Small-x Helicity Phenomenology

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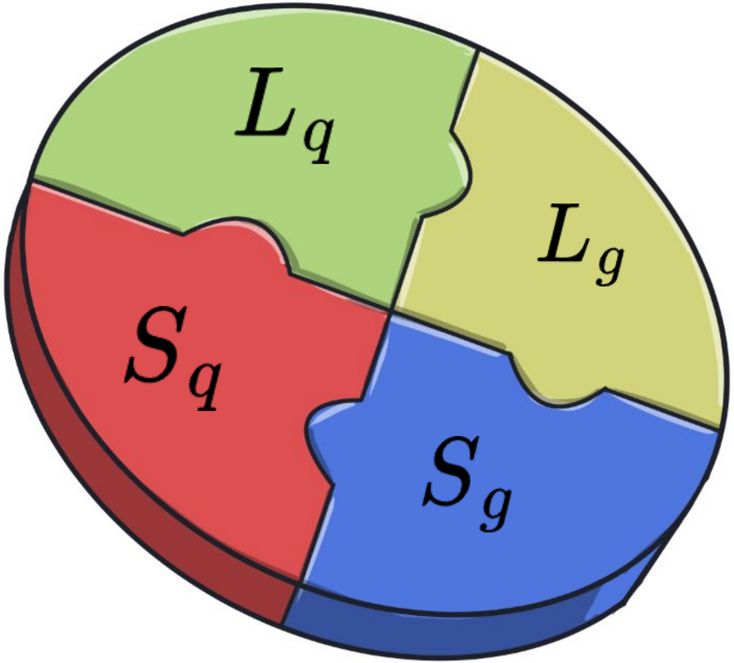
Beyond-Eikonal Methods in High-Energy Scattering

Proton Spin Puzzle

Jaffe-Manohar Spin Sum Rule: $\frac{1}{2} = S_q + L_q + S_g + L_g$

$$\frac{1}{2} =$$



$S_{q,g}$ = Helicity of quarks and gluons

$L_{q,g}$ = Orbital angular momentum

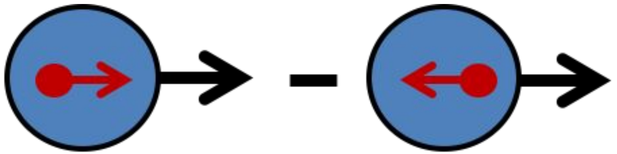
Quark Helicity Parton Distribution Functions (hPDFs)

Net quark spin　　Quark hPDF　　Anti-quark hPDF　　Singlet hPDF

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx \sum_q \left(\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2)\right) = \frac{1}{2}\int_0^1 \Delta\Sigma(x, Q^2)$$

Helicity PDFs:

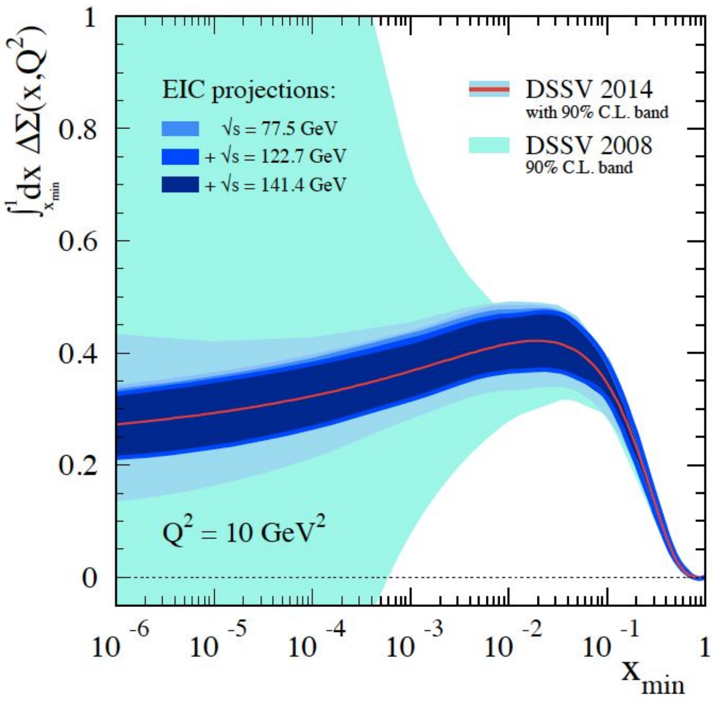
$$\Delta q =$$



- Q^2 = resolution at which we probe the proton
- Longitudinal momentum fraction, Bjorken $x \sim \frac{1}{s}$. We need theory to extrapolate to *x*=0

Quark hPDF - DGLAP extraction

2 x (quark spin)



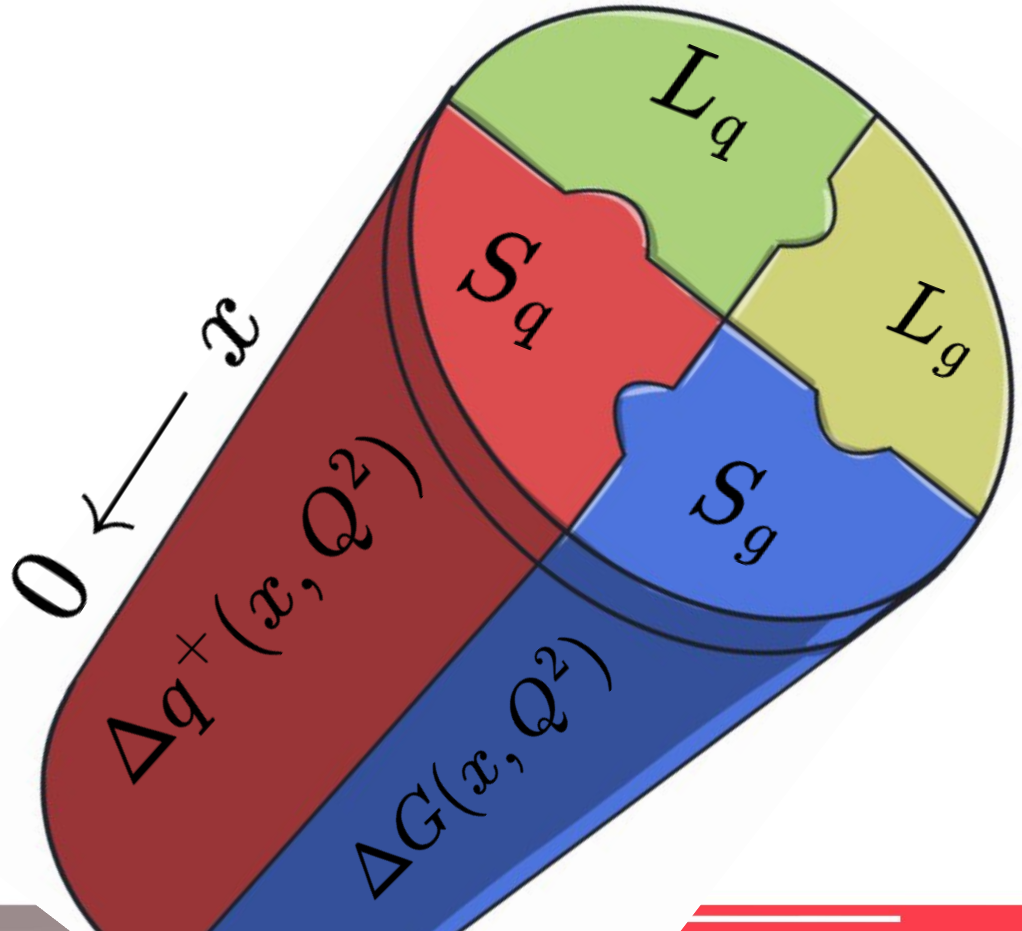
$$\Delta\Sigma = \sum_q (\Delta q + \Delta \bar{q})$$

$$\Delta\Sigma(x, Q_0^2) = N x^{\alpha}(1 - x^{\beta})(1 + \eta x^{\kappa})$$

$$\Delta\Sigma(x, Q_0^2) \xrightarrow{\text{DGLAP}} \Delta\Sigma(x, Q^2)$$

- E. Aschenauer et al, **arXiv:1509.06489 [hep-ph]**, (DSSV = de Florian, Sassot, Stratmann, Vogelsang, DGLAP-based helicity PDF extraction from data)
- **Large uncertainty at small-x!**

$$\frac{1}{2} =$$



$$\Delta q^+ = \Delta q + \Delta \bar{q}$$

$\Delta G =$ Gluon hPDF

If we want to resolve the proton spin puzzle through determining the helicity PDFs we **must** understand their small-x behaviour.

We need small-x theory

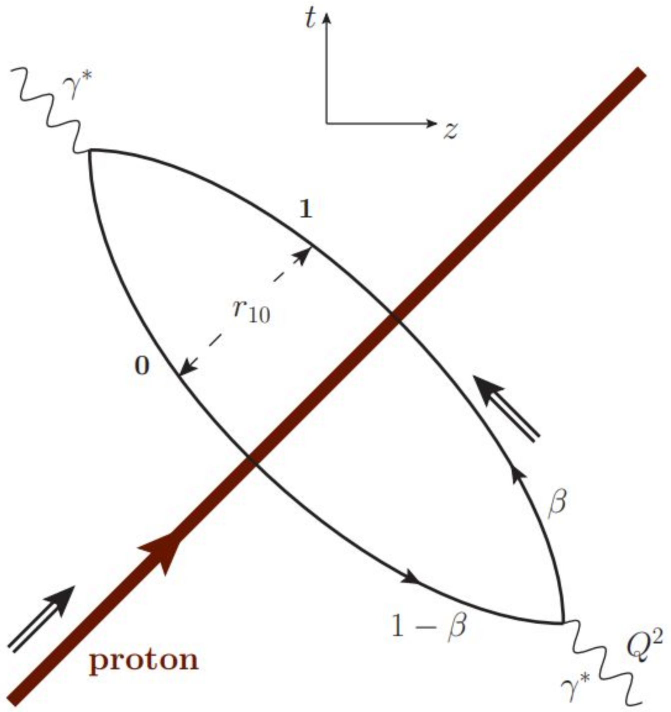
This theory must be validated through experiments capable of measuring small-x polarized observables - a job for the EIC

DIS in the Dipole Picture

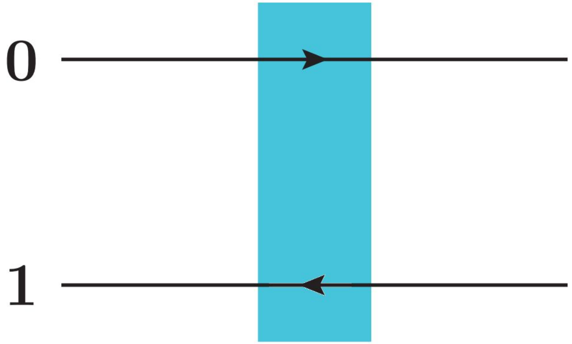
$$q^{\mu} = \left(\frac{-Q^2}{2q^-}, q^-, \underline{q} \right) \quad , \quad \boldsymbol{q} \cdot \boldsymbol{x} = \boldsymbol{q}^- \boldsymbol{x}^+ - \frac{\boldsymbol{Q}^2}{2\boldsymbol{q}^-} \boldsymbol{x}^- - \underline{\boldsymbol{q}} \cdot \underline{\boldsymbol{x}}$$

Large q^- □ large x^- separation

$$\Rightarrow \boldsymbol{F_2} \propto |\psi|^2 \otimes N(x, Q^2)$$

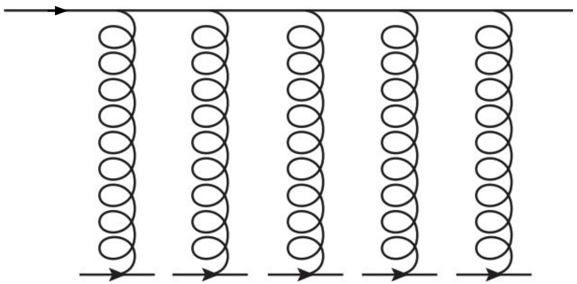


DIS in the Dipole Picture



- The proton is a shockwave
- Working in light-cone gauge $A^{-} = 0$
- Unpolarized structure functions F1, F2 proportional to Dipole Amplitude
- $N(s) = 1 - \frac{1}{N_c} \langle \mathrm{tr}[V_{\underline{1}} V_{\underline{0}}^{\dagger}] \rangle (s)$

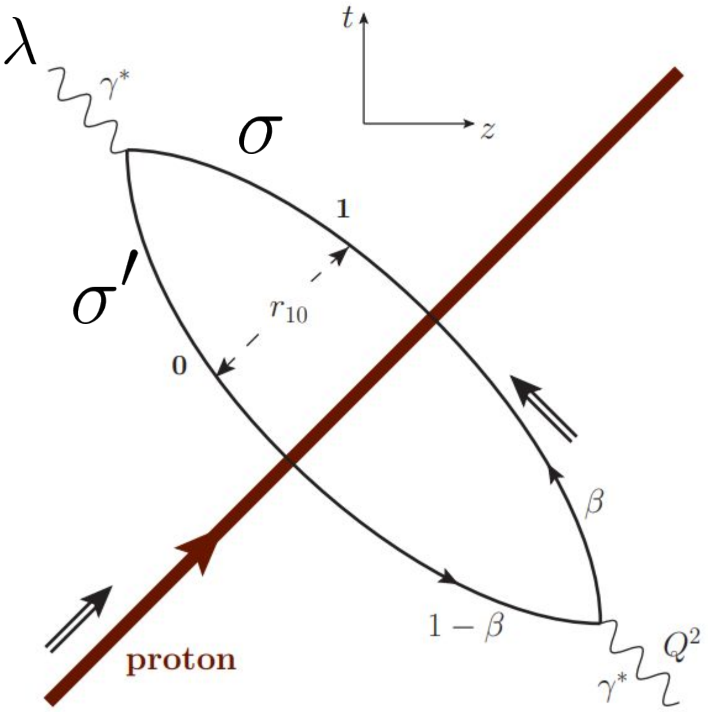
- Quarks undergo multiple eikonal re-scatterings
- Quark lines replaced with Wilson Lines



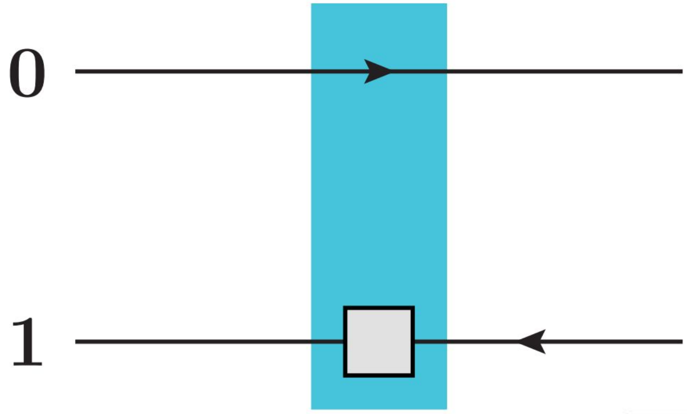
- $V_{\underline{1}}[x_f^-, x_i^-] \equiv V_{\underline{x}_1}[x_f^-, x_i^-] = \mathcal{P} \exp \left[ig \int\limits_{x_i^-}^{x_f^-} dx^- \, A^+(0^+, x^-, \underline{x}_1) \right]$

(Polarized) DIS in the (Polarized) Dipole Picture

$$g_1 \propto |\psi|^2 \otimes (Q + 2G_2)$$



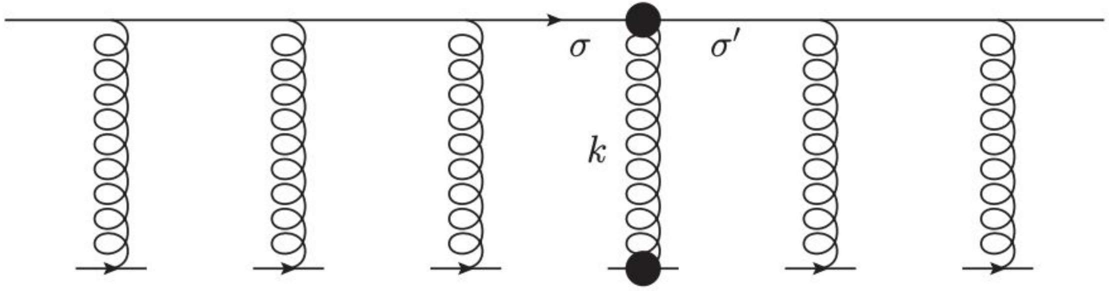
(Polarized) DIS in the (Polarized) Dipole Picture



- In pDIS, the electron and proton have their helicity specified
- Cross-section now dependent on **Polarized Dipole Amplitudes**:

$$\left\langle \mathrm{tr}\left[V_{\underline{1}} V_{\underline{0}}^{\mathrm{pol}\dagger}\right] \right\rangle (s)$$

- Quark line undergoes one extra exchange, containing helicity information, which is **sub-eikonal**



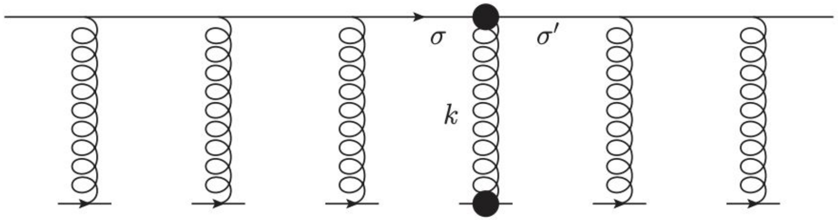
Sub-eikonal Expansion

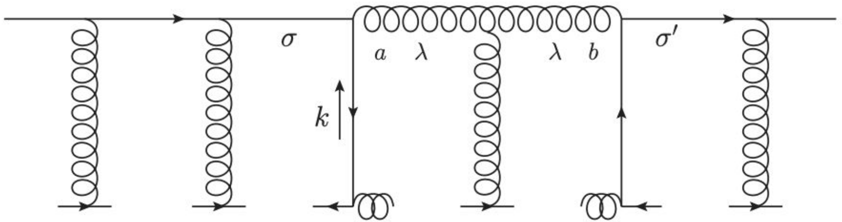
- Expansion in energy or in *x*

$1/x,$	$x^0,$	x^1
Eikonal	Sub-Eikonal	Sub-Sub-Eikonal
F_1, F_2	$g_1^{p,n}, \Delta q, \Delta \bar{q}$	Transversity

- No eikonal terms contain any helicity information - Wilson lines are helicity independent
- Must calculate sub-eikonal terms to access helicity

Polarized Wilson Lines





$$V_{\mathbf{x}}^{\mathrm{pol}} = \int dz^- V_{\mathbf{x}}(\infty, z^-)\Gamma V_{\mathbf{x}}(z^-, -\infty)$$

$\Gamma :$

$\bar{\psi}\gamma^+\gamma^5\psi$

- Axial Current

$\sim F_{12}$

- Chromo-magnetic field

$e^{-ix^-\frac{k_\perp^2}{2k^-}} \approx 1 - ix^-\frac{k_\perp^2}{2k^-}$

- Sub-eikonal phase expansio

$A_\perp$

- Polarized gluon vertex

For helicity, 3 polarized dipoles enter:

G_2 "Gluon dipole". Contains $e^{-ix^- \frac{k_\perp^2}{2k^-}}$, $A_\perp$

Q_q "Quark dipole". Contains $\bar{\psi}\gamma^+\gamma^5\psi$, $\sim F_{12}$

$\widetilde{G}$ "Adjoint dipole". Contains the same operators as Q, but inserted into adjoint Wilson lines instead of fundamental

+ neighbour dipoles, $\Gamma_2, \Gamma_q, \widetilde{\Gamma}$, that enforce lifetime ordering

Operator definition of quark helicity TMD

$$g_{1L}^{q}(x,k_T^2) = \frac{1}{(2\pi)^3}\frac{1}{2}\sum_{S_L} S_L \int d^2r\,dr^- e^{ik\cdot r} \times \langle pS_L|\bar{\psi}(0)\mathcal{U}[0,r]\frac{\gamma^+\gamma^5}{2}\psi(r)|pS_L\rangle_{r^+=0}$$

$$\left\langle \mathrm{tr}\left[V_{\underline{1}}V_{\underline{0}}^{\mathrm{pol}\dagger}\right]\right\rangle(s)$$

- The gauge link in $A^- = 0$ gauge can be written as an infinite Wilson line
- The contraction of the quark fields gives a quark propagator which contains a Wilson line
- Expand one of the Wilson lines in eikonality and contract against Dirac matrix to pick out relevant polarized dipoles

Quark Singlet Helicity Distribution

At small-*x*, the operator definitions of the gluon, singlet and non-singlet helicity distributions coincide with the operator definitions of the polarized dipole amplitudes

$$\Delta\Sigma(x, Q^2) = -\frac{N_c N_f}{2\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2}, \frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \left[Q(x_{10}^2, zs) + 2\, G_2(x_{10}^2, zs)\right]$$

Q, G_2 are Polarized Dipole Amplitudes defined in terms of the quark axial current and covariant derivative operators respectively.

Gluon Helicity Distribution

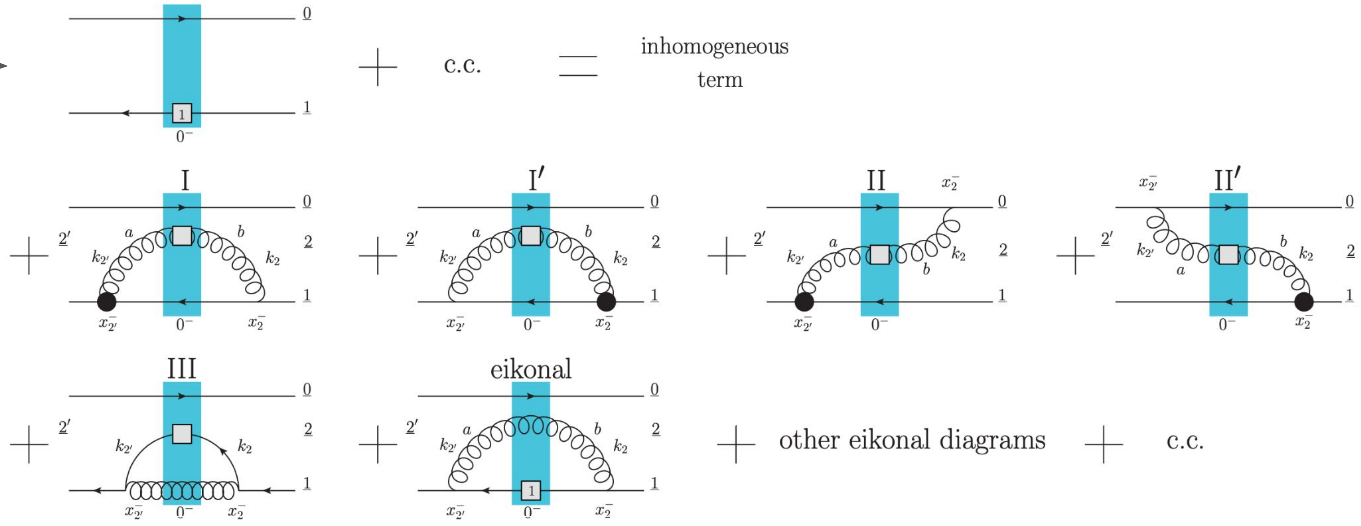
$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s(Q^2)\pi^2} G_2 \left(x_{10}^2 = \frac{1}{Q^2}, zs = \frac{Q^2}{x} \right)$$

- G_2 satisfies the Jaffe-Manohar Gluon Helicity Distribution definition

Small-*x* / Rapidity Evolution

- Relate Polarized Dipole Amplitude to themselves at higher energies by resumming emission diagrams - resumming Double Log (DLA) contributions: $\alpha_s \ln^2(1/x)$

$Q_q(x_{10}^2, zs)$ →



Double Logarithm Approximation

Using Light-Cone Operator Treatment, we need to resum all gluon exchanges that exchange helicity information

$\Rightarrow$ Resumming all terms containing: $$\alpha_s \int_x^1 \frac{dz}{z} \int_{1/s}^{1/Q^2} \frac{d^2 x_{21}}{x_{21}^2}$$

Longitudinal part. Present in un-polarized evolution

Transverse part. UV exactly cancelled in un-polarized evolution

Resum double log (DLA) terms:

$$\alpha_s \ln^2(1/x)$$

Large Nc&Nf, Linearized Helicity Evolution

In the large Nc&Nf, Nc/Nf fixed limit, the evolution equations for the polarized dipole amplitudes close:

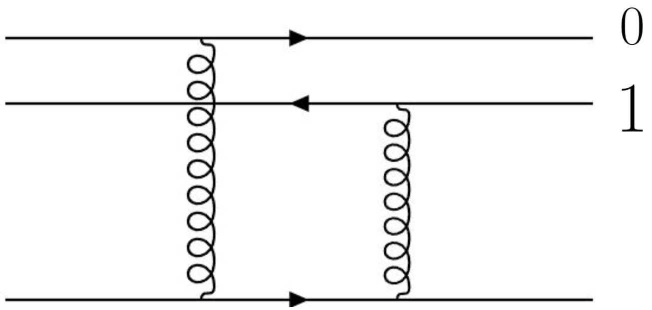
$$Q_q(s_{10},\eta) = Q_q^{(0)}(s_{10},\eta) + \int_{s_{10}+y_0}^{\eta} d\eta' \int_{s_{10}}^{\eta'-y_0} ds_{21}\, \alpha_s(s_{21}) \Big[Q_q(s_{21},\eta') + 2\tilde{G}(s_{21},\eta') + 2\tilde{\Gamma}(s_{10},s_{21},\eta') - \bar{\Gamma}_q(s_{10},s_{21},\eta') + 2G_2(s_{21},\eta') + 2\Gamma_2(s_{10},s_{21},\eta') \Big] + \frac{1}{2}\int_{y_0}^{\eta} d\eta' \int_{\max\{0,s_{10}+\eta'-\eta\}}^{\eta'-y_0} ds_{21}\, \alpha_s(s_{21}) \Big[Q_q(s_{21},\eta') + 2G_2(s_{21},\eta') \Big]$$

+ 9 more

- 5 Polarized dipole amplitudes mix under evolution: $Q_{u,d,s}, \tilde{G}, G_2$
- With 5 auxiliary dipoles: $\Gamma_{u,d,s}, \tilde{\Gamma}, \Gamma_2$ - which impose lifetime ordering
- Small-*x* cutoff, $y_0 \propto \ln 1/x_0$

Inhomogeneous term

The inhomogeneous term is given by a Born-inspired ansatz:



$$\propto \int_0^s \frac{dk_\perp^2}{k_\perp^2}\left(1 - e^{-\underline{k}\cdot\underline{x}_{10}}\right) = \pi \ln(s x_{10}^2)$$

$$\propto \eta - s_{10}$$

$$\Gamma_q^{(0)} = Q_q^{(0)} = a_q \eta + b_q s_{10} + c_q$$

- Same form of the other Dipole Amplitudes
- Parameters a,b,c need to be extracted from data
- 15 parameters for singlet hPDFS

Large Nc&Nf Helicity Evolution

- At large Nc&Nf and in the Double Log approximation, a finite number of polarized dipole amplitudes mix in a **closed system** of equations
- These are amenable to **numeric solutions**
- Undetermined initial conditions: $Q_q^{(0)} = a_q \eta + b_q s_{10} + c_q$
- Initial conditions extracted from **data**

Recap:

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx \sum_q (\Delta q(x, Q^2) + \Delta\bar{q}(x, Q^2))$$

$$S_g(Q^2) = \int_0^1 dx \Delta G(x, Q^2)$$

$$\Delta q + \Delta\bar{q} = -\frac{1}{\pi}\int_0^{\eta_{max}} d\eta \int_{s_{10}^{min}}^{\eta} ds_{10} \left[Q_q(s_{10}, \eta) + 2G_2(s_{10}, \eta)\right]$$

$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s(Q^2)\pi^2} G_2\left(x_{10}^2 = \frac{1}{Q^2}, zs = \frac{Q^2}{x}\right)$$

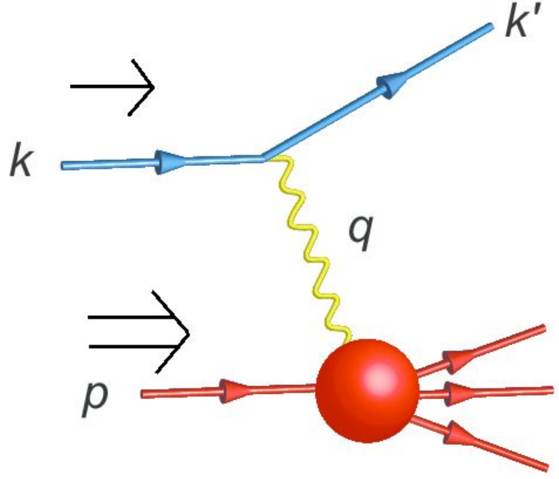
Large $N_c \& N_f$ Helicity Evolution

$$Q_q^{(0)}, \tilde{G}^{(0)}, G_2^{(0)}$$

Phenomenology

Observables - Double Spin Asymmetries in DIS

$$A_{||} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} \propto A_1 \propto g_1^{p,n}$$



- $\uparrow (\downarrow)$ is positive (negative) helicity electron
- $\Uparrow (\Downarrow)$ is positive (negative) helicity proton
- A_1 is virtual photoproduction asymmetry

Describing Observables - pDIS

What enters into observables are linear combinations of hPDFs

$$\Delta q^+ = \Delta q + \Delta \bar{q}$$
$$\Delta q^- = \Delta q - \Delta \bar{q}$$

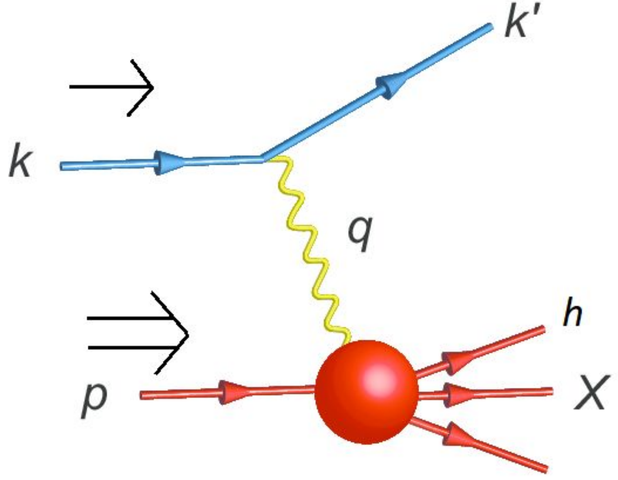
- Three relevant hPDFs in DIS: $\Delta u^+, \Delta d^+, \Delta s^+$
- Data exist for two observables that contain these hPDFs in linearly independent combinations: g_1^p and g_1^n

$$g_1^p(x, Q^2) = \frac{1}{2} \sum_q Z_q^2 \Delta q^+(x, Q^2)$$

- Z_q is the quark charge fraction

Observables - Double Spin Asymmetries in SIDIS

$$A_{||}(z) = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} \propto g_1^h(z)$$



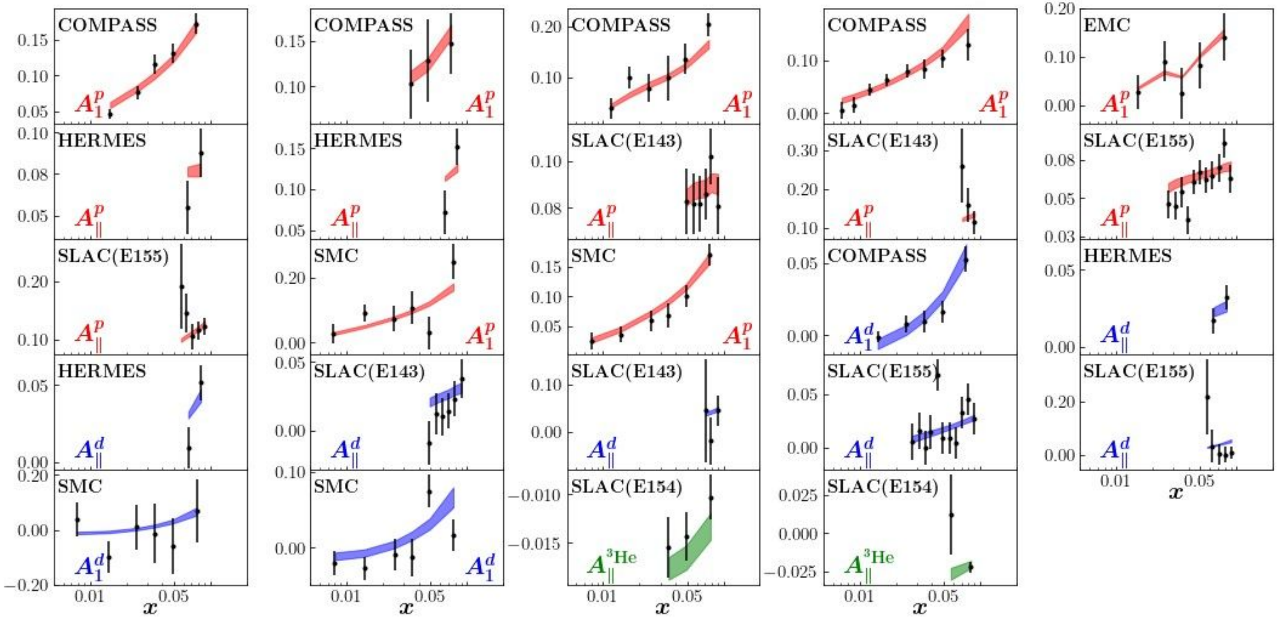
- *h* is the tagged hadron
- *z* is the momentum fraction of the virtual photon carried by the tagged hadron

g_1^h Structure Functions

$$g_1^h(x, z, Q^2) = \frac{1}{2}\sum_q Z_q^2 \Delta q(x, z, Q^2) D_q^h(z, Q^2)$$

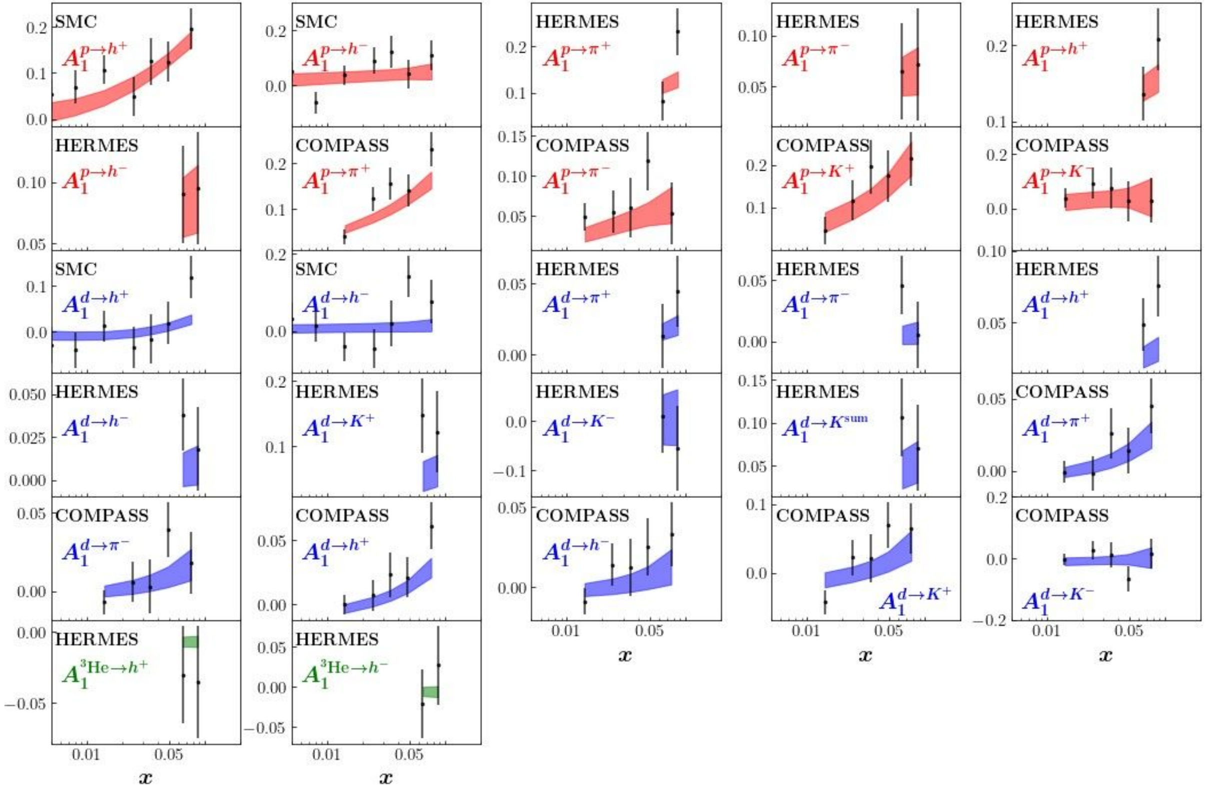
- D_q^h are fragmentation functions - giving the probability quark *q* fragments into hadron *h*
- z Is the fraction of the virtual photons momentum carried by the hadron
- The flavour hPDF is obtained via $\Delta q = \frac{1}{2}(\Delta q^+ + \Delta q^-)$
- $\boldsymbol{\Delta q^-}$ is obtained from non-singlet evolution

Global fit of DIS - Data vs Theory



- Red curves - our theory
- Black dots - data
  - COMPASS
  - EMC
  - SMC
  - SLAC
  - HERMES

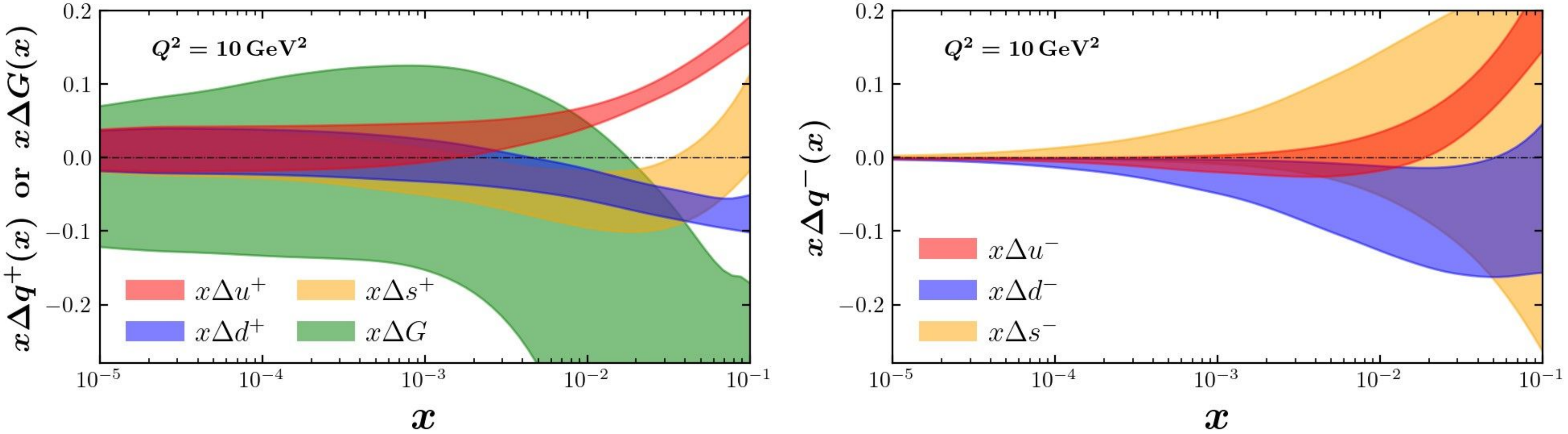
Global fit of SIDIS - Data vs Theory



χ^2 and Data Cuts

- First simultaneous fit of small-*x* theory to polarized DIS & SIDIS data
- Cut of 0.005< x < x_0 =0.1, given by $\alpha_s \ln 1/x \sim \mathcal{O}(1)$
- Cut of $1.69\, GeV^2 < Q^2 < 10.5\, GeV^2$
- Cut of 0.2 < z <0.8
- 24 fit parameters
- Describing 226 data points
- With a $\chi^2/npts$ = 1.03

Extraction of Helicity Parton Distribution Functions



Contribution to spin of the proton from spin of quarks and gluons

Quark helicity:

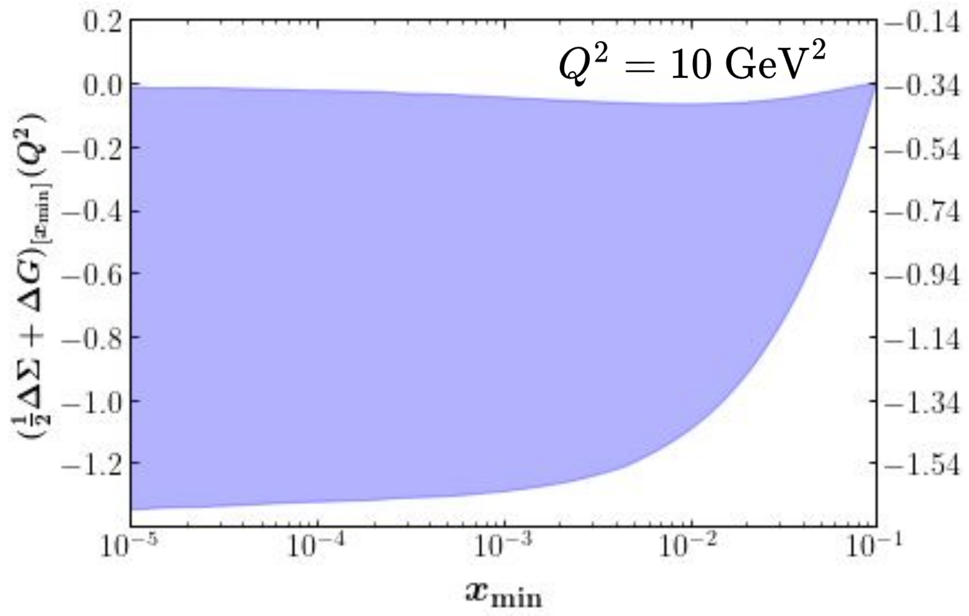
$$\Delta\Sigma(x,Q^2) \equiv \Delta u^+(x,Q^2) + \Delta d^+(x,Q^2) + \Delta s^+(x,Q^2)$$

Gluon helicity:

$$\Delta G(x,Q^2)$$

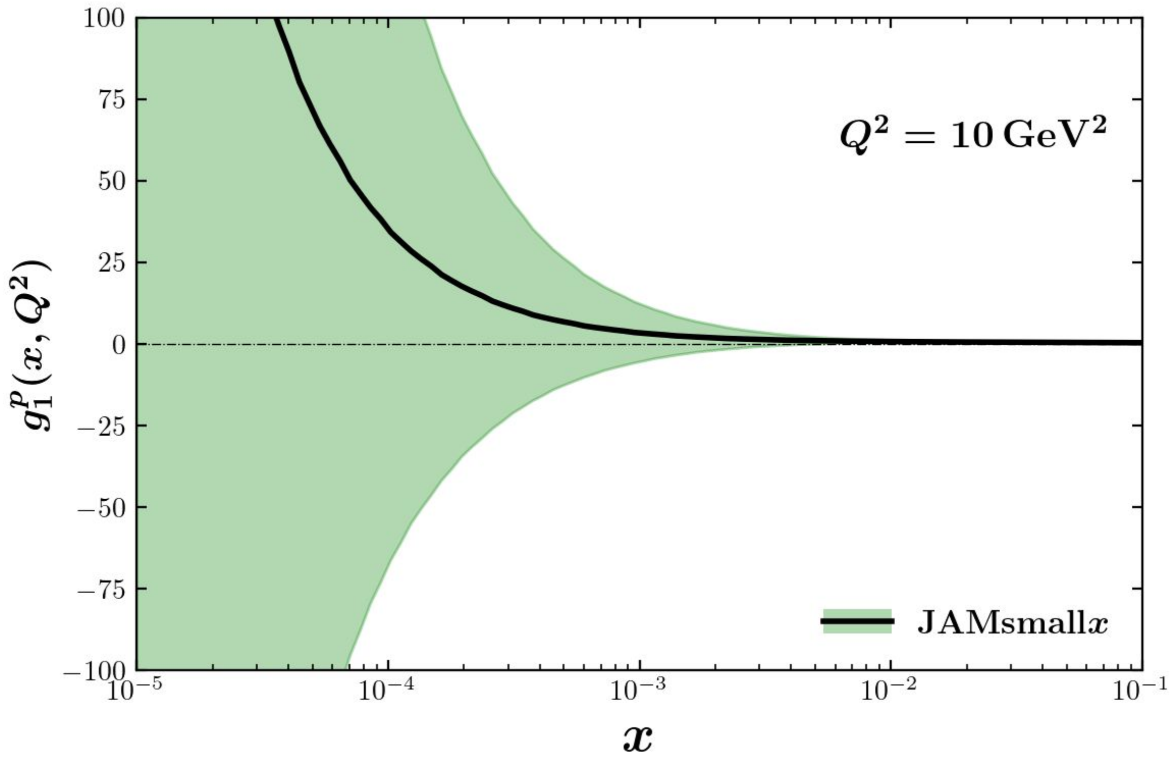
Partial moment of parton helicity:

$$\left(\tfrac{1}{2}\Delta\Sigma + \Delta G\right)_{[x_{\min}]} \equiv \int_{x_{\min}}^{0.1} dx\ \left(\tfrac{1}{2}\Delta\Sigma + \Delta G\right)(x,Q^2)$$



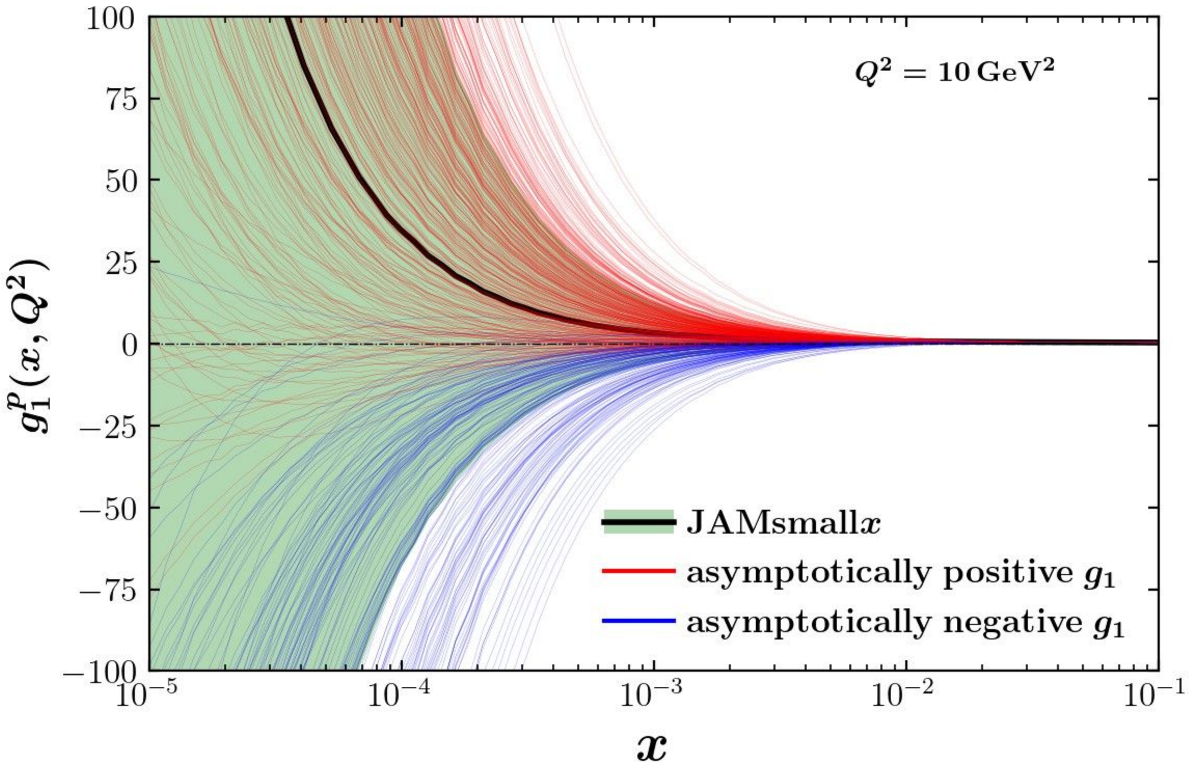
$$\int_{10^{-5}}^{0.1} dx\ \left(\tfrac{1}{2}\Delta\Sigma + \Delta G\right)(x,Q^2) = -0.64 \pm 0.60$$

Extraction of g1 structure function

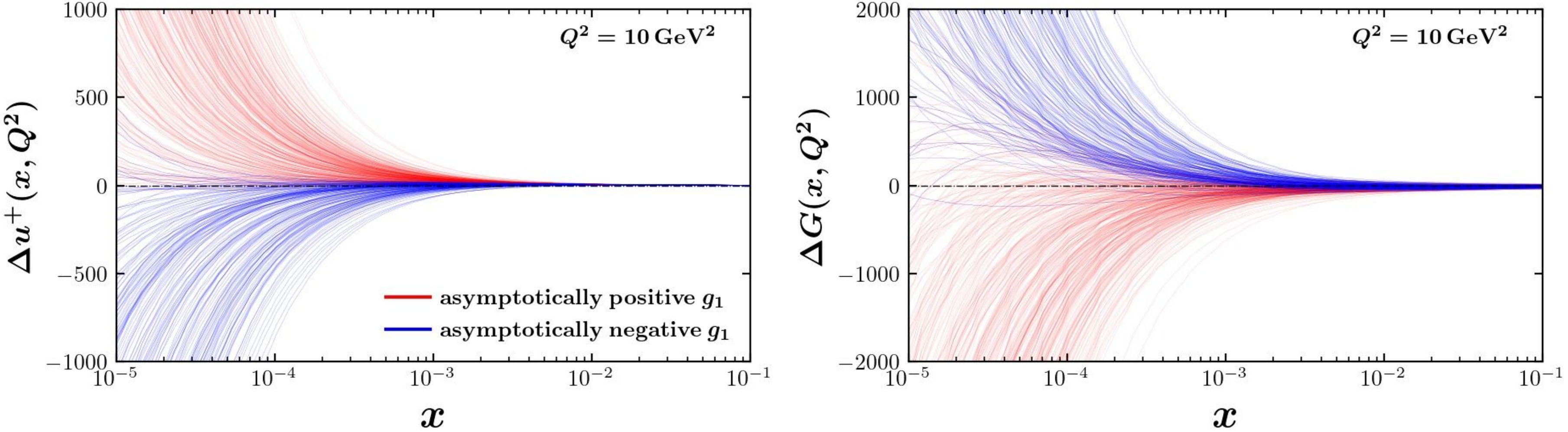


Understanding the uncertainty in g1

Lets keep track of the sign of the replicas



Correlation between g1 and hPDFs



$$\sim -(Q_u + 2G_2)$$

$$\sim G_2$$

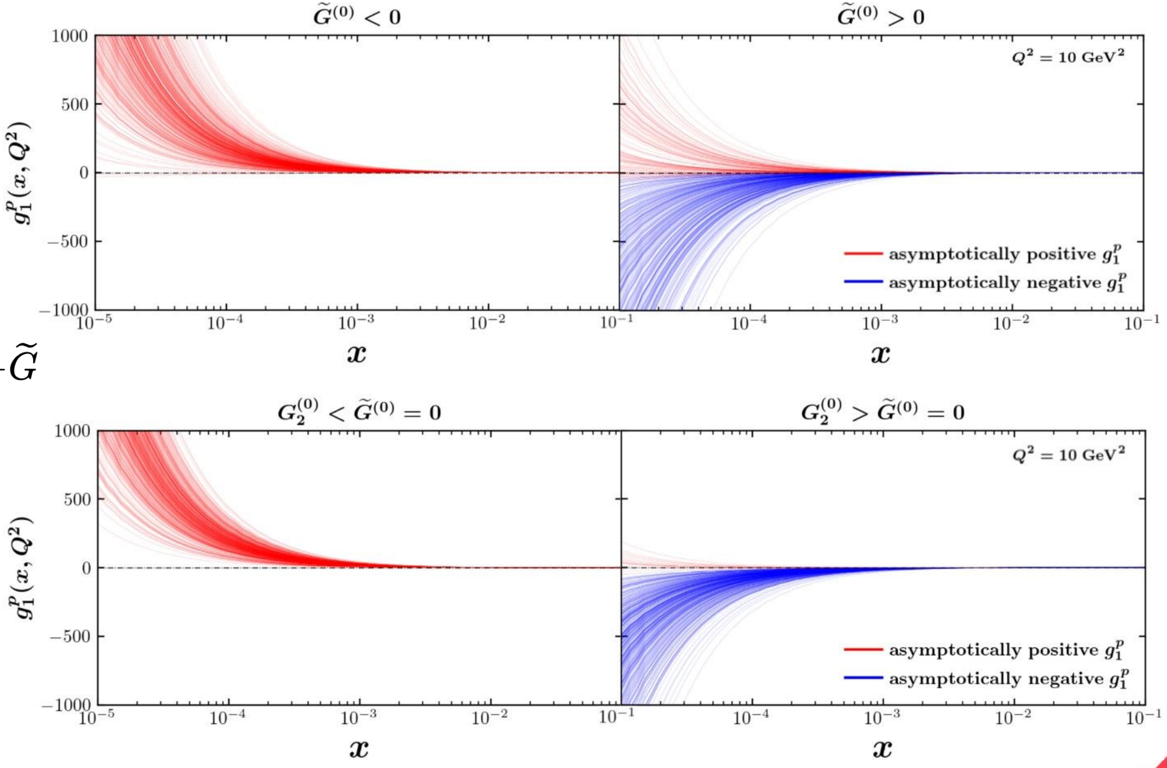
Anti-correlation between g1 and gluon helicity

Understanding the uncertainty in g1

g1 is driven by the polarized dipole amplitudes

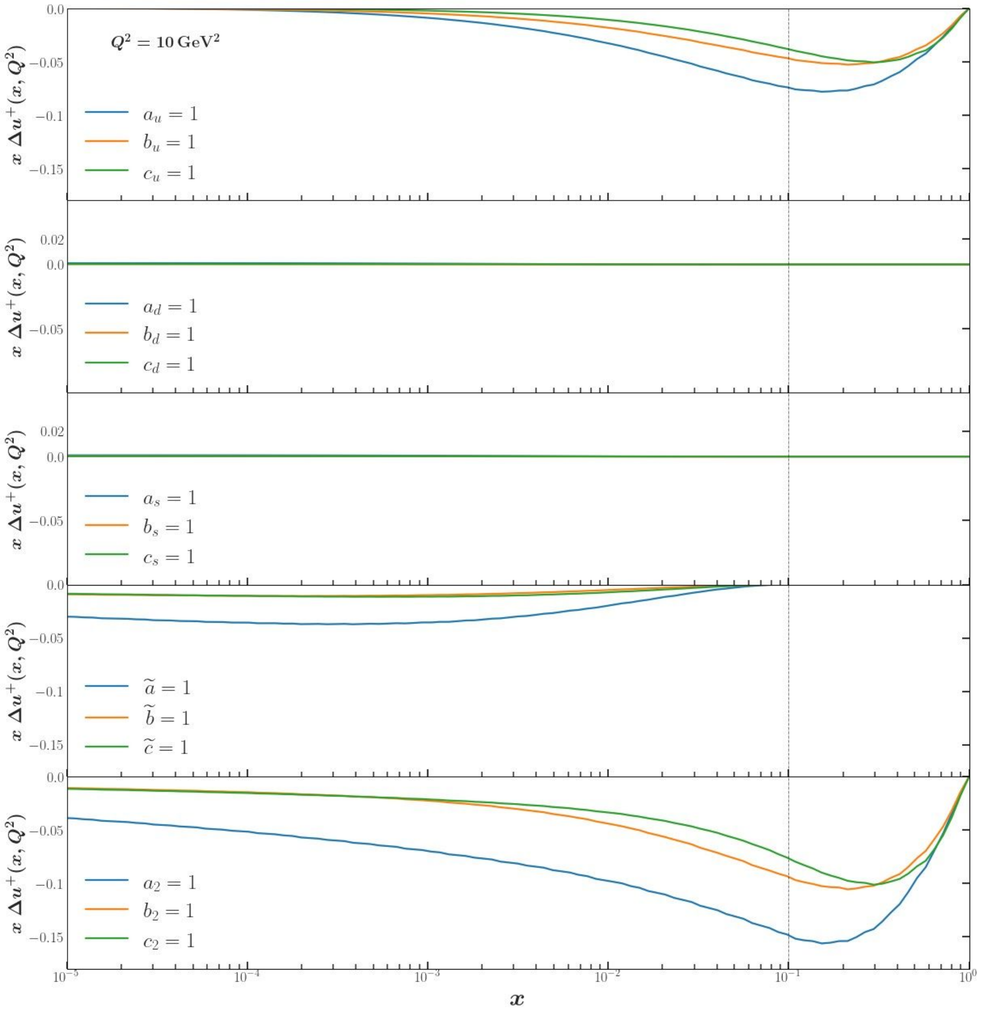
$$g_1^p(x) \propto \Delta q^+(x) \sim -(Q_q + 2G_2) \to -\widetilde{G}$$

which at, small-*x*, are driven by $\widetilde{G}$



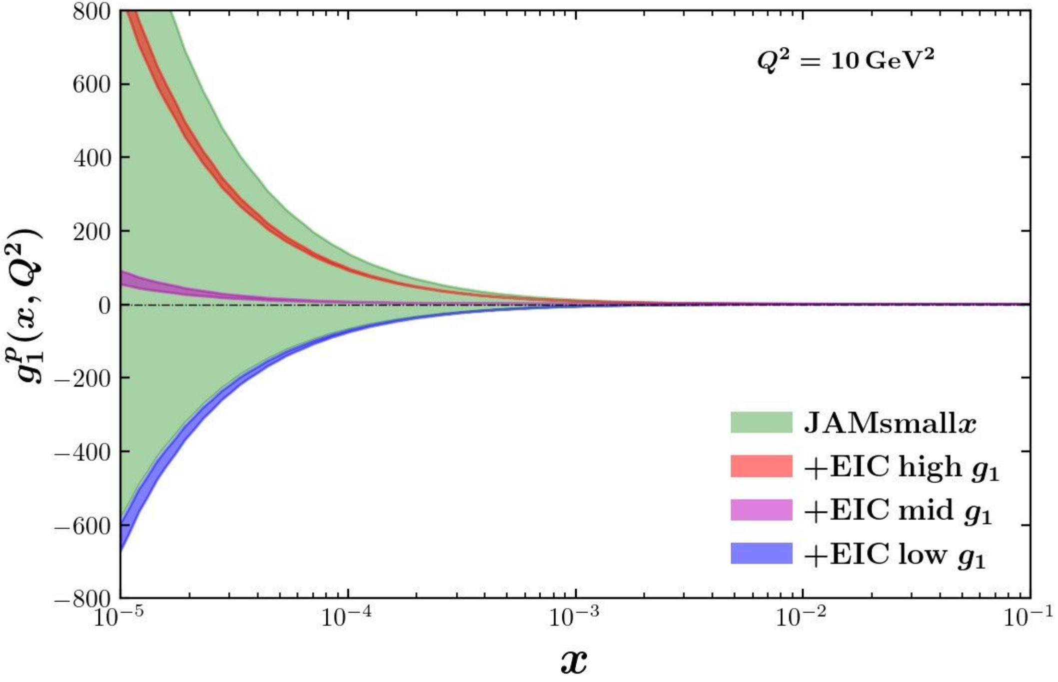
Understanding the uncertainty in g1 - Basis hPDFs

The true hPDF should be a linear combination of these curves

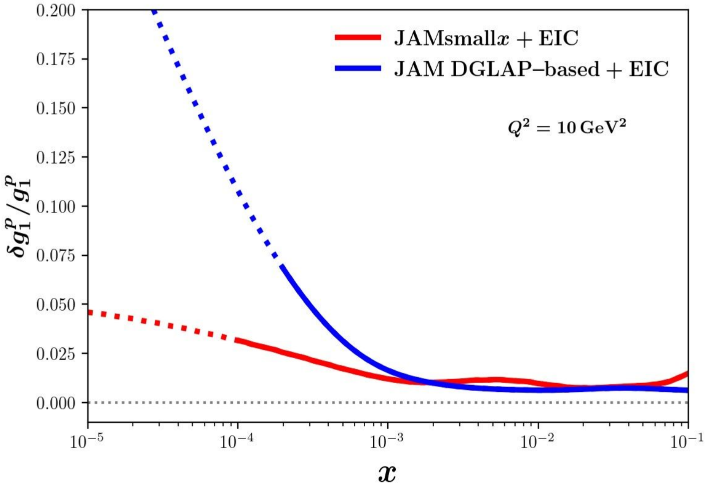


- At small-*x*, the hPDFs are dominated by G_2 and $\widetilde{G}$
  - Need smaller-*x* data
- At fixed *x*, one cannot disentangle Q and G_2
  - Need more *x* data or additional observables
- In the region of data, $\widetilde{G}$ is small
  - Need smaller-*x* data or larger x_0

Electron Ion Collider Impact



Uncertainty ratios

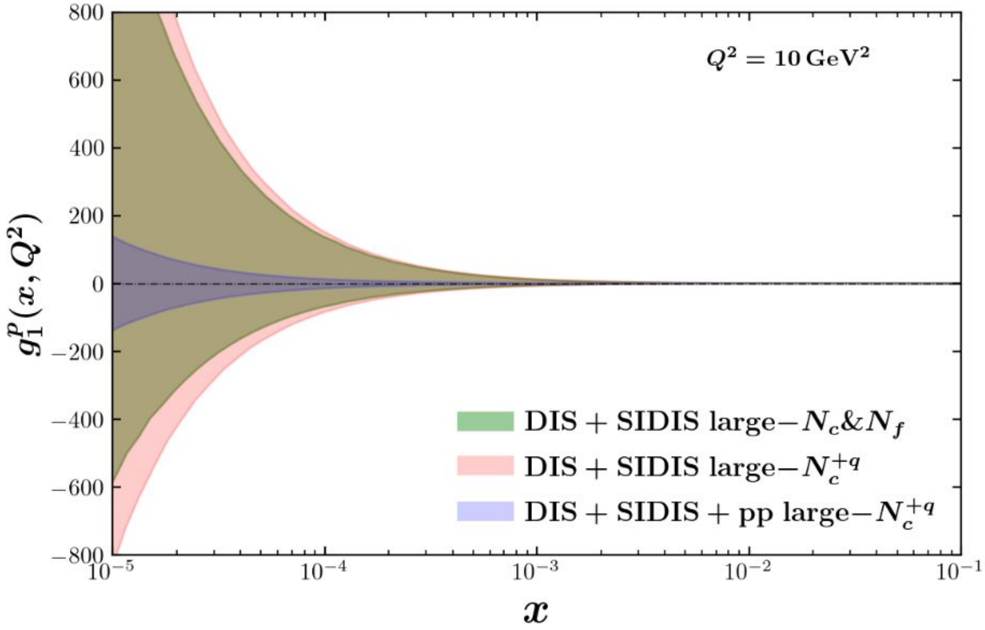


EIC pseudo data consistent with detector design in Yellow Report

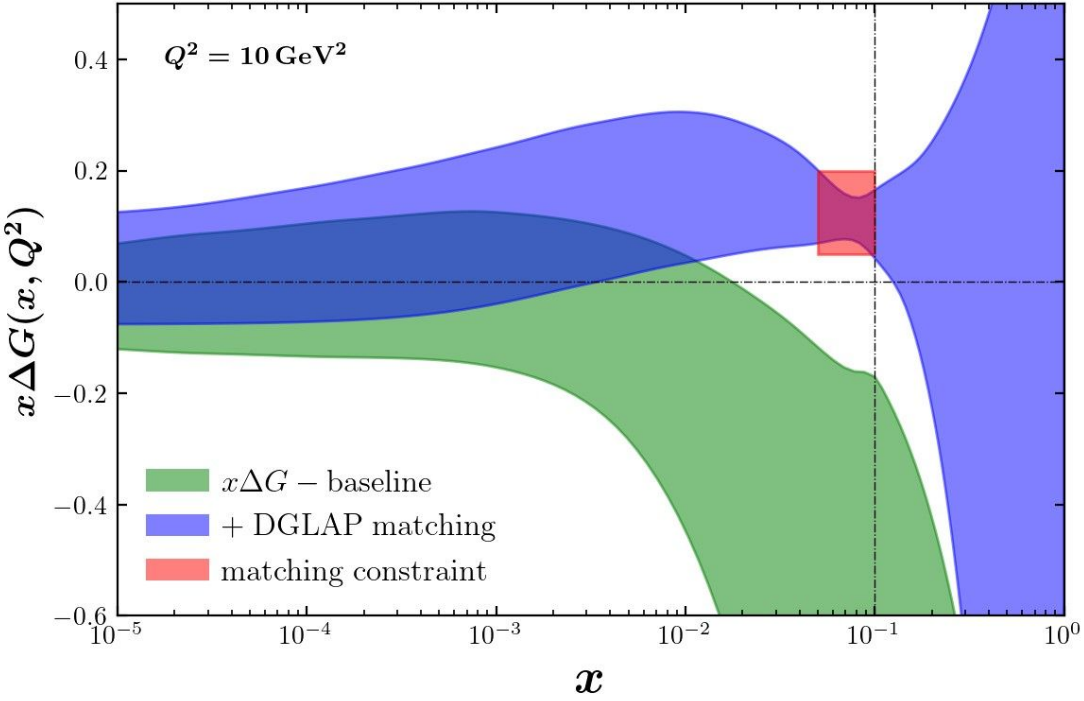
Gluon production in proton-proton scattering

- Gluon production in *pp* scattering provides an independent channel to probe gluon helicity
- See Ming Li's talk for more details

- Preliminary fit to *pp* data, performed by Nick Baldonado
- Using modified formalism, describes data and leads to a reduction in uncertainty



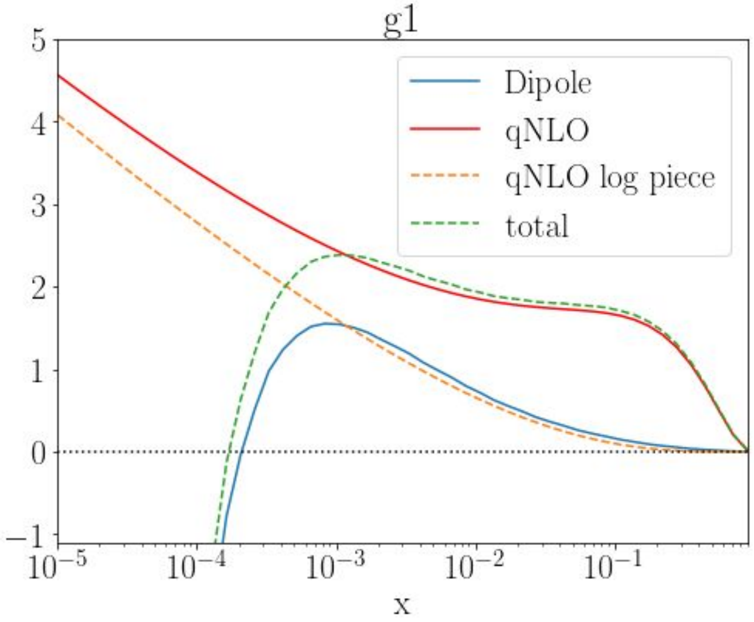
Another approach to reducing uncertainty: Matching to large-*x* DGLAP



Method of matched asymptotic expansions

Matching between small-*x* (Dipole) and large-*x* (Collinear Factorization)

$$g_q(x) = g_q^{\mathrm{Dipole}}(x) + g_1^{\mathrm{CF}} - g_1^{Asy}(x)$$



Conclusions

- To solve the proton spin puzzle through the helicity PDFs, their **small-*x*** behaviour must be understood

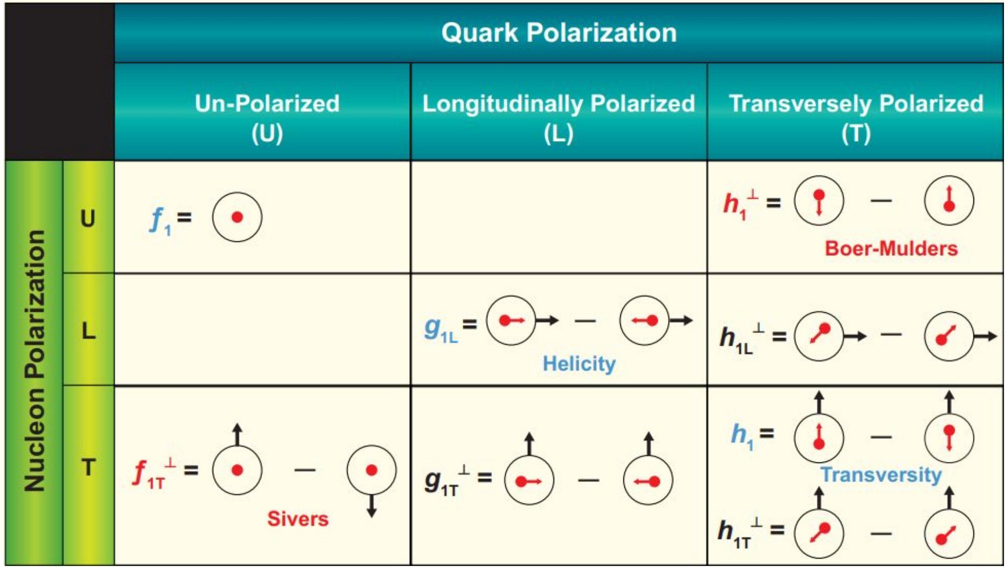
- Our dipole formalism **describes existing data** very well, but currently struggles to make predictions due to **few small-*x* data**

- The **EIC would provide sufficient data**
- Theoretical work is still being done to improve our predictions - the goal is to use EIC data as validation instead of a constraint

Backup Slides

Transverse Momentum Distributions

$$\sim \int d^2r dr^- e^{ik\cdot r} \left\langle PS|\bar{\psi}(0)\mathcal{U}[0,r]\Gamma\psi(r)|PS\right\rangle$$



Gauge Link: $\mathcal{U}[0,r]$

Γ :

$$\frac{1}{2}\gamma^+\gamma^5 \Rightarrow g_{1L}$$

$$\frac{1}{2}\gamma^5\gamma^+\gamma^\perp \Rightarrow h_1$$

Analytic solution comparison with Bartels, Ermolaev and Ryskin (BER)

$$\Delta G(x,Q^2) = \int \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} \left(\frac{Q^2}{\Lambda^2}\right)^{\Delta\gamma_{GG}(\omega)} \Delta G_\omega(\Lambda^2)$$

$$\Delta\gamma_{GG}^{BER}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s \frac{1-\frac{3\,\bar{\alpha}_s}{\omega^2}}{1-\frac{\bar{\alpha}_s}{\omega^2}}}\right] \qquad \Delta\gamma_{GG}^{us}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s\sqrt{1-\frac{4\,\bar{\alpha}_s}{\omega^2}}}\right]$$

- The intercept (largest power Re[ω]) is given by the right-most singularity (branch point) of the anomalous dimension.
- For BER this gives $\alpha_h = \sqrt{\frac{17+\sqrt{97}}{2}}\sqrt{\frac{\alpha_s\,N_c}{2\pi}} \approx 3.664\sqrt{\frac{\alpha_s\,N_c}{2\pi}}$
- For us $\alpha_h = \frac{4}{3^{1/3}}\sqrt{\mathrm{Re}\left[(-9+i\,\sqrt{111})^{1/3}\right]}\sqrt{\frac{\alpha_s\,N_c}{2\pi}} \approx 3.661\sqrt{\frac{\alpha_s\,N_c}{2\pi}}$

J. Borden, YK, 2304.06161 [hep-ph].