Brookhaven National Laboratory

RBRC RIKEN BNL Research Center

Spin-orbit correlation at small-x

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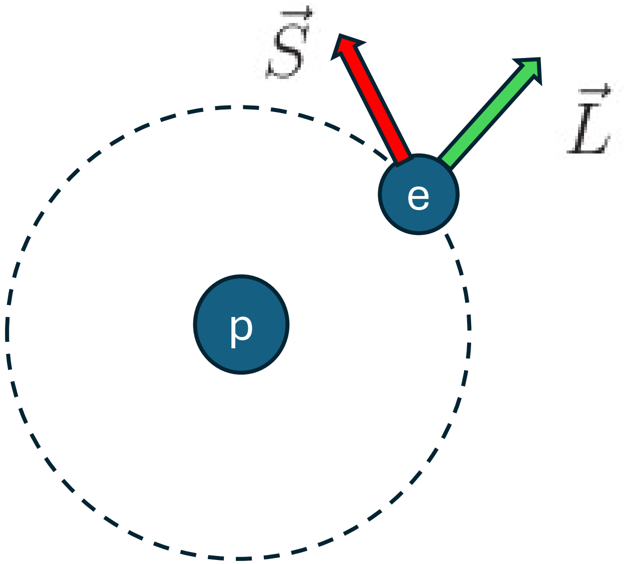
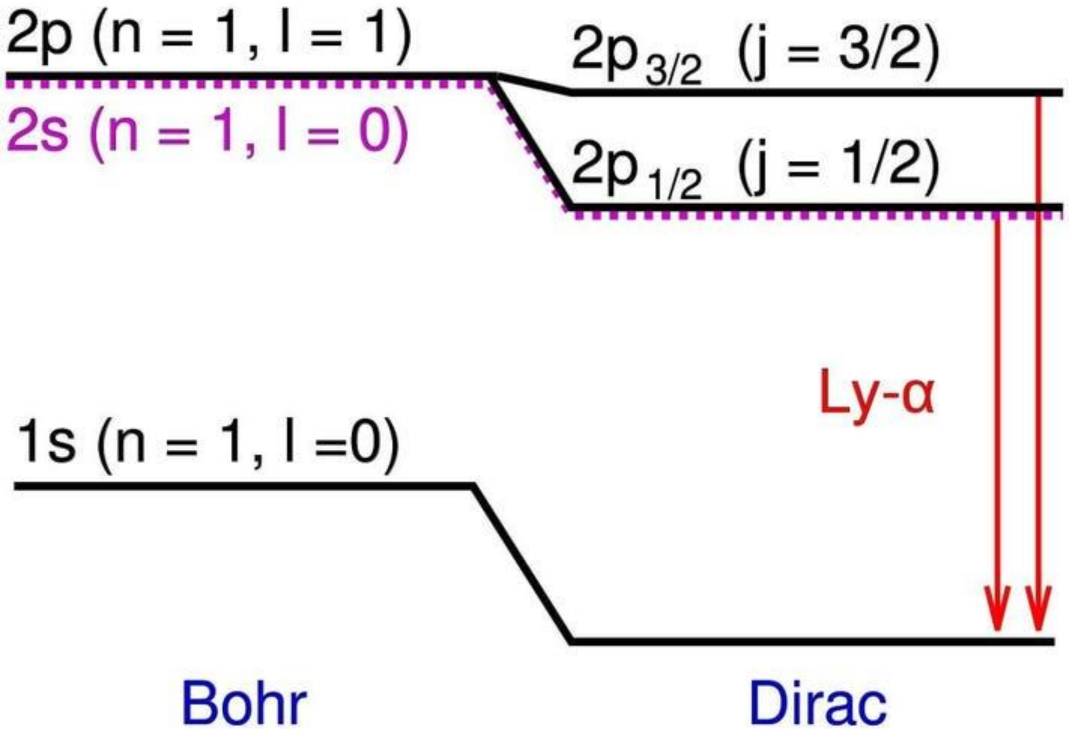
2404.04208; 2404.04209 with Shohini Bhattacharya, Renaud Boussarie,

2404.18872 with Jakob Schoenleber

Spin-orbit coupling in atoms

$$V = -\frac{\mu_B e}{mc^2 r^3} \vec{S} \cdot \vec{L}$$





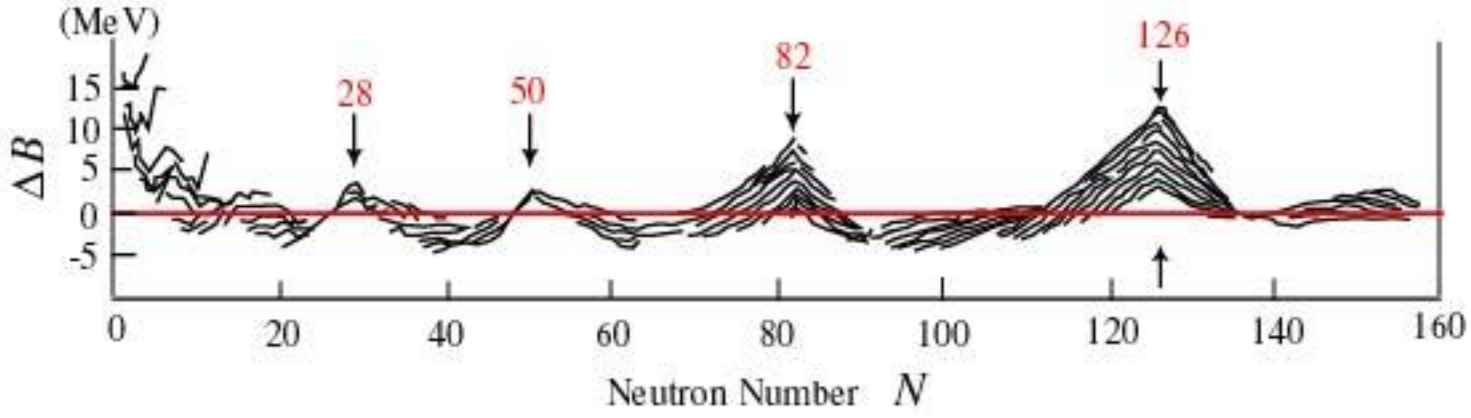
$\vec{\mu} \cdot \vec{B}$ in the electron rest frame + relativistic effects contributes to the hydrogen fine structure

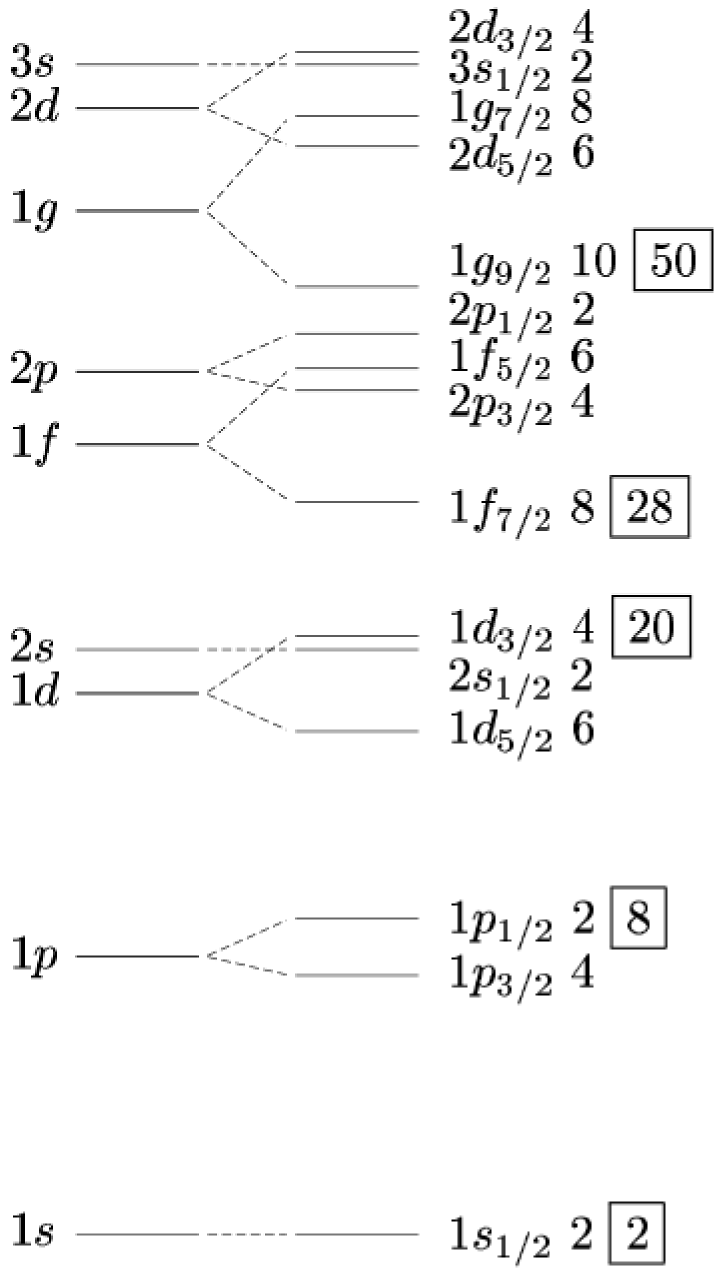
Spin-orbit coupling in nuclei

In the nuclear shell model, nucleons orbiting inside a nucleus feel a spin-orbit potential

Strong spin-orbit coupling → magic numbers

Mayer, Jensen Nobel prize (1963)





Spin-orbit coupling in nucleons?

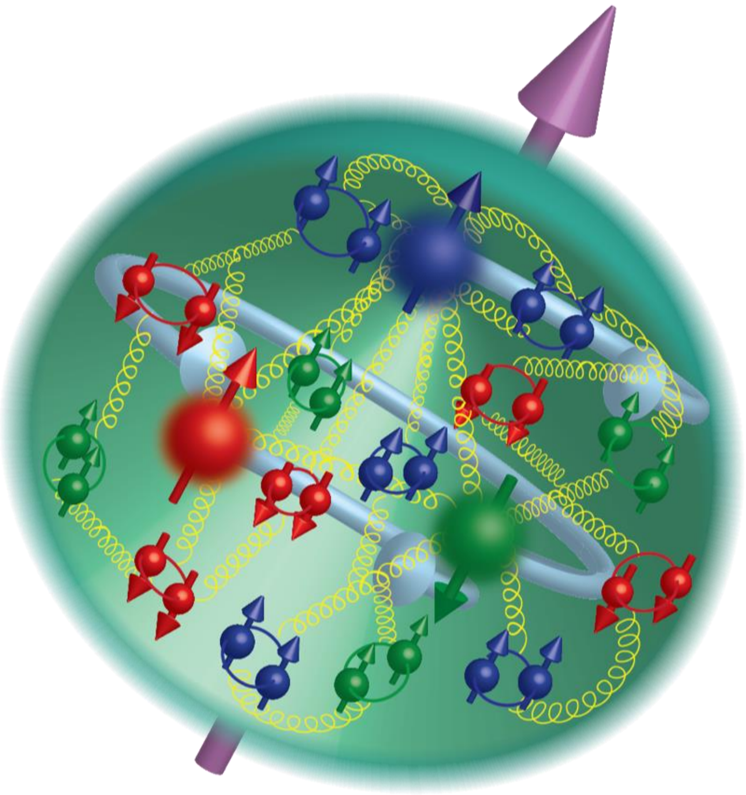
Quarks and gluons carry spin (helicity) and OAM
Naturally there should be spin-orbit coupling

Numbers of quarks and gluons indefinite

Gluon spin and OAM need to be carefully defined
→ Jaffe Manohar (canonical) scheme

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

spin spin orbit orbit



However, the whole discussion does not require polarization.
I will assume an unpolarized proton in the following.

Quark spin-orbit correlation

Polarized quark GTMD

$$\tilde{f}_q(x,\xi,k_\perp,\Delta_\perp) = \int \frac{d^3z}{2(2\pi)^3} e^{ixP^+z^- - ik_\perp\cdot z_\perp} \langle p's'|\bar{q}(-z/2)W_\pm\gamma^+\gamma_5 q(z/2)|ps\rangle$$

$$= \frac{-i}{2M}\bar{u}(p's')\left[\frac{\epsilon_{ij}k_\perp^i\Delta_\perp^j}{M^2}G_{1,1}^q + \frac{\sigma^{i+}\gamma_5}{P^+}(k_\perp^i G_{1,2}^q + \Delta_\perp^i G_{1,3}^q) + \sigma^{+-}\gamma_5 G_{1,4}^q\right]u(ps)$$

Meissner, Metz, Schlegel (2008)

Quark spin-orbit correlation Lorce, Pasquini (2011)

$$C_q = \int_{-1}^{1} dx \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^q(x,k_\perp,0) \sim \langle S^z L^z\rangle$$

x-distribution

$$C_q(x) = \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^q(x,k_\perp,0)$$

$C_q > 0$ if helicity and OAM are aligned

$C_q < 0$ if they are anti-aligned

Gluon spin-orbit correlation

Polarized gluon GTMD

$$x\tilde{f}_g(x,\xi,k_\perp,\Delta_\perp) = i\int \frac{d^3z}{(2\pi)^3 P^+} e^{ixP^+z^- - ik_\perp\cdot z_\perp} \langle p'|\tilde{F}^{+\mu}(-z/2)\widetilde{W}_\pm F^+_{\ \mu}(z/2)|p\rangle$$
$$= \frac{-i}{2M}\bar{u}(p')\left[\frac{\epsilon_{ij}k^i\Delta^j}{M^2}G^g_{1,1} + \frac{\sigma^{i+}\gamma_5}{P^+}(k^iG^g_{1,2} + \Delta^iG^g_{1,3}) + \sigma^{+-}\gamma_5 G^g_{1,4}\right]u(p)$$

$$xC_g(x) = \int d^2k_\perp \frac{k_\perp^2}{M^2} G^g_{1,1}(x,k_\perp,0)$$

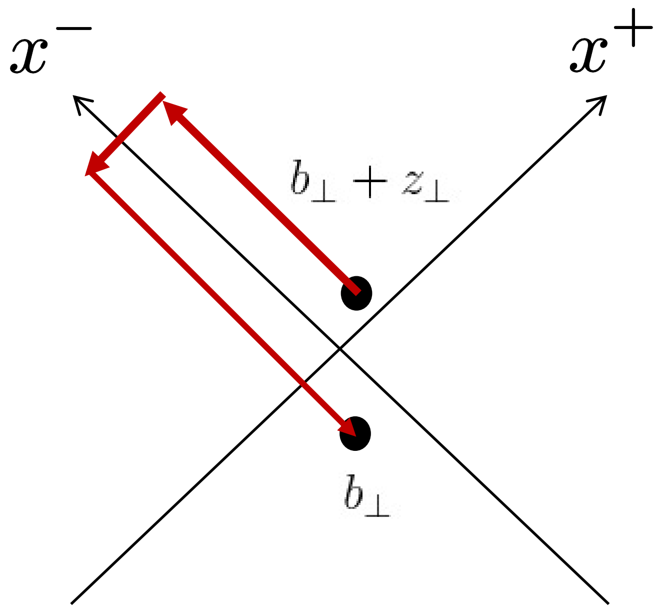
$C_g(x)$ is odd. The first moment vanishes $\int dx C_g(x) = 0$

OAM and spin-orbit correlation

$$L_q(x) = \int dk_\perp db_\perp b_\perp \times k_\perp f_q(x, k_\perp, b_\perp)$$

$$C_q(x) = \int dk_\perp db_\perp b_\perp \times k_\perp \tilde{f}_q(x, k_\perp, b_\perp)$$

γ_5 rotation



Staple-shaped Wilson line

$$k_\perp \to \partial^\mu \to D^\mu - i\frac{1}{D^+}F^{+\mu}$$

Staple-shaped Wilson line

→ Gauge invariant canonical OAM YH (2011)

Twist structure of OAM PDF

YH, Yoshida (2012)

$$L^q_{can}(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'}(H_q(x') + E_q(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\tilde{H}_q(x')$$

Wandzura-Wilczek part

$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \Phi_F(x_1,x_2)\mathcal{P}\frac{3x_1 - x_2}{x_1^2(x_1-x_2)^2}$$

$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Phi}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^2(x_1-x_2)} .$$

genuine twist-3

$$\Phi_F \sim \langle P'|\bar{\psi}\gamma^+ F^{+i}\psi|P\rangle$$

$$M_F \sim \langle P'|F^{+\mu}F^{+i}F^+_\mu|P\rangle$$

$$L^g_{can}(x) = \frac{x}{2}\int_x^{\epsilon(x)} \frac{dx'}{x'^2}(H_g(x') + E_g(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta G(x')$$

$$+2x\int_x^{\epsilon(x)} \frac{dx'}{x'^3}\int dX \Phi_F(X,x') + 2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{M}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^3(x_1-x_2)}$$

$$+2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 M_F(x_1,x_2)\mathcal{P}\frac{2x_1 - x_2}{x_1^3(x_1-x_2)^2}$$

Twist structure of spin-orbit correlation

YH, Schoenleber (2024)

See also,
Rajan, Engelhardt, Liuti (2017)
for the quark part

$$C_q(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'^2} x'\Delta q(x') - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2} q(x')$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \frac{\Psi_{qF}(x_1,x_2)}{x_1-x_2} P\frac{3x_1-x_2}{x_1^2(x_1-x_2)}$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Psi}_{qF}(x_1,x_2) P\frac{1}{x_1^2(x_1-x_2)},$$

$$C_g(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'^2} x'\Delta G(x') - 2x\int_x^{\epsilon(x)} \frac{dx'}{x'^2} G(x') - 4x\sum_q \int_x^{\epsilon(x)} \frac{dx'}{x'^3} \int dX \tilde{\Psi}_{qF}(X,x')$$
$$+4x\int_x^{\epsilon(x)} dx_1 \int dx_2 P\frac{\tilde{N}_F(x_1,x_2)}{x_1^3(x_1-x_2)} + 4x\int_x^{\epsilon(x)} dx_1 \int dx_2 \frac{N_F(x_1,x_2)}{x_1^3(x_1-x_2)} P\frac{2x_1-x_2}{x_1-x_2}$$

Ψ_F, N_F partly related to ETQS and three-gluon distributions relevant to transverse SSA

2 spin sum rules, 1 momentum sum rule?

Spin

$$\frac{1}{2} = \frac{1}{2}\sum_q (A_{q+\bar{q}} + B_{q+\bar{q}}) + \frac{1}{2}(A_g + B_g)$$ Ji (1996)

$$= \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$ Jaffe, Manohar (1990)

Momentum

$$1 = \sum_q A_{q+\bar{q}} + A_g$$ Feynman (1969)

2 spin sum rules, 2 momentum sum rules!

Spin

$$\frac{1}{2} = \frac{1}{2}\sum_q (A_{q+\bar{q}} + B_{q+\bar{q}}) + \frac{1}{2}(A_g + B_g)$$

Ji (1996)

$$= \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Jaffe, Manohar (1990)

Momentum

$$1 = \sum_q A_{q+\bar{q}} + A_g$$

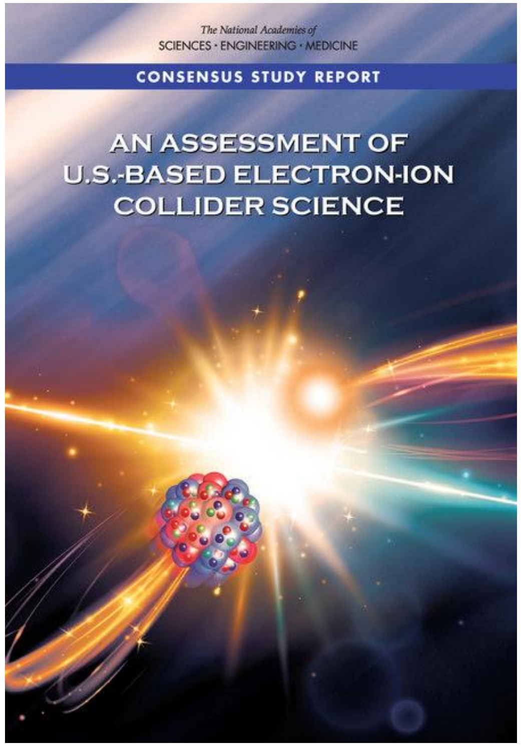
Feynman (1969)

$$= \Delta\Sigma^{(3)} + \frac{1}{2}\Delta G^{(3)} - 3C_q^{(2)} - \frac{3}{2}C_g^{(2)}$$

$$-\frac{3}{2}\int dx dx' \left[\sum_q \left(\frac{2x}{x-x'}\tilde{\Psi}_{Fq}(x,x') + \Psi_{Fq}(x,x')\right) - \frac{\tilde{N}_F(x,x')}{x-x'}\right] + \sum_q \frac{m_q}{M} H_{1q}^{(2)}$$

YH, Schoenleber (2024)

Spin-orbit correlation at small-x



Gluon saturation at small-x:
one of the core topics of EIC

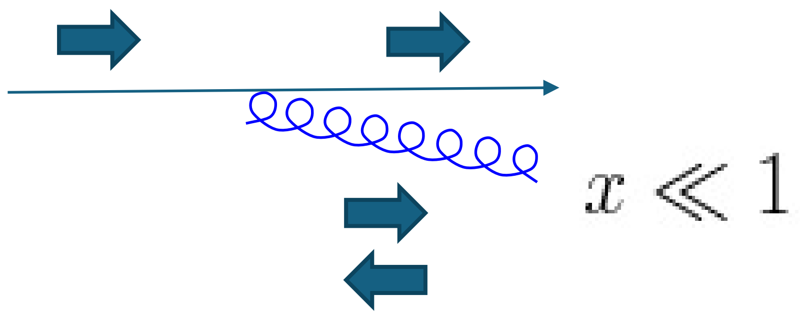
Naively, anything related to helicity, OAM are sub-eikonal at small-x

Finding 1: An EIC can uniquely address three profound questions about nucleons-protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

Intuitive argument

Imagine a very energetic quark emits a soft gluon

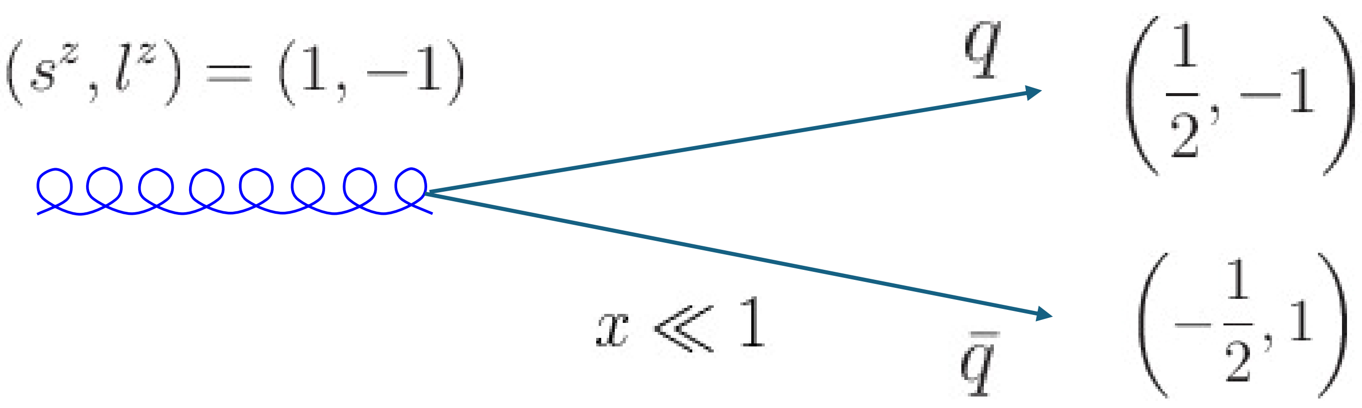


Quark spin and momentum unchanged.

From angular momentum conservation, the total angular momentum of the emitted gluon must be zero

$$(s^z, l^z) = (\pm 1, \mp 1)$$

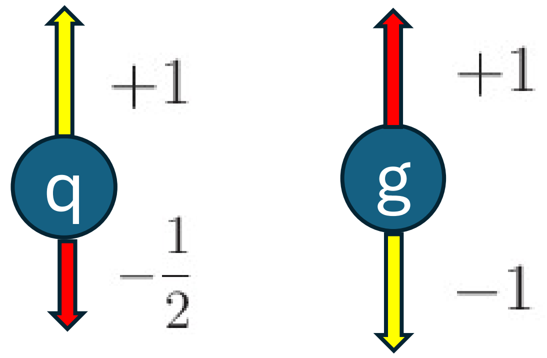
Imagine the emitted soft gluon further splits into a $q\bar{q}$ pair



Same handedness, same OAM

Opposite handedness,
OAM from AM conservation

Helicity and OAM are always in opposite directions
Only $L^z = \pm 1$ states appear at small-x!



Perfect spin-orbit anti-correlation

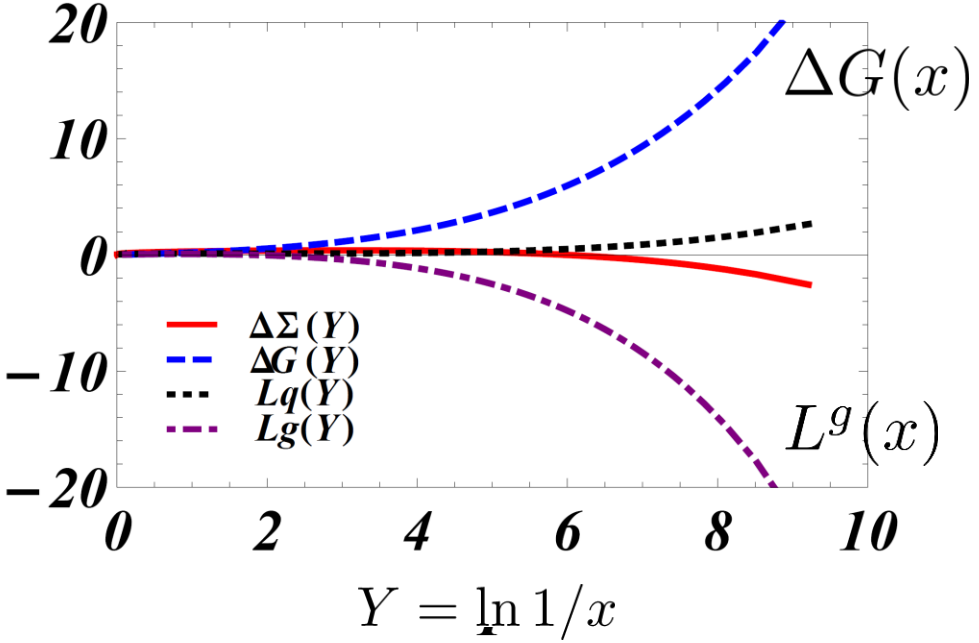
Helicity-OAM cancellation at small-x

If $\Delta q(x) \sim \Delta G(x) \sim \frac{1}{x^c}$, then

Boussarie, YH, Yuan (2019)

$$\Delta q(x) \approx -\frac{1}{1+c} L_q(x), \qquad \Delta G(x) \approx -\frac{2}{1+c} L_g(x),$$

YH, Yang (2018)



More complete treatment Kovchegov, Manley (2023)
Manley (2024)

The correlation exists even in unpolarized/spinless hadrons

Gluon spin-orbit coupling at small-x

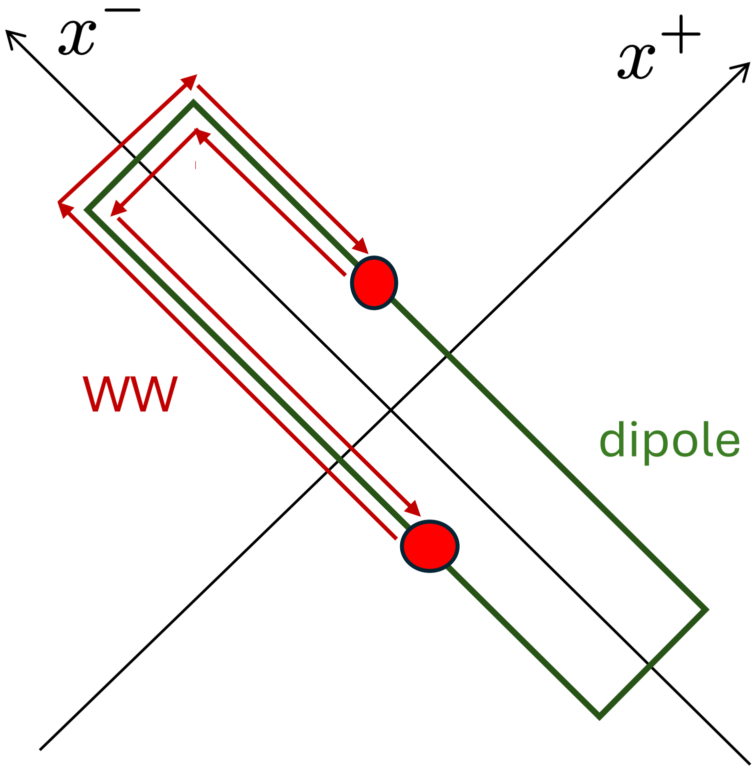
$$\frac{i}{x}\int \frac{d^3z}{(2\pi)^3 P^+} e^{ixP^+z^- - ik_\perp\cdot z_\perp}\langle p'|2\mathrm{Tr}[W_+\tilde{F}^{+\mu}(-z/2)W_\pm F^+_\mu(z/2)]|p\rangle = -i\frac{\epsilon_{ij}k^i_\perp \Delta^j_\perp}{M^2}C_g^{[+\pm]}(x,\xi,k_\perp,\Delta_\perp),$$

There are two inequivalent configurations of Wilson lines

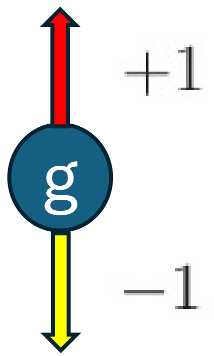
Weiszacker-Williams type
Dipole type

Bomhof, Mulders, Pijlman (2006)
Dominguez, Marquet, Xiao, Yuan (2011)

Approximate $e^{ixP^+z^-} \approx 1$ (eikonal approximation)



Gluon spin-orbit correlation (Dipole type)



$$\epsilon_{ij}\, e^{-i\left(k_\perp-\frac{\Delta_\perp}{2}\right)\cdot z_\perp+i\left(k_\perp+\frac{\Delta_\perp}{2}\right)\cdot w_\perp}\,\mathrm{Tr}\partial^i U(w_\perp)\partial^j U^\dagger(z_\perp)$$

$$\longrightarrow\quad \epsilon_{ij}k_\perp^i\Delta_\perp^j\,\mathrm{Tr}U(w_\perp)U^\dagger(z_\perp)$$

$$\frac{xC_g^{\mathrm{dip}}(x,k_\perp)}{M^2}=-\frac{2N_c}{\alpha_s}\int\frac{d^2w_\perp d^2z_\perp}{(2\pi)^4}e^{-ik_\perp\cdot(z_\perp-w_\perp)}\frac{\langle p|\frac{1}{N_c}\mathrm{Tr}U(w_\perp)U^\dagger(z_\perp)-1|p\rangle}{\langle p|p\rangle}$$

cf. Boer, van Daal, Mulders, Petreska (2018)

$$C_g^{\mathrm{dip}}(x)=-G(x)\qquad -1\times 1=-1$$ times the number of gluons

Spin-orbit correlation survives the eikonal approximation

Gluon spin-orbit correlation (WW type)

$$\epsilon_{ij}\,\mathrm{Tr}[U^\dagger(w_\perp)\partial_i U(w_\perp)U^\dagger(z_\perp)\partial_j U(z_\perp)]$$

Work in the MV model cf. Jamal-Jalilian, Kovner, McLerran, Weigert (1997)

$$k_\perp^2 \frac{C_g^{\mathrm{WW}}(x,k_\perp)}{M^2} = -f_g^{\mathrm{WW}}(x,k_\perp) - \frac{C_F}{\pi\alpha_s x}\int \frac{d^2b_\perp d^2r_\perp}{(2\pi)^3} e^{-ik_\perp\cdot r_\perp}\partial_i^r D(r_\perp)\partial_i^r\left(\frac{1-e^{\frac{N_c}{C_F}D(r_\perp)}}{D(r_\perp)}\right)$$

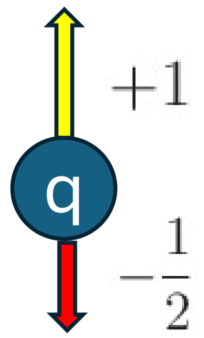
$$C_g^{\mathrm{WW}}(x) = -G(x)$$

$C_g^{\mathrm{WW}}(x) = C_g^{\mathrm{dip}}(x)$ exactly for any x

cf. YH, Nakagawa, Xiao, Yuan, Zhao (2016)

$$L_g^{\mathrm{dip}}(x) = L_g^{\mathrm{WW}}(x)$$

Quark spin-orbit coupling at small-x

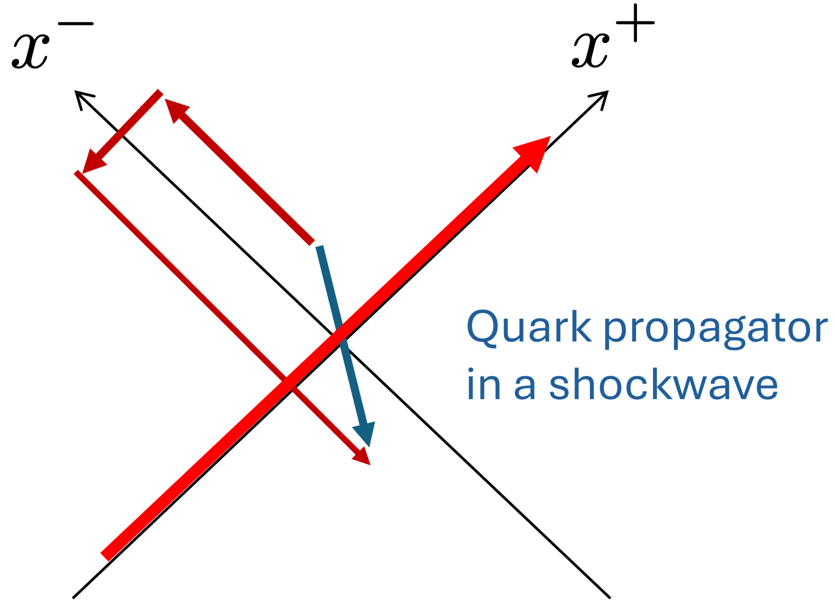


$$\int \frac{d^3z}{2(2\pi)^3} e^{ixP^+z^- - ik_\perp\cdot z_\perp} \langle p'|\bar{\psi}(-z/2)\gamma^+\gamma_5 W_\pm \psi(z/2)|p\rangle = -i\frac{\epsilon_{ij}k_\perp^i \Delta_\perp^j}{M^2} C_q(x,\xi,k_\perp,\Delta_\perp)$$

$$\frac{C_q(x,k_\perp)}{M^2} = \frac{N_c S_\perp}{8\pi^4 x k_\perp^2}\int d^2k_{g\perp}(k_\perp - k_{g\perp})\cdot k_\perp \frac{\ln\frac{k_\perp^2}{(k_\perp - k_{g\perp})^2}}{k_\perp^2-(k_\perp-k_{g\perp})^2} \frac{\langle p|\left(\frac{1}{N_c}\mathrm{Tr}UU^\dagger - 1\right)(k_{g\perp})|p\rangle}{\langle p|p\rangle}$$

$$C_q(x) = -\frac{1}{2}q(x)$$

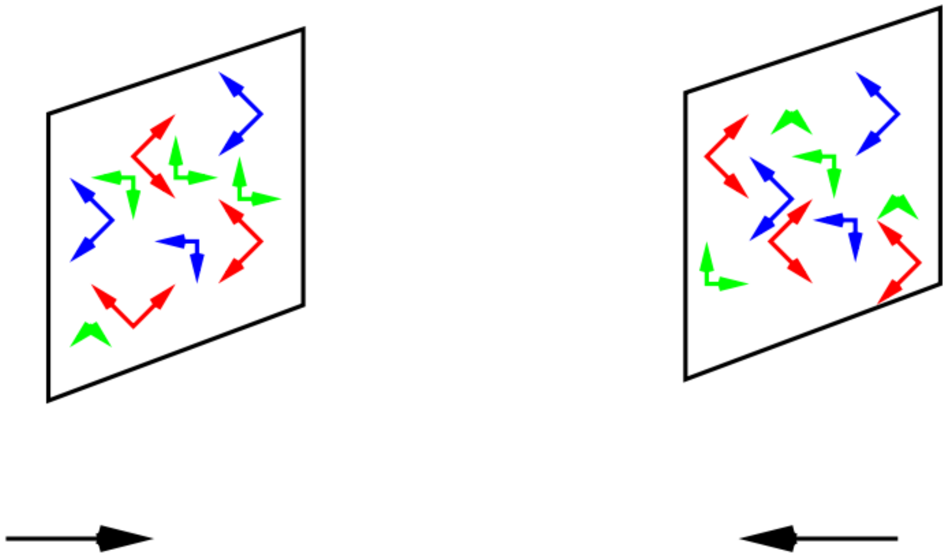
$-\frac{1}{2}\times 1 = -\frac{1}{2}$ times the number of quarks



Linearly polarized gluons

Small-x gluons are Weiszacker-Williams fields (equivalent photon approximation, boosted Coulomb field) of large-x quarks

McLerran, Venugopalan (1993)



Electric & magnetic field confined in the transverse plane.

Soft gluons are linearly polarized

$$\frac{1}{\sqrt{2}}\left(|+\rangle_s + |-\rangle_s\right), \qquad \frac{1}{\sqrt{2i}}\left(|+\rangle_s - |-\rangle_s\right)$$

Quantum entanglement of spin and OAM

Implement perfect spin-orbit anti-correlation

` Bell states'

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_s|-\rangle_l + |-\rangle_s|+\rangle_l\right), \qquad |\Psi^-\rangle = \frac{1}{\sqrt{2}i}\left(|+\rangle_s|-\rangle_l - |-\rangle_s|+\rangle_l\right)$$

Maximally entangled state realized on each soft gluon!

$\langle S^z\rangle = \langle L^z\rangle = 0$ but $\langle S^z L^z\rangle = -1$

True nature of the system encoded in correlations

Quantum entanglement of the spin and orbital angular momentum of photons using metamaterials

TOMER STAV, ARKADY FAERMAN, ELHANAN MAGUID, DIKLA OREN, [...], AND MORDECHAI SEGEV +2 authors Authors Info & Affiliations

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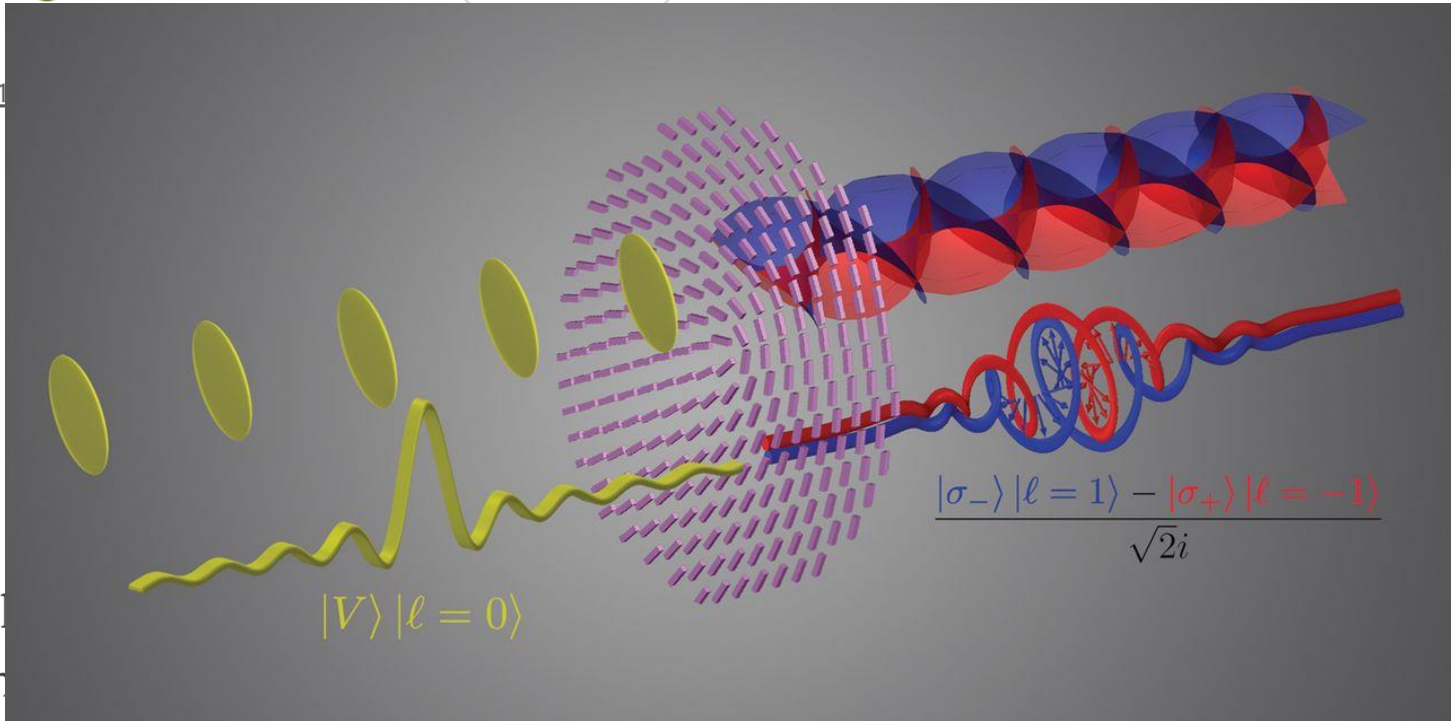


Abstract

Metamaterials constructed from deep subwavel
phenomena ranging from negative refractive in
eral relativity, and superresolution imaging. More recently, metamaterials have been suggested as a new platform for quantum optics. We present the use of a dielectric metasurface to generate entanglement between the spin and orbital angular momentum of photons. We demonstrate the genera-

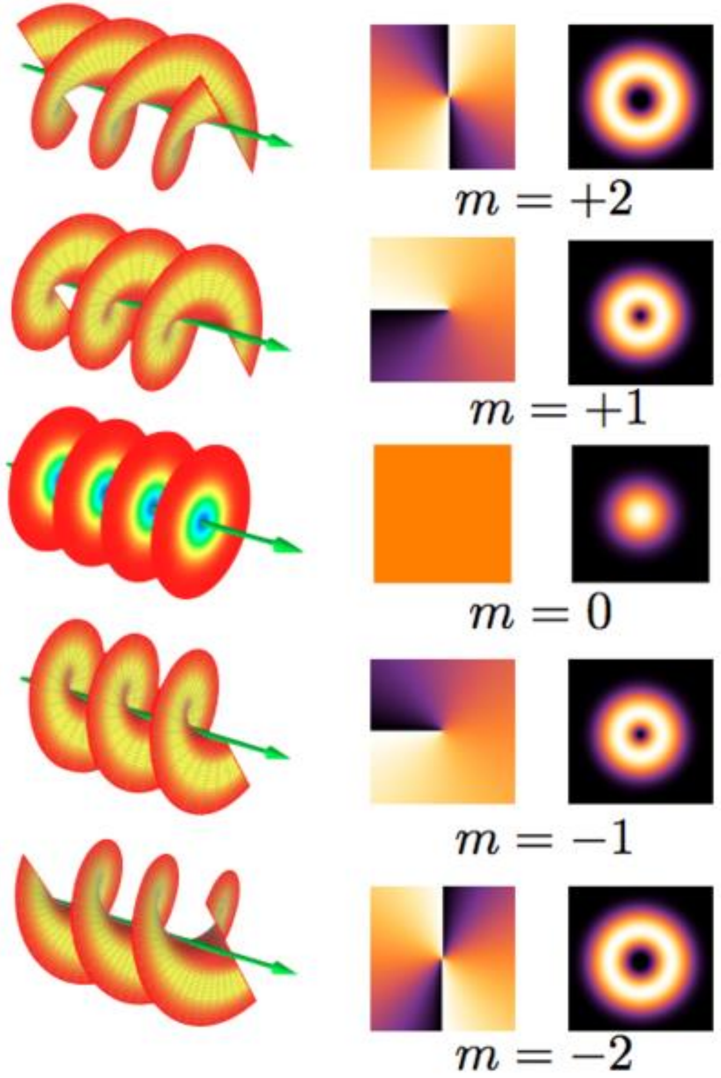
In QCD, spin-orbit entanglement is a default property of soft gluons!

QED example

Photon OAM

$$|\pm\rangle_l \sim e^{\pm i\phi}$$

e.g., Laguerre-Gaussian beam



$$|\Psi^+\rangle \sim (1,i)e^{-i\phi} + (1,-i)e^{i\phi} \sim (\cos\phi, \sin\phi)$$

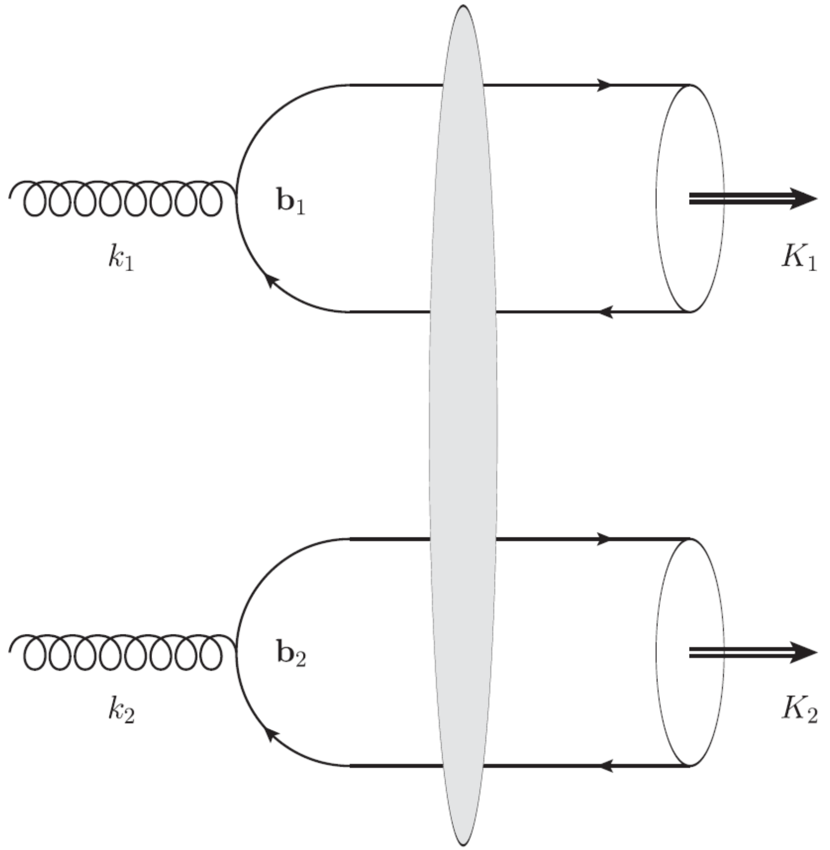
$$|\Psi^-\rangle \sim -i\left((1,i)e^{-i\phi} - (1,-i)e^{i\phi}\right) \sim (-\sin\phi, \cos\phi)$$

Experimental observable

Quark and gluon GTMDs $G_{1,1}$ appeared in certain exclusive reactions, but no quantitative estimate made.

e.g., double quarkonia production in pp

Boussarie, YH, Xiao, Yuan (2017)



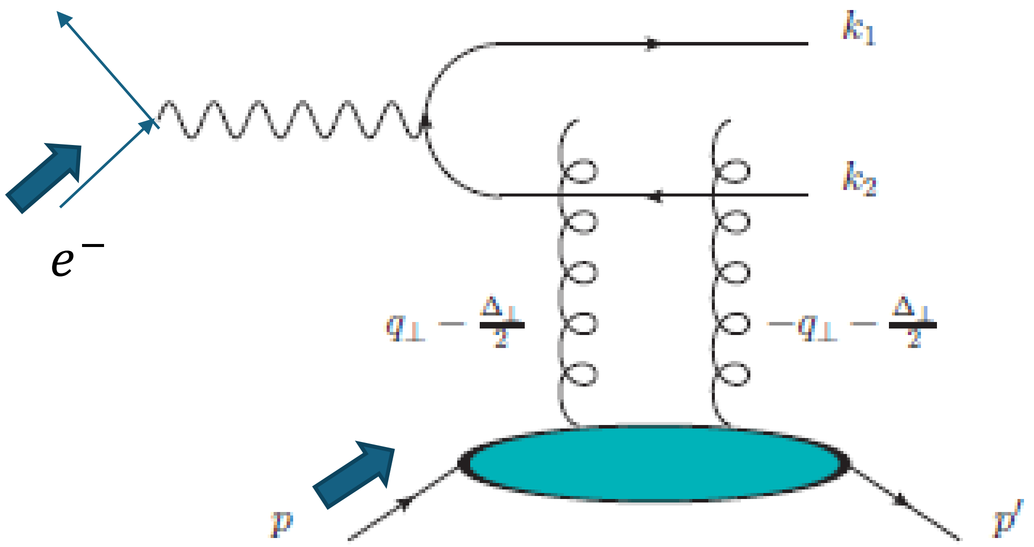
$$\int_0^{2\pi} \frac{d\phi}{2\pi} \frac{d\sigma(\chi_1, \chi_0)}{dY_1 dY_2 d^2\boldsymbol{K} d\boldsymbol{\Delta}^2} = \frac{3\alpha_s^4 x^2 \boldsymbol{K}^2}{32 m_1^9 m_2^7 N_c^4 (N_c^2 - 1)^2} x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2) \langle \mathcal{O}_{\chi_1}(^3P_1^1)\rangle \langle \mathcal{O}_{\chi_0}(^3P_0^1)\rangle$$

$$\times \left[\left(\mathcal{G}_1 + \frac{\boldsymbol{K}^2}{2M^2} \mathcal{G}_2 \right)^2 + \frac{\boldsymbol{\Delta}^2}{4\boldsymbol{K}^2} \left(\mathcal{G}_1^2 + \frac{\boldsymbol{K}^4}{4M^4} (\mathcal{G}_2^2 - 8\mathcal{G}_2\mathcal{G}_4 + 8\mathcal{G}_4^2) \right) \right]$$

Longitudinal double spin asymmetry in diffractive dijets

Bhattacharya, Boussarie, YH, (2022)

previously proposed as a signal of gluon OAM



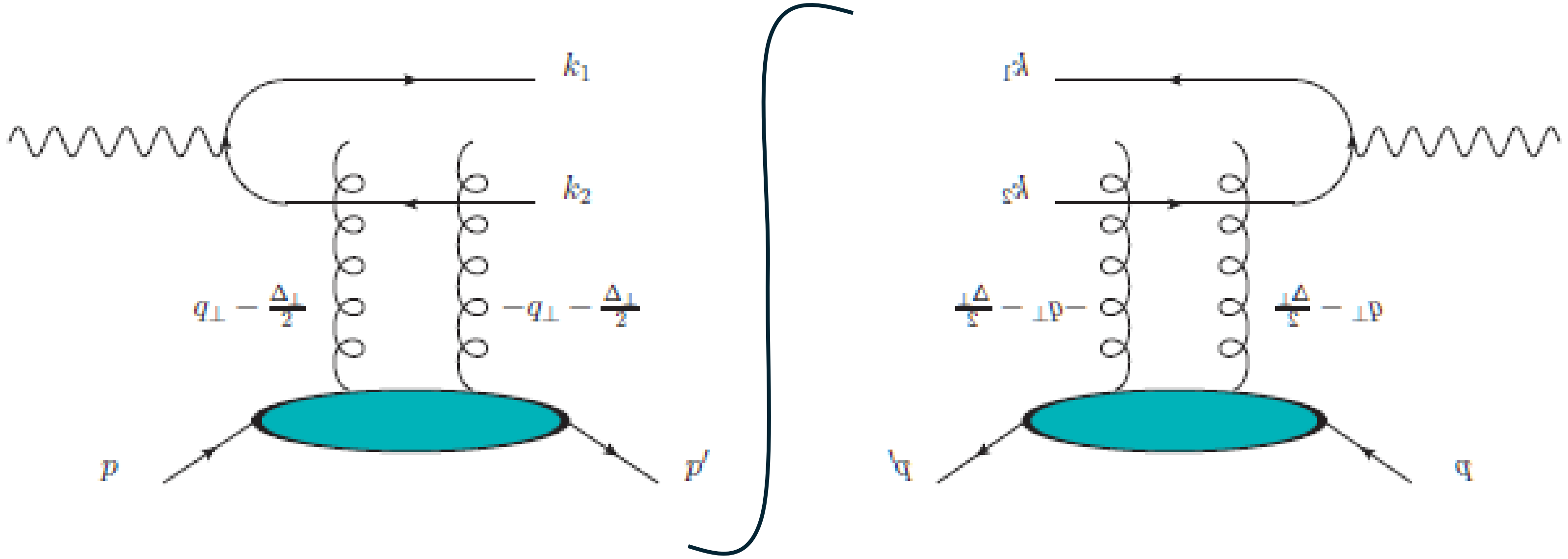
$$L^z \sim b_\perp \times k_\perp$$

conjugate to $\Delta_\perp$ proton recoil momentum

correlated with jet transverse momentum

$$d\sigma^{h_p h_l} \sim h_p h_l \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \mathrm{Re}(iA_L^{2*} A_T^{3i} - iA_T^{2i*} A_L^3)$$

Spin, orbit, and spin-orbit



$\mathcal{H}_g$	$\mathcal{L}_g$	OAM	computed in our 2022 paper
$\mathcal{H}_g$	$\tilde{\mathcal{H}}_g$	helicity	
$\tilde{\mathcal{H}}_g$	$\mathcal{C}_g$	spin-orbit correlation	→ previously neglected

Cross section

$$\frac{d\sigma_{\rm DSA}}{dydQ^2d\phi_{l_\perp}dzdq_\perp^2d^2\Delta_\perp} = \frac{e_q^2\alpha_{em}^2\alpha_s^2 y}{2\pi^3 Q^2 N_c}\frac{\xi(1+\xi)|l_\perp||\Delta_\perp|\cos(\phi_{l_\perp}-\phi_{\Delta_\perp})}{z\bar{z}(W^2+Q^2)(W^2-M_J^2)(q_\perp^2+\mu^2)^2}\mathrm{Re}(\Sigma_L+\Sigma_O+\Sigma_h+\Sigma_C)$$

$$\Sigma_L = -\left(\mathcal{F}_g^{(1)*}+4(1-\beta)\mathcal{F}_g^{(2)*}\right)\mathcal{L}_g^{(2)}+(z-\bar{z})^2\mathcal{F}_g^{(1)*}\left(\mathcal{L}_g^{(2)}+8(1-\beta)\mathcal{L}_g^{(3)}\right)$$

$$\Sigma_h = (1-\xi)\mathcal{F}_g^{(1)*}\tilde{\mathcal{F}}_g^{(2)}-(1-\xi)(z-\bar{z})^2\Big[8(1-\beta)\left(\mathcal{F}_g^{(1)*}+4(1-\beta)\mathcal{F}_g^{(2)*}\right)\tilde{\mathcal{F}}_g^{(3)} + \left((4\beta-3)\mathcal{F}_g^{(1)*}+16(1-\beta)\mathcal{F}_g^{(2)*}+32(1-\beta)^2\mathcal{F}_g^{(3)*}\right)\tilde{\mathcal{F}}_g^{(2)}\Big],$$

$$\Sigma_C = 2(1-\beta)\left(2\mathcal{C}_g^{(2)*}\tilde{\mathcal{F}}_g^{(2)}+\xi\mathcal{C}_g'^{(2)*}\tilde{\mathcal{E}}_g^{(2)}\right)$$

GPD moments with third pole at $x=\pm\xi$

$$\mathcal{H}_g^{(3)}(\xi) = \int_{-1}^{1}dx\frac{\xi^4 H_g(x,\xi)}{(x^2-\xi^2)^3},$$

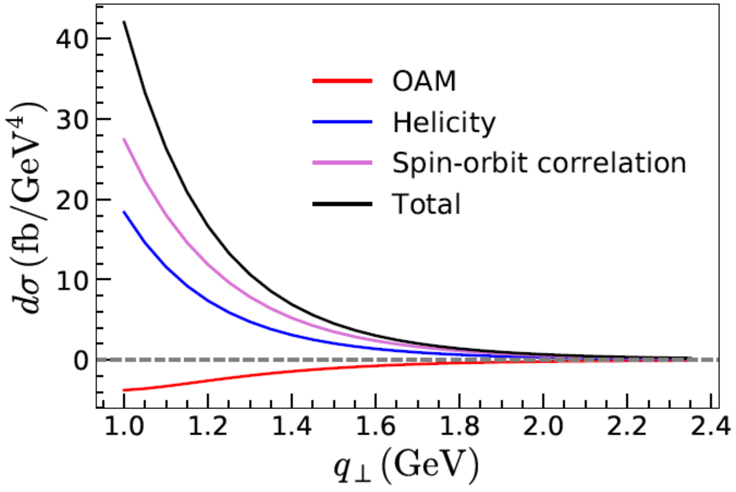
→ Potential factorization breaking

Proportional to $(z-\bar{z})^2$

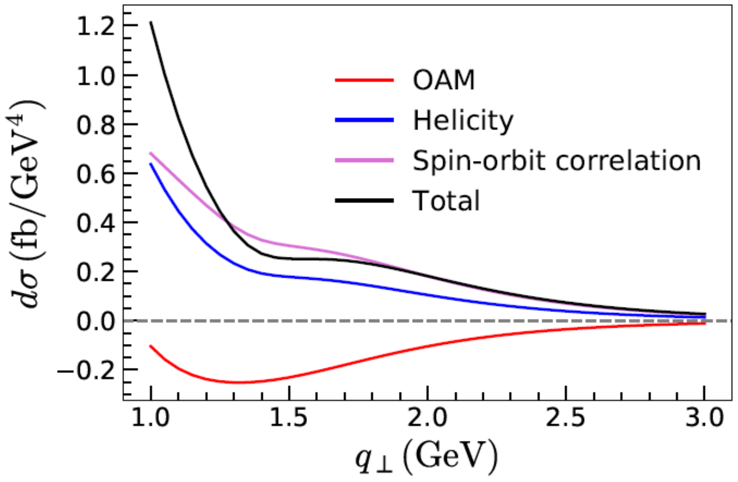
Focus on the region $z\approx\bar{z}$

Prediction at the EIC (revised)

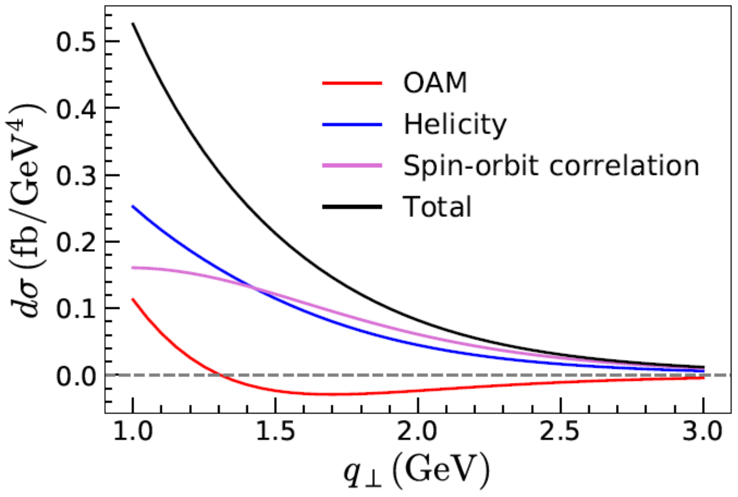
Bhattacharya, Boussarie, YH (2024)



$$Q^2 = 2.7\,\mathrm{GeV}^2$$



$$Q^2 = 4.8\,\mathrm{GeV}^2$$

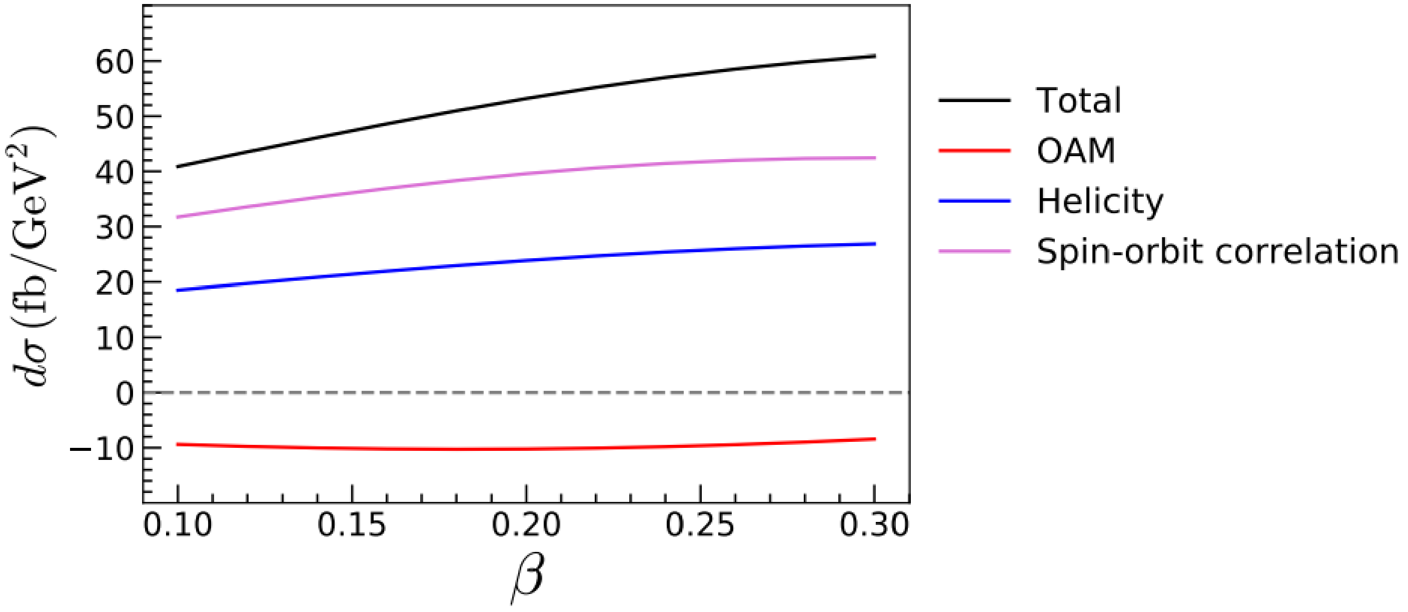


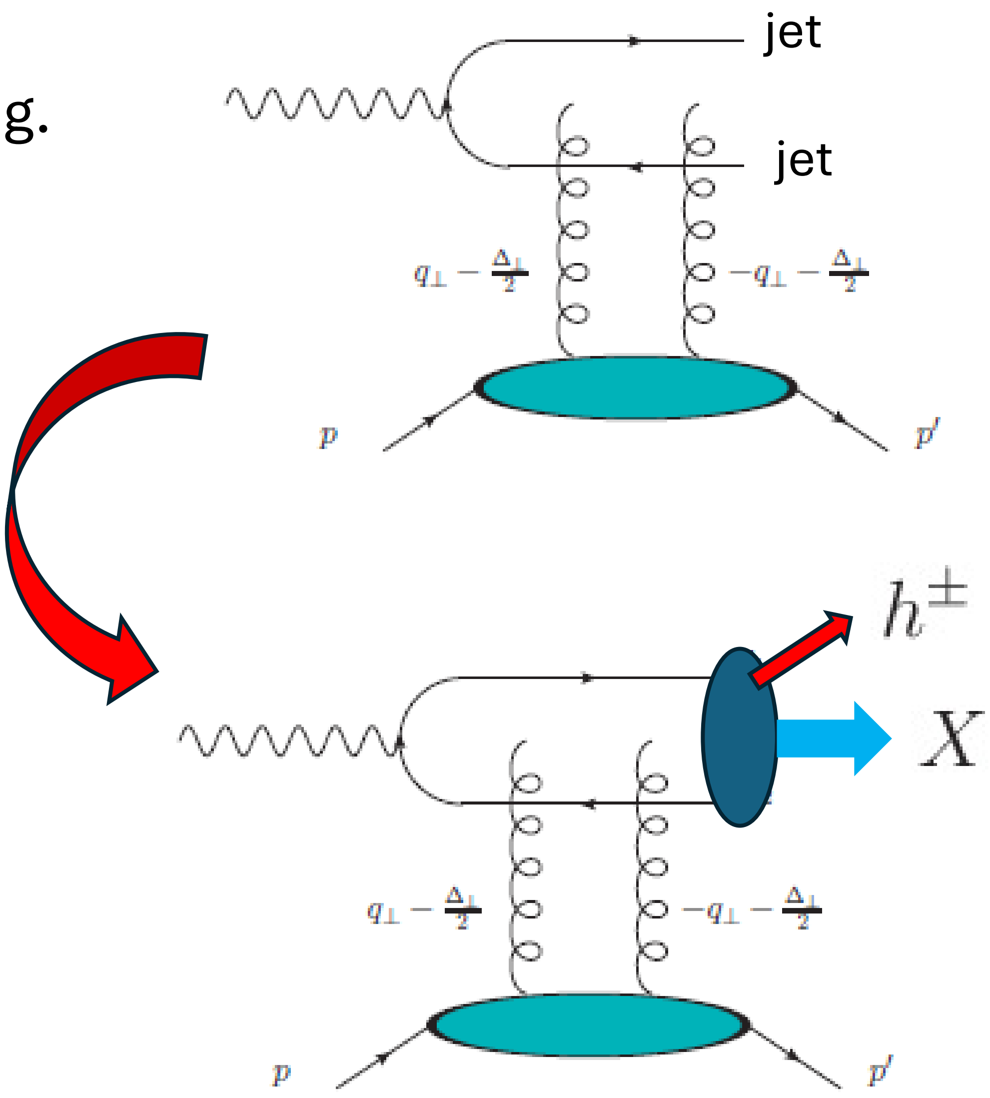
$$Q^2 = 10\,\mathrm{GeV}^2$$

Semi-inclusive diffractive DIS (SIDDIS)

In practice, jet reconstruction at low-Pt is challenging.
Re-interpret as SIDDIS

YH, Xiao, Yuan (2022)





Conclusions

Quark and gluon spin-orbit correlations analyzed in QCD

New momentum sum rule, analog of Jaffe-Manohar spin sum rule

Novel emergent property of dense systems of gluons uncovered

Quantum entanglement between spin and OAM

→ Connection to QIS? EIC?

Finding 1: An EIC can uniquely address three profound questions about nucleons-protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?