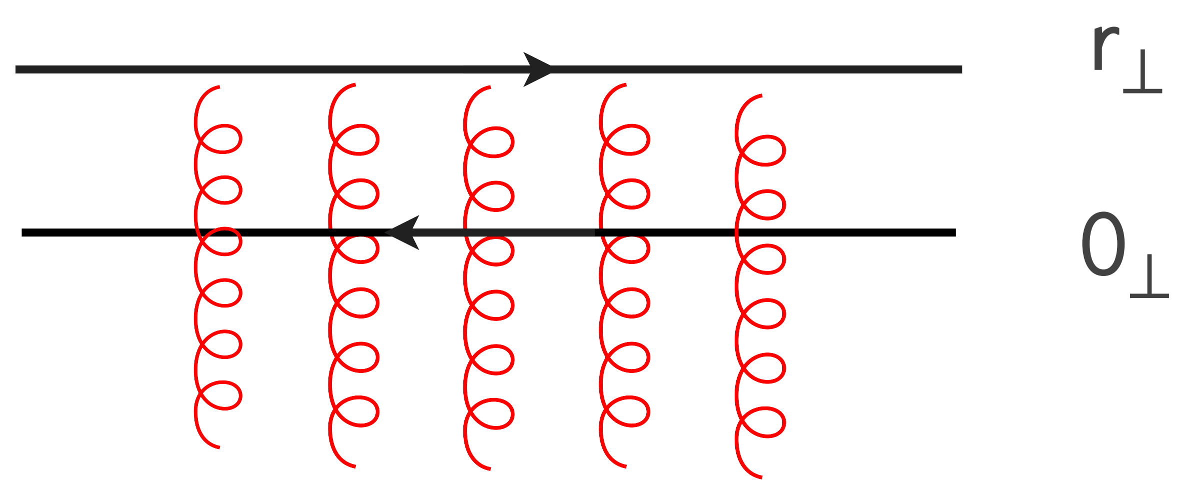
Dipole

Regge limit

- Setting $x = 0$ in the 3D gluon operator

- We recover the dipole operator at small x:

$$r^i r^j \, G^{ij}(x = 0\,, r_\perp) \;\rightarrow\; \langle P | \,\mathrm{Tr}\, U_{r_\perp} \, U^\dagger_{0_\perp} \, | P \rangle$$



BK with kinematic constraint

X-dependent dipole operator (definition)

[R. Boussarie, Y. M. T. , 2309.16576 [hep-ph] 2112.01412 [hep-ph] 2006.14569 [hep-ph]

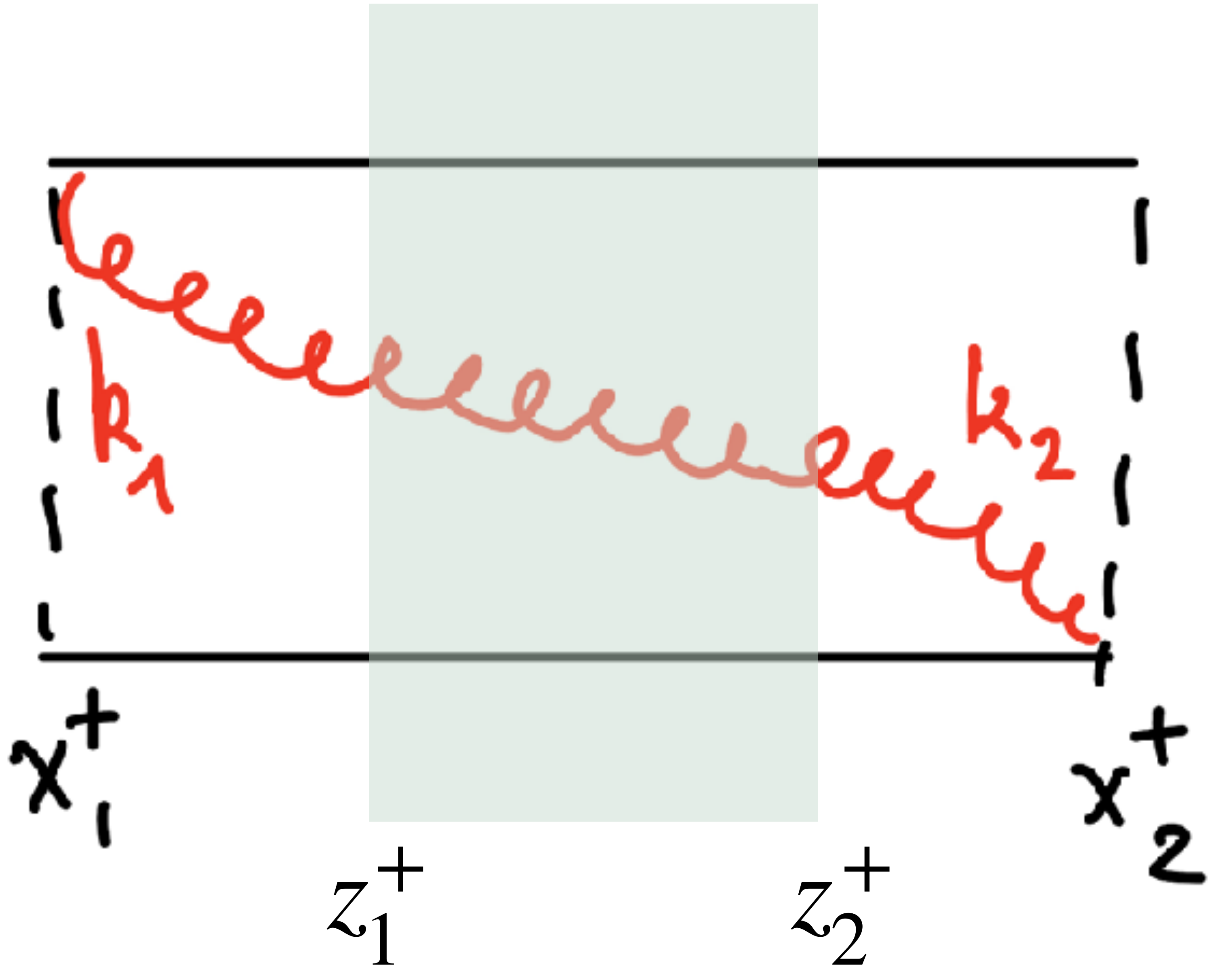
$$\int \mathrm{d}\boldsymbol{b}\, S(\boldsymbol{r},x) \equiv g^2(2\pi)^3 2P^- \langle P|P\rangle\, \boldsymbol{r}^i \boldsymbol{r}^j x G^{ij}(\boldsymbol{r},x) = \int \mathrm{d}\boldsymbol{b} \int \mathrm{d}z^+ \int \mathrm{d}z'^+\, \mathrm{e}^{ixP^-(z^+-z'^+)}$$
$$\times \frac{\partial^2}{\partial z^+ \partial z'^+} \langle P|\, \mathrm{tr}\, U^\dagger_{\boldsymbol{0}}(z'^+,z^+) U_{\boldsymbol{r}}(z'^+,z^+)|P\rangle\,.$$

Finite Wilson lines

Quantum phase: x dependence

Small-x evolution with dynamical shock wave

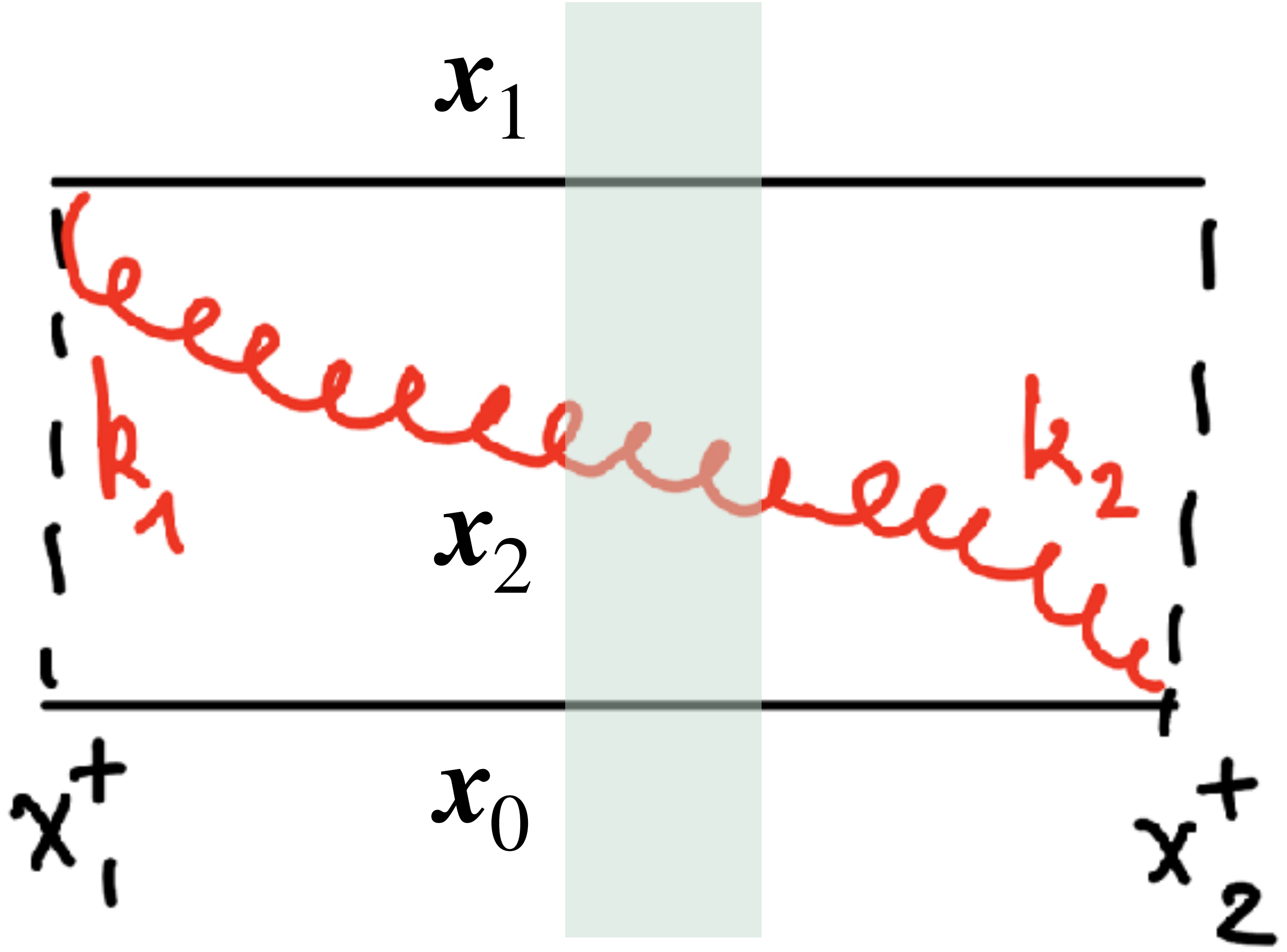
$$\langle a^-(x_2^+,\boldsymbol{x}_2)a^-(x_1^+,\boldsymbol{x}_1)\rangle = \frac{1}{2}\int\frac{\mathrm{d}k^+}{k^+}\int\mathrm{d}\boldsymbol{k}_2\int\mathrm{d}\boldsymbol{k}_1\,\frac{\boldsymbol{k}_2\cdot\boldsymbol{k}_1}{(k^+)^2}\,(\boldsymbol{k}_2|\mathcal{G}_{k^+}(x_2^+,x_1^+)|\boldsymbol{k}_1)\,\mathrm{e}^{i\boldsymbol{k}_2\cdot\boldsymbol{x}_2-i\boldsymbol{k}_1\cdot\boldsymbol{x}_1}$$



$$A^\mu \;\to\; a^\mu + A^\mu$$

$$\to \mathrm{e}^{-i\frac{\boldsymbol{k}_2^2}{2k^+}(x_2-\xi_2)^+}\mathrm{e}^{-i\frac{\boldsymbol{k}_1^2}{2k^+}(z_1-x_1)^+}\,\mathrm{e}^{-i\frac{(\boldsymbol{k}_2+\boldsymbol{k}_1)^2}{2k^+}(z_2-z_1)^+}\,\frac{\partial^2}{\partial z_1^+\partial z_2^+}\left[U^{ab}_{\boldsymbol{k}_2-\boldsymbol{k}_1}\,\mathrm{tr}(t^aU_0^\dagger t^bU_{\boldsymbol{r}})(x_2^+,x_1^+)\right]$$

Small-x evolution with dynamical shock wave



$Y = \log \Lambda^+$

$Y' = \log k^+$

$$\delta S_Y(\boldsymbol{x}_{10}, x) = g^2 \int_0^Y \mathrm{d}Y' \int_x^1 \mathrm{d}x' \int \mathrm{d}\boldsymbol{z}\, K_{Y'}(\boldsymbol{x}_{12}, \boldsymbol{x}_{20}, x, x') \times S_{Y'}(\boldsymbol{x}_{12}, \boldsymbol{x}_{21}, x')$$

N-body operator

$$S_Y^{(n)}(\boldsymbol{x}_1, ..., \boldsymbol{x}_2, x) = \int_{-\infty}^{+\infty} \mathrm{d}z^+ \int_{-\infty}^{z^+} \mathrm{d}z'^+ \, \mathrm{e}^{ixP^-(z'-z)^+} \frac{\partial^2}{\partial z^+ \partial z'^+} U_1 \otimes U_2 \otimes ... \otimes U_n(z'^+, z^+)$$

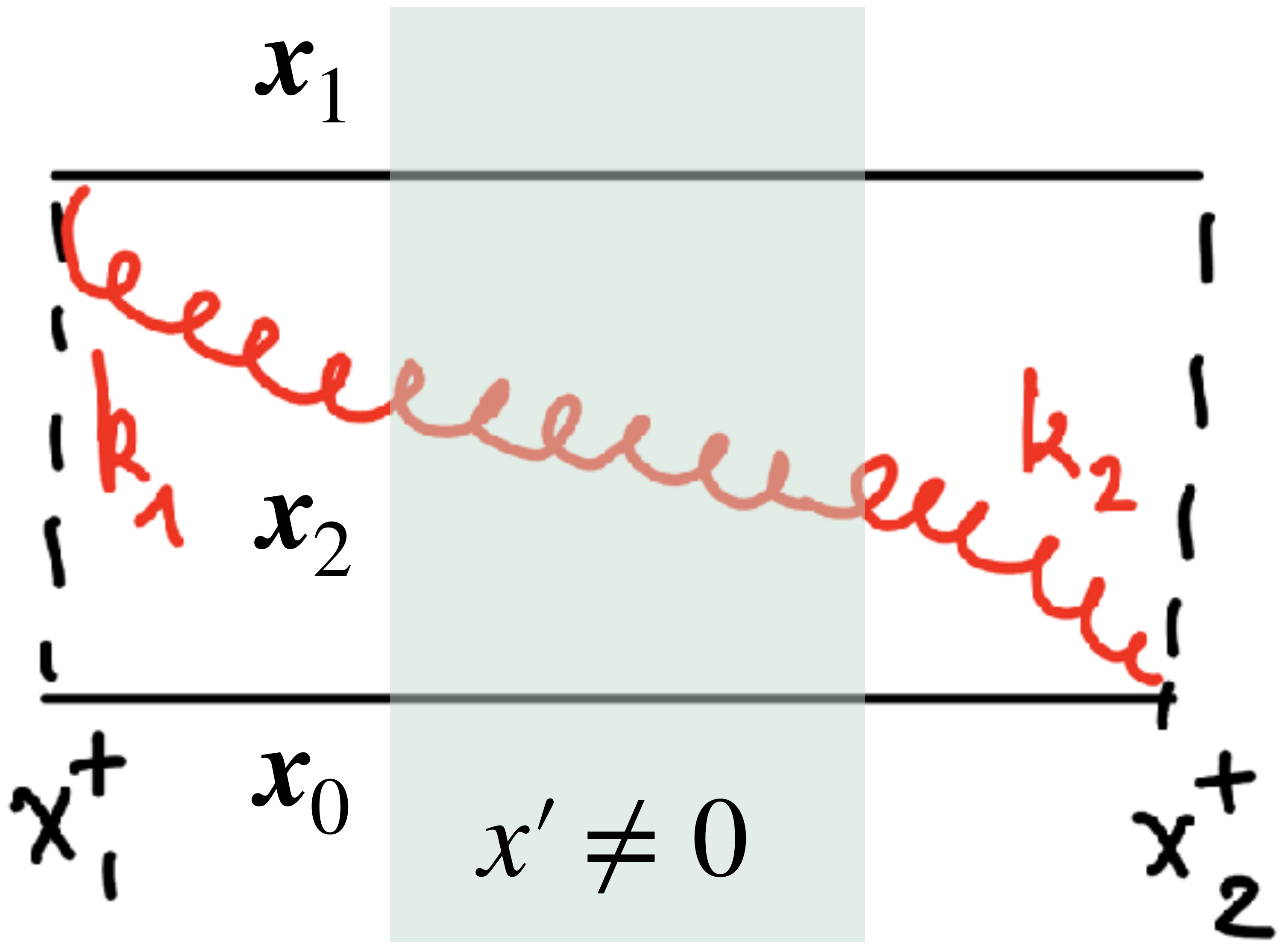
Quantum phase: x dependence

Finite Wilson lines

Evolution Kernel

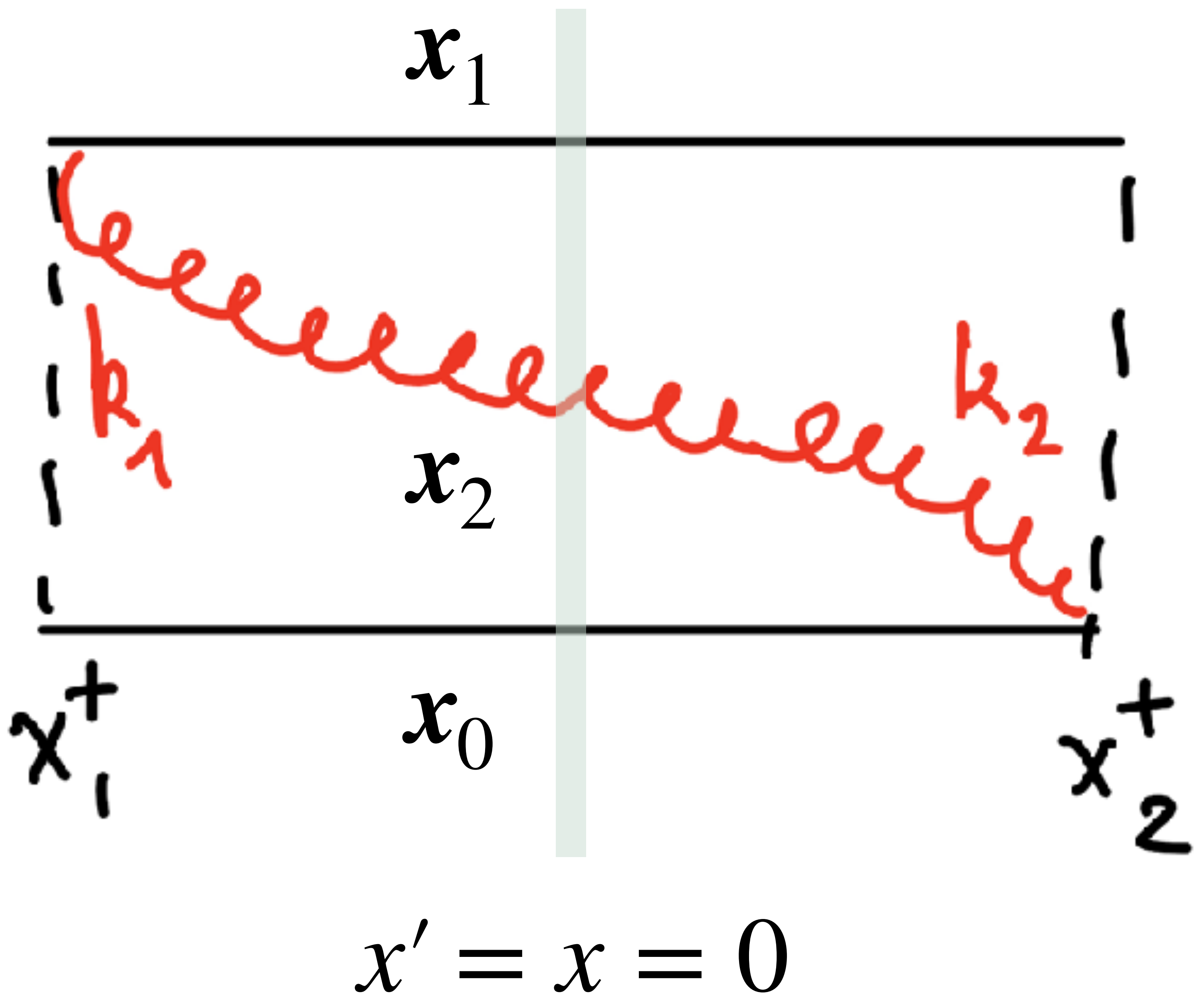
$$K_{k^+}(\boldsymbol{x}_{12}, \boldsymbol{x}_{20}, x, x') = \int \mathrm{d}\boldsymbol{k}_2 \int \mathrm{d}\boldsymbol{k}_1 \, \delta\left(x' - x - \frac{(\boldsymbol{k}_2 + \boldsymbol{k}_1)^2}{2k^+P^-}\right)$$

$$\times \frac{\boldsymbol{k}_2 \cdot \boldsymbol{k}_1}{(\boldsymbol{k}_2^2 + 2xk^+P^-)(\boldsymbol{k}_1^2 + 2xk^+P^-)} \left(\mathrm{e}^{-i\boldsymbol{k}_2\cdot\boldsymbol{x}_{12}} - \mathrm{e}^{-i\boldsymbol{k}_2\cdot\boldsymbol{x}_{20}}\right)\left(\mathrm{e}^{i\boldsymbol{k}_1\cdot\boldsymbol{x}_{12}} - \mathrm{e}^{i\boldsymbol{k}_1\cdot\boldsymbol{x}_{20}}\right)$$



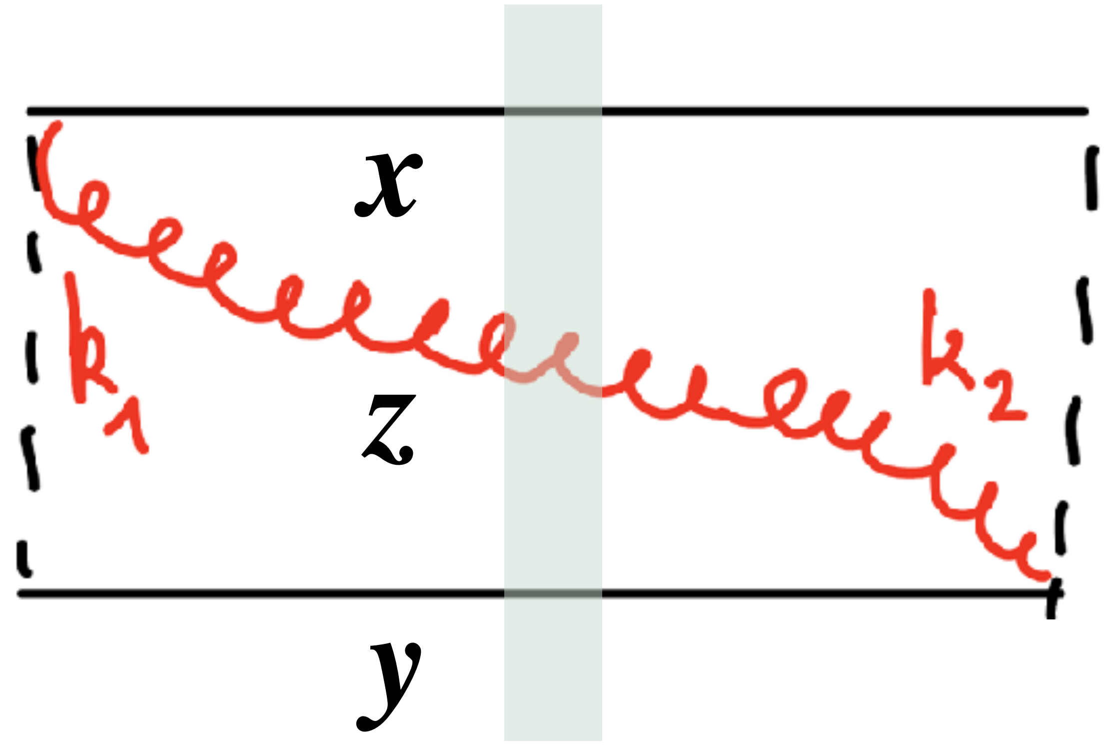
In the limit $x \to 0$ and $P^- \to \infty$ it reduces to BK

$$K_{k^+}(\boldsymbol{x}_{12}, \boldsymbol{x}_{20}, x = 0, x') \to \frac{\boldsymbol{x}_{10}^2}{\boldsymbol{x}_{12}^2 \boldsymbol{x}_{20}^2} \delta(x')$$



Collinearly improved BK (real term): $x \ll x'$

$$S(\rho_{\boldsymbol{xy}}, \eta, Y) = \bar{\alpha} \int_0^Y \mathrm{d}Y' \int_0^\eta \mathrm{d}\eta' \int \mathrm{d}\boldsymbol{z} K^{BK}_{\boldsymbol{xz},\boldsymbol{zy}} \times \delta(Y' - \eta' - \hat{\rho}) \; [S(\rho_{\boldsymbol{xz}}, \eta', Y') + S(\rho_{\boldsymbol{zy}}, \eta', Y') + ...]$$



where $\hat{\rho} = \ln(\boldsymbol{k}_1 + \boldsymbol{k}_2)^2 / Q_0^2$

- Dimensional reduction: $Y = \rho + \eta$

$$S(\rho, \eta, \rho + \eta) = \bar{S}(\rho.\eta)\,,$$

$$S(\rho, Y - \rho, Y) \equiv \tilde{S}(\rho, Y)$$

Rapidity shift operator

η evolution

$$\frac{\mathrm{d}}{\mathrm{d}\eta} \bar{S}(\rho, \eta) = \mathbb{K} \; \mathrm{e}^{-\Theta(\hat{\rho} - \rho)(\rho - \hat{\rho})\partial_\eta} \; \Theta(\eta) \, \bar{S}(\rho', \eta)\,,$$

Y evolution

$$\frac{\mathrm{d}}{\mathrm{d}Y} \tilde{S}(\rho, Y) = \mathbb{K} \; \mathrm{e}^{-\Theta(\rho - \hat{\rho})(\rho - \hat{\rho})\partial_Y} \; \Theta(Y - \hat{\rho}) \, \tilde{S}(\rho', Y)$$

Comparing with the literature:

- Several forms of the equation exist [Beuf (2014) Ducloué, Iancu, Mueller, Soyez, Triantafyllopoulos (2015-2019)]

$$\frac{\partial S_{\boldsymbol{xy}}(Y)}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2} \Theta(Y-\rho_{\min})\left[S_{\boldsymbol{xz}}(Y-\Delta_{\boldsymbol{xyz}})S_{\boldsymbol{zy}}(Y-\Delta_{\boldsymbol{xyz}})-S_{\boldsymbol{xy}}(Y)\right],$$

$$\Theta(Y-\rho)\Theta\left(Y-\Theta(\rho_1-\rho)\rho_1\right) = \Theta(Y-\rho_{\min}), \qquad \rho_{\min} \equiv \ln\frac{1}{r_{\min}^2 Q_0^2} \quad \text{with} \quad r_{\min} = \min\{|\boldsymbol{x}-\boldsymbol{y}|, |\boldsymbol{x}-\boldsymbol{z}|, |\boldsymbol{y}-\boldsymbol{z}|\}.$$

$$\Delta_{\boldsymbol{xyz}} \equiv \Theta(\rho-\rho_1)(\rho-\rho_1) = \Theta(r_<-r)\ln\frac{r_<^2}{r^2} = \max\left\{0, \ln\frac{\min\{(\boldsymbol{x}-\boldsymbol{z})^2,(\boldsymbol{z}-\boldsymbol{y})^2\}}{(\boldsymbol{x}-\boldsymbol{y})^2}\right\}$$

- Equivalent to our formulation after converting $\hat{\rho} = \log(\boldsymbol{k}_1+\boldsymbol{k}_2)^2 \to \log(1/\min[(\boldsymbol{x}-\boldsymbol{z})^2,(\boldsymbol{y}-\boldsymbol{z})^2])$

$$\frac{\mathrm{d}}{\mathrm{d}Y}\tilde{S}(\rho,Y) = \mathbb{K}\,\mathrm{e}^{-\Theta(\rho-\hat{\rho})(\rho-\hat{\rho})\partial_Y}\;\Theta(Y-\hat{\rho})\,\tilde{S}(\rho',Y)$$

Summary

- New 3D-gluon distribution that encodes dipole operator and PDF at finite x
- Provides systematic approach to resum large collinear double logs at small x
- At small x, after dimensional reduction, quantum evolution reduces to the two forms non-local forms of collinearly improved BK (including kinematic constraint)
- Outlook: investigate corrections beyond BK, are there other logarithmic