Sudakov Resummation for Small-x Physics

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EIC Science: from quark/gluon to cosmo

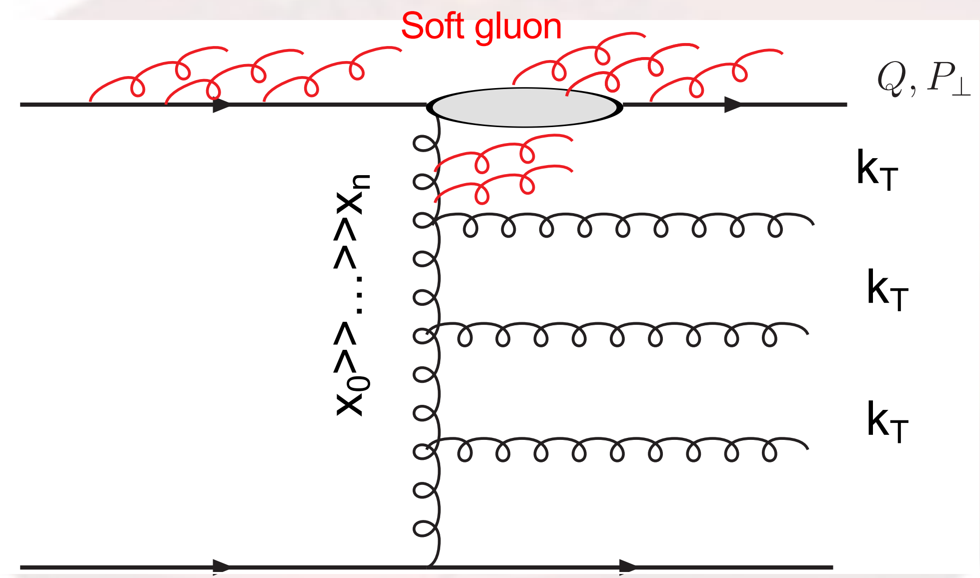
- How do the nucleonic properties such as mass and spin emerge from partons and their underlying interactions?
- How are partons inside the nucleon distributed in both momentum and position space?
- What happens to the gluon density in nucleons and nuclei at small *x*? Does it saturate at high energy, giving rise to gluonic matter with universal properties in all nuclei (and perhaps even in nucleons)?
- How do color-charged quarks and gluons, and jets, interact with a nuclear medium? How do confined hadronic states emerge from these quarks and gluons? How do the quark-gluon interactions generate nuclear binding?
- Do signals from beyond-the-standard-model physics manifest in electron-proton/ion collisions? If so, what can we learn about the nature of these new particles and forces?

EIC Whitepaper for LRP
QCD Whitepaper, 2303.02579

Transverse Momentum Distributions at small-x

- Consistency between the collinear TMD definitions and the small-x dipole calculations have been established
  - Dominguez-Marquet-Xiao-Yuan 2011
- They are the most studied subjects in small-x phenomenology: inclusive, semi-inclusive processes
- Unique predictions of the TMDs from small-x formalism
  - Significant linear polarization for the gluon (Metz-Zhou 2011)
  - Spin (of hadron) dependence offers nontrivial QCD dynamics (Zhou et al, 2015; Kovchegov et al, 2016-2021)

Beyond leading order picture: additional dynamics comes in



- BFKL vs Sudakov resummations (LL)

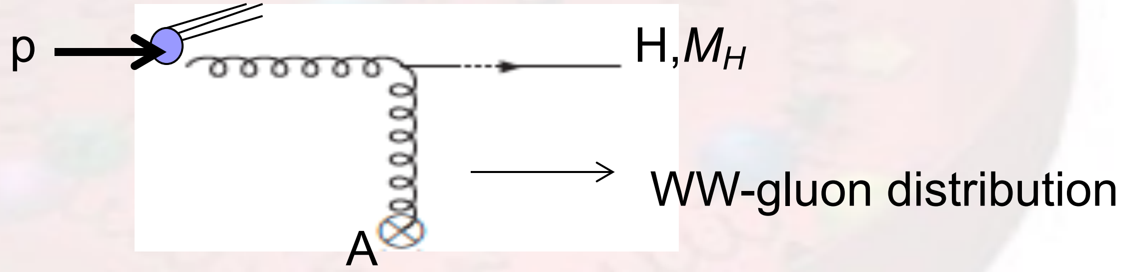
QCD evolution at high energy

- BFKL/BK-JIMWLK (small-x)
- Sudakov (TMD)

Mueller-Xiao-Yuan 2013
Balistky-Tarasov 2014
Xiao-Yuan-Zhou 2016

Sudakov resummation at small-x

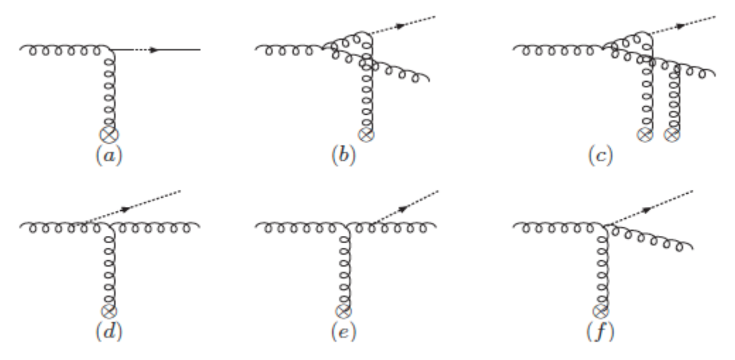
- Take massive scalar particle production $p+A\to H+X$ as an example to demonstrate the double logarithms, and resummation



$$\frac{d\sigma^{(\mathrm{LO})}}{dy d^2k_\perp} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp\cdot r_\perp} x_0 g_p(x_0) S^{(WW)}(x_\perp, x'_\perp)$$

$$S_Y^{WW}(x_\perp, y_\perp) = -\left\langle \mathrm{Tr}\left[\partial_\perp^\beta U(x_\perp) U^\dagger(y_\perp) \partial_\perp^\beta U(y_\perp) U^\dagger(x_\perp)\right]\right\rangle_Y$$

Explicit one-loop calculations



+

$$x_0 g_p(x_0) \int \frac{d\xi}{\xi} \mathbf{K}_{DMMX} \otimes S^{WW}(x_\perp, y_\perp) + \left(-\frac{1}{\epsilon}\right) S^{WW}(x_\perp, y_\perp) \mathcal{P}_{g/g} \otimes x_0 g(x_0) \ ,$$

- Collinear divergence ➔ DGLAP evolution
- Small-x divergence ➔ BK-type evolution

Dominguiz-Mueller-Munier-Xiao, 2011

Soft vs Collinear gluons

- Radiated gluon momentum

$$k_g = \alpha_g p_1 + \beta_g p_2 + k_{g\perp} \ ,$$

- Soft gluon, α~β<<1
- Collinear gluon, α~1, β<<1
- Small-x collinear gluon, 1-β<<1, α→0
  - Rapidity divergence

Final result with resummation

- Double logs at one-loop order

$$\frac{d\sigma^{(\mathrm{LO+NLO})}}{dyd^2k_\perp}|_{k_\perp\ll Q}=\sigma_0\int\frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2}e^{ik_\perp\cdot r_\perp}S^{WW}_{Y=\ln 1/x_a}(x_\perp,x'_\perp)xg_p(x,\mu^2=\frac{c_0^2}{r_\perp^2})$$
$$\left\{1+\frac{\alpha_s}{\pi}C_A\left[\beta_0\ln\frac{Q^2r_\perp^2}{c_0^2}-\frac{1}{2}\left(\ln\frac{Q^2r_\perp^2}{c_0^2}\right)^2+\frac{\pi^2}{2}\right]\right\}\ ,$$

- Inc

$$\frac{d\sigma^{(\mathrm{resum})}}{dyd^2k_\perp}|_{k_\perp\ll Q}=\sigma_0\int\frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2}e^{ik_\perp\cdot r_\perp}e^{-S_{sud}(Q^2,r_\perp^2)}S^{WW}_{Y=\ln 1/x_a}(x_\perp,x'_\perp)$$
$$\times xg_p(x,\mu^2=c_0^2/r_\perp^2)\left[1+\frac{\alpha_s}{\pi}\frac{\pi^2}{2}N_c\right]\ ,$$

Mueller-Xiao-Yuan 2012

Sudakov leading double logs+small-x logs in hard processes, e.g., dijet

- Each incoming parton contributes to a half of the associated color factor in Sudakov
  - Initial gluon radiation, aka, TMDs

$$\frac{d\sigma}{dy_1 dy_2 dP_\perp^2 d^2k_\perp} \propto H(P_\perp^2) \int d^2x_\perp d^2y_\perp e^{ik_\perp\cdot(x_\perp - y_\perp)} \widetilde{W}_{x_A}(x_\perp, y_\perp)$$

Sudakov

$$\Rightarrow H(P_\perp^2) \int d^2x_\perp d^2y_\perp e^{ik_\perp\cdot R_\perp} e^{-S_{sud}(P_\perp, R_\perp)} \widetilde{W}_{x_A}(x_\perp, y_\perp)$$

Mueller-Xiao-Yuan 2013

TMD at small-x: Sudakov and BFKL (BK)

- Start with the factorized TMDs, with full operator definitions
- Calculate the high order corrections in dipole formalism
  - With proper subtraction
- Solve the TMD evolution with BK-evolved dipole (quadrupole) amplitude

See also Skokov's talk Friday

Subtracted TMD at small-x

$$f_g^{(sub.)}(x, r_\perp, \mu_F, \zeta_c) = f_g^{unsub.}(x, r_\perp)\sqrt{\frac{S^{\bar{n},v}(r_\perp)}{S^{n,\bar{n}}(r_\perp)S^{n,v}(r_\perp)}}$$

CGC: WW-gluon
Dipole gluon

Subtract the endpoint
Singularity (Collins 2011)

$$\zeta_c^2 = x^2(2v\cdot P)^2/v^2$$

- TMD evolution follows Collins 2011
  - with resummation, doesn't depend on scheme
- Small-x evolution follows the relevant BK-evolution, respectively
  - Dipole: BK
  - WW: DMMX

Small-x gluon distribution with TMD resummation

Mueller-Xiao-Yuan, 2012;Xiao-Yuan-Zhou 2016
Caucal-Salazar-Schenke-Venugopalan 2022-23
Taels-Altinoluk-Beuf-Marquet 2022
Mukherjee-Skokov-Tarasov-Tiwari, 2023

$$xG^{(1)}(x,k_\perp,\zeta_c=\mu_F=Q)$$

→ Hard scale entering TMD Factorization, e.g., Higgs

$$-\frac{2}{\alpha_S}\int\frac{d^2x_\perp d^2y_\perp}{(2\pi)^4}e^{ik_\perp\cdot r_\perp}\mathcal{H}^{WW}(\alpha_s(Q))e^{-S_{sud}(Q^2,r_\perp^2)} \times\mathcal{F}^{WW}_{Y=\ln 1/x}(x_\perp,y_\perp)\ ,$$

Small-x evolution

Pert. corrections

Sudakov resum.

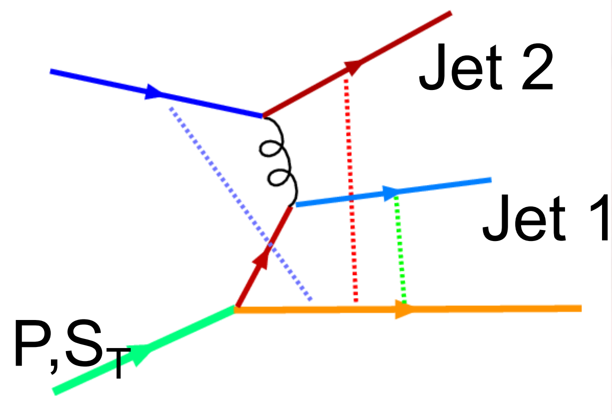
Prediction Power!!

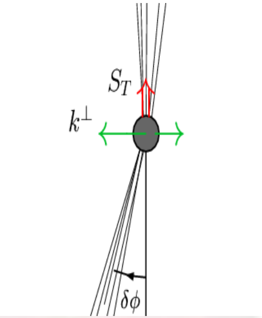
Comments/open questions

- Compare (in some kinematics, match) to TMD gluon distribution in the collinear framework?
- Unambiguously determine gluon saturation
  - Both predict x-dependence, hard to distinguish them
  - Need a big nucleus to enhance the saturation effects, i.e., EIC!

Dijet-correlation in pp is much more complicated

- Initial state and/or final state interactions





Boer-Vogelsang 03

Standard (naïve) Factorization breaks!

Becchetta-Bomhof-Mulders-Pijlman, 04-06
Collins-Qiu 08; Vogelsang-Yuan 08
Rogers-Mulders 10; Xiao-Yuan, 10

Modified factorization

- Dilute system on a dense target, in the large Nc limit,

$$\frac{d\sigma^{(pA\to \text{Dijet}+X)}}{d\mathcal{P}.\mathcal{S}.} = \sum_q x_1 q(x_1)\frac{\alpha_s^2}{\hat{s}^2}\left[\mathcal{F}_{qg}^{(1)} H_{qg\to qg}^{(1)} + \mathcal{F}_{qg}^{(2)} H_{qg\to qg}^{(2)}\right] + x_1 g(x_1)\frac{\alpha_s^2}{\hat{s}^2}\left[\mathcal{F}_{gg}^{(1)}\left(H_{gg\to q\bar{q}}^{(1)} + H_{gg\to gg}^{(1)}\right) + \mathcal{F}_{gg}^{(2)}\left(H_{gg\to q\bar{q}}^{(2)} + H_{gg\to gg}^{(2)}\right) + \mathcal{F}_{gg}^{(3)} H_{gg\to gg}^{(3)}\right],$$

Dominguez-Marquet-Xiao-Yuan 2011

- Kt-dependent gluon distributions

$$\mathcal{F}_{qg}^{(1)} = xG^{(2)}(x, q_\perp), \quad \mathcal{F}_{qg}^{(2)} = \int xG^{(1)}(q_1) \otimes F(q_2) \ ,$$

$$\mathcal{F}_{gg}^{(1)} = \int xG^{(2)}(q_1) \otimes F(q_2), \quad \mathcal{F}_{gg}^{(2)} = \int \frac{q_{1\perp} \cdot q_{2\perp}}{q_{1\perp}^2} xG^{(2)}(q_1) \otimes F(q_2)$$

$$\mathcal{F}_{gg}^{(3)} = \int xG^{(1)}(q_1) \otimes F(q_2) \otimes F(q_3) \ ,$$

Dominguez-Marquet-Xiao-Yuan 2011

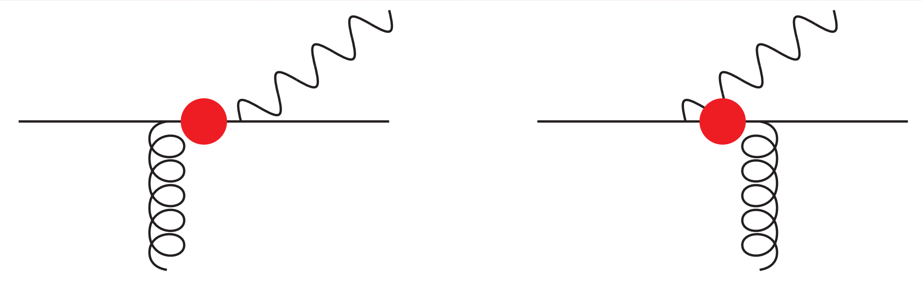
Extend to dijet in hadronic processes: count the leading double logs

- Each incoming parton contributes to a half of the associated color factor
  - Initial gluon radiation, aka, TMDs
- Soft gluon radiation in collinear calculation also demonstrates this rule
  - Sterman, et al
  - Sub-leading logs will be much complicated, usually a matrix form

Mueller, Xiao, Yuan 2013

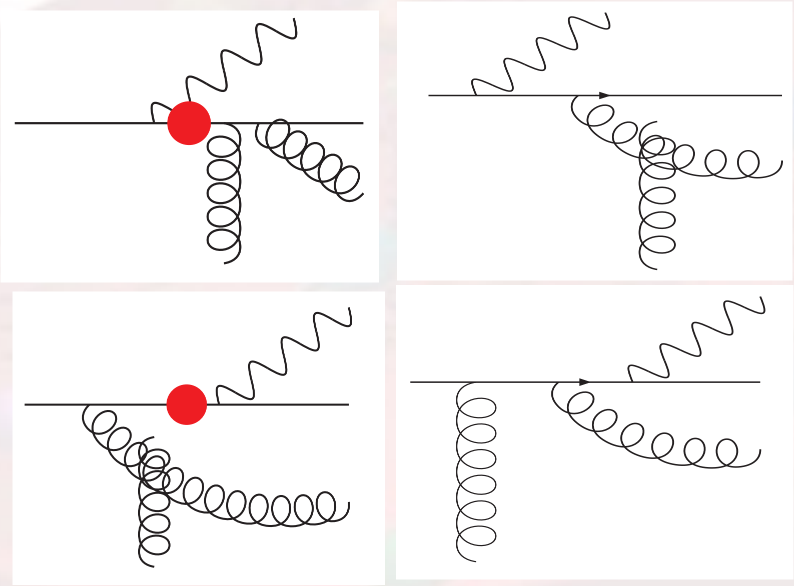
Photon-Jet correlation

- Leading order



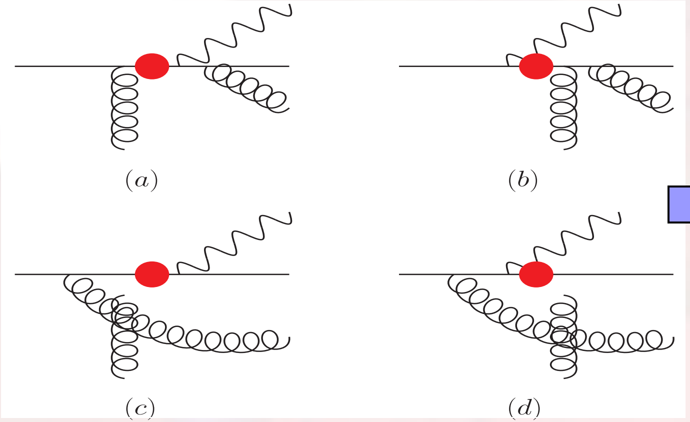
Dipole gluon distribution

BK-evolution



$$\frac{\partial}{\partial Y}S_Y^{(2)}(x_\perp,y_\perp) = -\frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2b_\perp\,(x_\perp-y_\perp)^2}{(x_\perp-b_\perp)^2(y_\perp-b_\perp)^2}\left[S_Y^{(2)}(x_\perp,y_\perp) - S_Y^{(4)}(x_\perp,b_\perp,y_\perp)\right]$$

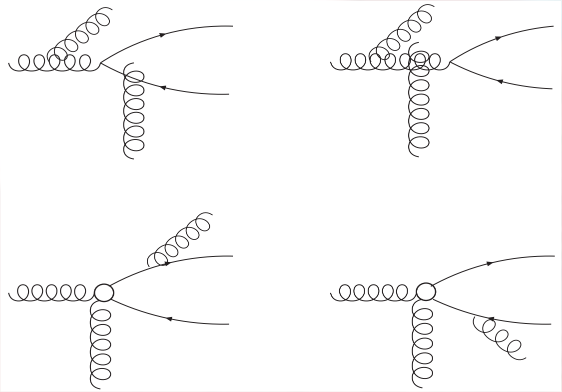
Soft gluon radiation



$$\Rightarrow \left(-\frac{\alpha_s}{2\pi}\frac{C_F+C_A}{2}\right)\ln^2\left(\frac{Q^2(x_\perp-y_\perp)^2}{c_0^2}\right)$$

- A^2 from (a,b) contribute to $C_F/2$ (jet)
- A^2 from (c,d) contribute to C_F
- Interference contribute to 1/2Nc

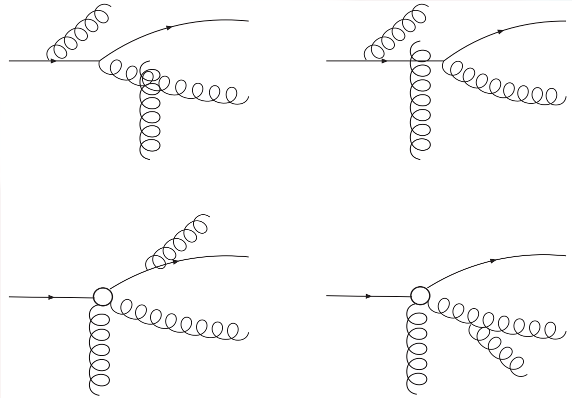
gg→qq



$$\Rightarrow \left(-\frac{\alpha_s}{2\pi}N_c\right)\ln^2\left(\frac{Q^2(x_\perp - y_\perp)^2}{c_0^2}\right)$$

- $|A_1|^2 \to C_A$, $|A_2|^2 \to C_F/2$, $|A_3|^2 \to C_F/2$
- $2A_1^*(A_2+A_3) \to -Nc/2$
- $2A_2^*A_3$, 1/Nc suppressed

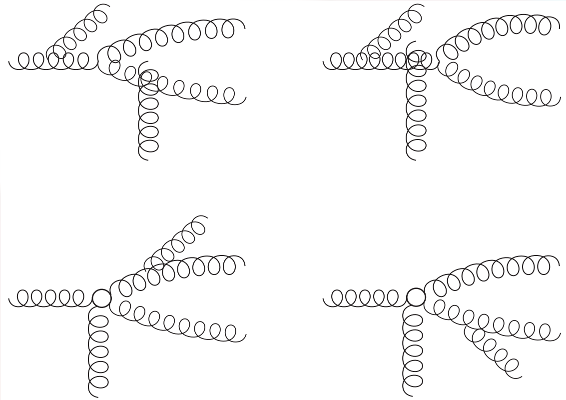
$qg \to qg$



$$\left(-\frac{\alpha_s}{2\pi}\frac{C_F+C_A}{2}\right)\ln^2\left(\frac{Q^2(x_\perp-y_\perp)^2}{c_0^2}\right)$$

- $|A_1|^2 \to C_F$, $|A_2|^2 \to C_F/2$, $|A_3|^2 \to C_A/2$
- $2A_3^*(A_1+A_2) \to -Nc/2$
- $2A_1^*A_2$, large Nc suppressed

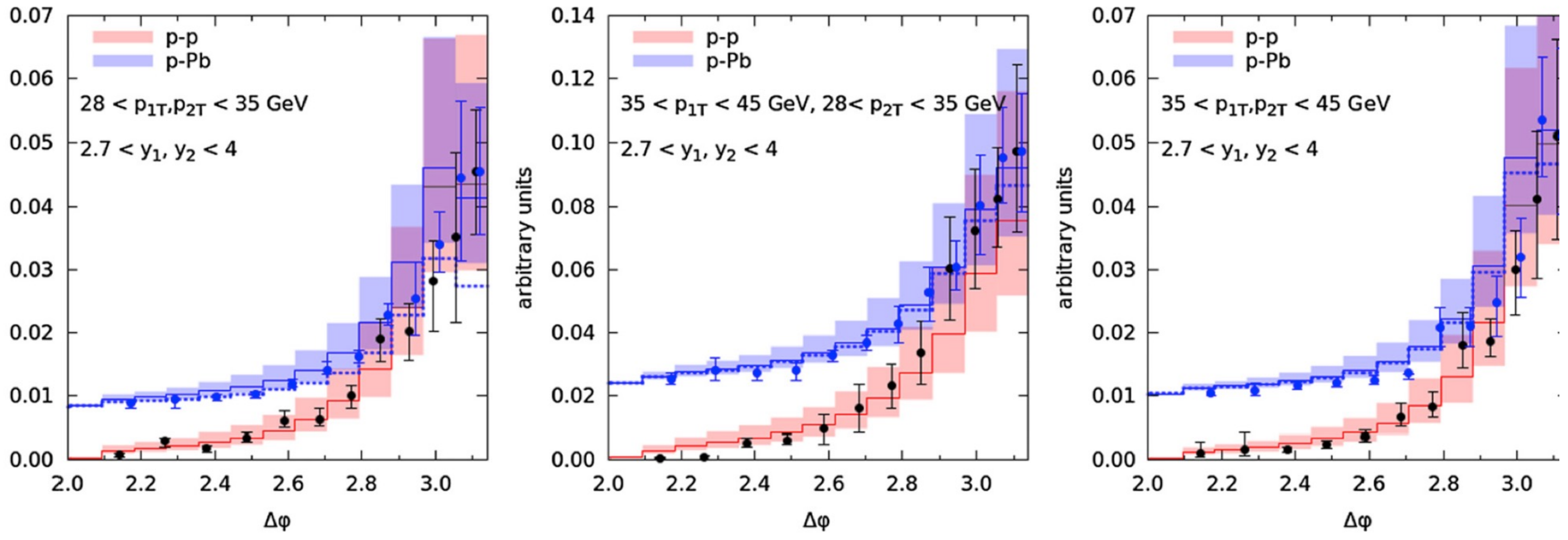
gg→gg



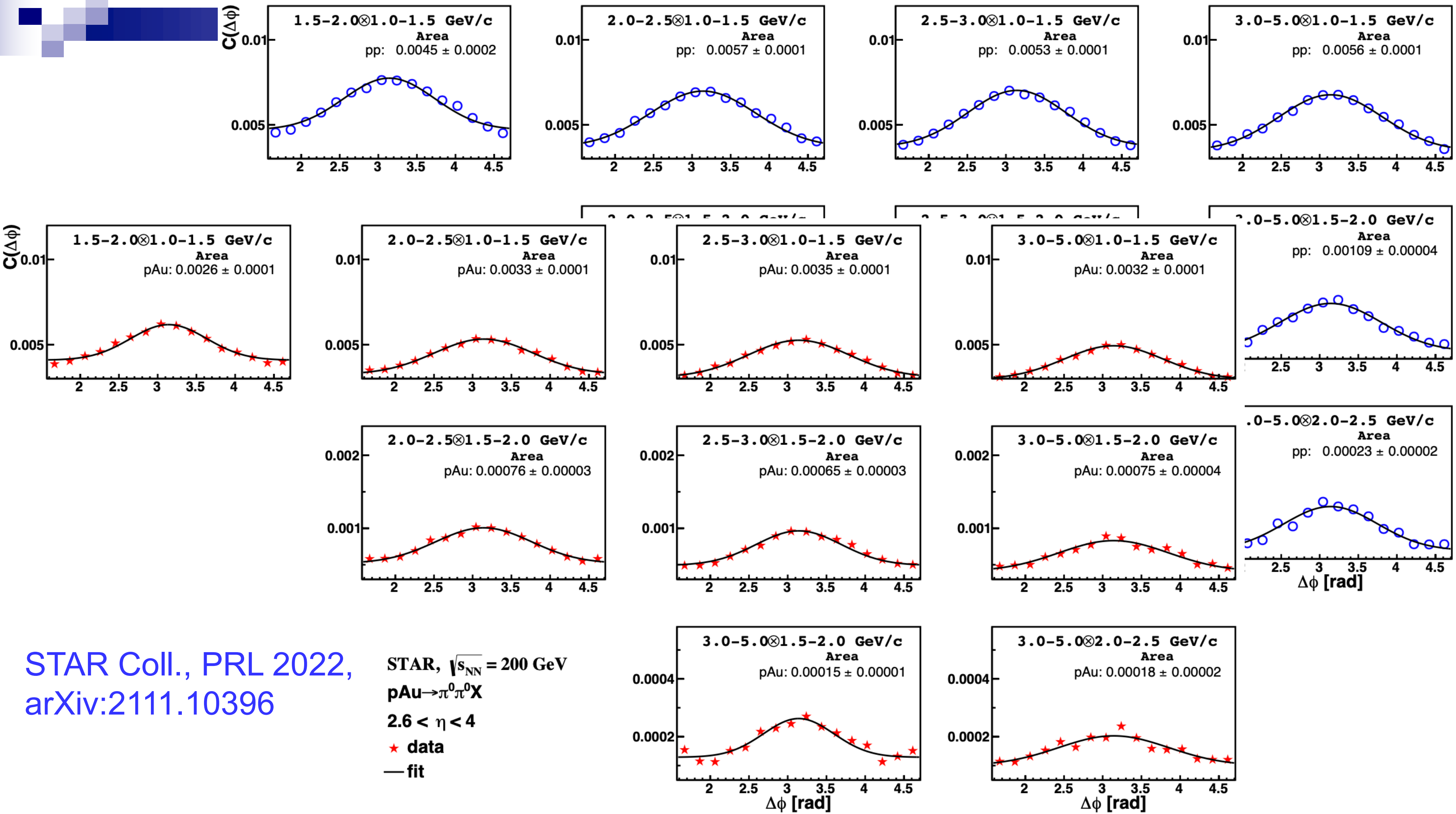
$$\Rightarrow \left(-\frac{\alpha_s}{2\pi}N_c\right)\ln^2\left(\frac{Q^2(x_\perp-y_\perp)^2}{c_0^2}\right)$$

- $|A_1|^2 \to C_A$, $|A_2|^2 \to C_A/2$, $|A_3|^2 \to C_A/2$
- $2A_1^*(A_2+A_3)+2A_2^*A_3 \to -Nc$

pA collisions at the LHC



van Hameren, Kotko, Kutak, Sapeta, 1903.0136

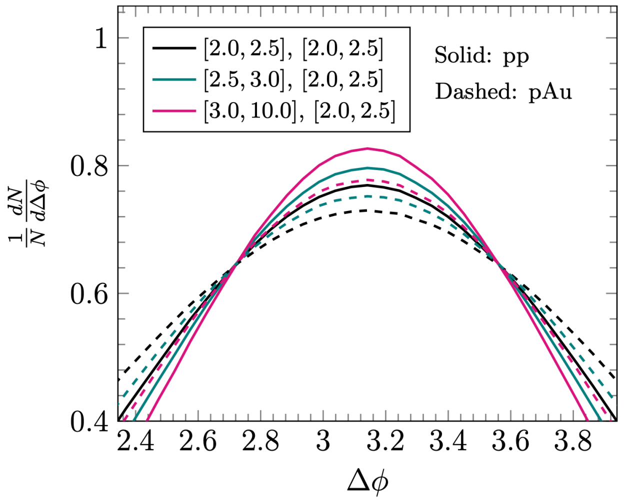


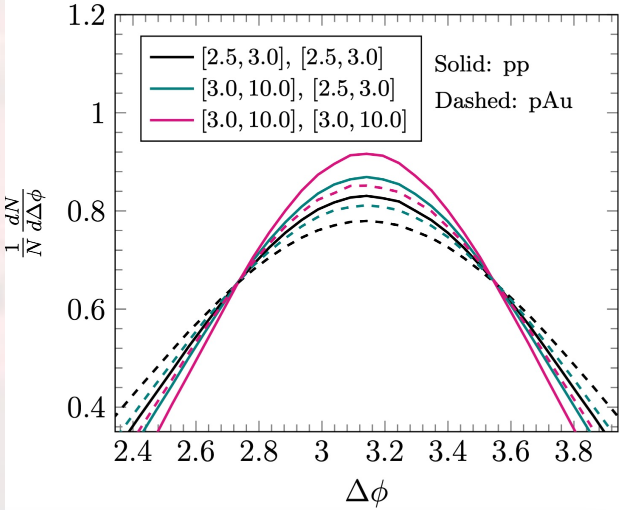
STAR Coll., PRL 2022, arXiv:2111.10396

Include Sudakov effects in the CGC for di-hadron correlations

- Unintegrated gluon distributions w/ Sudakov, e.g.,

$$\mathcal{F}_{qg}^{(a)}(x_g, q_\perp) = \frac{-N_c S_\perp}{2\pi^2 \alpha_s} \int_0^\infty \frac{b_\perp db_\perp}{2\pi} J_0(q_\perp b_\perp) e^{-S_{\rm Sud}^{q+g\to q+g}(b_\perp)} \nabla_{b_\perp}^2 S_{x_g}(b_\perp)$$

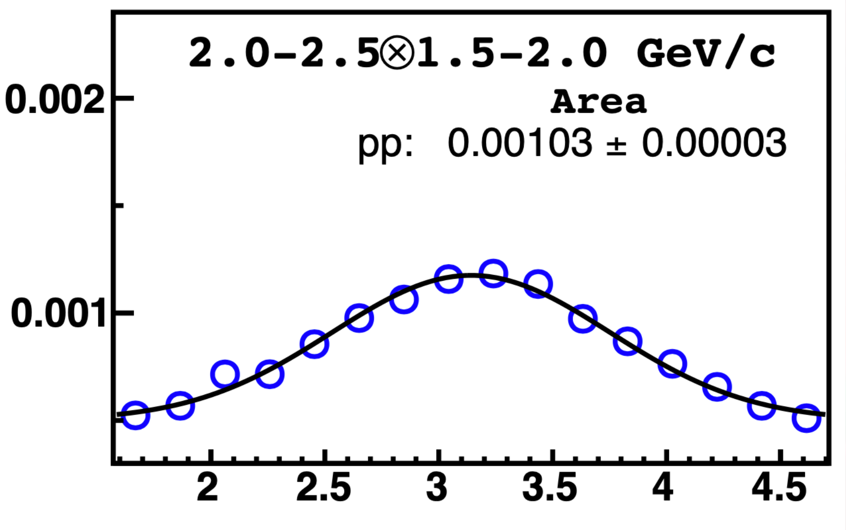


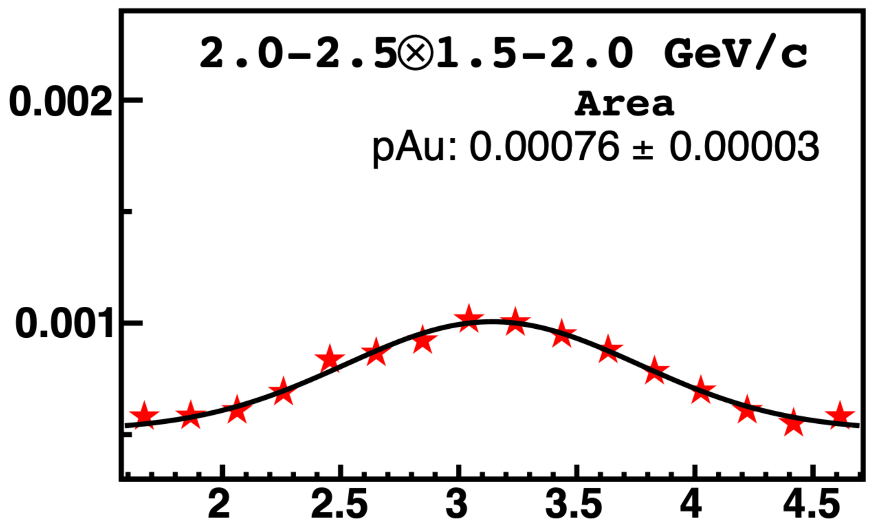


Stasto, Wei, Xiao, Yuan, 2018, 1805.05712

Real data teach us more on the physics

- Compare pp to pA

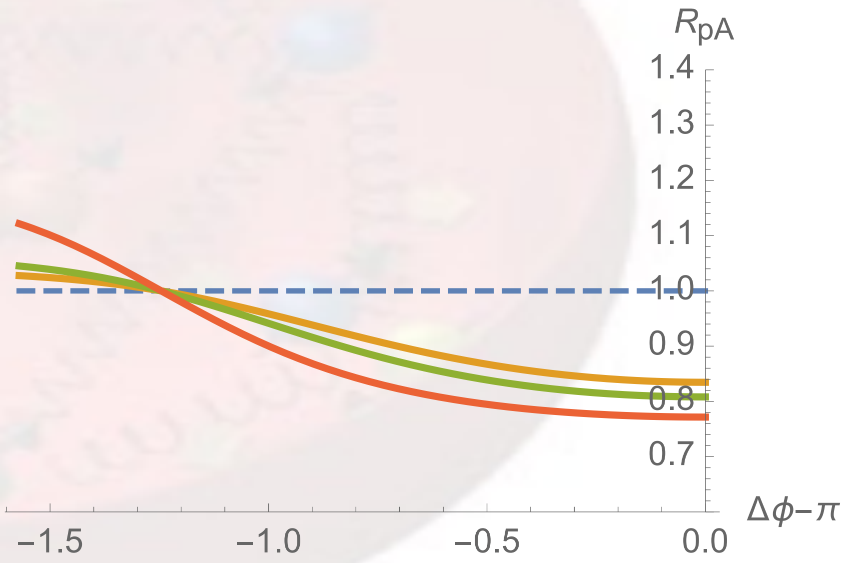




STAR Coll., PRL 2022, arXiv:2111.10396

Simple extraction of nuclear suppression indicates a Pt-broadening effects

- Suppression factor depends on the background subtraction
  - STAR fit: constant background+simple Gaussian shows no Pt-broadening
- Pt-broadening is not as profound as our previous predictions
  - It may change if different background subtraction used



Looking forward

- We need more data
  - Cross check the background! E.g., through charged particle pairs, mixed pairs etc., and photon+hadron correlations
- We need theory developments
  - Complete NLL resummation for dijet in hadronic collisions in CGC (collinear framework done)
  - Need BK-JIMWLK evolution for all different UGDs, at least qualitatively

Beyond the leading double logs: collinear

- Jet size-dependence is computed by averaging the azimuthal angle between the soft gluon and leading jet
- Matrix form due to colored final state   Kidonakis-Sterman 1997

$$x_1 f_a(x_1,\mu=b_0/b_\perp)x_2 f_b(x_2,\mu=b_0/b_\perp)e^{-S_{\rm Sud}(Q^2,b_\perp)}$$
$$\mathrm{Tr}\left[\mathbf{H}_{ab\to cd}\exp[-\int_{b_0/b_\perp}^{Q}\frac{d\mu}{\mu}\gamma^{s\dagger}]\mathbf{S}_{ab\to cd}\exp[-\int_{b_0/b_\perp}^{Q}\frac{d\mu}{\mu}\gamma^{s}]\right]$$

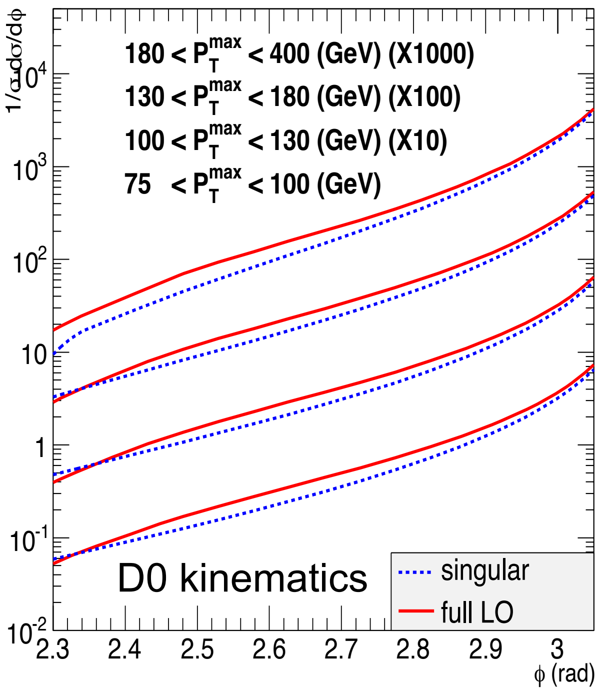
(Sun, C.-P. Yuan, F. Yuan, PRL 2014)

$$S_{\rm Sud}(Q^2,b_\perp)=\int_{b_0^2/b_\perp^2}^{Q^2}\frac{d\mu^2}{\mu^2}\left[\ln\left(\frac{Q^2}{\mu^2}\right)A+B+D_1\ln\frac{Q^2}{P_T^2R_1^2}+D_2\ln\frac{Q^2}{P_T^2R_2^2}\right]$$

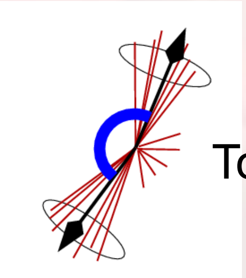
D: color-factor for the jet

R: jet size

Compare to the full calculations



$$\frac{\alpha_s}{2\pi^2}\frac{1}{q_\perp^2}\sum_{ab,a'b'}\sigma_0\int\frac{dx_1'}{x_1'}\frac{dx_2'}{x_2'}x_1' f_a(x_1',\mu)x_2' f_b(x_2',\mu)$$
$$\times\left\{h^{(0)}_{a'b'\to cd}\left[\xi_1\mathcal{P}_{a'/a}(\xi_1)\delta(1-\xi_2)+\xi_2\mathcal{P}_{b'/b}(\xi_2)\delta(1-\xi_1)\right.\right.$$
$$\left.+\delta(1-\xi_1)\delta(1-\xi_2)\delta_{aa'}\delta_{bb'}\left((C_a+C_b)\ln\frac{Q^2}{q_\perp^2}+C_c\ln\frac{1}{R_1^2}+C_d\ln\frac{1}{R_2^2}\right)\right]$$
$$\left.+\delta(1-\xi_1)\delta(1-\xi_2)\delta_{aa'}\delta_{bb'}\Gamma^{ab\to cd}_{sn}\right\}\ , \quad (10$$

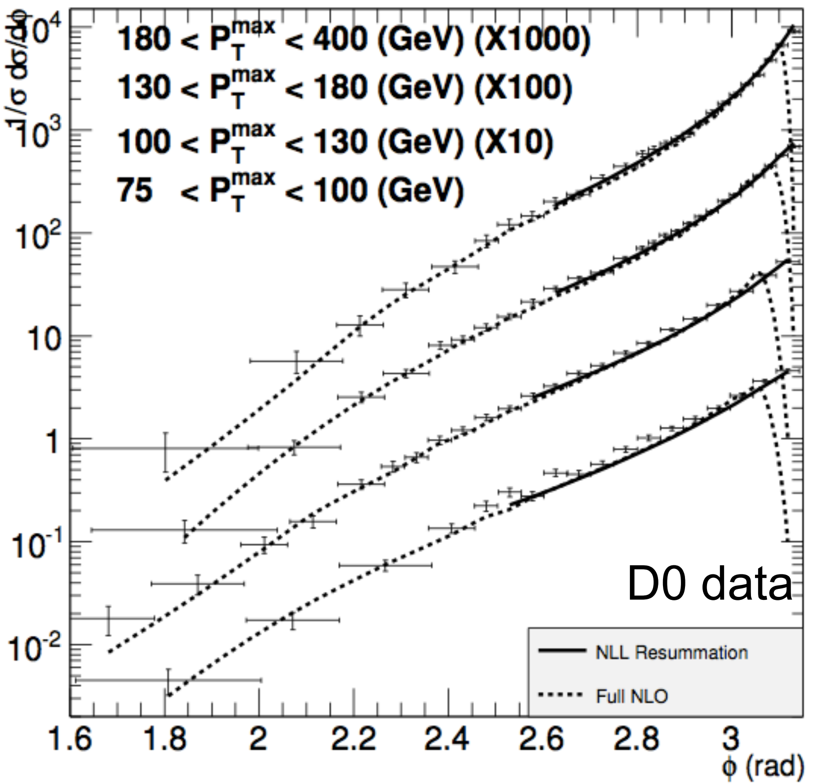


Leading P_T

Total $q_T \approx P_T \mathrm{Sin}(\Delta\phi)$

full LO: Nagy 2002, NLOJET++

Compare to the data



NLL Resummation:
Sun, C.P. Yuan, F.Yuan, PRL2014

$$x_1\, f_a(x_1, \mu = b_0/b_\perp) x_2\, f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\rm Sud}(Q^2, b_\perp)}$$
$$\mathrm{Tr}\left[\mathbf{H}_{ab\to cd} \exp[-\int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s\dagger}] \mathbf{S}_{ab\to cd} \exp[-\int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s}]\right]$$

Full NLO: Nagy 2002, NLOJET++