NON-EIKONAL EFFECTS IN DIJET PRODUCTION AT THE ELECTRON-ION COLLIDER

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Beyond-Eikonal Methods in High-Energy Scattering
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HIGH-ENERGY SCATTERING

- At high-energy, the medium is composed by a dense ensemble of gluons: **semi-classical approximation**:
  - Classical gluon background field (small-x): $A^{\mu}(x)$
  - Quantum source (large-x): $J^{\mu}(x)$

- At high-energy we perform the **eikonal approximation**:
  - The rapidity difference between the source and the probe is infinity
  - Power suppressed corrections of the CoM energy are neglected

THE EIKONAL APPROXIMATION

- **Only gluons** contribute to the low-x (classical) regime of the medium
- The classical field is **infinitely boosted** with respect to the (right-moving) probe:

$$A^\mu(x) = \Lambda^\mu_\nu A^\nu_0(\Lambda^{-1}x) = (\gamma A^-_0, \gamma^{-1}A^+_0, \mathbf{A}^\perp_0)$$

  - Only the longitudinal (-) component is probed

$$A^-(x) = \gamma A^-_0(\Lambda^{-1}x) = \gamma A^-_0(\gamma x^+, \gamma^{-1}x^-, \mathbf{x}^\perp)$$

  - The field can only be probed at $x^+ = 0$ (***shockwave approximation***)
  - The probe is not sensitive to the x^- dependence of the field (***frozen gluons***)

$$A^\mu(x) = \delta^{\mu -}\delta(x^+)a^-(\mathbf{x}^\perp)$$

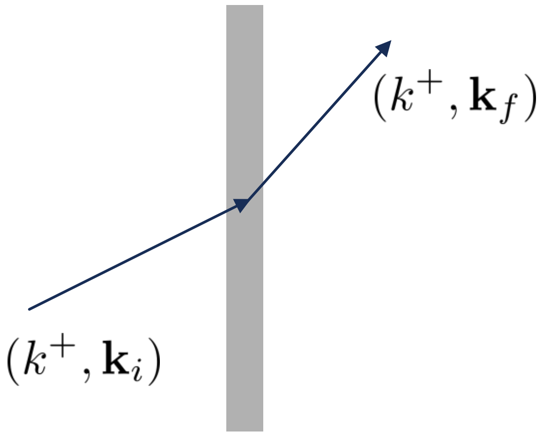
EIKONAL SCATTERING

- Color particle - background field scattering:
  - The projectile propagator can be solved exactly:

$$\Delta(x,y) \propto \delta^{(2)}(\mathbf{x}-\mathbf{y})U_{[x^+,y^+]}(\mathbf{x})$$

  - It is **diagonal** in the transverse coordinates
  - There is **no exchange of longitudinal** (+) **momentum** in the interaction
  - There is **no spin transfer**
  - The particle is only color rotated through the eikonal **Wilson line**:

$$U_{[x^+,y^+]}(\mathbf{x}) = \mathcal{P}^+ \exp\left\{ -ig\int_{y^+}^{x^+} dz^+ A^-(z^+,\mathbf{z})\right\}$$



- CGC observables depend on $\langle A^- \, A^- \rangle$ for a Gaussian distribution

BEYOND EIKONAL SCATTERING – FINITE LENGTH

- Relaxing the shockwave approximation:

$$\Delta_R(x,y) = \int_{k^+} \frac{\Theta(k^+)}{2k^+} e^{-ik^+(x^- - y^-)} \int_{\mathbf{y}}^{\mathbf{x}} \mathcal{D}^2\mathbf{z}(z^+) \ \exp\left\{ i \int_{y^+}^{x^+} dz^+ \frac{k^+}{2}\dot{\mathbf{z}}^2 \right\} U_{[\frac{L^+}{2}, -\frac{L^+}{2}]}(\mathbf{z})$$

  - Used for computing medium induced radiation:

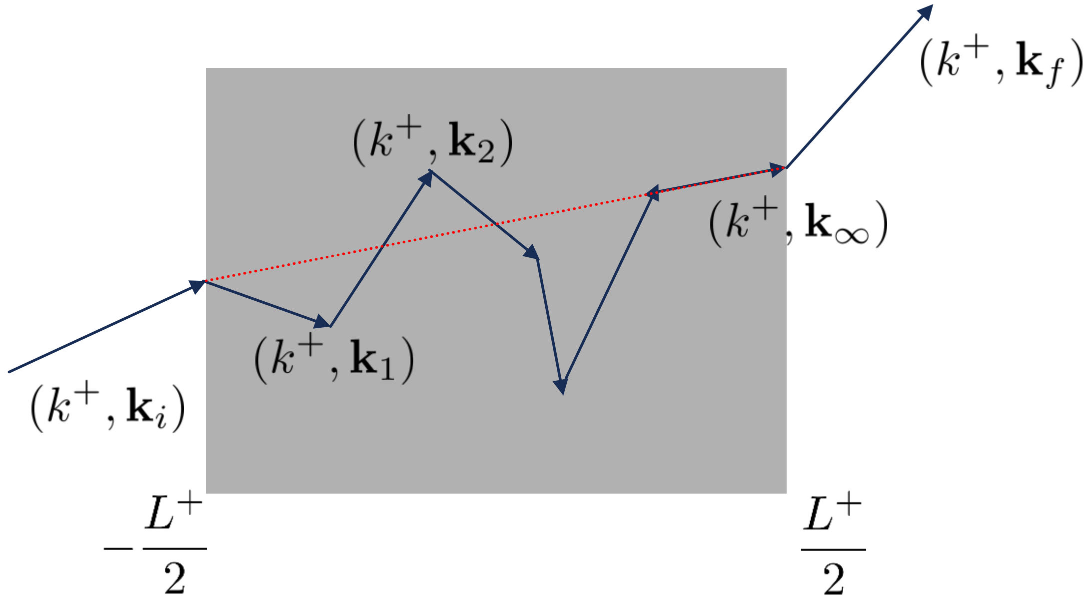
    Zakharov: 9607440

  - The eikonal approximation is the saddle-point solution of the path integral where the particle moves in a straight line:

$$\mathbf{z}_{\rm cl}(z^+) = \mathbf{y} + \frac{z^+ - y^+}{x^+ - y^+}(\mathbf{x} - \mathbf{y})$$

    and the angle of (transverse) propagation is small:

$$k^+ \to \infty$$



BEYOND EIKONAL SCATTERING – FINITE LENGTH

- Relaxing the shockwave approximation:
  - To get rid of the path integral one can **expand around the saddle point** to get an eikonal expansion.
    - First order eikonal correction:

$$\Delta_R(x,y) = \int_{k^+} \frac{\Theta(k^+)}{2k^+} e^{-ik^+(x^- - y^-)} \int_{\mathbf{k}_i,\mathbf{k}_f} e^{i\mathbf{k}_f\cdot\mathbf{x} - i\mathbf{k}_i\cdot\mathbf{y}} \int_{\mathbf{z}} e^{-i\mathbf{z}\cdot(\mathbf{k}_f - \mathbf{k}_i)} \Bigg\{ U_{[\frac{L^+}{2}, -\frac{L^+}{2}]}(\mathbf{z}) - \int_{\frac{L^+}{2}}^{-\frac{L^+}{2}} dz^+ U_{[\frac{L^+}{2}, z^+]}(\mathbf{z}) \left[\frac{\mathbf{k}_f^i + \mathbf{k}_i^i}{2k^+} \overleftrightarrow{\partial}_{\mathbf{z}^i} + \frac{i}{2k^+} \overleftarrow{\partial}_{\mathbf{z}^i} \overrightarrow{\partial}_{\mathbf{z}^i} \right] U_{[z^+, -\frac{L^+}{2}]}(\mathbf{z}) \Bigg\}$$

Altinoluk, Armesto, Beuf, Martínez, Salgado: 1404.2219

    - Second order eikonal correction:

Altinoluk, Armesto, Beuf, Moscoso: 1505.01400

BEYOND EIKONAL SCATTERING – TRANSVERSE FIELD

- Relaxing the **shockwave approximation** + including **transverse field**:
  - First order correction (by using an alternative method):

$$\Delta_R(x,y) = \int_{k^+} \frac{\Theta(k^+)}{2k^+} e^{-ik^+(x^- - y^-)} \int_{\mathbf{k}_i,\mathbf{k}_f} e^{i\mathbf{k}_f\cdot\mathbf{x} - i\mathbf{k}_i\cdot\mathbf{y}} \int_{\mathbf{z}} e^{-i\mathbf{z}\cdot(\mathbf{k}_f - \mathbf{k}_i)} \Bigg\{ U_{[\frac{L^+}{2}, -\frac{L^+}{2}]}(\mathbf{z}) - \int_{\frac{L^+}{2}}^{-\frac{L^+}{2}} dz^+ U_{[\frac{L^+}{2}, z^+]}(\mathbf{z}) \left[\frac{\mathbf{k}_f^i + \mathbf{k}_i^i}{2k^+} \overleftrightarrow{D}_{\mathbf{z}^i} + \frac{i}{2k^+} \overleftarrow{D}_{\mathbf{z}^i} \overrightarrow{D}_{\mathbf{z}^i} \right] U_{[z^+, -\frac{L^+}{2}]}(\mathbf{z}) \Bigg\}$$

$$D_{\mathbf{z}^j}(v^+) = \partial_{\mathbf{z}^j} - ig\mathbf{A}^j(v^+, \mathbf{z})$$

Balitsky, Tarasov: 1505.02151

Chirilli: 1807.11435

Altinoluk, Beuf, Czajka, Tymowska: 2012.03886

BEYOND EIKONAL SCATTERING – z^- DEPENDENCE

- Relaxing the **shockwave approximation** + including **transverse field** + **time dependent** field:
  - First order correction (by using an iterative method):

$$\Delta_R(x,y) = \int_{k^+} \frac{\Theta(k^+)}{2k^+} e^{-ik^+(x^- - y^-)} \int_{\mathbf{k}_i,\mathbf{k}_f} e^{i\mathbf{k}_f\cdot\mathbf{x} - i\mathbf{k}_i\cdot\mathbf{y}} \int_{\mathbf{z}} e^{-i\mathbf{z}\cdot(\mathbf{k}_f - \mathbf{k}_i)} \Bigg\{ U_{[\frac{L^+}{2}, -\frac{L^+}{2}]}(\mathbf{z})$$

$$- \int_{\frac{L^+}{2}}^{-\frac{L^+}{2}} dz^+ U_{[\frac{L^+}{2}, z^+]}(\mathbf{z}) \left[\frac{\mathbf{k}_f^i + \mathbf{k}_i^i}{2k^+} \overleftrightarrow{D}_{\mathbf{z}^i} + \frac{i}{2k^+} \overleftarrow{D}_{\mathbf{z}^i} \overrightarrow{D}_{\mathbf{z}^i} \right] U_{[z^+, -\frac{L^+}{2}]}(\mathbf{z}) \Bigg\}$$

$$+ \frac{x^- + y^-}{2} \partial^+ U_{[\frac{L^+}{2}, -\frac{L^+}{2}]}(\mathbf{z})$$

Change of longitudinal momentum

Jalilian-Marian: 1708.07533

Altinoluk, Beuf: 2109.01620

BEYOND EIKONAL SCATTERING – z^- DEPENDENCE

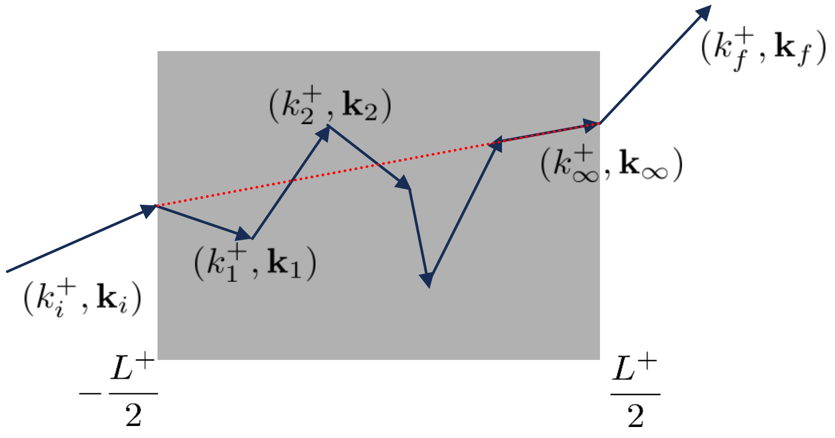
- The propagator can be generalized to a 4D path integral:

$$\Delta_R(x,y) = \int \mathcal{D}p^+(\tau^+)\frac{\Theta(\langle p^+\rangle)}{2\langle p^+\rangle}\int_{\vec{y}}^{\vec{x}} \mathcal{D}^3\vec{z}(\tau^+)\exp\left\{i\int_{y^+}^{x^+} d\tau^+\left[-\frac{m^2}{2\langle p^+\rangle}+\frac{\langle p^+\rangle}{2}\dot{\mathbf{z}}^2 - p^+\dot{z}^-\right]\right\}\mathcal{U}_{[x^+,y^+]}[z^+,\vec{z}]$$

PA: 2307.13573

- The interaction of the particle with the medium is given by the Wilson line:

$$\mathcal{U}_{[x^+,y^+]}[z^+(\tau^+),\vec{z}(\tau^+)] = \mathcal{P}_+\exp\left\{-ig\int_{y^+}^{x^+} d\tau^+\left[A^-(z^+,\vec{z})\frac{p^+}{\langle p^+\rangle}+\vec{A}(z^+,\vec{z})\cdot\dot{\vec{z}}\right]\right\}$$



CONVERTING THE FORMALISM INTO NUMBERS - EIKONAL

- CGC observables depend on $\langle O[U] \rangle$ (dipole, quadrupole, etc.)
- The approach:
  - Use a model for the distibution of $\rho(x)$ (Usually Gaussian White Noise)
  - Compute the stochastic field using the YM EoM

$$-\partial_\perp^2 A_a^-(\vec{x}) = \rho_a(\vec{x})$$

  - Simulate the operator average. (analytical for MV model)
  - Compute observable

CONVERTING THE FORMALISM INTO NUMBERS - SUBEIKONAL

- Now observables depend on $\langle O[A^-, A^+, A_\perp^i]\rangle$
- We still assume: target composed by a dense ensemble of gluons, **semi-classical approximation**:
  - Classical gluon background field: $A^\mu(x)$
  - Stochastic source current: $J^\mu(x)$
- Given a model for the current the field is given by the classical YM EoM:

$$D_\mu F^{\mu\nu} = J^\nu$$

- For a large nucleus we can still assume a Gaussian distribution, apart from the $\langle A^- \, A^- \rangle$ correlator, now we also need:

$\langle A^\mu(x)\, A^\nu(y)\rangle$ correlators

THE LARGE NUCLEAR ENSEMBLE

- We assume that:
  - The target is composed by $A \gg 1$ **nucleons**
  - Each nucleon is in a singlet state of N_c **quarks**
  - All the particles are **independent**, uncorrelated and have the **same momenta** $(P_q^-, 0_\perp)$

$$\langle \hat{O} \rangle_A = \prod_{i=1}^{A} \prod_{j=1}^{N_c} \int d^3\vec{b}_{ij} \frac{\rho_A(\vec{b}_{ij})}{A} \langle \hat{O} \rangle_q(\{\vec{b}\})$$

Cougoulic, Kovchegov: 2005.14688

$$\langle \hat{\mathcal{O}} \rangle_q(\{\vec{b}\}) = \prod_{i=1}^{A} \left[\frac{1}{N_c!} \left(\prod_{j=1}^{N_c} \int \frac{d^3\vec{q}_{ij}}{2(2\pi)^3 P_q^-} e^{i\vec{q}_{ij}\cdot\vec{b}_{ij}} \right) \epsilon_{\{\alpha_i\}} \epsilon_{\{\bar{\alpha}_i\}} \right] \left\langle \left\{ P_q^- - \frac{q^-}{2}, -\frac{\mathbf{q}}{2}, \bar{\alpha} \right\} \middle| \hat{\mathcal{O}} \middle| \left\{ P_q^- + \frac{q^-}{2}, \frac{\mathbf{q}}{2}, \alpha \right\} \right\rangle$$

CORRELATORS BEYOND THE EIKONAL APPROXIMATION

- We can compute $\langle J^{\mu}(x)\, J^{\nu}(y)\rangle$ by using:

$$J_a^{\mu}(\vec{x}) = g t^a_{\alpha\beta} \int_{\vec{p},\vec{p}'} e^{-i\vec{x}\cdot(\vec{p}-\vec{p}')}\Gamma^{\mu}_{\sigma\sigma'}(\vec{p},\vec{p}')\ \hat{b}^{\dagger}_{\alpha,\sigma'}(\vec{p}')\hat{b}_{\beta,\sigma}(\vec{p})$$

- How do we evaluate $\langle A^{\mu}(x)\, A^{\nu}(y)\rangle$?
  - We asume that $A^{\mu} \sim O(g^0)$ (valid only in the **dilute limit**)
  - We neglect the x^- dependence of the field
  - We neglect the non-linear terms in the YM equation. In the covariant gauge:

$$-\partial_\perp^2 A_a^{\mu}(\vec{x}) = J_a^{\mu}(\vec{x}) \qquad\qquad A_a^{\mu}(\vec{x}) = \int_{\mathbf{z}}\int_{\mathbf{q}} e^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{z})}\frac{J_a^{\mu}(x^+,\mathbf{z})}{\mathbf{q}^2}$$

CORRELATORS BEYOND THE EIKONAL APPROXIMATION

- Assuming a **homogeneous target**

$$\left\langle A_a^\mu\left(\vec{x}\right)A_b^\nu\left(\vec{y}\right)\right\rangle = \delta^{ab}\delta(x^+ - y^+)\frac{\tilde{\mu}^2}{L^+}G^{\mu\nu}(\mathbf{x}-\mathbf{y})$$

$\tilde{\mu}$: transverse color charge density
L^+: target width

$$G^{--}(\mathbf{r}) = \int_{\mathbf{P}} e^{i\mathbf{P}\cdot\mathbf{r}}\frac{1}{\mathbf{P}^4}$$

MV MODEL

$$G^{i-}(\mathbf{r}) = \frac{1}{2P_q^-}\int_{\mathbf{P}} e^{i\mathbf{P}\cdot\mathbf{r}}\frac{\mathbf{P}^i}{\mathbf{P}^4}$$

$$G^{ij}(\mathbf{r}) = \frac{1}{(2P_q^-)^2}\int_{\mathbf{P}} e^{i\mathbf{P}\cdot\mathbf{r}}\frac{\mathbf{P}^i\mathbf{P}^j + \epsilon^{im}\epsilon^{jn}\mathbf{P}^m\mathbf{P}^n}{\mathbf{P}^4}$$

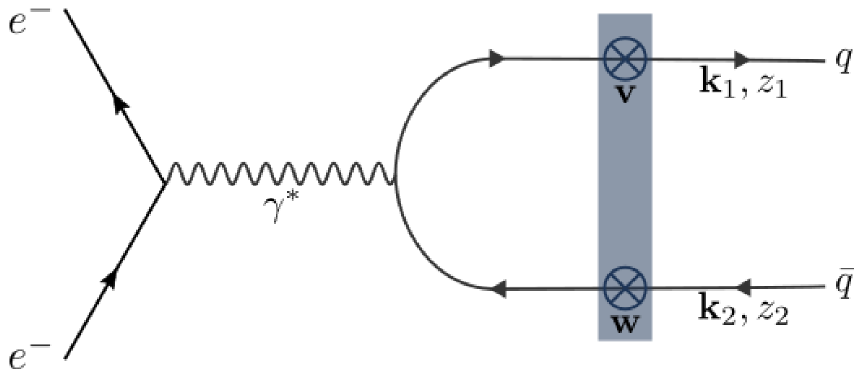
ENERGY SUPPRESED CORRELATORS

DIJET PRODUCTION IN DIS AT THE EIKONAL LEVEL

- The virtual photon splits into a quark-antiquark pair:
  - Depends on the polarization of the photon (L or T)
  - Can be computed using **perturbative QED**
- The pair interacts with the dense target:
  - Eikonal scattering depends only on **Wilson lines**

$$d(\mathbf{v},\mathbf{w}) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[\mathcal{U}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})\right]\right\rangle,$$

$$Q(\mathbf{w}',\mathbf{v}',\mathbf{v},\mathbf{w}) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[\mathcal{U}(\mathbf{w}')\mathcal{U}^\dagger(\mathbf{v}')\mathcal{U}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})\right]\right\rangle,$$

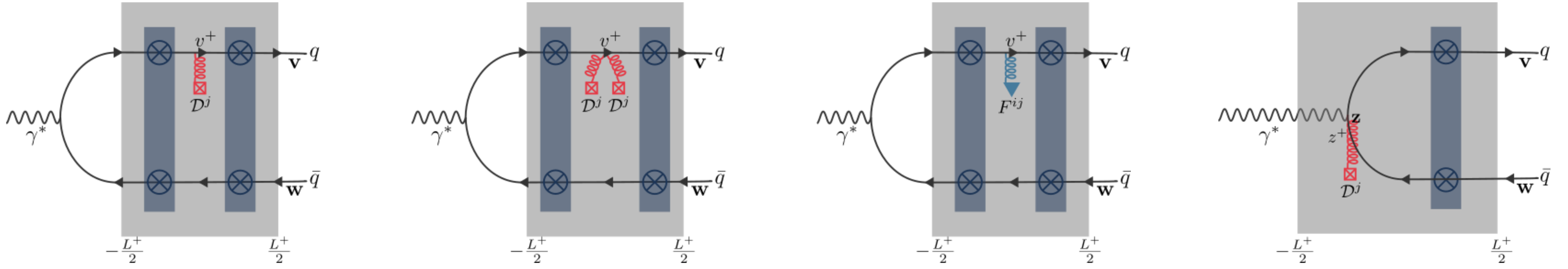


Dominguez, Marquet, Xiao, Yuan: 1101.0715

$$\left.\frac{d\sigma^{\gamma^*_\lambda+A\to q\bar{q}+X}}{d^2\mathbf{k}_1 d^2\mathbf{k}_2 d\eta_1 d\eta_2}\right|_{\mathrm{Eik.}} = \int_{\mathbf{v},\mathbf{v}',\mathbf{w},\mathbf{w}'} e^{i\mathbf{k}_1\cdot(\mathbf{v}'-\mathbf{v})+i\mathbf{k}_2\cdot(\mathbf{w}'-\mathbf{w})}\mathcal{C}_\lambda(\mathbf{w}'-\mathbf{v}',\mathbf{w}-\mathbf{v})$$
$$\times\left[Q(\mathbf{w}',\mathbf{v}',\mathbf{v},\mathbf{w}) - d(\mathbf{w}',\mathbf{v}') - d(\mathbf{v},\mathbf{w}) + 1\right]$$

DIJET PRODUCTION BEYOND THE EIKONAL APPROXIMATION

- Next-to-eikonal diagrams to the dijet production amplitude (**only transverse corrections**):



Altinoluk, Beuf, Czajka, Tymowska: 2212.10484

- Depend on objects like:

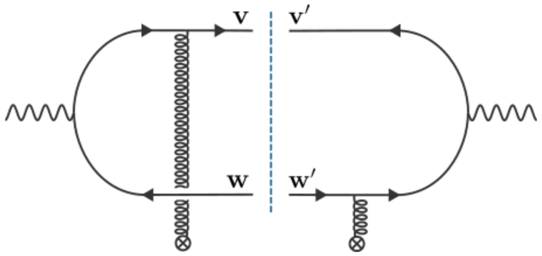
$$d_j^{(1)}(\mathbf{v}_*, \mathbf{w}) = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[\mathcal{U}_j^{(1)}(\mathbf{v})\mathcal{U}^\dagger(\mathbf{w})\right]\right\rangle$$

$$\mathcal{U}_j^{(1)}(\mathbf{z}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+ \mathcal{U}_{[\frac{L^+}{2}, v^+]}(\mathbf{z}) \overleftrightarrow{D_{\mathbf{z}^j}}(v^+) \mathcal{U}_{[v^+, -\frac{L^+}{2}]}(\mathbf{z})$$

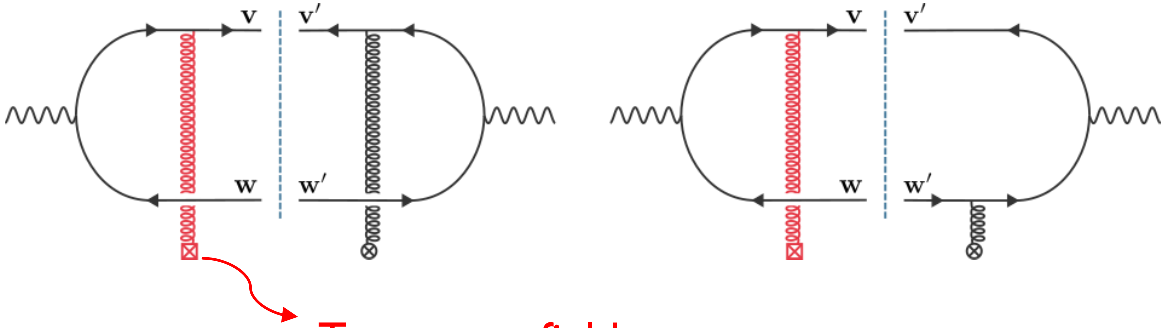
$$D_{\mathbf{z}^j}(v^+) = \partial_{\mathbf{z}^j} - ig\mathbf{A}^j(v^+, \mathbf{z})$$

DIJET PRODUCTION IN THE DILUTE LIMIT

- For a dense target we have 2 challenges:
  - Resum the non-eikonal field correlators
  - Include non-linear terms in the YM equations
- We study the case where $k_{1\perp}, k_{2\perp} \gg Q_s$ (**dilute limit**):
  - $A^{\mu} \sim O(g^0)$: **perturbative expansion**
  - Only 2-gluon exchange



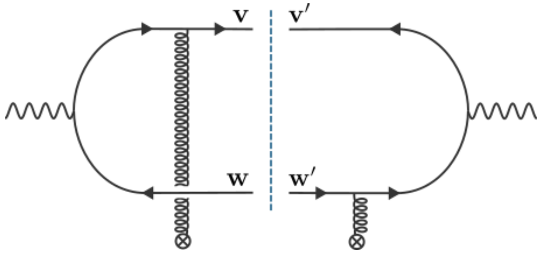
Eikonal term



Transverse field

EIKONAL DIJET PRODUCTION IN THE DILUTE LIMIT

- Only 4 diagrams contribute



+ permutations

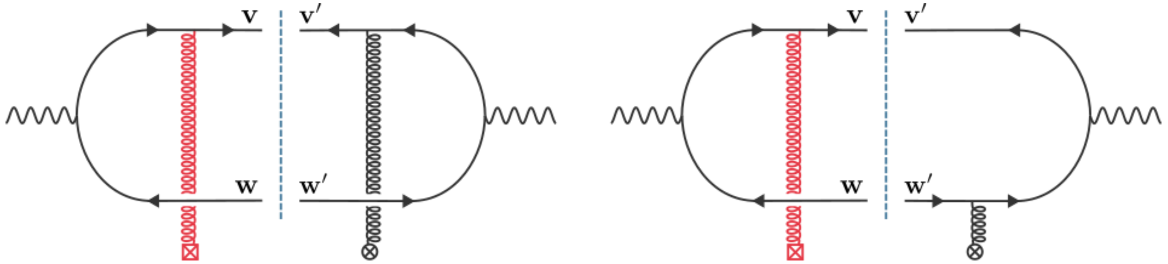
- The result is analytical

$$\frac{d\sigma^{\gamma_L^*+A\to q\bar{q}+X}}{d^2\mathbf{k}_1 d\eta_1 d^2\mathbf{k}_2 d\eta_2} = \frac{8N_c\alpha_{\text{em}}e_f^2 Q_s^2 S_\perp}{(2\pi)^3}\delta_z z_1^3 z_2^3 \frac{Q^2(\mathbf{k}_1^2-\mathbf{k}_2^2)^2}{(\mathbf{k}_1+\mathbf{k}_2)^4(\mathbf{k}_1^2+\epsilon_f^2)^2(\mathbf{k}_1^2+\epsilon_f^2)^2}$$

  - (Similar expression for transversely polarized photon)

SUBEIKONAL DIJET PRODUCTION IN THE DILUTE LIMIT

- Only 4 diagrams contribute



+ permutations

- The result is analytical

$$\frac{d\sigma^{\gamma_L^*+A\to q\bar{q}+X}}{d^2\mathbf{k}_1 d\eta_1 d^2\mathbf{k}_2 d\eta_2} = \left.\frac{d\sigma^{\gamma_L^*+A\to q\bar{q}+X}}{d^2\mathbf{k}_1 d\eta_1 d^2\mathbf{k}_2 d\eta_2}\right|_{\text{eik.}} \times \frac{N_c}{W^2}\left(\frac{\mathbf{k}_1^2+\epsilon_f^2}{z_1} - \frac{\mathbf{k}_2^2+\epsilon_f^2}{z_2}\right)$$

- W: Center of Mass energy of the $\gamma^* A$ system:
  - $W \to \infty$ in the eikonal approximation

RESULT

- Analytical solution for the differential cross section:

$$\frac{d\sigma^{\gamma_L^*+A\to q\bar{q}+X}}{d^2\mathbf{k}_1 d\eta_1 d^2\mathbf{k}_2 d\eta_2} = \frac{8N_c\alpha_{\rm em}e_f^2Q_s^2S_\perp}{(2\pi)^3}\delta_z z_1^3 z_2^3$$

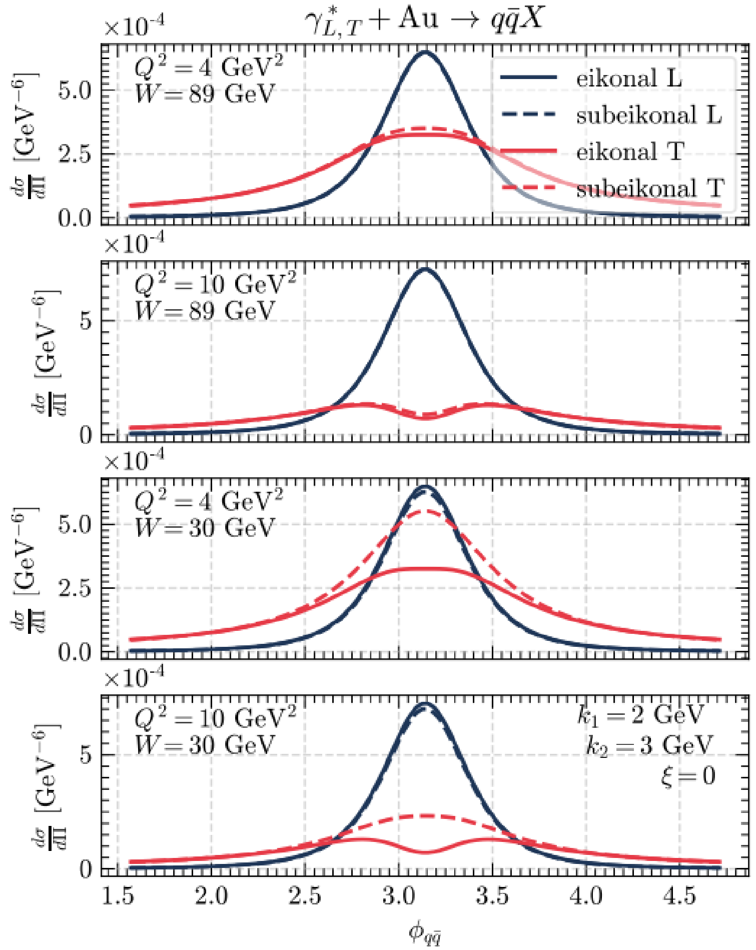
$$\frac{Q^2(\mathbf{k}_1^2-\mathbf{k}_2^2)^2}{(\mathbf{k}_1+\mathbf{k}_2)^4(\mathbf{k}_1^2+\epsilon_f^2)^2(\mathbf{k}_1^2+\epsilon_f^2)^2}\left[1+\frac{N_c}{W^2}\left(\frac{\mathbf{k}_1^2+\epsilon_f^2}{z_1}-\frac{\mathbf{k}_2^2+\epsilon_f^2}{z_2}\right)\right]$$

subeikonal correction

  - Similar expression for a transversely polarized photon

- Longitudinal momentum fraction asymmetry:

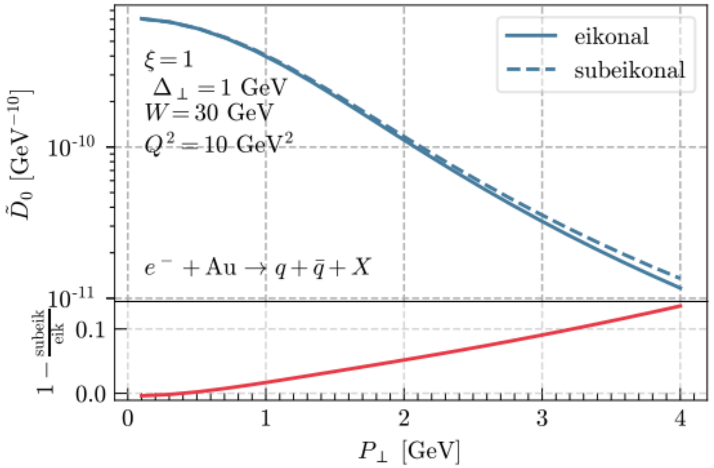
$$\xi = \ln\frac{1-z_1}{z_1}$$

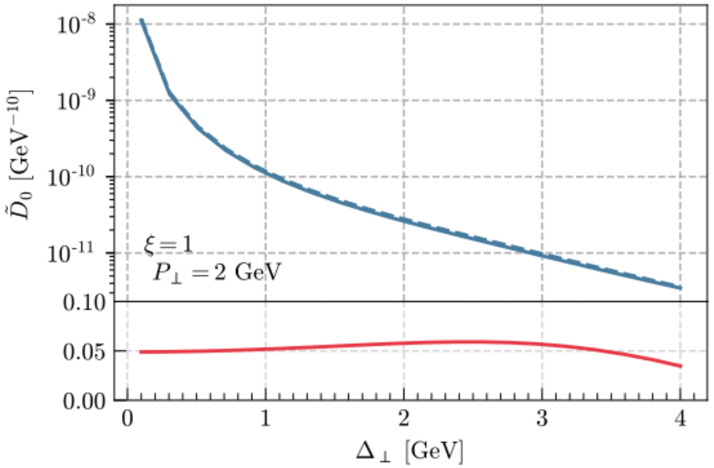


NUMERICAL RESULTS

- Momentum Imbalance: $\mathbf{\Delta} = \mathbf{k}_1 + \mathbf{k}_2$
- Relative momentum: $\mathbf{P} = z_2\mathbf{k}_1 - z_1\mathbf{k}_2$
- Harmonic expansion w.r.t. the angle ϕ between P and Δ:

$$\frac{d\sigma_\lambda^{\gamma^* A\to q\bar{q}X}}{d\Pi} = D_{0,\lambda}(P_\perp, \Delta_\perp)\left[1 + 2\sum_{n=1}^{\infty} v_{n,\lambda}(P_\perp, \Delta_\perp)\cos\phi\right]$$

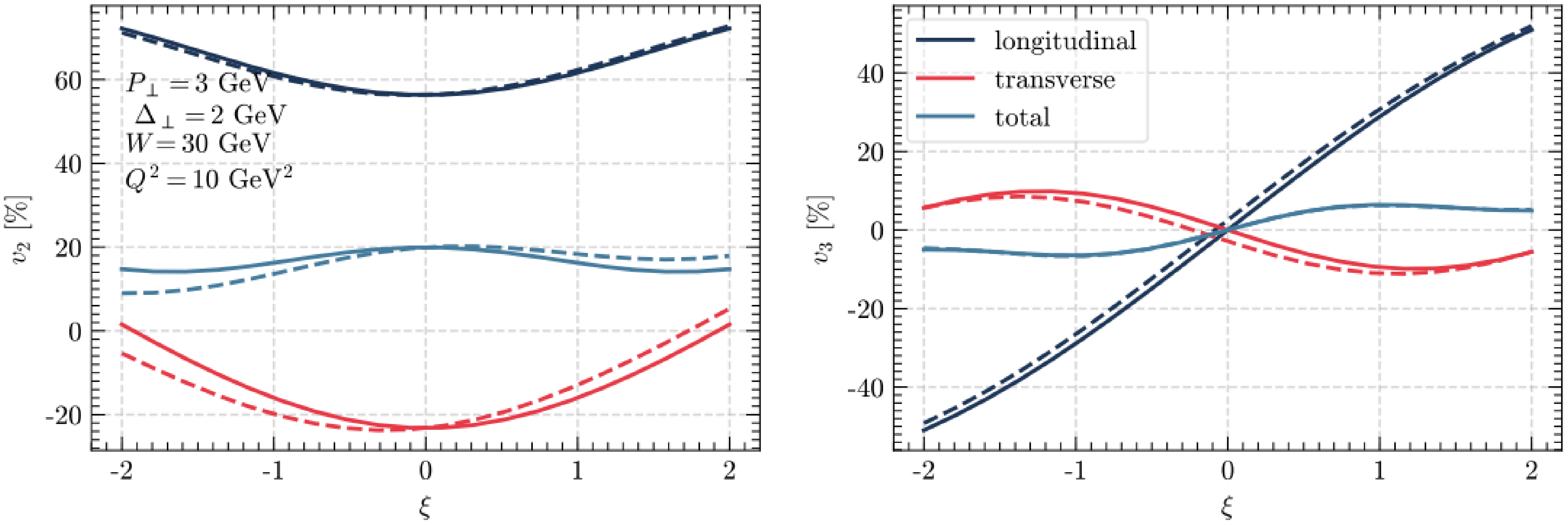




- ❖ EIC energy ($\sqrt{s} = 90$ GeV)
- ❖ 10% correction at relatively large $P_\perp$
- ❖ Weak dependence with $\Delta_\perp$

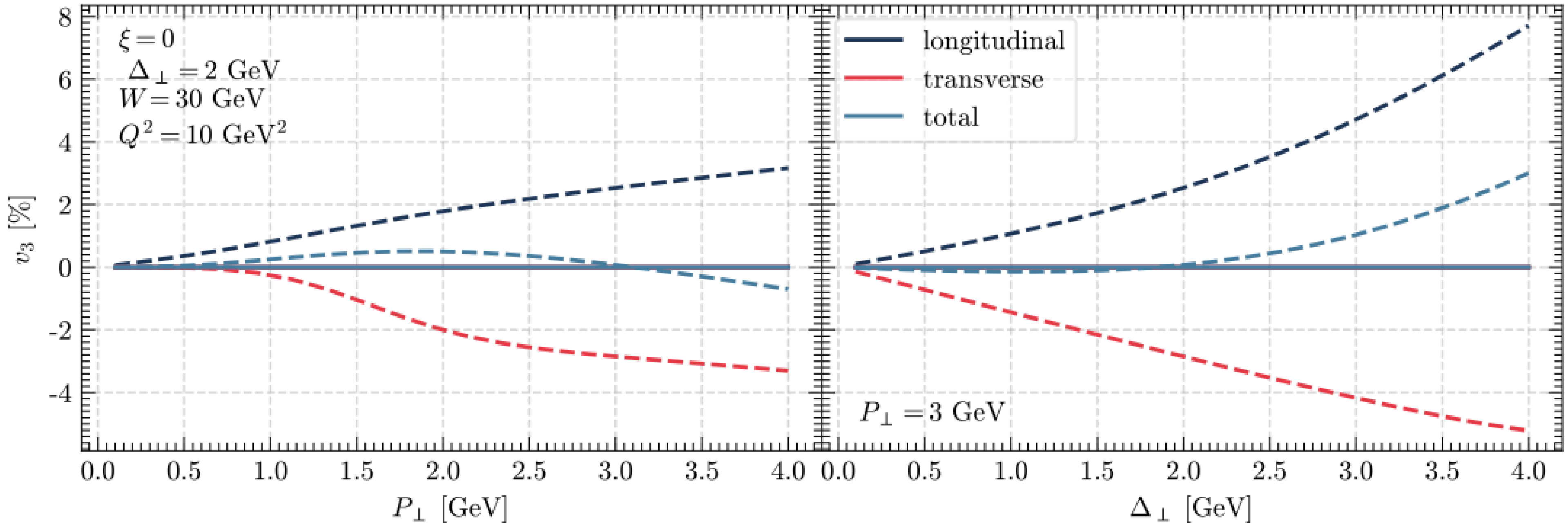
NUMERICAL RESULTS

- Non-eikonal corrections break the (odd) even harmonics (anti)symmetry w.r.t. to ξ:



NUMERICAL RESULTS

- Finite odd harmonics when the particles have same longitudinal momenta ($\xi = 0$)



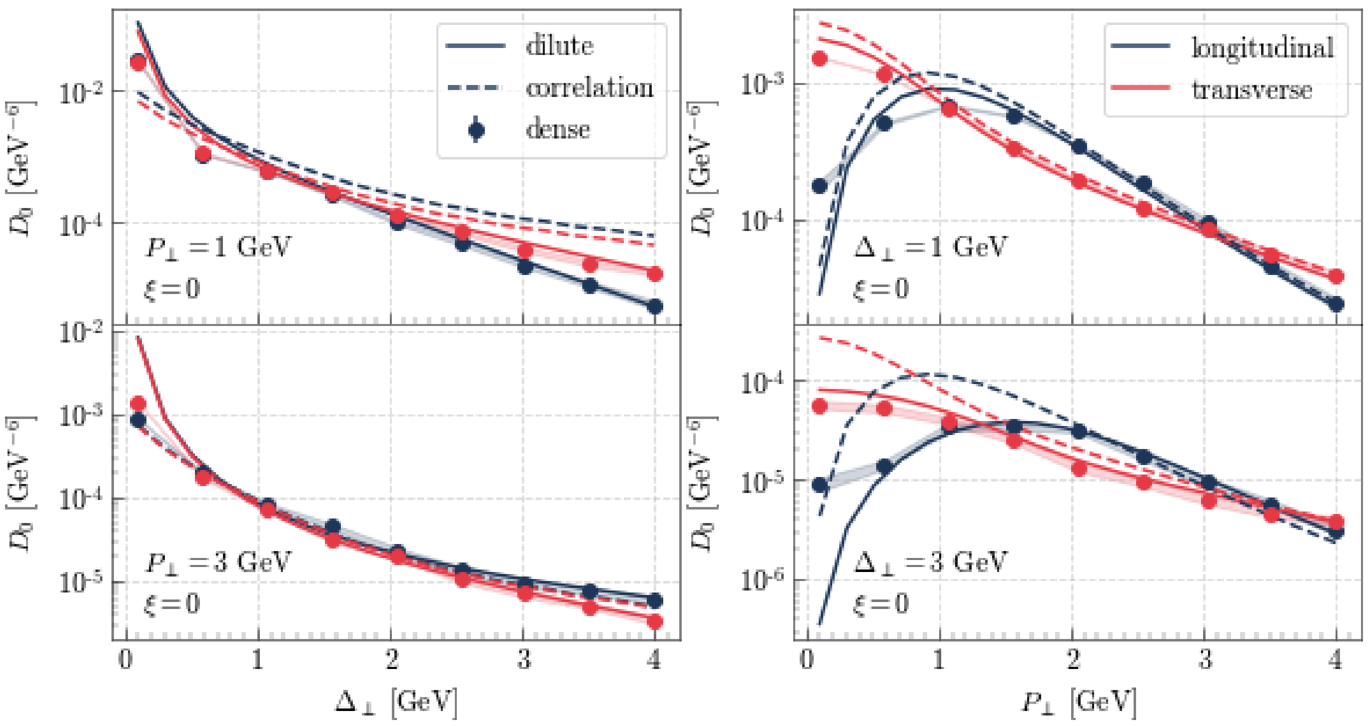
Analogous to double particle production in pA collisions

COMPARISON WITH THE DENSE AND CORRELATION LIMIT AT EIKONAL ACCURACY

- Our analysis is valid when $\Delta_\perp, P_\perp > Q_s$
- It matches the correlation limit when

$$P_\perp \gg \Delta_\perp > Q_s$$

- Very good agreement with the dense limit despite the simplicity of the model



CONCLUSIONS AND OUTLOOK

- We have computed the **field correlators** including all components
- We have analyzed the dijet differential cross section in the **dilute limit** and **beyond the eikonal approximation:**
  - **Analytical solution**
  - **O(10%) corrections** at relatively high momenta
  - **Non-zero odd harmonics**
- **Non-eikonal corrections are relevant at the EIC energies!**
- There is still room for improvement:
  - Studying the dense limit
  - Including other non-eikonal sources (z^- dependence, A^+, classical quarks)
  - More involved model for the target (include correlation, finite A effects, inhomogeneous nucleus, …)