The Three-Nucleon Correlation function

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Introduction

- When a high-energy pp or p–nucleus collision occurs, particles are produced and emitted at relative distances of the order of the nuclear force
- The effect of the mutual interaction between hadrons is reflected as a correlation signal in the momentum distributions of the detected particles which can be studied using correlation functions
- The correlation function incorporate information on the emission process as well as on the final state interaction of the emitted pairs or triplets
- By measuring correlated particle pairs or triplets at low relative energies and comparing the yields to theoretical predictions, it is possible to study the hadron dynamics.

The two-particle correlation function

- The two-particle correlation function is defined as the ratio of the yield of a particle pair to the product of the single-particle yields.

$$C\left(\vec{p}_1, \vec{p}_2\right) = \frac{\mathcal{P}\left(\vec{p}_1, \vec{p}_2\right)}{\mathcal{P}\left(\vec{p}_1\right)\mathcal{P}\left(\vec{p}_2\right)}$$

- $\mathcal{P}(\vec{p}_1, \vec{p}_2)$ is the probability of finding a pair with momenta $\vec{p}_1$ and $\vec{p}_2$
- $\mathcal{P}(\vec{p}_i)$ is the probability of finding each particle with momentum $\vec{p}_i$.
- In absence of correlations, the two-particle probability factorizes, $\mathcal{P}(\vec{p}_1, \vec{p}_2) = \mathcal{P}(\vec{p}_1)\mathcal{P}(\vec{p}_2)$, and the correlation function is equal to unity.

The two-particle correlation function

- The correlation between the pair is related to the particle emission and the subsequent interaction of the pair

$$C\left(\vec{p}_1, \vec{p}_2\right) = \frac{1}{\Gamma} \sum_{m_1, m_2} \int d^3 r_1 \, d^3 r_2 S_1\left(r_1\right) S_1\left(r_2\right) \times \left|\Psi_{m_1, m_2}(\vec{p}_1, \vec{p}_2, \vec{r}_1, \vec{r}_2)\right|^2$$

- $S_1(r)$ describes the spatial shape of the source for single-particle emissions. It can be approximated as a Gaussian probability distribution with a width R_M

$$S_1(r) = \frac{1}{(2\pi R_M^2)^{\frac{3}{2}}} e^{-r^2/2R_M^2}$$

- The integration on the CM coordinates leads to the Koonin-Pratt relation for two-particle correlation function

$$C(k) = \frac{1}{\Gamma} \int d^3 r \, S(r) |\psi_k\left(\vec{r}\right)|^2$$

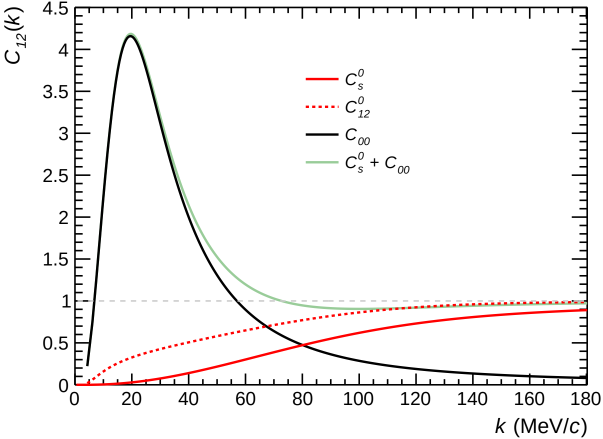
The pp correlation function: $C(k) = \frac{1}{\Gamma}\int d^3r\, S(r)|\psi_k(\vec{r})|^2$

$\psi_k(\vec{r})$ is the two-particle scattering wave function at $E = \hbar^2 k^2/2\mu$

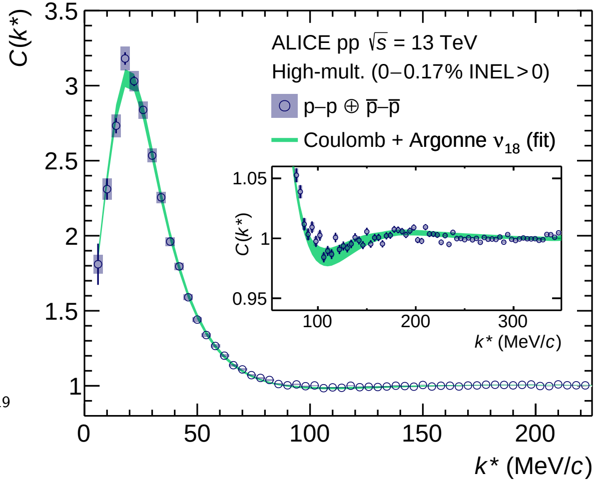
$$\psi_k = 4\pi \sum_{\ell m S S_z J J_z} i^\ell\, \Psi^{JJ_z}_{\ell S}\, \mathcal{Y}^{JJ_z}_{[\ell S]}(\hat{k})^*$$

with $\Psi^{JJ_z}_{\ell S} = \sum_{\ell' S'} \frac{u^{\ell' S'}_{\ell S}(k,r)}{kr} \mathcal{Y}^{JJ_z}_{\ell' S'}(\hat{r})$

In the case of two protons $u^{\ell'}_{\ell}(kr) \rightarrow \delta_{\ell\ell'} F_{\ell'}(\eta, kr) + T_{\ell\ell'} \mathcal{O}_{\ell'}(kr)$



The pp Correlation Function



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The pd Correlation Function

- We now consider the pd correlation function:

$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2,m_1} \int d^3r_1 d^3r_2 d^3r_3 \; S_1(r_1) S_1(r_2) S_1(r_3) |\Psi_{m_2,m_1}|^2$$

- the probability of deuteron formation

$$A_d = \frac{1}{3} \sum_{m_2} \int d^3r_1 d^3r_2 \; S_1(r_1) S_1(r_2) |\phi_{m_2}|^2$$

- the single particle source function

$$S_1(r) = \frac{1}{(2\pi R_{\mathrm{M}}^2)^{\frac{3}{2}}} e^{-r^2/2R_{\mathrm{M}}^2}$$

The pd Correlation Function

- the pd correlation function results

$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2,m_1} \int \rho^5 d\rho d\Omega \, \frac{e^{-\rho^2/4R_M^2}}{(4\pi R_M^2)^3} |\Psi_{m_2,m_1}|^2$$

$$\Psi_{m_2,m_1} = \sum_{LSJ} \sqrt{4\pi} i^L \sqrt{2L+1} e^{i\sigma_L} (1 m_2 \frac{1}{2} m_1 \mid S J_z)(L 0 S J_z \mid J J_z) \Psi_{LSJJ_z}(\mathbf{x}, \mathbf{y})$$

- the Jacobi coordinates: $\mathbf{x}_\ell = \mathbf{r}_j - \mathbf{r}_i$, $\mathbf{y}_\ell = \mathbf{r}_\ell - \frac{\mathbf{r}_i + \mathbf{r}_j}{2}$
- the hyperspherical coordinates $\rho = \sqrt{x_1^2 + (4/3) y_1^2}$, $\Omega \equiv [\alpha_1, \hat{x}_1, \hat{y}_1]$
- The scattering wave function, $\Psi_{LSJJ_z}(\mathbf{x}, \mathbf{y})$ is expanded using the HH basis

The pd wave function

$$\Psi_{LSJJ_z} = \sum_{n,\alpha} \frac{u_{n,\alpha}(\rho)}{\rho^{5/2}} \mathcal{Y}_{n,\alpha}(\Omega) + \frac{1}{\sqrt{3}} \sum_{sym} \left\{ Y_L(\hat{y}_\ell) \left[\varphi^d(i,j)\chi(\ell) \right]_S \right\}_{JJ_z} \frac{F_L(\eta, ky_\ell)}{ky_\ell}$$

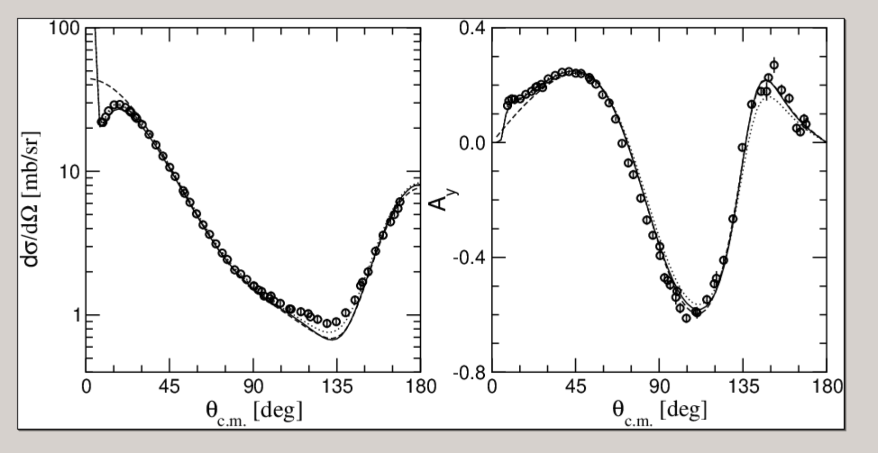
$$+ \sum_{L'S'} T^J_{LS,L'S'} \frac{1}{\sqrt{3}} \sum_{sym} \left\{ Y_{L'}(\hat{y}_\ell) \left[\varphi^d(i,j)\chi(\ell) \right]_{S'} \right\}_{JJ_z} \frac{\overline{G}_{L'}(\eta, ky_\ell) + iF_{L'}(\eta, ky_\ell)}{ky_\ell}$$

For energies below the deuteron breakup $u_{[K]}(\rho \to \infty) \to 0$ whereas for energies above the deuteron breakup it describes the breakup amplitude. The elastic amplitude is

$$M^{SS'}_{S_zS'_z}(\theta) = f_c(\theta)\delta_{SS'}\delta_{S_zS'_z} + \frac{\sqrt{4\pi}}{k} \sum_{LL'J} C_{LL'J}\, T^J_{LS,L'S'}\, Y_{L'M'}(\theta, 0)$$

with f_c the Coulomb amplitude. The cross section and analyzing power are
$d\sigma/d\Omega(\theta) = tr(MM^\dagger)/6$ and $A_y(\theta) = tr(M\sigma_y M^\dagger)/6$

pd elastic observables at $E_p = 65\,\mathrm{MeV}$



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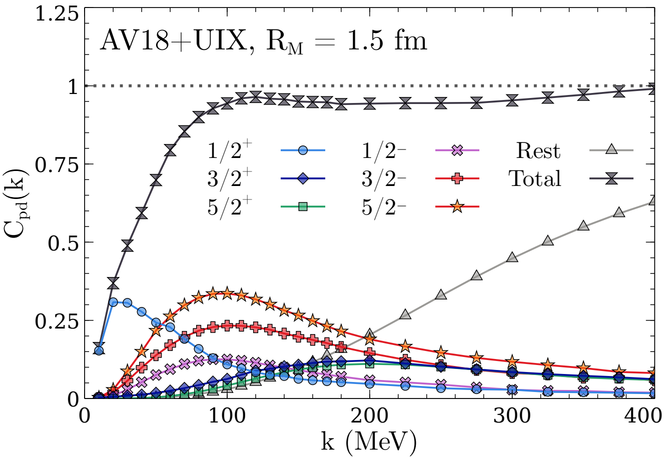
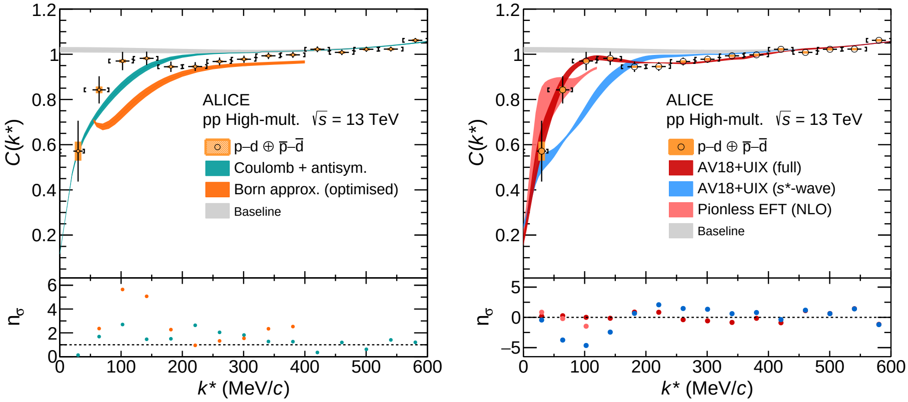
The pd Correlation Function: partial-wave contributions



The pd Correlation Function: comparison to experiment



M. Viviani, S. König, A. Kievsky, L.E. Marcucci, B. Singh, O. Vázquez Doce, Phys. Rev. C 108, 064002 (2023)
ALICE collaboration, arXiv:2308.16120 [nucl-ex]

The ppp correlation function

$$C_{ppp}(Q) = \int \rho^5 d\rho d\Omega\, S_{\rho_0}(\rho)|\Psi_{ppp}|^2$$

with Q the hyper-momentum, S_{ρ_0} the source function defined as

$$S_{\rho_0}(\rho) = \frac{1}{\pi^3 \rho_0^6} e^{-(\rho/\rho_0)^2}$$

Ψ_{ppp} is the ppp scattering wave function

$$\Psi_{ppp} = \frac{(2\pi)^3}{(Q\rho)^{5/2}} \sum_{JJ_z,K\gamma} \Psi^{JJ_z}_{K\gamma} \sum_{M_L M_S} (LM_L SM_S|JJ_z)\mathcal{Y}^{l_x l_y}_{KLM_L}(\Omega_Q)^*$$

with the coordinate wave function taken the fomr

$$\Psi^{JJ_z}_{K\gamma} = \sum_{K'\gamma'} u^{K'\gamma'}_{K\gamma}(Q,\rho) \sum_{M_L M_S} (L'M_L S'M_S|JJ_z)\mathcal{Y}^{l'_x l'_y}_{K'L'M_L}(\Omega_\rho)\chi^{s_x}_{S'M_S}$$

The ppp Wave Function

In a compact form, the ppp wave function is

$$\Psi^J_{[K]}(\vec{x},\vec{y}) = \sum_{[K']} u^{[K]}_{[K']}(\rho)\mathcal{B}^J_{[K']}(\Omega)$$

with $\mathcal{B}^J_{[K]}$ antisymmetric HH-spin functions

The ppp wave is completely determined from the hyperradial functions $u_{[K]}(\rho)$. And they are determined from the boundary conditions as $\rho \to \infty$.

For a given energy, $E = \hbar^2 Q^2/m$, and in the nnn case,

$$u_{[K]}(\rho \to \infty) \to \sqrt{Q\rho}\,[J_{K+2}(Q\rho) + \tan\delta_K\, Y_{K+2}(Q\rho)]$$

In the ppp case the asymptotic equations are coupled not allowing this simple picture

ppp wave function analysis

Using the property of the HH functions

$$\Psi_s^0 = e^{i\vec{Q}\cdot\vec{\rho}} = \frac{(2\pi)^3}{(Q\rho)^2} \sum_{[K]} i^K J_{K+2}(Q\rho) \mathcal{Y}_{[K]}(\Omega) \mathcal{Y}^*_{[K]}(\hat{Q})$$

where $\vec{Q}\cdot\vec{\rho} = \vec{k}_1\cdot\vec{x} + \vec{k}_2\cdot\vec{y}$ and J_{K+2} a Bessel function.

- For the case of three nucleons we have to include the correct symmetrization.

The nnn case:

$$\Psi_s^0 = \frac{(2\pi)^3}{(Q\rho)^2} \sum_{[K]} i^K J_{K+2}(Q\rho) \mathcal{B}_{[K]}(\Omega) \mathcal{B}^*_{[K]}(\hat{Q})$$

with $\mathcal{B}_{[K]}(\Omega)$ antisymmetric in the hyperangle-spin space.

- For the case of three protons we have to include the correct asymptotics

The ppp wave function analysis

For three protons the asymptotic form it is not known in a close form
The Coulomb interaction coupled the asymptotic equations through the term

$$\sum_{ij} \frac{e^2}{r_{ij}}$$

In a first step we have performed an average of the Coulomb interaction on the hyperangles

$$V_c(\rho) = \int d\Omega \sum_{ij} \frac{e^2}{r_{ij}} |\mathcal{Y}_0(\Omega)|^2 = \frac{16}{\pi} \frac{e^2}{\rho}$$

and the plane wave takes the form

$$e^{i\vec{Q}\cdot\vec{\rho}} \to \Psi_s^0 = \frac{1}{C_{3/2}(0)} \frac{(\pi)^3}{(Q\rho)^{5/2}} \sum_{[K]} i^K F_{K+3/2}(\eta, Q\rho) \mathcal{B}_{[K]}(\Omega) \mathcal{B}^*_{[K]}(\hat{Q})$$

ppp Correlation Analysis

The ppp wave function is

$$\Psi_{ppp} = \sum_{[K]} u_{[K]}(\rho)\mathcal{B}_{[K]}(\Omega) = \Psi^0 + \sum_{J,K}^{\overline{J},\overline{K}} \Psi_K^J$$

To determine Ψ^J we use the Adiabatic Hyperspherical Harmonic basis:

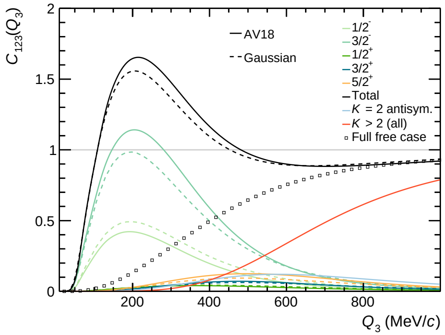
$$-\frac{\hbar^2}{m}\frac{\Lambda^2(\Omega)}{\rho^2} = H_\Omega\phi_\nu(\rho,\Omega) = U_\nu(\rho)\phi_\nu(\rho,\Omega)$$

$$\Psi_K^J = \rho^{-5/2}\sum_\nu w_\nu^{J,K}(\rho)\phi_\nu(\rho,\Omega)$$

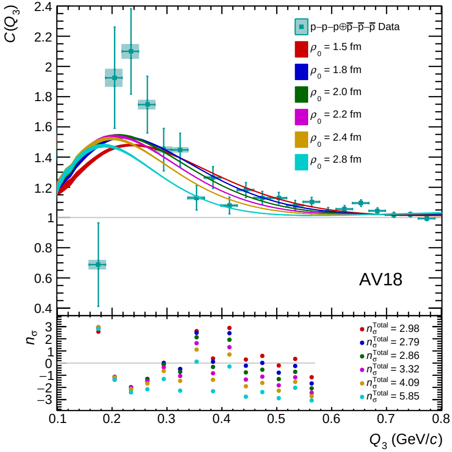
with the adiabatic functions $\phi_\nu(\rho\to\infty,\Omega)\to\mathcal{B}_{[K]}(\Omega)$

and the hyperradial functions $w_\nu^{J,K}(\rho)\to\sqrt{Q\rho}[\delta_{KK'}F_{K+3/2}(Q\rho)+T_{KK'}\mathcal{O}_{K'+3/2}(Q\rho)]$

The ppp correlation function



Comparison to data



A. Kievsky, E. Garrido, M.Viviani, L.E. Marcucci, L. Šerkšnytė, R. Del Grande, Phys. Rev. C109, 034006 (2024)

Some remarks

- To compare the experimental and the theoretical correlation functions some corrections have been considered
- For the pp case the corrected correlation function is defiend as

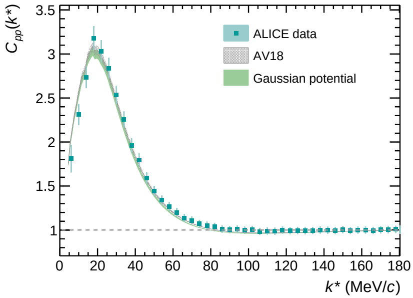
$$C(k) = \lambda_{pp} C_{pp}(k) + \lambda_{pp_\Lambda} C_{pp_\Lambda}(k) + \lambda_{\mathrm{X}} C_{\mathrm{X}}(k)$$

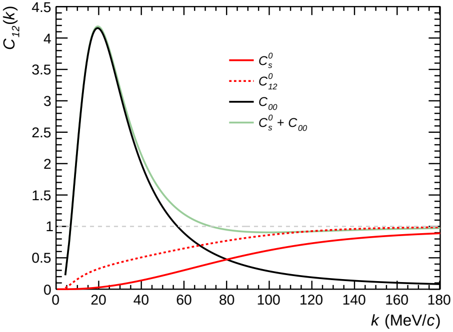
- primary protons $\lambda_{pp} = 0.67$, secondary protons produced mainly in the decay of the Λ, $\lambda_{pp_\Lambda} = 0.203$, misidentification contributions $\lambda_x = 0.127$
- For the ppp case the corrected correlation function is defiend as

$$C(Q_3) = \lambda_{ppp} C_{ppp}(Q_3) + \lambda_{ppp_\Lambda} C_{ppp_\Lambda}(Q_3) + \lambda_{\mathrm{X}} C_{\mathrm{X}}(Q_3)$$

- primary protons $\lambda_{ppp} = 0.618$, secondary protons produced mainly in the decay of the Λ, $\lambda_{ppp_\Lambda} = 0.196$, misidentification contributions $\lambda_x = 0.186$

The pp correlation function





$$C_{12}(k) = C^0_s + C_{00} = \int d\mathbf{r} S_{12}(r) \left[|\Psi^0_s|^2_\Omega - \frac{1}{2}\left(\frac{F_0(\eta,kr)}{kr}\right)^2 + \frac{1}{2}\left(\frac{u_0(kr)}{kr}\right)^2 \right]$$

$$C_{pp}(k) = \lambda_{pp} C_{12}(k) + \lambda_{pp_\Lambda} C_{pp_\Lambda}(k) + \lambda_{\rm X} C_{\rm X}(k)$$

The $p\Lambda$ and $pp\Lambda$ correlation functions

- The $p\Lambda$ correlation function is defined as

$$C(k) = \int d^3r\, S(r) |\psi_{p\Lambda}(\vec{r})|^2$$

- $\psi_{p\Lambda}$ is the scattering $p\Lambda$ wave function. It is governed by the $p\Lambda$ interaction which is not very well kown
- The few $p\Lambda$ scattering data can be described in the context of the EFT at different orders
- At different cutoffs different sets of low-energy scattering parameters appear

	NLO13						NLO19				SMS N2LO		
C(MeV)	450	500	550	600	650	700	500	550	600	650	500	550	600
a_0 (fm)	-2.90	-2.91	-2.91	-2.91	-2.90	-2.90	-2.91	-2.90	-2.91	-2.90	-2.80	-2.79	-2.80
r_e^0 (fm)	2.64	2.86	2.84	2.78	2.65	2.56	3.10	2.93	2.78	2.65	2.82	2.89	2.68
a_1 (fm)	-1.70	-1.61	-1.52	-1.54	-1.51	-1.48	-1.52	-1.46	-1.41	-1.40	-1.56	-1.58	-1.56
r_e^1 (fm)	3.44	3.05	2.83	2.72	2.64	2.62	2.62	2.61	2.53	2.59	3.16	3.09	3.17

The $N\Lambda$ and $NN\Lambda$ interaction

Using EFT concepts we define a low energy (two-parameter) $N\Lambda$ interaction

$$V_{N\Lambda}(r) = \sum_{S=0,1} V_S \; e^{-(r/r_S)^2} \mathcal{P}_S$$

$\mathcal{P}_S$ is a spin projector and V_S and r_S fixed to describe a_S and r_e^S.

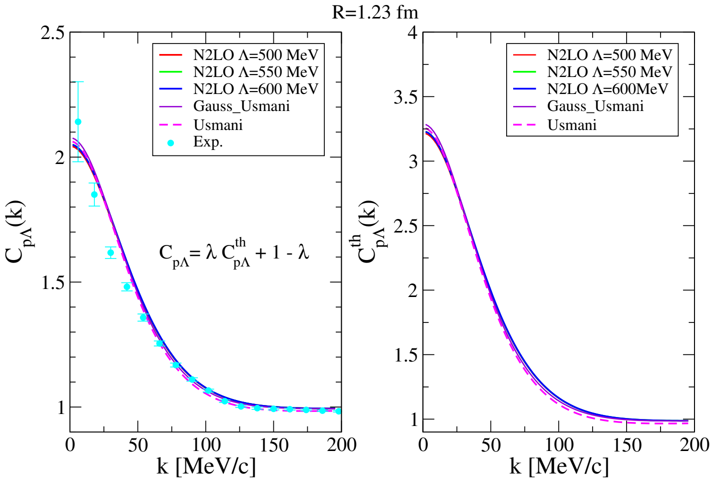
We define a two-parameter $NN\Lambda$ interaction as

$$W_{NN\Lambda}(r_{13}, r_{23}) = W_3 e^{-(r_{13}^2 + r_{23}^2)/r_3^2}$$

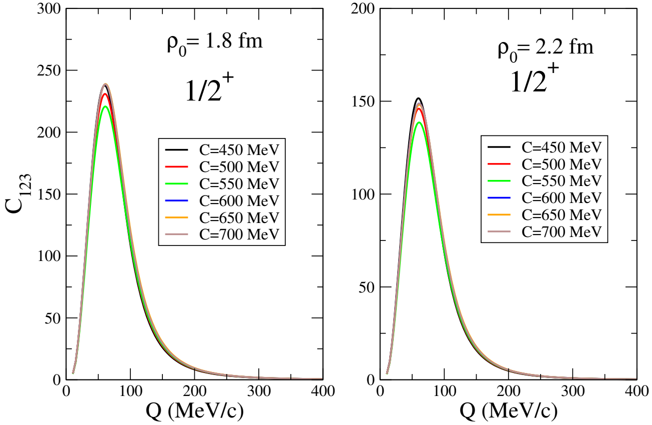
with W_3 fixed to give the hypertriton binding energy $E_3 = 2.390\,\text{MeV}$

	NLO13					NLO19				SMS N2LO		
C(MeV)	500	550	600	650	700	500	550	600	650	500	550	600
V_0 (MeV)	-30.180	-30.574	-31.851	-34.831	-37.198	-25.954	-28.817	-31.851	-34.831	-31.140	-29.753	-34.273
r_0 (fm)	1.467	1.459	1.434	1.380	1.342	1.563	1.495	1.434	1.380	1.439	1.466	1.382
V_1 (MeV)	-29.205	-33.839	-36.258	-38.455	-39.143	-38.984	-39.470	-42.055	-49.373	-27.544	-28.609	-27.392
r_1 (fm)	1.338	1.247	1.216	1.183	1.170	1.178	1.163	1.126	1.143	1.361	1.344	1.364
E_3 (MeV)	2.873	2.879	2.925	2.985	3.027	2.792	2.839	2.904	3.255	2.819	2.799	2.878
W_3 (MeV)	11.83	11.83	12.32	12.873	13.224	10.545	11.056	11.795	16.8	10.65	10.375	11.4
r_3 (fm)	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0

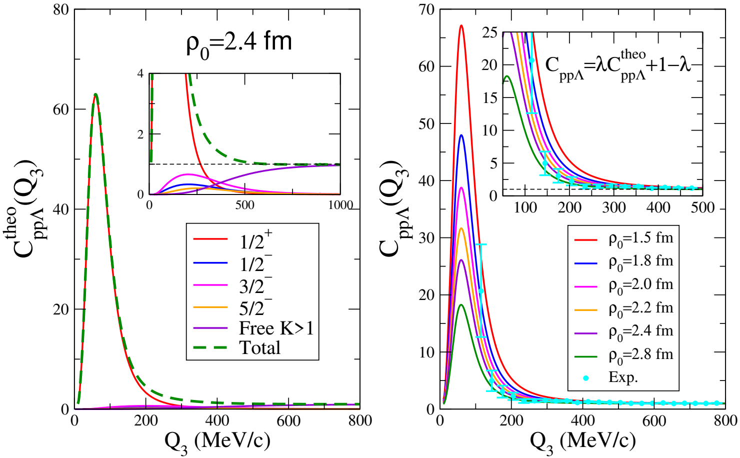
The $p\Lambda$ correlation function



The $pp\Lambda$ correlation function (preliminary)



The $pp\Lambda$ correlation function (preliminary)



- A $NN\Lambda$ three-body force is included fixed to describe the $B(^3_\Lambda\text{H})$

Summary

- Although its apparent simplicity, the three-nucleon problem is of great complexity
- Measurements of the correlation function allow for new tests of the NN and NNN interactions
- In the ppp case the Coulomb interaction couples the asymptotic dynamics increasing the difficulties of the numerical treatment
- The corrections of the computed pp and ppp correlation functions needs the kwonledge of the $p\Lambda$ and $pp\Lambda$ correlation functions
- However the $p\Lambda$ interaction is not well known

- Studies on the $p\Lambda$ and $pp\Lambda$ correlation functions have been started
- The $pp\Lambda$ correlation functions could be sensitive to the $NN\Lambda$ three-body force, an important ingredient in the studies of compact systems

Screened Coulomb potential

To study the effect of screening on the Coulomb potential, we introduce the following short-range potential

$$V_{sc}(r) = \frac{e^2}{r}\, e^{-(r/r_{sc})^n}$$

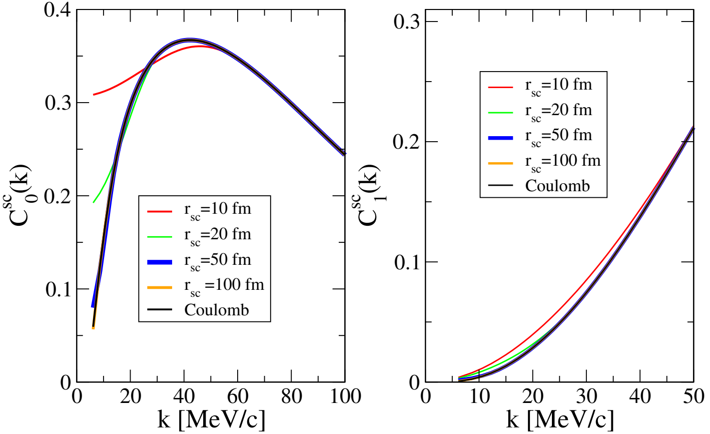
where r_{sc} is the screening radius and the parameter $n = 4$ allows for a sufficient fast cut of the Coulomb potential. With the above potential the radial functions $u_\ell(kr)$ have the following asymptotic form:

$$u_\ell(kr \to \infty) \longrightarrow kr[j_\ell(kr) + T_{\ell\ell}\mathcal{O}(kr)]$$

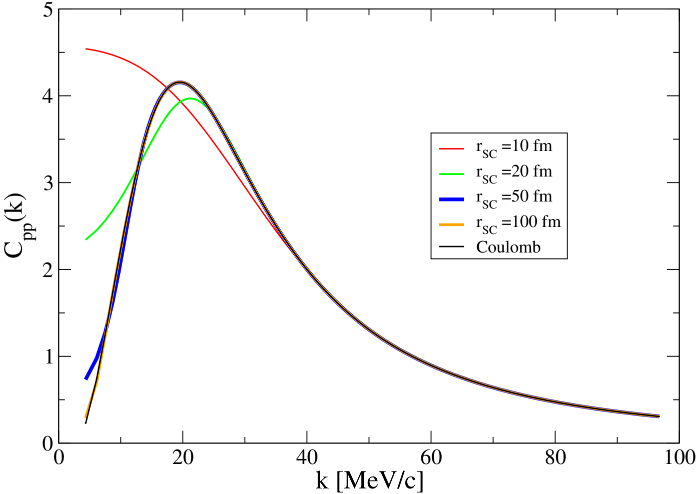
with $\mathcal{O}(kr) = \eta_\ell(kr) + ij_\ell(kr)$.
The correlation function using the screened Coulomb potential results

$$C^{sc}_{pp}(k) = \frac{1}{4\sqrt{\pi}R^3}\frac{1}{k^2}\int dr e^{-(r^2/4R^2)}\left(\sum_{\ell\equiv\text{even}} u^2_\ell(kr)(2\ell+1) + 3\sum_{\ell\equiv\text{odd}} u^2_\ell(kr)(2\ell+1)\right).$$

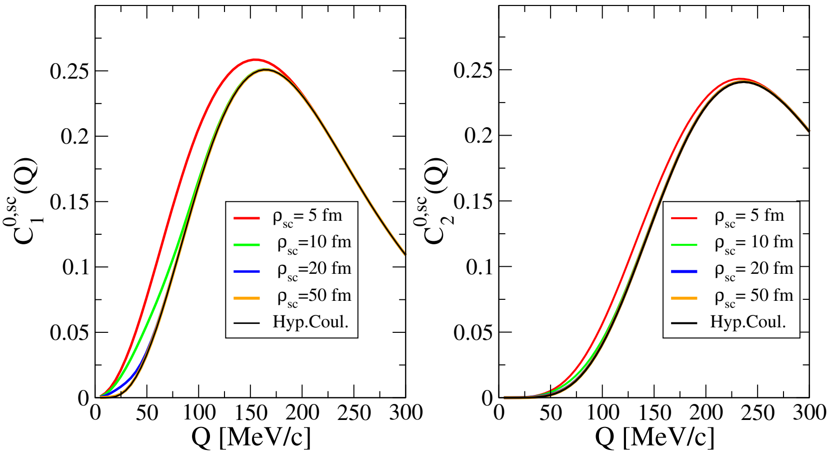
The *pp* Screened Correlation function



The *pp* Screened Correlation function



The *ppp* Screened Correlation function



The *ppp* Screened Correlation function