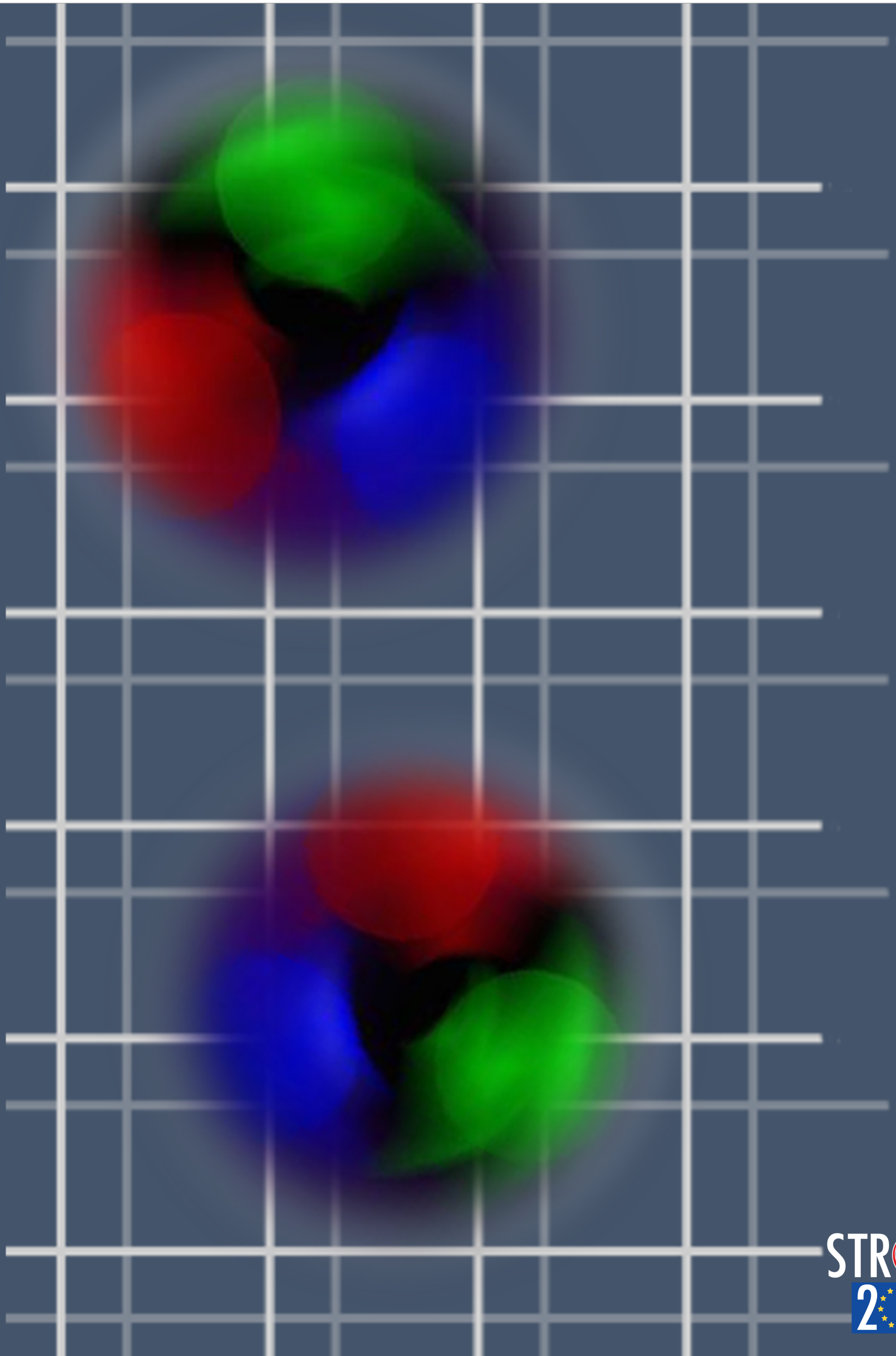


Variational studies of (hyper)nuclear interactions from Lattice QCD

Assumpta Parreño
Universitat de Barcelona

NPLQCD Collaboration
www.ub.edu/nplqcd

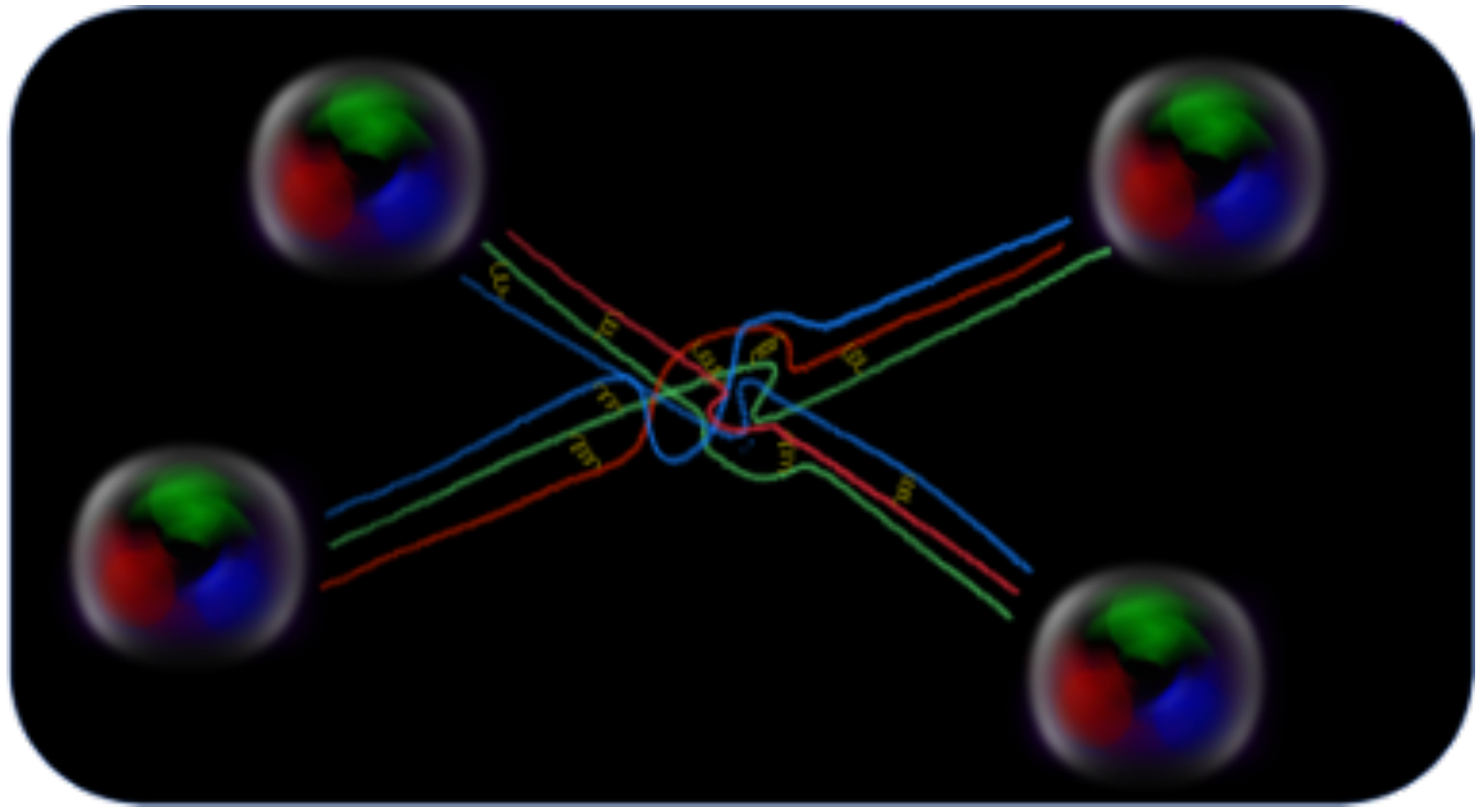
ECT* 2024
SPICE workshop:
Strange hadrons as a precision tool for strongly interacting systems
May 13-17, 2023

STRONG 2020 · UNIVERSITAT DE BARCELONA · ICCUB Institut de Ciències del Cosmos UNIVERSITAT DE BARCELONA · EXCELENCIA MARÍA DE MAEZTU

Solving QCD

$$\mathcal{L}_{QCD} = \bar{q}_{ij}\left(i\gamma^{u}\partial_{u} - m_{j}\right)q_{ij} + g(\bar{q}_{ij}\gamma^{u}\lambda_{a}q_{ij})F_{u}^{a} - \frac{1}{4}F_{uv}^{a}F_{a}^{uv}$$

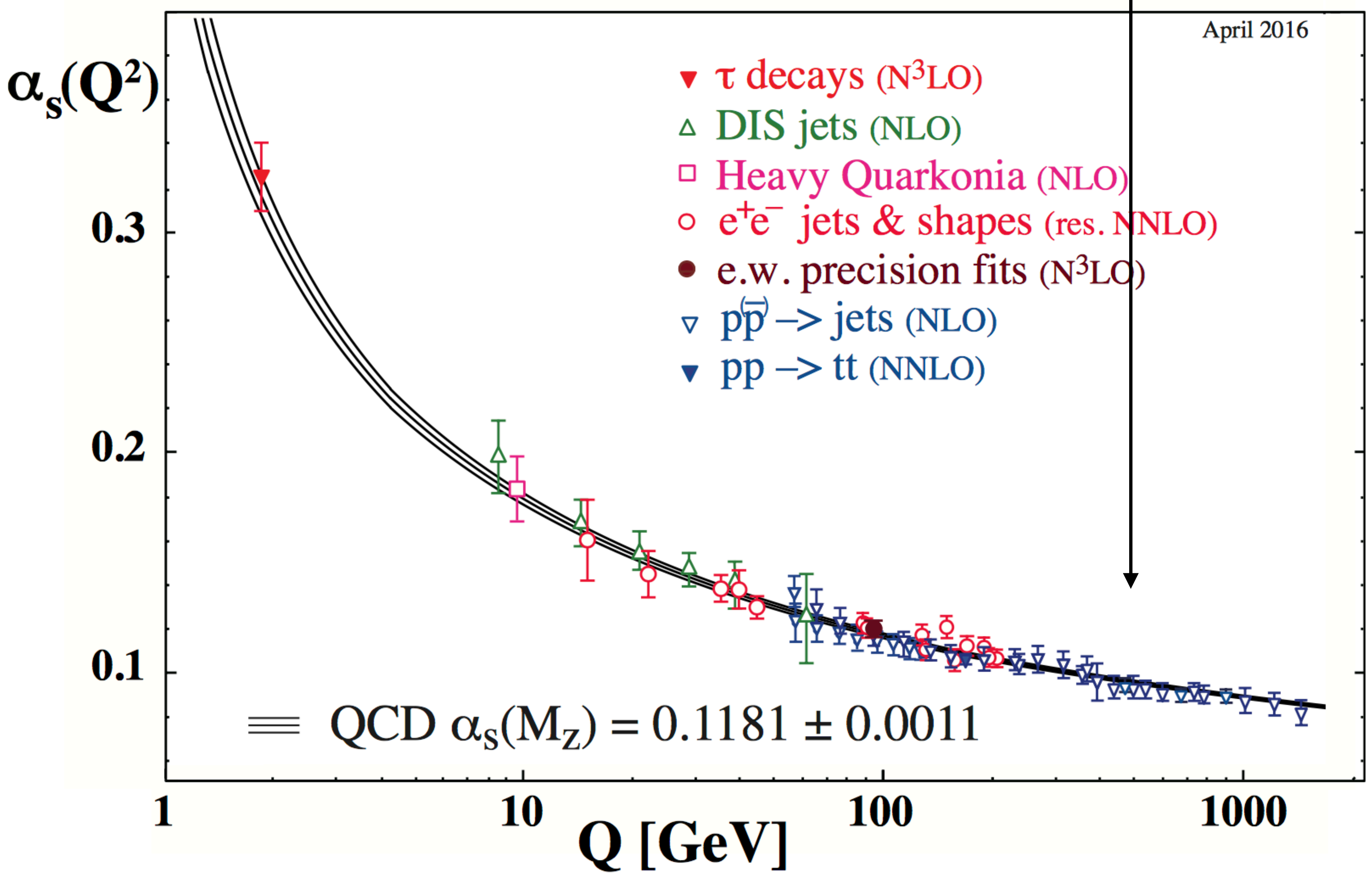
$$i = r,g,b \quad j = u,d,c,s,t,b$$



Nuclear physics, the non-perturbative regime of QCD

Perturbation theory applicable

strong coupling constant *vs* energy



S. Bethke, G.Dissertori, G.P. Salam
EPJ Web of Conferences 120 07005 (2016)

Solving QCD at low-energies. LQCD + EFT

$$\mathcal{L}_{QCD} = \bar{q}_{ij}\left(i\gamma^{u}\partial_{u} - m_{j}\right)q_{ij} + g(\bar{q}_{ij}\gamma^{u}\lambda_{a}q_{ij})F_{u}^{a} - \frac{1}{4}F_{uv}^{\;a}F_{\;a}^{uv}$$

$$i = r,g,b \quad j = u,d,c,s,t,b$$

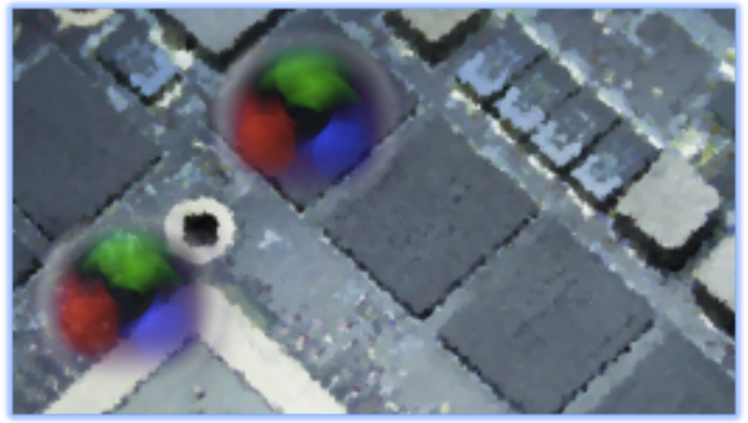
Nuclear physics, the non-perturbative regime of QCD

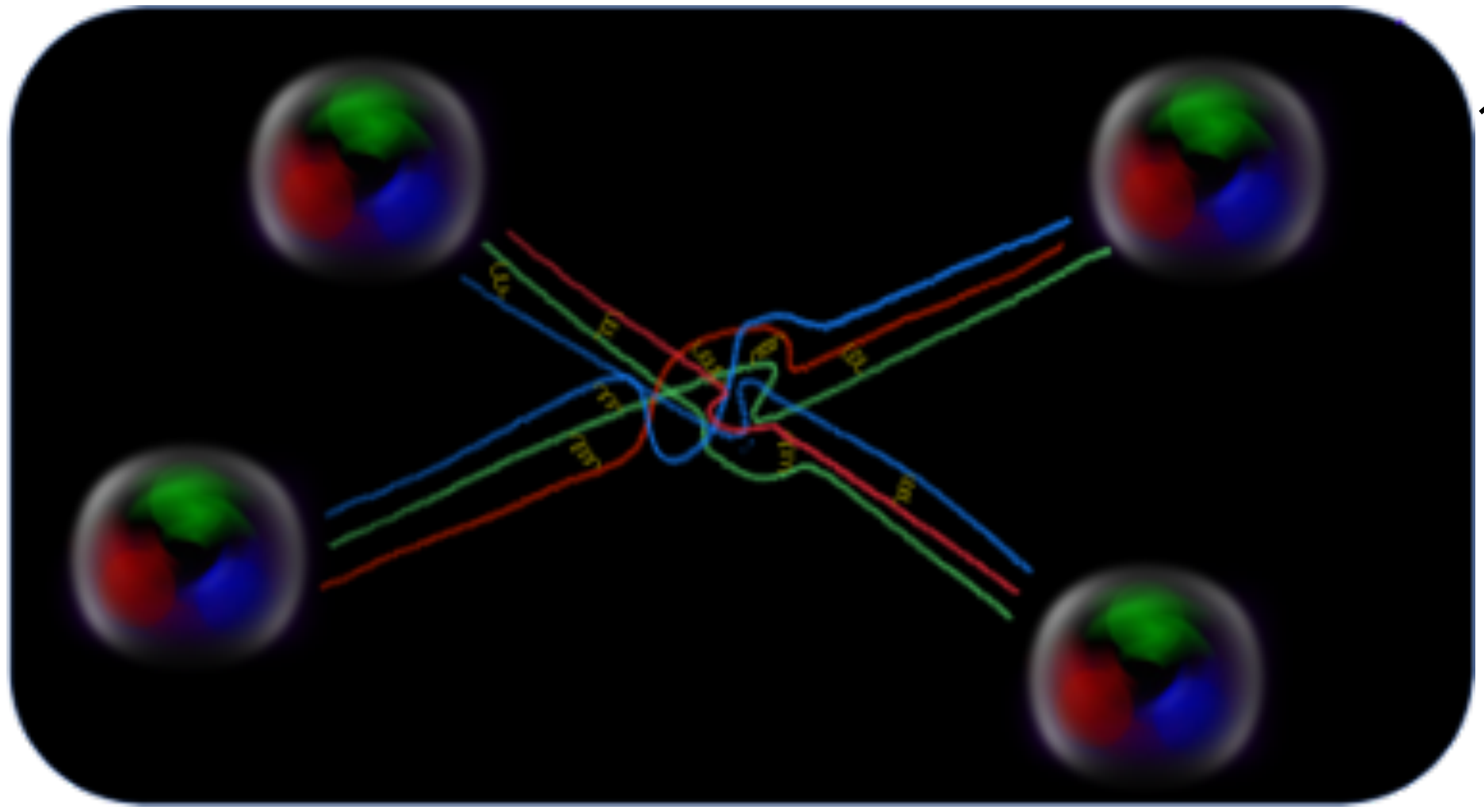
The quantum propagation is expressed as a weighted sum over paths

PATH INTEGRAL
Feynman, 1948

$$A = \int D(q)\ \exp\left(i\int_{i}^{f} dt\ L(q(t))\right)$$

go to **Euclidean space**
(numerical methods/important sampling)





Solving QCD at low-energies. LQCD + EFT

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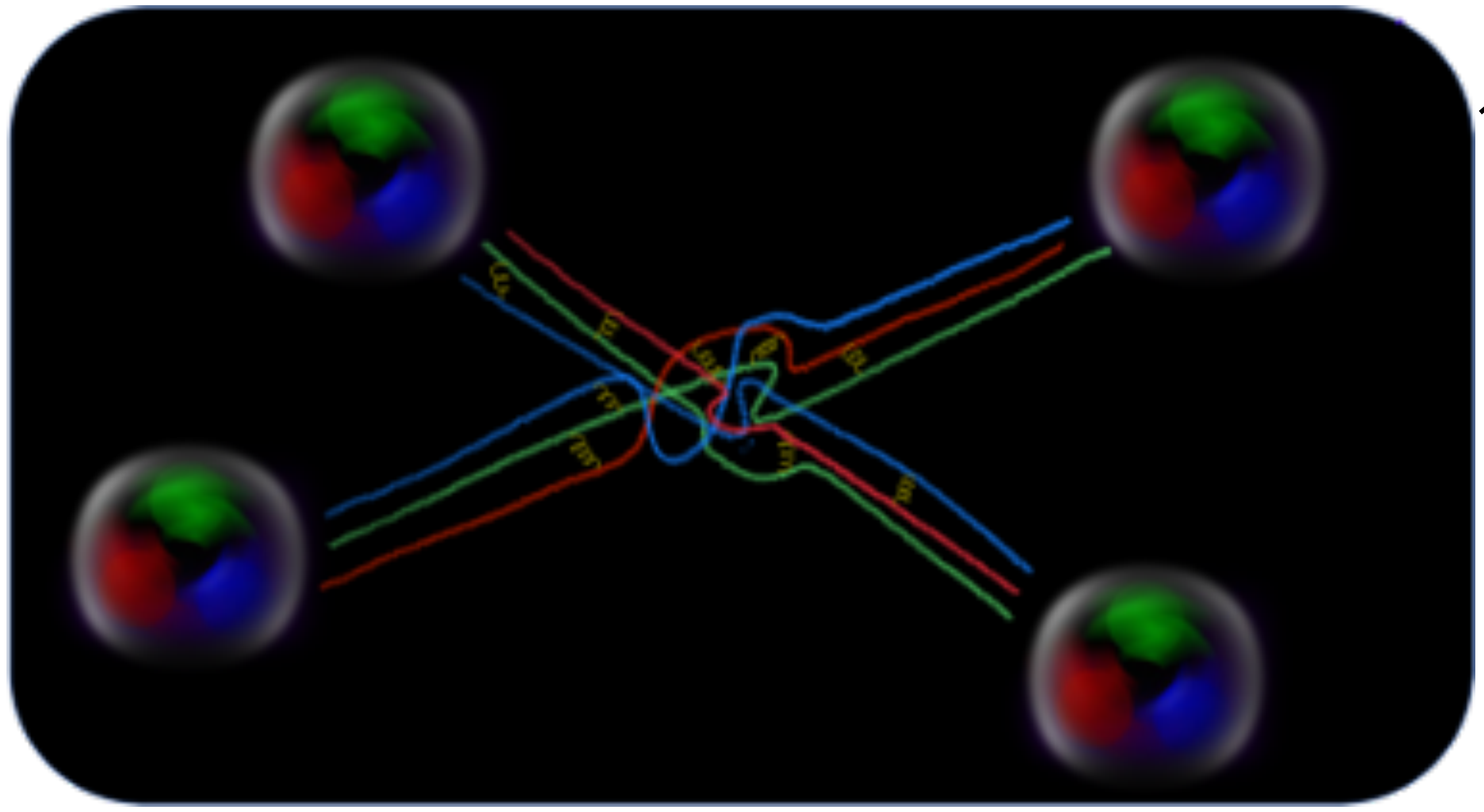
Nuclear physics, the non-perturbative regime of QCD



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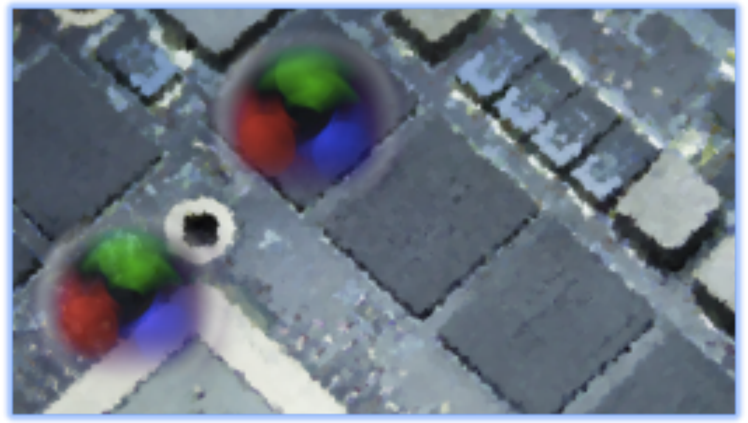
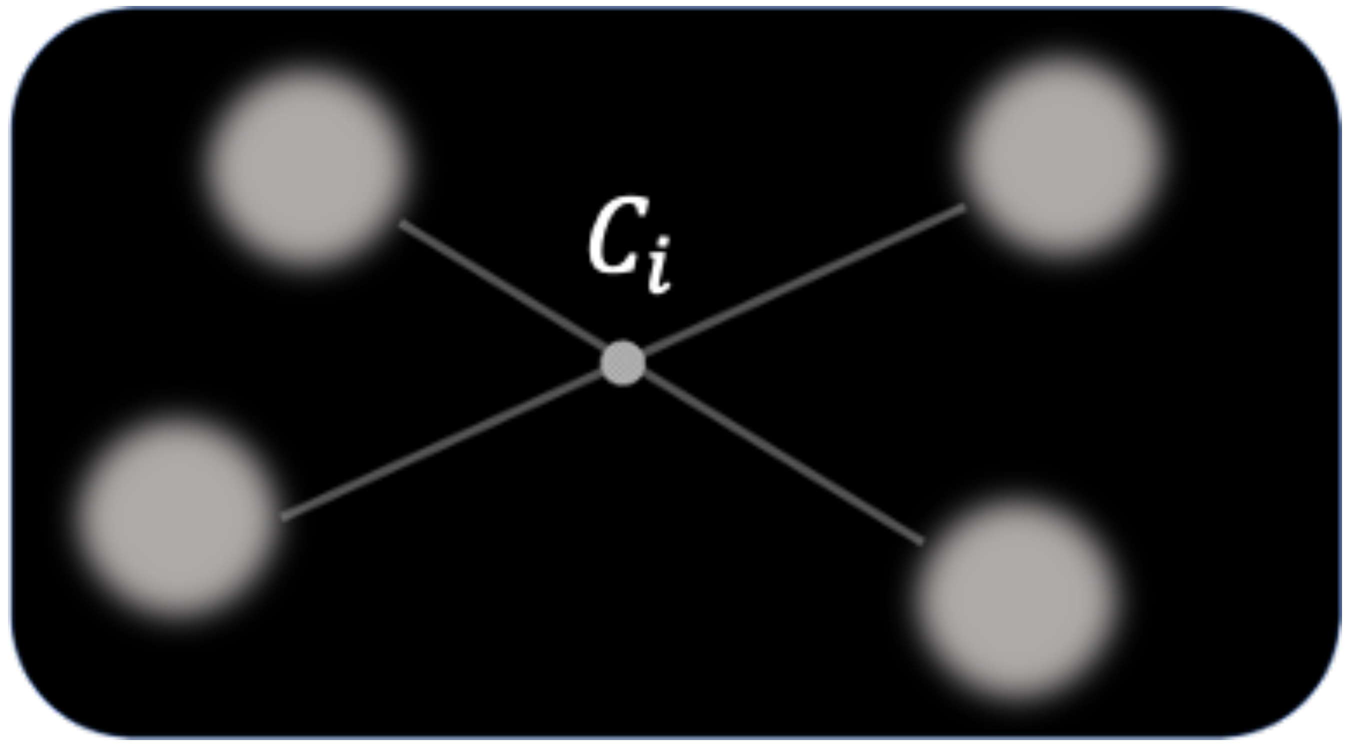




$$\mathcal{L}_{\rm EFT}\,[\pi, N, \ldots; m_\pi, m_N, \ldots; C_i]$$

LECs

Solving QCD at low-energies. LQCD + EFT

$$\mathcal{L}_{QCD} = \bar{q}_{ij}\left(i\gamma^{u}\partial_{u} - m_{j}\right)q_{ij} + g(\bar{q}_{ij}\gamma^{u}\lambda_{a}q_{ij})F_{u}^{a} - \frac{1}{4}F^{a}_{uv}F^{uv}_{a}$$

$$i = r,g,b \quad j = u,d,c,s,t,b$$

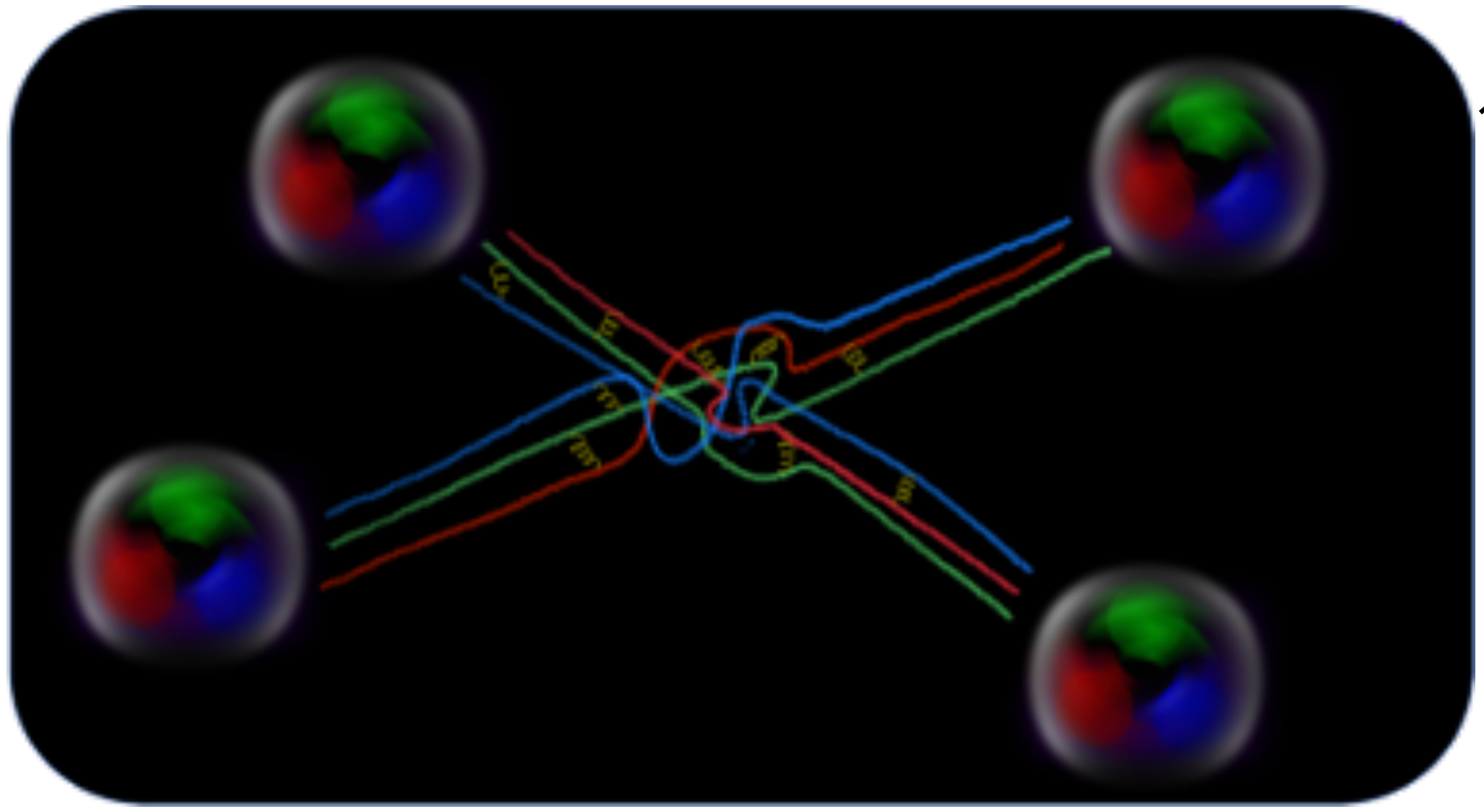
Nuclear physics, the non-perturbative regime of QCD

Lattice QCD calculations allow for:

Connection to QCD

Systematically improve the calculation

Control the uncertainties



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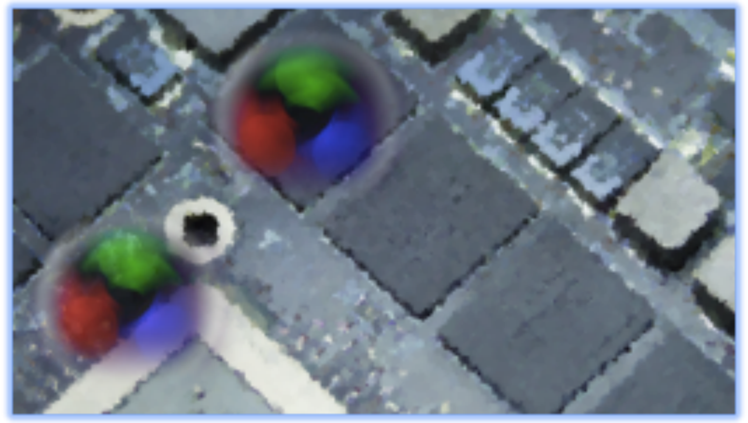

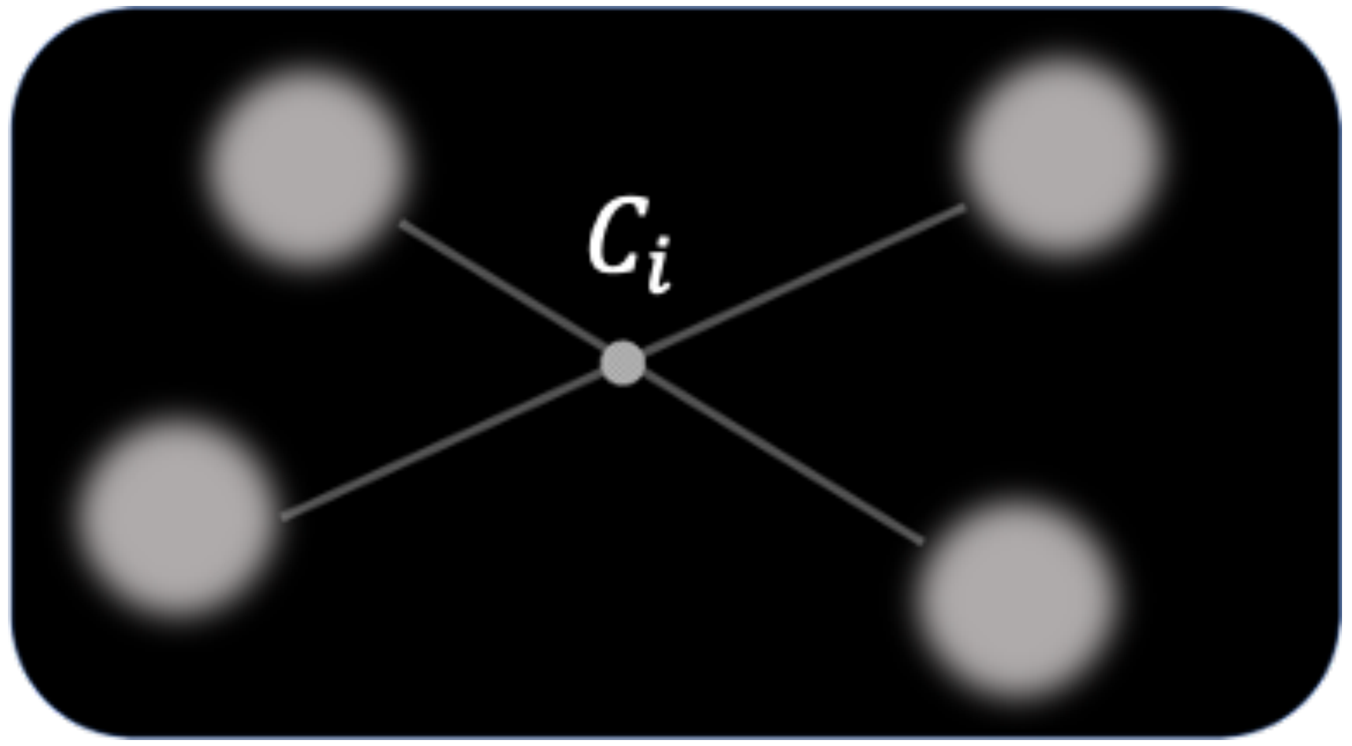

$$\mathcal{L}_{\mathrm{EFT}}\left[\pi, N, \dots; m_{\pi}, m_{N}, \dots; C_{i}\right]$$

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$$\mathcal{L}_{QCD} = \bar{q}_{ij}\left(i\gamma^{u}\partial_{u} - m_{j}\right)q_{ij} + g(\bar{q}_{ij}\gamma^{u}\lambda_{a}q_{ij})F_{u}^{a} - \frac{1}{4}F^{a}_{uv}F^{uv}_{a}$$

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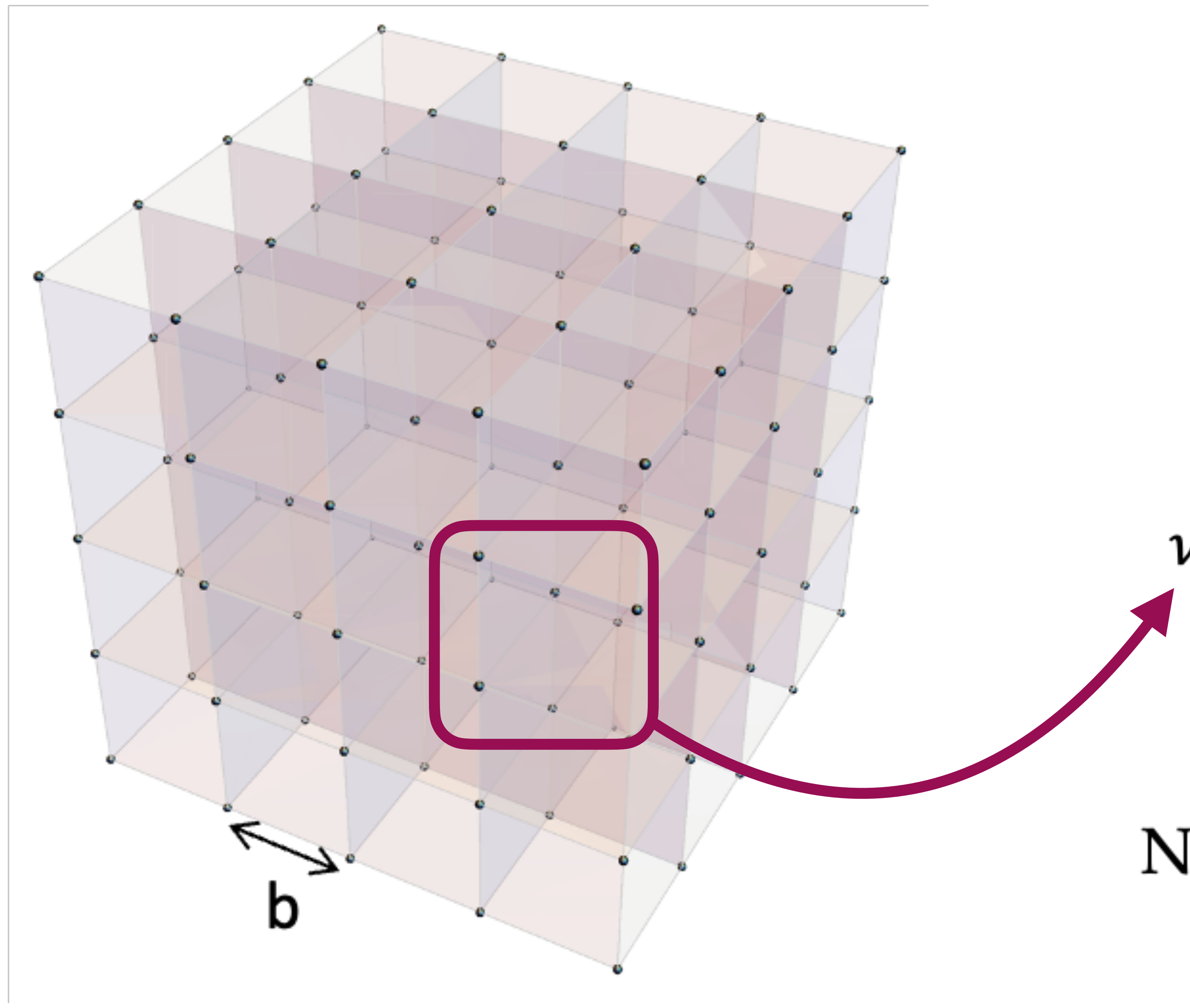

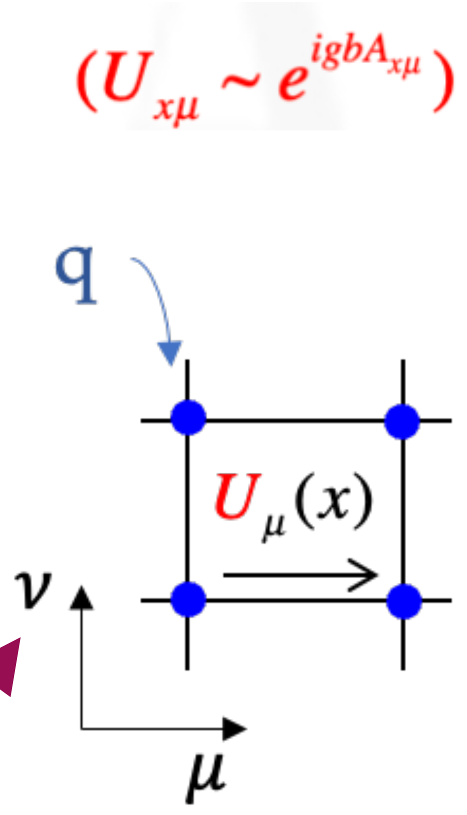

$L_x \times L_y \times L_z \times T$

$L >> m_\pi^{-1}$ Finite volume

$b << \Lambda_{QCD}^{-1}$ discretize spacetime

The quantum propagation is expressed as a weighted sum over paths

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Feynman, 1948

$$A = \int D(q)\, \exp\left(i\int_i^f dt\, L(q(t))\right)$$

go to **Euclidean space**
(numerical methods/important sampling)

Nonperturbative (numerical) solution

expectation values

$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}q\, \mathcal{D}\bar{q}\, \mathcal{D}A_\mu\, \hat{\mathcal{O}}[q, \bar{q}, A]\, e^{iS_{QCD}}$$

$t \to i\tau$

go to **Euclidean space**
numerical methods/important sampling

Solving QCD at low-energies. LQCD + EFT

$$\mathcal{L}_{QCD} = \bar{q}_{ij}\left(i\gamma^u \partial_u - m_j\right) q_{ij} + g(\bar{q}_{ij}\gamma^u \lambda_a q_{ij}) F_u^a - \frac{1}{4} F^{\ a}_{uv} F^{\ uv}_{a}$$

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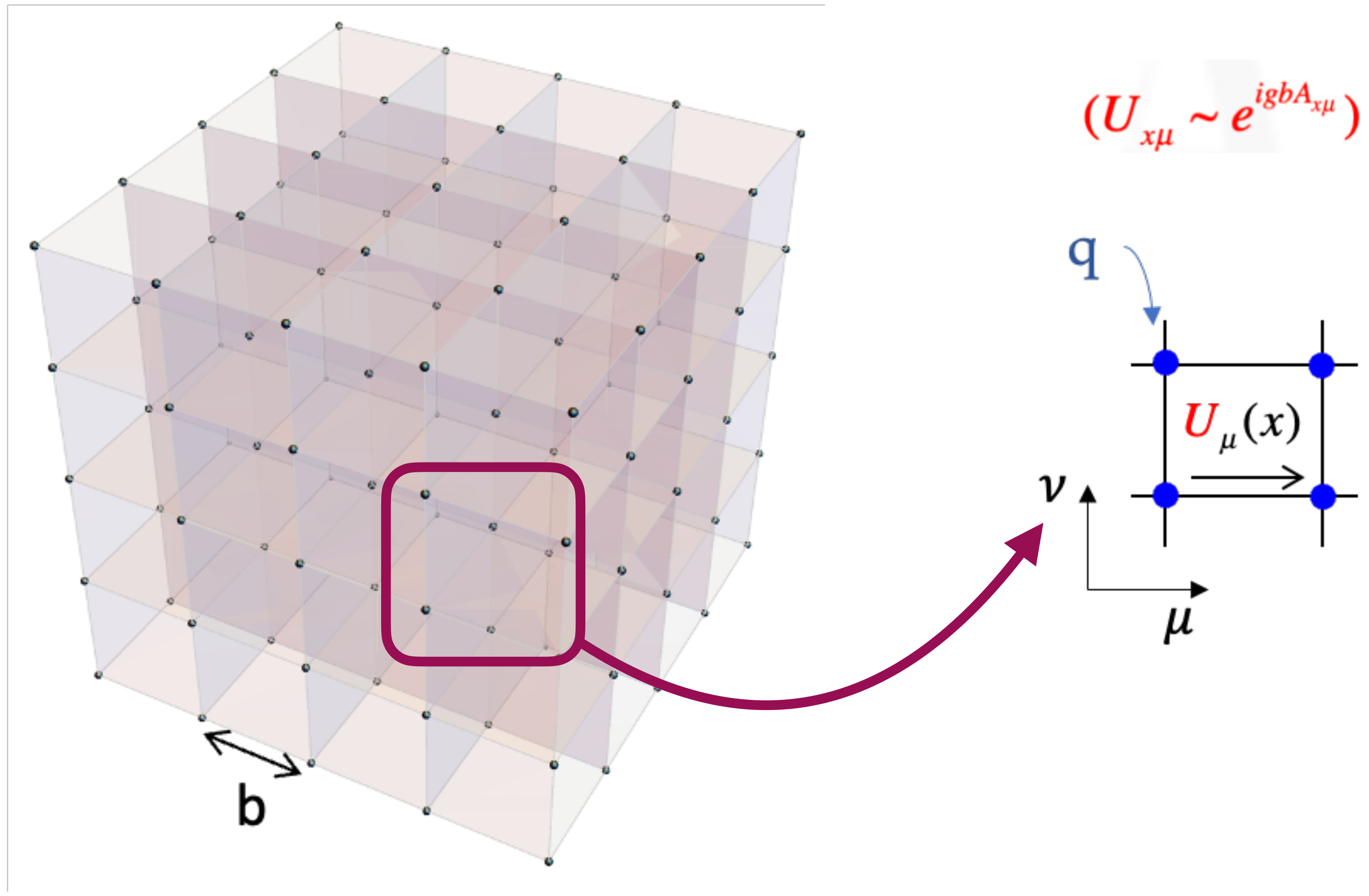

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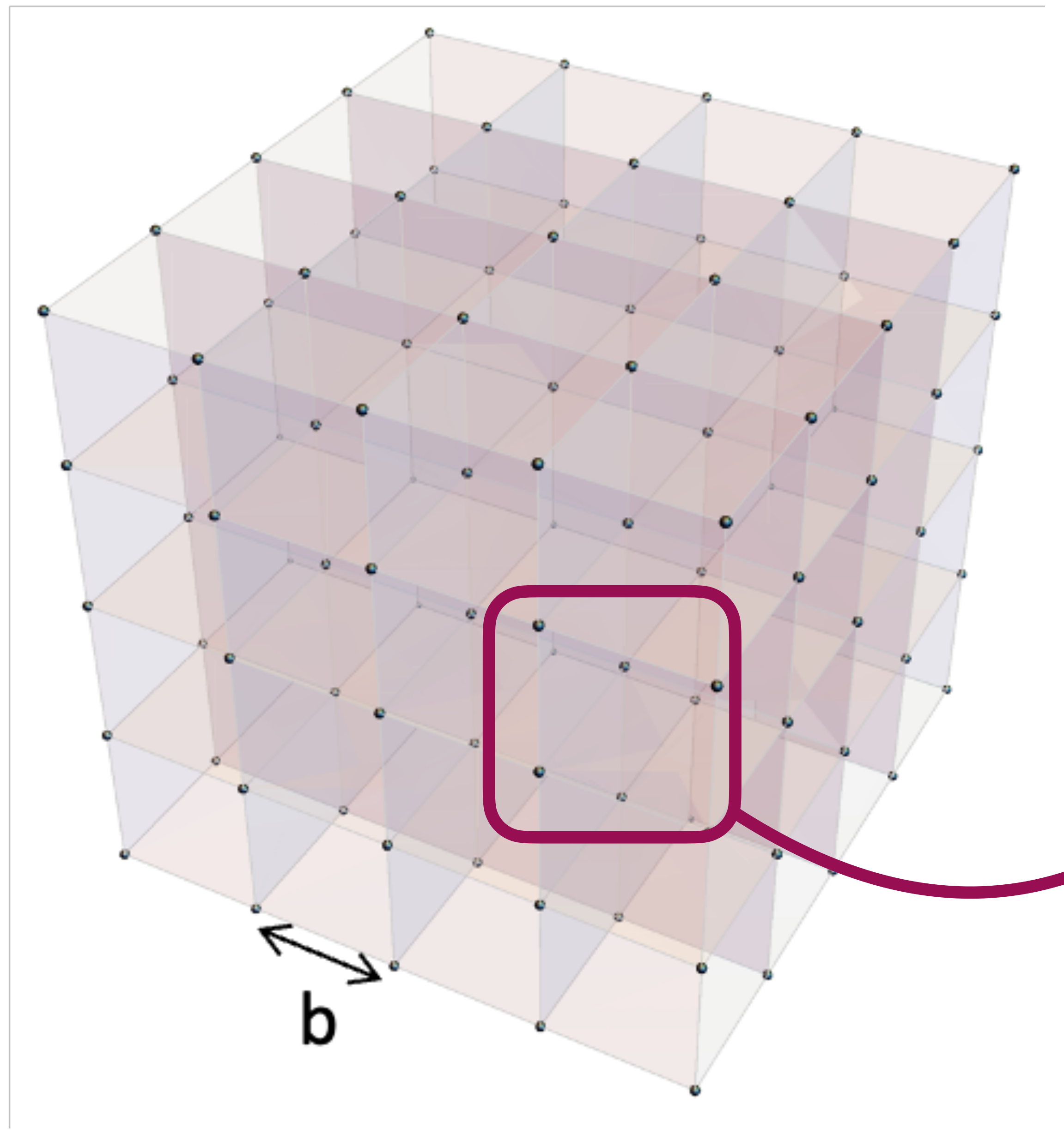

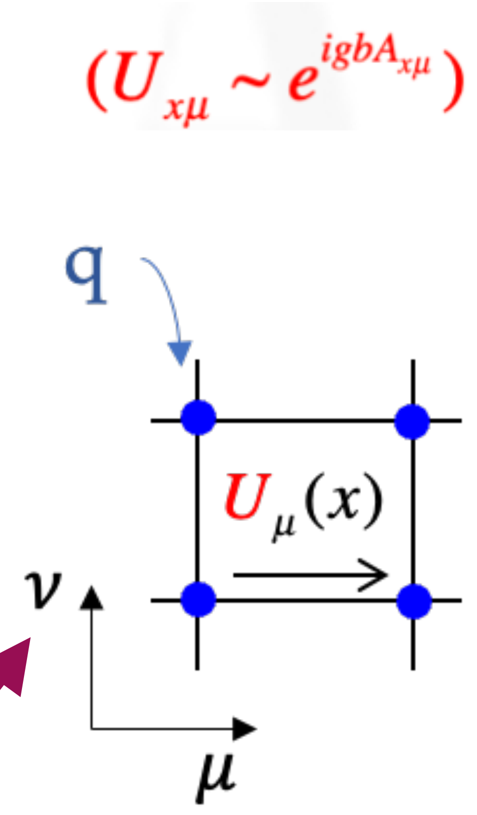

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$t \to i\tau$

go to **Euclidean space**
numerical methods/important sampling

$$\langle\hat{O}\rangle = \frac{1}{Z}\int DU\ D\bar{\psi}\ D\psi\,\hat{O}[\psi,\bar{\psi},U]\,e^{-\bar{\psi}Q(U)\psi - S_g[U]}$$

$$\langle\hat{O}\rangle = \frac{1}{Z}\int DU\ \hat{O}[Q(U)^{-1}]\det(Q(U))e^{-S_g[U]}$$

propagators — configurations $(\sim P(U))$

$$\langle\hat{\mathcal{O}}\rangle \approx \frac{1}{N_{\rm cfg}}\sum_{n=1}^{N_{\rm cfg}}\hat{\mathcal{O}}(\{U\}_n)$$

Algorithm

$\{U^{[i]}\}$, (Markov process)

each configuration is created by the preceding one :

$$\mathrm{P}\left(U^{[i-1]} \to U^{[i]}\right) P\left(U^{[i-1]}\right) = \mathrm{P}\left(U^{[i]} \to U^{[i-1]}\right) P\left(U^{[i]}\right)$$

Basic Monte Carlo algorithm

$$\langle \hat{O} \rangle = \frac{1}{Z} \int DU \; \hat{O}\,[Q(U)^{-1}]\,\det(Q(U))\,e^{-S_g[U]}$$

$\underbrace{\hat{O}\,[Q(U)^{-1}]}_{\text{propagators}}$ $\underbrace{\det(Q(U))\,e^{-S_g[U]}}_{\text{configurations}}$ (~ P (U))

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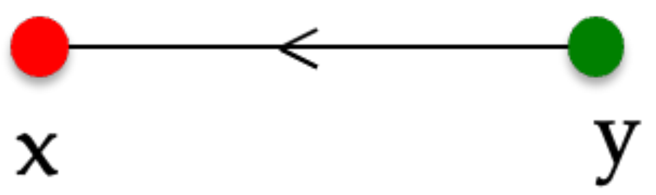
Basic Monte Carlo algorithm

For each gauge-field configuration, calculate the quark propagator
(inverse of the fermion matrix)



$$Q_u^{-1}(x,y) = u(y)\bar{u}(x)$$

$$\left\langle \hat{O} \right\rangle = \frac{1}{Z} \int DU \; \underbrace{\hat{O}\,[Q(U)^{-1}]}_{\text{propagators}} \; \underbrace{\det(Q(U))\, e^{-S_g[U]}}_{\text{configurations} \;\; (\sim \mathrm{P}\,(\mathrm{U}))}$$

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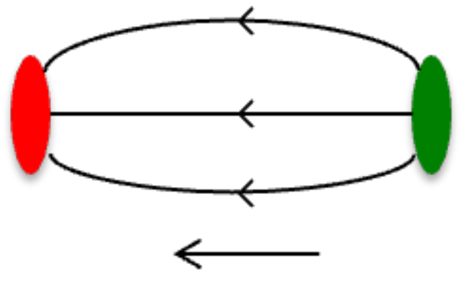
Basic Monte Carlo algorithm

In order to study hadrons, we need to contract propagators onto correlation functions $C_i(t)$

$$C(\Gamma^{\nu}, \vec{p}, t) = \sum_{\vec{x}_1} e^{-i\vec{p}\vec{x}_1}\, \Gamma^{\nu} \left\langle J(\vec{x}_1, t)\bar{J}(\vec{x}_0, 0)\right\rangle$$



for ex. $\bar{u}(x_1)\gamma_5 d(x_1)$ $\bar{d}(x_0)\gamma_5 u(x_0)$

$$\pi^{+} = \bar{d}\gamma_5 u$$

$$\langle \pi^{\dagger}(x_1)\, \pi(x_0)\rangle = \langle \overline{u}(x_1)\, \gamma_5\, d(x_1)\, \overline{d}(x_0)\, \gamma_5\, u(x_0)\rangle$$

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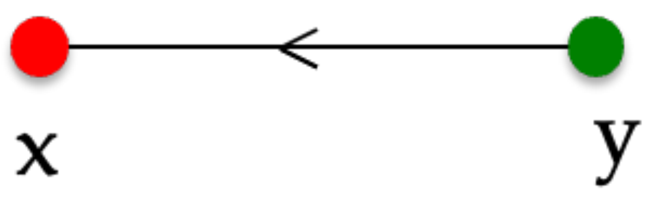

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$$\langle \hat{O}\rangle = \frac{1}{Z}\int DU\ \hat{O}\,[Q(U)^{-1}]\,\det(Q(U))\,e^{-S_g[U]}$$

propagators configurations (~ P (U))

Algorithm

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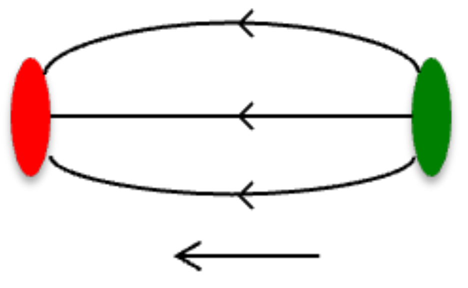

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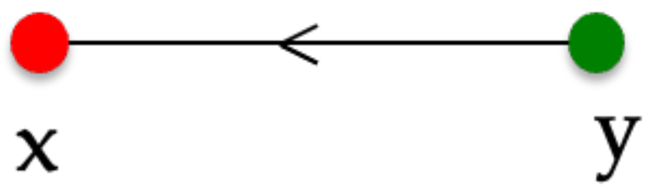

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$$\langle \hat{O} \rangle = \frac{1}{Z} \int DU\ \hat{O}\,[Q(U)^{-1}]\, \det(Q(U))\, e^{-S_g[U]}$$

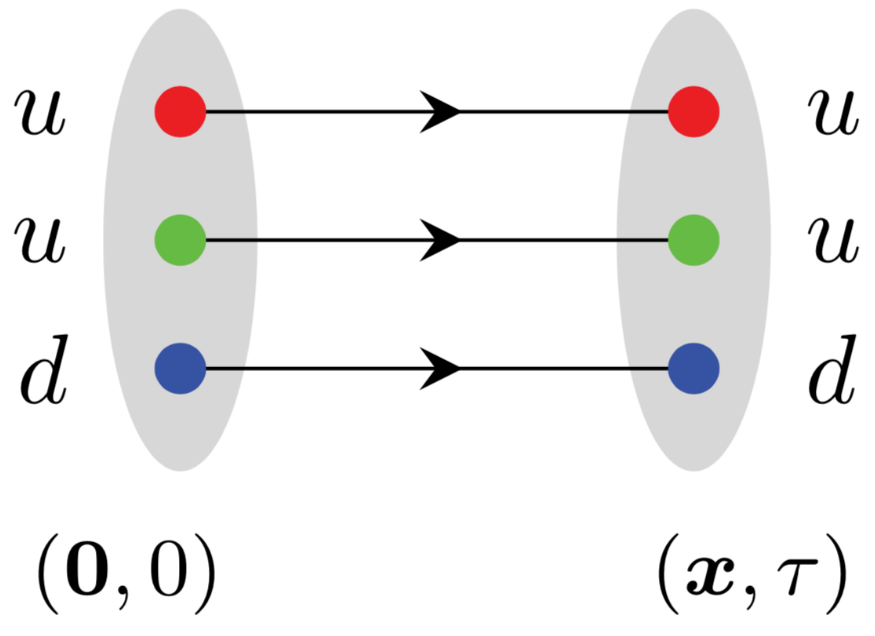
propagators configurations (~ P (U))

REQUIRE MULTIPLE VOLUMES AND LATTICE SPACINGS
to recover the continuum infinite limit

$$\langle \hat{O} \rangle \approx \frac{1}{N_{\mathrm{cfg}}} \sum_{n=1}^{N_{\mathrm{cfg}}} \hat{O}(\{U\}_n)$$

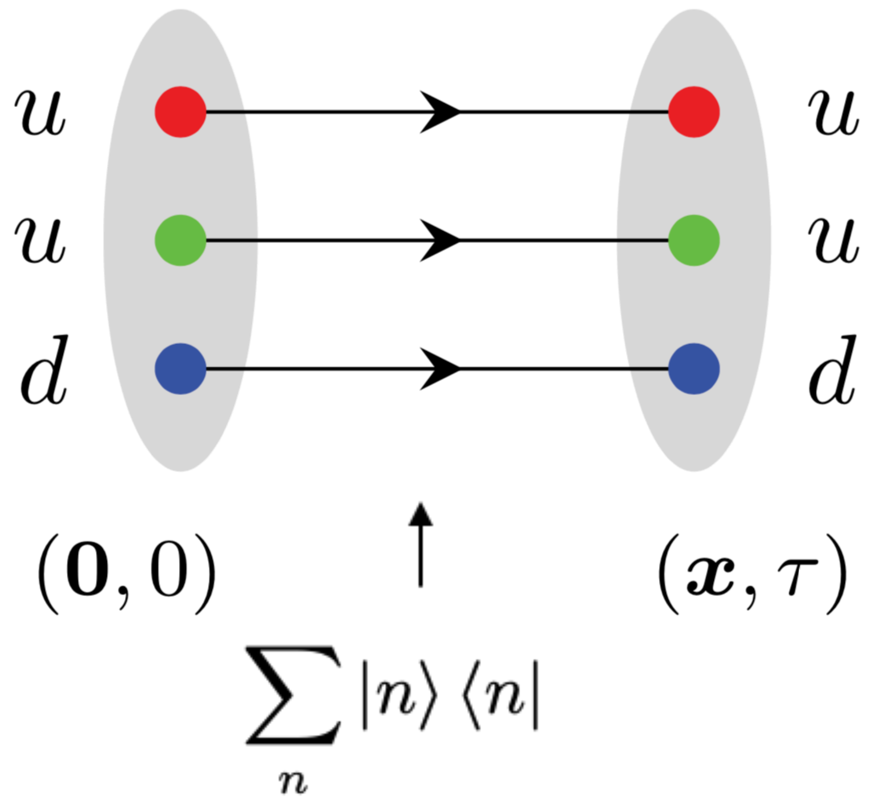
Energy levels

$$C_{2pt}(\tau, \boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{-\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{p}}\, \Gamma_{\beta\alpha} \langle \mathcal{X}_\alpha(\boldsymbol{x}, \tau) \bar{\mathcal{X}}_\beta(\mathbf{0}, 0) \rangle$$



LQCD DIRECT METHOD:
FV Energy levels from two-point correlation functions

Energy levels



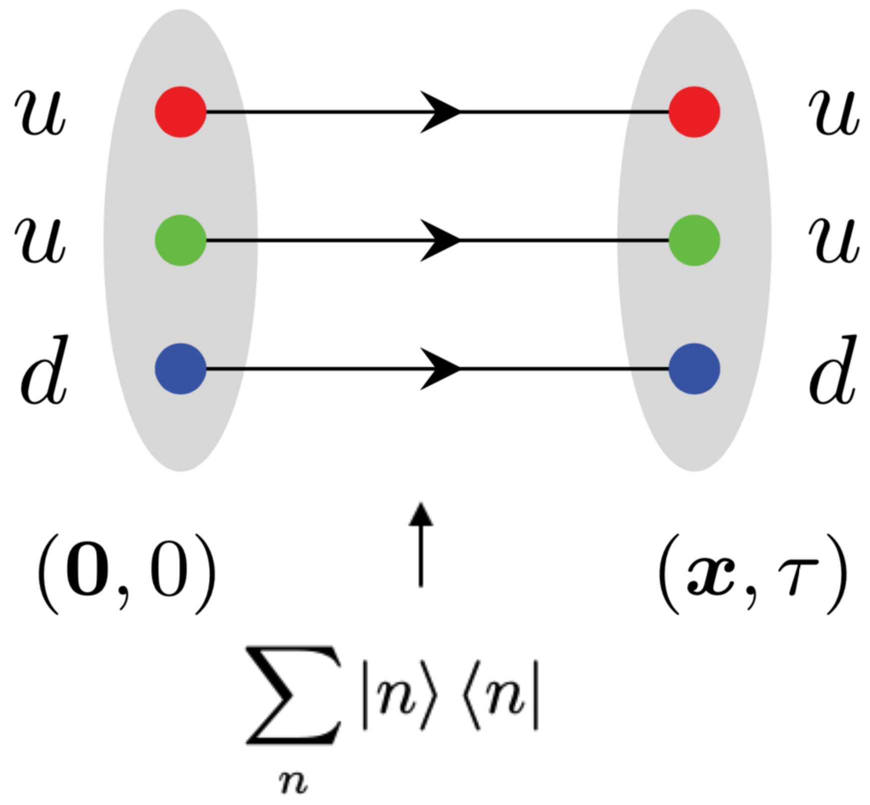
$$C_{2pt}(\tau,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{-\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{p}}\,\Gamma_{\beta\alpha}\langle\mathcal{X}_\alpha(\boldsymbol{x},\tau)\bar{\mathcal{X}}_\beta(\mathbf{0},0)\rangle$$

$$= Z_0^{snk}\,Z_0^{\dagger src}\,e^{-E^{(0)}t} + Z_1^{snk}\,Z_1^{\dagger src}\,e^{-E^{(1)}t} + \ldots$$

Tower of energy eigenstates
of the system
in the finite volume

LQCD DIRECT METHOD:
FV Energy levels from two-point correlation functions

Energy levels



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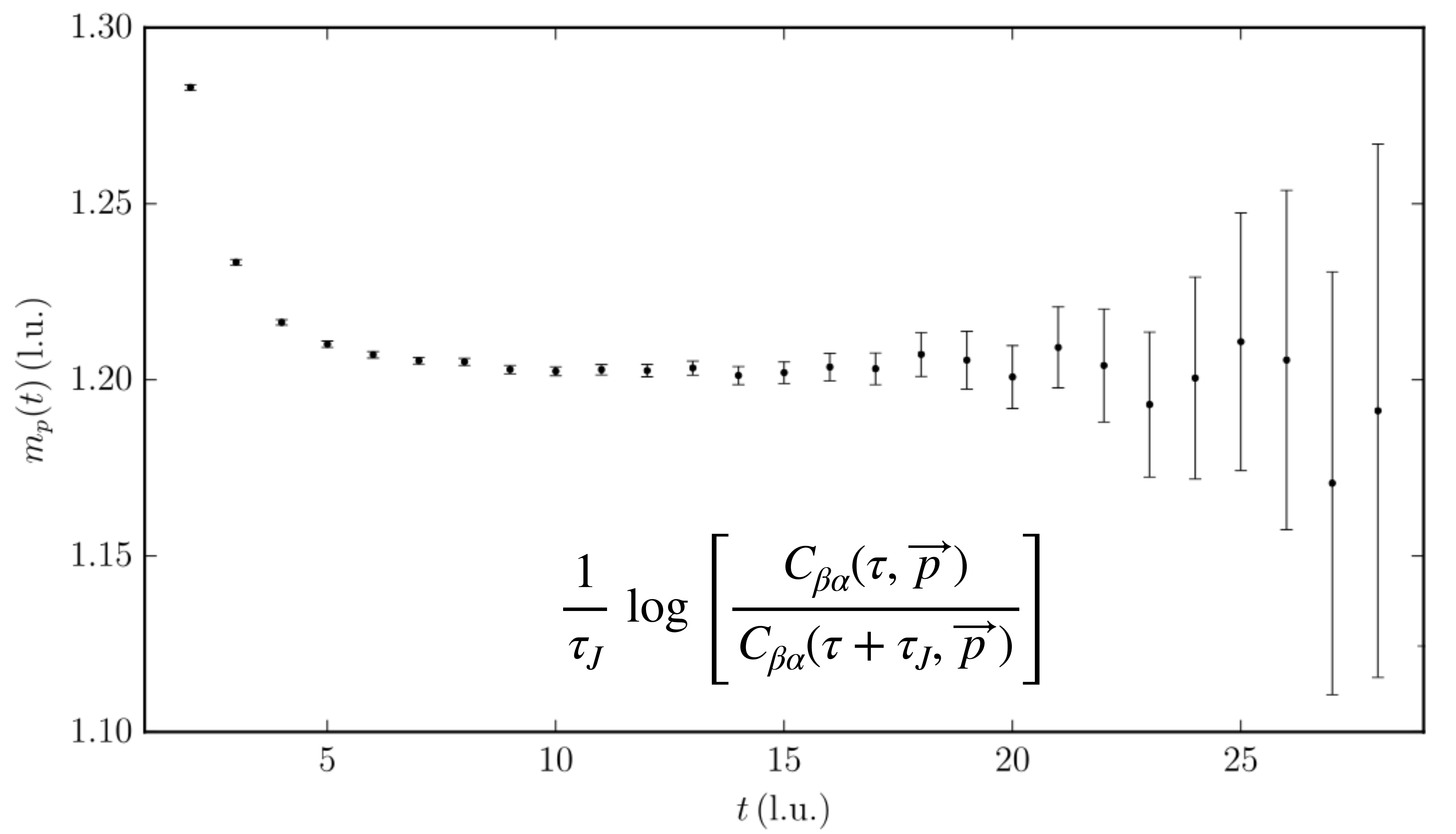
$$= \underbrace{Z_0^{snk} Z_0^{\dagger src} e^{-E^{(0)}t}}_{\text{dominates at large t}} + Z_1^{snk} Z_1^{\dagger src} e^{-E^{(1)}t} + \dots$$

Tower of energy eigenstates
of the system
in the finite volume

E_n

$$p_\alpha(\mathbf{x}, t) = \epsilon^{ijk} u^i_\alpha(\mathbf{x}, t) \left(u^{j\mathsf{T}}(\mathbf{x}, t) C \gamma_5 d^k(\mathbf{x}, t) \right)$$



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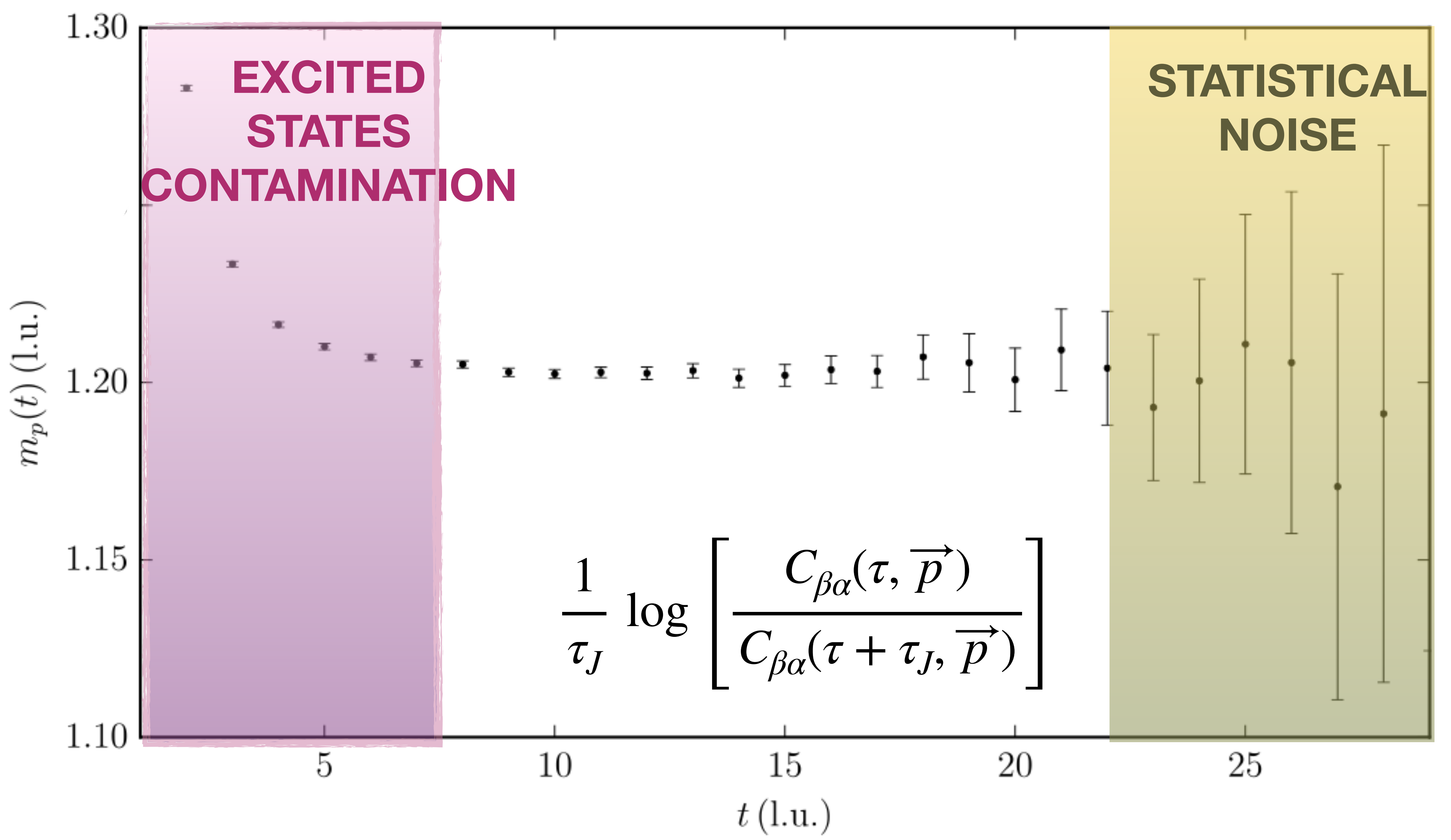
$$= Z_0^{snk} Z_0^{\dagger src} e^{-E^{(0)}t} + Z_1^{snk} Z_1^{\dagger src} e^{-E^{(1)}t} + \ldots$$

dominates at large t

signal-to-noise degradation

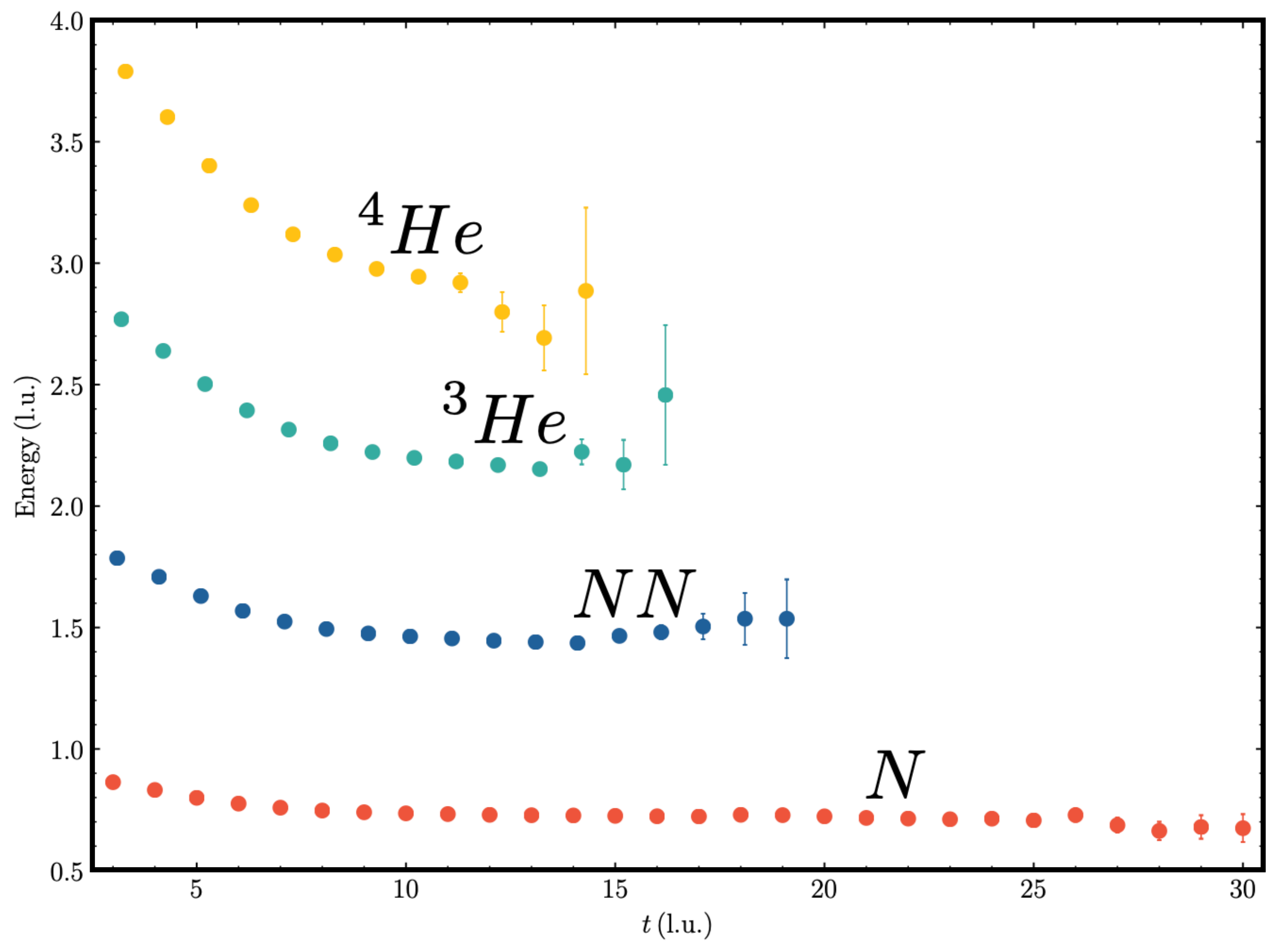
$$p_\alpha(\mathbf{x}, t) = \epsilon^{ijk} u^i_\alpha(\mathbf{x}, t) \left(u^{j\mathsf{T}}(\mathbf{x}, t) C\gamma_5 d^k(\mathbf{x}, t) \right)$$



Expectation is that for A nucleons:

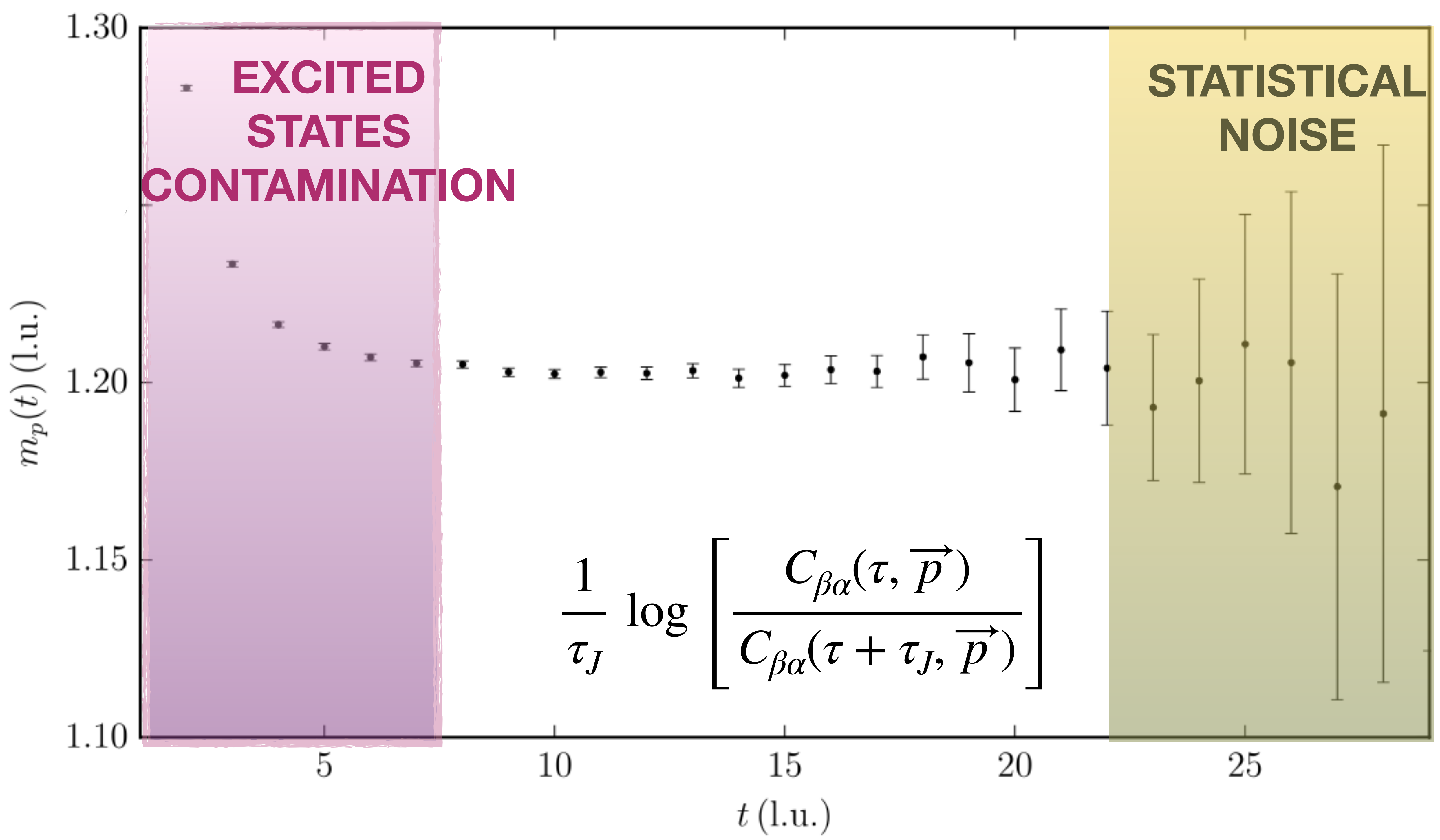
$$\frac{\sigma}{\langle C \rangle} \sim \frac{\exp\left[A\left(M_N - \frac{3m_\pi}{2}\right)t\right]}{\sqrt{N}}$$

G. Parisi, Phys.Rept. 103 (1984)
G.P. Lepage, Boulder TASI (1989)
M.L. Wagman, M.J. Savage, Phys.Rev.D 96 (2017)



signal-to-noise degradation

$$p_\alpha(\mathbf{x},t) = \epsilon^{ijk} u^i_\alpha(\mathbf{x},t) \left(u^{j\top}(\mathbf{x},t) C\gamma_5 d^k(\mathbf{x},t)\right)$$



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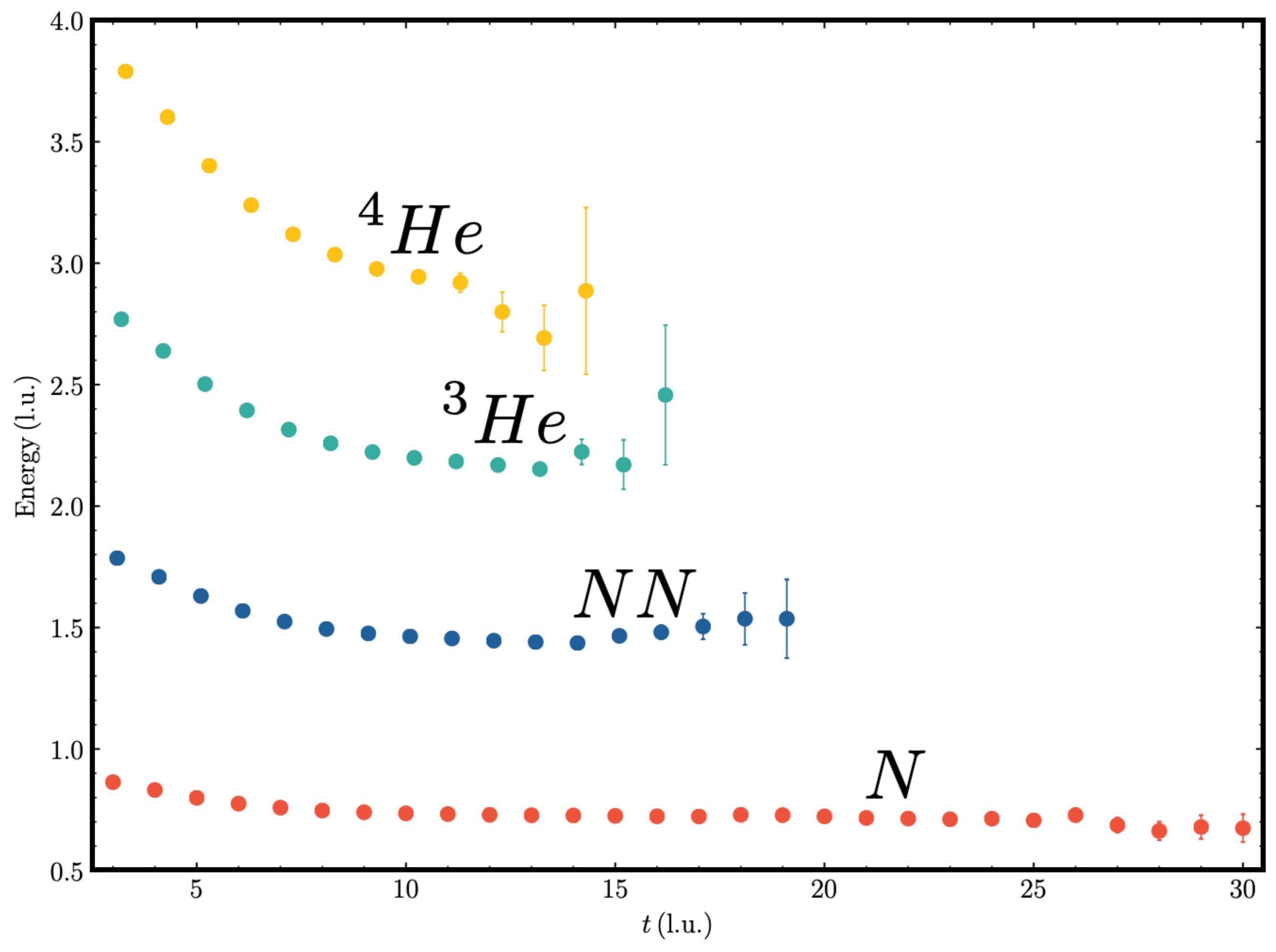
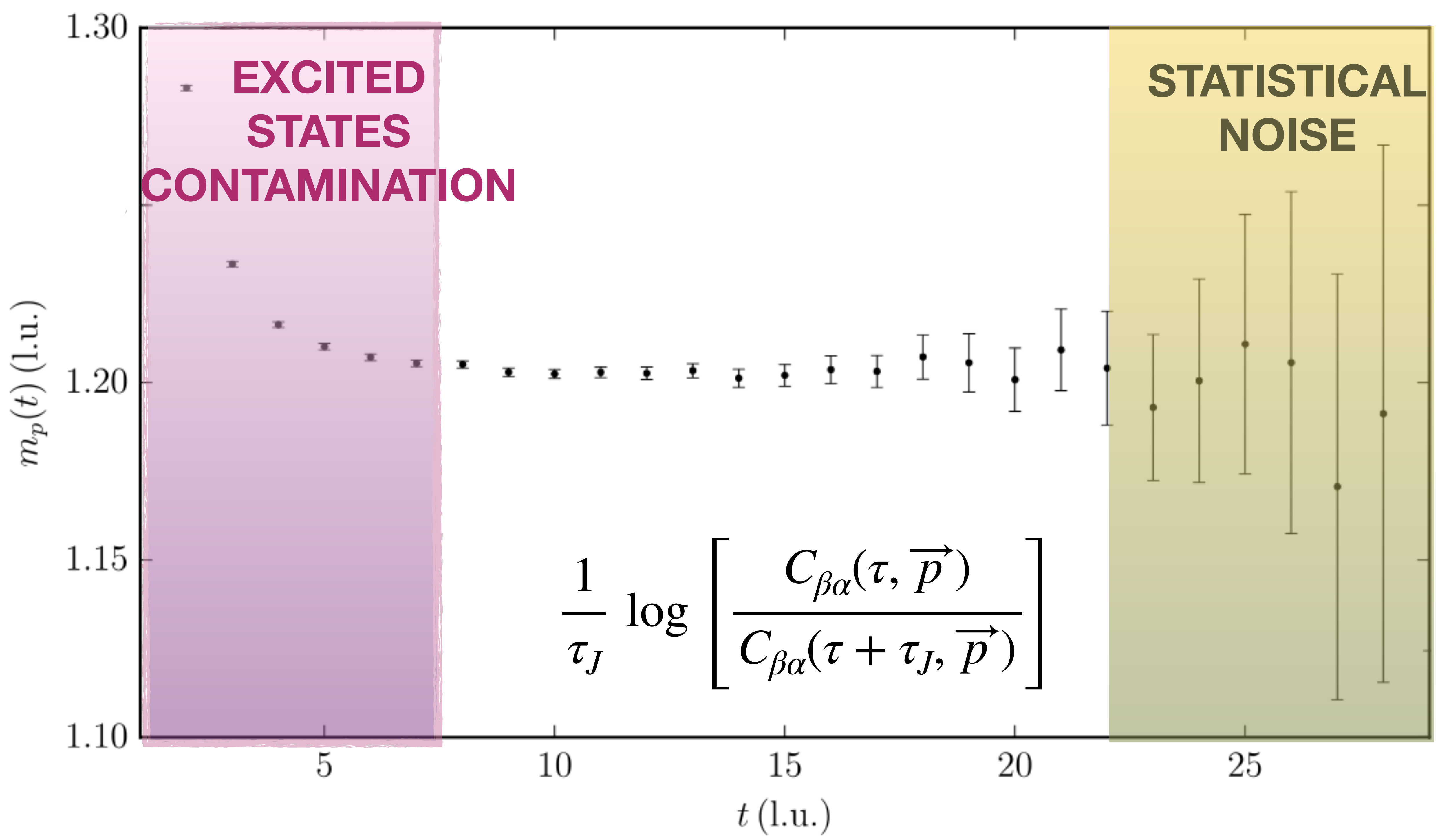


Increase the statistics / Increase the pion mass

Construct operators with a better overlap with the ground state

signal-to-noise degradation

$$p_\alpha(\mathbf{x},t) = \epsilon^{ijk} u^i_\alpha(\mathbf{x},t) \left(u^{j\mathsf{T}}(\mathbf{x},t) C\gamma_5 d^k(\mathbf{x},t)\right)$$

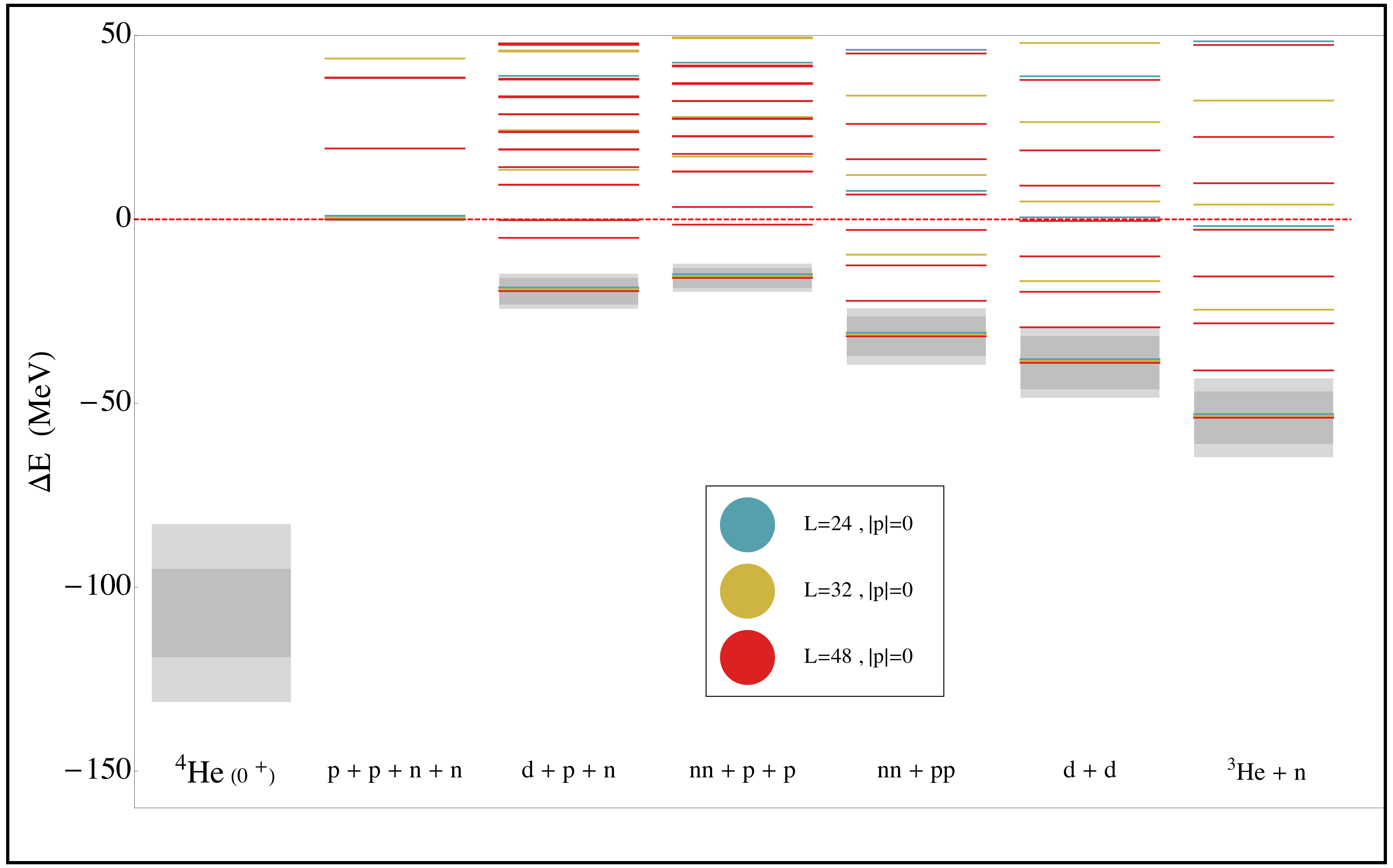


LQCD DIRECT METHOD:
FV Energy levels from two-point correlation functions

"*Challenges*" with LQCD studies of nuclear systems

Small excited-state gaps may lead to incorrect identification of ground-state energy

$$\Delta E = E_1 - E_0 \sim \frac{1}{L^2}$$



S.R. Beane et al. [NPLQCD], Phys.Rev.D 87 (2013)

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Construct operators with a better overlap with the ground state

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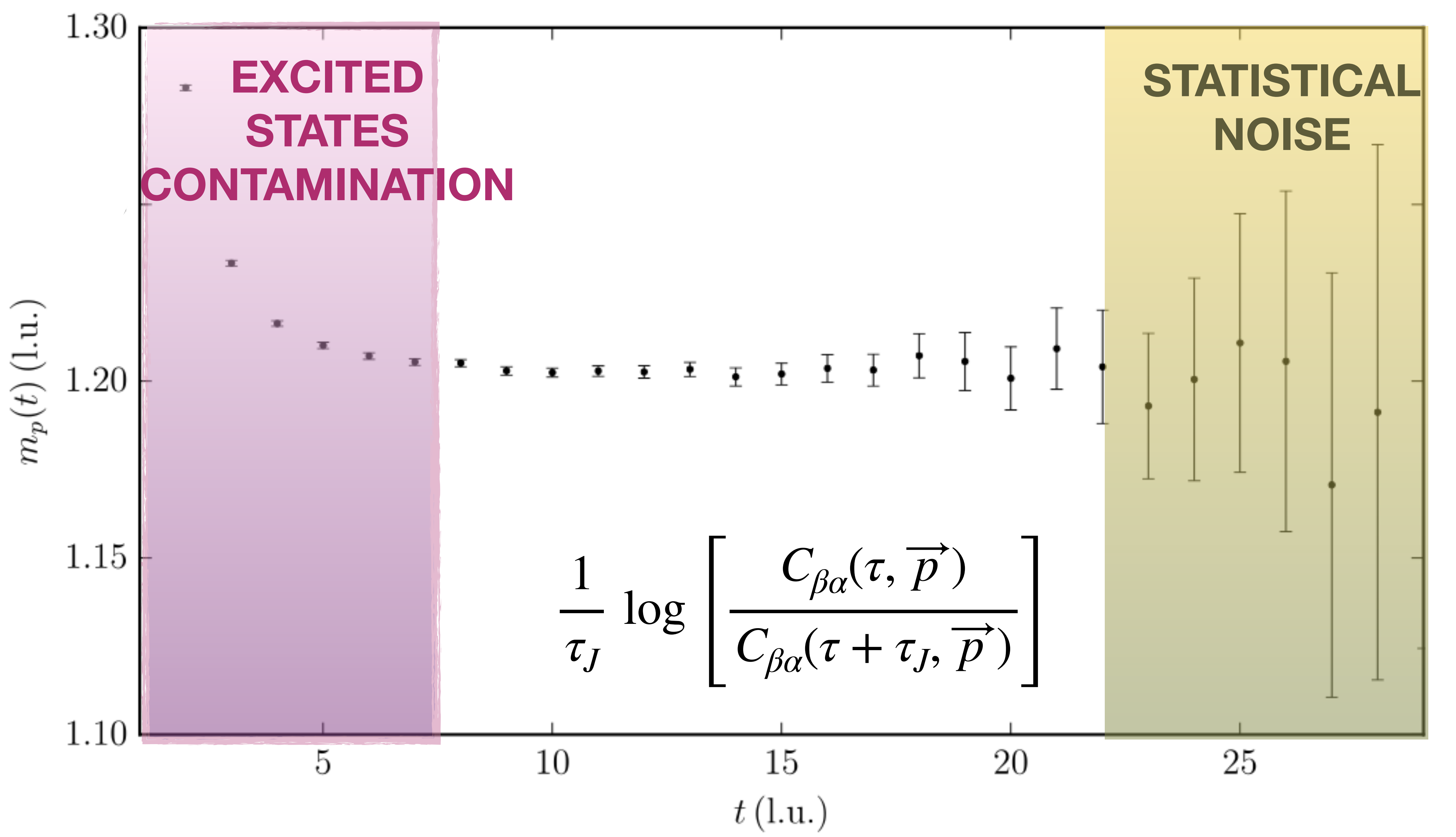
$$p_\alpha(\mathbf{x},t) = \epsilon^{ijk} u^i_\alpha(\mathbf{x},t)\left(u^{j\mathsf{T}}(\mathbf{x},t) C\gamma_5 d^k(\mathbf{x},t)\right)$$



Direct method

Misidentification of the plateau

E. Berkowitz et al. [CalLat], Phys.Lett.B 765 (2017)
S.R. Beane et al. [NPLQCD], arXiv:1705.09239 [hep-lat]
T. Yamazaki et al. [PACS], EPJ Web Conf. 175 (2018)

$$R(\tau, \boldsymbol{p}) = \frac{C_{B_1 B_2}(\tau, \boldsymbol{p})}{C_{B_1}(\tau, \boldsymbol{p}) C_{B_2}(\tau, \boldsymbol{p})}$$

$\downarrow$ Operator dependence?

$$\Delta E_{\mathfrak{n}}$$

$\downarrow$ Lüscher's method

$$k^* \cot\delta = \frac{2}{\sqrt{\pi}L}\mathcal{Z}_{00}(1; (\tfrac{k^* L}{2\pi})^2)$$

$$k^* \cot\delta$$
$$B$$

pre-variational → bound
variational → ???

Potential method

Only applicable at the energy of the calculation/system

Test convergence expansion

Ground-state saturation requirement

Short-distance operator dependence

T. Iritani et al. [HAL QCD], Phys.Rev.D 99 (2019)

$$R(\tau, \boldsymbol{r}) = \frac{C_{B_1 B_2}(\tau, \boldsymbol{r})}{C_{B_1}(\tau, \boldsymbol{r}) C_{B_2}(\tau, \boldsymbol{r})}$$

$\downarrow$

$$\left(\frac{\partial_\tau^2}{4m_B} - \partial_\tau - H_0\right) R(\tau, \boldsymbol{r}) = \int d^3\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}') R(\tau, \boldsymbol{r}')$$

$\downarrow$

$$U(\boldsymbol{r}, \boldsymbol{r}') = V(\boldsymbol{r})\delta(\boldsymbol{r} - \boldsymbol{r}') + \mathcal{O}(\nabla_{\boldsymbol{r}}^2/\Lambda^2)$$

$\downarrow$

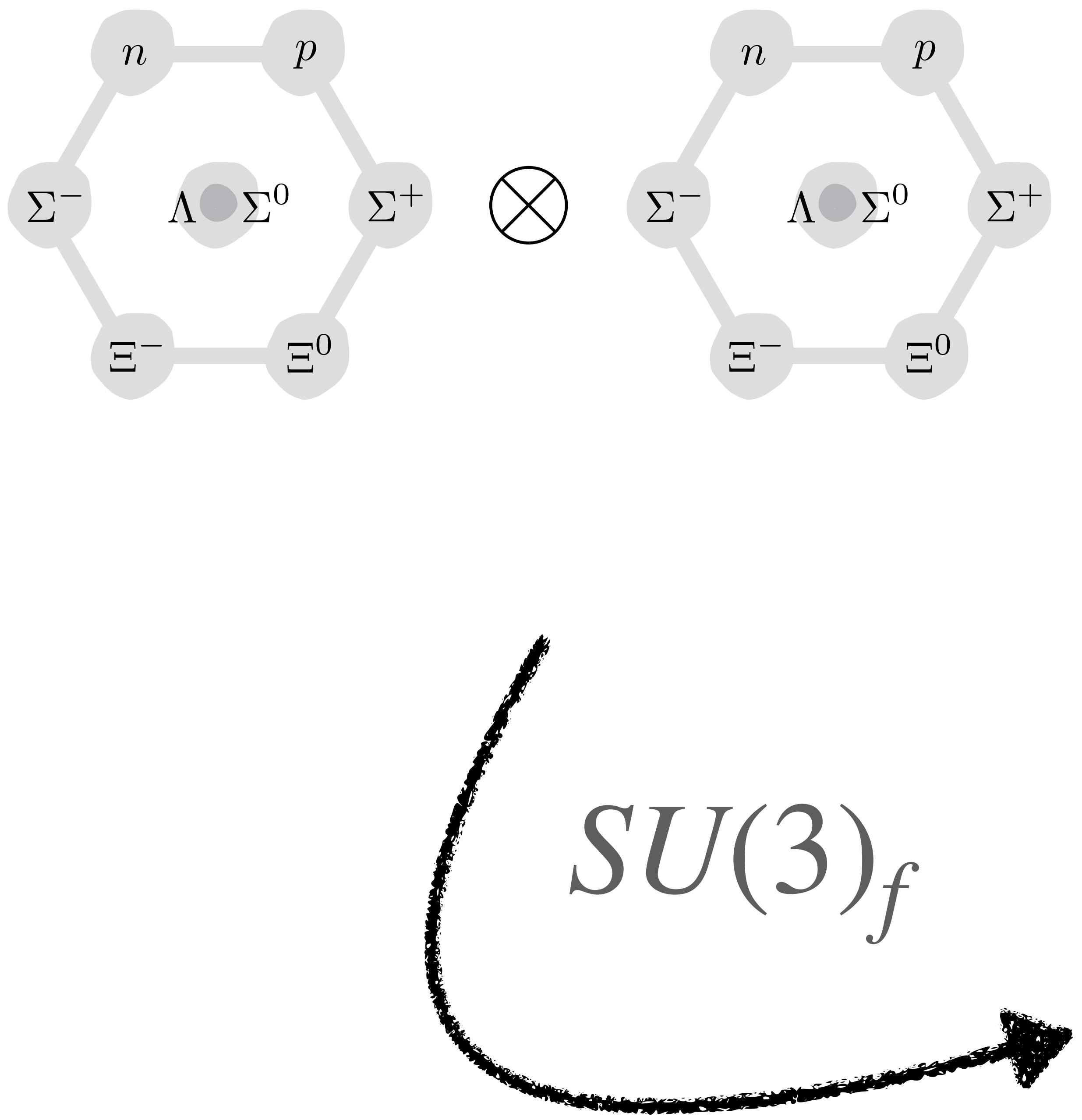
$$k^* \cot\delta$$
$$B$$

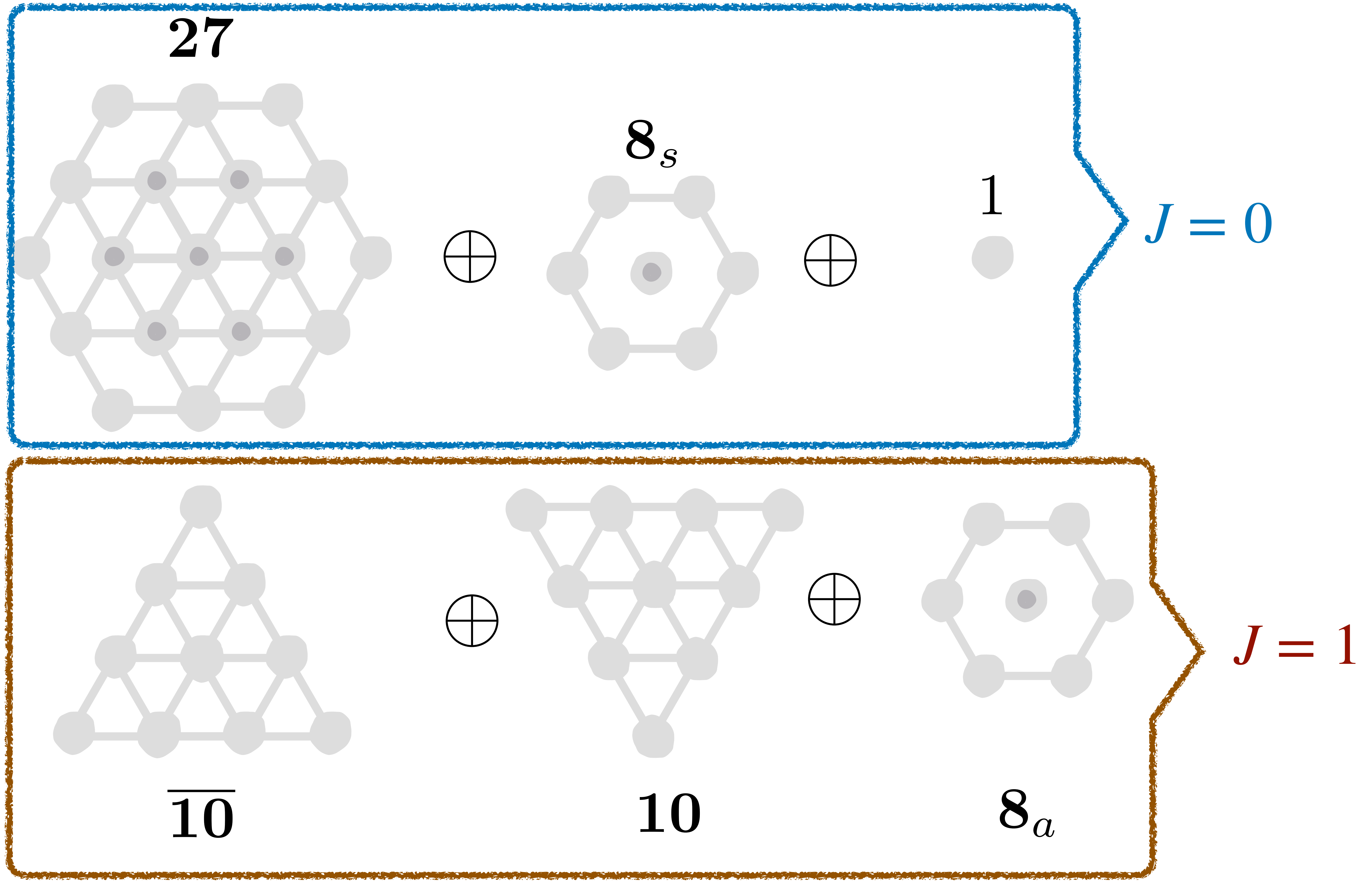
not bound

NN systems at unphysical m_π

$\mathbf{8} \otimes \mathbf{8}$

Physical quark masses ⟶ 64 flavor states





$8 \otimes 8 = 27 \oplus 8_S \oplus 1 \oplus \overline{10} \oplus 10 \oplus 8_A$

Finite Volume LQCD energy eigenstates can be classified by:

- their baryon number B,
- total isospin I,
- strangeness and
- cubic irrep Γ_J , which plays the role of the continuum, infinite-volume total angular momentum, J.

For ex, for I=1 NN,

Γ_J	Γ_ℓ	Γ_S	ℓ
A_1^+	A_1^+	A_1^+	$0, 4, 6, \ldots$
E^+	E^+	A_1^+	$2, 4, 6, \ldots$
T_2^+	T_2^+	A_1^+	$2, 4, 6, \ldots$
T_1^+	T_1^+	A_1^+	$4, 6, \ldots$
A_2^+	A_2^+	A_1^+	$6, \ldots$

$$\Gamma_J \subseteq \Gamma_\ell \otimes \Gamma_S$$

$$\Gamma_S \in \{A_1^+, T_1^+\}$$

S=0 ↗ (A_1^+)

S=1 ↑ (T_1^+)

for I=0 NN

Γ_J	Γ_ℓ	Γ_S	ℓ
T_1^+	$A_1^+ \oplus E^+ \oplus T_1^+ \oplus T_2^+$	T_1^+	$0, 2, 4, 6, \ldots$
E^+	$T_1^+ \oplus T_2^+$	T_1^+	$2, 4, 6, \ldots$
T_2^+	$A_2^+ \oplus E^+ \oplus T_1^+ \oplus T_2^+$	T_1^+	$2, 4, 6, \ldots$
A_2^+	T_2^+	T_1^+	$2, 4, 6, \ldots$
A_1^+	T_1^+	T_1^+	$4, 6, \ldots$

L [fm]	T [fm]
3.4	6.7
4.5	6.7
6.7	9

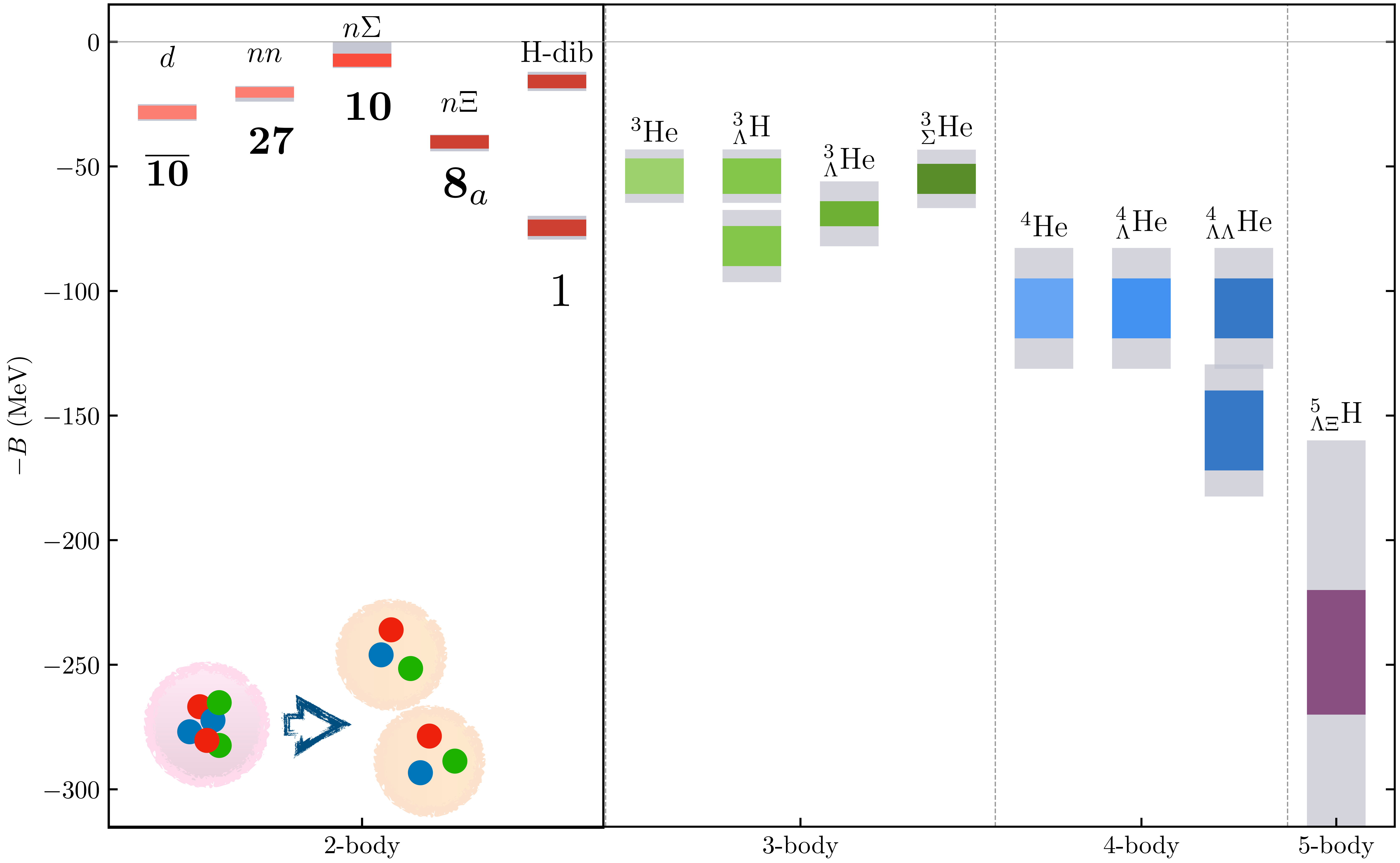
$b[fm] = 0.1453(16)$

$SU(3)_f$

$m_\pi \sim 800$ MeV

no e.m. interactions

NPLQCD, Phys.Rev. D87 (2013) no.3, 034506; Phys.Rev. D96 (2017) no.11, 114510



Baryon-baryon interaction @ $m_\pi \sim 450$ MeV

away from the $SU(3)_f$ limit

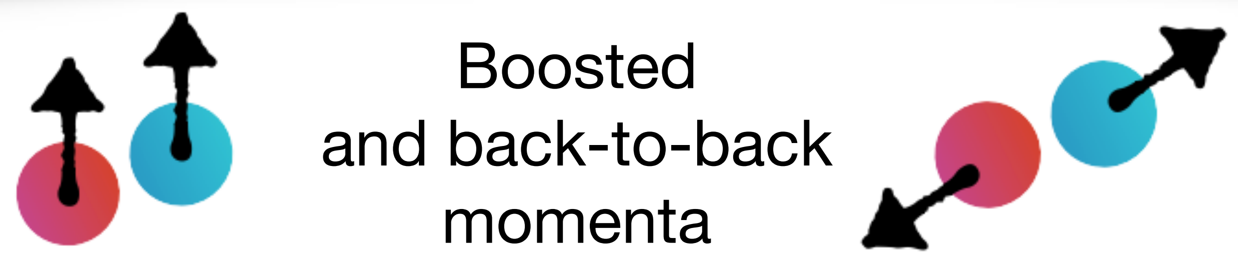
$n_f = 2 + 1$

$m_\pi = 450(5)\text{MeV}$

$b = 0.117(2)\ \text{fm}$

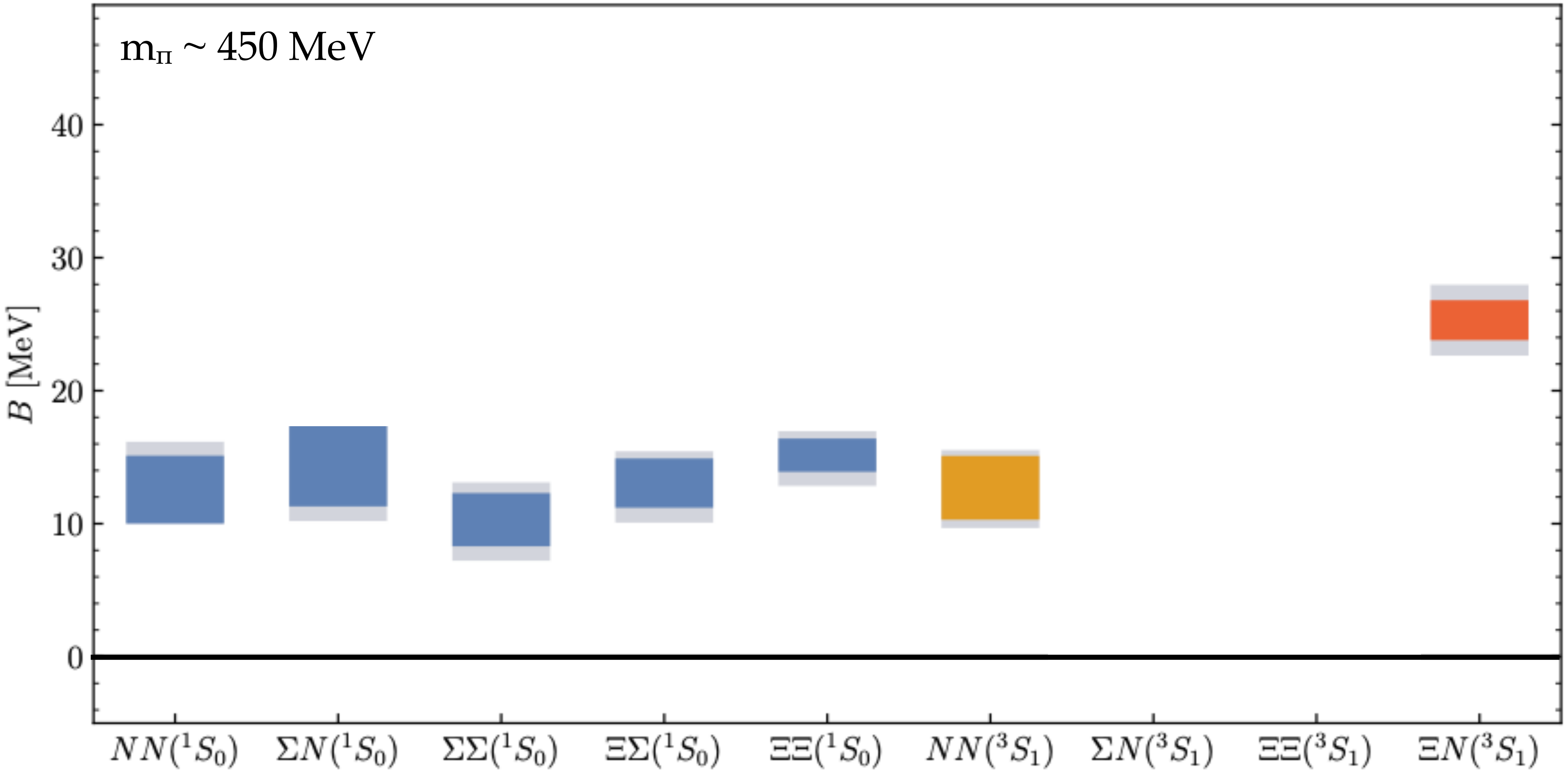
$L = 2.8, 3.7, 5.6\ \text{fm}$

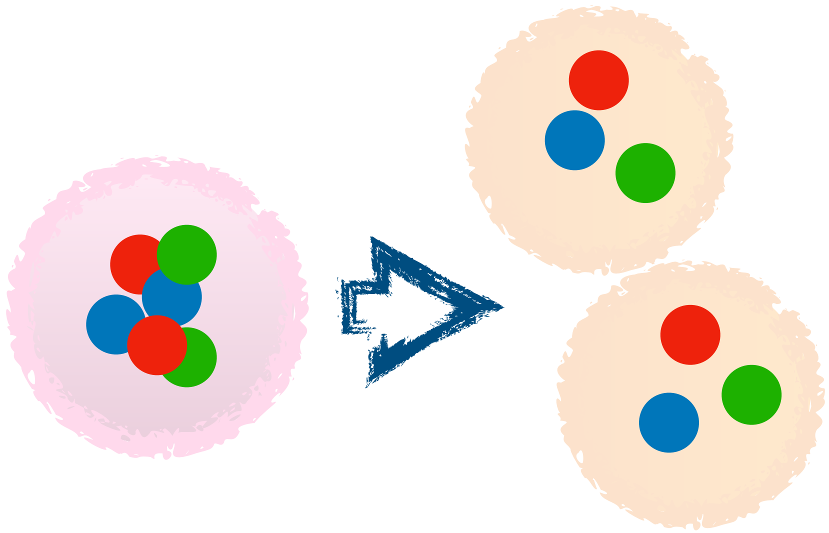
$T = 7.5, 11.2, 11.2\ \text{fm}$



no e.m. interactions

Marc Illa et al (NPLQCD) PRD 103 (2021) 5, 054508





away from the $SU(3)_f$ limit

$n_f = 2 + 1$

$m_\pi = 450(5)\mathrm{MeV}$

$b = 0.117(2)\ \mathrm{fm}$

$L = 2.8, 3.7, 5.6\ \mathrm{fm}$

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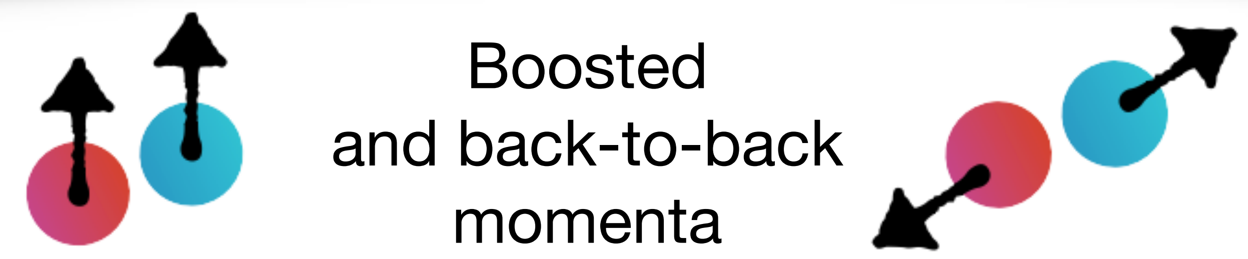
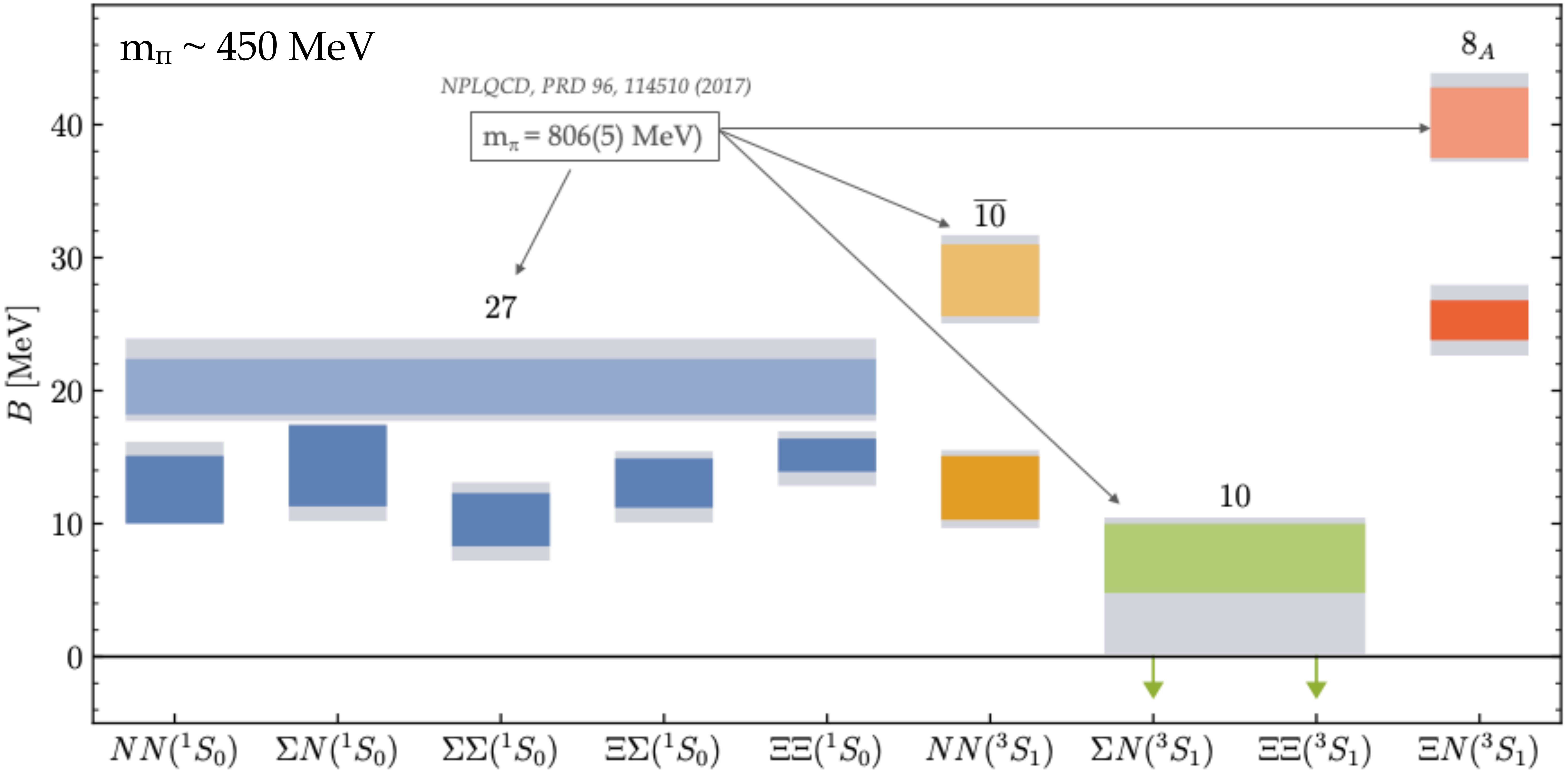
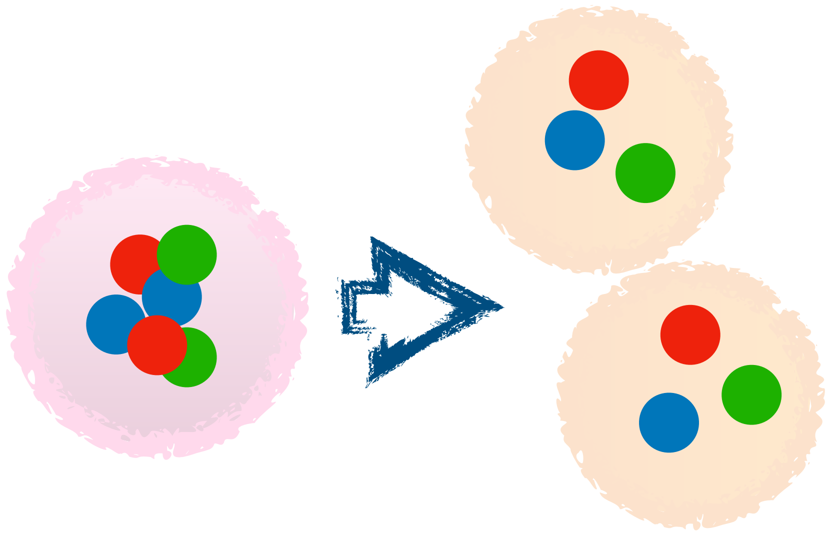


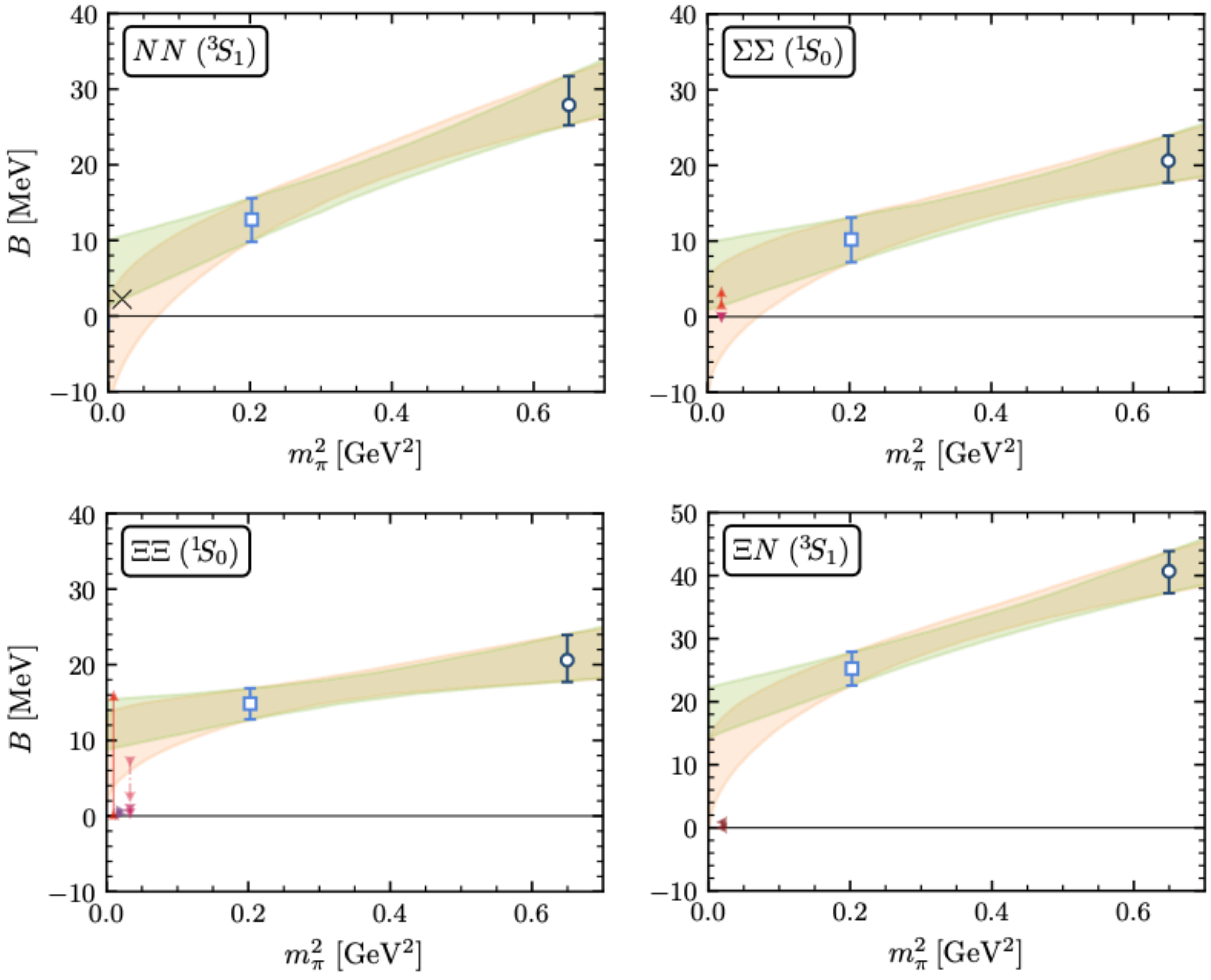
no e.m. interactions

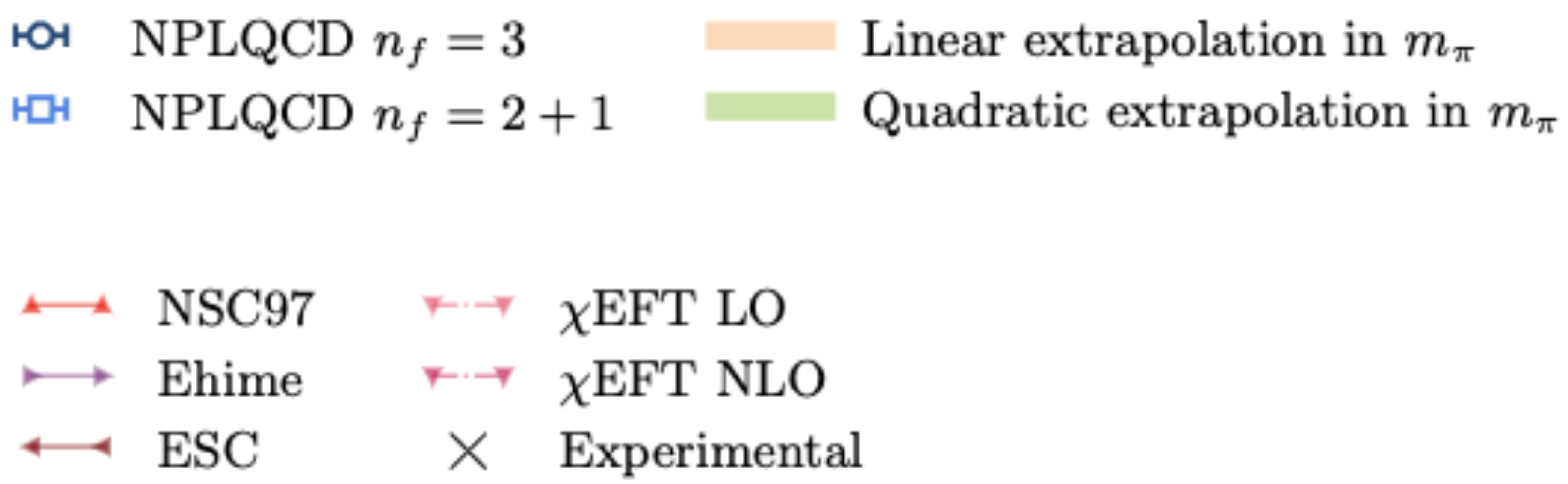
Marc Illa et al (NPLQCD) PRD 103 (2021) 5, 054508





Marc Illa et al (NPLQCD) PRD 103 (2021) 5, 054508



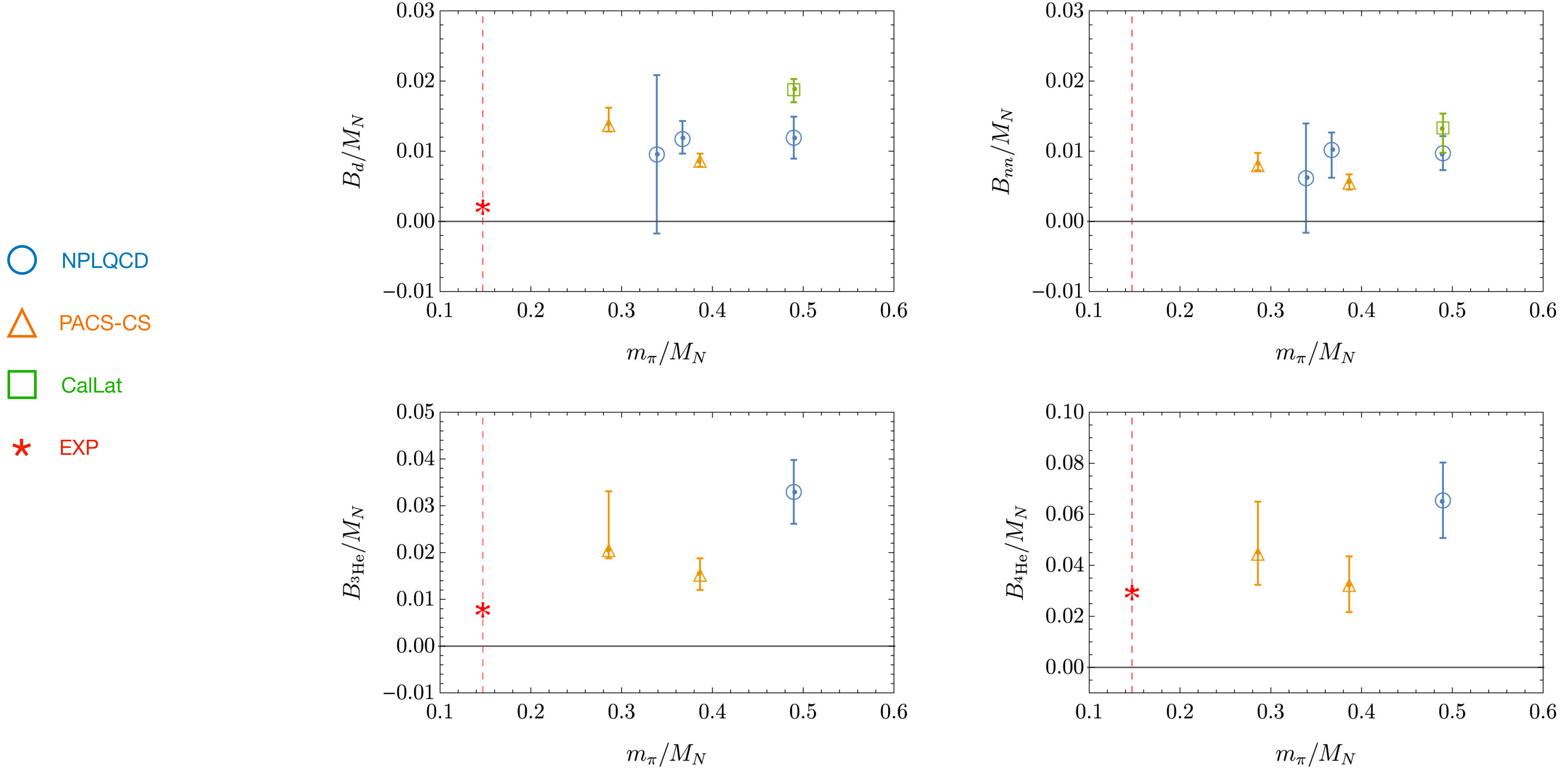


$$B_{\text{lin}}(m_\pi) = B_{\text{lin}}^{(0)} + B_{\text{lin}}^{(1)} m_\pi$$

$$B_{\text{quad}}(m_\pi) = B_{\text{quad}}^{(0)} + B_{\text{quad}}^{(1)} m_\pi^2$$

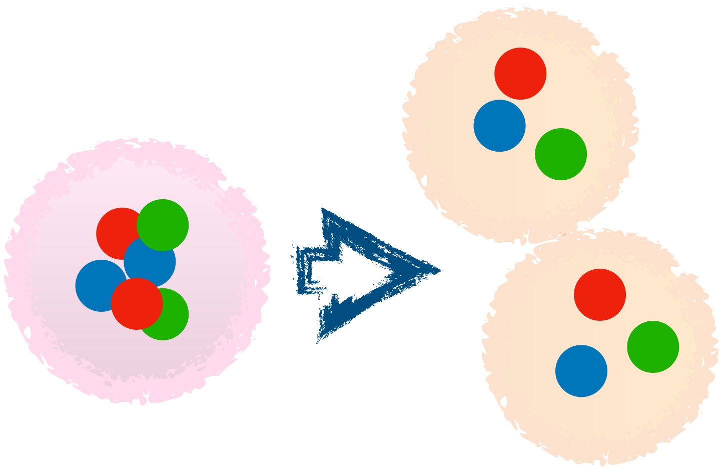
Nuclear physics with LQCD - Controversy

Davoudi, Detmold, Shanahan, Orginos, Parreño, Savage, Wagman, Physics Reports 900 (2021) 1–74

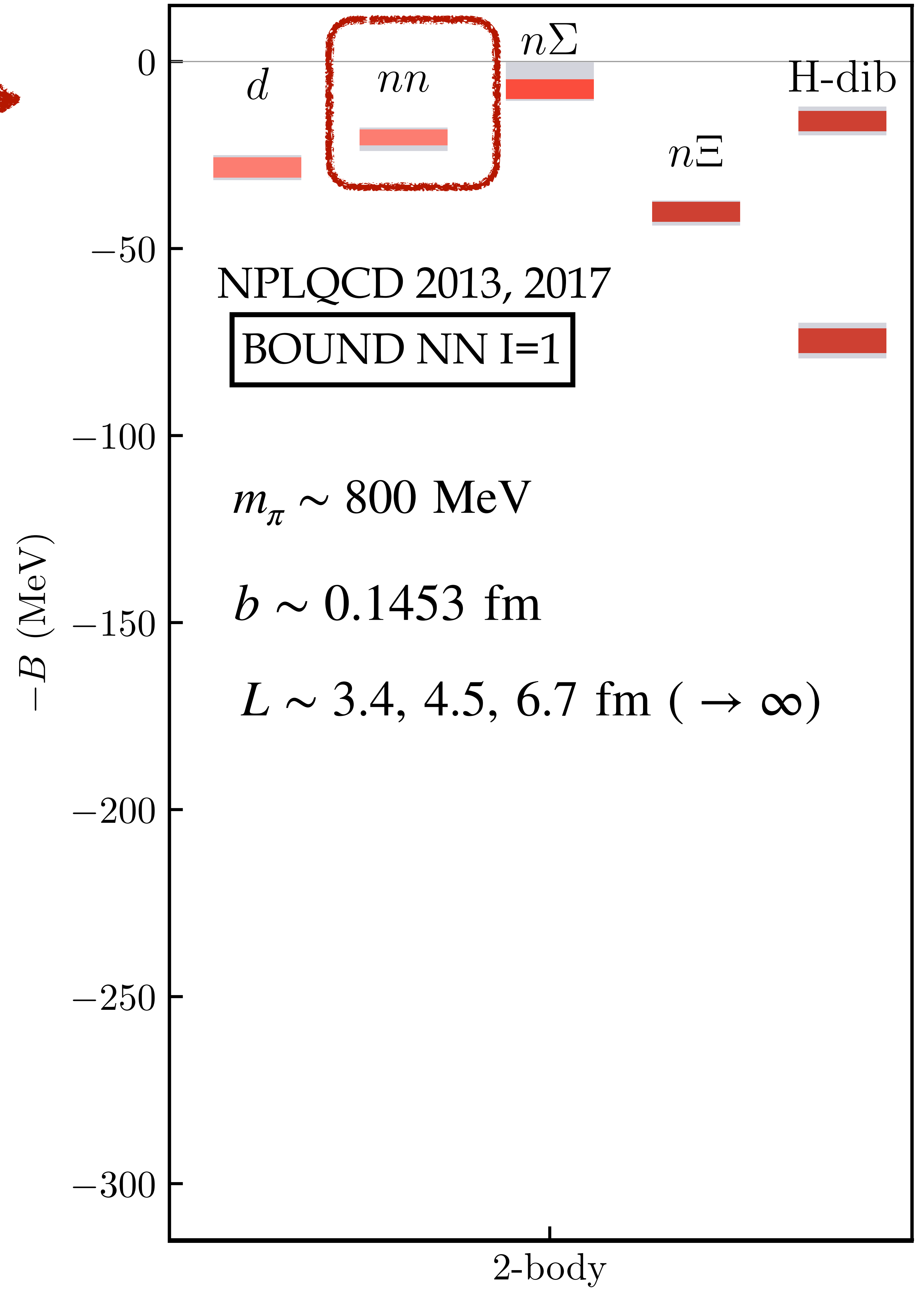


Misidentification of the plateau?

E. Berkowitz et al. [CalLat], Phys.Lett.B 765 (2017)
S.R. Beane et al. [NPLQCD], arXiv:1705.09239 [hep-lat]
T. Yamazaki et al. [PACS], EPJ Web Conf. 175 (2018)



Deep bound states



Small excited-state gaps may lead to incorrect identification of the ground-state energy

- Is the fitting interval correctly identified?
- Are we missing excited state contributions?
- Is there an operator dependence on the energy levels extracted?

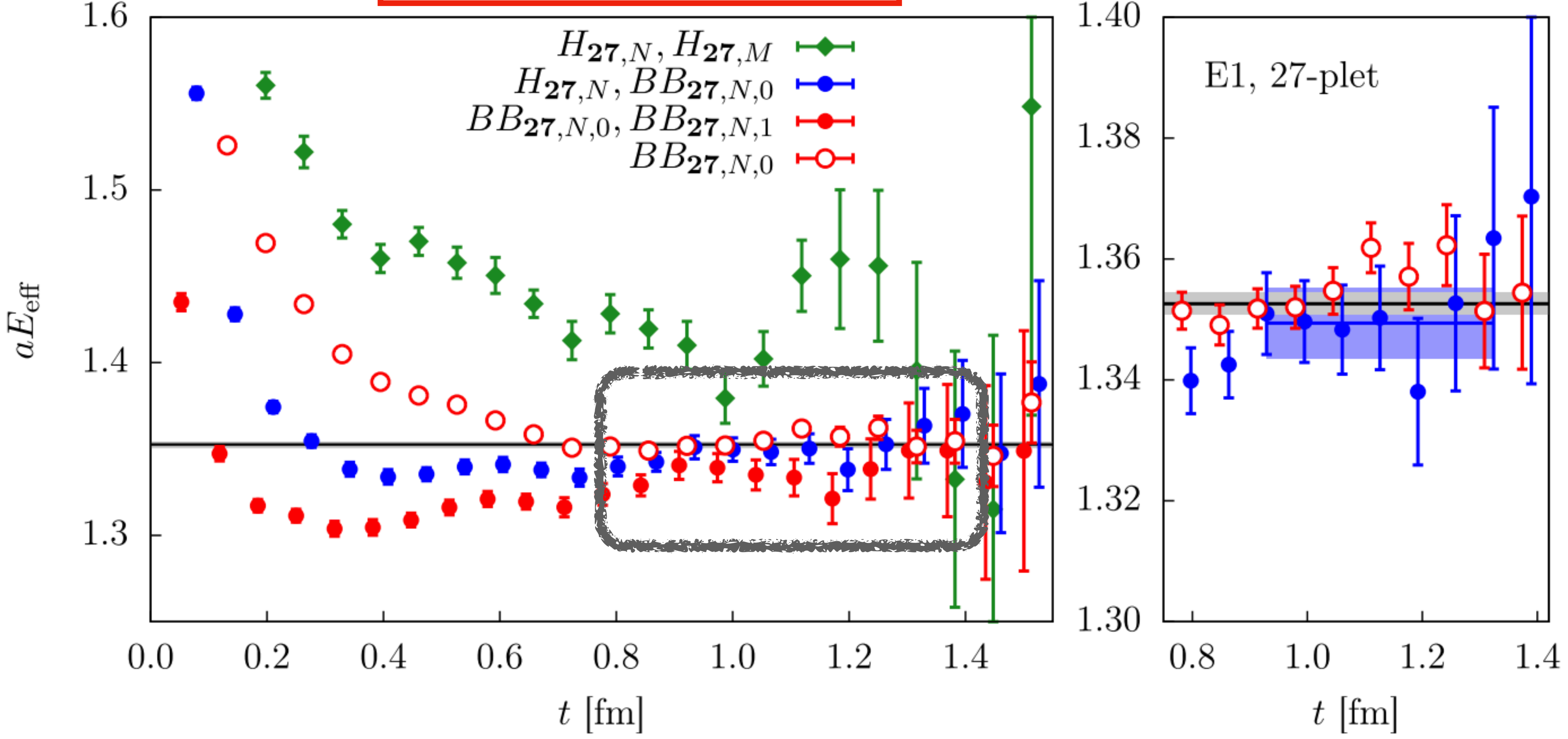
Reduce uncertainty at small time: GPoF, matrix Prony, variational

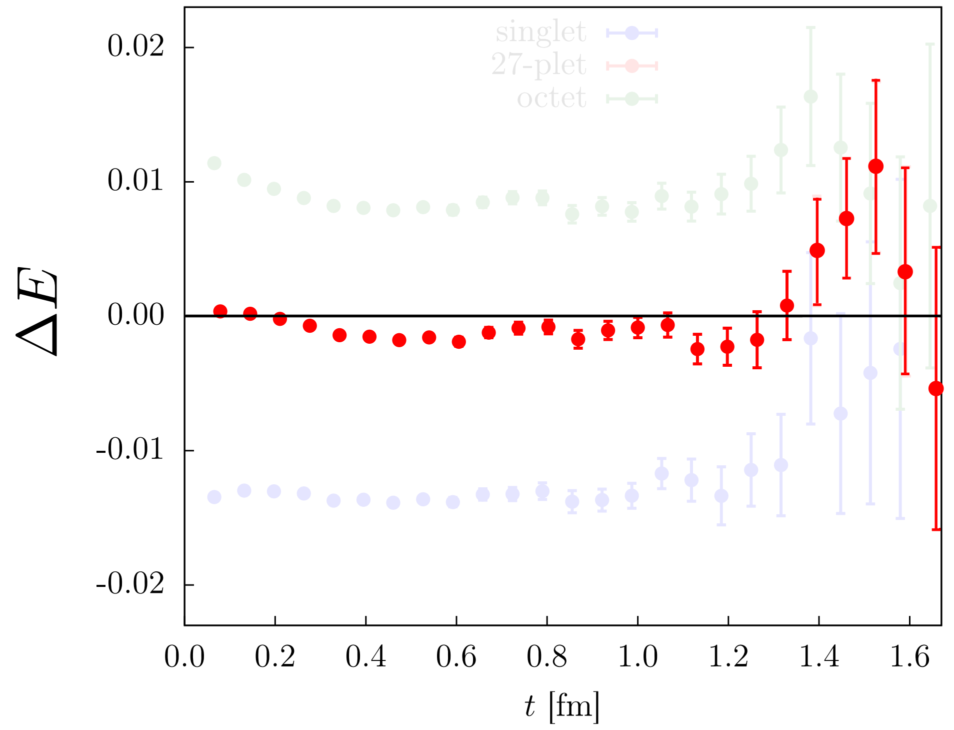
First variational calculation

A. Francis et al., Phys.Rev.D 99 (2019)

$m_\pi \sim 960$ MeV

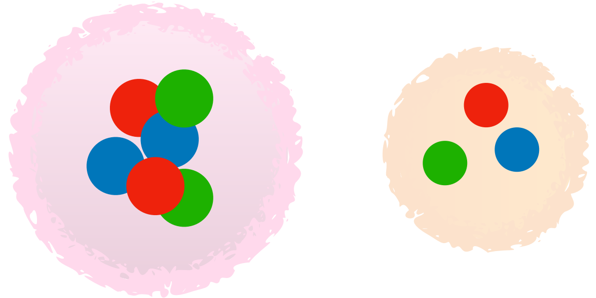
UNBOUND NN I=1



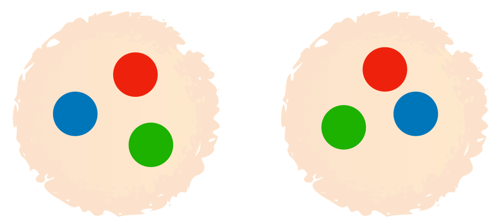


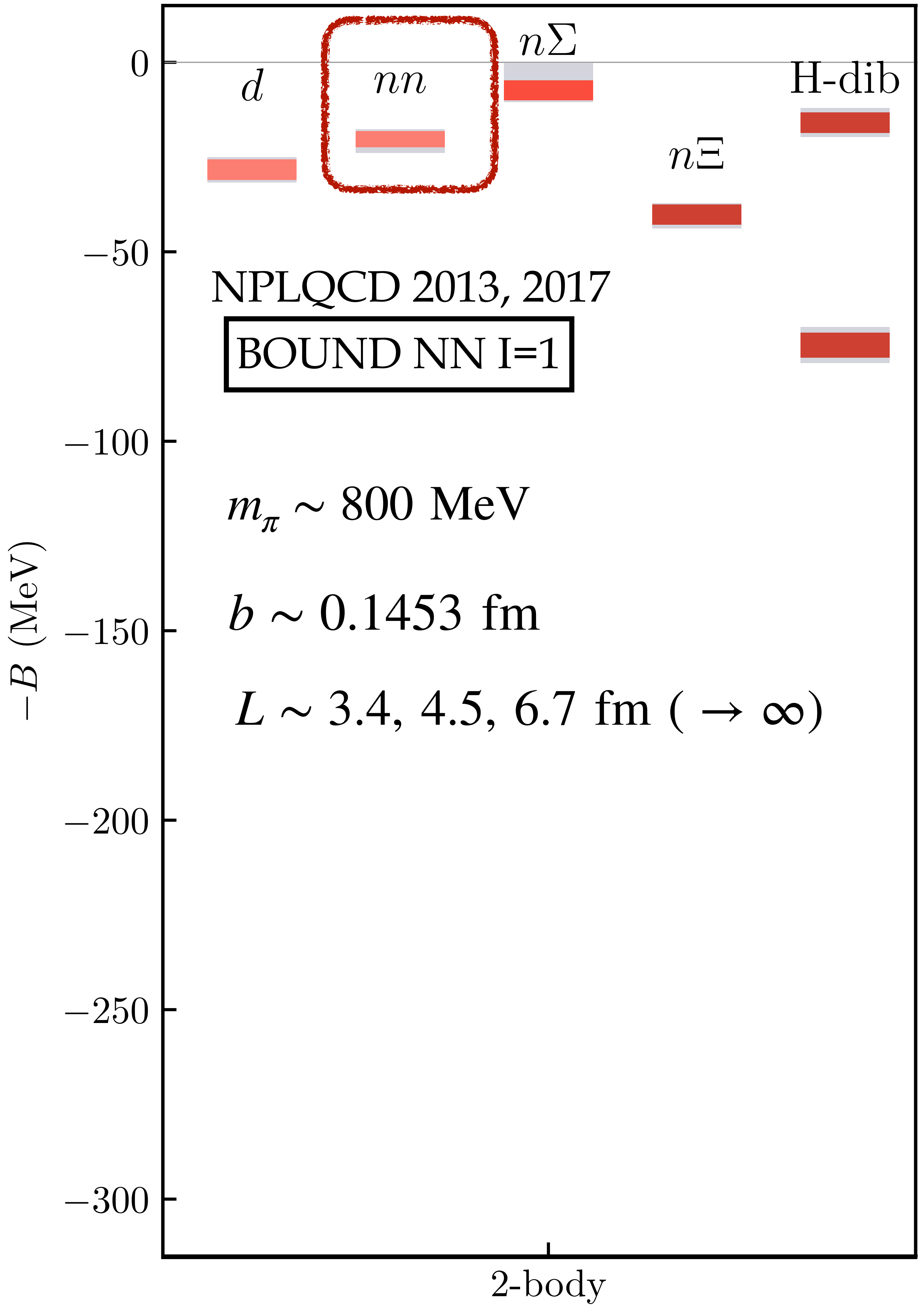
Ground-state energy close to threshold

Hermitian 2x2 matrix with hexaquark and dibaryon-like operators



Non-hermitian 2x2 matrix with dibaryon-like operators

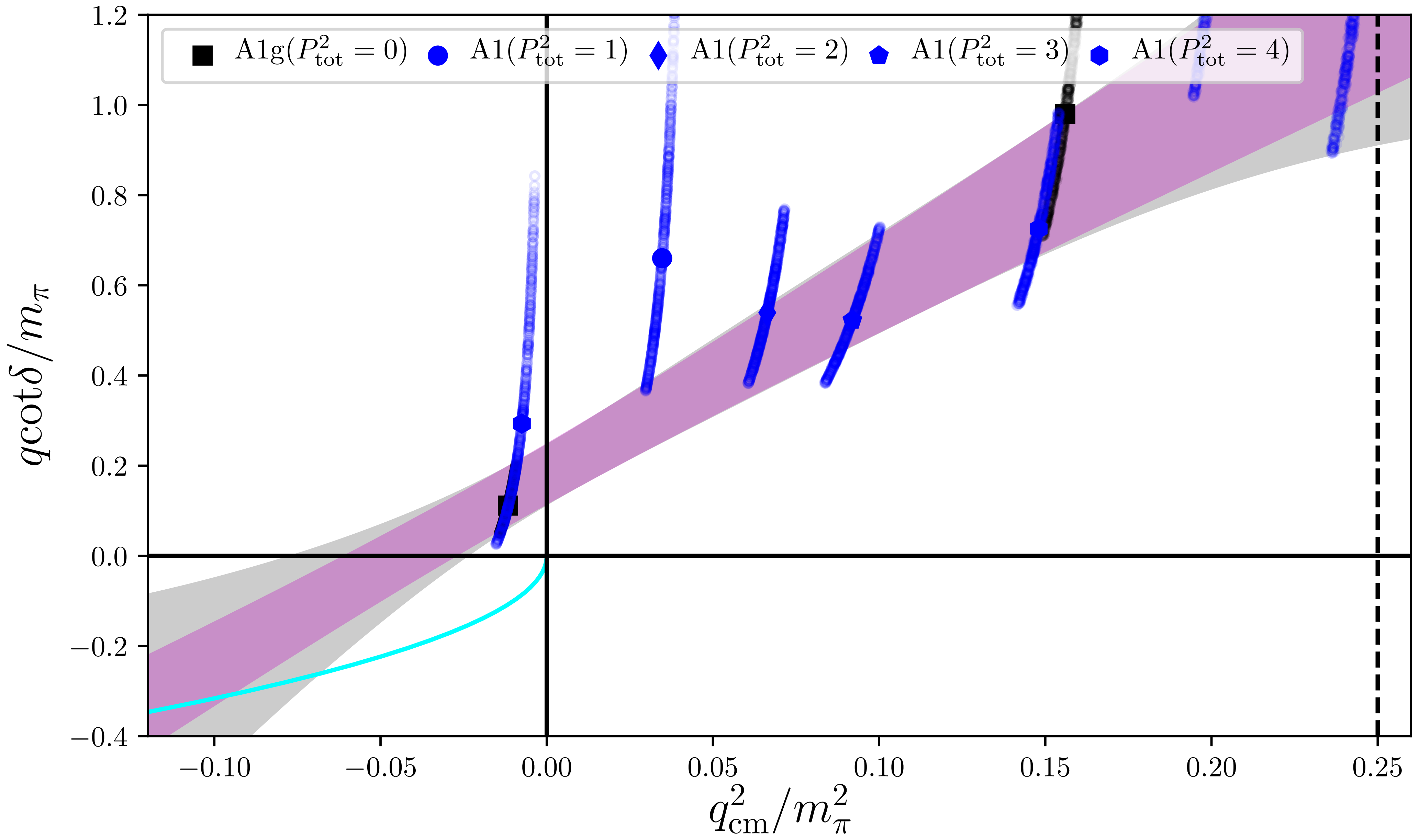




Stochastic Laplacian Heaviside method

CalLat B. Hörz et al., Phys.Rev.C 103 (2021)

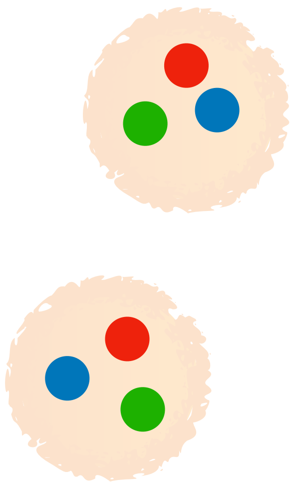
$m_\pi \sim 714$ MeV



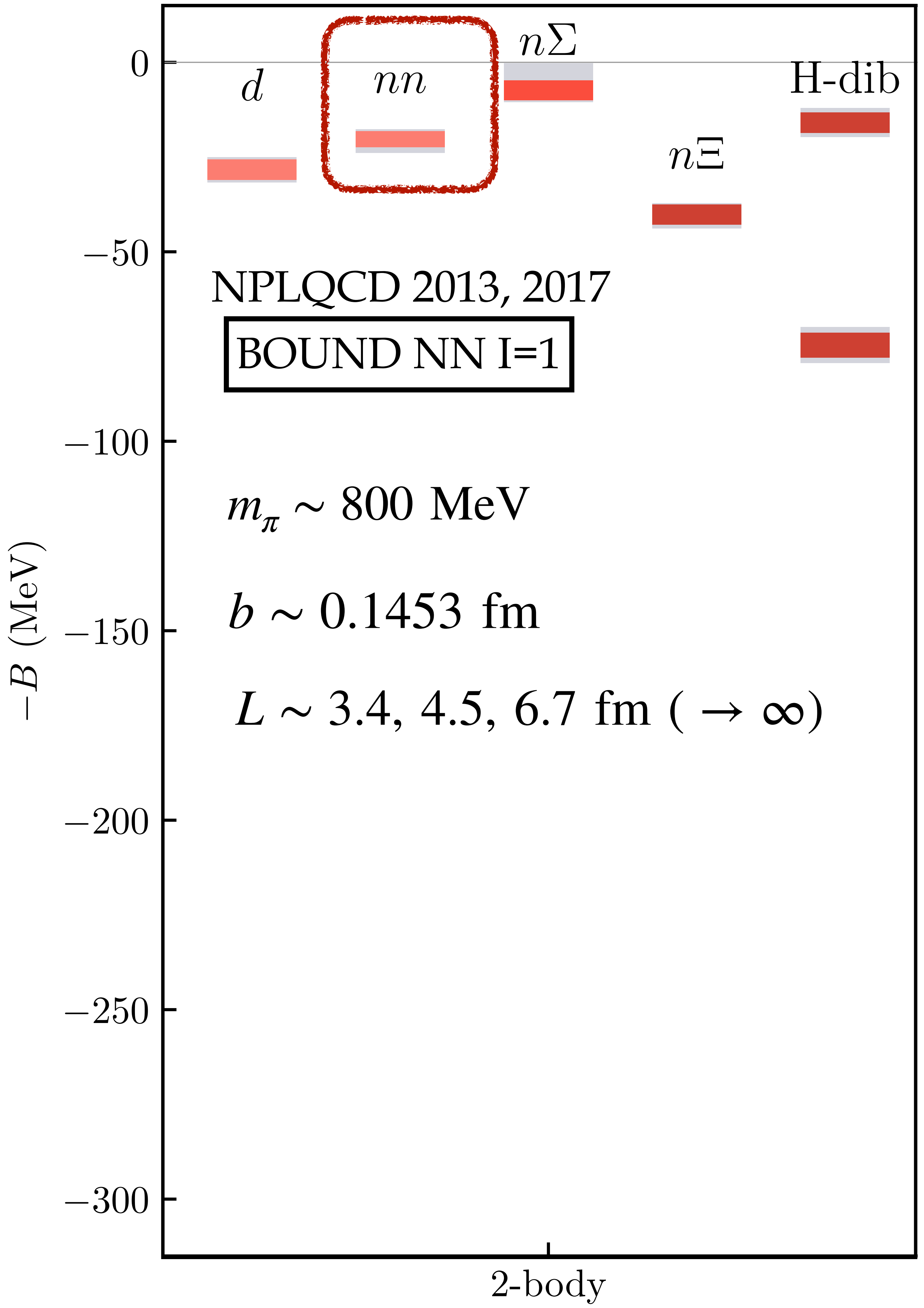
UNBOUND NN I=1

$b \sim 0.086$ fm

$L = 48\,b \sim 4.1$ fm



Hermitian 2x2 matrix with dibaryon-like operators



NPLQCD 2013, 2017

BOUND NN I=1

$m_\pi \sim 800$ MeV

$b \sim 0.1453$ fm

$L \sim 3.4,\ 4.5,\ 6.7$ fm ($\to \infty$)

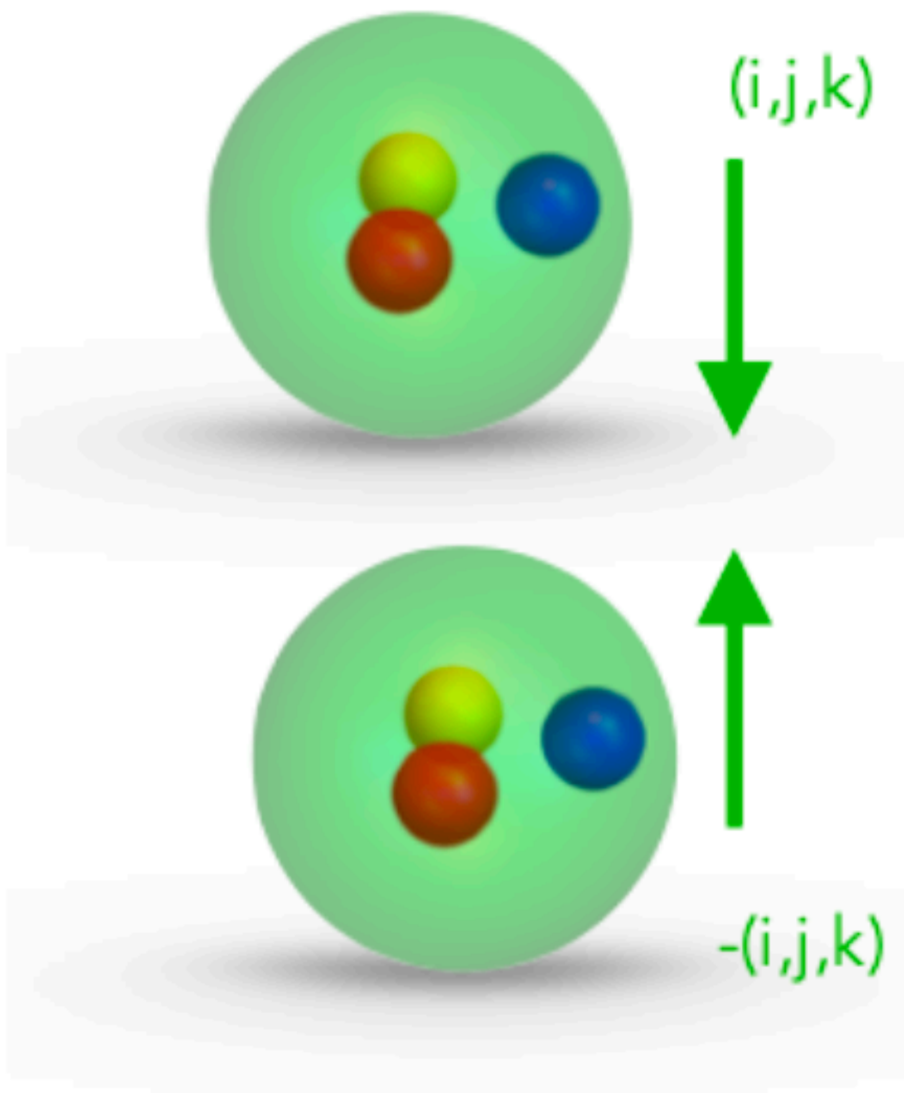
Variational calculation - Types of operators

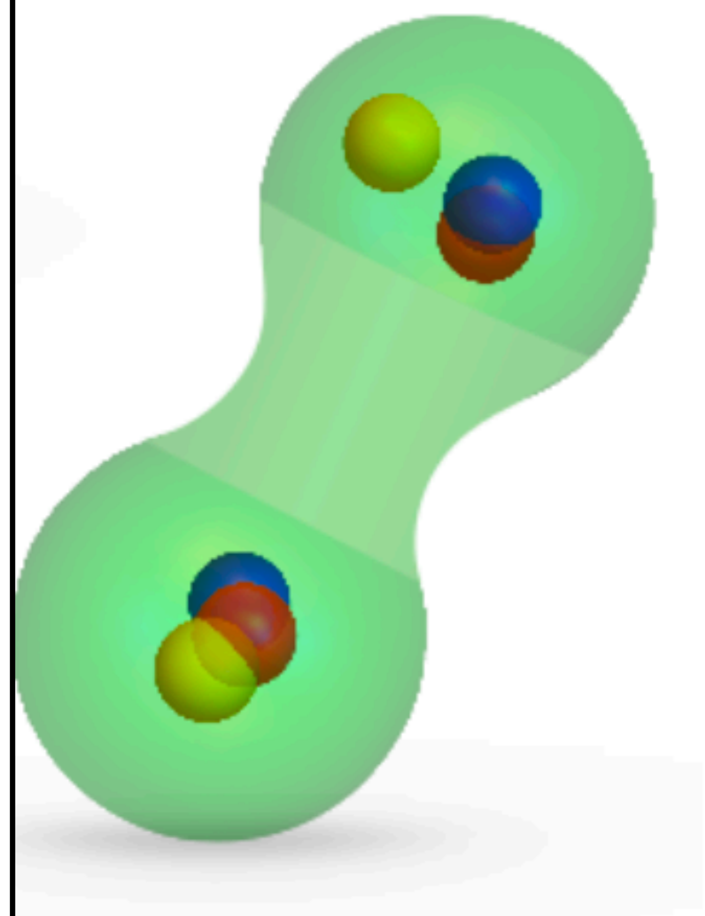
Local hexaquark operators

- Six Gaussian smeared quarks at a point

Dibaryon Operators

- Two spatially-separated plane-wave baryons with relative momenta
- Relative momentum: up to four units → 5 operators





Quasi-local Operators

- Two exponentially localized baryons
- NN -EFT motivated deuteron-like structure

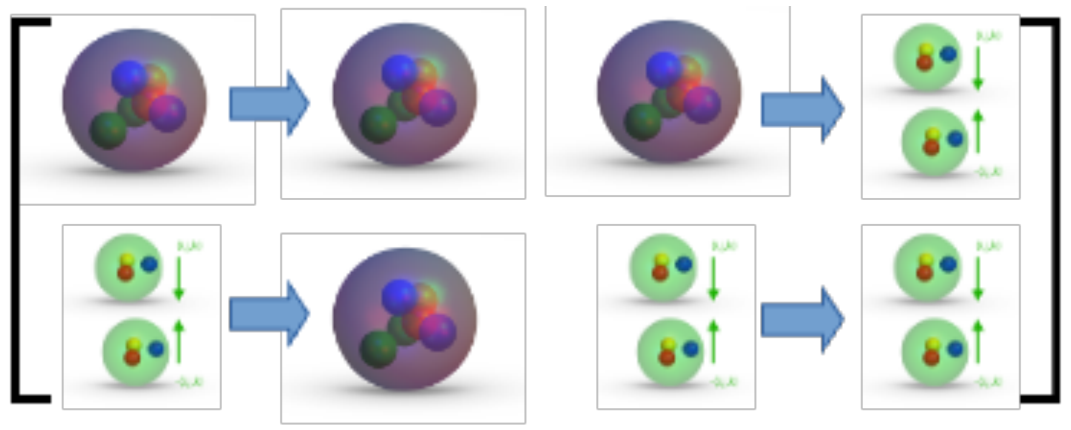
NPLQCD, PRD 107 (2023) 9, 094508: Largest set of operators to date
b=0.145 fm , L/b=32 (4.7 fm aprox)

Nuclear physics with LQCD - Variational calculation

Our first variational results added dibaryon operators at the source and sink through the (Hermitian) matrix of correlators:

S. Amarasinghe et al (NPLQCD) PRD 107 (2023) 9, 094508

$$\mathbf{C}(t) =$$



Solve **Generalized Eigenvalue Problem** (GEVP): $$C(t)\,\vec{v}_n(t,t_0) = \lambda_n(t,t_0)\,C(t_0)\,\vec{v}_n(t,t_0)$$

GEVP Eigenvalues provide **rigorous (stochastic) variational upper bounds on energy levels**

Variational methods lead to correlation functions with positive definite spectral representations

$$C(t) = \sum_n |Z_n|^2 e^{-E_n t}$$

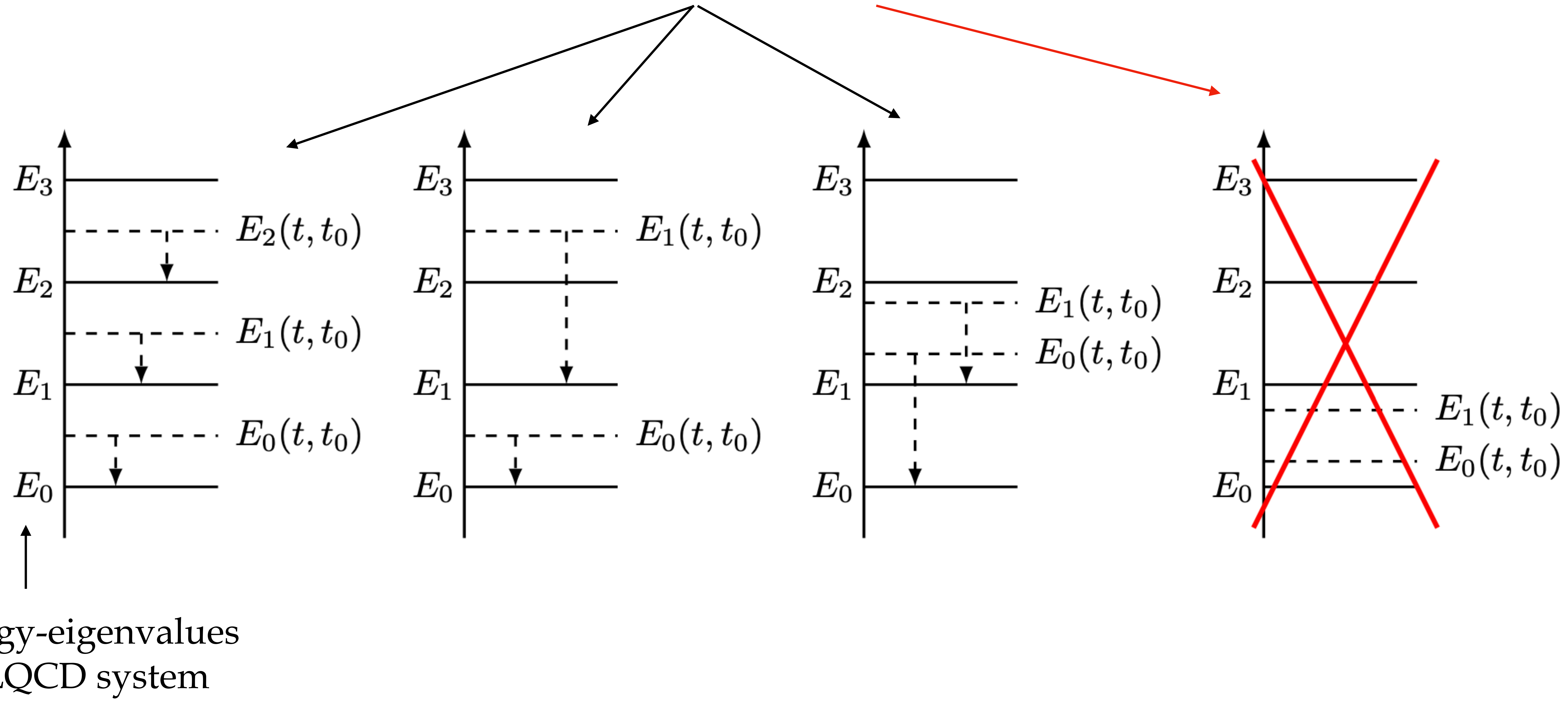
For ex., the effective mass provides a genuine upper bound on the g.s. energy:

$$E(t) = -\ln\left[\frac{C(t+1)}{C(t)}\right] = -\ln\left[\frac{\sum_n e^{-E_n} e^{-E_n t}}{\sum_n e^{-E_n t}}\right] \geq -\ln\left[\frac{e^{-E_0}\sum_n e^{-E_n t}}{\sum_n e^{-E_n t}}\right] = E_0 \qquad \text{where } E_n \geq E_0$$

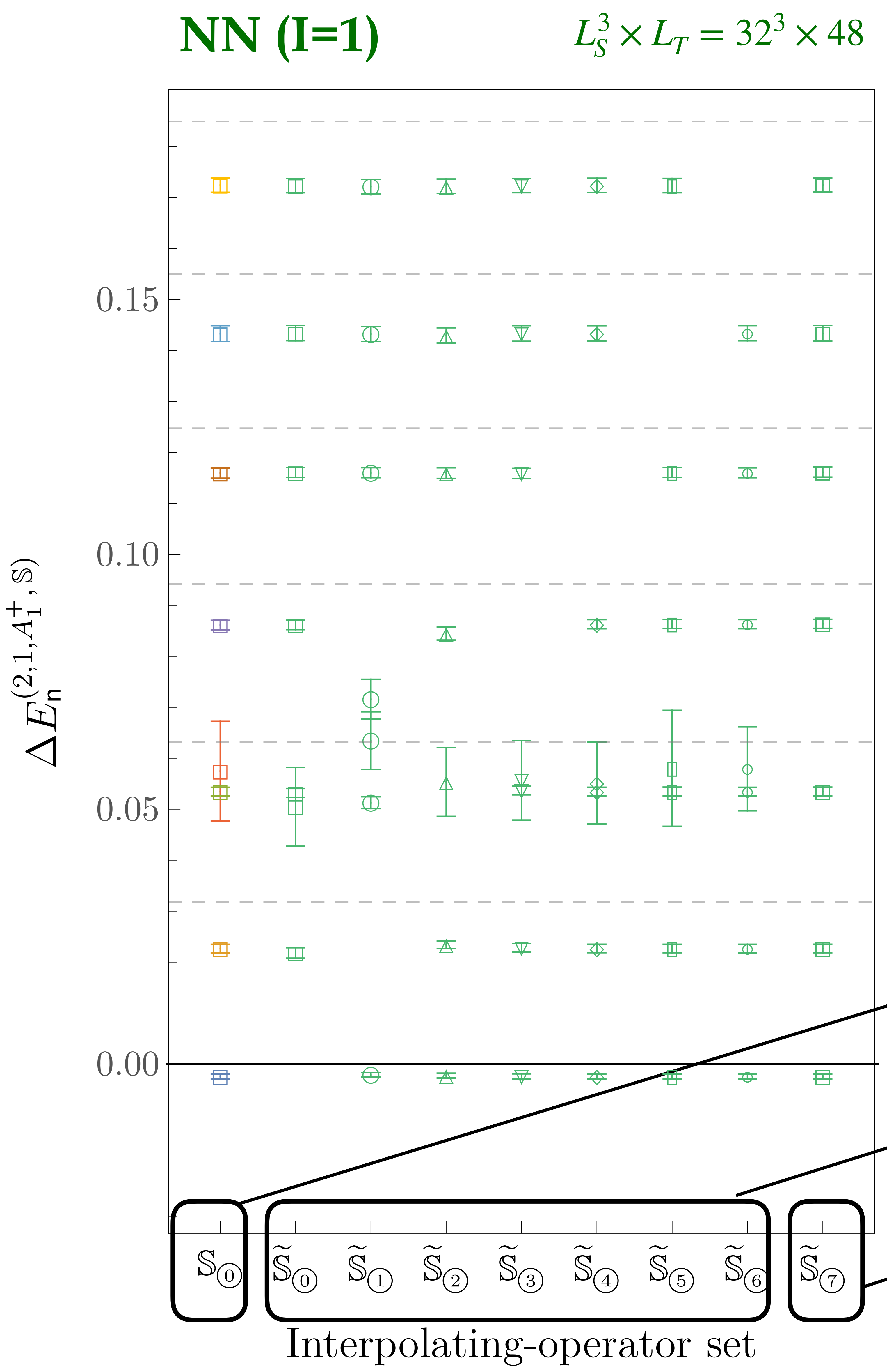
Cauchy Interlacing theorem

$$C(t)\,\vec{v}_n(t,t_0) = \lambda_n(t,t_0)\,C(t_0)\,\vec{v}_n(t,t_0)$$

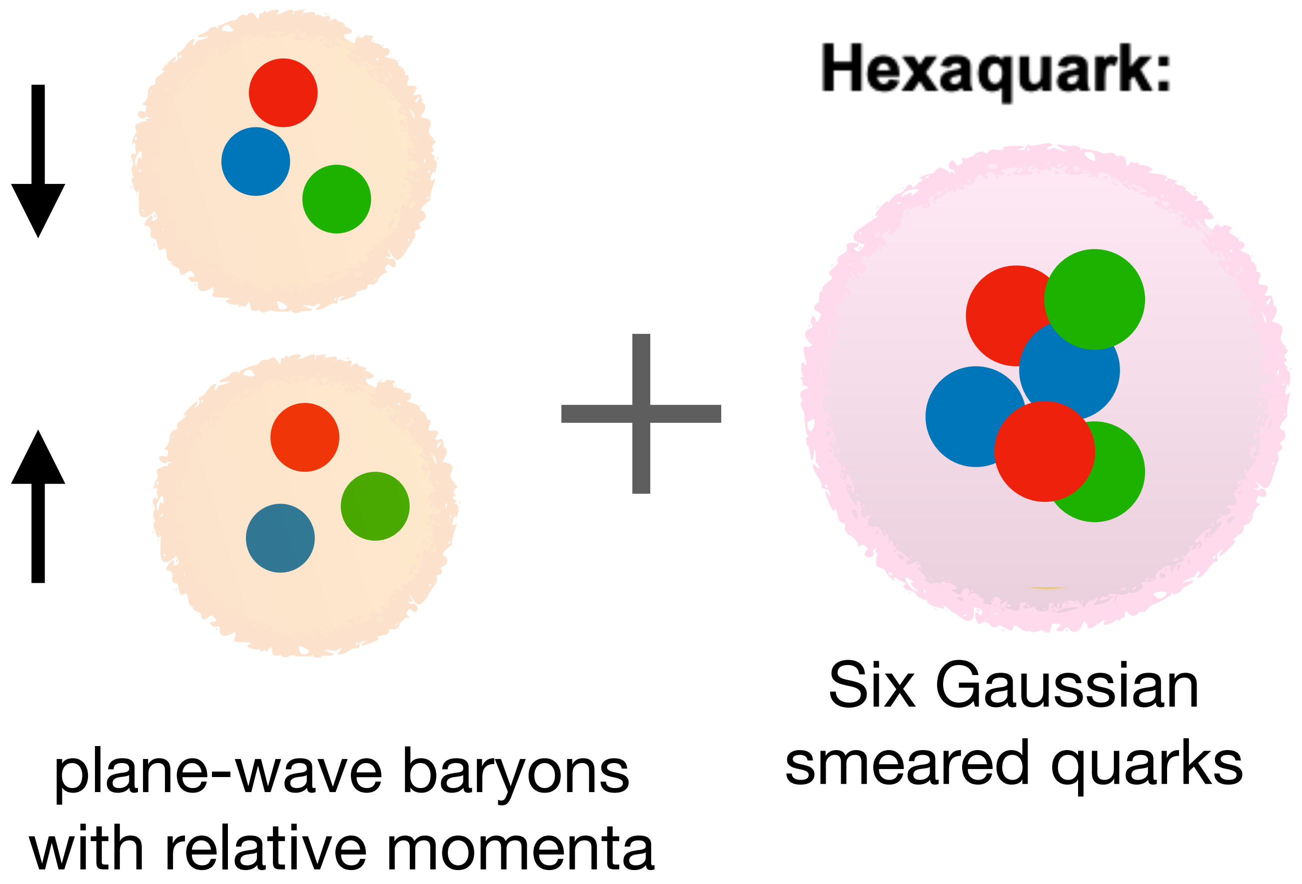
locations of observed effective masses
which are consistent or inconsistent with the interlacing theorem



It tell us the minimum number of energy eigenvalues below a particular effective mass extracted from the GEVP



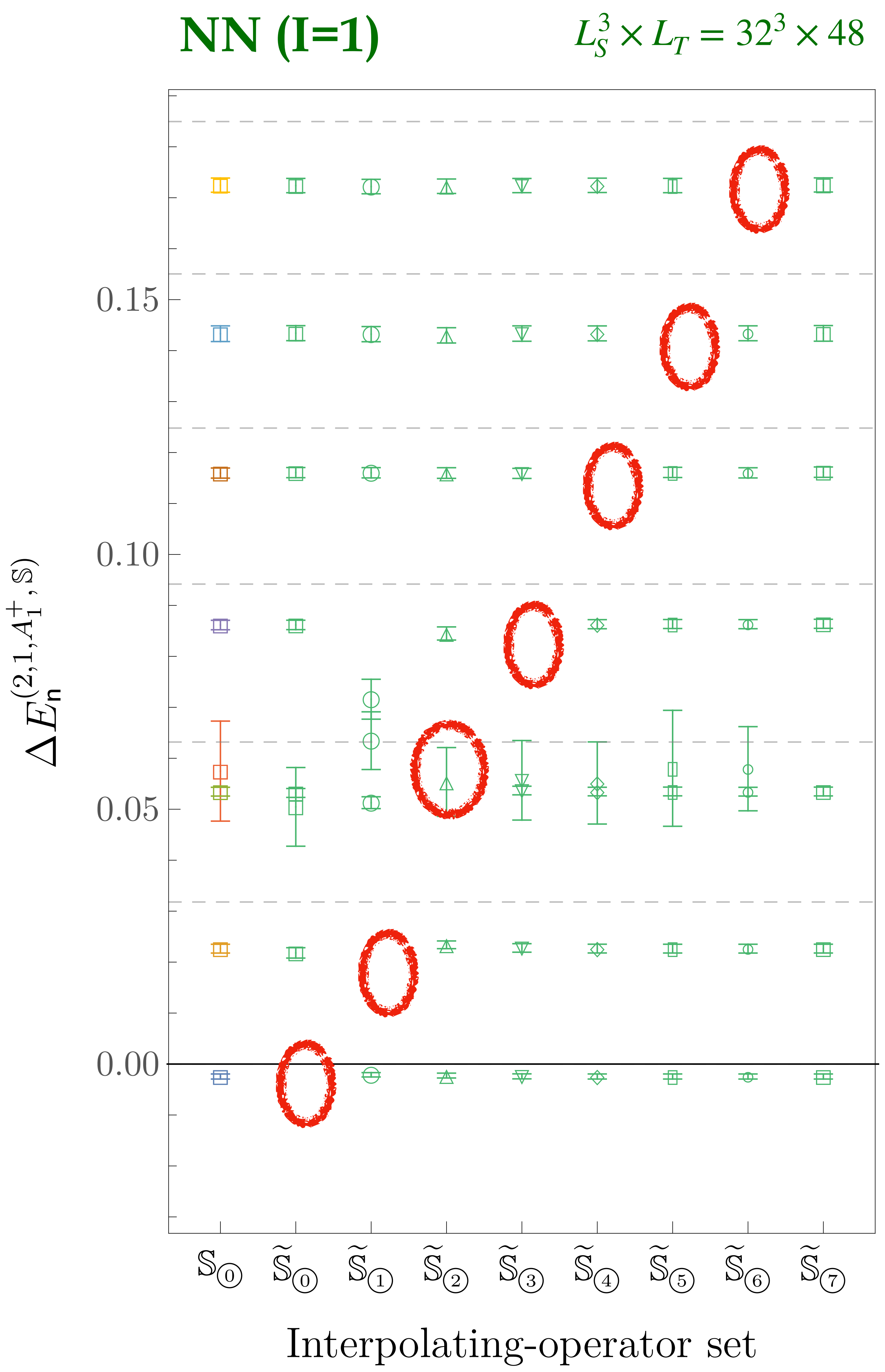
S. Amarasinghe et al (NPLQCD) PRD 107 (2023) 9, 094508



S_0 contains all operators except the quasi-locals
(hexaquark and dibaryons ops with different relative momentum)

Set without a particular dibaryon operator
(taking out a dibaryon op with a given value of the relative momentum)

Set with only the whole set of dibaryon operators (NO hexaquark)



S. Amarasinghe et al (NPLQCD) PRD 107 (2023) 9, 094508

Similarly with what happens in the meson sector, removing the operator structure with maximum overlap on to a given energy level leads to missing energy levels

Importance of using an interpolating-operator set with significant overlap onto all energy levels of interest.

Having a large interpolating-operator set is not sufficient to guarantee that a set will have good overlap onto the ground state or a particular excited state

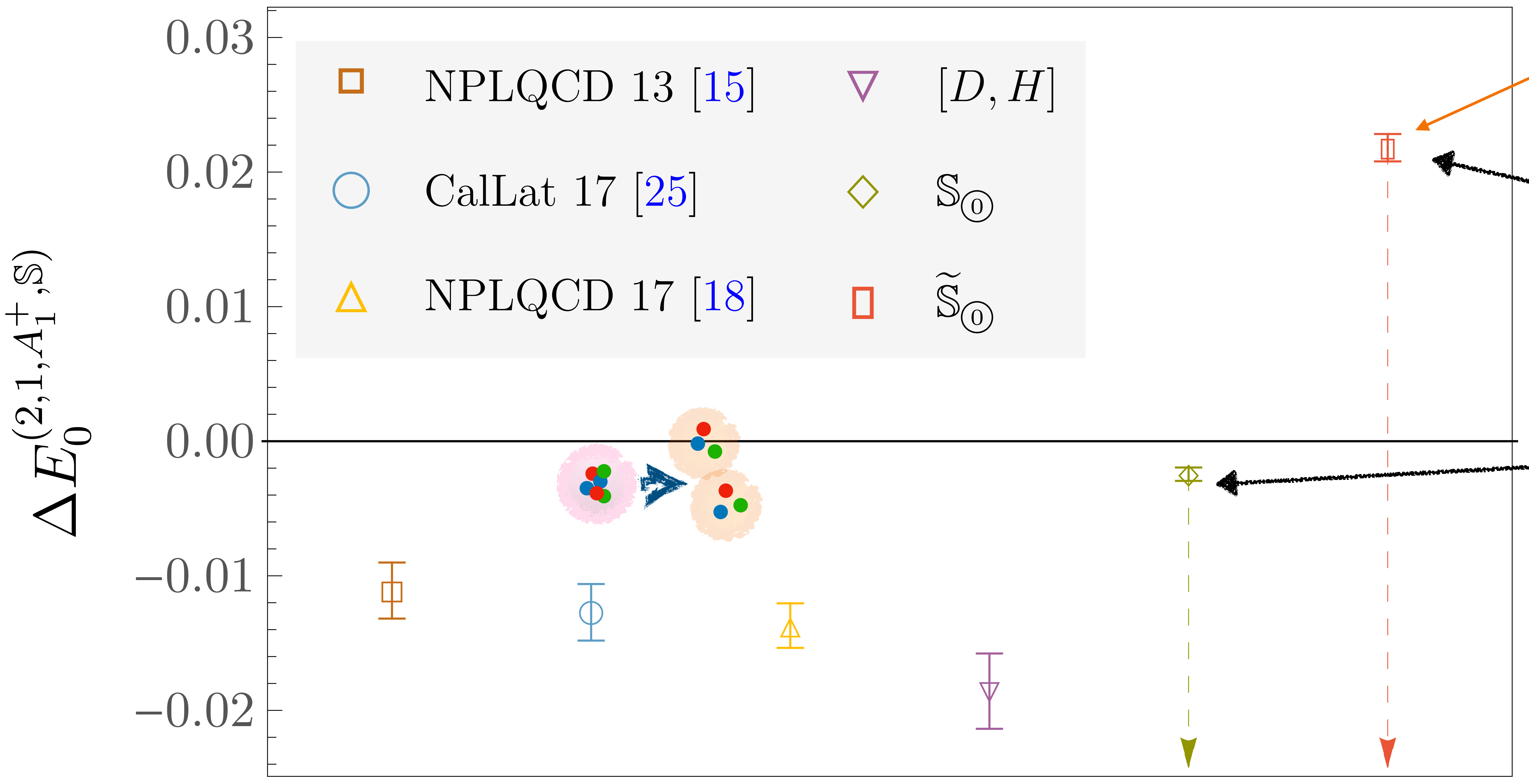
Variational upper bounds. No evidence for (or against) bound states

Operator dependence on variational bounds

"Additional bound": large overlap to Hex

S. Amarasinghe et al (NPLQCD) PRD 107 (2023) 9, 094508

NN (I=1)



taking out a dibaryon op
with zero relative momentum

variational method provides (stochastic) upper bounds on energy levels

calculations made on the same set of configurations

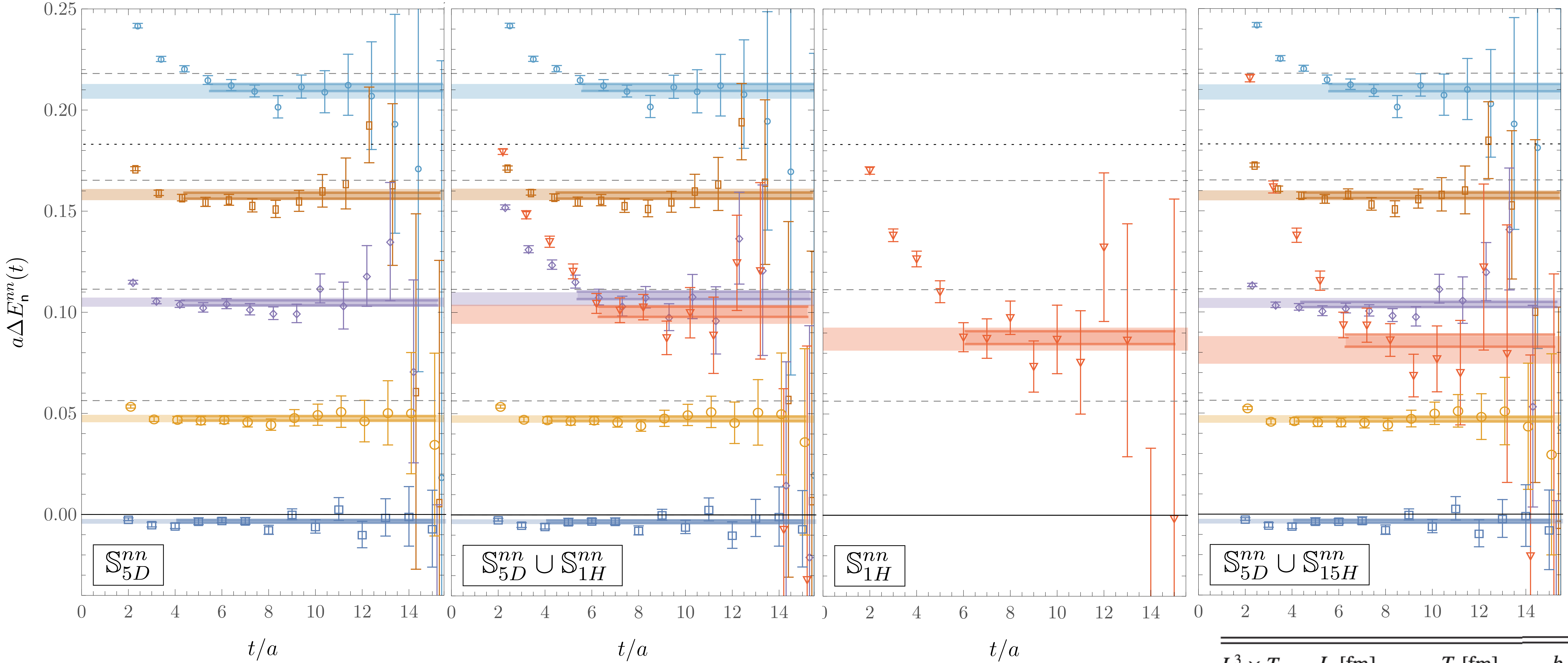
The (upper bounds on the) gs energies obtained from the resulting GEVP solutions for these correlation-function matrices are significantly closer to threshold than the those obtained using hexaquark sources and dibaryon sinks (in this work and in previous studies using the same gauge-field ensemble)

NN spectroscopy @ 800 MeV - Variational calculation

NPLQCD, e-Print: 2404.12039 [hep-lat]

NN (I=1)

$L_S^3 \times L_T = 24^3 \times 48$



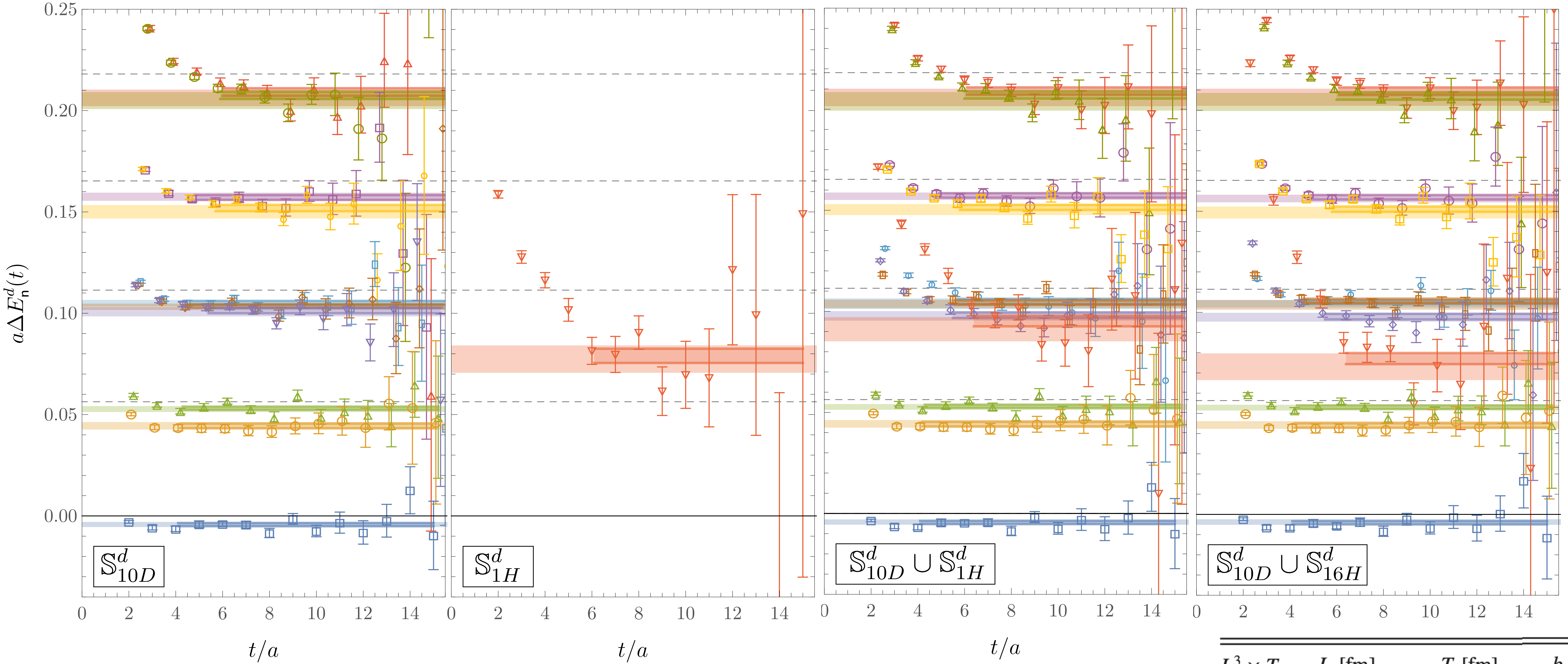
CALCULATIONS ON A SECOND VOLUME

$L^3 \times T$	L [fm]	T [fm]	b [fm]
$24^3 \times 48$	3.4	6.7	0.1453(16)
$32^3 \times 48$	4.5	6.7	0.1453(16)

NPLQCD, e-Print: 2404.12039 [hep-lat]

NN (I=0)

$L_S^3 \times L_T = 24^3 \times 48$



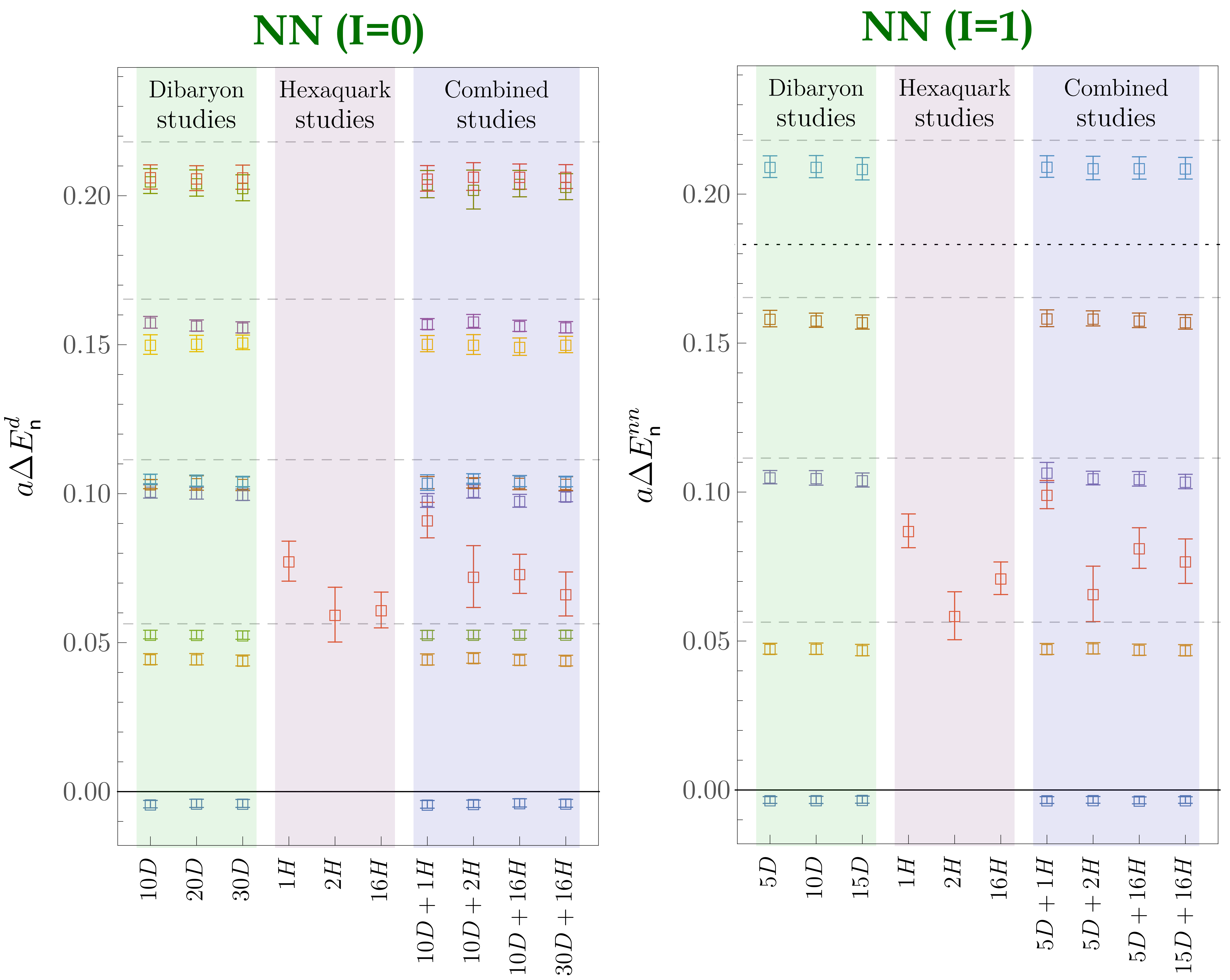
CALCULATIONS ON A SECOND VOLUME

$L^3 \times T$	L [fm]	T [fm]	b [fm]
$24^3 \times 48$	3.4	6.7	0.1453(16)
$32^3 \times 48$	4.5	6.7	0.1453(16)

Summary of variational bounds

$$L_S^3 \times L_T = 24^3 \times 48$$

NPLQCD, e-Print: 2404.12039 [hep-lat]



NPLQCD, e-Print: 2404.12039 [hep-lat]

Updated variational calculation:

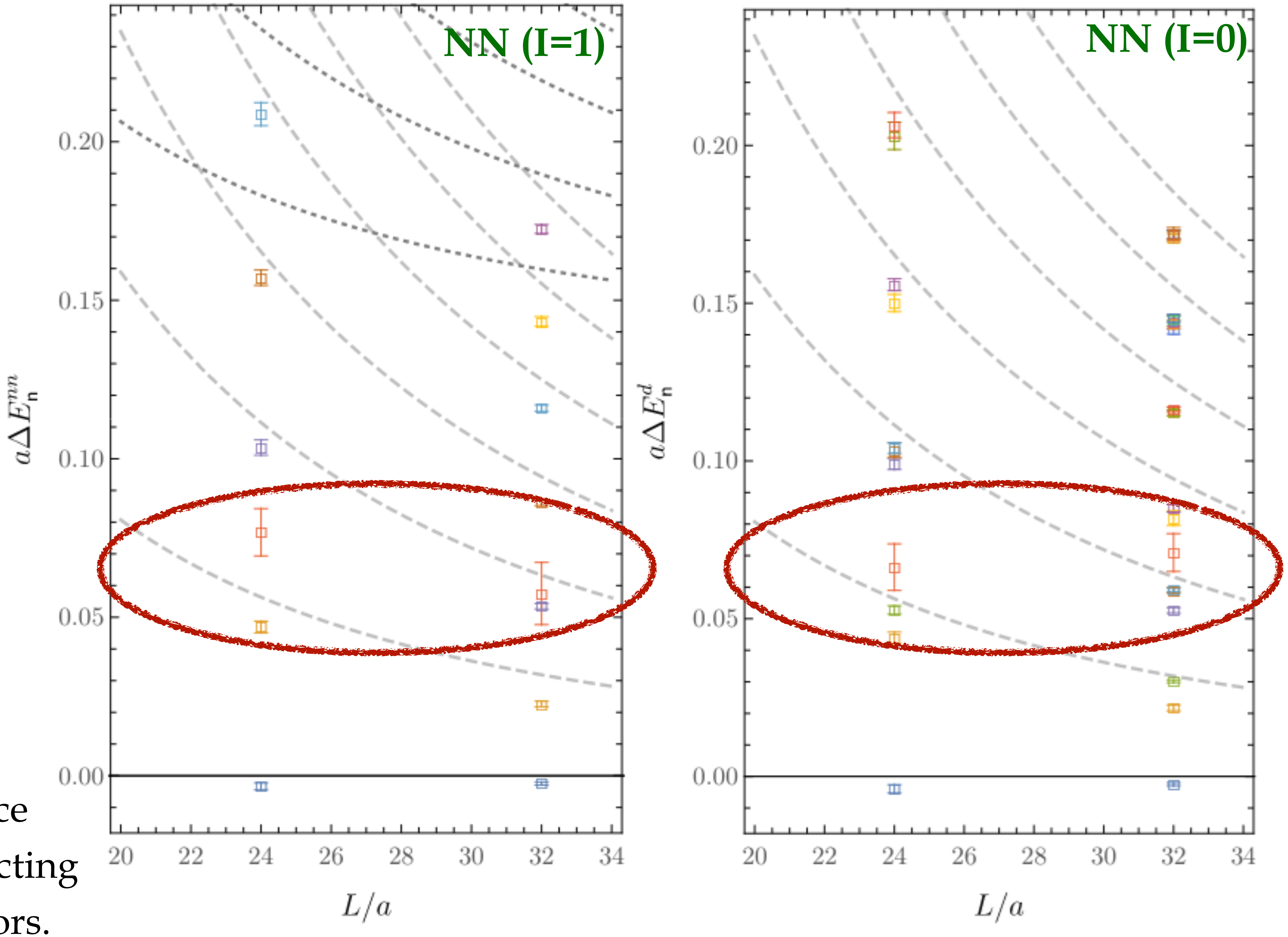
A complete basis of local NN hexaquark operators is included

No evidence for (or against) bound states

Operator dependence on variational bounds

The additional bound is observed at two lattice volumes

$L_S^3 \times L_T = 32^3 \times 48$ and $24^3 \times 48$



The additional state exhibits weak volume dependence compared with the states that fall near the non-interacting levels and overlap strongly with the dibaryon operators.

M. Lüscher, Nucl. Phys. B 364, 237 (1991) **resonance?**

Experimental evidence supports the existence of a resonance :

$$I = 0,\ J^P = 3^+ \ \rightarrow \ d^*(2380)$$

Bashkanov et al., PRL 102, 052301 (2009)

Adlarson et al. (WASA-at-COSY), PRL 106, 242302 (2011)

Adlarson et al. (WASA-at-COSY), PLB 743, 325 (2015)

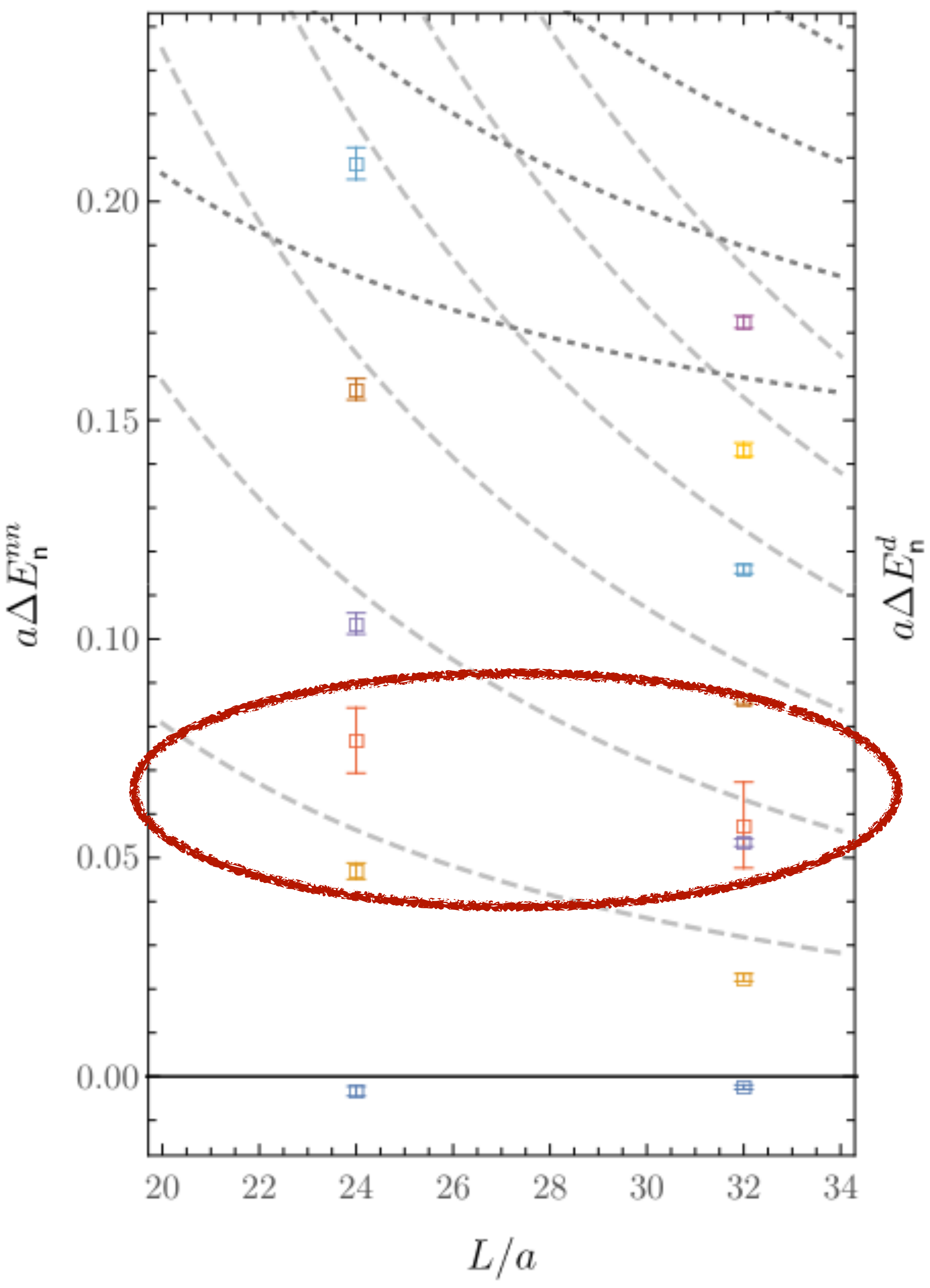
Bashkanov et al., Eur. Phys. J. A51, 87 (2015)

Adlarson et al., Eur. Phys. J. A52, 147 (2016)

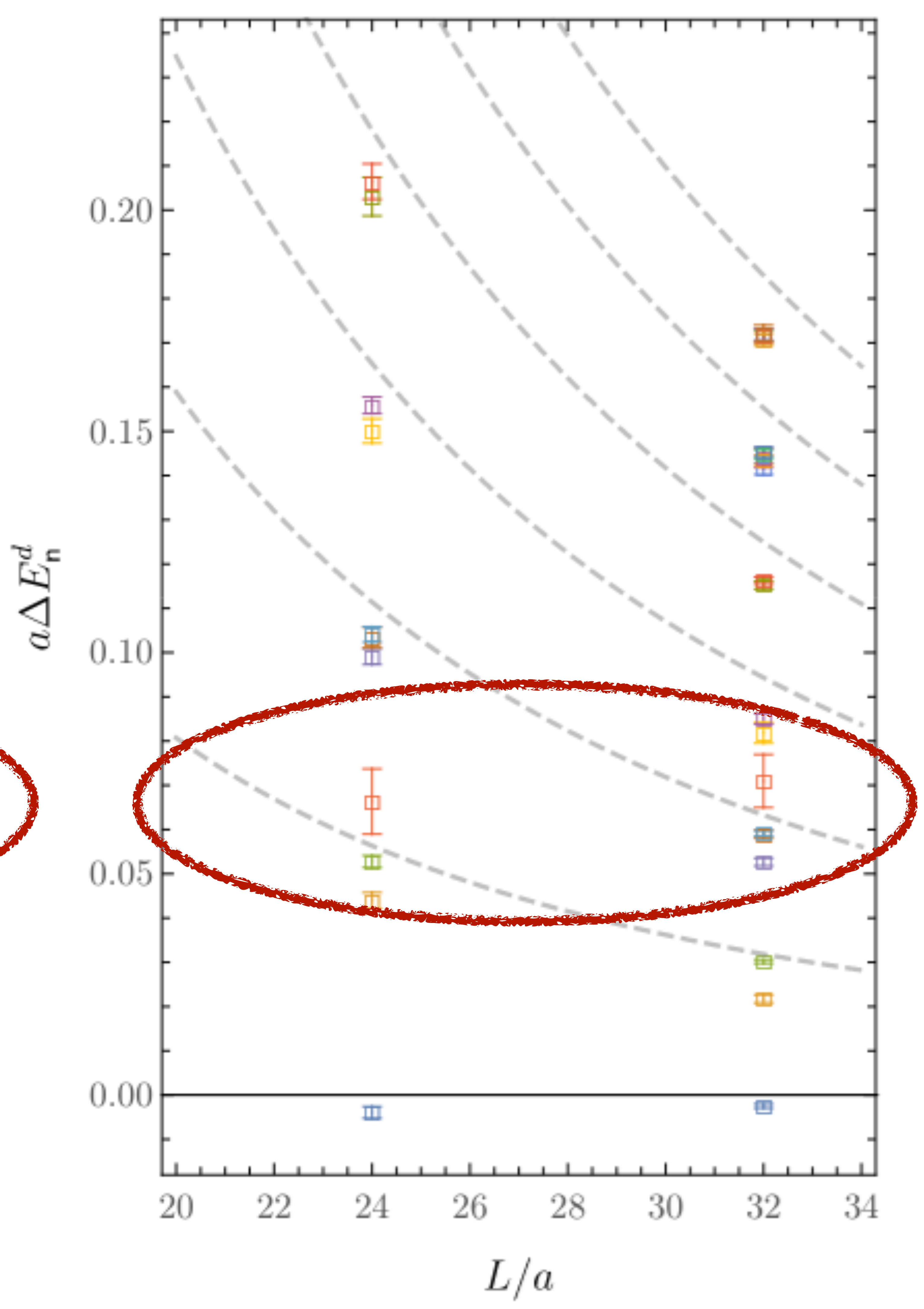
$$\Gamma_J = \Gamma_\ell \otimes \Gamma_S, \ \ \text{where } \Gamma_S \in \{A_1^+, T_1^+\}$$

S=0 ↗ ↑ S=1

$L_S^3 \times L_T = 32^3 \times 48$ and $24^3 \times 48$

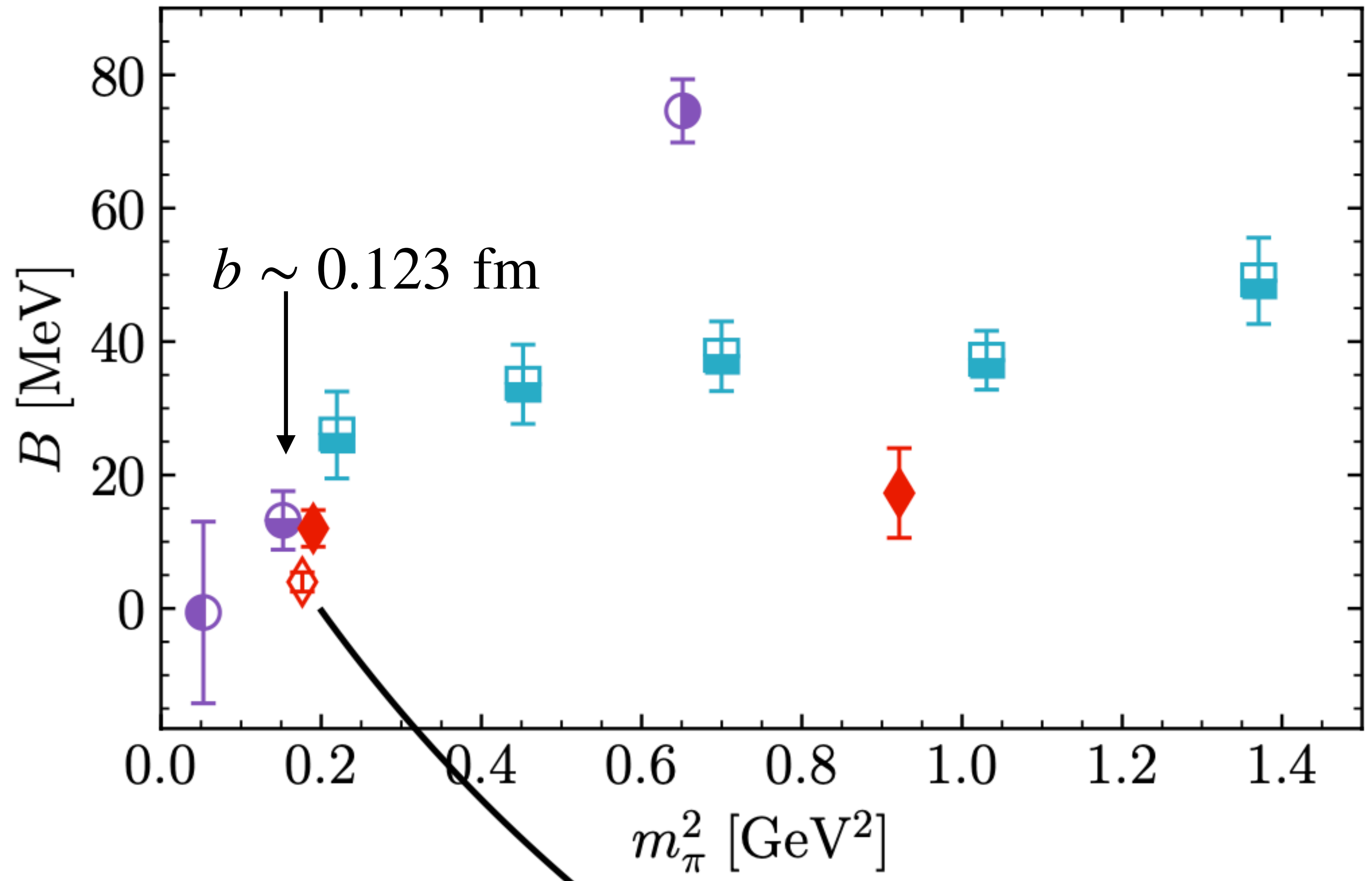


??
interesting target for future study



heavy-quark-mass analog of the d* resonance ?

Potentially important



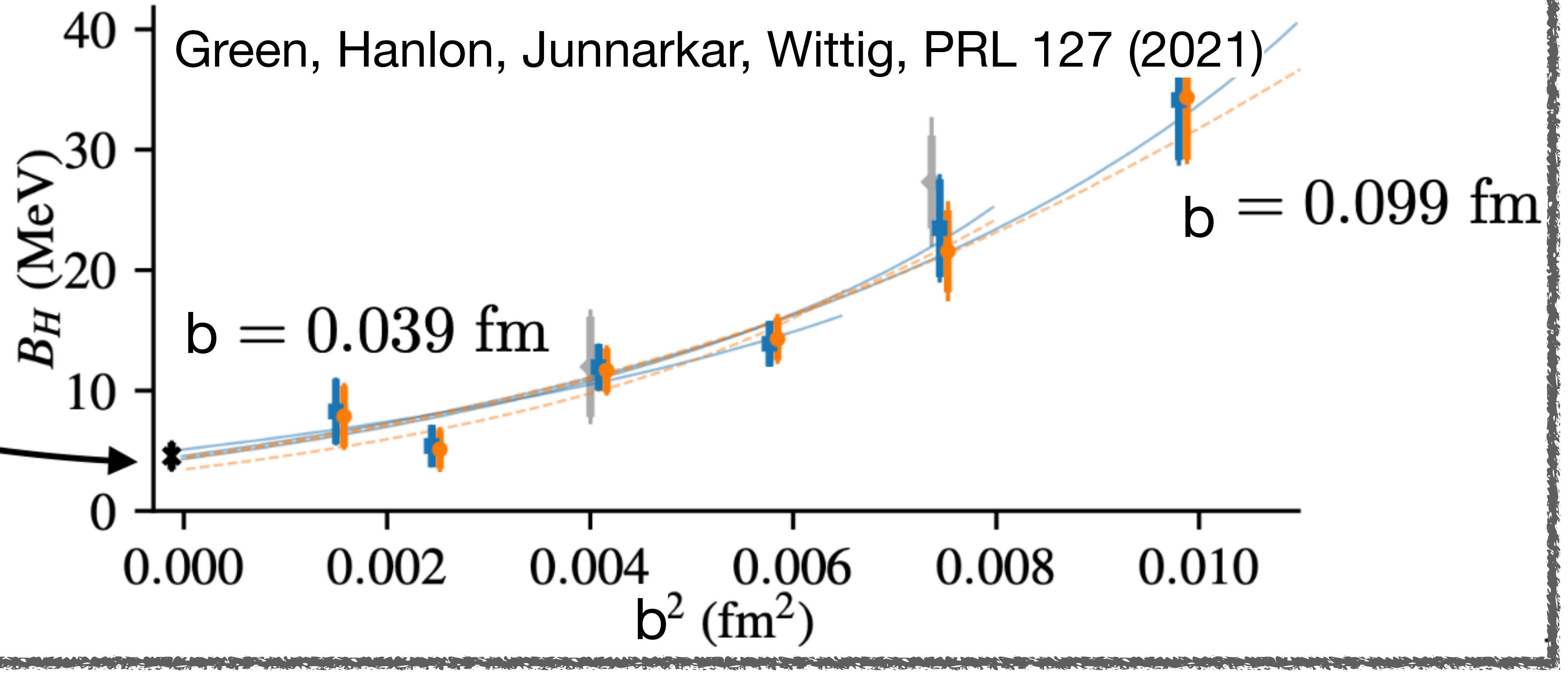
NPLQCD

HAL — Hadrons to Atomic nuclei from Lattice QCD

JG|U → first application of variational approach to BB (with only dibaryon-like operators)

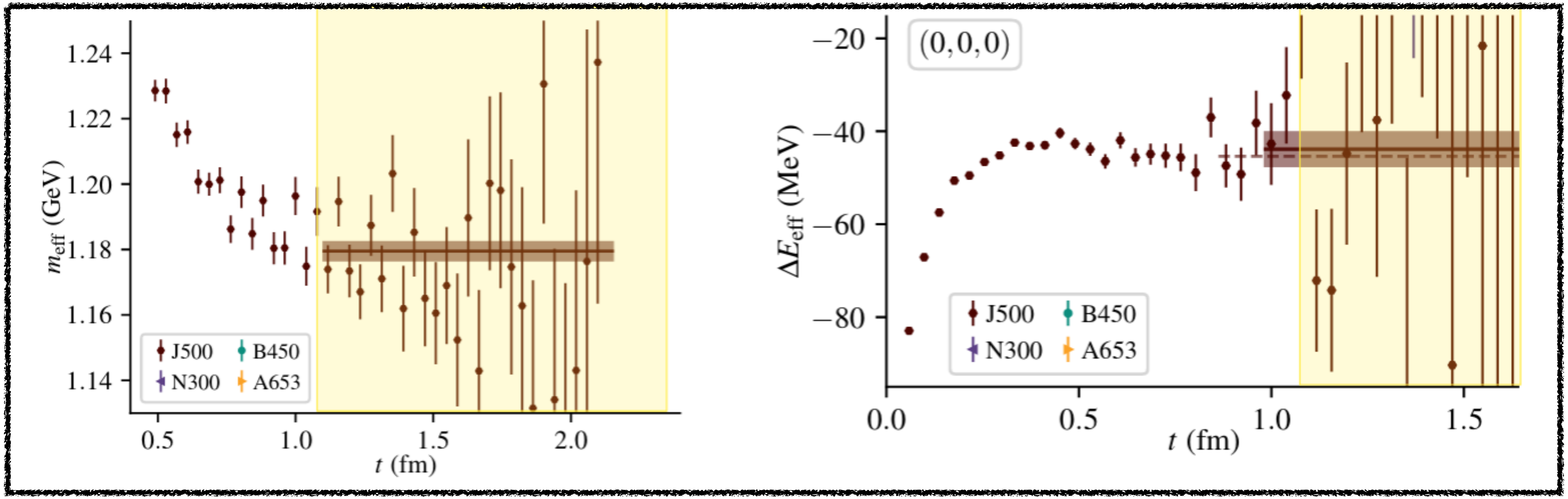
$m_\pi \sim m_K \sim 420$ MeV

first extrapolation to the continuum for BB systems



large statistical uncertainties in the region where
single-state dominates the nucleon correlator

Green, Hanlon, Junnarkar, Wittig, PRL 127 (2021)



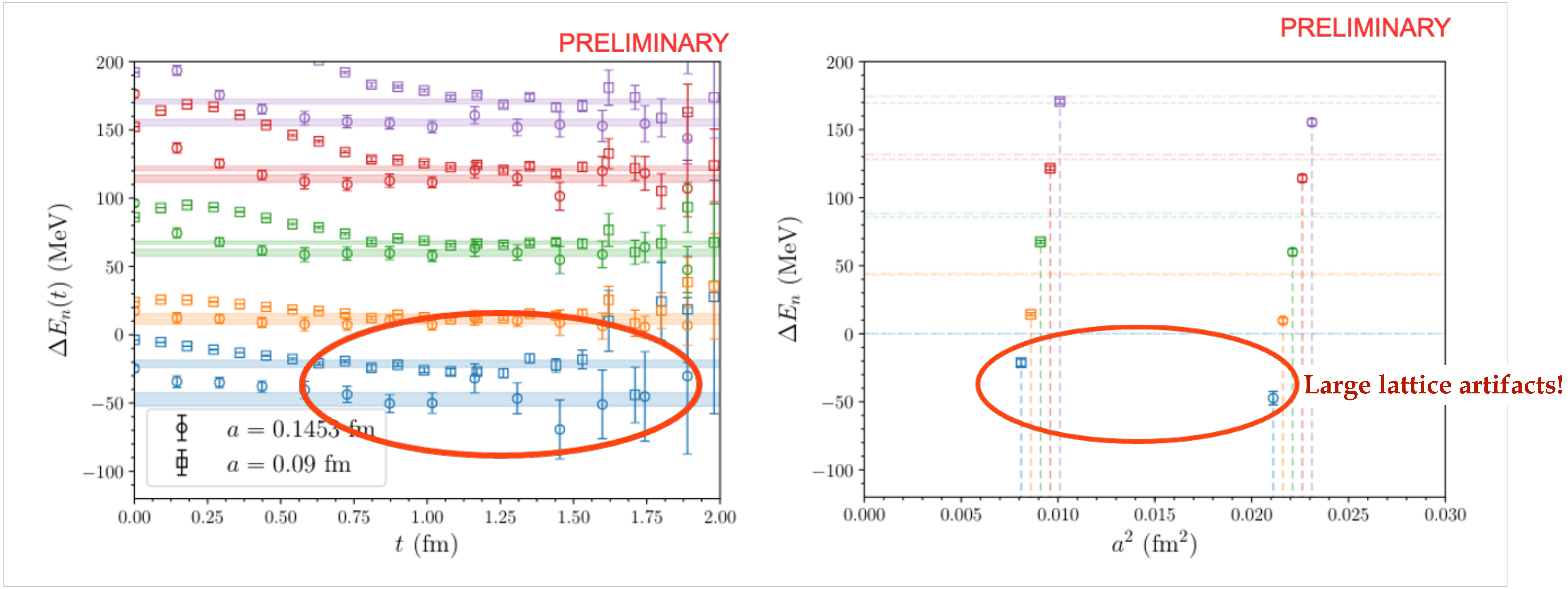
single baryon

two baryons

We have started a study of all SU(3) irreps at two different lattice spacings

L/a	T/a	L (fm)	T (fm)	a (fm)
48	64	4.13	5.50	0.086
32	48	4.64	6.96	0.145

Ex: flavor singlet channel (H-dibaryon)



- Variational studies have revealed significant interpolating-operator dependence in LQCD calculations of NN energy spectrum with unphysical quark masses

- Variational bounds don't provide conclusive evidence for (or against) bound states

- The combination of dibaryon and hexaquark operators provides strong evidence for the presence of an additional energy level below in both the deuteron and dineutron channels

- Similar analysis in the strange sector are underway

- Are lattice artefacts important? Very preliminary results seem to indicate that this is the case, but more statisticsis needed to answer this question

We have started a study of all SU(3) irreps at two different lattice spacings

In order to make physical statements and predictions, quantities determined from LQCD calculations must be extrapolated to the continuum limit.

- Calculations near the physical pion mass (coarse extrapolations at the moment) are under way

We have started studies of Octet Baryon - Octet Baryon @ m_π = 170 MeV