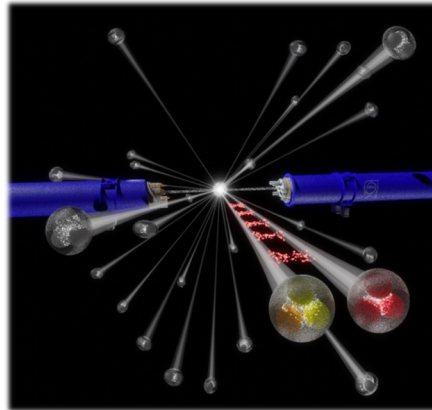




Accessing hadron dynamics in many-body systems with femtoscopy

Raffaele Del Grande^{1,*}

¹Physik Department E62, Technische Universität München, 85748 Garching, Germany



16th May 2024, SPICE Workshop, ECT*, Trento, Italy

*raffaele.del-grande@tum.de

Hadronic interactions in many-body systems

- Properties of nuclei and hypernuclei cannot be described satisfactorily with two-body forces only

H. Hammer, A. Nogga, A. Schwenk, RMP 85, 197 (2013)
L.E. Marcucci et al., Front. Phys. 8:69 (2020)

- NNN interaction contributes ~10% to the binding energies of ^3H and ^4He

- Many-body scattering requires three-body calculations (e.g. **neutron-deuteron**)

L. Girlanda et al., PRC 102, 064003 (2020)
E. Epelbaum et al., PRC 99, 024313 (2019)

^3H and ^4He binding energies and n-d scattering length

Potential(NN)	^3H [MeV]	^4He [MeV]	$^2a_{nd}$ [fm]
AV18	7.624	24.22	1.258
CDBonn	7.998	26.13	
N3LO-Idaho	7.854	25.38	1.100

Potential(NN+NNN)	^3H [MeV]	^4He [MeV]	$^2a_{nd}$ [fm]
AV18/UIX	8.479	28.47	0.590
CDBonn/TM	8.474	29.00	
N3LO-Idaho/N2LO	8.474	28.37	0.675
Exp.	8.48	28.30	0.645±0.010

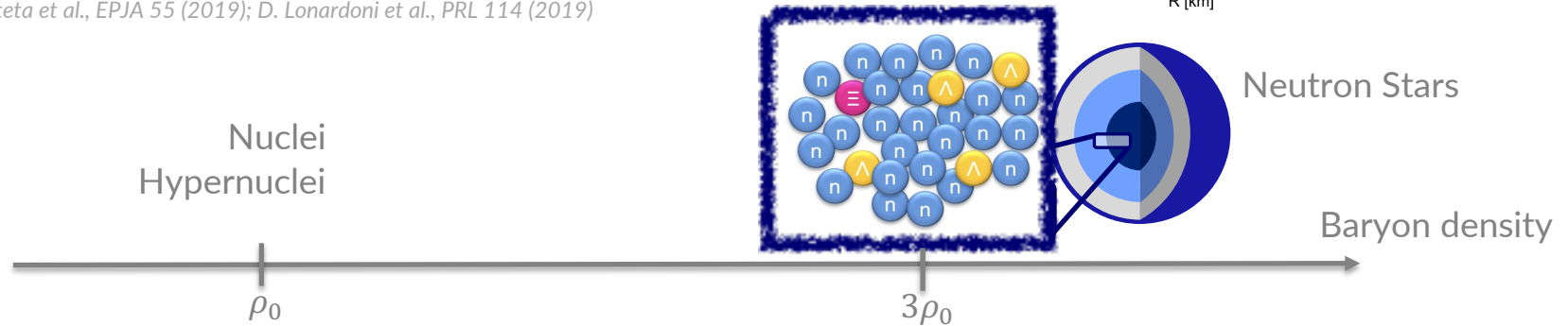
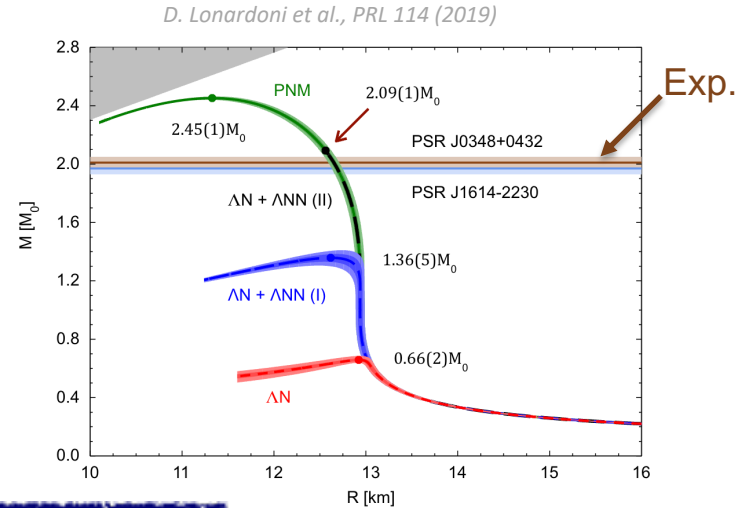
L.E. Marcucci et al., Front. Phys. 8:69 (2020)



Hadronic interactions in many-body systems

- Production of hyperons energetically favourable in neutron stars (NS) around 2-3 ρ_0
L. Tolos, L. Fabbietti, PPNP 112 (2020) 103770
- Only two-body ΛN
 → Too soft equation of state (EoS)
- Introduction of three-body ΛNN forces
 → Stiffens EoS, model-dependent
 → Need for additional experimental constraints

D. Logoteta et al., EPJA 55 (2019); D. Lonardoni et al., PRL 114 (2019)

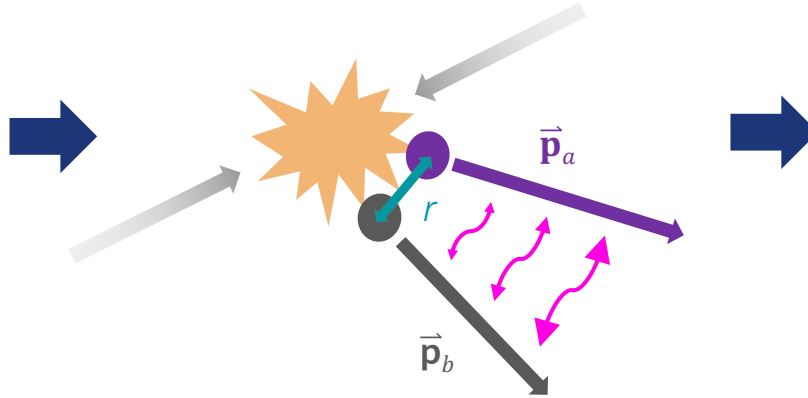


The femtoscopy technique at the LHC

ALICE at the LHC



Hadronic interaction

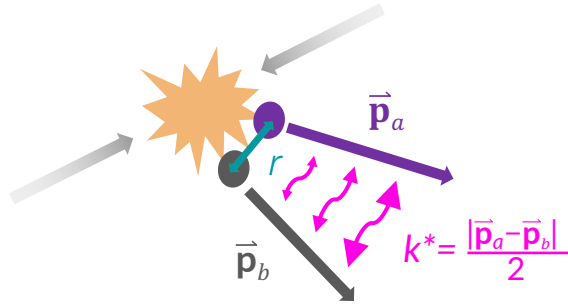


Femtoscopy technique

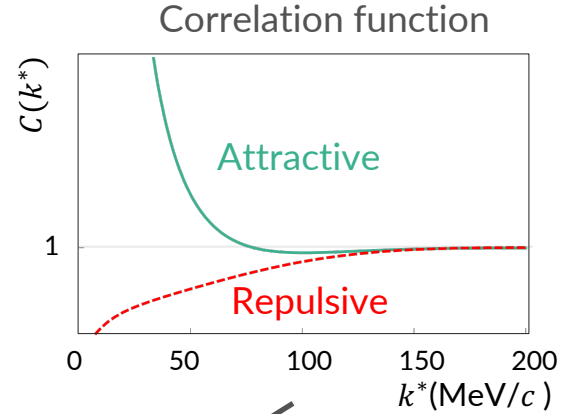
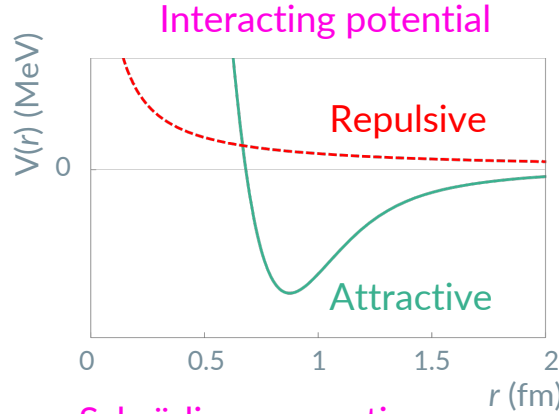
Correlation function

$$C(\vec{p}_a, \vec{p}_b) \equiv \frac{P(\vec{p}_a, \vec{p}_b)}{P(\vec{p}_a) P(\vec{p}_b)}$$

The femtoscopy technique at the LHC



Emission source $S(\vec{r})$



Schrödinger equation

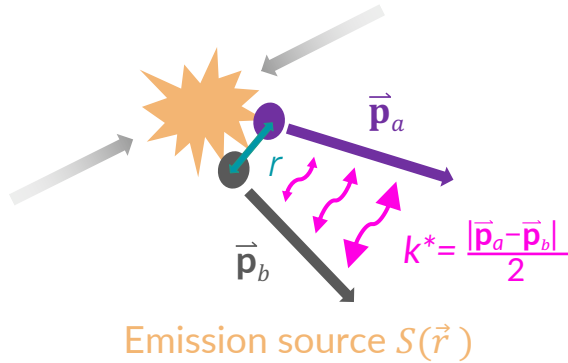
CATS Framework: D. Mihaylov et al., Eur. Phys. J. C78 (2018) 394

Two-particle wave function

$$C(k^*) = \int S(\vec{r}) |\psi(\vec{k}^*, \vec{r})|^2 d^3\vec{r} = \mathcal{N}(k^*) \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

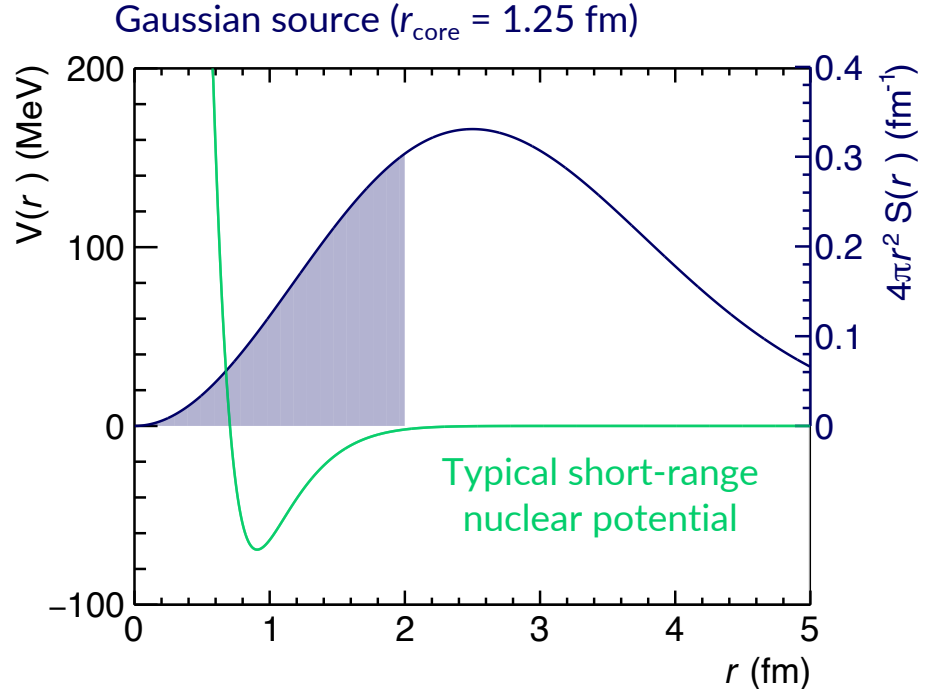
Measuring $C(k^*)$, fixing the source $S(\vec{r})$, study the interaction

The femtoscopy technique at the LHC



Small particle-emitting source created in pp and p-Pb collisions at the LHC:

- **Essential ingredient for detailed studies of the strong interaction**
- Source function determined using p-p correlation functions



Source determination

Femtoscscopy used in a “traditional” way: known interaction → determine the source size

p-p interaction:

Coulomb + Argonne v18

p- Λ interaction:

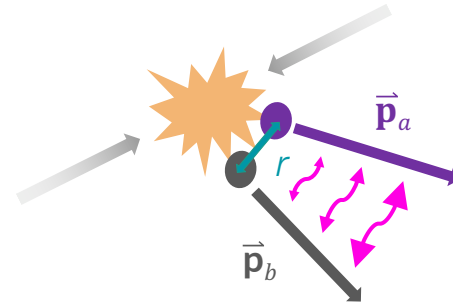
χ EFT **NLO** and **LO**

$$C(k^*) = \int S(\vec{r}) |\psi(\vec{k}^*, \vec{r})|^2 d^3\vec{r}$$

- Fit of the data as function of m_T
- Gaussian source

$$S(r) = G[r, r_{core}(m_T)]$$

$$= \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4r_{core}^2}\right)$$



Source determination

Femtoscopy used in a “traditional” way: known interaction → determine the source size

p-p interaction:

Coulomb + Argonne v18

p-Λ interaction:

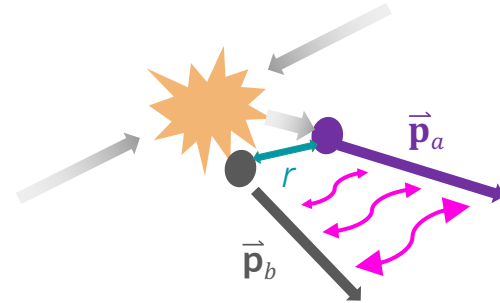
χEFT **NLO** and **LO**

$$C(k^*) = \int S(\vec{r}) |\psi(\vec{k}^*, \vec{r})|^2 d^3\vec{r}$$

- Fit of the data as function of m_T
- Gaussian source

$$S(r) = G[r, r_{core}(m_T)] = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4r_{core}^2}\right)$$

+ Resonances with $c\tau \sim r_{core} \sim 1\text{fm}$ ($\Delta^{++}, N^*, \Sigma^*$)



Particle	Primordial fraction	Resonances $\langle c\tau \rangle$
Proton	33 %	1.6 fm
Lambda	34 %	4.7 fm

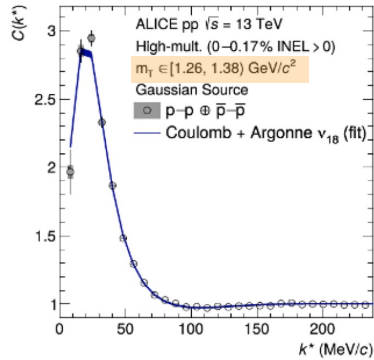
U. Wiedemann and U. Heinz PRC 56 (1997)

Source determination

Femtoscopy used in a “traditional” way: known interaction → determine the source size

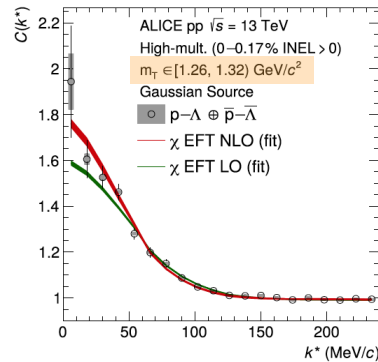
p-p interaction:

Coulomb + Argonne v18

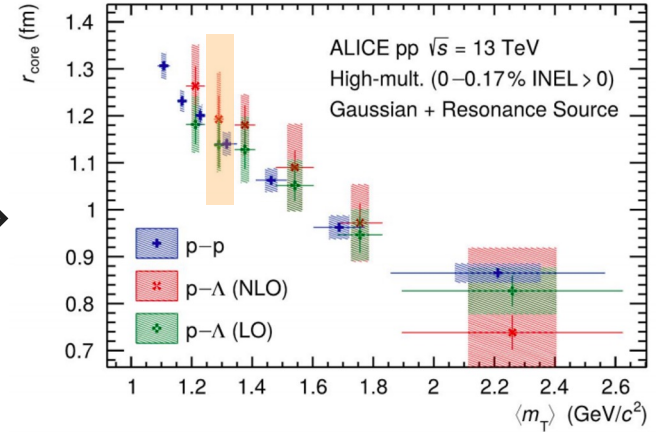


p-Λ interaction:

χEFT NLO and LO



ALICE Coll., PLB, 811 (2020)



- Gaussian source

$$S(r) = G[r, r_{core}(m_T)] = \frac{1}{(4\pi r_{core}^2)^{3/2}} \exp\left(-\frac{r^2}{4r_{core}^2}\right)$$

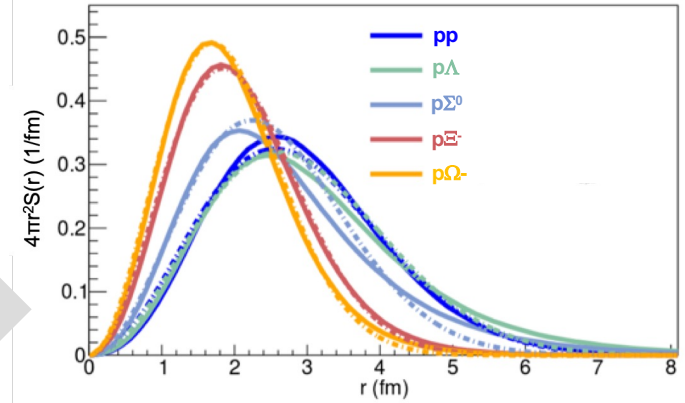
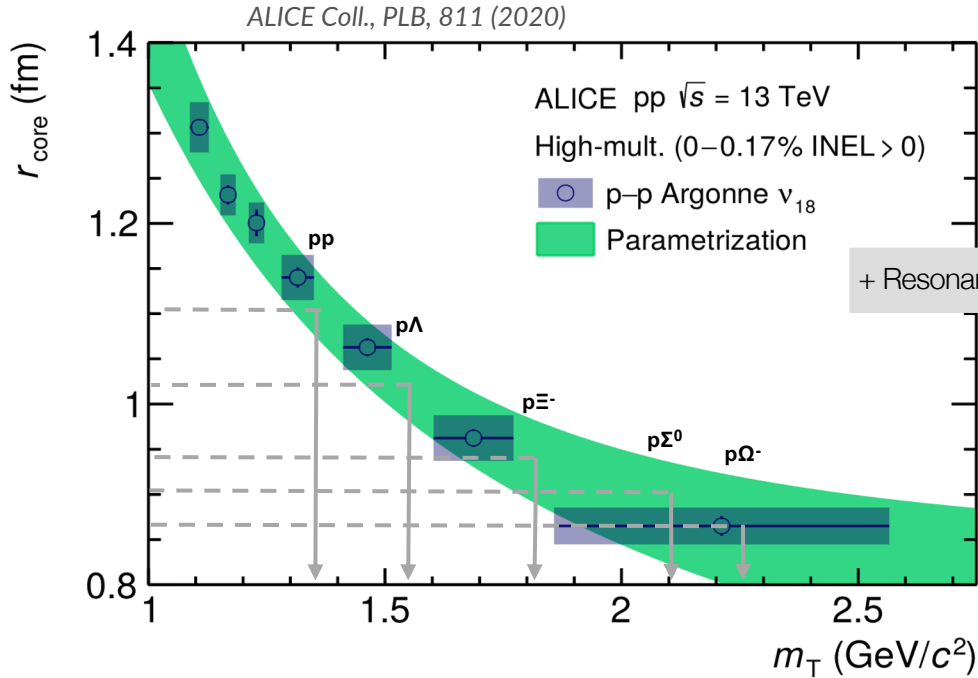
+ Resonances with $\tau \sim r_{core} \sim 1\text{fm}$ ($\Delta^{++}, N^*, \Sigma^*$)

One universal source for all hadrons

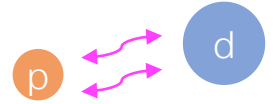
Same results in $K^+p, \pi\pi$

ALICE Coll. arXiv:2311.14527v1 (2023)

Source determination



Pair	r_{Core} [fm]	r_{Eff} [fm]
p-p	1.1	1.2
p- Λ	1.0	1.3
p- Σ^0	0.87	1.02
p- Ξ^-	0.93	1.02
p- Ω^-	0.86	0.95



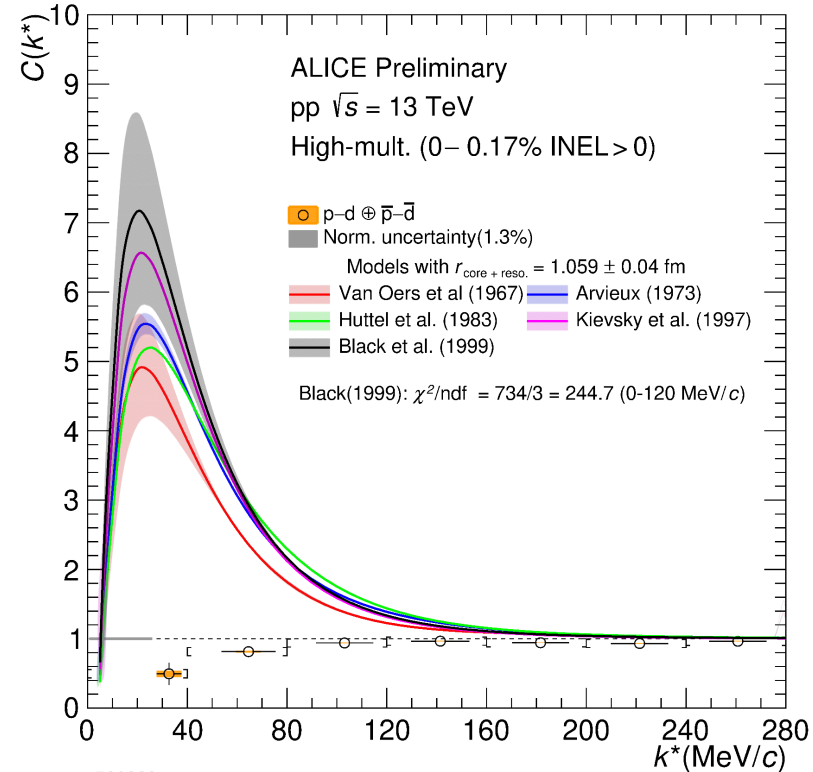
- Point-like particle models anchored to scattering experiments

	$S = 1/2$		$S = 3/2$	
	$f_0(\text{fm})$	$d_0(\text{fm})$	$f_0(\text{fm})$	$d_0(\text{fm})$
Van Oers et al. (1967)	$-1.30^{+0.20}_{-0.20}$	—	$-11.40^{+1.20}_{-1.80}$	$2.05^{+0.25}_{-0.25}$
Arvieux (1973)	$-2.73^{+0.10}_{-0.10}$	$2.27^{+0.12}_{-0.12}$	$-11.88^{+0.10}_{-0.40}$	$2.63^{+0.01}_{-0.02}$
Huttel et al. (1983)	-4.0	—	-11.1	—
Kievsky et al. (1997)	-0.024	—	-13.7	—
Black et al. (1999)	$0.13^{+0.04}_{-0.04}$	—	$-14.70^{+2.30}_{-2.30}$	—

W. T. H. Van Oers, & K. W. Brockman Jr, *NPA* 561 (1967);
 J. Arvieux et al., *NPA* 221 (1973); E. Huttel et al., *NPA* 406 (1983);
 A. Kievsky et al., *PLB* 406 (1997); T. C. Black et al., *PLB* 471 (1999);

- Coulomb + strong interaction using the Lednický model
- Only s-wave interaction
- Source radius evaluated using the hadron-hadron universal MT scaling

Point-like particle description doesn't work for p-d



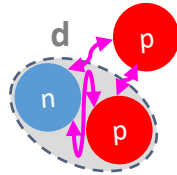
ALI-PREL-501009

Proton-deuteron correlations

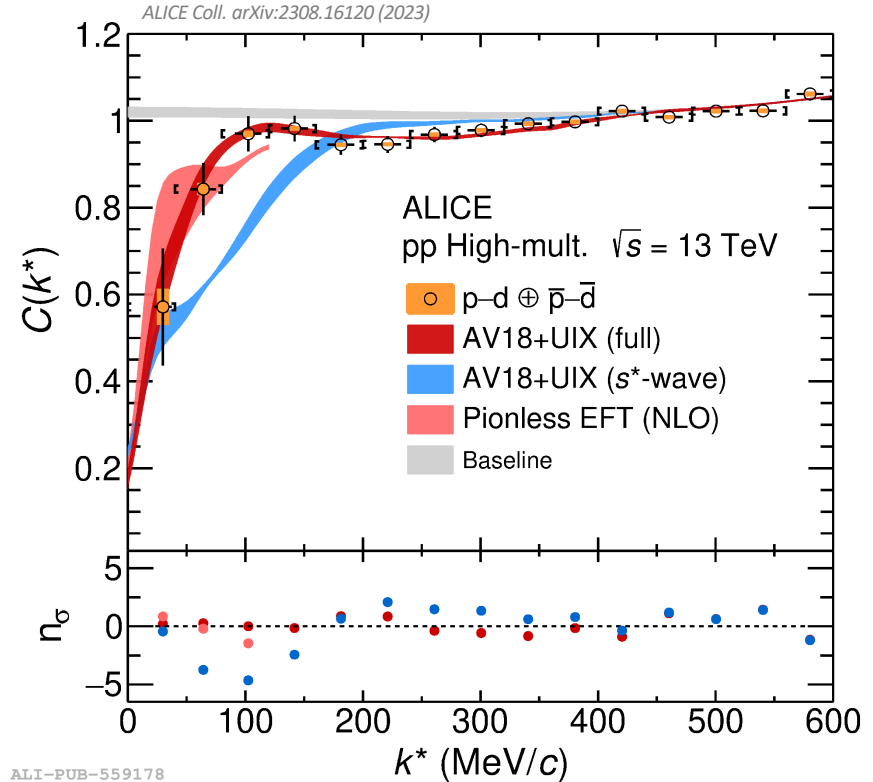
The measured p-d correlation function reflects the full three-nucleon dynamics:

Coulomb + strong interaction (NN and NNN) +
Quantum Statistics

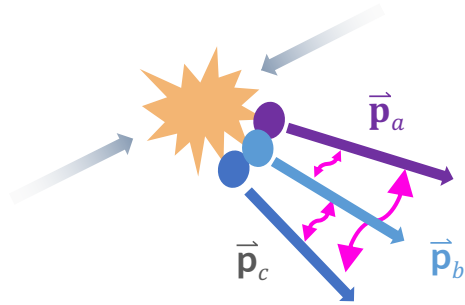
M. Viviani et al., PRC 108 (2023) 6, 064002



- Sensitivity to the short inter-particle distances
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics



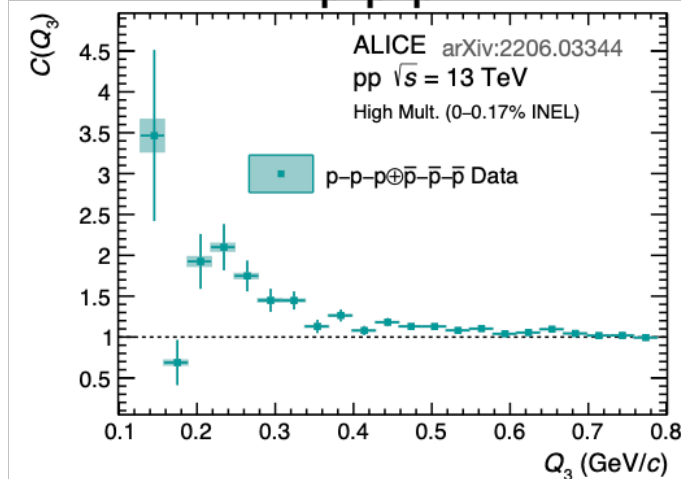
Three-particle femtoscopy



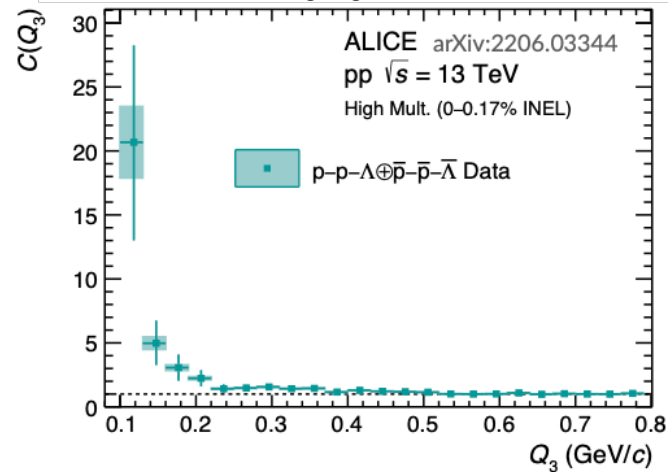
$$C(\vec{p}_a, \vec{p}_b, \vec{p}_c) = \int S(\vec{x}_a, \vec{x}_b, \vec{x}_c) |\psi_{\vec{p}_a, \vec{p}_b, \vec{p}_c}(\vec{x}_a, \vec{x}_b, \vec{x}_c)|^2 d^3\vec{x}_a d^3\vec{x}_b d^3\vec{x}_c$$

Three-body scattering
wave function

p-p-p ALICE Coll., EPJ A 59, 145 (2023)



p-p- Λ



Interpretation of the measurements

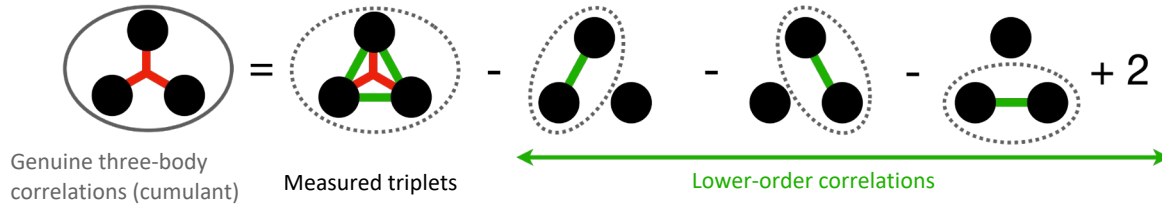
- Step 0: Cumulant expansion method, to answer the question:
“Is there any genuine three-body effect in the measured correlation function?”
- Step 1: Calculate the three-particle scattering wave function and compute the correlation function

Step 0: Cumulants

Cumulants in femtoscopy

Genuine three-body effects can be isolated using the Kubo's cumulant expansion method

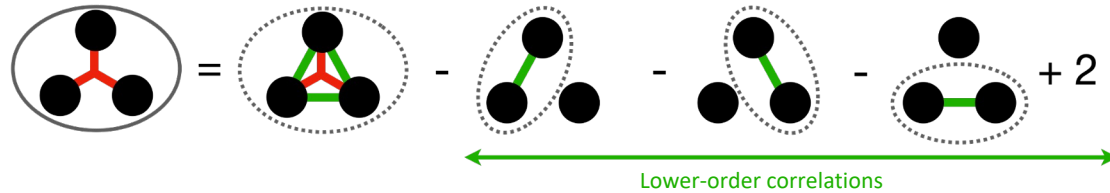
J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)



Cumulants in femtoscopy

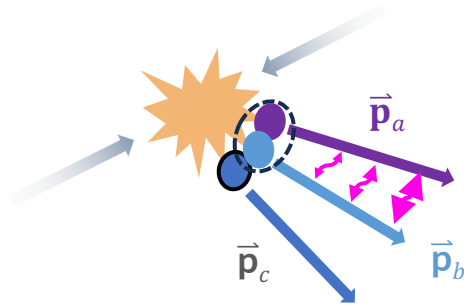
Genuine three-body effects can be isolated using the Kubo's cumulant expansion method

J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)



Data-driven method:

One particle from another collision



Projector method:

Each term of the lower-order correlations can be calculated using two-particle correlation functions

R. Del Grande, L. Serksnyte et al, EPJC 82 (2022)

$$\psi_{\vec{p}_a, \vec{p}_b, \vec{p}_c}(\vec{x}_a, \vec{x}_b, \vec{x}_c) = \psi_{\vec{p}_a, \vec{p}_b}(\vec{x}_a, \vec{x}_b) e^{i \vec{p}_c \cdot \vec{x}_c}$$

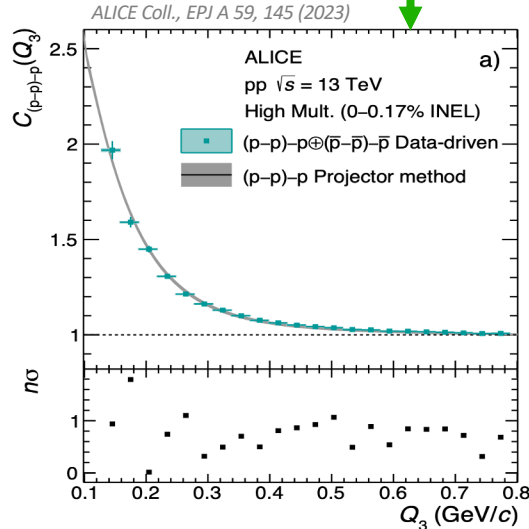
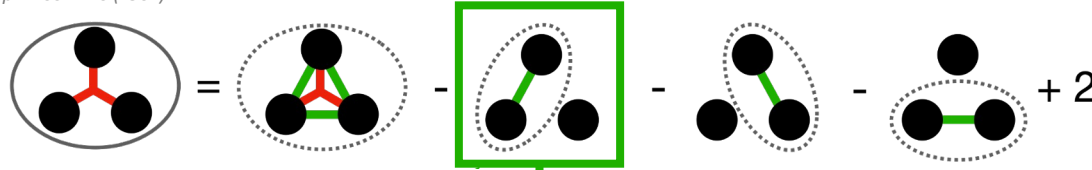
Correlated pair

Uncorrelated particle

Cumulants in femtoscopy

Genuine three-body effects can be isolated using the Kubo's cumulant expansion method

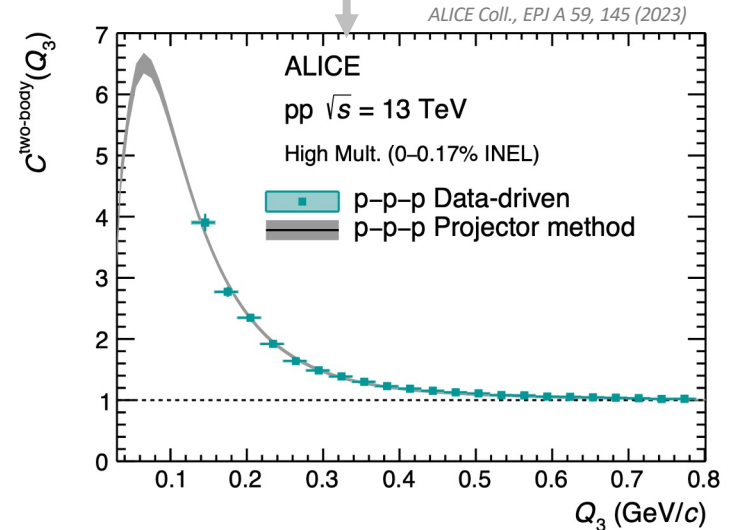
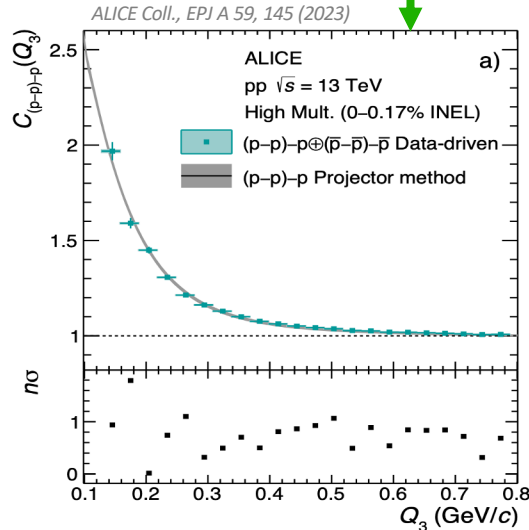
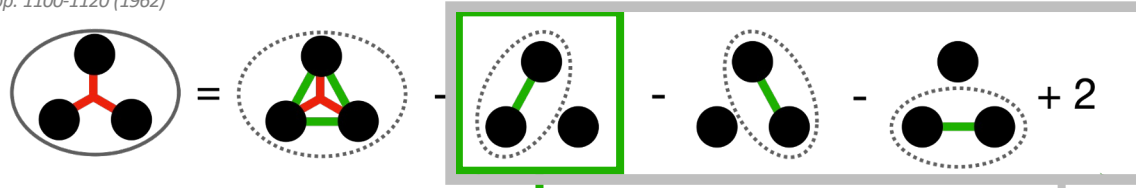
J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)



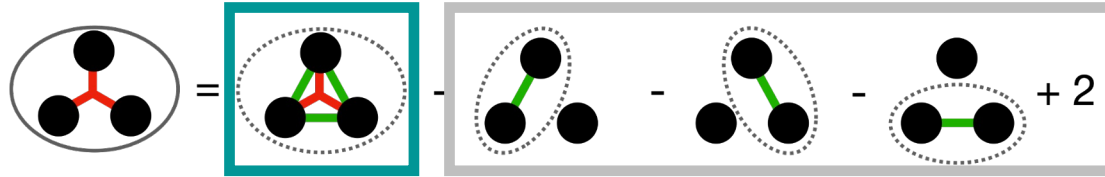
Cumulants in femtoscopy

Genuine three-body effects can be isolated using the Kubo's cumulant expansion method

J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)

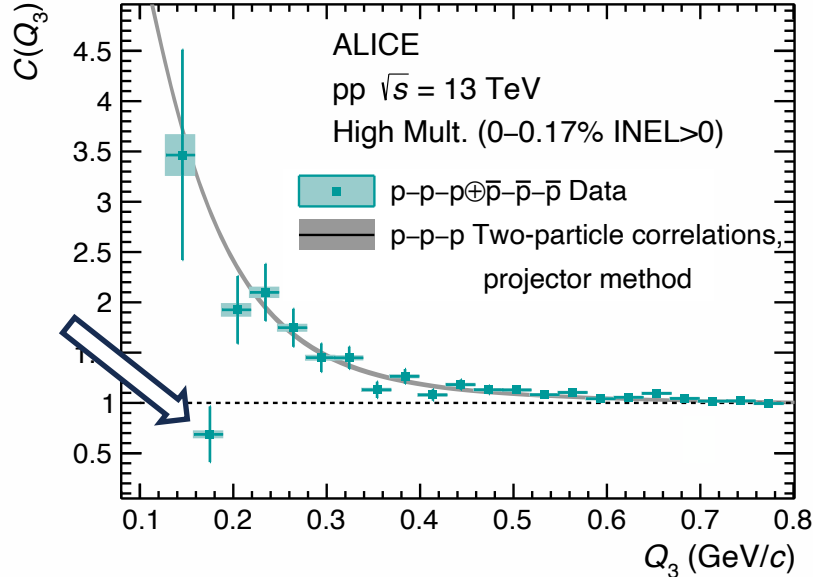


p-p-p and p-p- Λ correlations



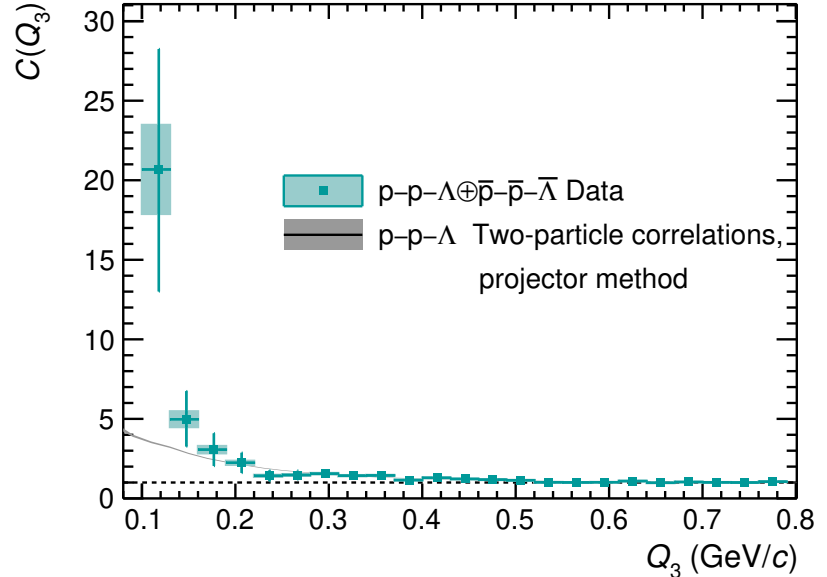
p-p-p

ALICE Coll., EPJ A 59, 145 (2023)

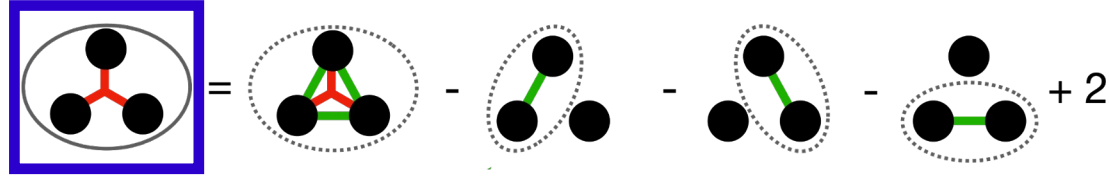


p-p- Λ

ALICE Coll., EPJ A 59, 145 (2023)

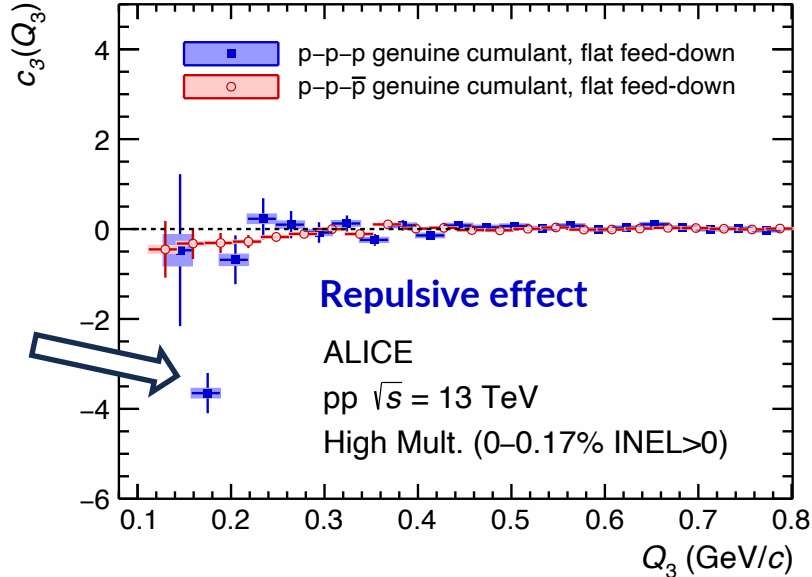


p-p-p and p-p- Λ cumulants



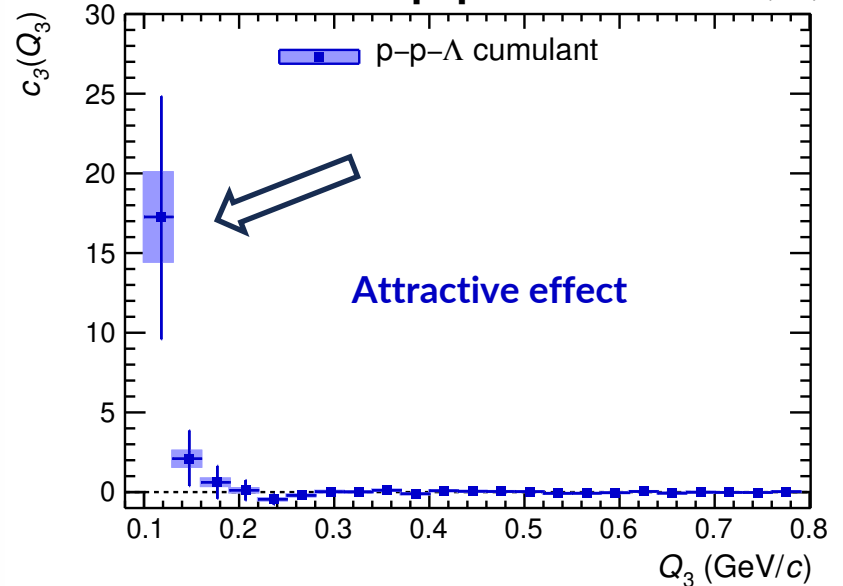
p-p-p

ALICE Coll., EPJ A 59, 145 (2023)



p-p- Λ

ALICE Coll., EPJ A 59, 145 (2023)



Step 1: Calculation of the three-body wave function

Hyperspherical coordinates

- Decomposition of the wave function in hyperspherical harmonics:

$$\psi = \sum_{[K]} \psi_{[K]} = \sum_{[K]} \rho^{-5/2} u_{[K]}(\rho) Y_{[K]}(\Omega)$$

- Schrödinger equation with the interaction:

$$-\frac{\hbar^2}{2m} \left(\frac{\partial^2 u_{[K]}(\rho)}{\partial \rho^2} - \frac{(K + 3/2)(K + 5/2)}{\rho^2} u_{[K]}(\rho) \right) + \sum_{[K']} U_{[K][K']}(\rho) u_{[K']}(\rho) = E u_{[K]}(\rho)$$

- The hypercentral potential is obtained as

$$U_{[K][K']}(\rho) = \int d\Omega Y_{[K]}^*(\Omega) [V_{12} + V_{23} + V_{31} + V_{123}] Y_{[K']}(\Omega)$$

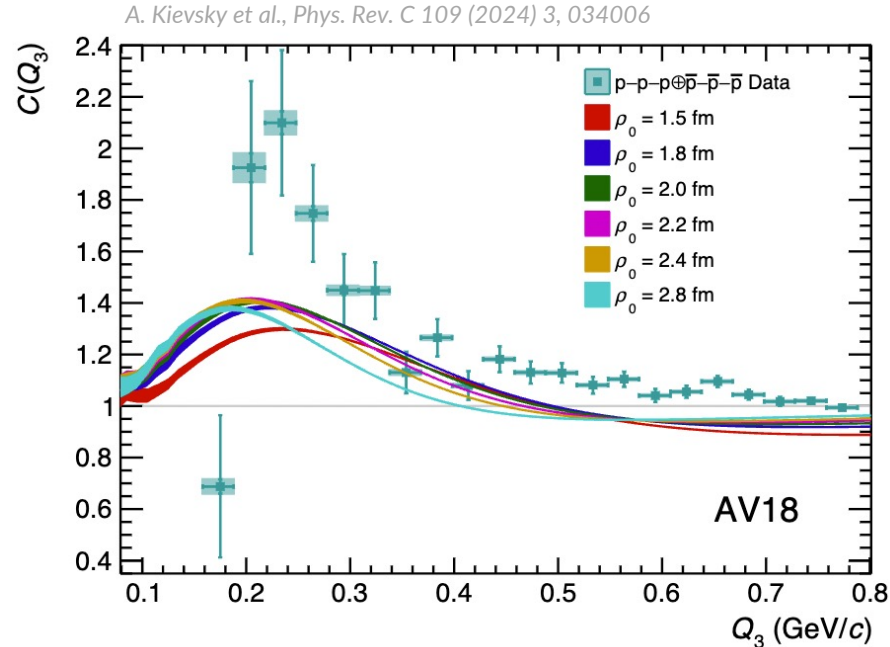
- Three-body correlation function:

$$C(Q_3) = \int S(\rho) |\psi(Q_3, \rho)|^2 \rho^5 d\rho$$

in collaboration with A. Kievsky, M. Viviani, L. Marcucci and E. Garrido.

A. Kievsky et al., Phys. Rev. C 109 (2024) 3, 034006

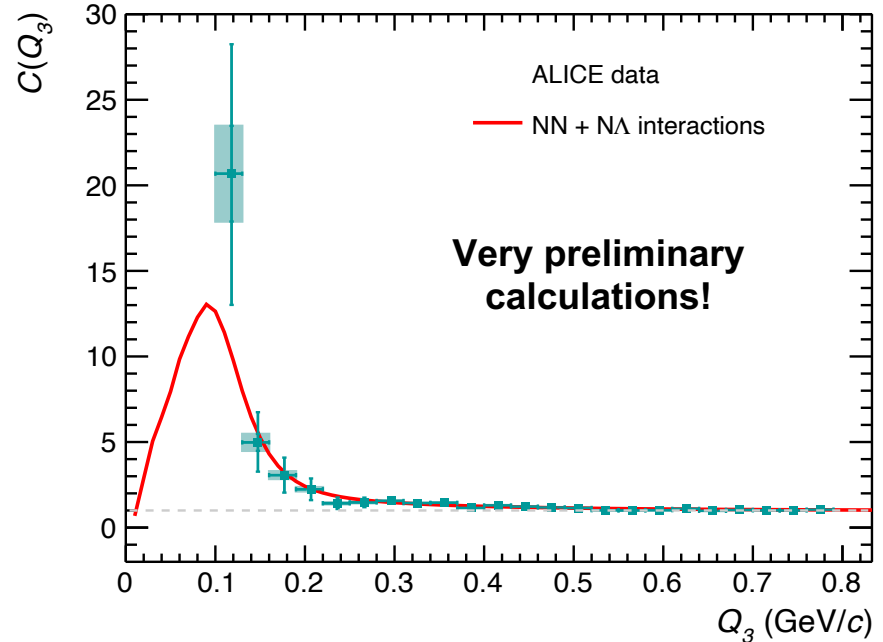
- Hyper-Source radius: $\rho_0 = 2 R_M$
- Wave function:
AV18 + Coulomb + Antisymmetrisation
- Interaction at $K \leq 2$, free solution for $K > 2$



- Three-body correlation function:

$$C(Q_3) = \int S(\rho) |\psi(Q_3, \rho)|^2 \rho^5 d\rho$$

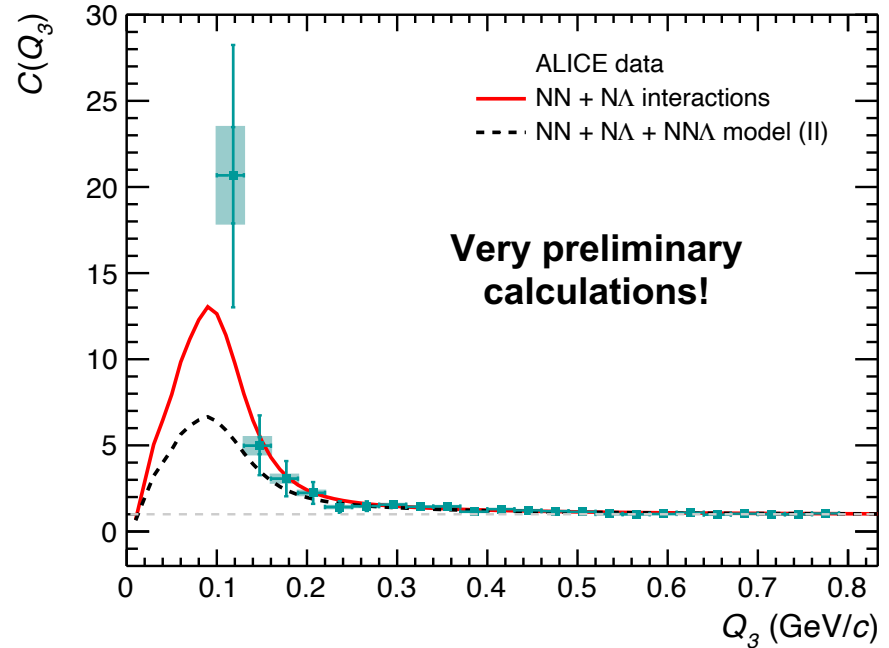
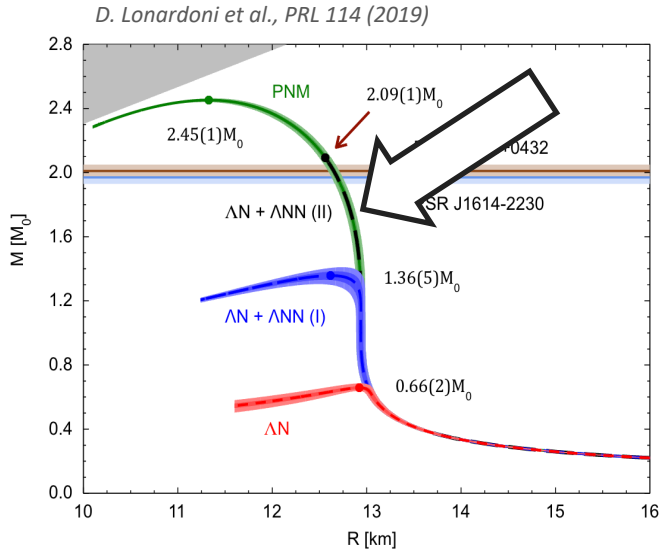
- Two-body interactions + Coulomb + Antisymmetrisation
- Hyper-source radius determined from the two-body p-p and p- Λ sources
- Calculations include the interaction for $K \leq 10$



- Three-body correlation function:

$$C(Q_3) = \int S(\rho) |\psi(Q_3, \rho)|^2 \rho^5 d\rho$$

- Inclusion of the Λ NN (model II)



Larger statistics is needed to test three-body interaction models

Femtoscopy in pp collisions at the LHC

- small particle source size (about 1 fm)
- alternative method to study many-body systems

Results achieved by ALICE:

- p-d correlation function shows sensitivity to the full three-nucleon dynamics
- Free scattering of three hadrons measured for the first time:
 - p-p-p correlation function: evidence of antisymmetrisation effects
 - p-p- Λ correlation function: no strong evidence of a three-body repulsion (modelling has to be improved)

Backup

- X_i denotes the general i -th stochastic variable
- The most general decomposition of 2-particle correlation is:

$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c$$

- By definition, the 2nd term on the right is the 2-particle cumulant
- Cumulants cannot be measured directly, however:

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

- The most general decomposition of 3-particle correlation is:

$$\begin{aligned} \langle X_1 X_2 X_3 \rangle &= \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \\ &+ \langle X_1 X_2 \rangle_c \langle X_3 \rangle + \langle X_1 X_3 \rangle_c \langle X_2 \rangle + \langle X_2 X_3 \rangle_c \langle X_1 \rangle \\ &+ \langle X_1 X_2 X_3 \rangle_c \end{aligned}$$

- Using the 2-particle cumulant: $\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$
- Working recursively from higher to lower orders, we have 3-particle cumulant expressed in terms of the measured 3-, 2-, and 1-particle averages:

$$\begin{aligned} \langle X_1 X_2 X_3 \rangle_c &= \langle X_1 X_2 X_3 \rangle \\ &- \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle \\ &+ 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \end{aligned}$$

In terms of the probability distribution functions we have:

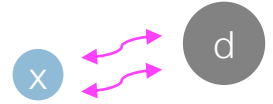
$$\begin{aligned}
 \langle X_1 X_2 \rangle_c &= \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle = \\
 &= \int X_1 X_2 f(X_1, X_2) dX_1 dX_2 - \int X_1 f_1(X_1) dX_1 \int X_2 f_1(X_2) dX_2 \\
 &= \int X_1 X_2 \left[f(X_1, X_2) - f_1(X_1) f_1(X_2) \right] dX_1 dX_2 \\
 &= \int X_1 X_2 f_c(X_1, X_2) dX_1 dX_2
 \end{aligned}$$

with

$$f_c(X_1, X_2) = f(X_1, X_2) - f_1(X_1) f_1(X_2)$$

For three particles:

$$f_c(X_1, X_2, X_3) = f(X_1, X_2, X_3) - f(X_1, X_2) f(X_3) - f(X_1, X_3) f(X_2) - f(X_3, X_2) f(X_1) + 2f(X_1) f(X_2) f(X_3)$$



- Coulomb + strong interaction using the Lednický model

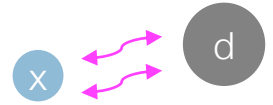
$$\psi(\vec{k}^*, \vec{r}^*) = e^{i\delta_c} \sqrt{A_c(\eta)} \left[e^{-i\vec{k}^* \cdot \vec{r}^*} F(-i\eta, 1, i\xi) + f_C(k^*) \frac{\tilde{G}(\rho, \eta)}{r^*} \right]$$

$$f_C(k^*) = \left(\frac{1}{f_0} + \frac{d_0 \cdot k^{*2}}{2} - \frac{2h(k^*)}{a_c} - ik^* A_C(k^*) \right)^{-1}$$

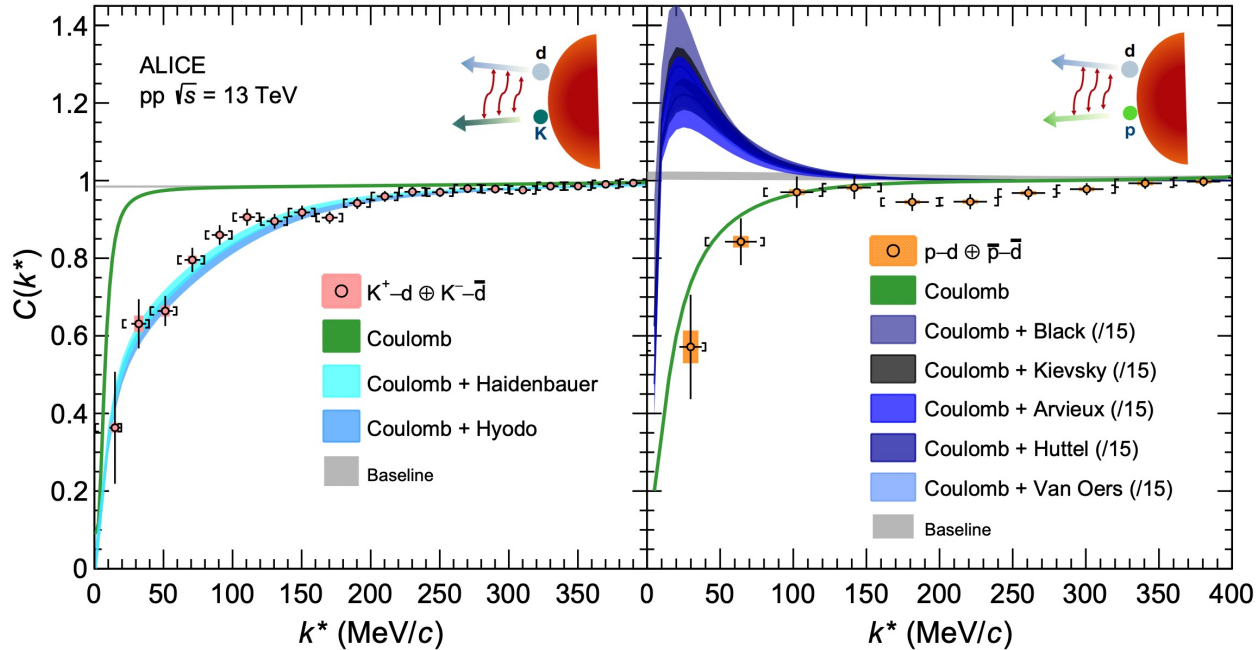
- Point-like particle models anchored to scattering experiments

System	Spin averaged		$S = 1/2$		$S = 3/2$	
	$f_0(\text{fm})$	$d_0(\text{fm})$	$f_0(\text{fm})$	$d_0(\text{fm})$	$f_0(\text{fm})$	$d_0(\text{fm})$
p-d			$-1.30^{+0.20}_{-0.20}$	—	$-11.40^{+1.20}_{-1.80}$	$2.05^{+0.25}_{-0.25}$
			$-2.73^{+0.10}_{-0.10}$	$2.27^{+0.12}_{-0.12}$	$-11.88^{+0.10}_{-0.40}$	$2.63^{+0.01}_{-0.02}$
			-4.0	—	-11.1	—
			-0.024	—	-13.7	—
			$0.13^{+0.04}_{-0.04}$	—	$-14.70^{+2.30}_{-2.30}$	—
K^+ -d	-0.470	1.75				
	-0.540	0.0				

- Only s-wave interaction
- Source radius evaluated using the hadron-hadron universal m_T scaling



- Coulomb + strong interaction using the Lednický model

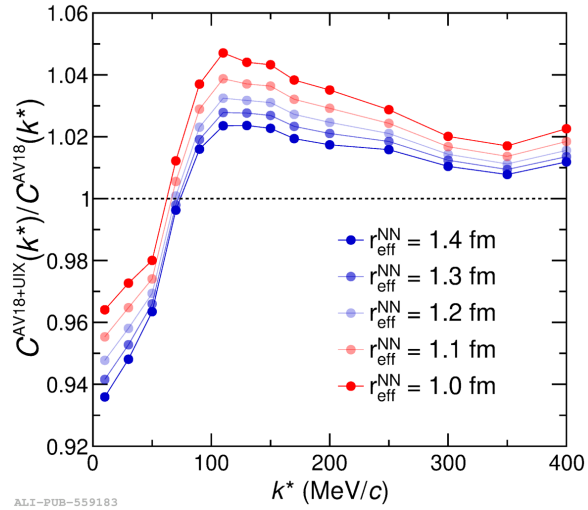
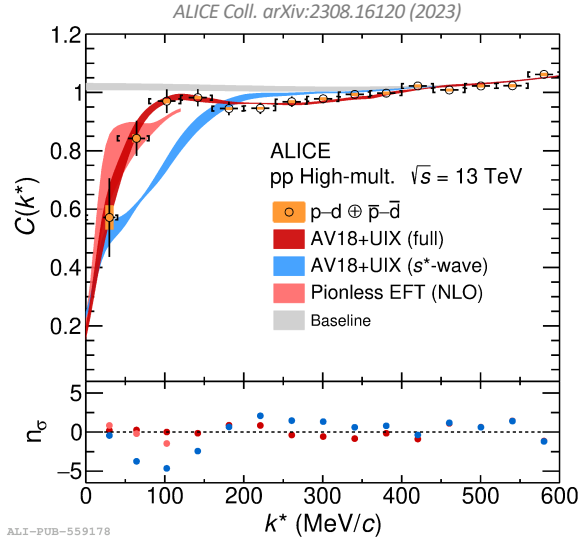
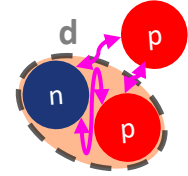


Point-like particle description doesn't work for p-d

Proton-deuteron correlation

The measured p-d correlation function reflects the full three-nucleon dynamics:
Coulomb + strong interaction (NN and NNN) + Quantum Statistics

M. Viviani et al., arXiv:2306.02478 (2023)



- Sensitivity to the short inter-particle distances
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics

