Lambda potential in dense matter examined from hypernuclei

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A. Jinno, K. Murase, Y. Nara, & A. Ohnishi, PRC 108, 065803 (2023).

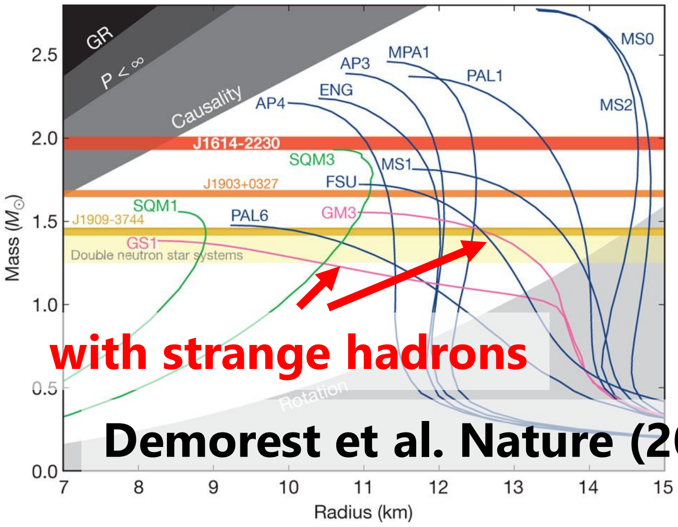
2024/5/13-17 SPICE workshop @ ECT*

- **Introduction: hyperon puzzle of neutron stars and Λ potential**
- **Verifying Λ potentials from <u>hypernuclear data</u>**
- **Model independent analysis for constraining Λ potentials**
- **Summary**

Introduction: hyperon puzzle of neutron stars and Λ potential

Hyperon puzzle of neutron stars

➢ Most of the equations of state in which hyperons (e.g. Λ) appear become too soft to support massive neutron stars with $2M_\odot$ (solar mass).



Demorest et al. Nature (2010)

➢ Many solutions have been proposed for avoiding softening.

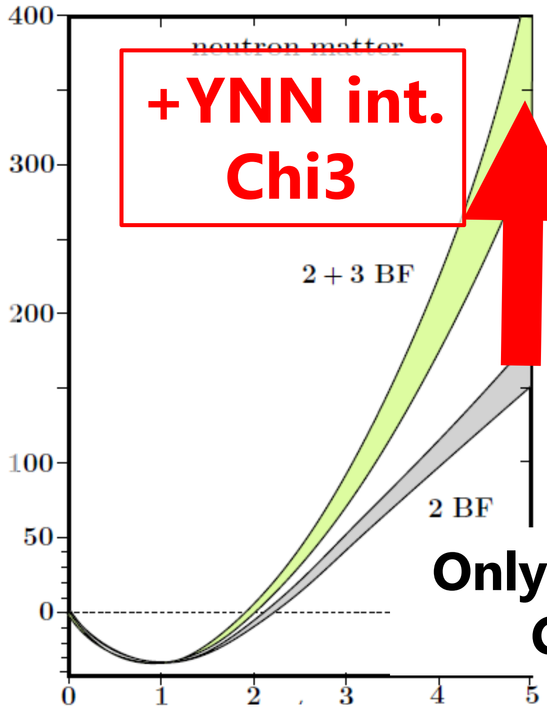
- Many-baryon repulsions (e.g. ΛNN): e.g. Nishizaki, Yamamoto, & Takatsuka (2002); Togashi, Hiyama, Yamamoto, & Takano (2016); Gerstung, Kaiser, & Weise (2020).
- YY repulsions (e.g. ΛΛ): e.g. Weissenborn, Chatterjee, Schaffner-Bielich (2012); Fortin, Avancini, Providencia, & Vidana (2017).
- Transition to quark matter without phase transition (QH continuity): e.g. Baym, Hatsuda, Kojo, Powell, Song, & Takatsuka (2018); Kojo, Baym, & Hatsuda (2022).

YNN three-body repulsion from Chiral EFT

- **YNN three-body force in dense matter:** Nishizaki, Yamamoto, & Takatsuka (2002); Lonardoni et al. (2015); Togashi, Hiyama, Yamamoto, & Takano (2016); Friedman & Gal (2023) etc.
- **Chiral effective field theory** (decuplet saturation model)

Kohno(2018), <u>D. Gerstung, N. Kaiser, and W. Weise (2020)</u>

Λ potential in nuclear matter U_Λ (MeV)

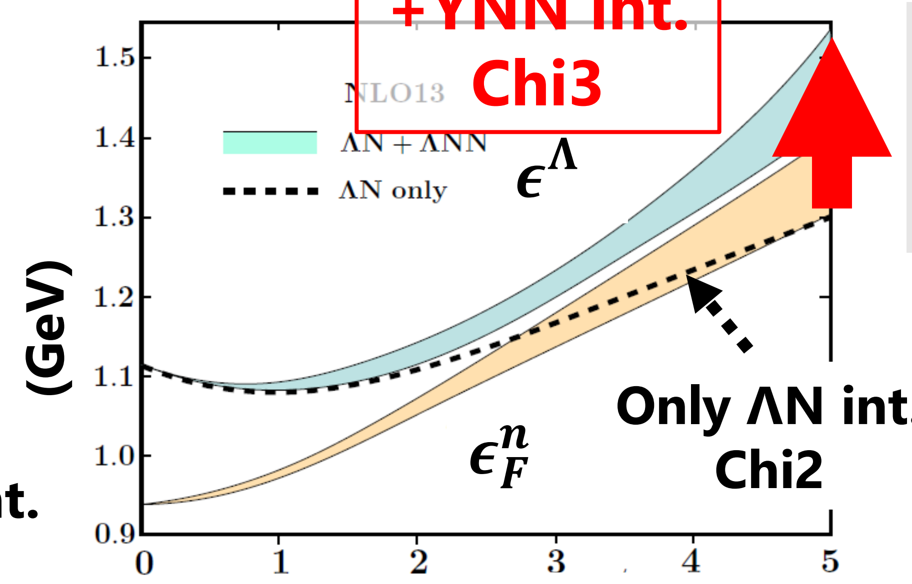


+YNN int. Chi3

Only ΛN int. Chi2

Density ρ/ρ_0

$$\epsilon^\Lambda(\rho) = m_\Lambda + U_\Lambda(\rho)$$



+YNN int. Chi3

Only ΛN int. Chi2

Density ρ/ρ_0

Λ does not appear even at high densities!

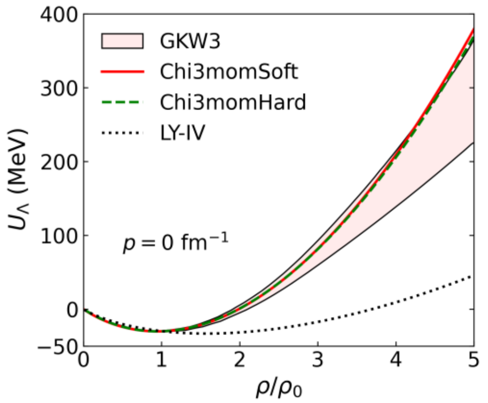
Avoiding hyperon puzzle!?

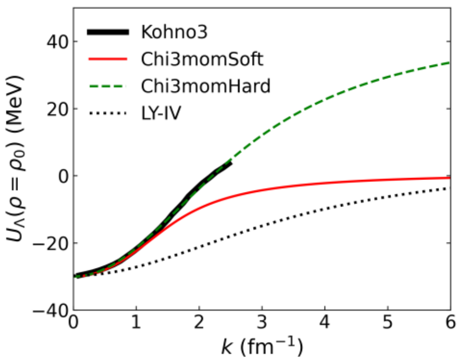
(Our previous work) Λ directed flow v_1

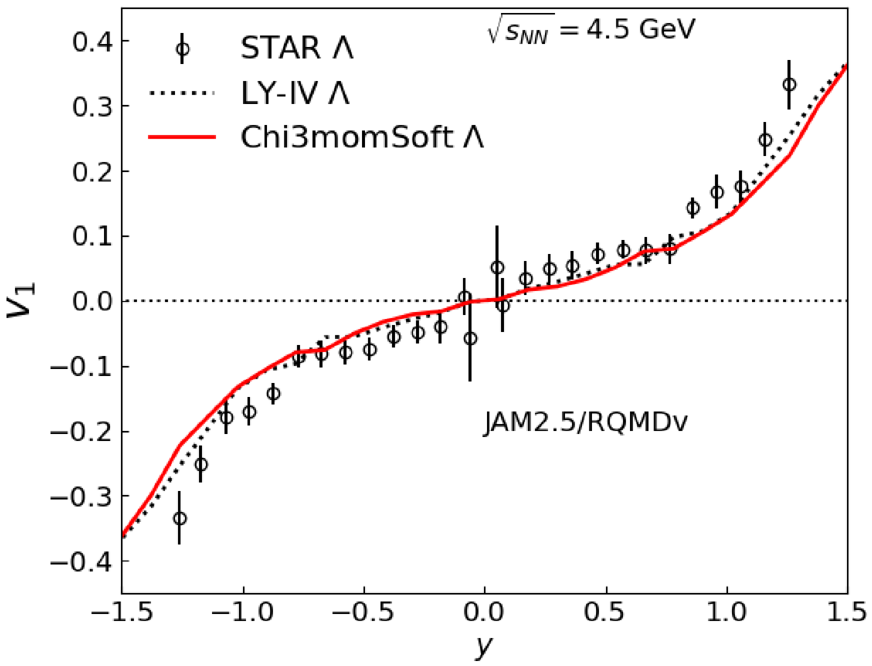
We have used Λ directed flow $v_1 = \langle p_x/p_T \rangle$ data of heavy-ion collision to verify the repulsive Λ potential from chiral EFT.

transverse momentum $p_T = \left(p_x^2 + p_y^2\right)^{1/2}$

Y. Nara, AJ, K. Murase, and A. Ohnishi, Phys. Rev. C 106 (2022) 044902





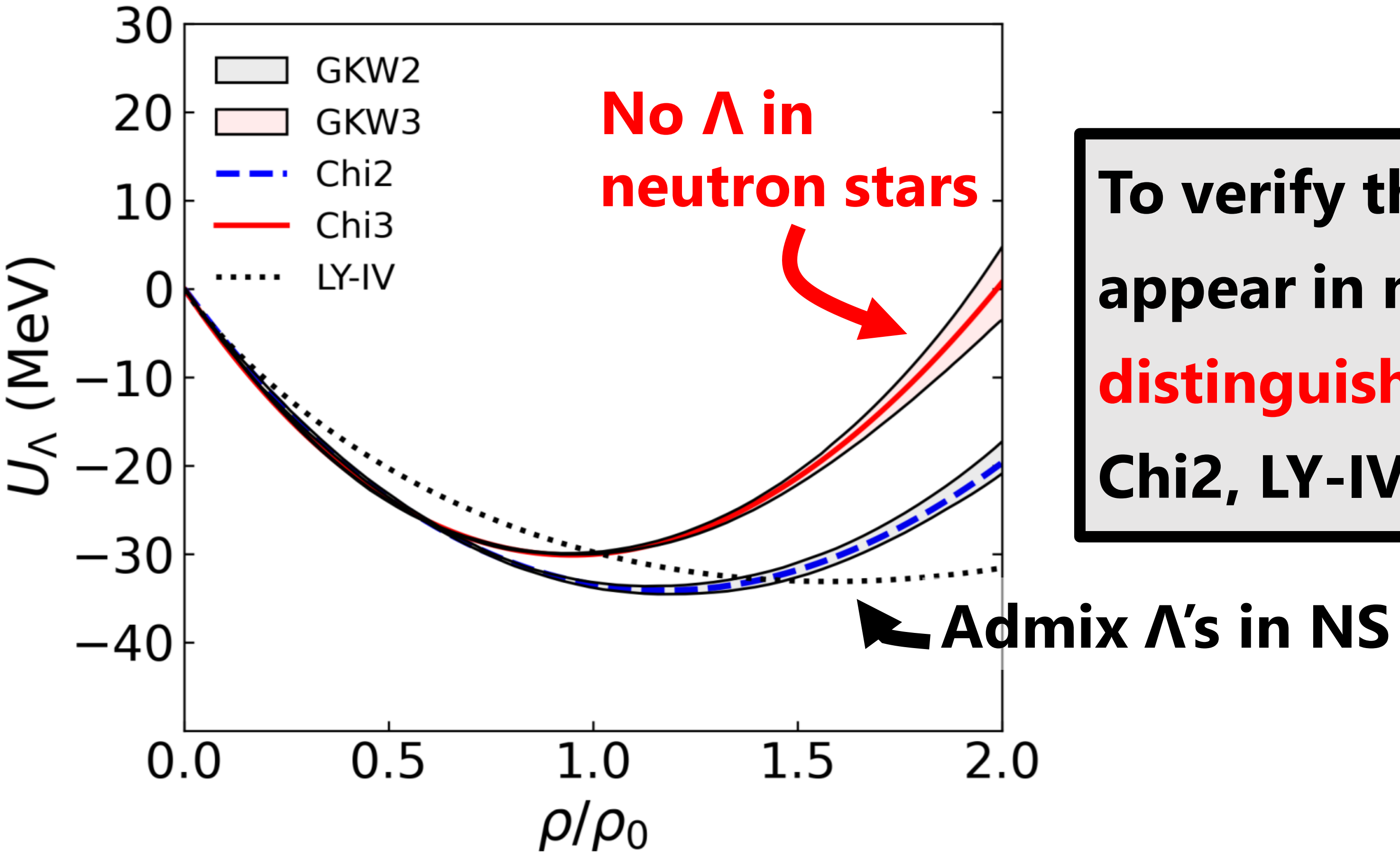


Chi3 (YN+YNN int.) reproduces the Λ v_1 data ($\sqrt{s_{NN}} \geq 4.5$ GeV).

On the other hand, more attractive Λ potential also reproduces the data.

There remain two scenarios in which Λ appears or does not in dense neutron star matter.

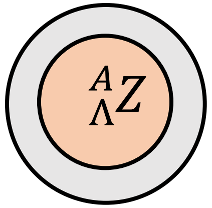
Purpose of this study



To verify the scenario which Λ does not appear in neutron stars (NS) by distinguishing three Λ potentials (Chi3, Chi2, LY-IV) using Λ hypernuclear data.

GKW2 (GKW3): Gerstung, Kaiser, and Weise (2020).
Chiral EFT calculation including YN (YN+YNN) interaction.
LY-IV: Lanskoy and Yamamoto (1997).
Skyrme-type Λ potential reproducing Λ binding energies.

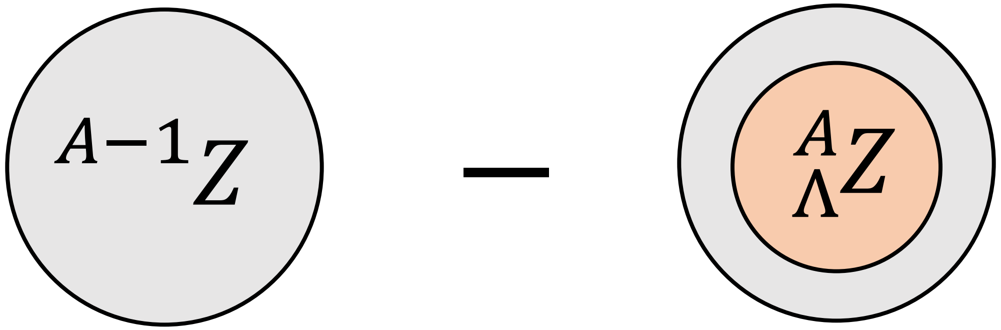
Verifying the Λ potential from hypernuclear data

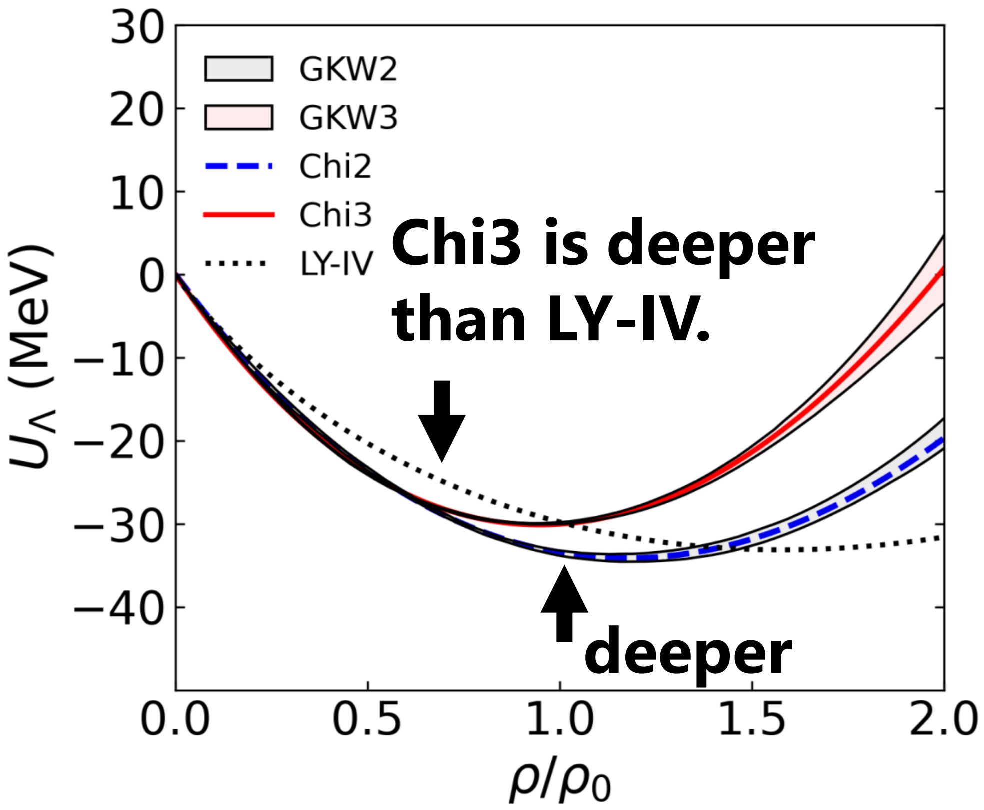


Can Λ potentials reproduce Λ binding energies?

Λ binding energy

B_Λ =





Expected to be sensitive to the Λ potential in $\rho \lesssim \rho_0$.

- **Can Chi2 and Chi3 reproduce the Λ binding energy data?**
- **If they reproduce the data, how is the level of accuracy compared to a conventional attractive model (LY-IV)?**

Spherical Skyrme-Hartree-Fock method

Rayet (1976) & (1981); Lanskoy and Yamamoto (1997); Guleria et al. (2012), Choi, Hiyama et al. (2022) etc…

density $\rho_N = \sum_{B=p,n} \sum_i |\psi_{B,i}|^2$

kinetic density $\tau_N = \sum_{B=p,n} \sum_i |\nabla\psi_{B,i}|^2$

Λ kinetic density $\tau_\Lambda = |\nabla\psi_\Lambda|^2$

- **Total energy of hypernuclei:** $\mathcal{E}({}^{A}_{\Lambda}Z) = \mathcal{E}_N + \mathcal{E}_\Lambda - \mathcal{E}_{\mathrm{c.m.}}$
- **Total energy of Λ:**

$$\mathcal{E}_\Lambda = \int d^3r \, \underline{\frac{\hbar^2}{2m_\Lambda}}\tau_\Lambda + a_1^\Lambda \rho_\Lambda \rho_N + \underline{a_2^\Lambda(\tau_\Lambda\rho_N + \tau_N\rho_\Lambda)}$$
$$- a_3^\Lambda(\rho_\Lambda \nabla^2 \rho_N) + a_4^\Lambda \rho_\Lambda \rho_N^{4/3} + a_5^\Lambda \rho_\Lambda \rho_N^{5/3}$$

: kinetic term with eff. mass

: density-dependent term

: surface term

- **Solving self-consistently the HF eq.** $\delta\mathcal{E}_{\mathrm{hyp}}/\delta\psi_{B,i} = 0$**, then we obtain**
  Λ binding energy $B_\Lambda = \mathcal{E}_{\mathrm{core}} - \mathcal{E}_{\mathrm{hyp}}$ **.**
- **We are ignoring the deformation, the spin-orbit force, the charge symmetry breaking effect, and the pair correlation.**

Fitting of the Λ potential from chiral EFT

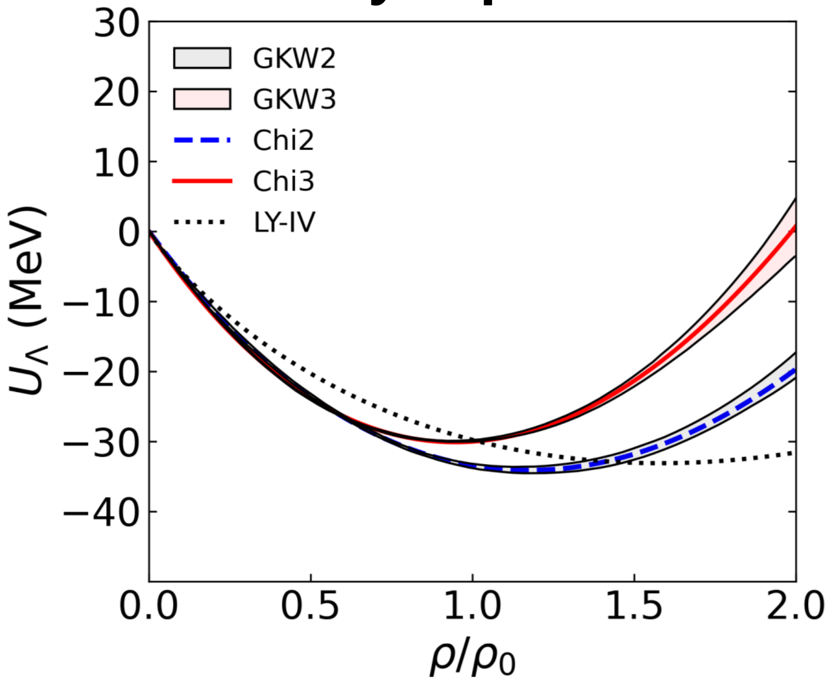
A. Jinno, K. Murase, Y. Nara, & A. Ohnishi, PRC 108, 065803 (2023).

Skyrme-type Λ potential in nuclear matter

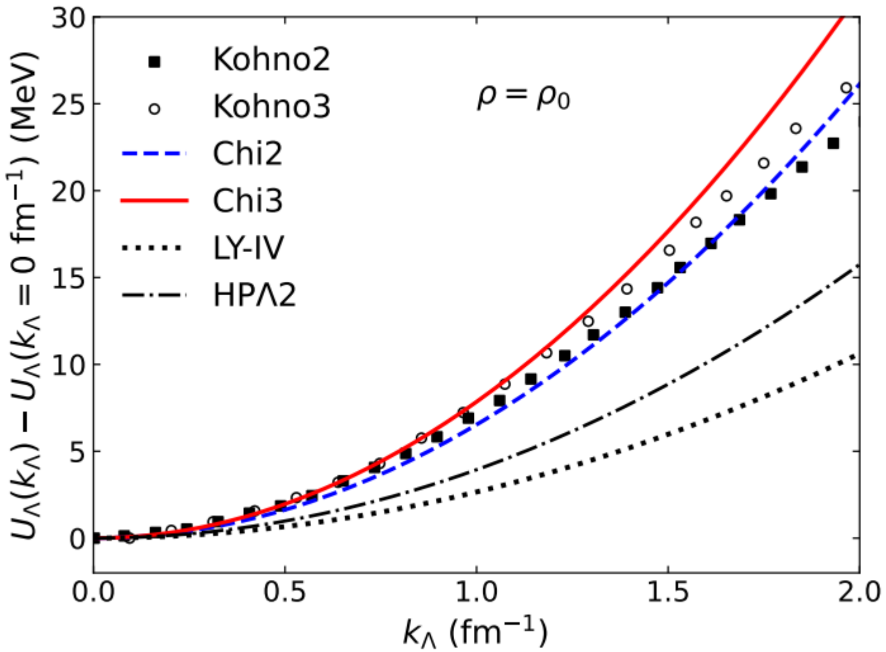
$$U_\Lambda = a_1^\Lambda \rho_N + \underline{a_2^\Lambda (k_\Lambda^2} \rho_N + \tau_N) - a_3^\Lambda \Delta\rho_N + a_4^\Lambda \rho_N^{4/3} + a_5^\Lambda \rho_N^{5/3}$$

Λ kinetic density $\tau_\Lambda = |\nabla\psi_\Lambda|^2$

Density Dependence



Momentum Dependence



GKW2 (GKW3): Gerstung, Kaiser, and Weise (2020).

LY-IV: Lanskoy and Yammoto (1997).

Kohno2 (Kohno3): Kohno (2018)

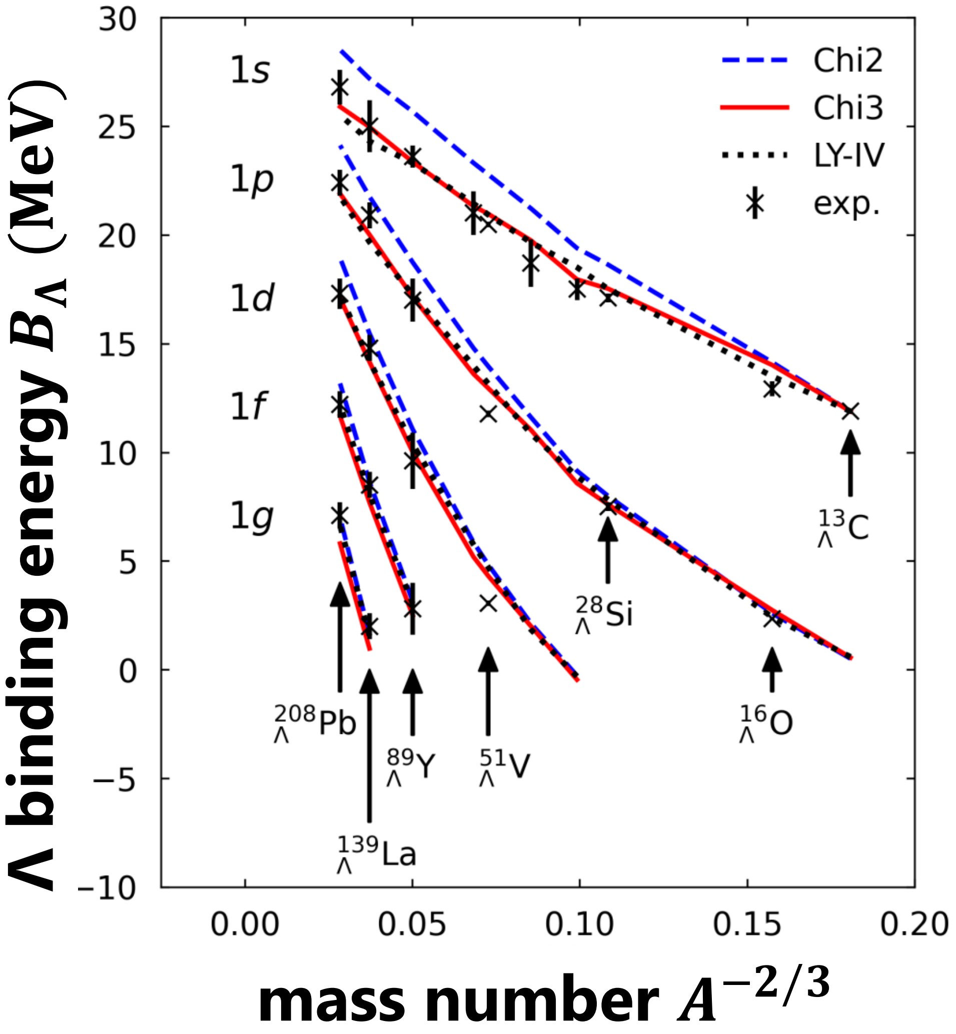
* The value of a_3^Λ is determined to reproduce the Λ binding energy of ${}^{13}_{\Lambda}\mathrm{C}$ (11.88 MeV).

(∵ Surface terms have a large effect. even-even nuclei)

Λ binding energies

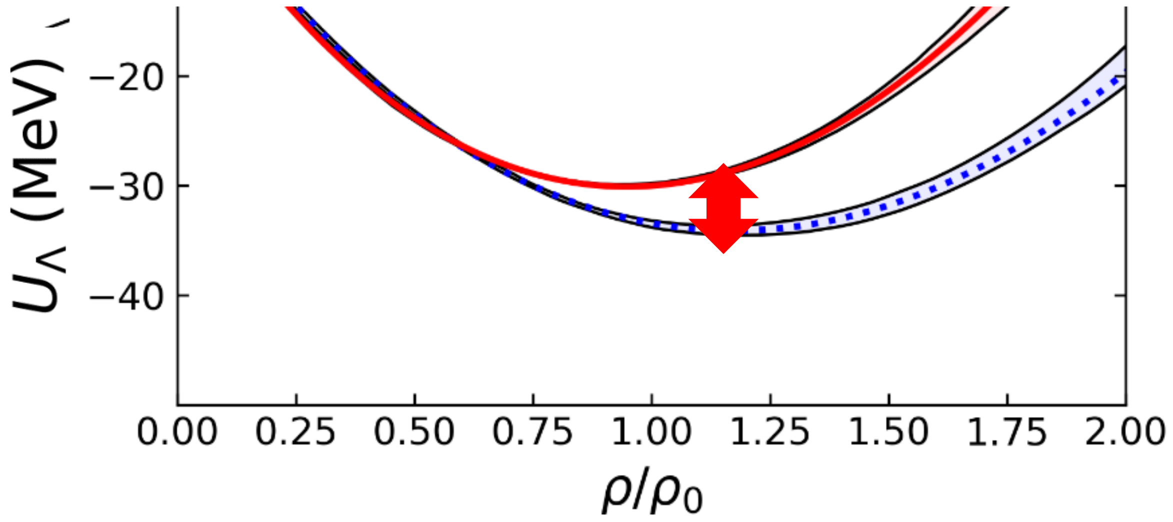
A. Jinno, K. Murase, Y. Nara, & A. Ohnishi, PRC 108, 065803 (2023).

(a_3^Λ in LY-IV model is also tuned to reproduce ${}^{13}_{\Lambda}$C data)



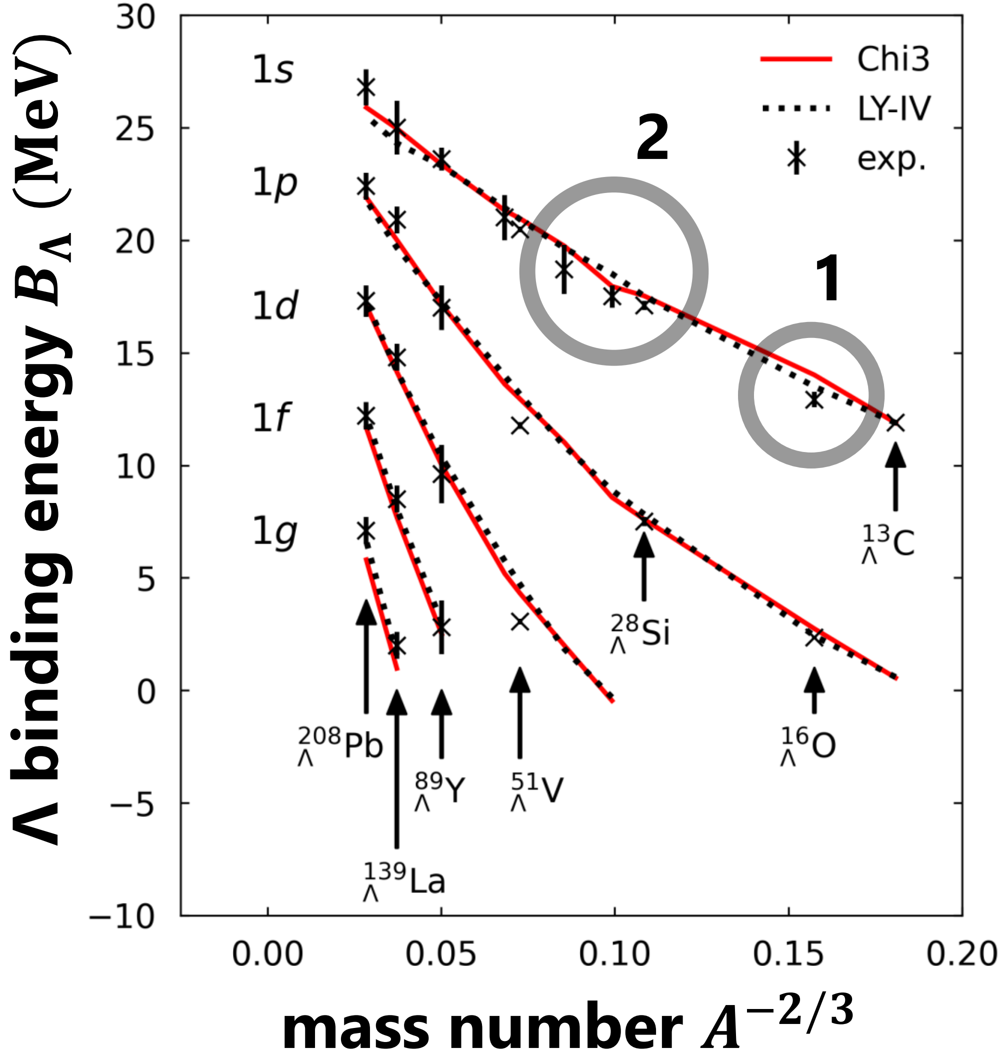
- **Chi2 overbounds a few MeV for *s*-wave.**

  ∵ Λ potential depth of Chi2 is too deep.



- **Chi3 reproduces the data, at the same level of accuracy as LY-IV.**

Differences between Chi3 and LY-IV



Chi3 and LY-IV differ for some hypernuclei.

1. $^{16}_{\Lambda}\mathrm{O}$ binding energy

LY-IV is preferred?

2. Kink at $^{32}_{\Lambda}S$

- **Why do they differ?**
- **Can we distinguish them from the current data?**

Difference in the Λ energy dominates.

Def. of the Λ binding energy

$$B_\Lambda = \mathcal{E}_{\text{core}} - \mathcal{E}_{\text{hyp}}$$

$$\mathcal{E}_{\text{hyp}} = \mathcal{E}_N + \underline{\mathcal{E}_\Lambda} - \mathcal{E}_{\text{c.m.}}$$

Def. of the Λ total energy

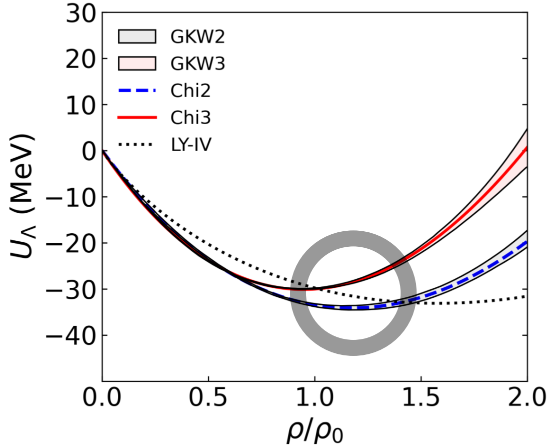
$$\boxed{\begin{aligned}
\mathcal{E}_\Lambda &= \mathcal{E}_{\Lambda,\text{kin}} + \mathcal{E}_{\Lambda,\rho} + \mathcal{E}_{\Lambda,\text{surf}} \\
\mathcal{E}_{\Lambda,\text{kin}} &= \int \mathrm{d}^3 r \left[\frac{\hbar^2}{2m_\Lambda}\tau_\Lambda + a_2^\Lambda(\tau_\Lambda \rho_N + \tau_N \rho_\Lambda)\right] \\
\mathcal{E}_{\Lambda,\rho} &= \int \mathrm{d}^3 r \left(a_1^\Lambda \rho_N + a_4^\Lambda \rho_N^{4/3} + a_5^\Lambda \rho_N^{5/3}\right) \\
\mathcal{E}_{\Lambda,\text{surf}} &= -\int \mathrm{d}^3 r\, a_3^\Lambda \rho_\Lambda \left(\nabla^2 \rho_N\right)
\end{aligned}}$$

$$\Delta\mathcal{E}_{\Lambda,i} = \mathcal{E}_{\Lambda,i}(\mathbf{LY-IV}) - \mathcal{E}_{\Lambda,i}(\mathbf{Chi3})$$

	$\Delta\mathcal{E}_\Lambda$, kin	$\Delta\mathcal{E}_\Lambda$, ρ	$\Delta\mathcal{E}_\Lambda$, surf
13C_Λ	-1.25	**2.57**	-1.10
16O_Λ	-1.45	**3.25**	-1.05
	-0.20 MeV	**+0.68 MeV**	+0.05 MeV

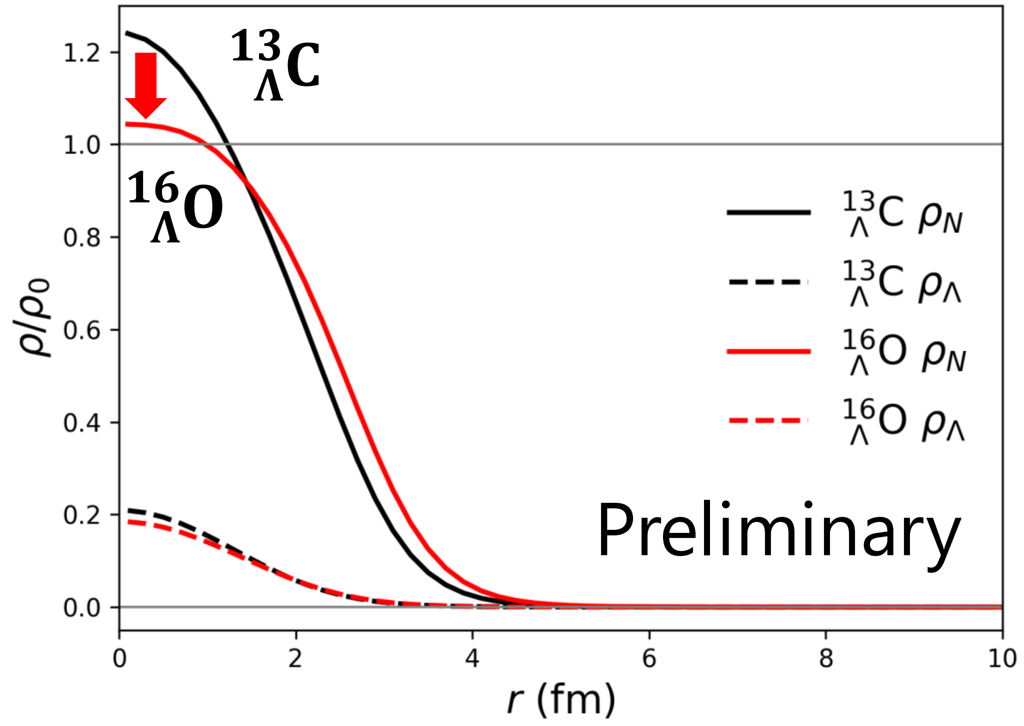
	$\Delta\mathcal{E}_\Lambda$, kin	$\Delta\mathcal{E}_\Lambda$, ρ	$\Delta\mathcal{E}_\Lambda$, surf
28Si_Λ	-2.63	**3.59**	-0.96
32S_Λ	-2.46	**2.90**	-0.93
	-0.17 MeV	**-0.69 MeV**	+0.03 MeV

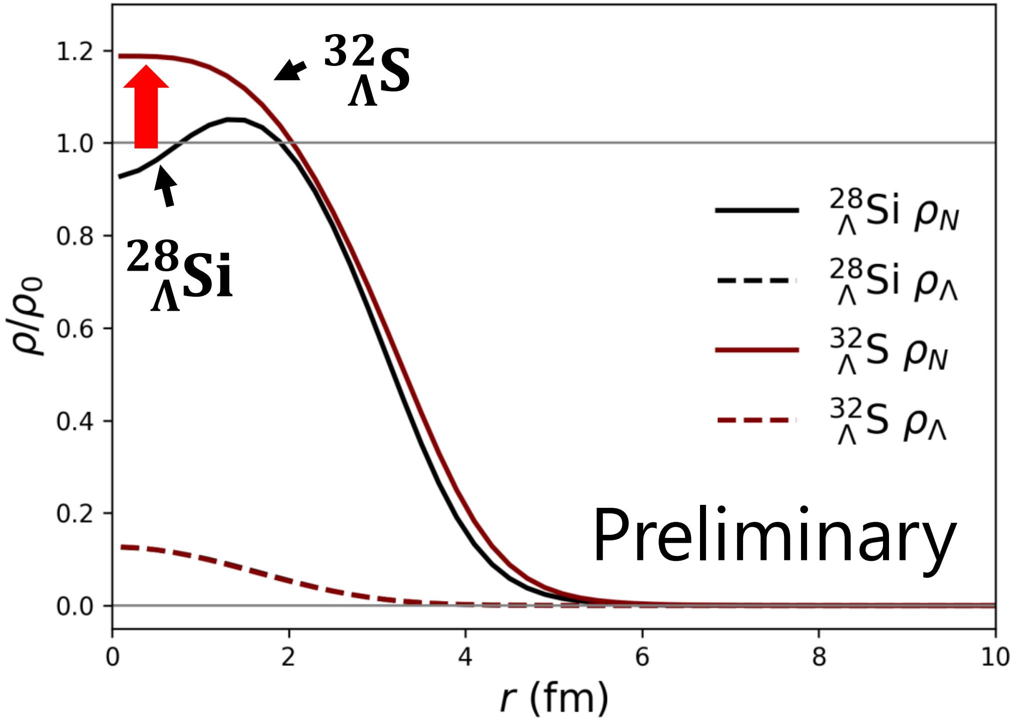
The Λ potential at $\rho > \rho_0$ makes the difference!



The nucleon density differs near the center.

The Λ potential at $\rho > \rho_0$ mainly makes the difference in B_Λ.

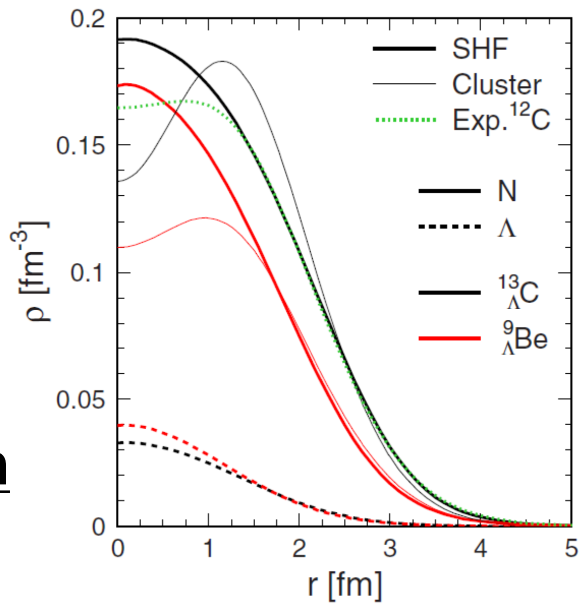




Then, can we distinguish them from the current data…?

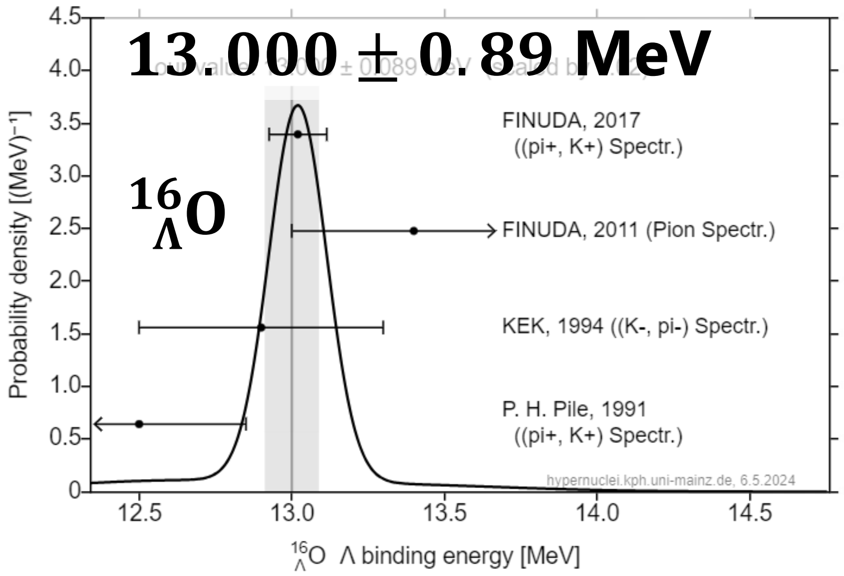
We have to discuss...

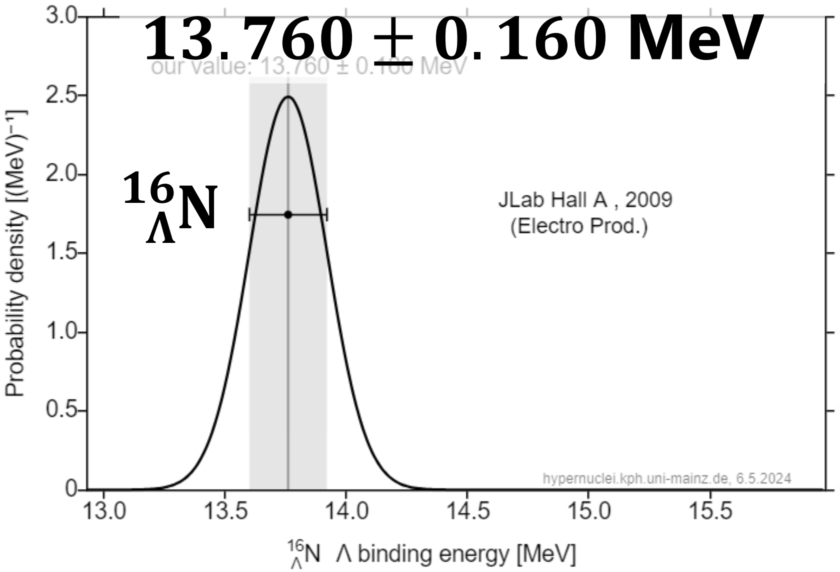
- **Feasibility of the calculated nucleon density**
  The nucleon density distribution of 12C is different among Skyrme-HF, cluster calc., and the electron scattering exp. data.



Schulze and Hiyama (2014)

- **Difference btw. $^{16}_{\Lambda}$O and $^{16}_{\Lambda}$N experimental data**
  → Analysis incorporating CSB is needed.



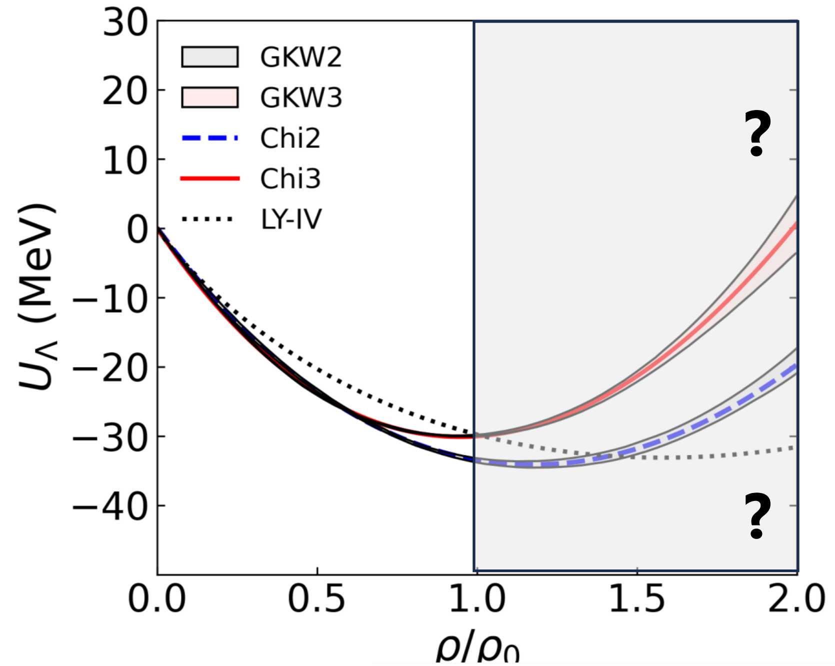


From *Chart of Hypernuclides*

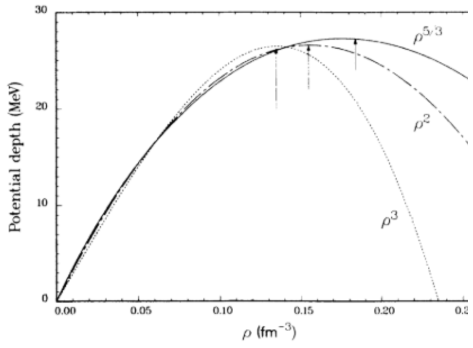
Model independent analysis for constraining Λ potentials

Motivation

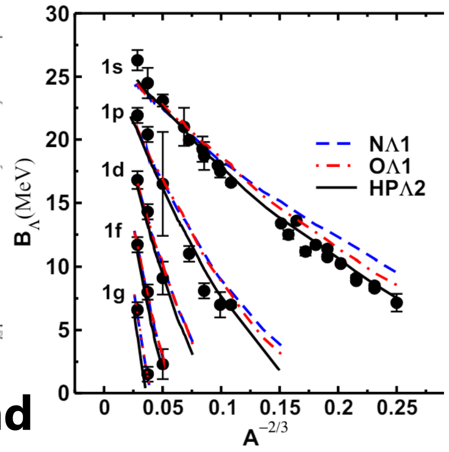
- **We cannot distinguish the repulsive and attractive Λ potentials from hypernuclear data.**
- **<u>To what extent can we constrain the Λ potential from the current hypernuclear data?</u>**



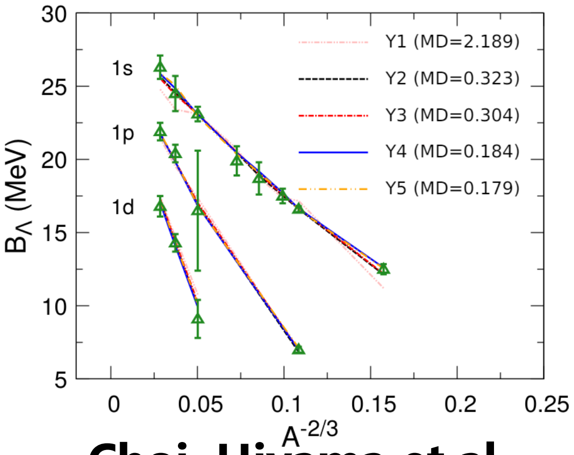
- **Best fitting study to the hypernuclear data is done by many.**
  But <u>estimation on the uncertainty of the Λ potential has not been done.</u>



Millener, Dover, and Gal (1988).



Guleria et al. (1988).



Choi, Hiyama et al. (2022).

How to analysis

$$J_\Lambda = U_\Lambda(\rho=\rho_0), L_\Lambda = 3\rho_0 \frac{\partial U_\Lambda}{\partial \rho}(\rho=\rho_0), K_\Lambda = 9\rho_0^2 \frac{\partial^2 U_\Lambda}{\partial \rho^2}(\rho=\rho_0), m_\Lambda^*/m_\Lambda(\rho=\rho_0)$$

cf. symmetry energy

Determining $a_1^\Lambda, a_2^\Lambda, a_4^\Lambda$, and a_5^Λ

$$U_\Lambda = a_1^\Lambda \rho_N + a_2^\Lambda \tau_N - a_3^\Lambda \Delta\rho_N + a_4^\Lambda \rho_N^{4/3} + a_5^\Lambda \rho_N^{5/3}$$

* The value of a_3^Λ is tuned for the Λ binding energy of ${}^{13}_{\Lambda}$C.

(∵ Surface terms have large effect. even-even nuclei)

$$\frac{\hbar^2}{2m_\Lambda^*} = \frac{\hbar^2}{2m_\Lambda} + a_2^\Lambda \rho_N$$

Skyrme-Hartree-Fock calculation to obtain Λ binding energy $B_\Lambda^{\rm cal}$

Comparison with the experimental data using RMSD

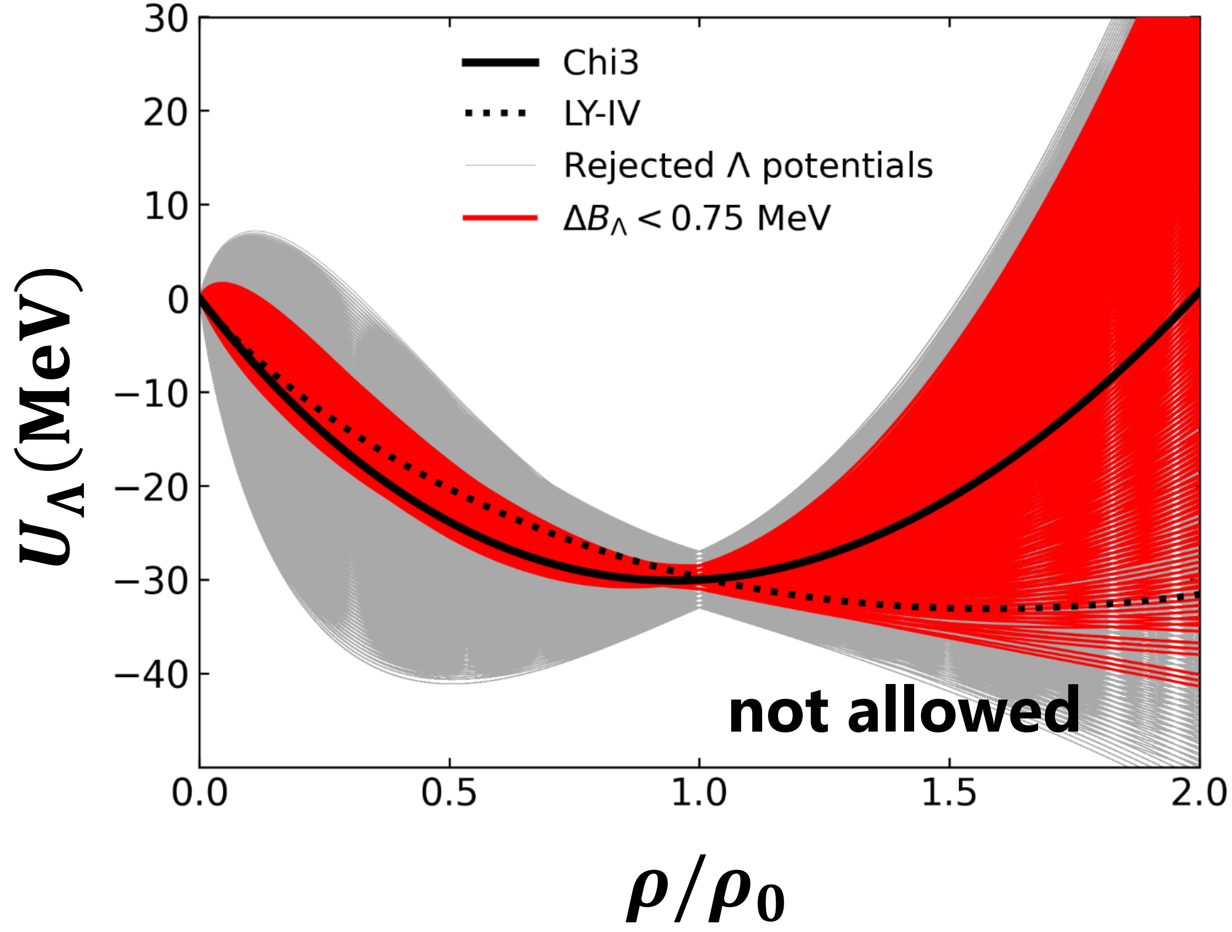
$$\Delta B_\Lambda = \sqrt{\frac{1}{N}\sum\left(B_\Lambda^{\rm exp} - B_\Lambda^{\rm cal}\right)^2}$$

${}^{208}_{\Lambda}$Pb, ${}^{139}_{\Lambda}$La, ${}^{89}_{\Lambda}$Y, ${}^{56}_{\Lambda}$Fe, ${}^{51}_{\Lambda}$V, ${}^{40}_{\Lambda}$Ca, ${}^{32}_{\Lambda}$S, ${}^{28}_{\Lambda}$Si, ${}^{16}_{\Lambda}$O
in *s, p, d, f*, and *g* orbitals ($N = 24$)
(0.5 MeV correction for (π^+,K^+) is included. Gogami et al. (2016))

What Λ potentials / parameters ($J_\Lambda, L_\Lambda, K_\Lambda, m_\Lambda^*/m_\Lambda$) have small ΔB_Λ?

Accepted Λ potentials

A. Jinno, K. Murase, Y. Nara, & A. Ohnishi, PRC 108, 065803 (2023).



gray lines: RMSD $\Delta B_\Lambda \geq 0.75$ MeV

red lines: RMSD $\Delta B_\Lambda < 0.75$ MeV

- **$\rho \leq \rho_0$: constrained**
- **$\rho > \rho_0$: Too attractive Λ potentials cannot reproduce the data.**

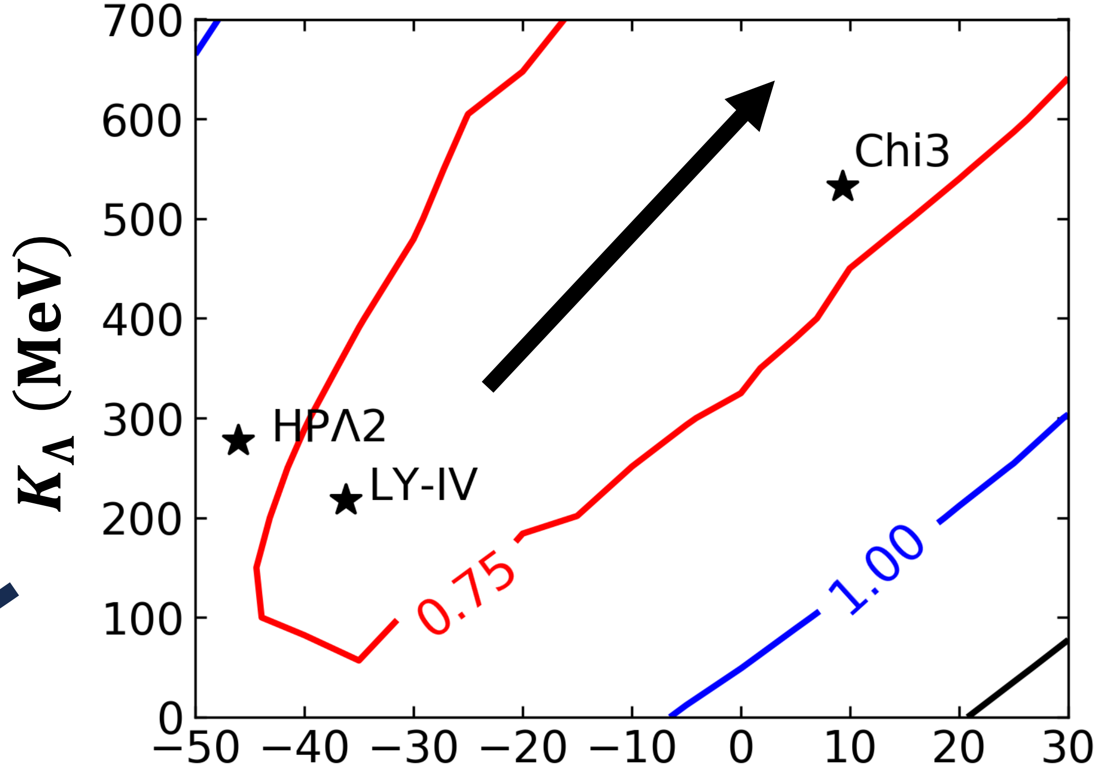
Chi3: Fitted to Chiral EFT results including ΛNN+ΣNN, Gerstung, Kaiser, and Weise (2020).
LY-IV: Skyrme-HF, Lanskoy and Yamamoto (1998).
HPΛ2: Skyrme-HF, Guleria et al. (2012).

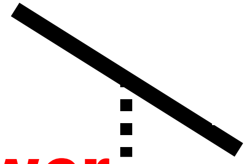
1st derivative L_Λ and 2nd derivative K_Λ

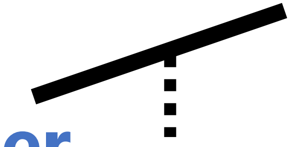
A. Jinno, K. Murase, Y. Nara, & A. Ohnishi, PRC 108, 065803 (2023).

J_Λ and m^*_Λ are chosen for minimizing ΔB_Λ by golden-section search.

Contour line: ΔB_Λ







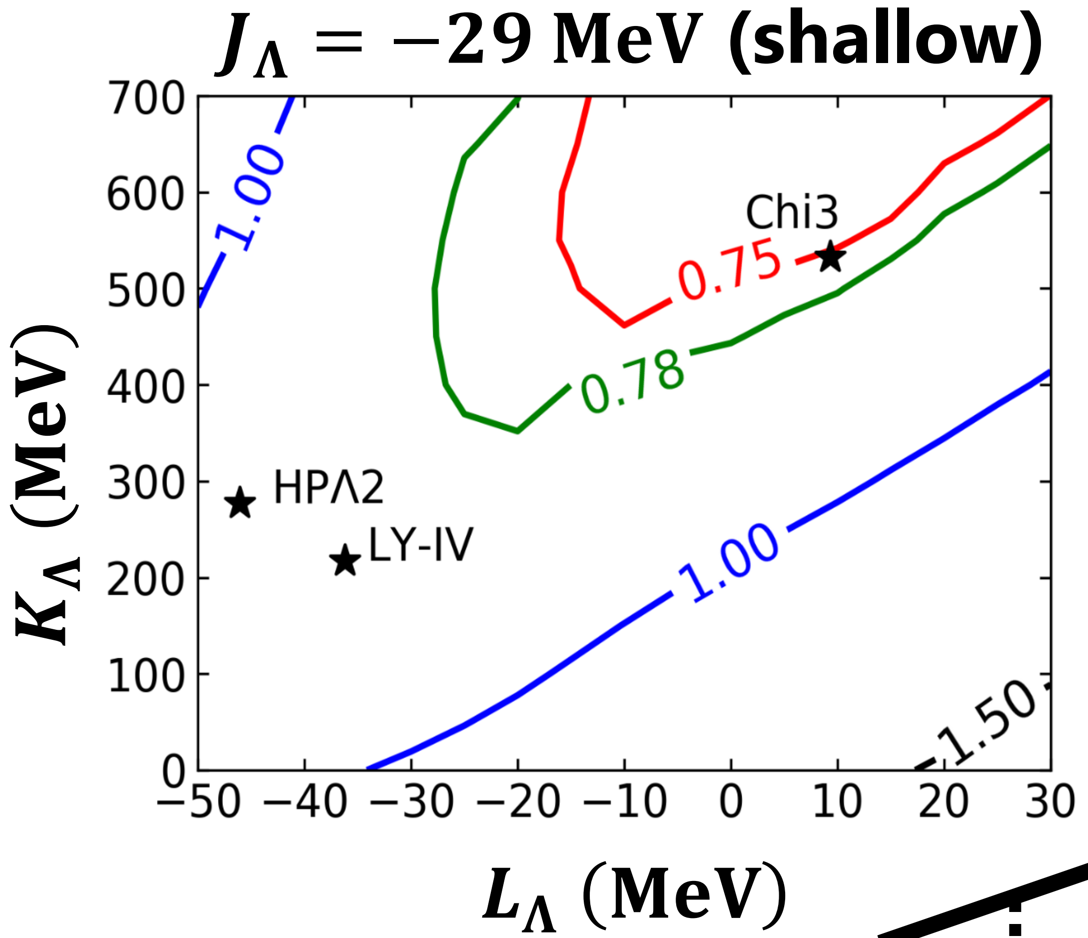
$\rho \le \rho_0$: constrained

Positive correlation btw. L_Λ and K_Λ

If $J_\Lambda = U_\Lambda(\rho_0)$ is well constrained from future data…

L_Λ and K_Λ at $J_\Lambda = -29$ MeV

A. Jinno, K. Murase, Y. Nara, & A. Ohnishi, PRC 108, 065803 (2023).



Λ potentials with $K_\Lambda > 350$ MeV,
or repulsive potentials at high densities are favored.

$U_\Lambda(\rho_0) = J_\Lambda$ is large.

⬇

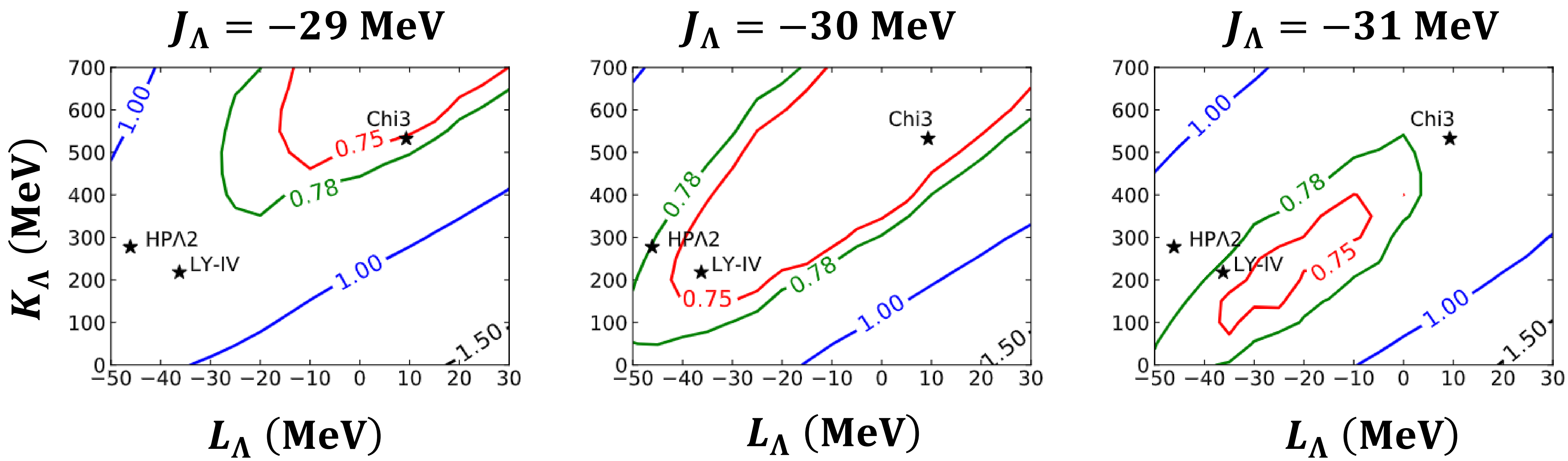
U_Λ at $\rho \lesssim \rho_0$ should be deeper,

or L_Λ should be larger.

⬇

Positive correlation between L_Λ and K_Λ

L_Λ and K_Λ for three different J_Λ



large $K_\Lambda (\gtrsim 350\ \mathrm{MeV})$ (wide range of K_Λ) **small** $K_\Lambda (\lesssim 550\ \mathrm{MeV})$

The range of K_Λ, or the degree of the repulsion at $\rho > \rho_0$, can be constrained by the precise measurements of heavier hypernuclei.

A. Jinno, K. Murase, Y. Nara, and A. Ohnishi, PRC 108, 065803 (2023).

Summary

Firstly, we have verified whether the repulsive Λ potential based on chiral EFT derived by Gerstung et al. can explain the Λ hypernuclear data.

- The Λ potential based on the chiral two-body force overbounds for 1*s* orbital.
- Difference in $U_\Lambda(\rho > \rho_0)$ may appear in $B_\Lambda({}^{16}_{\Lambda}\mathrm{O}) - B_\Lambda({}^{13}_{\Lambda}\mathrm{C})$ and $B_\Lambda({}^{32}_{\Lambda}\mathrm{S}) - B_\Lambda({}^{28}_{\Lambda}\mathrm{Si})$.

Next, we examine to what extent the Λ potential is constrained now.

- Too attractive Λ potentials at $\rho > \rho_0$ cannot explain the data.
- The repulsion at $\rho > \rho_0$ could be well constrained if $J_\Lambda = U_\Lambda(\rho_0)$ is well determined from future high-resolution heavy hypernuclear data.

Future work

- Including the charge-symmetry breaking effect to discuss ${}^{16}_{\Lambda}\mathrm{O}$ and ${}^{16}_{\Lambda}\mathrm{N}$
- Comparing the model dependence on the nucleon density (e.g. Gogny-HF)

Backup

Used binding energy data

TABLE III. Experimental data of Λ binding energy (B.E.) for various hypernuclei used in this work.

Hypernuclei	B.E. (MeV)				
	$1s$	$1p$	$1d$	$1f$	$1g$
$^{16}_{\Lambda}$O [65]	12.92 ± 0.35	2.35 ± 0.05			
$^{28}_{\Lambda}$Si [65]	17.1 ± 0.2	7.5 ± 0.2			
$^{32}_{\Lambda}$S [79]	17.5 ± 0.5				
$^{40}_{\Lambda}$Ca [80]	18.7 ± 1.1				
$^{51}_{\Lambda}$V [65, 81]	20.47 ± 0.13	11.77 ± 0.16	3.05 ± 0.13		
$^{56}_{\Lambda}$Fe [79]	21.0 ± 1.0				
$^{89}_{\Lambda}$Y [65]	23.6 ± 0.5	17.0 ± 1.0	9.6 ± 1.3	2.8 ± 1.2	
$^{139}_{\Lambda}$La [65]	25.0 ± 1.2	20.9 ± 0.6	14.8 ± 0.6	8.5 ± 0.6	2.0 ± 0.6
$^{208}_{\Lambda}$Pb [65]	26.8 ± 0.8	22.4 ± 0.6	17.3 ± 0.7	12.2 ± 0.6	7.1 ± 0.6

From where come the differences?

Λ total energy term dominates the difference.

Def. of the binding energy

$$B_\Lambda = -(\mathcal{E}_{\mathrm{hyp}} - \mathcal{E}_{\mathrm{core}})$$

$$\mathcal{E}_{\mathrm{hyp}} = \mathcal{E}_N + \mathcal{E}_\Lambda - \mathcal{E}_{\mathrm{c.m.}}$$

$$\Delta B_\Lambda \coloneqq B_\Lambda(\mathrm{Chi3}) - B_\Lambda(\mathrm{LY-IV})$$

$$= \Delta\mathcal{E}_N + \Delta\mathcal{E}_\Lambda + \Delta\mathcal{E}_{\mathrm{c.m.}}$$