Squeezing femtoscopic data from vector-baryon pairs

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SPICE: Strange hadrons as Precision tool for strongly InteraCting systEms

ECT, Trento, May 13 -17, 2024*

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Study of the interaction between Vector Mesons (V) and Baryons (B) in a coupled-channel (CC) scheme within the Correlation Function (CF) framework

- $\boldsymbol{\phi - p}$ **CF** (femtoscopic data available)
- $\boldsymbol{\rho - p}$ **CF** (ongoing data analysis)

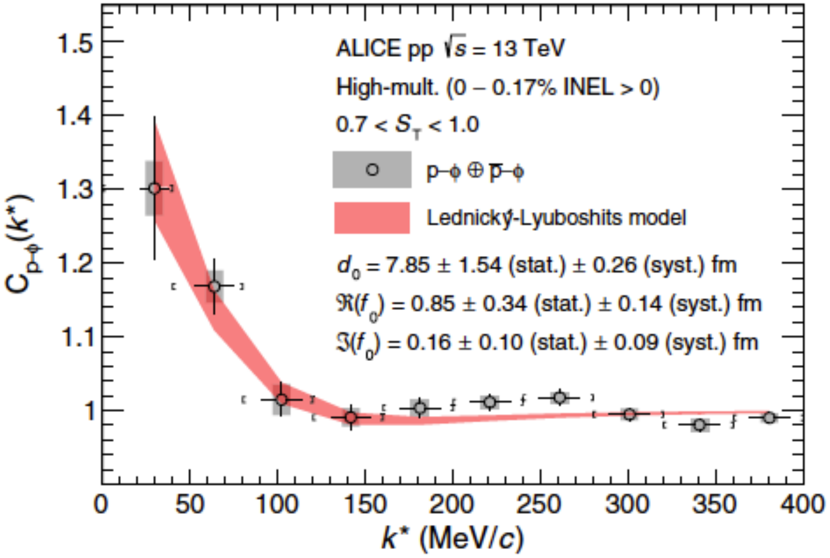
The nature of the interaction for these channels makes them the perfect testing ground to illustrate the relevance CC in the femto data analysis

The constraining power of the CF data on the interaction will be shown and we will extract the scattering information, as well as, get novel insights on the location of resonant states, couplings…

Experimental background

In 2021 the ALICE Collaboration released the experimental $\boldsymbol{\phi - p}$ **CF**

- Scattering parameters extracted from Lednický-Lyuboshits approach
- Conclusions: Interaction dominated by the elastic scattering
- From phenomenological Gaussian- and Yukawa-type potentials $g_{\phi N} = 0.14 \pm 0.03(stat) \pm 0.02(syst)$



ALICE Coll., Phys. Rev. Lett. 127 (2021) 232001

A subsequent publication, the possibility of finding $\boldsymbol{\phi - p}$ bound state was studied using the corresponding femto data

- Binding energy in the range of 12.8 – 56.1 MeV

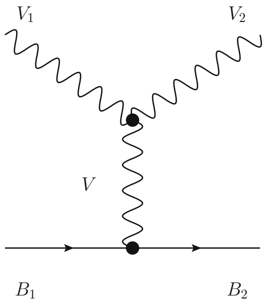
Chizzali, Kamiya et al., Phys. Lett. B 848 (2024) 138358

Theoretical background

Oset and Ramos, Eur. Phys. J. A 44, 445-454 (2010)

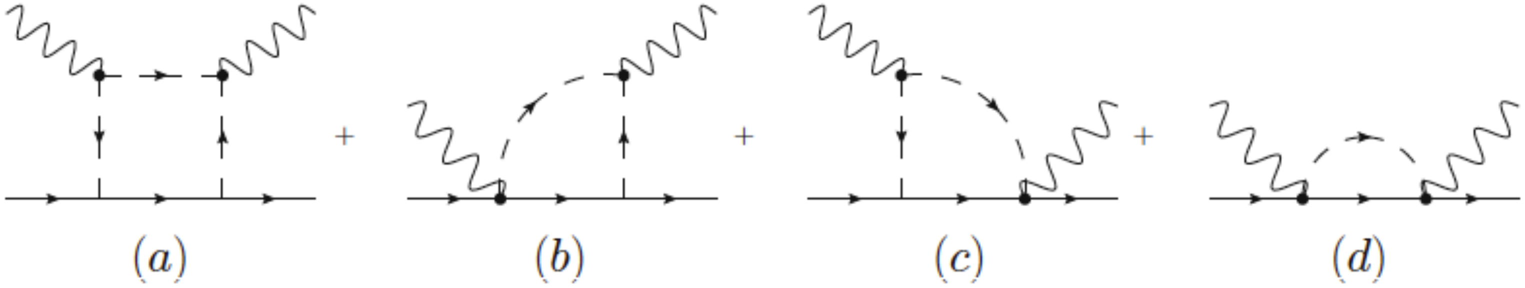
The interaction of V with the octet of stable B was studied, for the first time, within the local hidden gauge formalism using a coupled-channels unitary approach

- S-wave contribution, obtaining some states with degeneracy in 1/2-, 3/2-
- S=0, -1, -2 sectors



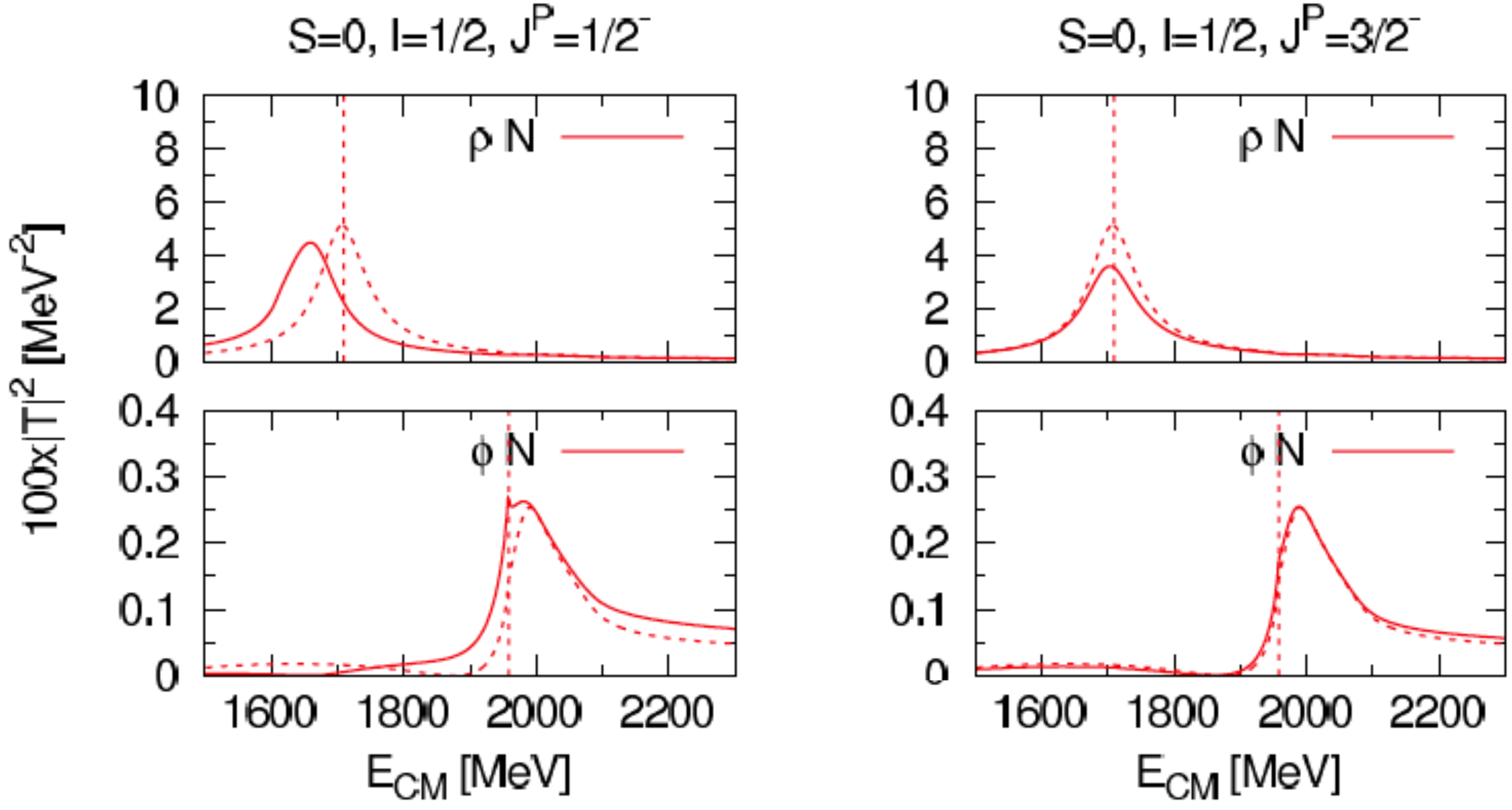
Garzón and Oset, Eur. Phys. J. A (2012) 48: 5

This work was followed incorporating to the V-B interaction other diagrams mediated by pseudoscalar channels



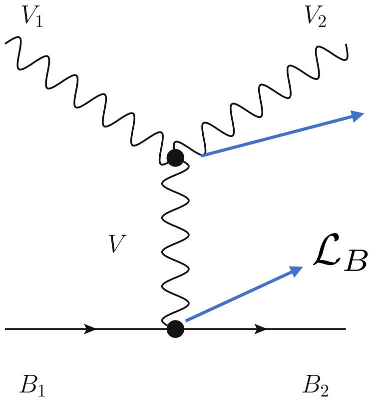
- (a), (b), (c) and (d) diagrams increase the width of the dynamically generated
- (b), (c) and (d) diagrams mixed with (a) break slightly the degeneracy only for J=1/2

Isospin basis (repulsive 3/2 components, no states):



Formalism: Interaction

Hidden gauge formalism



$$\mathcal{L}_{VVV} = ig\,\langle (V^\nu \partial_\mu V_\nu - \partial_\mu V^\nu V_\nu) V^\mu \rangle$$

$$\mathcal{L}_{BBV} = g\left(\langle \bar{B}\gamma_\mu [V^\mu, B]\rangle + \langle \bar{B}\gamma_\mu B\rangle \langle V^\mu \rangle\right)$$

$$V_\mu = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_\mu$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

Relativistic Interaction kernel projected onto s-wave

$$V_{ij} = -\frac{1}{4f^2} C_{ij} \sqrt{\frac{M_i + E_i}{2M_i}} \sqrt{\frac{M_j + E_j}{2M_j}} (2\sqrt{s} - M_i - M_j)$$

S=0, Q=+1 sector: 7 channels

C_{ij}	$\rho^0 p$	$\rho^+ n$	ωp	ϕp	$K^{*+}\Lambda$	$K^{*o}\Sigma^+$	$K^{*+}\Sigma^0$
$\rho^0 p$	0	$\sqrt{2}$	0	0	$-\sqrt{3}/2$	$1/\sqrt{2}$	$-1/2$
$\rho^+ n$		1	0	0	$-\sqrt{3}/\sqrt{2}$	0	$1/\sqrt{2}$
ωp			0	0	$-\sqrt{3}/2$	$-1/\sqrt{2}$	$-1/2$
ϕp				0	$\sqrt{3}/\sqrt{2}$	1	$1/\sqrt{2}$
$K^{*+}\Lambda$					0	0	0
$K^{*o}\Sigma^+$						1	$\sqrt{2}$
$K^{*+}\Sigma^0$							0

Formalism: T-matrix

The Bethe-Salpether equation is solved to calculate the scattering matrix

$$T_{ij} = (1 - V_{il} G_l)^{-1} V_{lj}$$

The vector meson – baryon loop after dimensional regularization:

$$G_l = \frac{2M_l}{(4\pi)^2}\left\{ a_l(\mu) + \ln\frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s}\ln\frac{m_l^2}{M_l^2} + \frac{q_{\rm cm}}{\sqrt{s}}\ln\left[\frac{(s+2\sqrt{s}q_{\rm cm})^2-(M_l^2-m_l^2)^2}{(s-2\sqrt{s}q_{\rm cm})^2-(M_l^2-m_l^2)^2}\right]\right\}$$

subtraction constants for the dimensional regularization scale $\mu = 630MeV$ in all the “l” channels.

isospin symmetry

$$a_{\rho^0 p} = a_{\rho^+ n} = a_{\rho N}$$

$$a_{\omega p} = a_{\omega N}$$

$$a_{\phi p} = a_{\phi N}$$

$$a_{K^{*+}\Lambda} = a_{K^*\Lambda}$$

$$a_{K^{*+}\Sigma^0} = a_{K^{*0}\Sigma^+} = a_{K^*\Sigma}$$

In principle, all vector mesons are assumed to be stable particles but…

Formalism: Interaction

… given the sizeable width of ***ρ*** and K*, we convolute the loop function *G* with their mass distribution:

$$\tilde{G}(s) = \frac{1}{N}\int_{(m_l-2\Gamma_l)^2}^{(m_l+2\Gamma_l)^2} dm^2 \left(-\frac{1}{\pi}\right) Im\left[\frac{1}{m^2 - m_l^2 + im\Gamma(m)}\right] G_l(s, m^2, M_l^2)$$

Ordinary loop function

$$N = \int_{(m_l-2\Gamma_l)^2}^{(m_l+2\Gamma_l)^2} dm^2 \left(-\frac{1}{\pi}\right) Im\left[\frac{1}{m^2 - m_l^2 + im\Gamma(m)}\right]$$

$$\Gamma(m) = \Gamma_l \frac{m_l^2}{m^2}\left(\frac{m^2-(m_1+m_2)^2}{m_l^2-(m_1+m_2)^2}\right)^{3/2} \theta(m-(m_1+m_2))$$

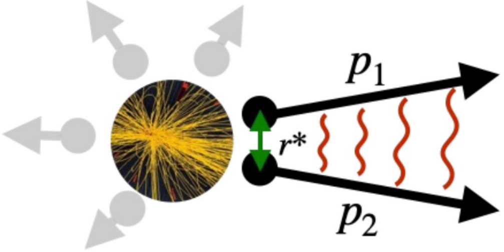
$$\rho \rightarrow m_1 = m_2 = m_\pi, \Gamma_\rho = 149.77MeV$$

$$K^* \rightarrow m_1 = m_K, m_2 = m_\pi, \Gamma_{K^*} = 48.3MeV$$

From PDG

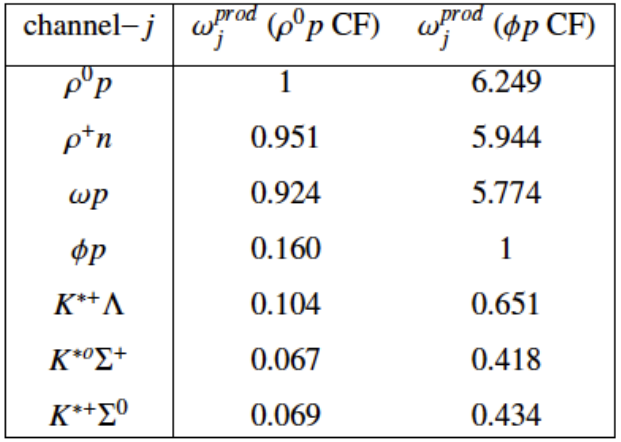
Masses of the decay products

Formalism: Correlation Function



The CF in multi-channel systems for a given pair “*i*” can be express as:

$$C_i(k^*) = \sum_j \omega_j^{\text{prod.}} \int d^3r^* S_j(r^*) |\psi_{ji}(k^*, r^*)|^2$$



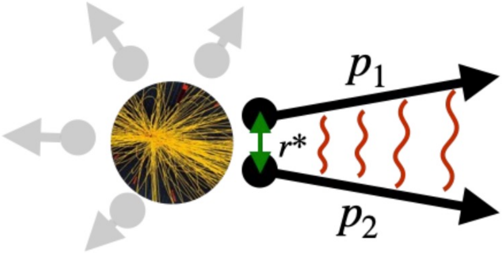
channel−j	ω_j^{prod} ($\rho^0 p$ CF)	ω_j^{prod} (ϕp CF)
$\rho^0 p$	1	6.249
$\rho^+ n$	0.951	5.944
ωp	0.924	5.774
ϕp	0.160	1
$K^{*+}\Lambda$	0.104	0.651
$K^{*o}\Sigma^+$	0.067	0.418
$K^{*+}\Sigma^0$	0.069	0.434

Values provided by **Maximilian Korwieser**

$\boldsymbol{\omega_j^{prod}}$: production weights take into account the initially produced *j* pairs that can convert to the measured *i* final state

- ✓ Depend on yields & kinematics: $\boldsymbol{\omega_j^{prod}} = \frac{N_j^{prim}}{N_i^{prim}} = \frac{N_{j_A}^{prim} N_{j_B}^{prim}}{N_{i_A}^{prim} N_{i_B}^{prim}}$
- ✓ Particle abundances estimated from **Thermal**[1] & Transport[2] moldels

Formalism: Correlation Function



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$$C_i(k^*) = \sum_j \omega_j^{\text{prod.}} \int d^3r^* S_j(r^*) |\psi_{ji}(k^*, r^*)|^2$$

$\boldsymbol{S_j(r^*)}$: emitting source describes the probability of emitting the particle pair *j* at a relative distance *r**.
For the present cases, it is taken to be the same for all channels:

$$S_j(r^*) = \frac{1}{\sqrt{4\pi}R_j^3} exp\left(- \frac{r^{*2}}{4R_j^2} \right)$$

Source size also fixed for all channels (and both CFs):
$\boldsymbol{R_j} = 1.08\ \text{fm}$

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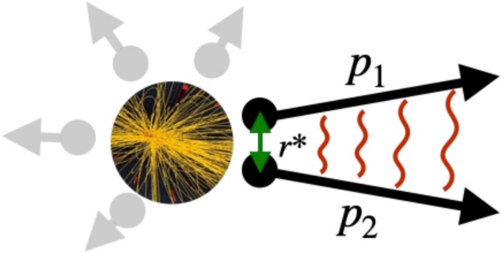
Formalism: Correlation Function



The CF in multi-channel systems for a given pair "*i*" can be express as:

$$C_i(k^*) = \sum_j \omega_j^{\text{prod.}} \int d^3r^* S_j(r^*)|\psi_{ji}(k^*, r^*)|^2$$

$\boldsymbol{\Psi_{ji}(k^*, r^*)}$: relative wave function describes the transition from a given inital channel *j* to the final observed channel *i* & it can be obtained from the scattering amplitude $\boldsymbol{T_{ji}}$ as

$$\psi_{ji}(k^*, r^*) = \delta_{ij} j_0(k^* r^*) + \int d^3q \frac{j_0(qr^*) T_{ji}(E, k^*, q)}{E - E_1^j(q) - E_2^j(q) + i\eta}$$

Finally, to compare to the genuine calculated by ALICE, we redefine our CF by adding a global normalizing prefactor $\mathbf{N_D}$:

$$C_i^{gen}(k^*) = N_D C_i(k^*)$$

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Fitting/bootstrap procedure:

The model depends on 6 parameters:

(the data set considered is the 13 points below k*=500 MeV in the gen. CF) *Rev. Lett. 127 (2021) 232001*

- 5 subtraction constants a_i's (already reduced employing isospin symmetry arguments)
  theoretical arguments establish their values to be constrain around -2
  we allow them to vary in the range of [-4,-1]
- Global normalization N_D
  taken to be within [0.8,1.2]

Oller and Meissner,
Phys. Lett. B 500 (2001) 263-272

But… a preliminary 6 parameter fit provides:

$$\boxed{\delta(a_{\rho N}) \approx \pm 2.5}$$

$$r_{ij} = \begin{array}{cccccc|c}
a_{\rho N} & a_{\omega N} & a_{\phi N} & a_{K^*\Lambda} & a_{K^*\Sigma} & N_D & \\
\hline
1.000 & 0.000 & 0.000 & 0.001 & 0.000 & 0.000 & a_{\rho N} \\
 & 1.000 & -0.683 & -0.081 & -0.392 & -0.360 & a_{\omega N} \\
 & & 1.000 & -0.056 & -0.283 & -0.267 & a_{\phi N} \\
 & & & 1.000 & -0.361 & 0.327 & a_{K^*\Lambda} \\
 & & & & 1.000 & 0.555 & a_{K^*\Sigma} \\
 & & & & & 1.000 & N_D
\end{array}$$

Covariance matrix

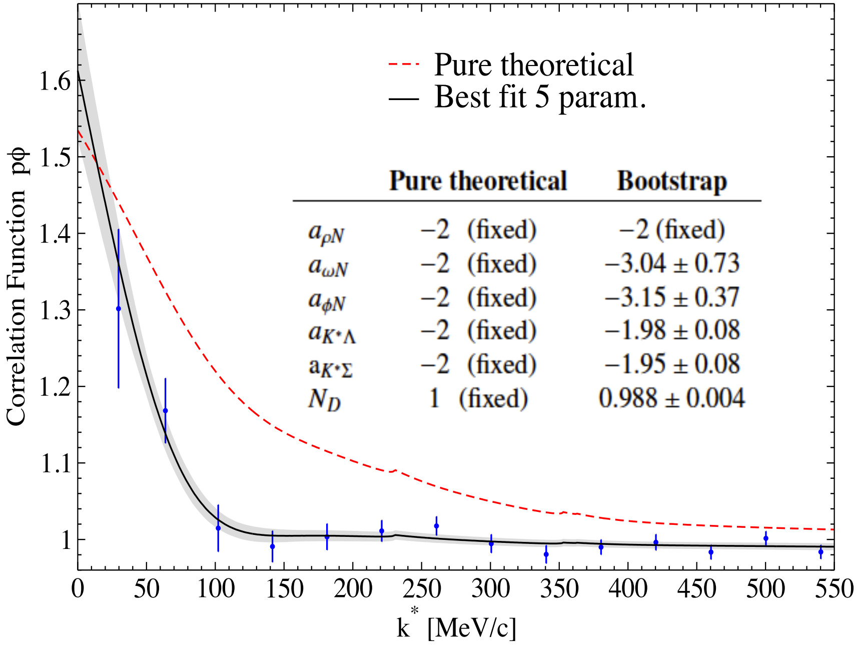
Uncorrelated parameter + big error compared to the range, data cannot constrain it (kept as -2)

$$\delta(a_{\omega N}), \delta(a_{\phi N}) \approx \pm 2$$

The size of these errors is associated with the notable correlation between them

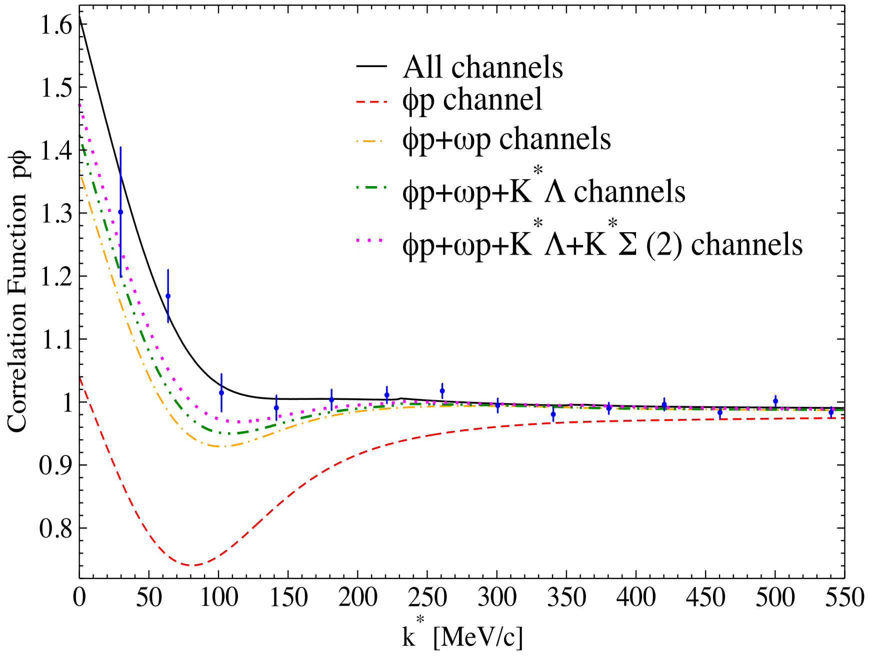
5 param analysis at the end!!!

Results: $\phi - p$ CF



	Pure theoretical	Bootstrap
$a_{\rho N}$	−2 (fixed)	−2 (fixed)
$a_{\omega N}$	−2 (fixed)	−3.04 ± 0.73
$a_{\phi N}$	−2 (fixed)	−3.15 ± 0.37
$a_{K^*\Lambda}$	−2 (fixed)	−1.98 ± 0.08
$a_{K^*\Sigma}$	−2 (fixed)	−1.95 ± 0.08
N_D	1 (fixed)	0.988 ± 0.004

Coupled channel contributions (best fit)



Results: $\phi - p$ scattering parameters

	Pure theoretical	Bootstrap	ALICE analysis
$a_0^{\phi p}$ (fm)	$0.272 + i\,0.189$	$(-0.034 \pm 0.035) + i\,(0.57 \pm 0.09)$	$(0.85 \pm 0.48) + i\,(0.16 \pm 0.19)$
$r_{eff}^{\phi p}$ (fm)	$-7.20 - i\,0.09$	$(-8.06 \pm 2.57) + i\,(0.05 \pm 0.53)$	7.85 ± 1.80

Other values found in literature:

- Analysis of the CLAS data

  $|a_0^{\phi p}| = (0.063 \pm 0.010)\mathrm{fm}$ I. I. Strakovsky, L. Pentchev, and A. I. Titov, Phys. Rev. C 101, 045201 (2020).

- LEPS measurements of ϕ cross section

  $a_0^{\phi p} = 0.15\mathrm{fm}$ A. I. Titov, T. Nakano, S. Daté, and Y. Ohashi, Phys. Rev. C 76, 048202 (2007).
  W. C. Chang et al., Phys. Lett. B 658, 209 (2008).

- SU(3) Effective Chiral Lagrangian with vector-meson dominance

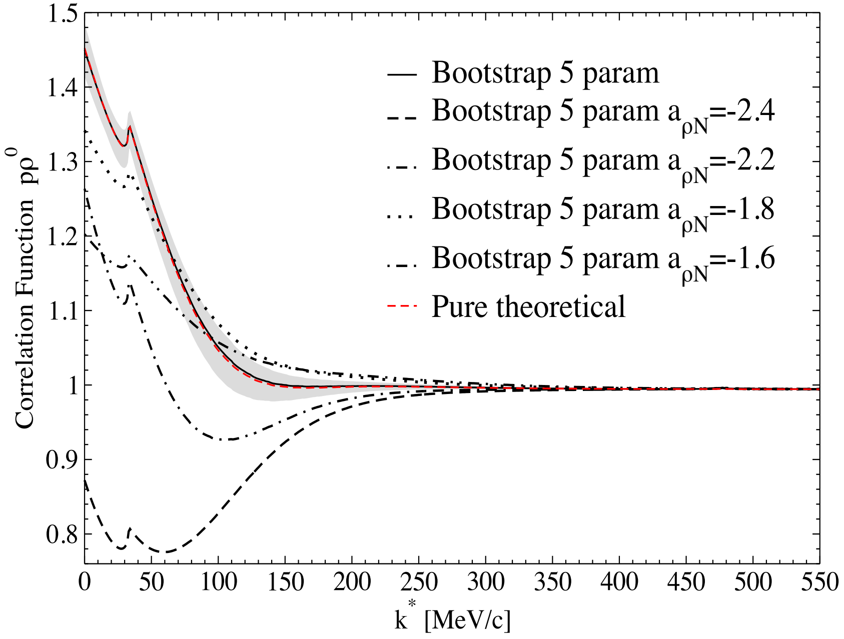
  $a_0^{\phi p} = (-0.01 + i0.08)\mathrm{fm}$ F. Klingl, N. Kaiser, and W. Weise, Nucl. Phys. A624, 527 (1997).

- QCD sum rule analysis

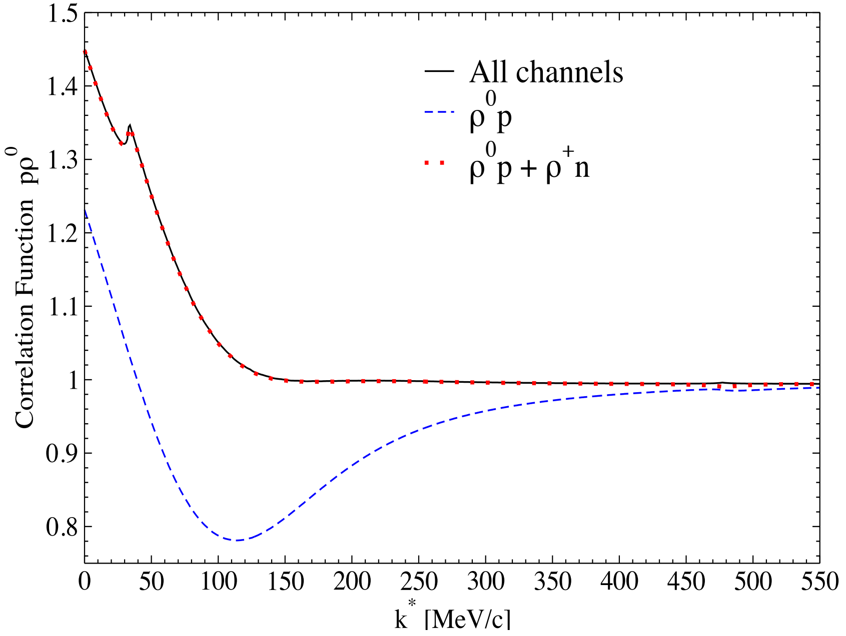
  $a_0^{\phi p} = (-0.15 \pm 0.02)\mathrm{fm}$ Y. Koike and A. Hayashigaki, Prog. Theor. Phys. 98, 631 (1997).

Results: limited prediction for $\rho - p$ CF

	Pure theoretical	Bootstrap
$a_{\rho N}$	−2 (fixed)	−2 (fixed)
$a_{\omega N}$	−2 (fixed)	−3.04 ± 0.73
$a_{\phi N}$	−2 (fixed)	−3.15 ± 0.37
$a_{K^*\Lambda}$	−2 (fixed)	−1.98 ± 0.08
$a_{K^*\Sigma}$	−2 (fixed)	−1.95 ± 0.08
N_D	1 (fixed)	0.988 ± 0.004

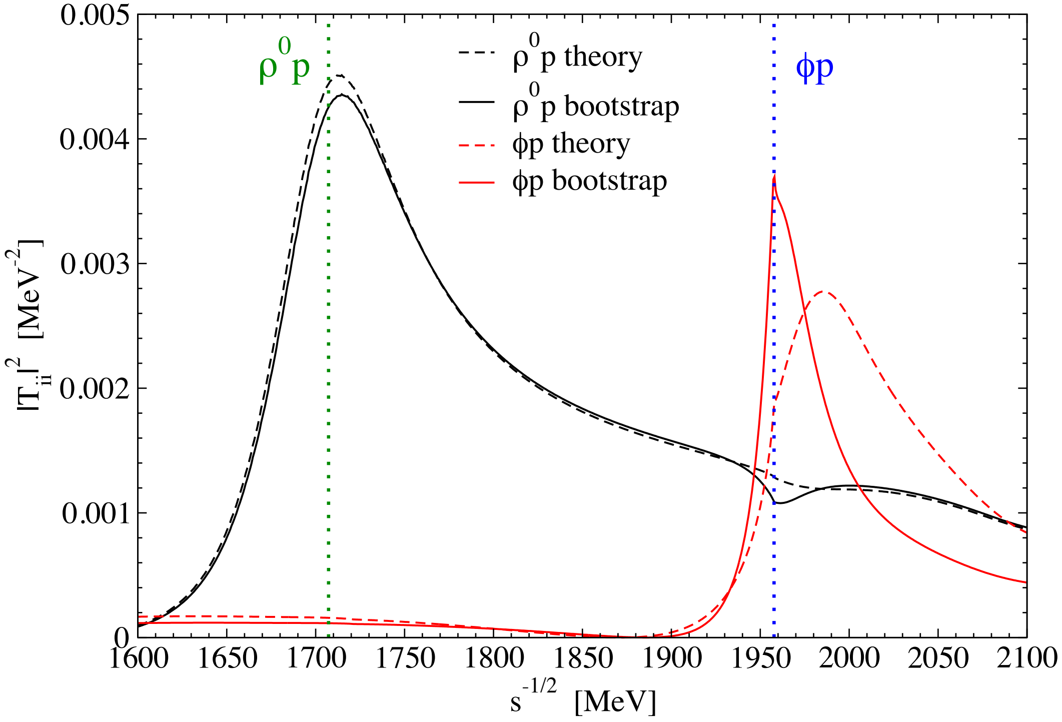


Coupled channel contributions



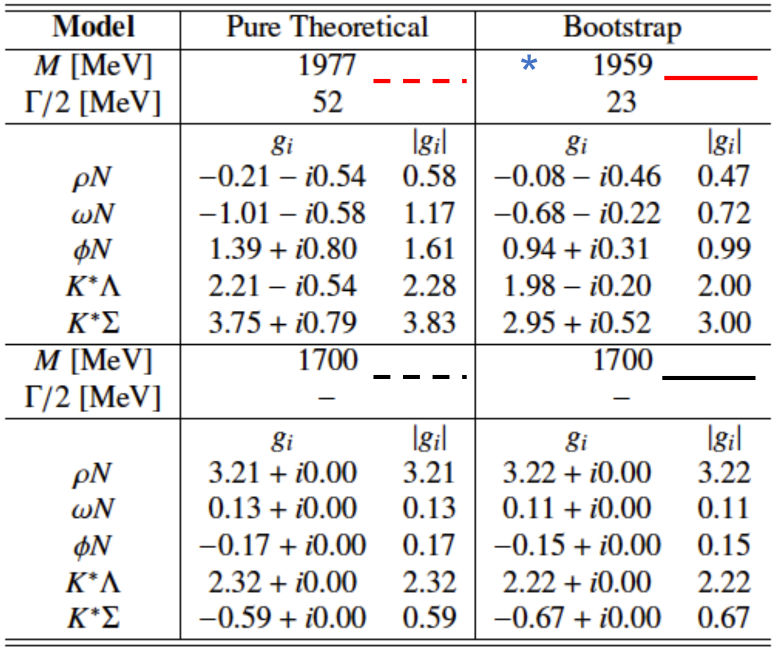
	Pure theoretical	Bootstrap
$d_0^{\rho^0 p}$ (fm)	$0.090 + i\,0.568$	$(0.09 \pm 0.03) + i\,(0.56 \pm 0.05)$
$r_{eff}^{\rho^0 p}$ (fm)	$-3.01 + i\,98.39$	$(-3.05 \pm 0.28) + i\,(98.40 \pm 0.12)$

Results: $\phi - p$ and $\rho - p$ elastic scattering amplitudes and pole content



I=1/2, S=0, Q=+1, J^P=1/2$^-$, 3/2$^-$

These quantum numbers qualify them as N* states



Model	Pure Theoretical		Bootstrap	
M [MeV]	1977	- - -	* 1959	——
Γ/2 [MeV]	52		23	
	g_i	$\lvert g_i \rvert$	g_i	$\lvert g_i \rvert$
ρN	$-0.21 - i0.54$	0.58	$-0.08 - i0.46$	0.47
ωN	$-1.01 - i0.58$	1.17	$-0.68 - i0.22$	0.72
ϕN	$1.39 + i0.80$	1.61	$0.94 + i0.31$	0.99
$K^*\Lambda$	$2.21 - i0.54$	2.28	$1.98 - i0.20$	2.00
$K^*\Sigma$	$3.75 + i0.79$	3.83	$2.95 + i0.52$	3.00
M [MeV]	1700	- - -	1700	——
Γ/2 [MeV]	–		–	
	g_i	$\lvert g_i \rvert$	g_i	$\lvert g_i \rvert$
ρN	$3.21 + i0.00$	3.21	$3.22 + i0.00$	3.22
ωN	$0.13 + i0.00$	0.13	$0.11 + i0.00$	0.11
ϕN	$-0.17 + i0.00$	0.17	$-0.15 + i0.00$	0.15
$K^*\Lambda$	$2.32 + i0.00$	2.32	$2.22 + i0.00$	2.22
$K^*\Sigma$	$-0.59 + i0.00$	0.59	$-0.67 + i0.00$	0.67

* In physical basis this state appears at 1957.75 MeV (just 20 KeV above the $\phi - p$ threshold)

CONCLUSIONS

The VM-B interaction (S=0, Q=+1) was studied within the hidden gauge formalism in a coupled channel scheme. The novel aspect of the present study is the use of the $\boldsymbol{\phi - p}$ CF data to constrain the theoretical model for the first time.

- The resulting model allowed us to reproduce the data in very good agreement with the experimental analysis, although the $\boldsymbol{\rho - p}$ SC showed to be unconstrained thereby providing a not very reliable prediction for the $\boldsymbol{\rho - p}$ CF (which relies completely in the $\boldsymbol{\rho N}$ amplitudes)

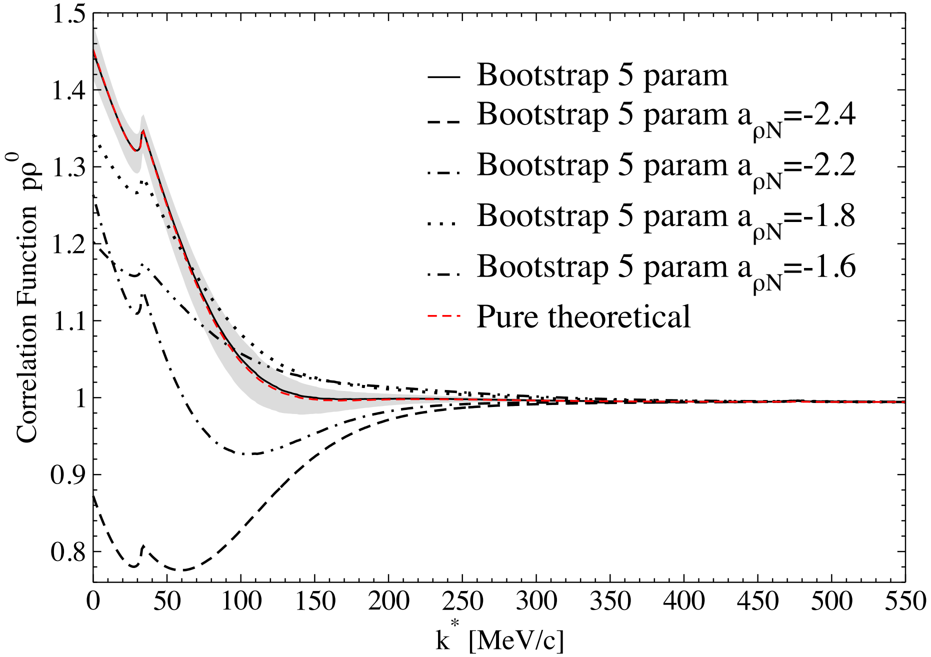
- From the constrained elastic $\boldsymbol{\phi - p}$ amplitude, we extracted very different scattering parameters compared to ALICE analysis due to the CC effects that play a fundamental role

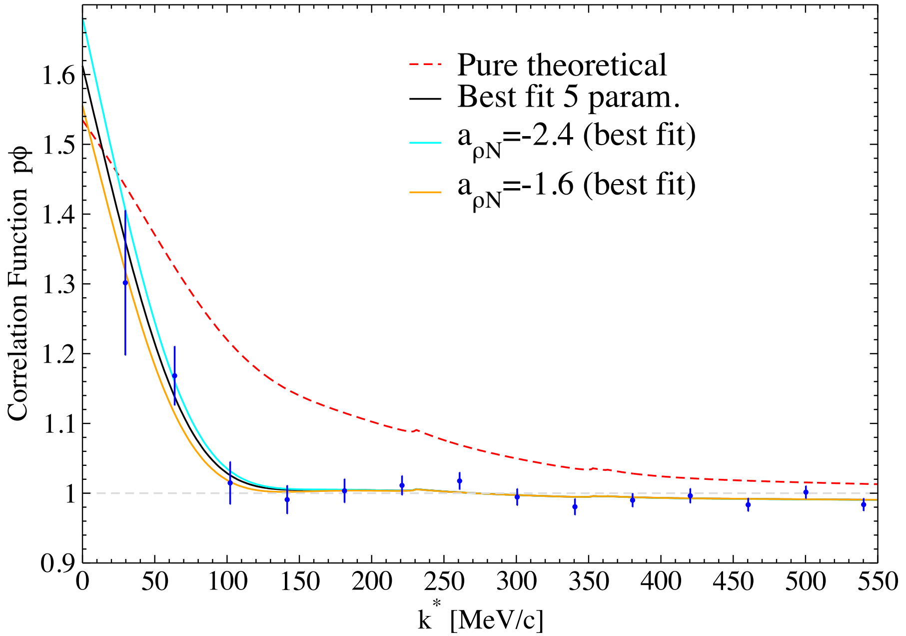
- As in the previous theoretical predictions, 2 dynamically generated states were found:
  a) one at 1959 MeV slightly above the $\boldsymbol{\phi - p}$ threshold
  b) another around 1700 (below $\boldsymbol{\rho - p}$ threshold) yet not well pin down by $\boldsymbol{\phi - p}$ femto data

- The present study shows that the ongoing $\boldsymbol{\rho - p}$ CF analysis will certainly help in order obtain a better understanding of the dynamics and will provide novel information about the low-lying pole.

Thank you for your attention

Results: limited prediction for $\rho - p$ CF





- These variations on aρN SC barely affect the $\phi - p$ **CF** confirming the lack of constraining effect from the data