Structure of light and medium-heavy Λ hypernuclei with antisymmetrized molecular dynamics

Masahiro Isaka, Hosei University

Grand challenges of hypernuclear physics

Interaction: To understand baryon-baryon interaction

- 2 body interaction between baryons (Y: hyperon, N: nucleon): YN, YY interactions

  → **Studied through hypernuclei due to difficulty of YN scattering exp.**

- Many-body interaction

  → **Important issue in recent studies**

Structure: To understand many-body system of nucleons and hyperon

Many-body force and "Hyperon puzzle" in neutron star

Hyperon puzzle

Massive ($2M_\odot$) neutron stars

2010 PSR J1614-2230 $(1.97 \pm 0.04) M_\odot$

2013 PSR J0348-0432 $(2.01 \pm 0.04) M_\odot$

?

Softening of EOS by hyperon mixing

How do we resolve?

Baryon many(three)-body force

If strong repulsion exists acting in hyperonic channels,
EOS of neutron star matter becomes stiff

Important issue in hypernuclear physics:
to reveal effects of hyperonic many-body force in Λ hypernuclei

Grand challenges of hypernuclear physics

Interaction: To understand baryon-baryon interaction

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- **Many-body interaction**

  → **Important issue in recent studies**

Structure: To understand many-body system of nucleons and hyperon

- **Addition of hyperon(s) shows new aspects of nuclear structure**

e.g.) structure change by hyperon(s)

- No Pauli exclusion between N and Y
- YN interaction is different from nuclear force

} Hyperon as an impurity in hypernuclei

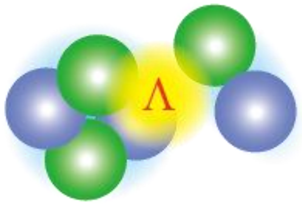
Structure of Λ hypernuclei

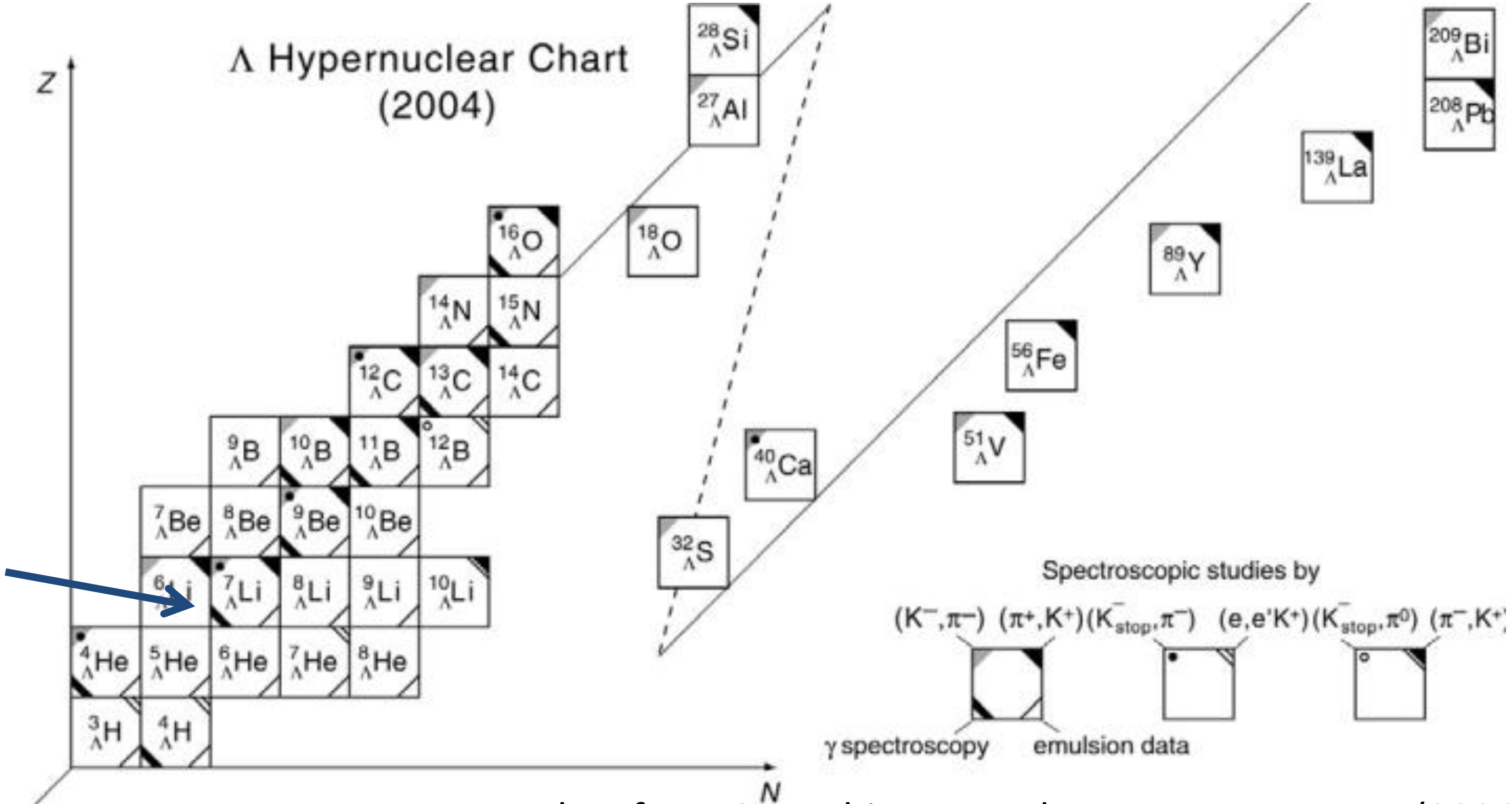
◆Λ hypernuclei observed so far

- **Concentrated in light Λ hypernuclei**
- **Most have well-developed cluster structure**

Light Λ hypernuclei

Developed cluster





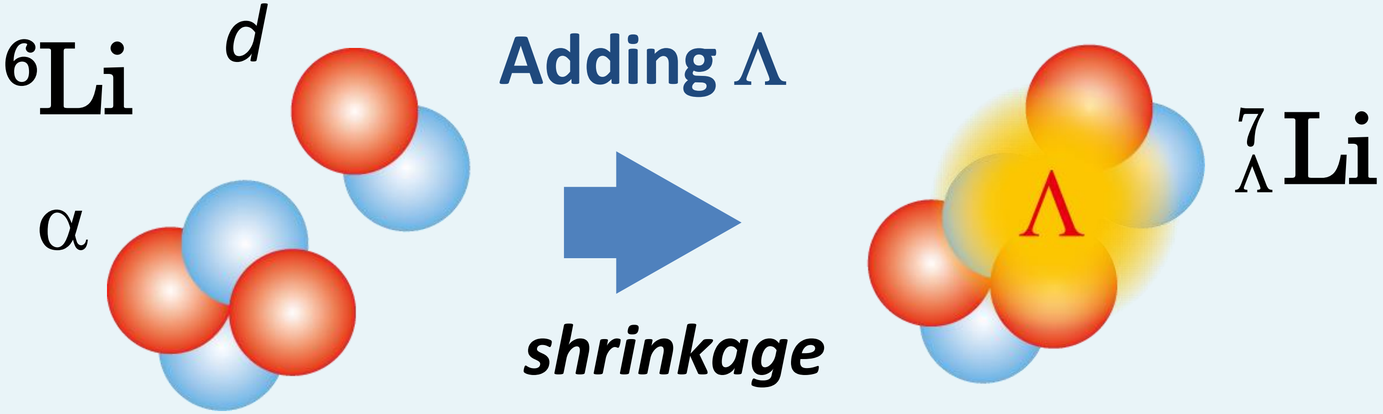
Taken from O. Hashimoto and H. Tamura, PPNP **57**(2006),564.

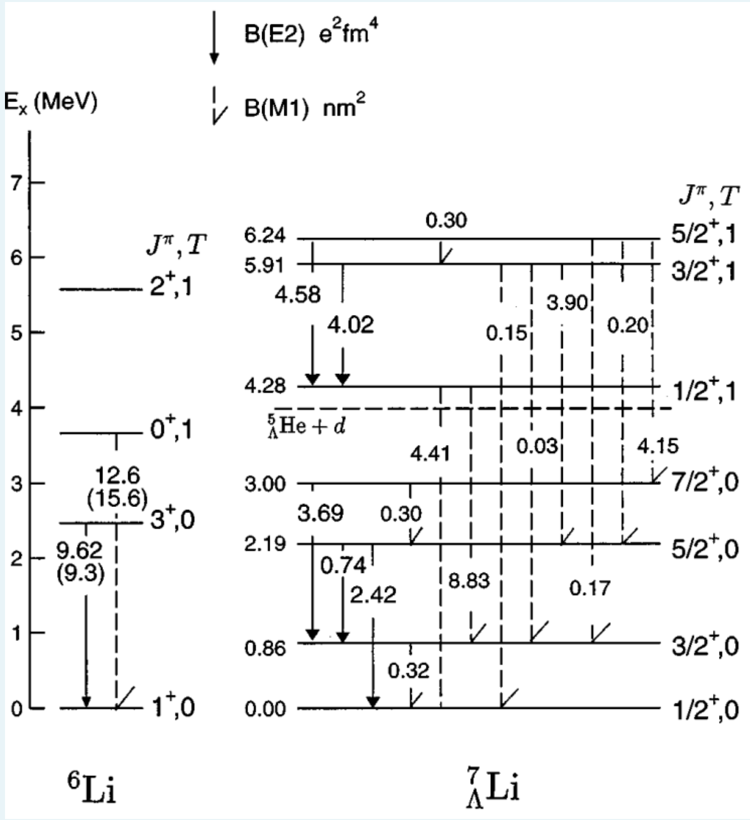
Cluster structure in light hypernuclei

◆Famous example of "impurity effects"

Example: $^{7}_{\Lambda}\mathrm{Li}$

Motoba, *et al.*, PTP**70**,189 (1983)
Hiyama, *et al.*, PRC**59** (1999), 2351.
Tanida, *et al.*, PRL**86** (2001), 1982.





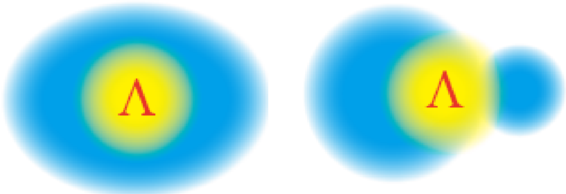
- Λ reduces inter-cluster distance b/w α + *d* of the core nucleus $^{6}\mathrm{Li}$
- Confirmed through B(E2) reduction

Toward heavier and exotic Λ hypernuclei

◆Experiments at J-PARC, JLab, … *etc.*

- **Heavier(*sd*-shell and more) hypernuclei can be produced**

Coexistence of structures



Light Λ hypernuclei

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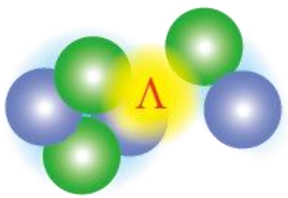
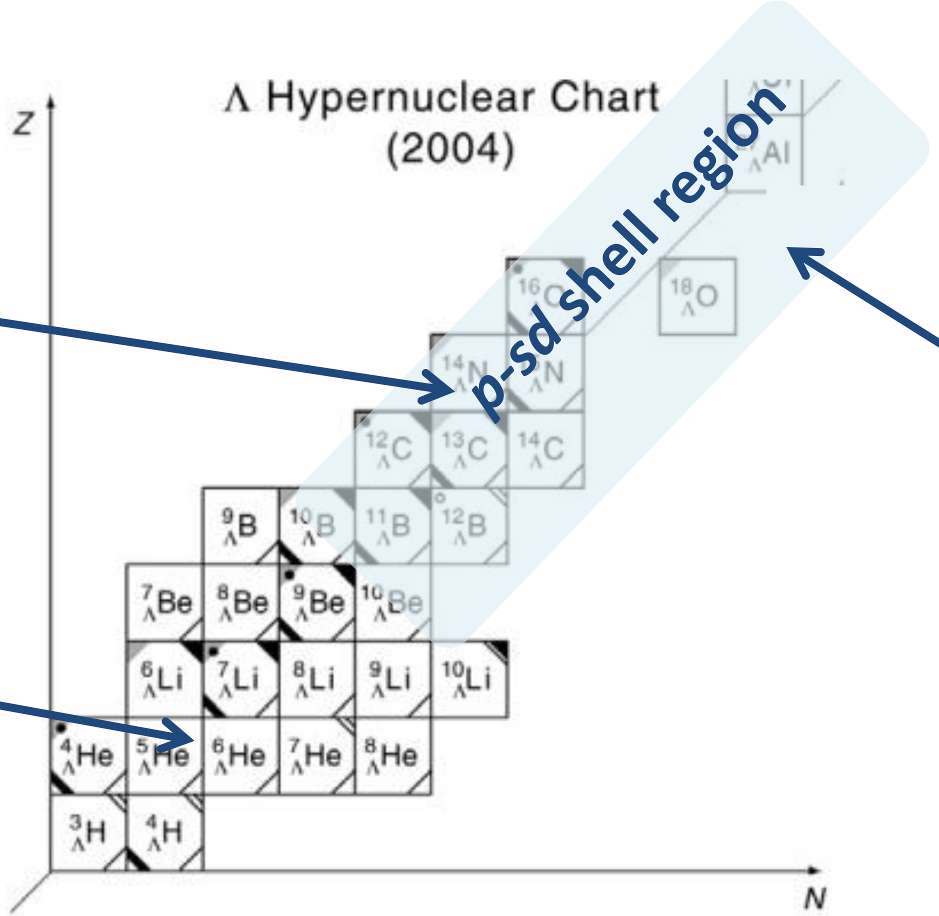
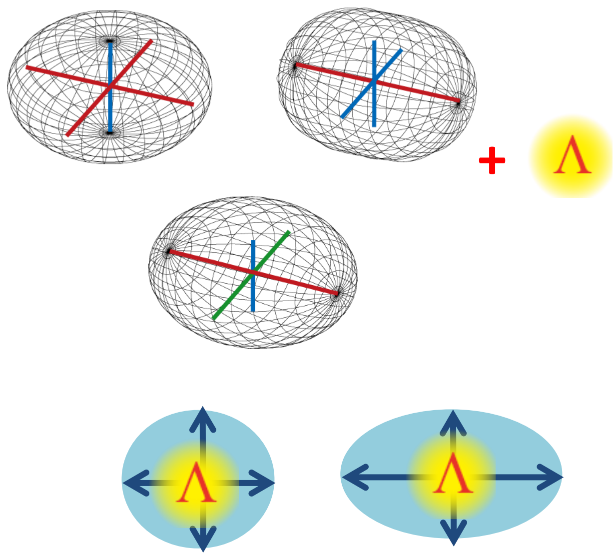




Various deformations



What will happen if a Λ is coupled to nuclei with various structures ?

What will happen if a Λ particle is coupled to nuclei ?

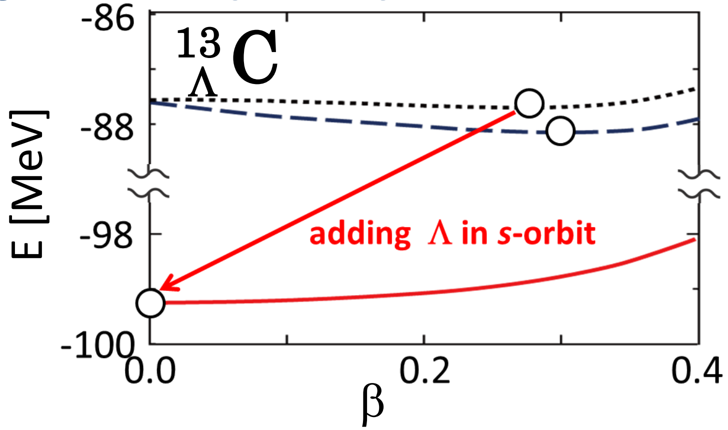
- **Dynamical changes of nuclear structure**
  - Changes of cluster structure
  - Deformation changes
- **Sensitivity of the Λ binding energy B_Λ on nuclear structure**
  - Dependence of B_Λ on nuclear deformation
  - Mass number A dependence & many-body force effects
- **Coupling of Λ particle in *p* orbit to clustering/deformed core nuclei**
  - Genuine hypernuclear states
  - Possibility to probe nuclear deformation using Λ particle

Deformation change by Λ particle

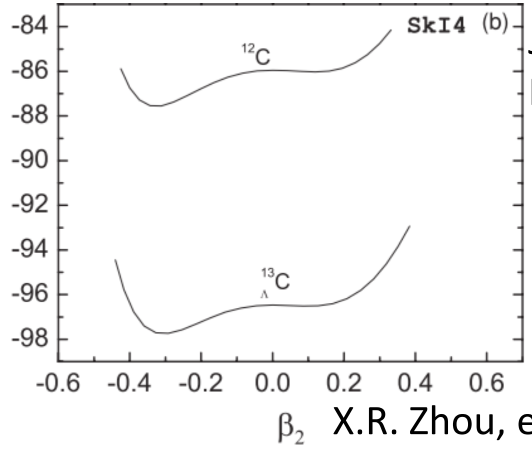
Many authors predict that Λ in *s*-orbit reduces nuclear deformation

Antisymmetrized molecular Dynamics (AMD)

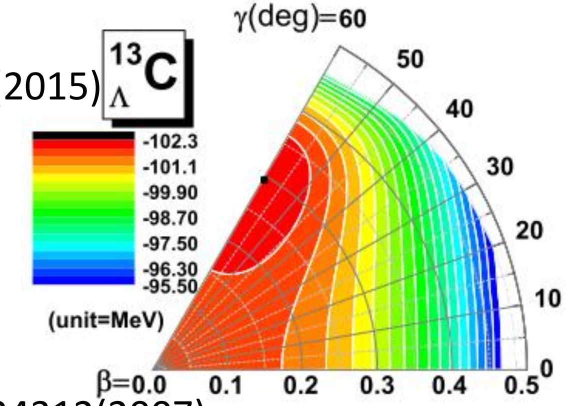
M.I, et al., PRC**83**, 044323(2011)



Skyrme-Hartree-Fock (SHF)



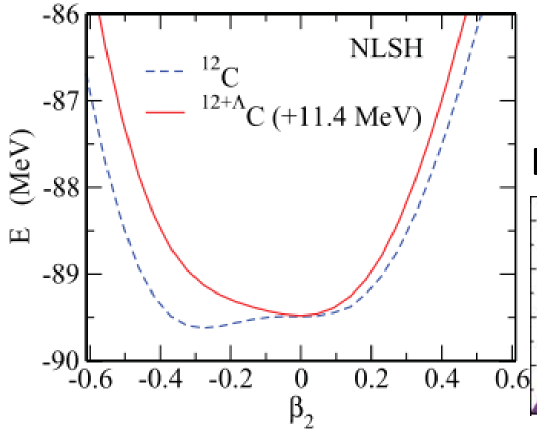
J.W. Cui, et al, PRC**91**,054306(2015)



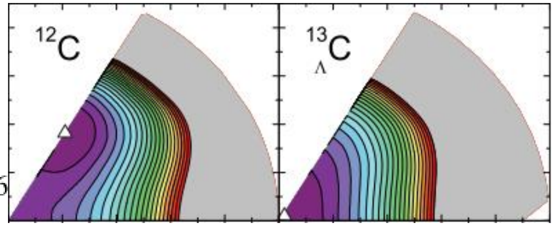
X.R. Zhou, et al., PRC**76**, 034312(2007)

Relativistic mean-field (RMF)

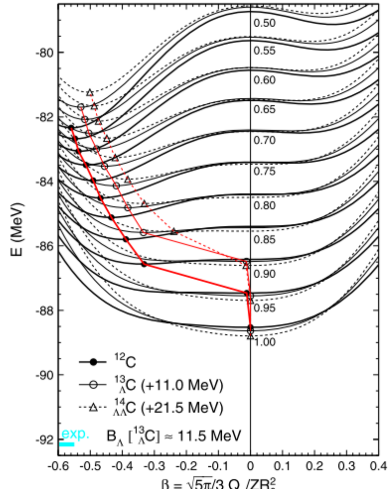
Win and Hagino, PR C**78**, 054311(2008)



B.N. Lu, et al., PRC**84**, 014328 (2011)



RMF & SHF



H. J. Schulze, et al., PTP**123**, 569('10)

Deformations/level structure with beyond-mean-field

J.W. Cui, X.R. Zhou, H.J. Schulze, PRC**91**,054306('15)

H. Mei, K. Hagino, J.M. Yao, T. Motoba, PRC**91**, 064305(2015); **97**, 064318(2018)

... and so on

Structure calculation of hypernuclei

Antisymmetrized Molecular Dynamics for hypernuclei (HyperAMD)

Hamiltonian

$$\hat{H} = \hat{T}_N + \hat{V}_{NN} + \hat{T}_\Lambda + \hat{V}_{\Lambda N} - \hat{T}_g$$

NN : Gogny D1S
ΛN : YNG interaction

Wave function

- **Nucleon part : Slater determinant**
  Spatial part of s.-p. w.f. is described as Gaussian packets

$$\varphi_N(\vec{r}) = \frac{1}{\sqrt{A!}} \det[\varphi_i(\vec{r}_j)]$$

$$\varphi_i(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \nu_\sigma (r - Z_i)_\sigma^2\right] \chi_i \eta_i$$

$$\chi_i = \alpha_i \chi_\uparrow + \beta_i \chi_\downarrow$$

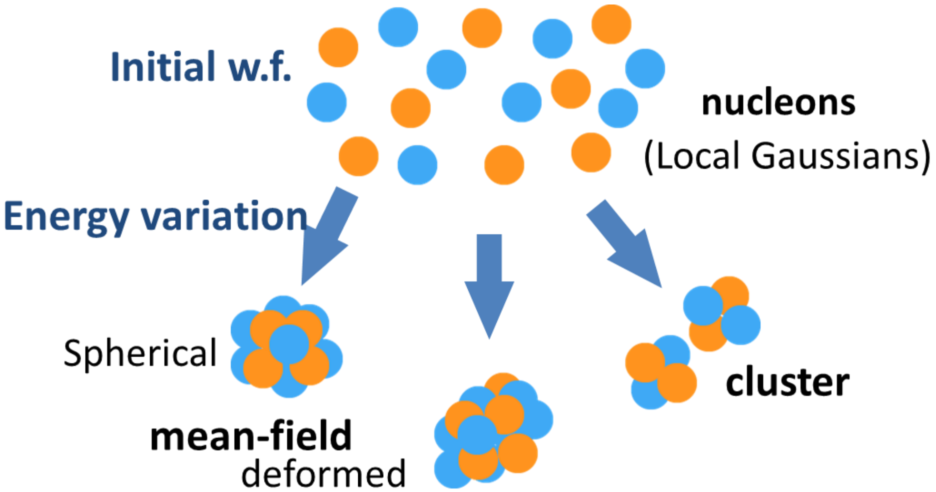
- **Single-particle w.f. of Λ hyperon:**
  Superposition of Gaussian packets

$$\varphi_\Lambda(r) = \sum_m c_m \varphi_m(r)$$

$$\varphi_m(r) \propto \exp\left[-\sum_{\sigma=x,y,z} \mu \nu_\sigma (r - z_m)_\sigma^2\right] \chi_m$$

$$\chi_m = a_m \chi_\uparrow + b_m \chi_\downarrow$$

- **Total w.f. :** $\psi(\vec{r}) = \sum_m c_m \varphi_m(r_\Lambda) \otimes \frac{1}{\sqrt{A!}} \det[\varphi_i(\vec{r}_j)]$



M.I., *et al.*, PRC**83**(2011) 044323, M. I., *et al.*, PRC**83**(2011) 054304

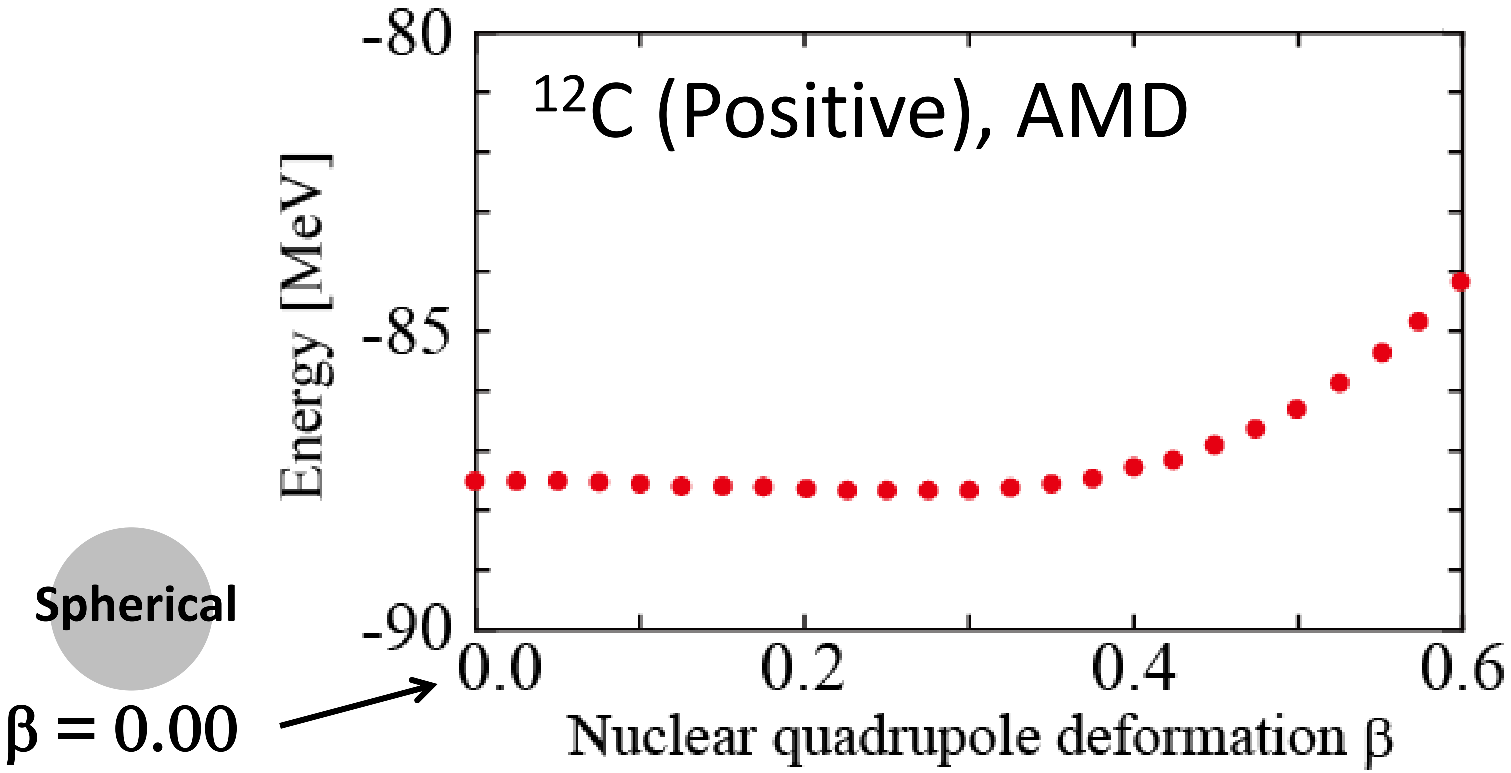
Deformation change by Λ particle

◆How to analyze from energy surface

Example: ^{12}C with AMD(antisymmetrized molecular dynamics)

- Energy variation at each β (and γ) ⟶ energy curve as a function of β
- Energy minimum at (β,γ)

M.I, et al., PRC**83**, 044323(2011)



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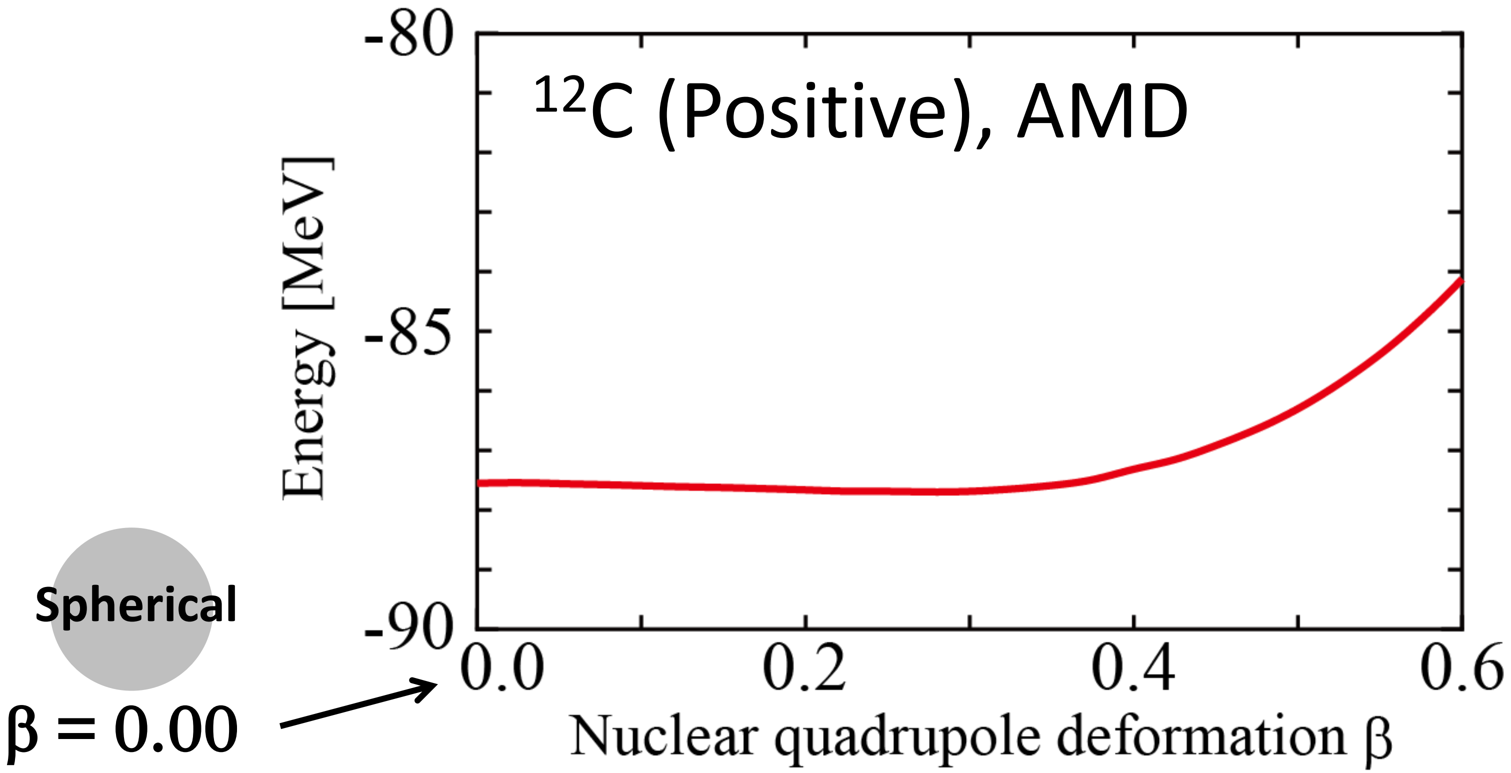

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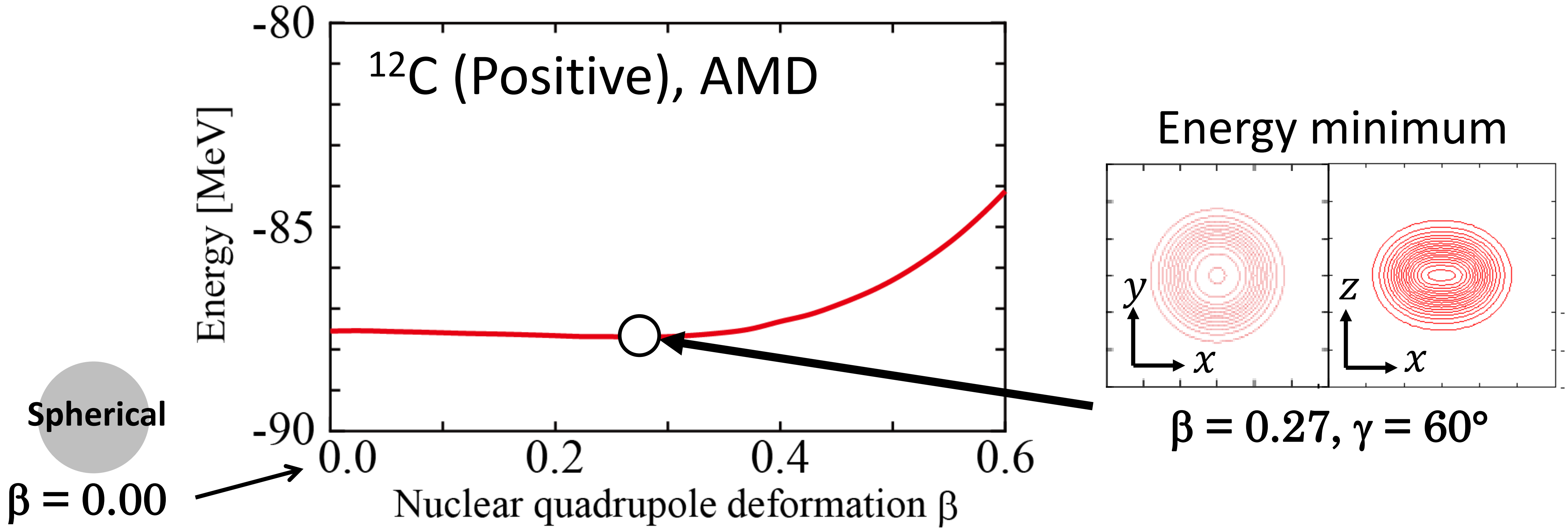


Deformation change by Λ in *s*-orbit

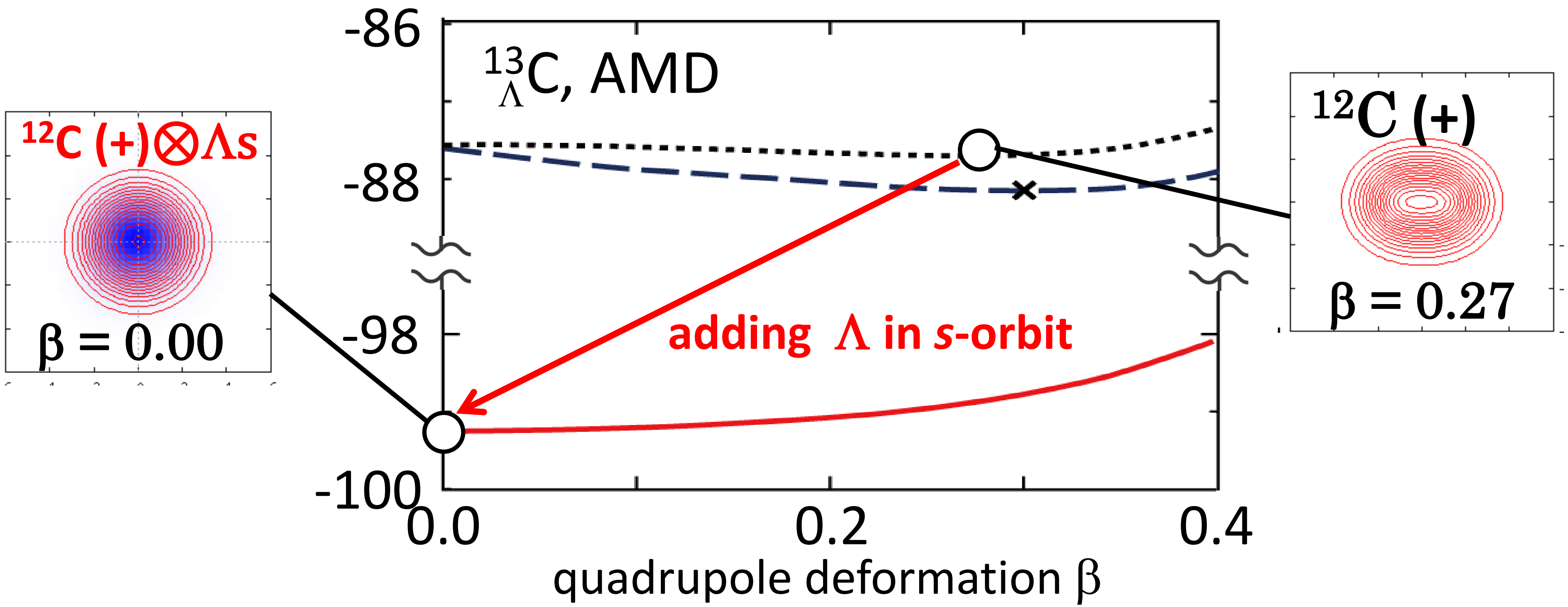
◆How to analyze from energy surface

M.I, et al., PRC**83**, 044323(2011)

Example: ^{12}C with AMD(antisymmetrized molecular dynamics)

- Energy variation of hypernucleus

→ energy minimum moves to smaller β with Λ in *s*-orbit

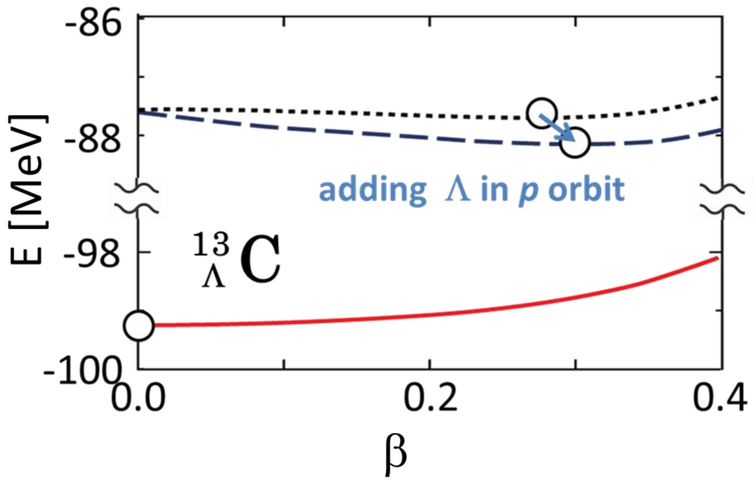


Deformation change by Λ particle

Λ in *p*-orbit enhances nuclear deformation

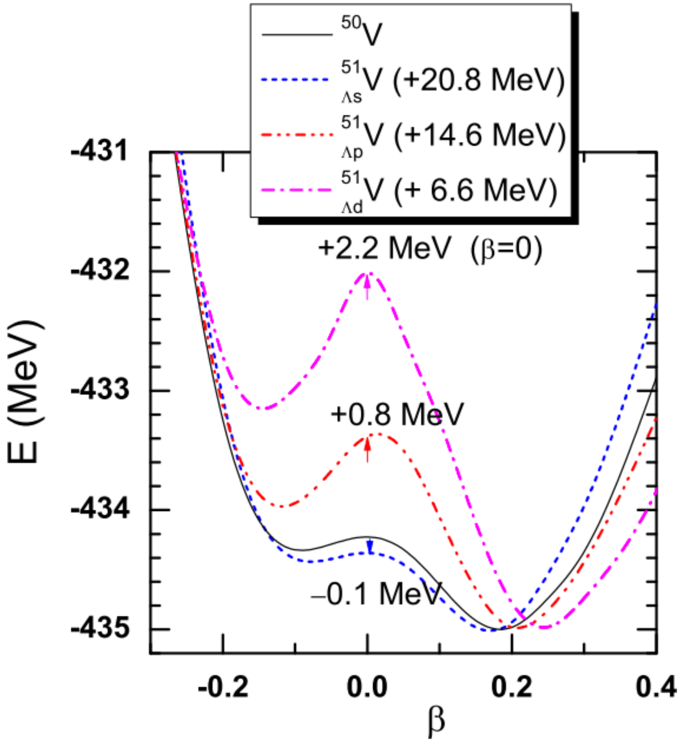
Antisymmetrized molecular dynamics (AMD)

M.I, et al., PRC**83**, 044323(2011)



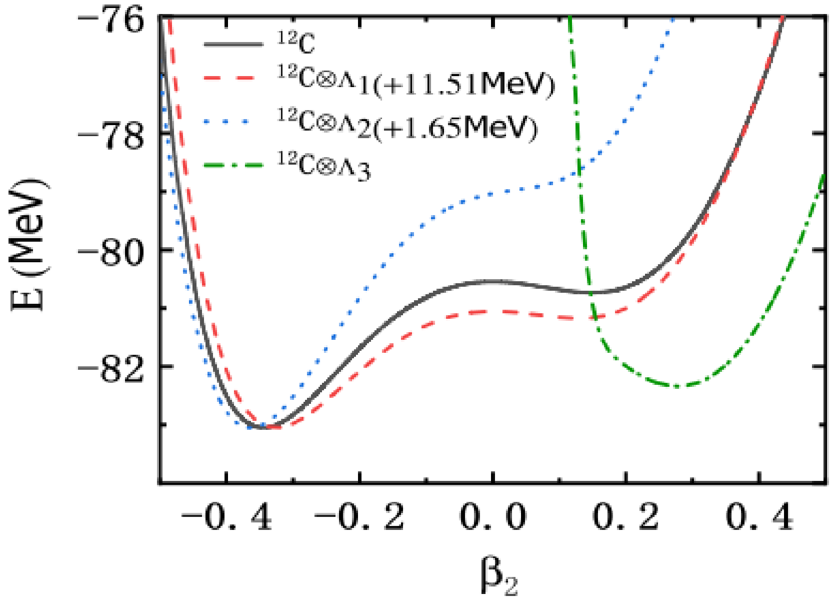
Triaxially deformed relativistic mean-field

W. X. Xue, et al., PRC**91**, 024327(2015)



Deformed Skyrme-Hartree-Fock (DSHF)

Bi-Cheng Fang et al., EPJA**56**,11(2020)

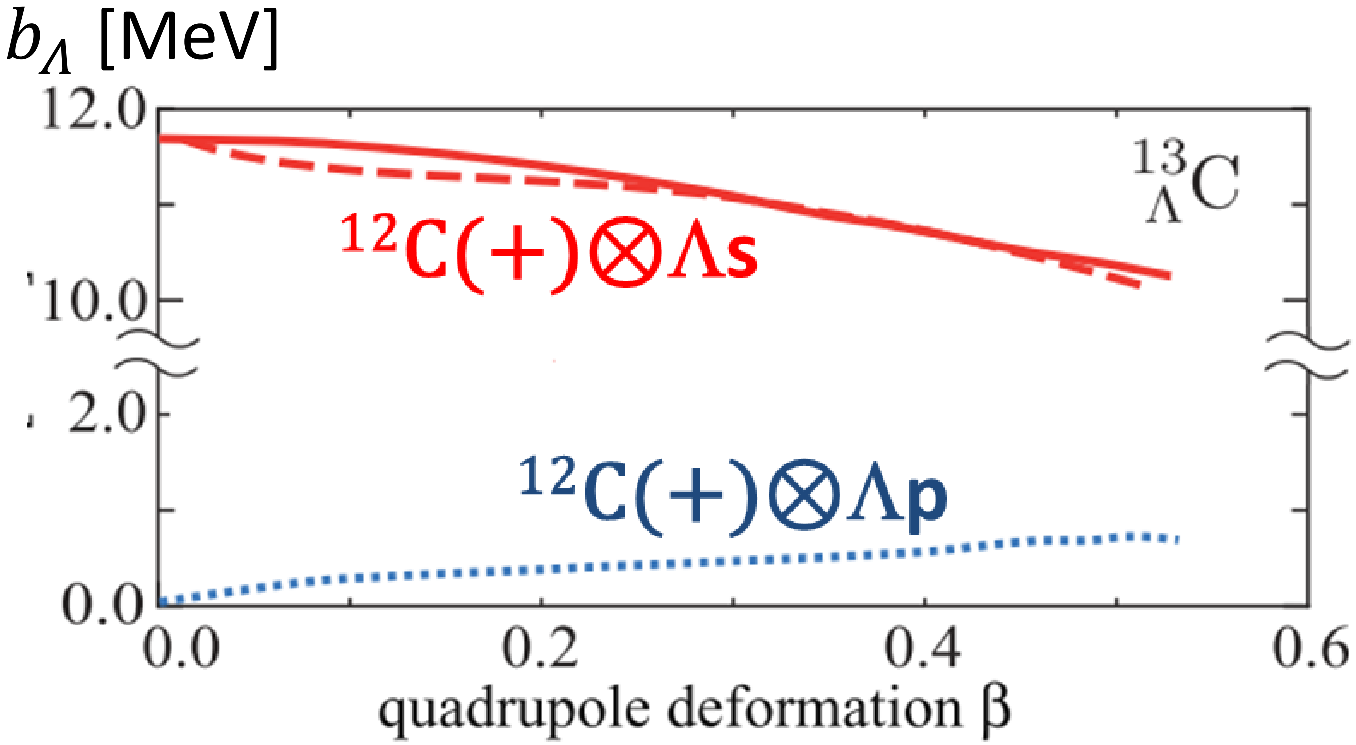


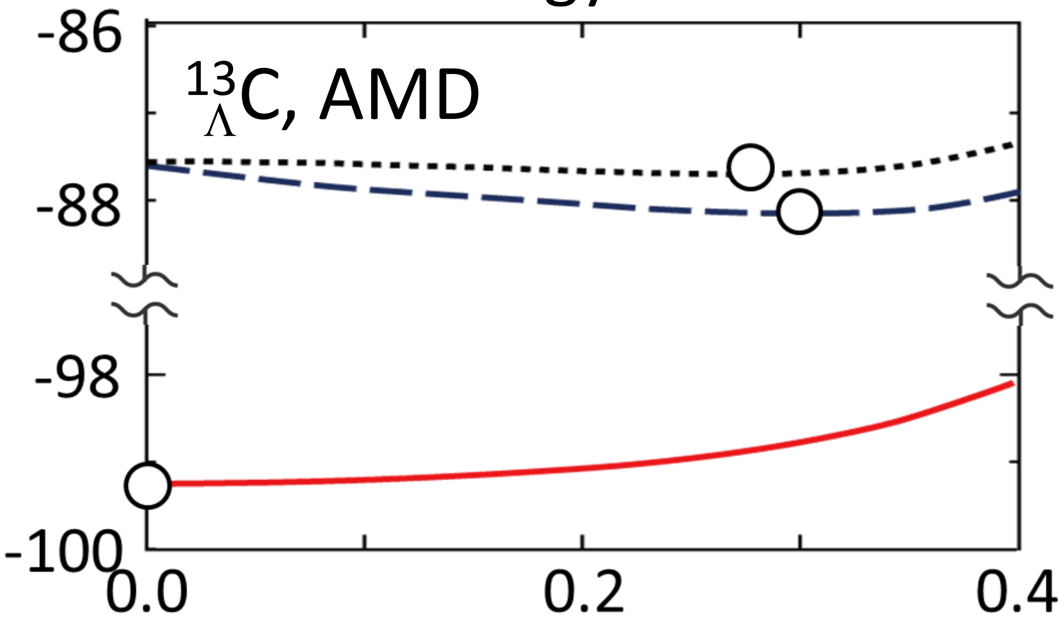
Deformation change of Λ in higher orbits such as *d*-orbit is also predicted by several papers:
W. X. Xue, et al., PRC**91**, 024327(2015), X. Y. Wu, et al., PRC**95**, 034309(2017)

Why does Λ change nuclear deformation?

- Λ in *s* orbit is deeply bound at small β, while Λ in *p* orbit prefers deformation
- Competition b/w Λ binding energy and energy surface of core nucleus

Λ binding energy: $b_\Lambda(\beta) = E_{core}(\beta) - E_\Lambda(\beta)$



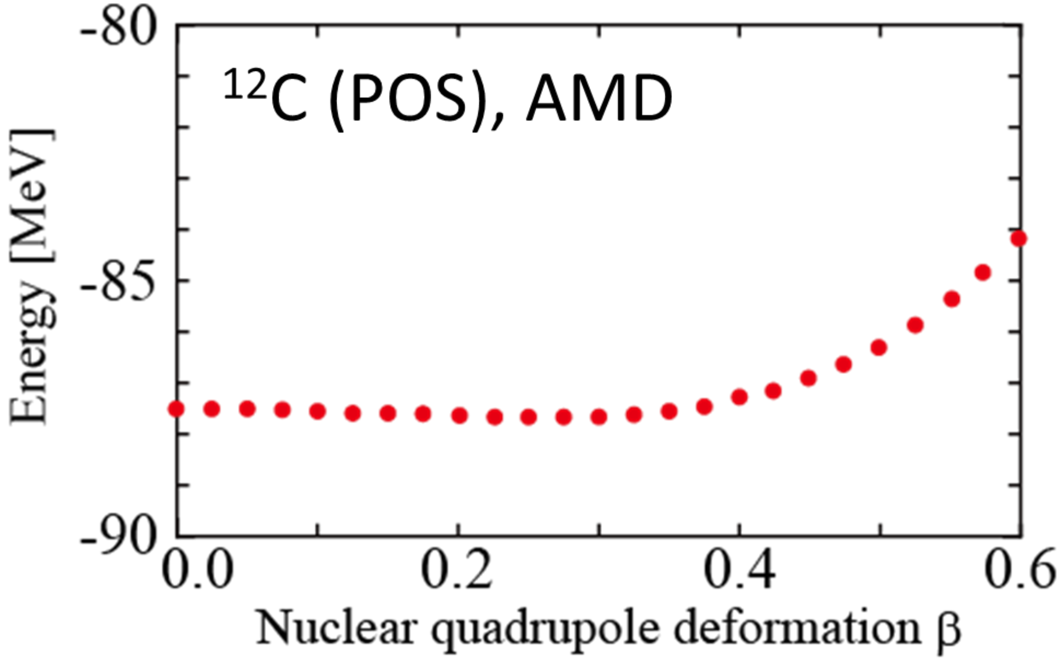


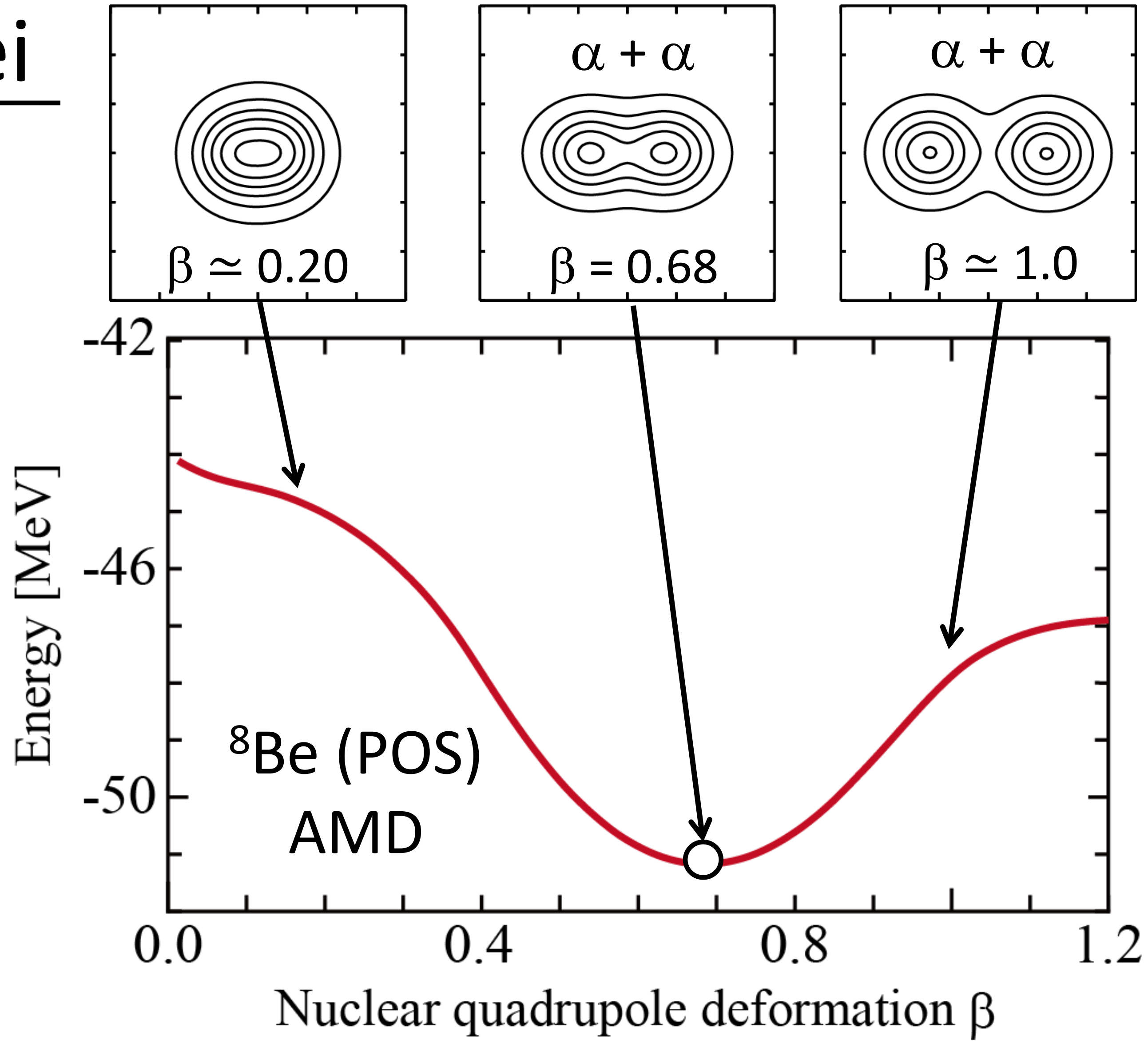
M.I, et al., PRC**83**, 044323(2011)

"binding energy of Λ" vs. "energy surface of the core nuclei"

Energy surface of core nuclei

Shape of energy surface is related to nuclear structure



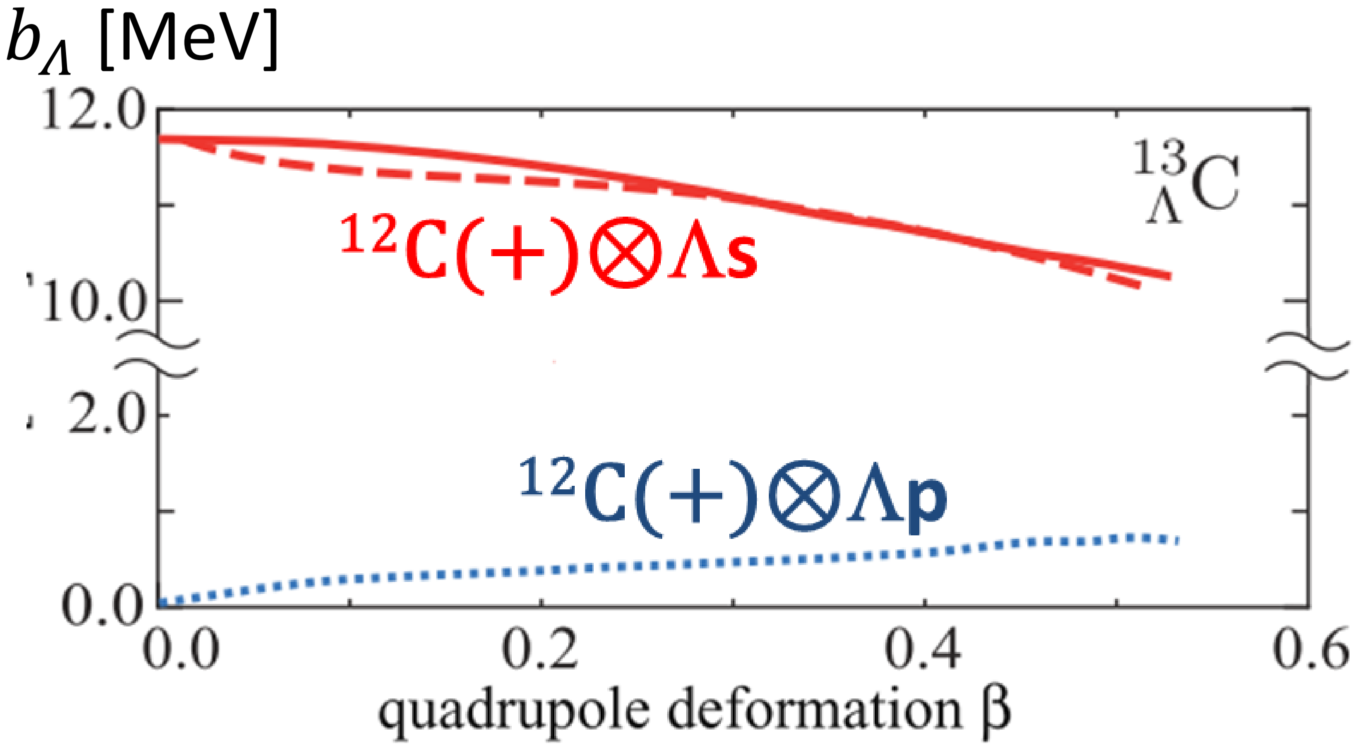


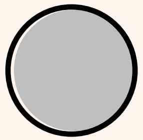
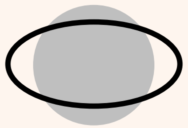
Energy increases when 2α clustering is reduced/enhanced from the minimum

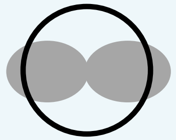
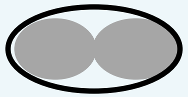
Deformation change by Λ particle

“*Overlap between Λ and N*” is the key!

Λ in *s*-orbit (*p*-orbit) is deeply bound with smaller β (larger β) due to larger overlap between Λ and nucleons



Λ in *s* orbit	 **Small β**	 **Large β**
Overlap b/w Λ & N	**Large**	**Small**
ΛN attraction	**Large**	**Small**

Λ in *p* orbit	 **Small β**	 **Large β**
Overlap b/w Λ & N	**Small**	**Large**
ΛN attraction	**Small**	**Large**

M.I, et al., PRC**83**, 044323(2011)

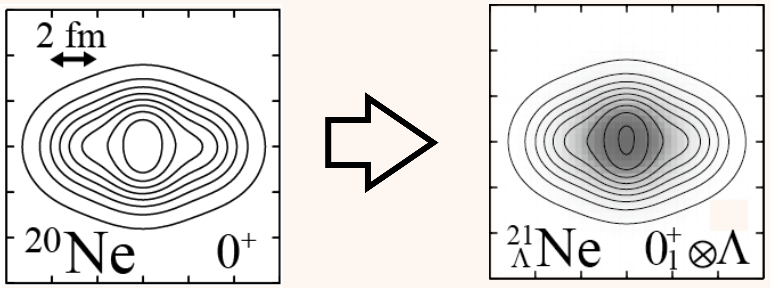
Structure dependence of “impurity effects”

◆Example: $^{21}_{\Lambda}$Ne (prediction by AMD calc)

M. Isaka, et al., PRC**83**, 054304(2011)

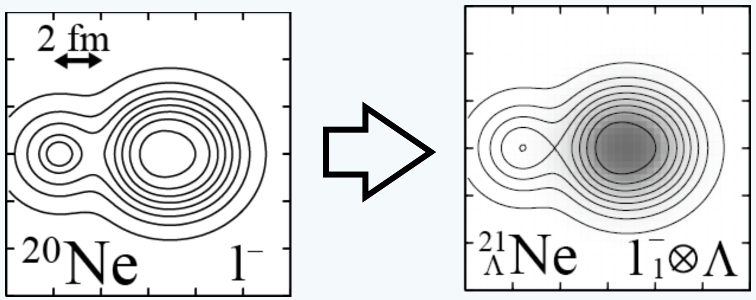
- **Shrinkage/deformation change are larger in α + ^{16}O + Λ cluster states, which appears as difference in intra-band B(E2) reduction**

Ground band



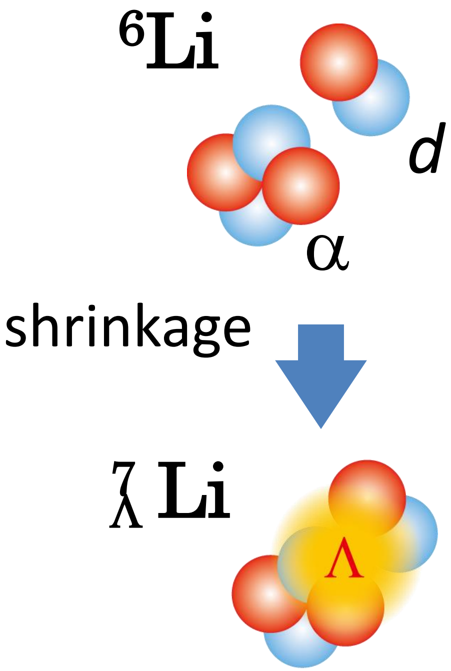
$K^\pi=0^+$ (^{20}Ne)	r_{RMS}(fm)	$0^+ \otimes \Lambda s$ ($^{21}_{\Lambda}$Ne)	r_{RMS}(fm)	Δr_{RMS}(fm)
0+	2.97	(1/2)+	2.92	−0.05
2+	2.96	(3/2)+	2.91	−0.05
		(5/2)+	2.91	−0.05
4+	2.93	(7/2)+	2.87	−0.06
		(9/2)+	2.88	−0.04
6+	2.87	(11/2)+	2.81	−0.05
		(13/2)+	2.83	−0.04

$K^\pi = 0^-$ (α + ^{16}O) band



$K^\pi=0^-$ (^{20}Ne)	r_{RMS}(fm)	$0^- \otimes \Lambda s$ ($^{21}_{\Lambda}$Ne)	r_{RMS}(fm)	Δr_{RMS}(fm)
1−	3.27	(1/2)−	3.15	−0.11
		(3/2)−	3.15	−0.11
3−	3.24	(5/2)−	3.13	−0.11
		(7/2)−	3.14	−0.10
5−	3.23	(9/2)−	3.11	−0.12
		(11/2)−	3.11	−0.13
7−	3.23	(13/2)−	3.06	−0.17
		(15/2)−	3.05	−0.18

cf. $^{7}_{\Lambda}$Li



What will happen if a Λ particle is coupled to nuclei ?

- **Dynamical changes of nuclear structure**
  - Changes of cluster structure
  - Deformation changes
- **Sensitivity of the Λ binding energy B_Λ on nuclear structure**
  - Dependence of B_Λ on nuclear deformation
  - Mass number A dependence & many-body force effects
- **Coupling of Λ particle in *p* orbit to clustering/deformed core nuclei**
  - Genuine hypernuclear states
  - Possibility to probe nuclear deformation using Λ particle

Dependence of B_Λ on nuclear deformation

B_Λ is smaller in largely deformed state than less deformed state

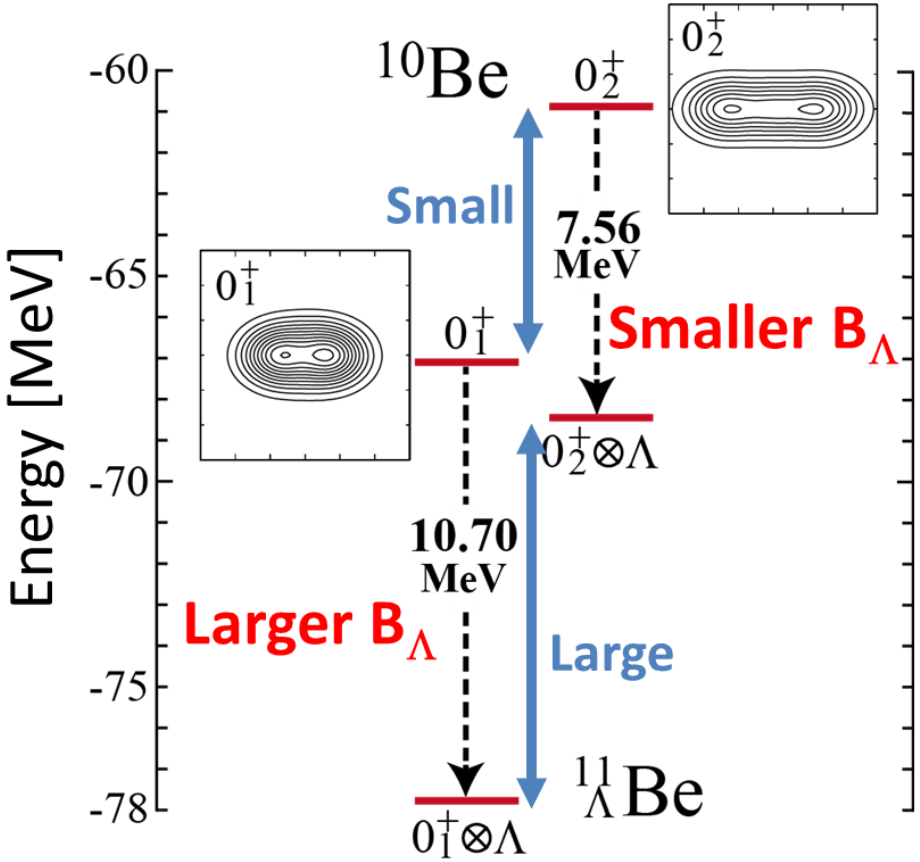
Example:

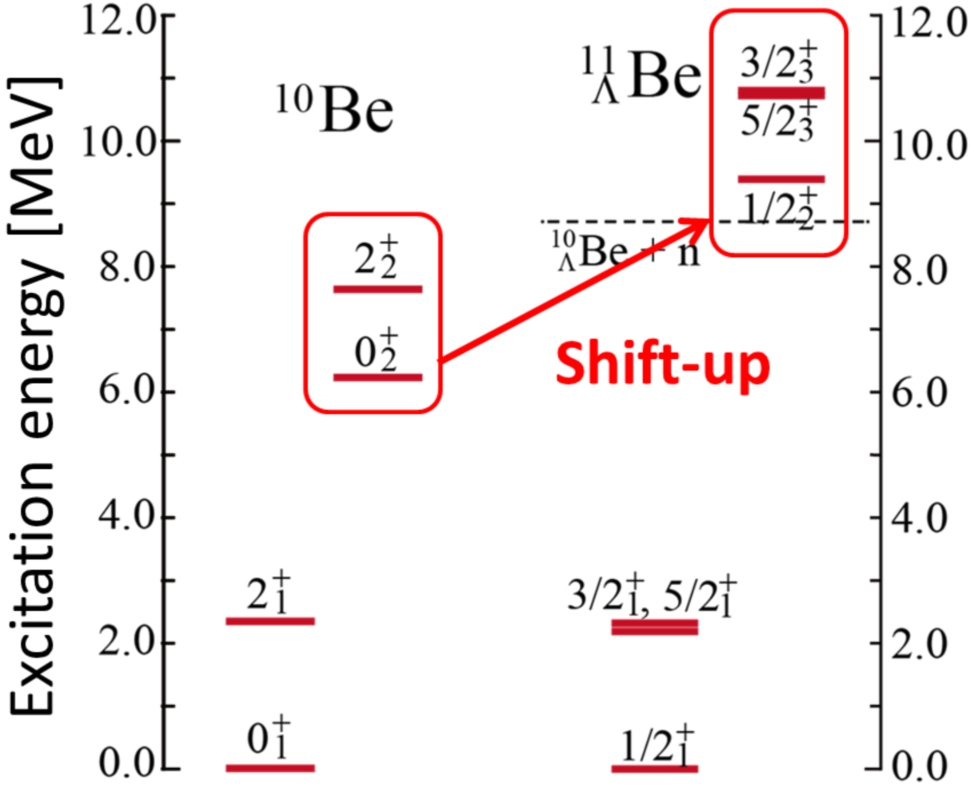
$^{11}_{\Lambda}$Be

AMD calc

M.I. and Kimura, PRC**92** (2015)

- Different deformations coexist near the ground state
- Λ in s orbit reduces it, but the difference remains
- B_Λ is different due to overlap → If different deformation coexist, shift-up/down in excitation spectra





Dependence of B_Λ on nuclear structure

B_Λ is smaller in cluster states than mean-field like states

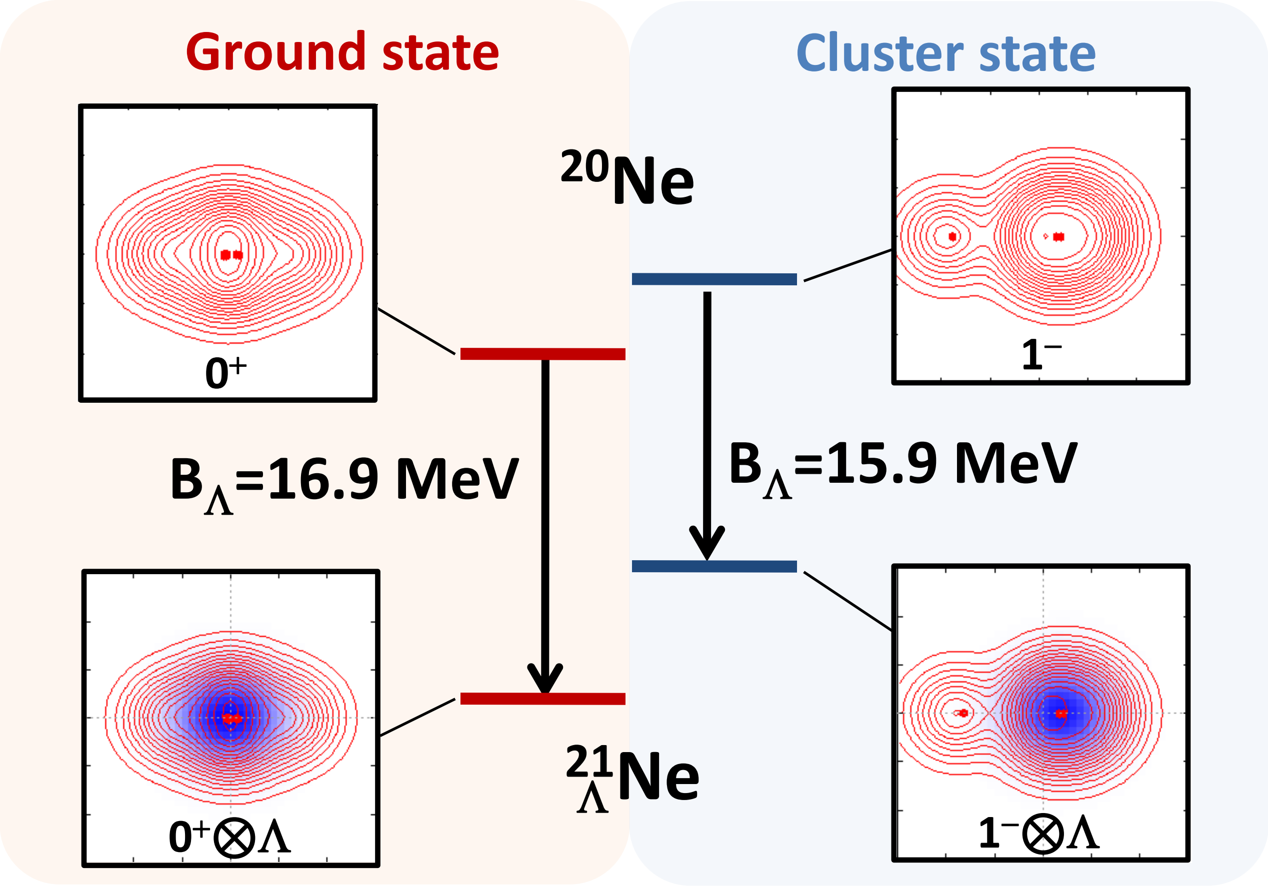
Example:

$^{21}_{\Lambda}\mathrm{Ne}$

- Different structures coexist near the ground state
- Λ in s orbit changes them, but the difference remains
- Λ is localized around ^{16}O in α + ^{16}O + Λ state → difference of B_Λ

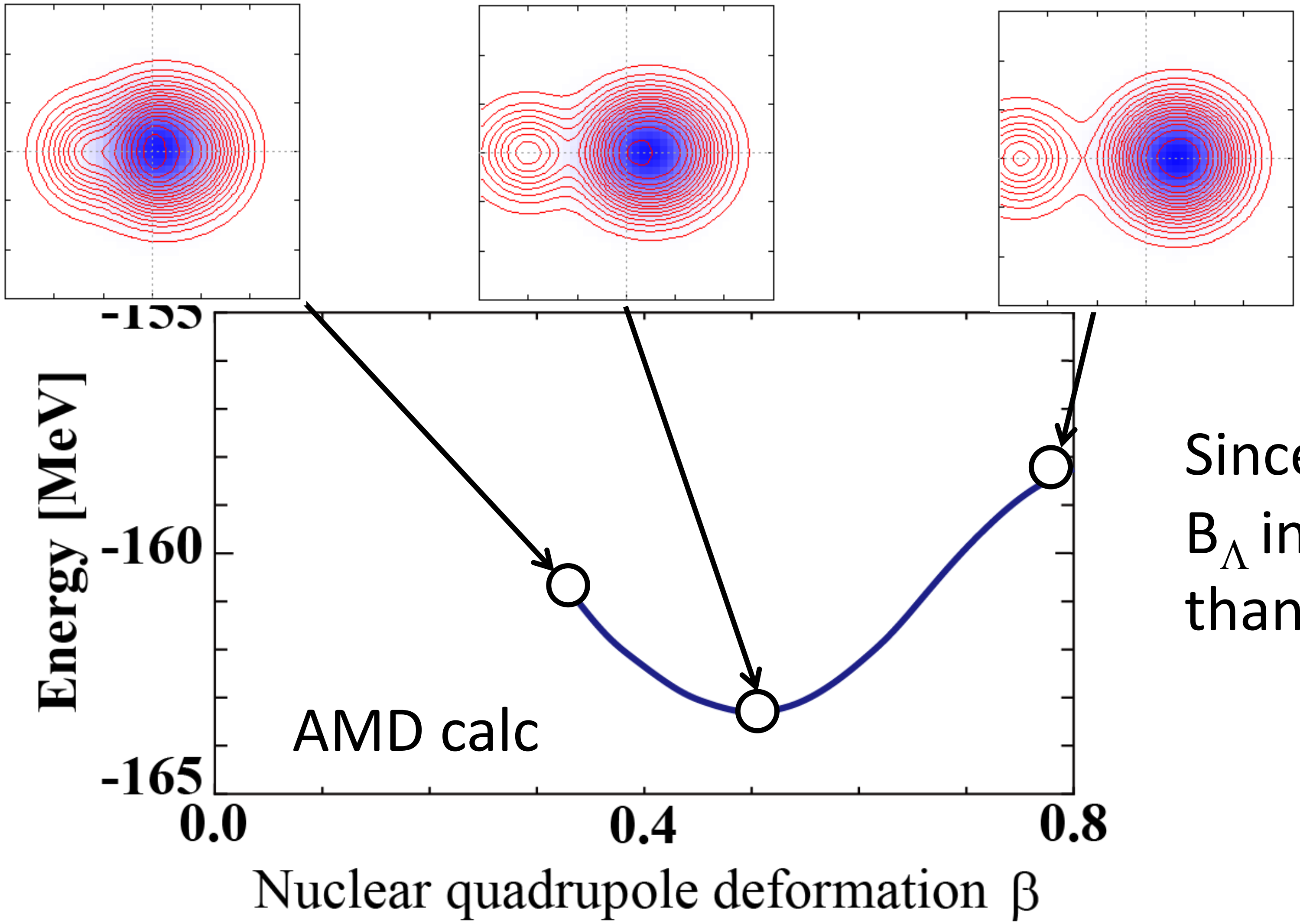
AMD calc

M. Isaka, et al., PRC**83**, 054304(2011)



Dependence of B_Λ on nuclear structure

Which cluster does a Λ particle prefer in $\alpha + {}^{16}O + \Lambda$ state?



"$\alpha + {}^{17}_{\Lambda}O$ like"

Λ prefers the ^{16}O cluster

Since $B_\Lambda({}^{21}_{\Lambda}Ne) > B_\Lambda({}^{17}_{\Lambda}O)$,
B_Λ in $\alpha + {}^{16}O + \Lambda$ states is smaller than the ground state in $^{21}_{\Lambda}Ne$

M. Isaka, et al., PRC**83**, 054304(2011)

A-dependence of B_Λ and many-body force effects

B_Λ: a most basic quantity

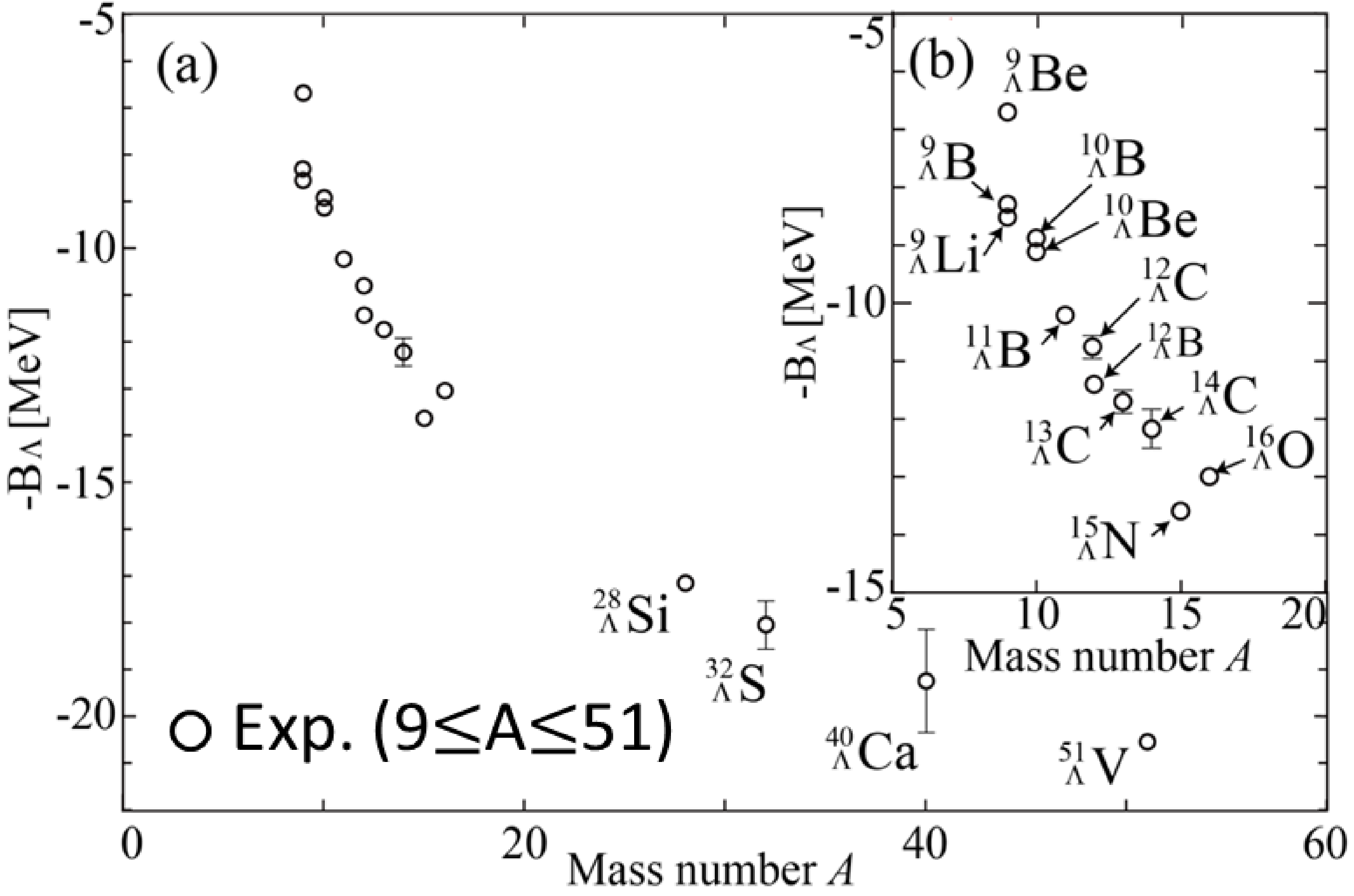
E

Energy of the core nuclei ^{A}Z

B_Λ: energy gain by Λ

Energy of corresponding hypernucleus $^{A+1}{}_{\Lambda}Z$

Exp. data: Hashimoto & Tamura, PPNP**57**,564(2006) and references therein, Tang, *et. al.*, PRC**90**,034320(2014).



To describe A dependence of B_Λ,

- Structure of hypernuclei
- YN and YNN interactions

should be essential

By describing structure of hypernuclei properly using appropriate ΛN interaction, many-body force (YNN) effects can be seen in A dep. of B_Λ

Short summary on many-body force effects

Energy difference between B_Λ^{cal} and B_Λ^{exp} is a room for many-body force

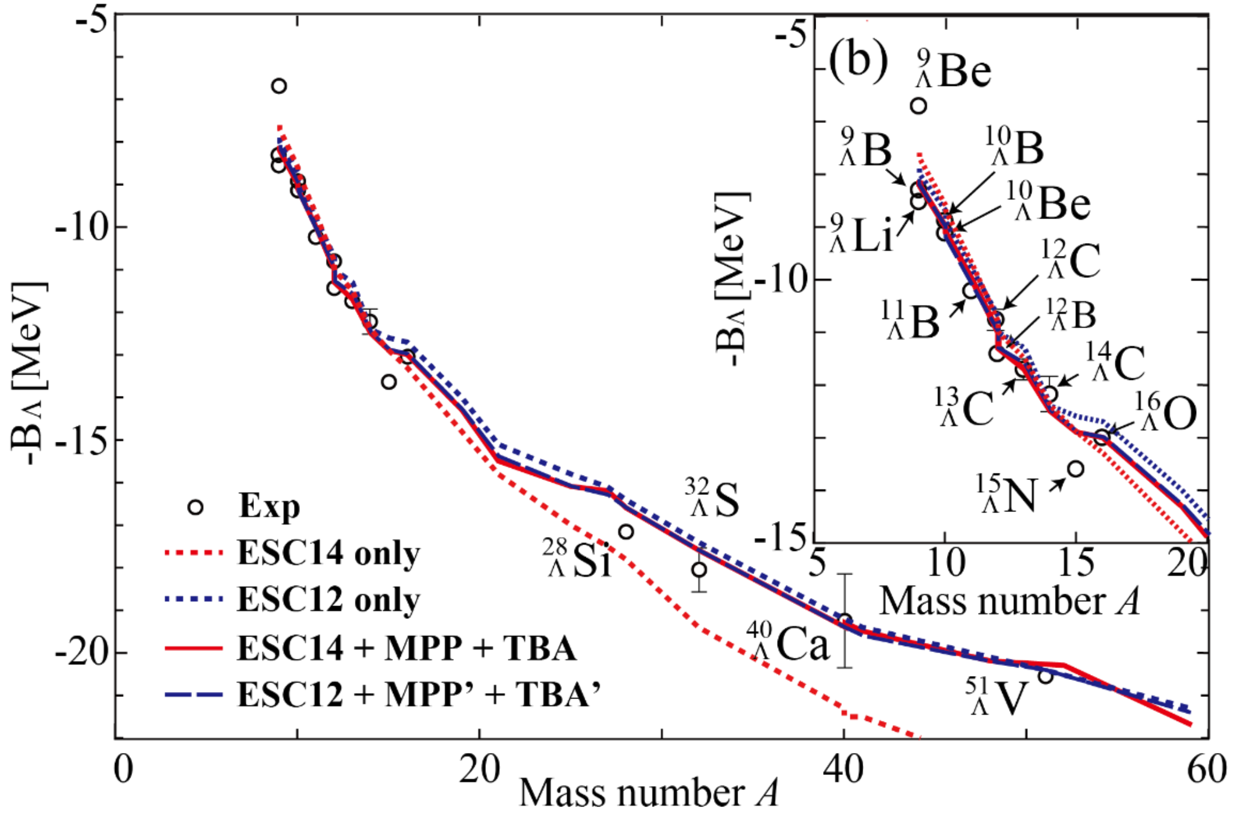
B_Λ values are different in ESC14 and ESC12 corresponding to odd force

In ESC14

- With weakly repulsive odd-state force
- Many body repulsion (MPP) determined by $^{16}O+^{16}O$ scattering is allowed, which explains massive NS

In ESC12

- With strongly repulsive odd-state force
- Weakened many-body force is allowed



Determining odd-state force will impose strong constraints on many-body force

What will happen if a Λ particle is coupled to nuclei ?

- **Dynamical changes of nuclear structure**
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- **Sensitivity of the Λ binding energy B_Λ on nuclear structure**
  - Dependence of B_Λ on nuclear deformation
  - Mass number A dependence & many-body force effects

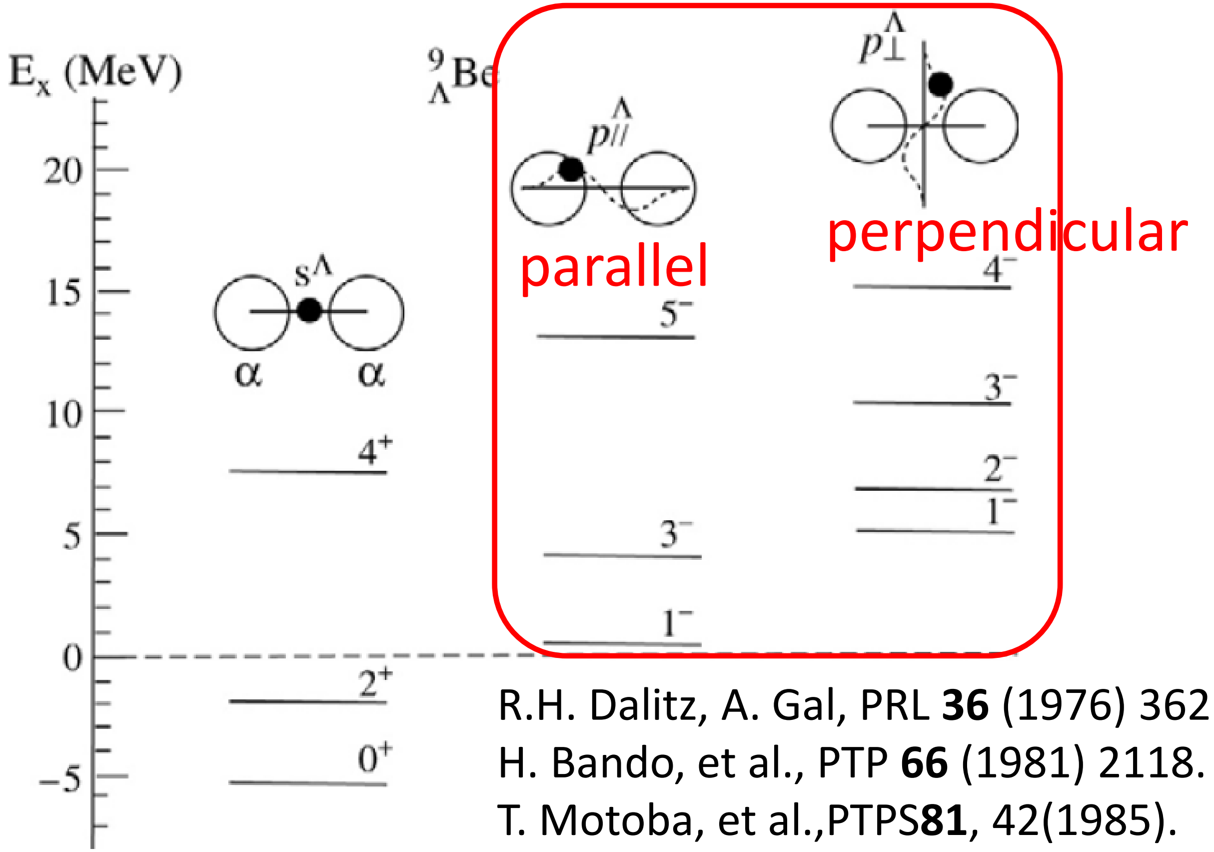
- **Coupling of Λ particle in *p* orbit to clustering/deformed core nuclei**
  - Genuine hypernuclear states
  - Possibility to probe nuclear deformation using Λ particle

Genuine hypernuclear states in $^{9}_{\Lambda}$Be

$^{9}_{\Lambda}$Be: axially symmetric 2α clustering

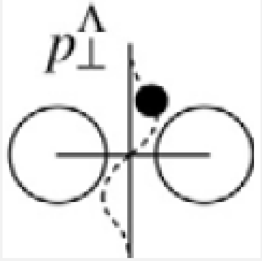
Two rotational bands as *p*-states
- Anisotropic *p* orbit of Λ hyperon
- Axial symmetry of 2α clustering

→ *p*-orbit parallel to/perpendicular to the 2α clustering



R.H. Dalitz, A. Gal, PRL **36** (1976) 362.
H. Bando, et al., PTP **66** (1981) 2118.
T. Motoba, et al.,PTPS**81**, 42(1985).

p states in $^{9}_{\Lambda}$Be



$\fallingdotseq$ ^{9}Be

"^{9}Be analog states"

Forbidden for n in ^{9}Be due to Pauli principle

"genuine hypernuclear states"

Genuine hypernuclear states in $^{9}_{\Lambda}$Be

$^{9}_{\Lambda}$Be: axially symmetric 2α clustering

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- Anisotropic *p* orbit of Λ hyperon
- Axial symmetry of 2α clustering

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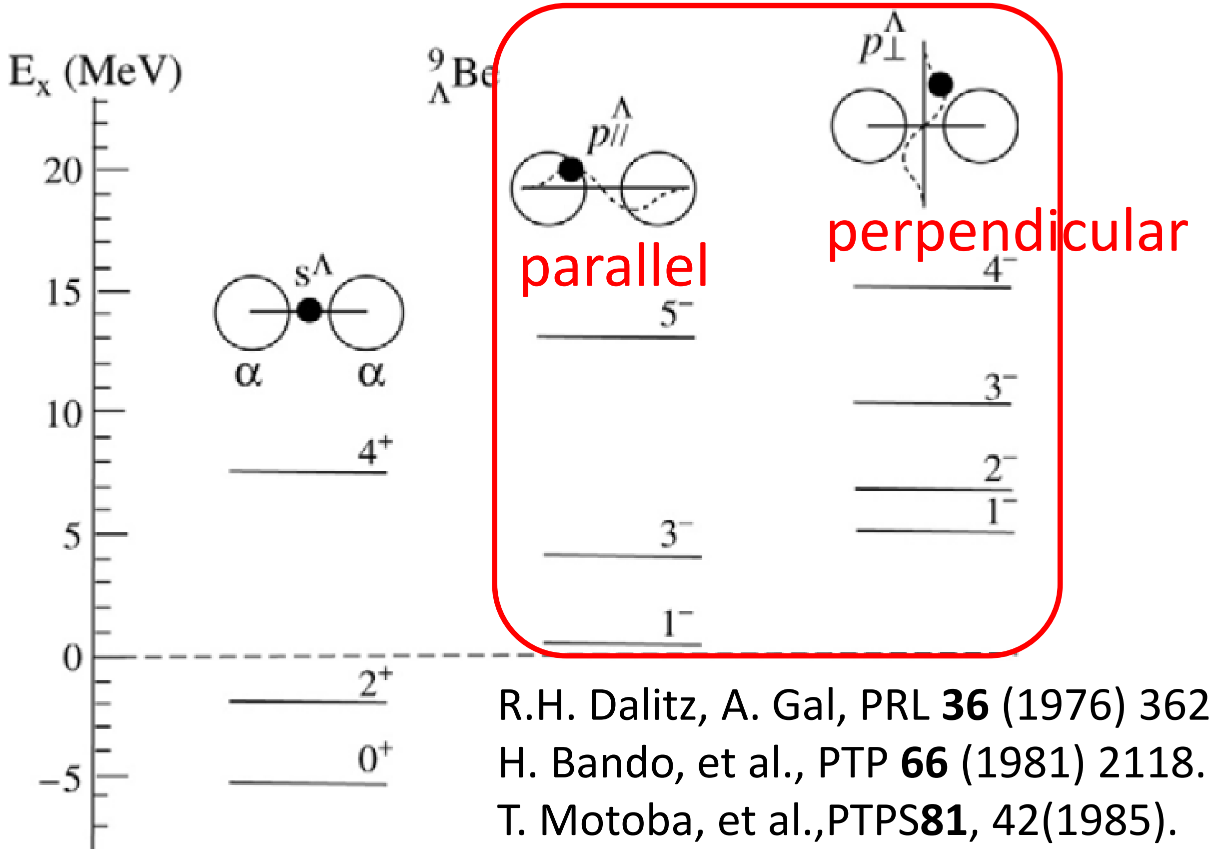

R.H. Dalitz, A. Gal, PRL **36** (1976) 362.
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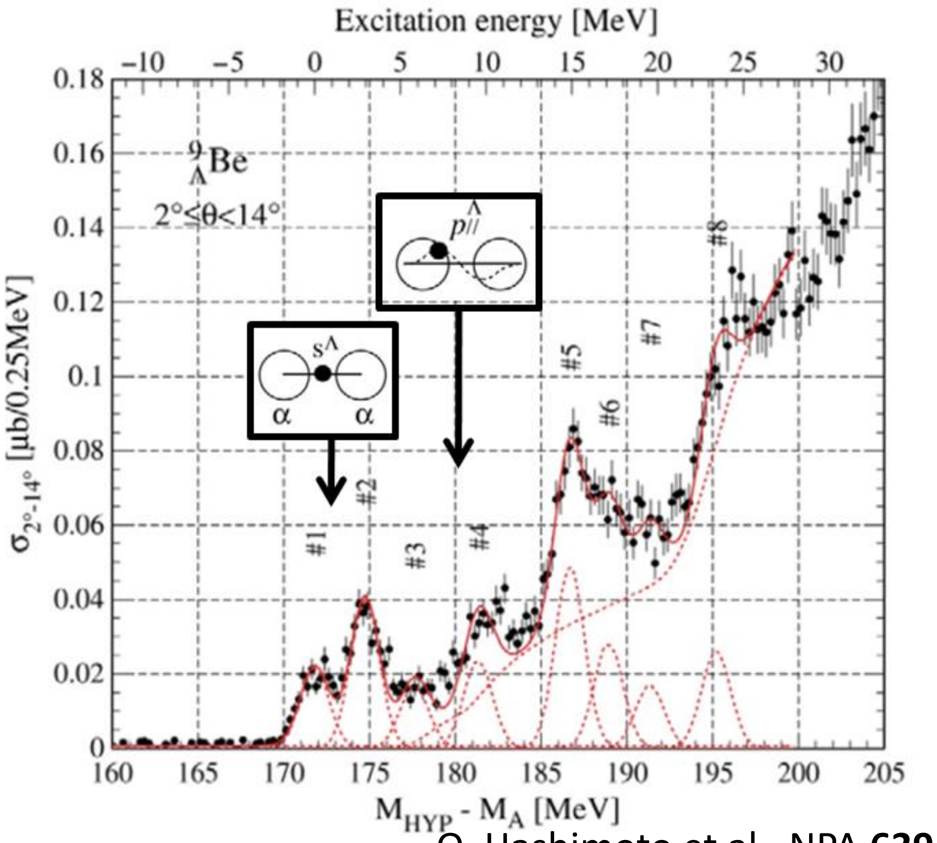


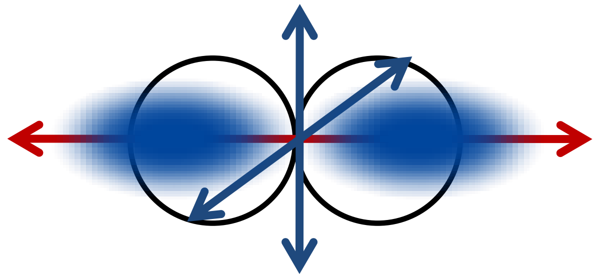
O. Hashimoto et al., NPA **639** (1998) 93c.

Split of *p*-state in $^{9}_{\Lambda}$Be

$^{9}_{\Lambda}$Be: axially symmetric 2α clustering

Genuine hypernuclear states:

p **orbit parallel to 2α (long axis)**

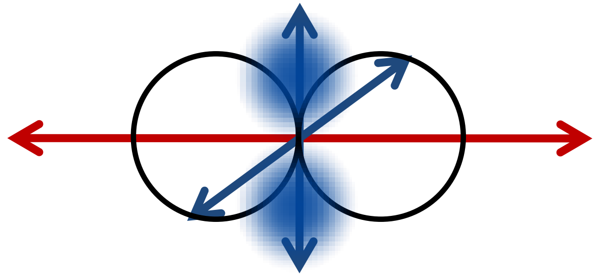


Large overlap

Deeply bound

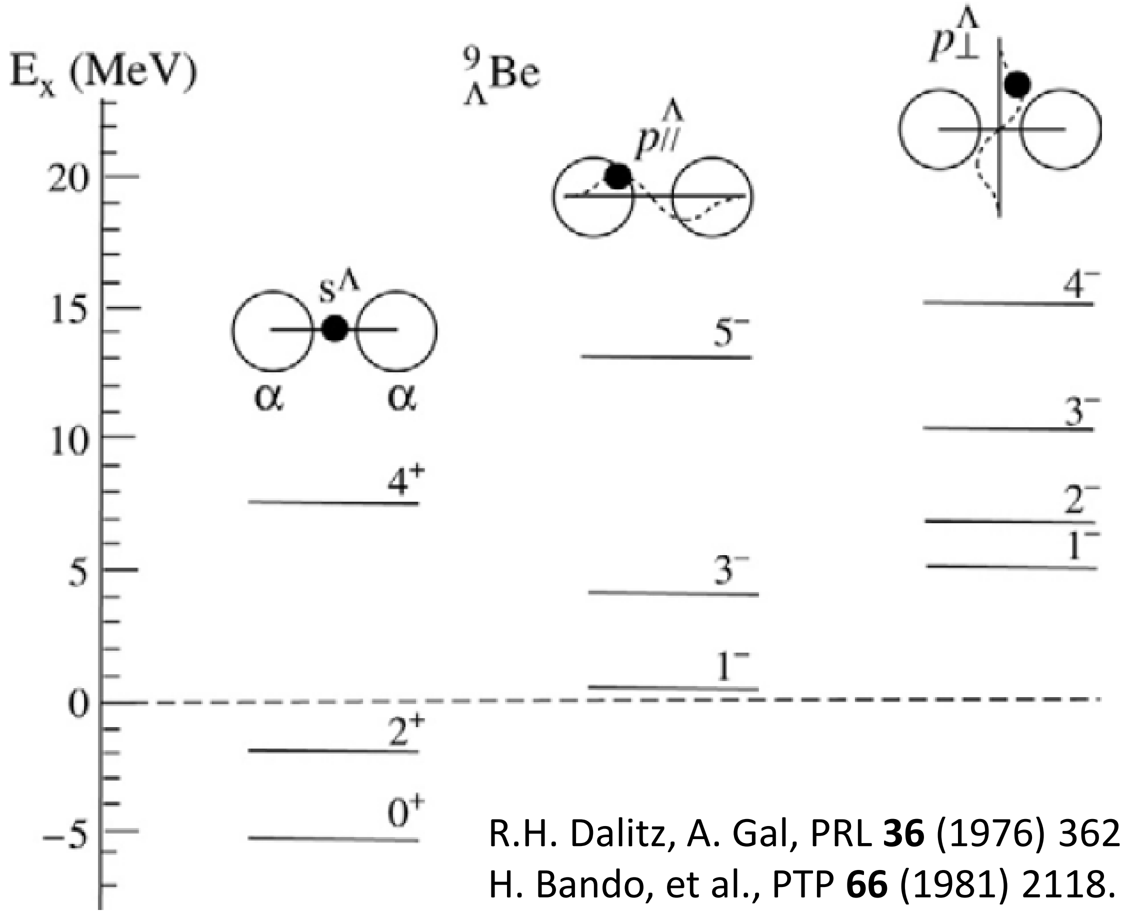
^{9}Be analog states:

p **orbit perpendicular to 2α (short axes)**



Small overlap

Shallowly bound



R.H. Dalitz, A. Gal, PRL **36** (1976) 362.
H. Bando, et al., PTP **66** (1981) 2118.
T. Motoba, et al.,PTPS**81**, 42(1985).

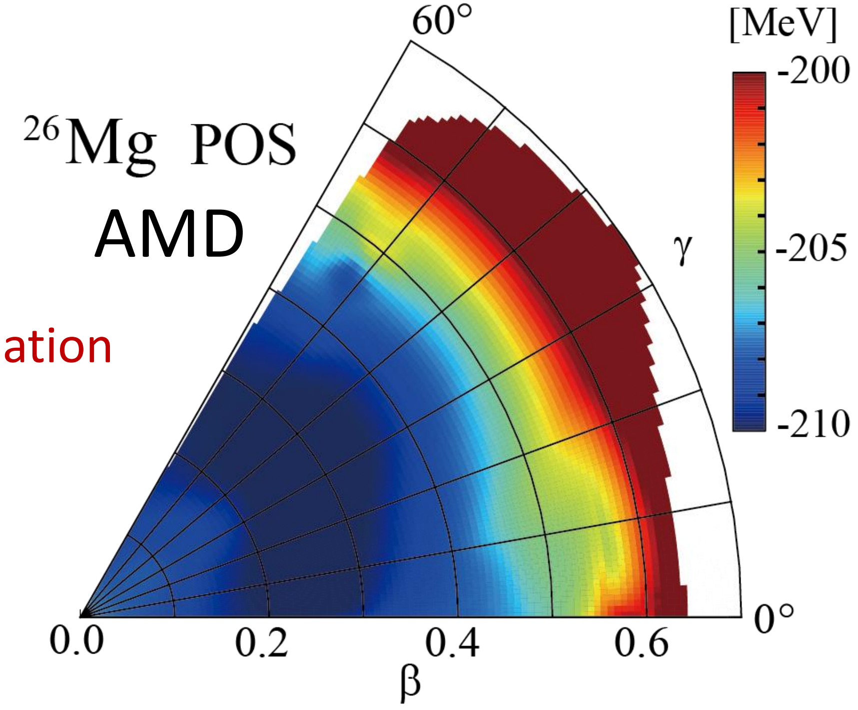
***p*-states splits into 2 bands depending on the direction of *p*-orbits**

Coupling of Λ in *p* orbit to triaxially deformed nuclei

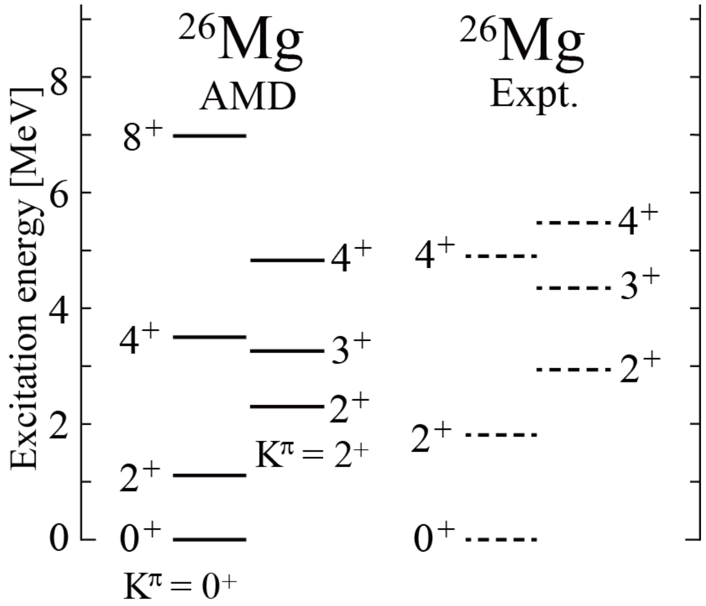
^{26}Mg: candidate of triaxially deformed nuclei, but identification is not easy

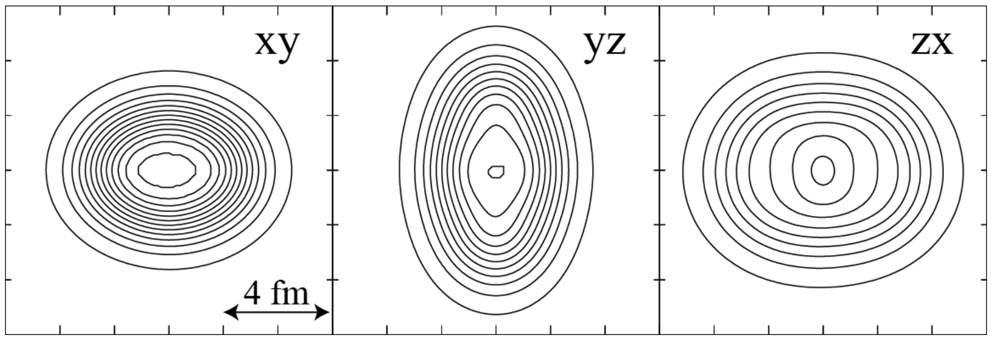
- Shell gap in Nilsson diagram: Z=12 (prolate) vs. N=14 (oblate) → triaxial
- β,γ-softness is discussed by several authors

Terasaki et al. NPA**621**(1997)
Rodrıguez-Guzman et al. NPA**709** (2002)
Peru et al PRC**77** (2008)
Hinohara, Kanada-En'yo PRC**83** (2011)



β: degree of deformation
γ: (tri)axiality

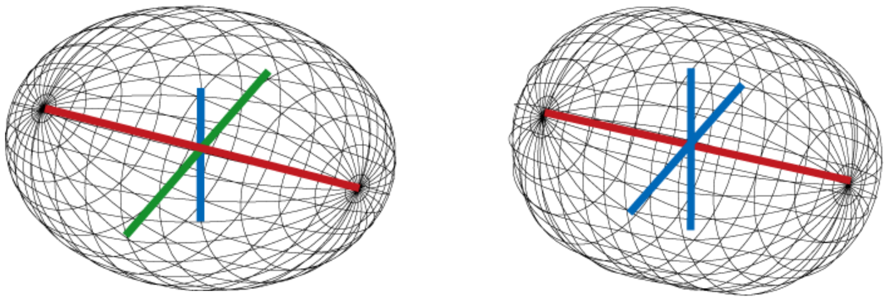


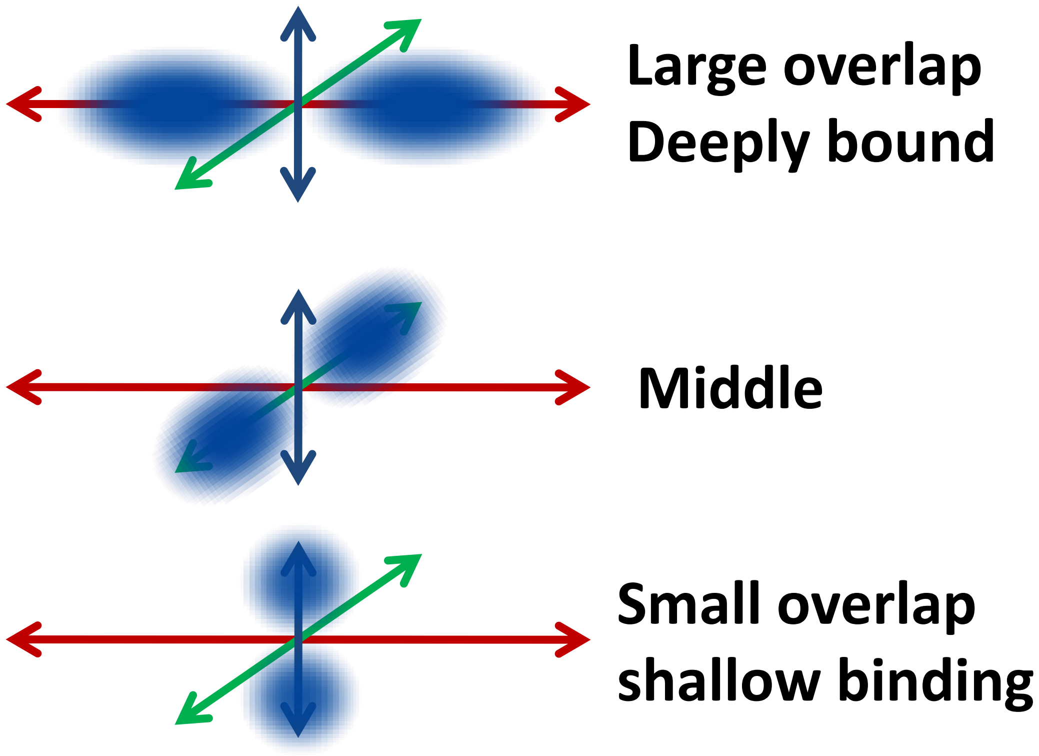


What will happen if Λ in p orbit is coupled to triaxially deformed ^{26}Mg ?

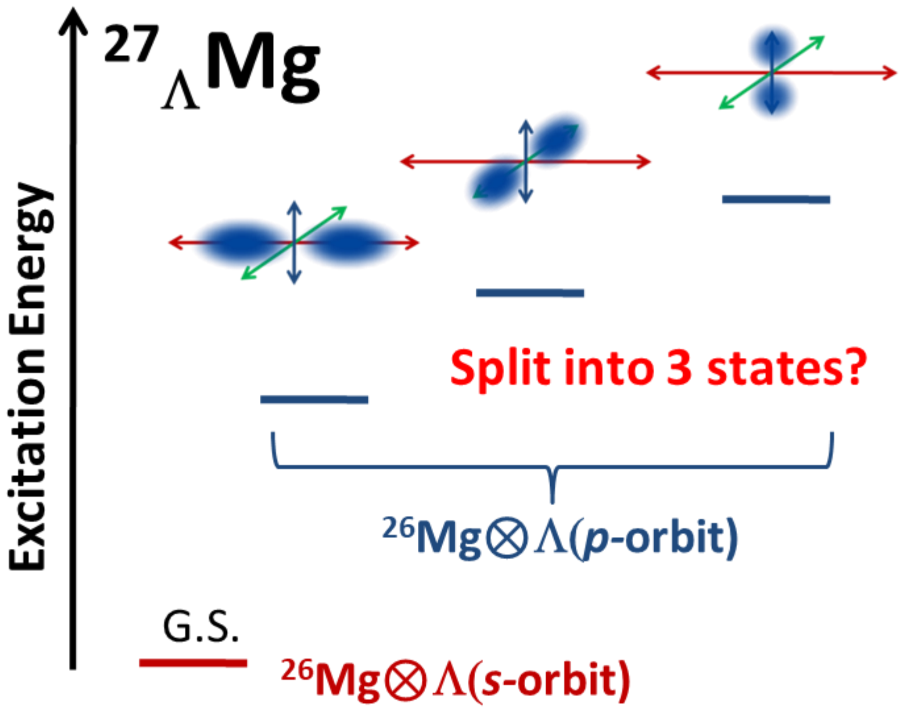
Triaxial deformation

If nucleus is triaxially deformed,
***p*-states can split into 3 different states**





Candidate: Mg hypernuclei

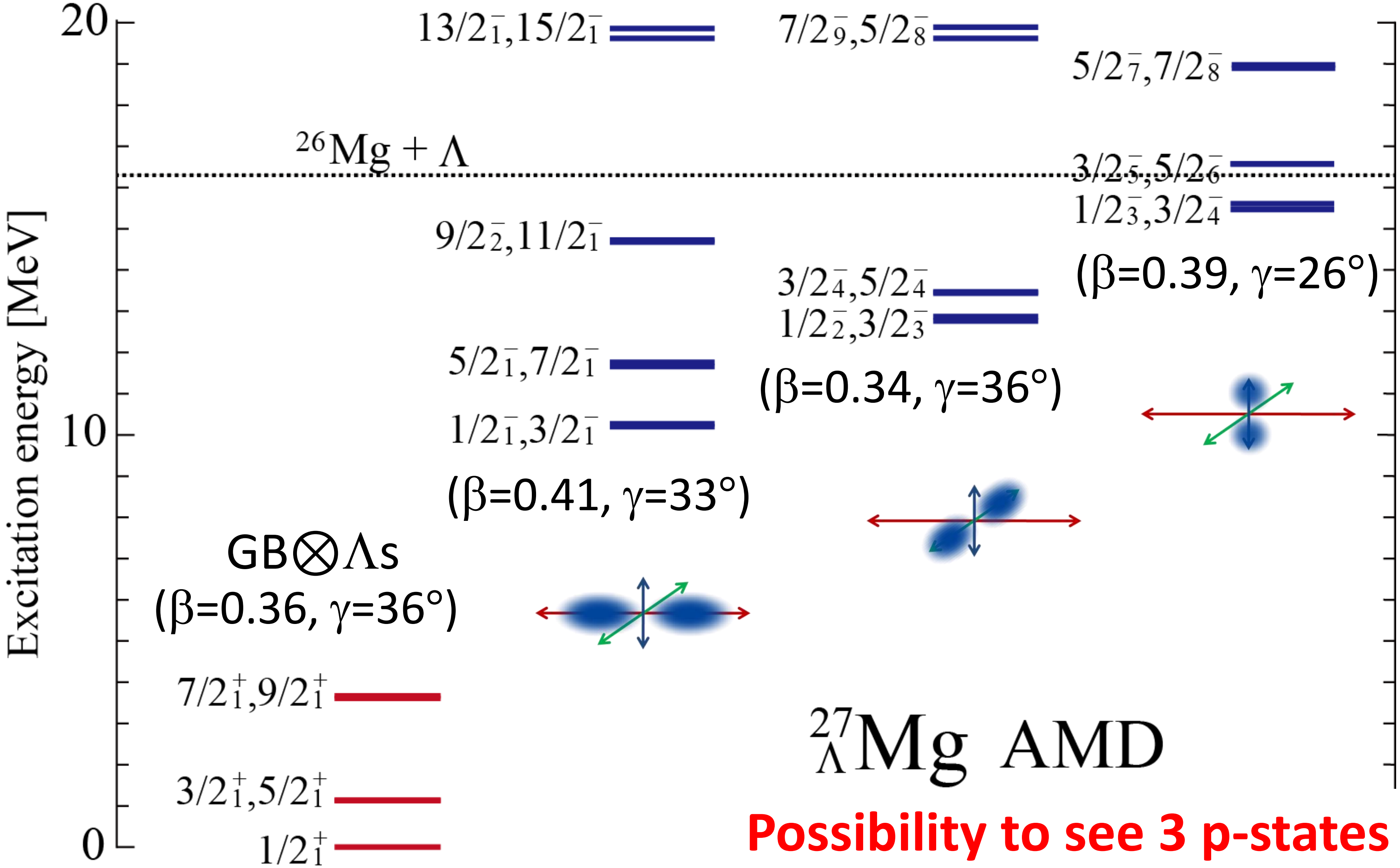


Observing the 3 different *p*-states is strong evidence of triaxial deformation

Results: $^{27}{}_{\Lambda}Mg$

M.I., et al., PRC**87**, 021304(R) (2013)
and calculation ongoing

- **3 bands are obtained by Λ in *p*-orbit** → **Splitting of the *p* states**

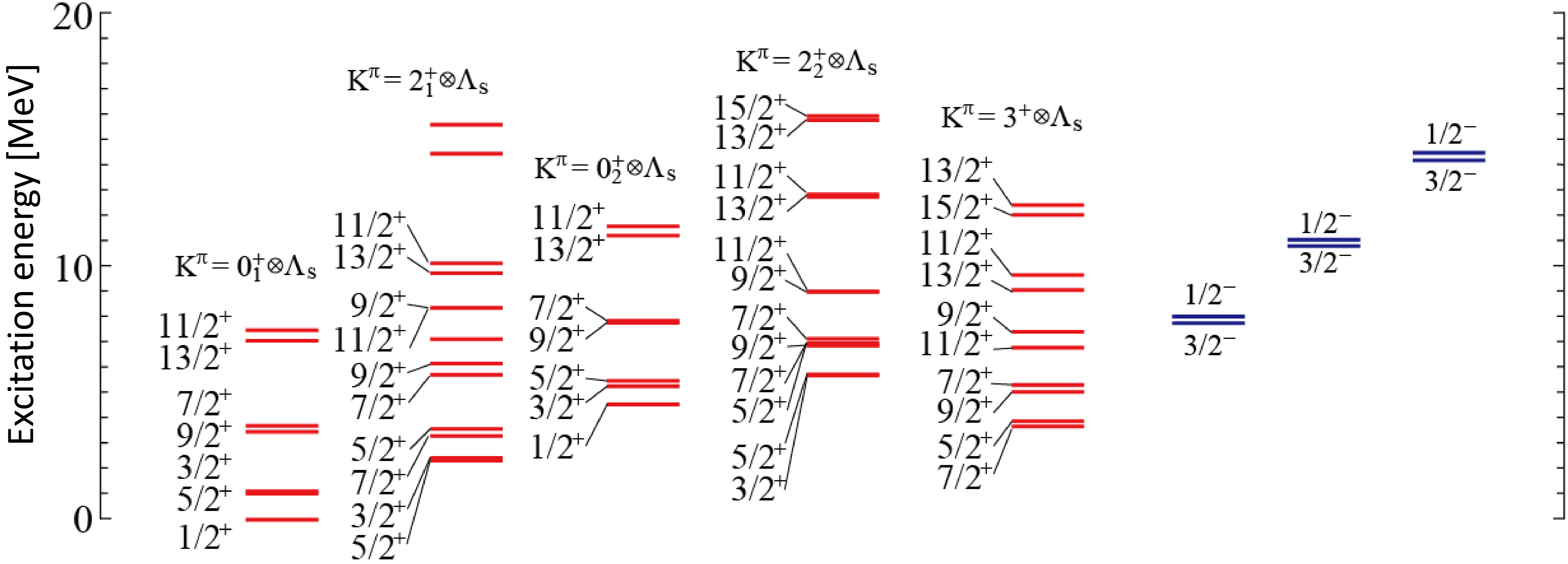


cf. ^{26}Mg ground state:
β=0.41, γ=33°

Possibility to see 3 p-states by $^{27}Al(e,e'K+)^{27}{}_{\Lambda}Mg$

Results: $^{27}{}_{\Lambda}Mg$

- Sophisticated structure calculation is ongoing
- We are now trying to predict production cross section of $^{27}Al(\gamma, K^+)^{27}{}_{\Lambda}Mg$



Summary

In Λ hypernuclei, by the coupling of Λ, we can expect following phenomena:

Dynamical changes of nuclear structure

- Changes of cluster structure
- Deformation changes

→ **Structure changes depending on core structure**

Sensitivity of B_Λ on nuclear structure

- B_Λ depending on nuclear deformation
- Both hyperonic interactions and structure of hypernuclei are important in A-dep. of B_Λ

→ **Many-body effects in Λ hypernuclei**

Coupling of Λ particle to clustering/deformed core nuclei

- Unique feature of Λ in *p* orbit such as genuine hypernuclear states
- Possibility to probe nuclear triaxial deformation using Λ in $^{27}{}_{\Lambda}Mg$

Backup

Many-body force effects

From studies of light Λ hypernuclei

- **Λ hypernuclei observed so far: concentrated in light Λ hypernuclei with A $\lesssim$ 10**
  - **Few-body calculation techniques**
  - **G-matrix calculation for ΛN interactions**
  - **Increases of experimental information**

E. Hiyama, NPA **805** (2008), 190c.
Y. Yamamoto, *et al.*, PTP Suppl. **117** (1994),361.
O. Hashimoto and H. Tamura, PPNP **57** (2006), 564.

→ Knowledge of ΛN interaction and development of interaction models

Developments of effective interactions

In this study, **G-matrix interaction derived from Nijmegen potential (YNG)**

- Nijmegen potential: a meson exchange model
- G-matrix calculation takes into account medium effects

YNG interaction depends on Fermi momentum k_F through nuclear density coming from ΛN-ΣN coupling effects

k_F can be calculated from density
e.g. Averaged Density Approximation (ADA)

$$\langle\rho\rangle = \int dr^3 \rho_N(\mathbf{r})\rho_\Lambda(\mathbf{r}) \quad k_F = \left(\frac{3\pi^2\langle\rho\rangle}{2}\right)^{1/3}$$

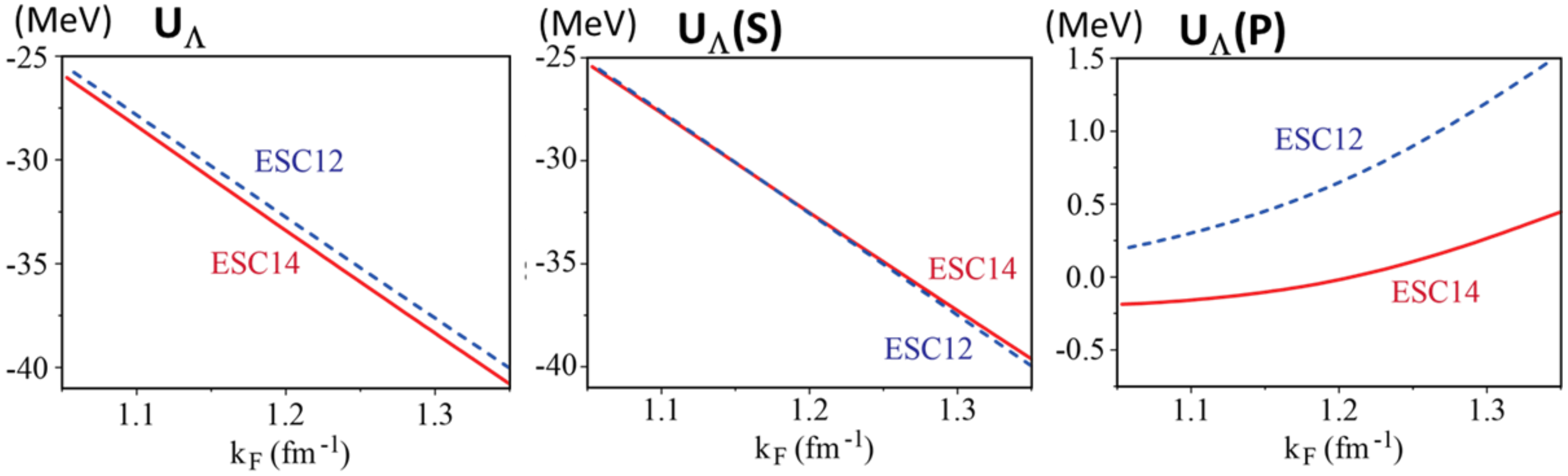
ΛN 2-body G-matrix effective interaction

◆G-matrix interaction from Nijmegen potential (YNG)

- **k_F (density) dependence from ΛN-ΣN coupling**
- **Ambiguity remains in spin-independent odd-state force**

Strongly repulsive: **ESC12**, NSC97
Weakly repulsive: ESC08a, **ESC14**

	U_Λ	$U_\Lambda(S)$	$U_\Lambda(P)$
ESC08a	−40.6	−39.5	+0.5
ESC08b	−39.4	−37.0	−0.6
ESC14	−40.8	−39.6	+0.4
ESC12	−40.0	−40.0	+1.5
ESC04a	−43.2	−38.4	−3.7
NSC97e	−37.7	−40.4	+4.0
NSC97f	−34.8	−39.1	+5.6



U_Λ : one-body potential energy of Λ
$U_\Lambda(S)$: S-state contribution in U_Λ
$U_\Lambda(P)$: P-state contribution in U_Λ

Many-body force in G-matrix interaction

Yamamoto, Furumoto, Yasutake and Rijken, PRC**88**,022801(2013); PRC**90**,045805(2014).

ESC(ESC12 or ESC14) + MPP + TBA

MPP: repulsion acting strongly at high density

TBA : phenomenological 3-body attraction

ESC: effective ΛN force including ΛN-ΣN effects

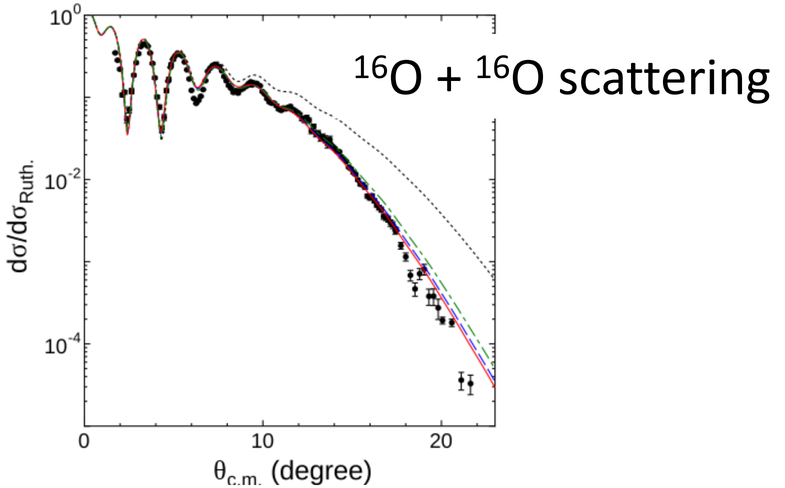
$$\rightarrow \quad V_{\Lambda N}(r;k_F) = \sum_{i=1}^{3}(a_i + b_i k_F + c_i k_F^2)\exp(-r^2/\beta_i^2)$$

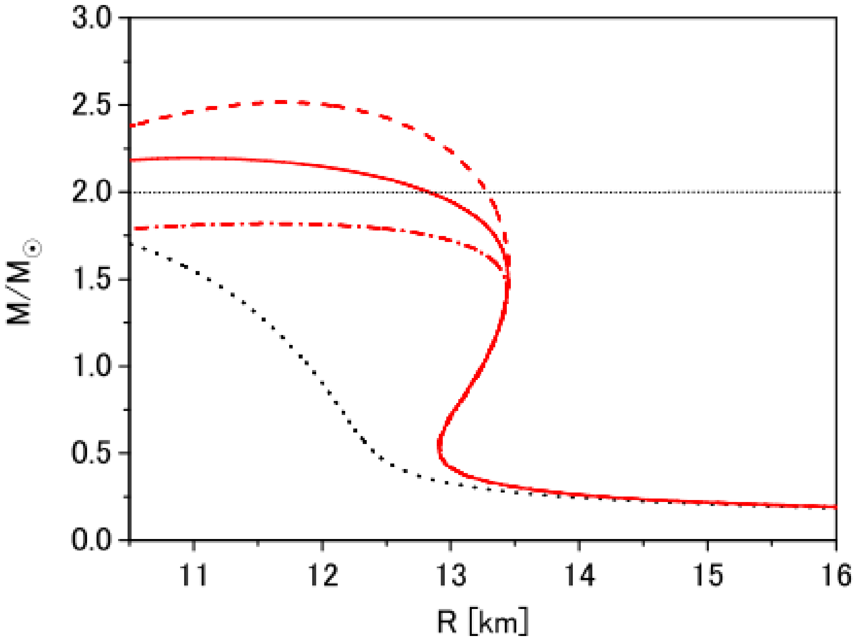
MPP: universal repulsion by $^{16}O + {}^{16}O$ scattering
and consistent with massive neutron star

TBA: phenomenologically determined by $^{89}_{\Lambda}Y$ data

$$\rightarrow \quad \Delta V_{\Lambda N}(r;k_F) = (a + b k_F + c k_F^2)\exp(-r^2/\beta_2^2)$$

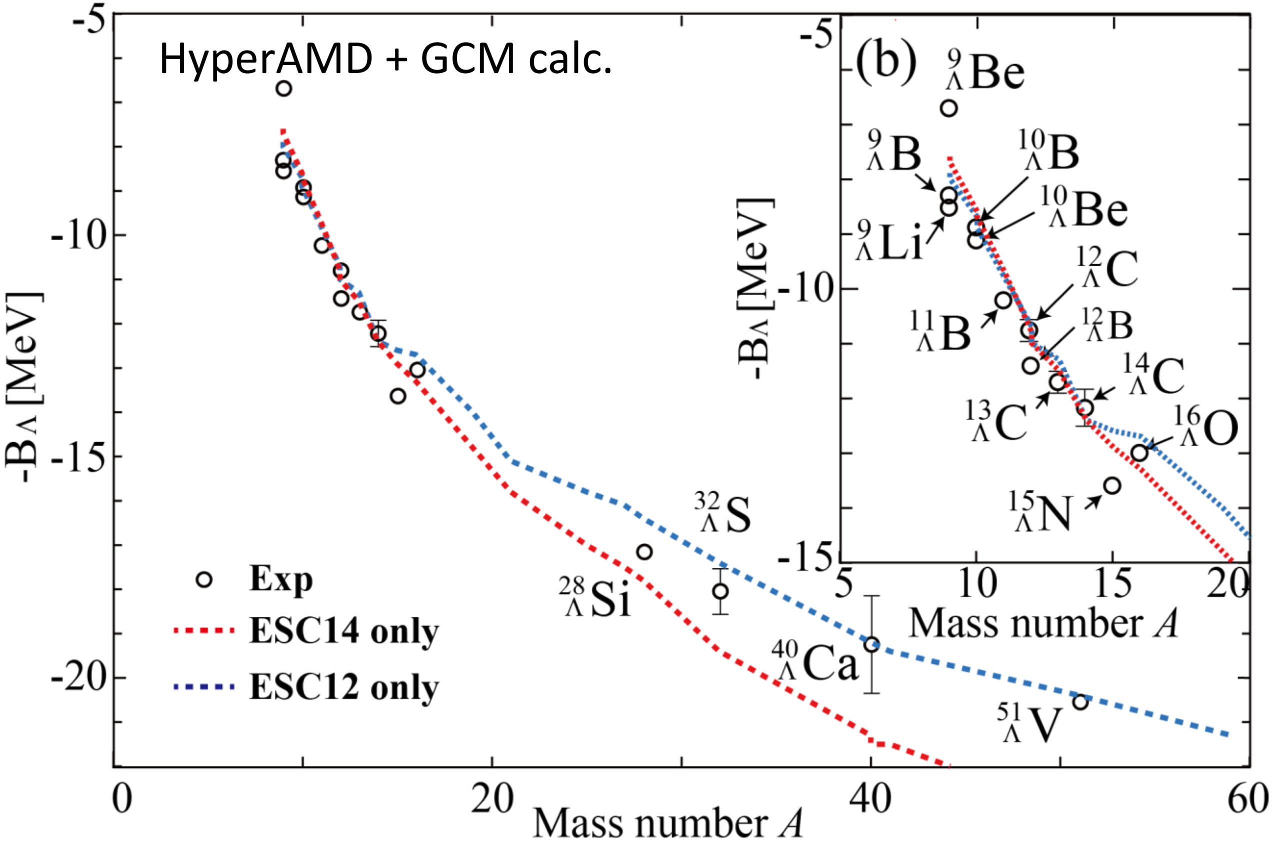
We use $V_{\Lambda N} + \Delta V_{\Lambda N}$ in HyperAMD calculation





Results: A dependence of B_Λ **without** many-body force

B_Λ values are different in ESC14 and ESC12 corresponding to odd force



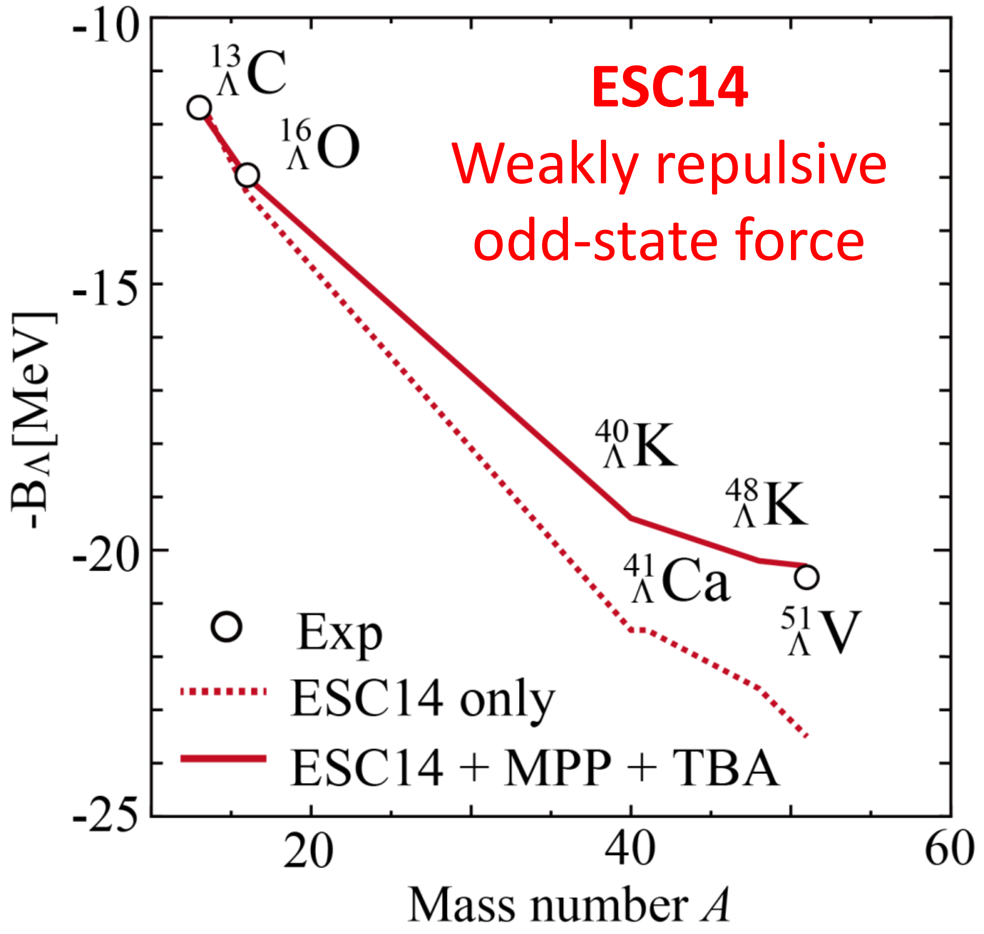
k_F is determined under ADA in each hypernucleus

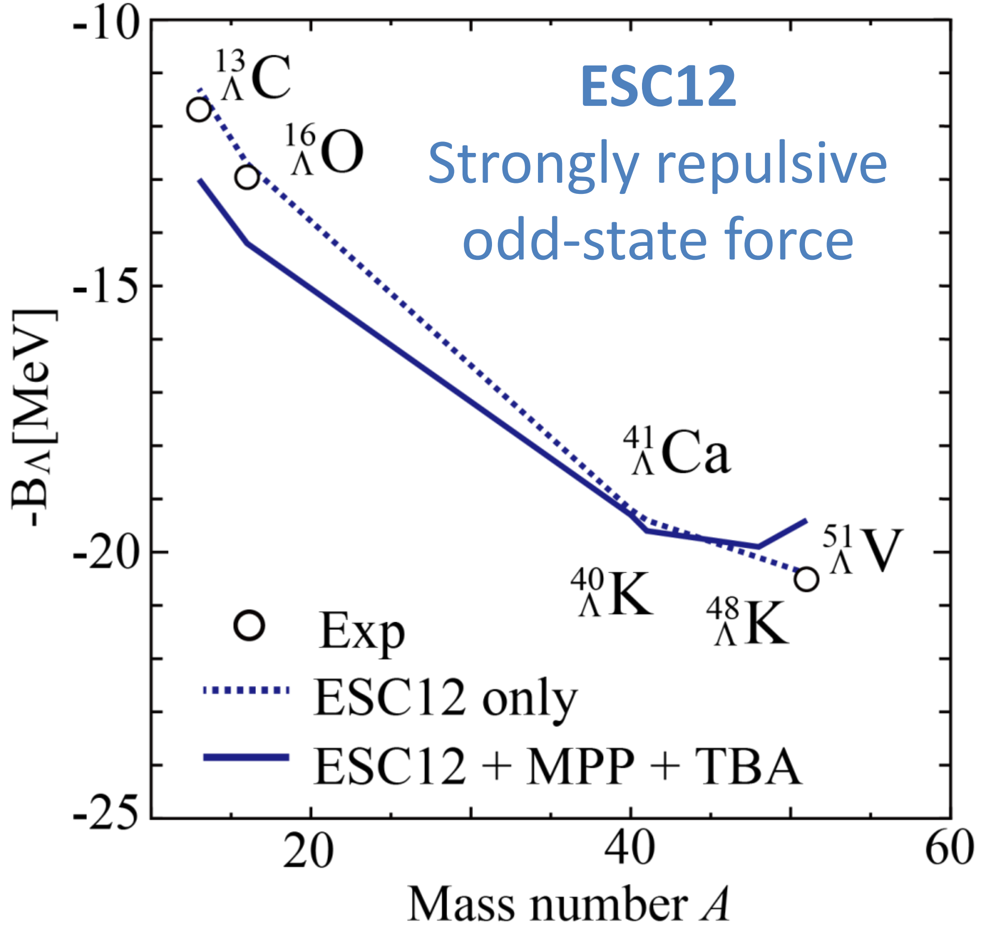
Spin-ind. odd-state force:
ESC14: weakly repulsive
ESC12: strongly repulsive

- ESC14 with weakly repulsive odd force overestimates B_Λ values
- ESC12 with stronger repulsive odd force gives smaller B_Λ values than ESC14, which are close to the observed data

Results：Many-body force effects

Many-body force { **MPP：repulsion by $^{16}O+^{16}O$ scattering, giving massive NS**
TBA：phenomenological attraction by $^{89}_{\Lambda}Y$ data

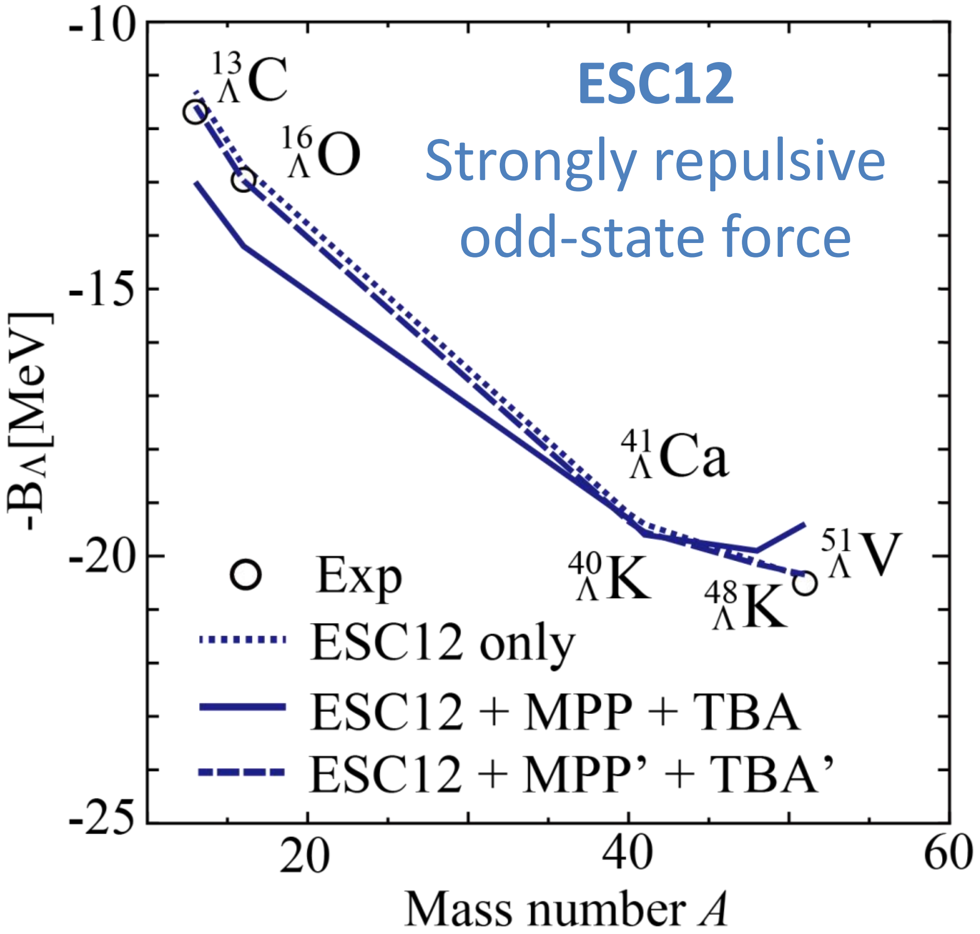




Adding MPP+TBA to ESC14 makes $B_{\Lambda}^{\mathrm{cal}}$ close to $B_{\Lambda}^{\mathrm{exp}}$, whereas ESC12 deviates

Results：What is possible many-body force with ESC12?

In ESC12, MPP should be weakened to reproduce $B_{\Lambda}^{\mathrm{exp}}$



MPP’ + TBA’: weakened for ESC12

- Weakened to reproduce $^{16}{}_{\Lambda}$O and $^{89}{}_{\Lambda}$Y
- MPP’ is too weak to explain massive NS

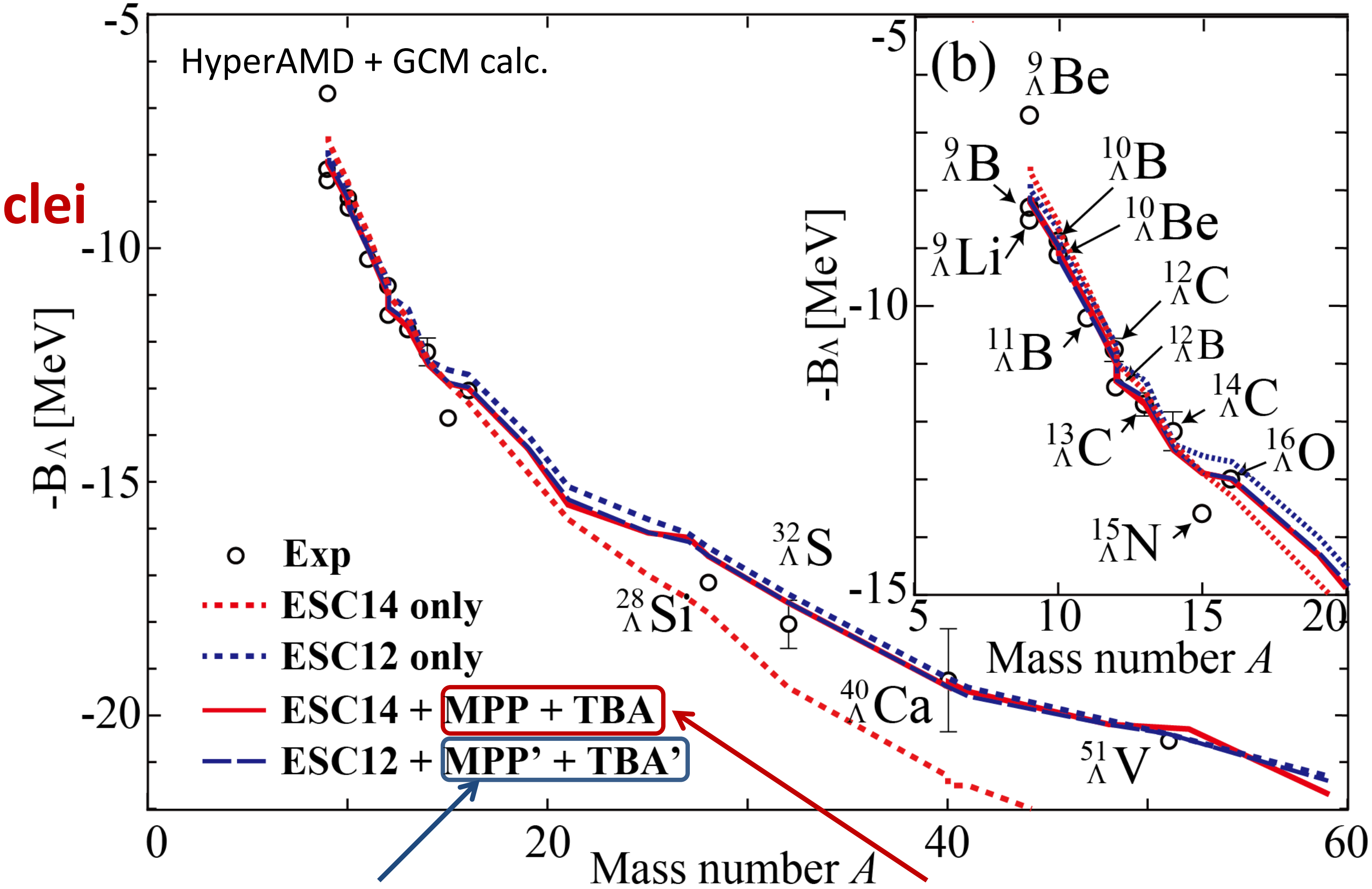
cf. Original MPP + TBA allowed with ESC14

- MPP gives maxim mass of NS
- TBA is determined by $^{89}{}_{\Lambda}$Y data

- **In ESC12 with strongly repulsive odd-force, no MPP is allowed to explain NS**
- **Determining odd-state force will impose strong constraints on many-body force**

Results：Many-body force effect

Same trend is found in $9 \leqq A \leqq 59$ Λ hypernuclei



Weakened to reproduce $^{16}_{\Lambda}$O & $^{89}_{\Lambda}$Y

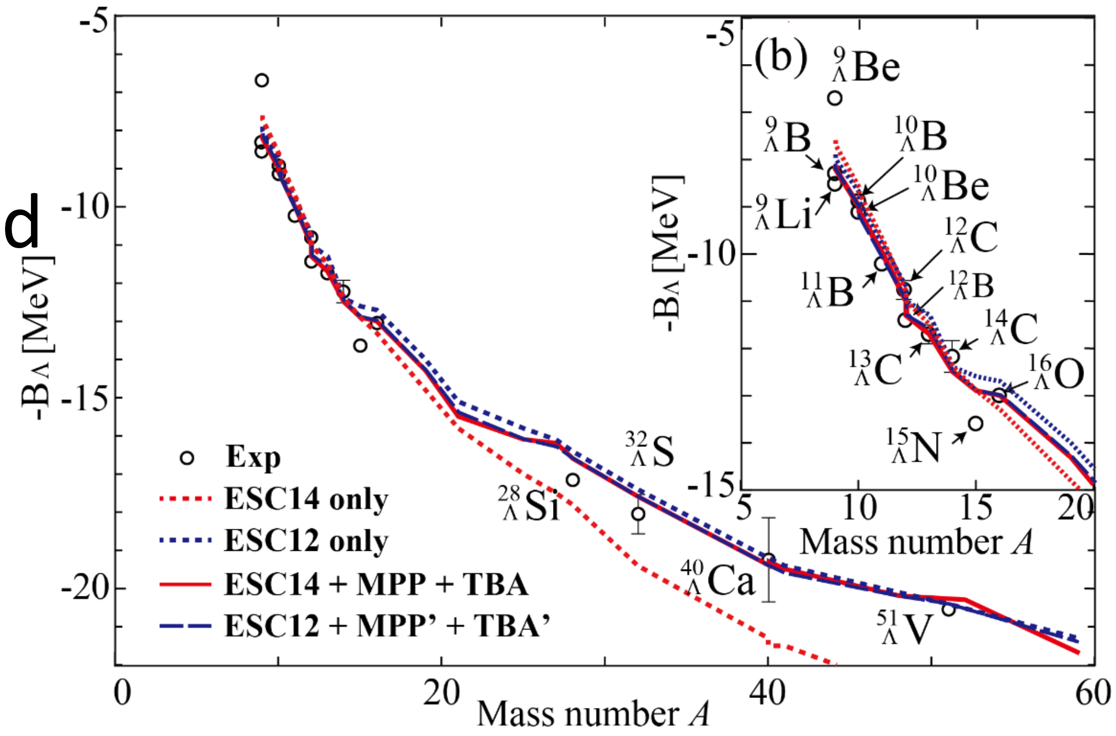
MPP from ^{16}O+^{16}O scattering explains massive NS

Many-body force effects with AMD

- **Energy difference between B_Λ^{cal} with ΛN force and B_Λ^{exp} is a room for many-body force**

<u>In ESC14</u>

- With weakly repulsive odd-state force
- Many body repulsion (MPP) determined by ^{16}O+^{16}O scattering is allowed, which explains massive NS



<u>In ESC12</u>

- With strongly repulsive odd-state force
- Weakened many-body force is allowed

Determining odd-state force will impose strong constraints on many-body force

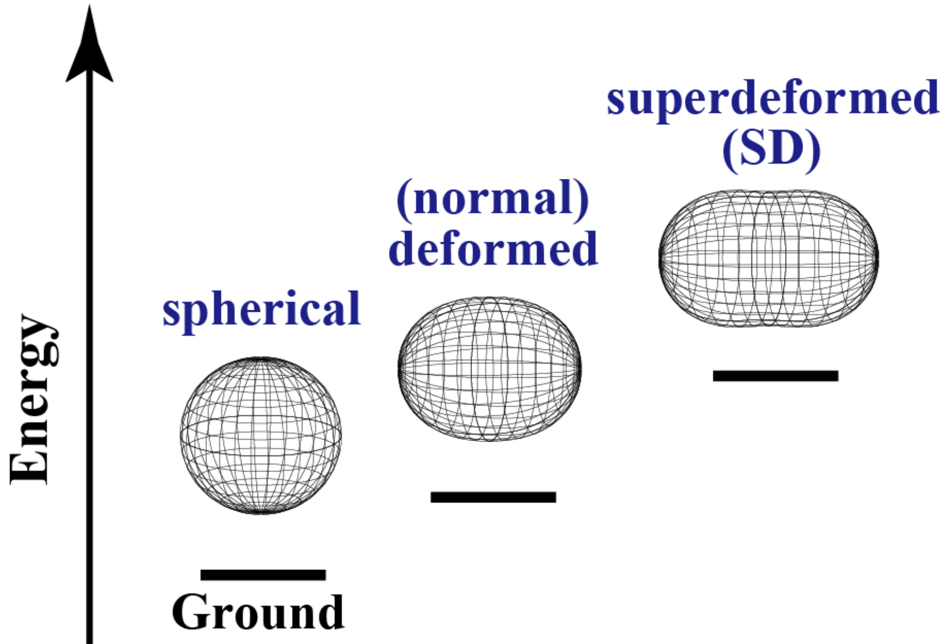
Backup

Overlap & BLmd

An extreme cases: superdeformation

Y. Kanada-En'yo, M. Kimura, PRC**72**, 064322(2005)
Y. Taniguchi, et al., PRC**76**, 044317 (2007)

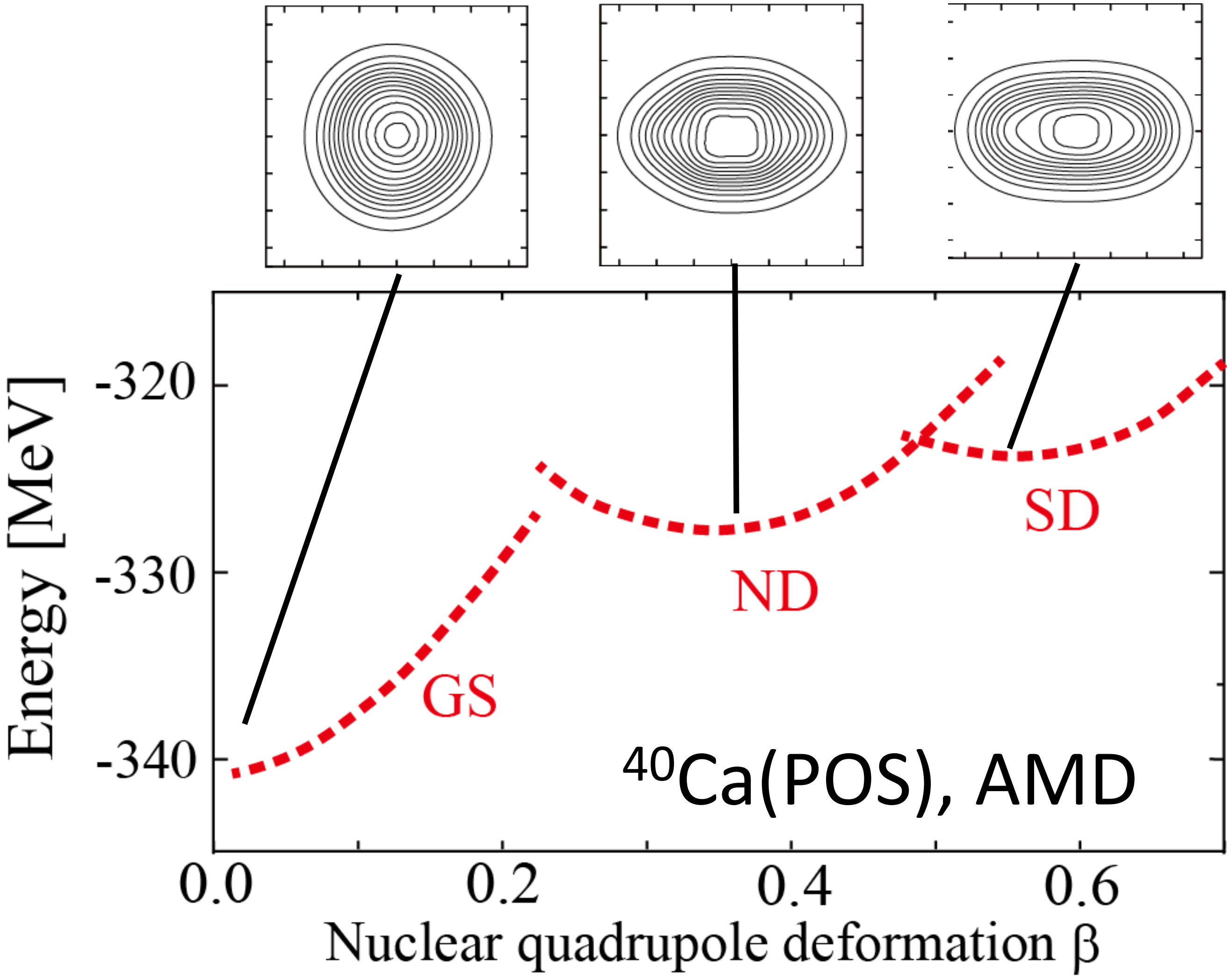
Different deformations could appear in the excited states in a nucleus



Superdeformed(SD) states

- With 1:2 axis ratio of deformation
- Observed in ^{40}Ca, ^{36}Ar and ^{44}Ti

Svensson *et al.*, PRC**63**, 061301R (2001).
Ideguchi, *et al.*, PRL **87**, 222501 (2001)
O'Leary, *et al.*, PRC**61**, 064314 (2000).



If a Λ particle is added, what happen?

An extreme cases: superdeformation

Y. Kanada-En'yo, M. Kimura, PRC**72**, 064322(2005)
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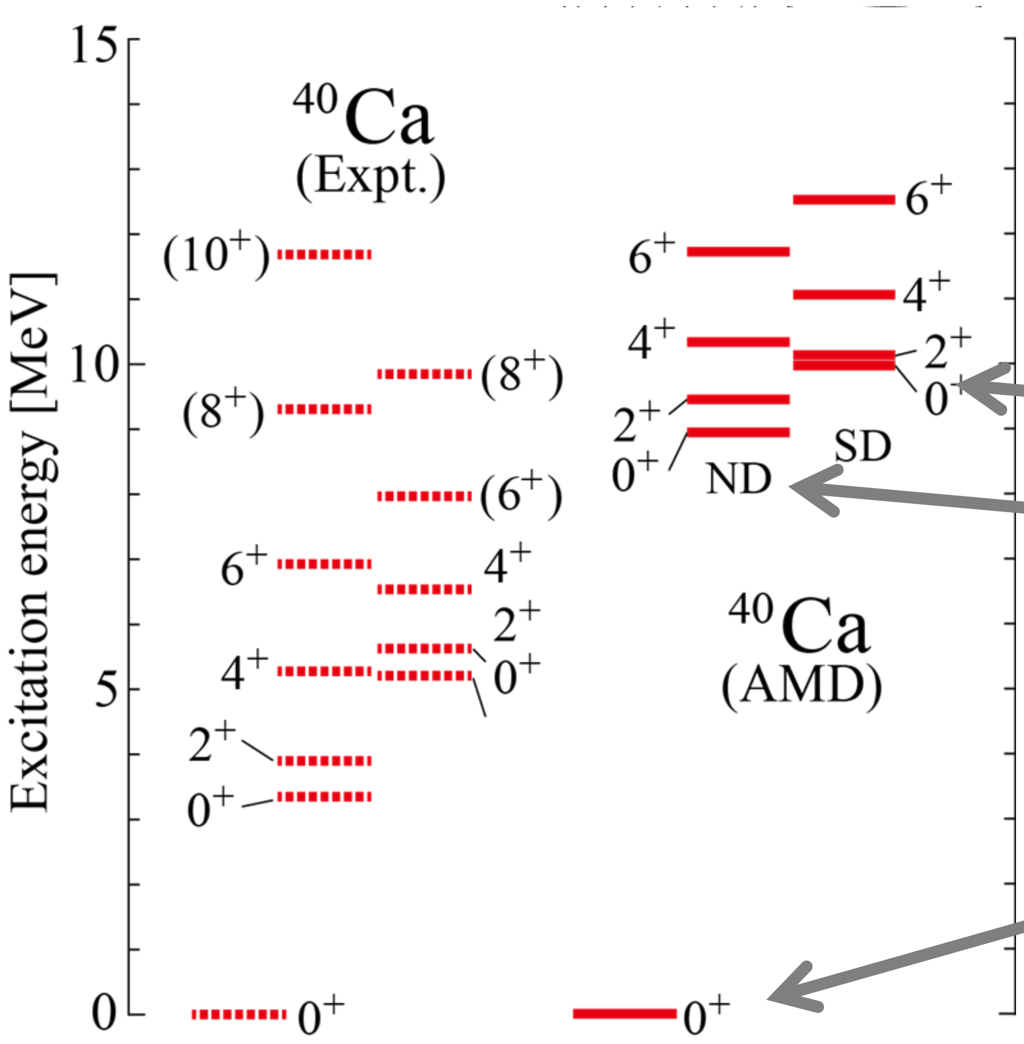
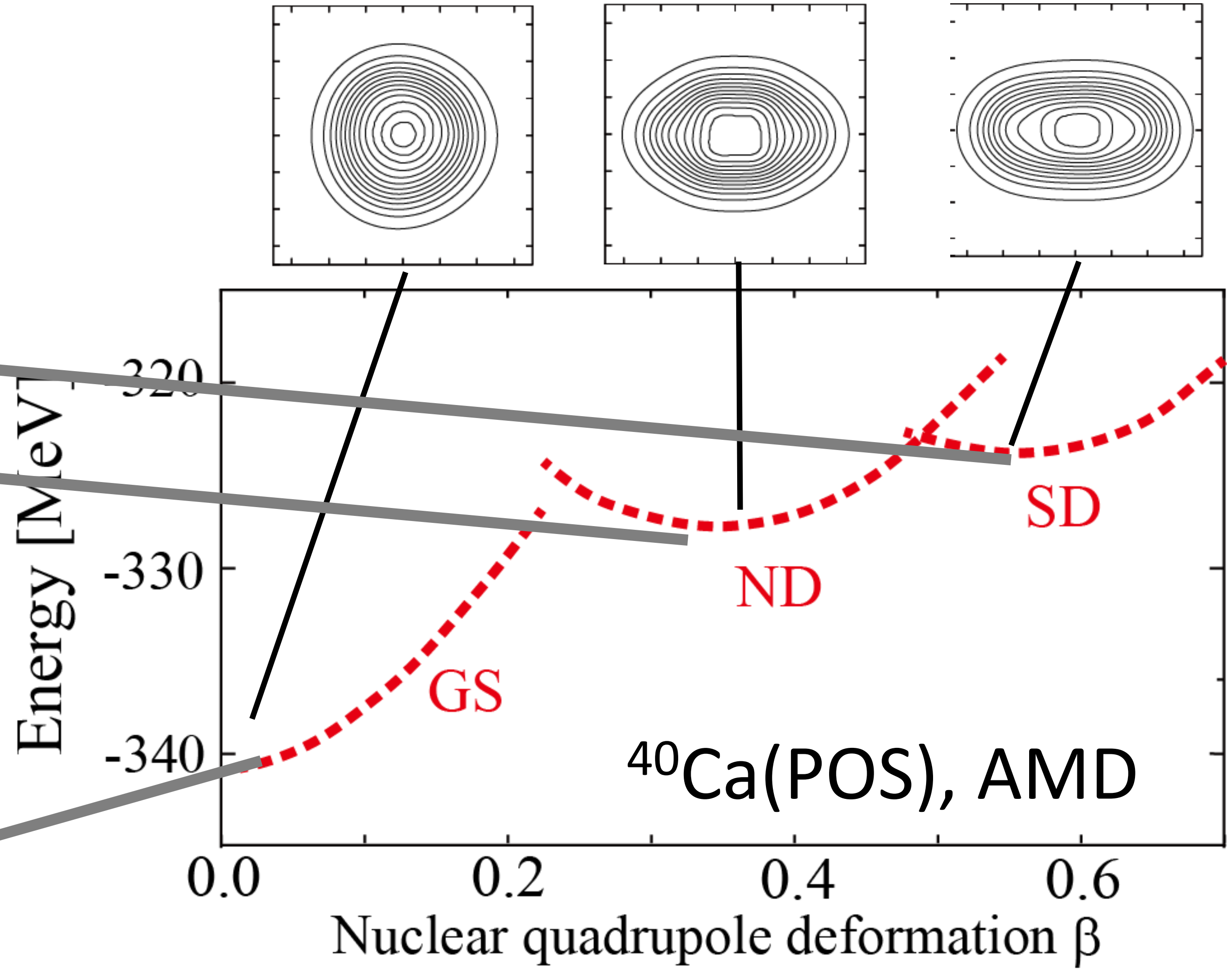
Different deformations could appear in the excited states in a nucleus

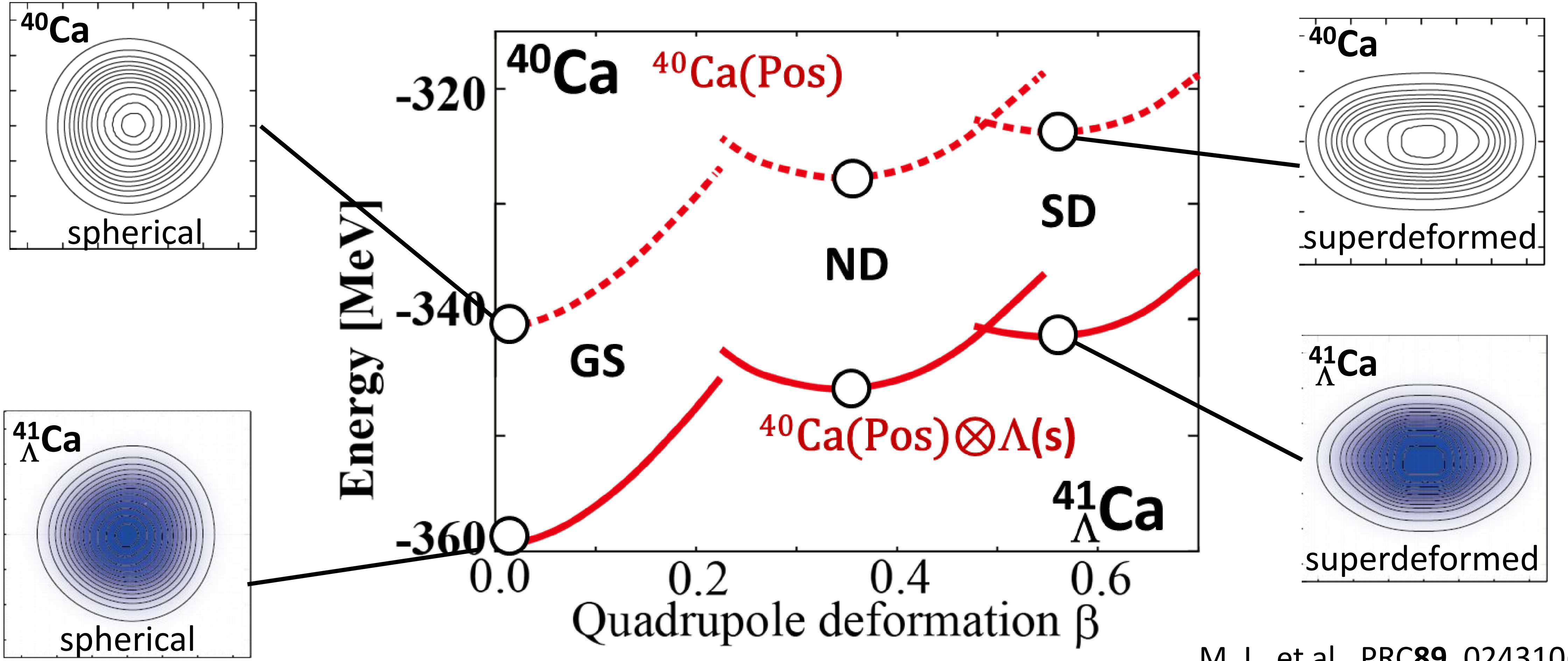




If a Λ particle is added, what happen?

An extreme cases: superdeformation

- Corresponding energy surface appear in $^{41}{}_{\Lambda}Ca$
- Energy (local) minima are almost unchanged

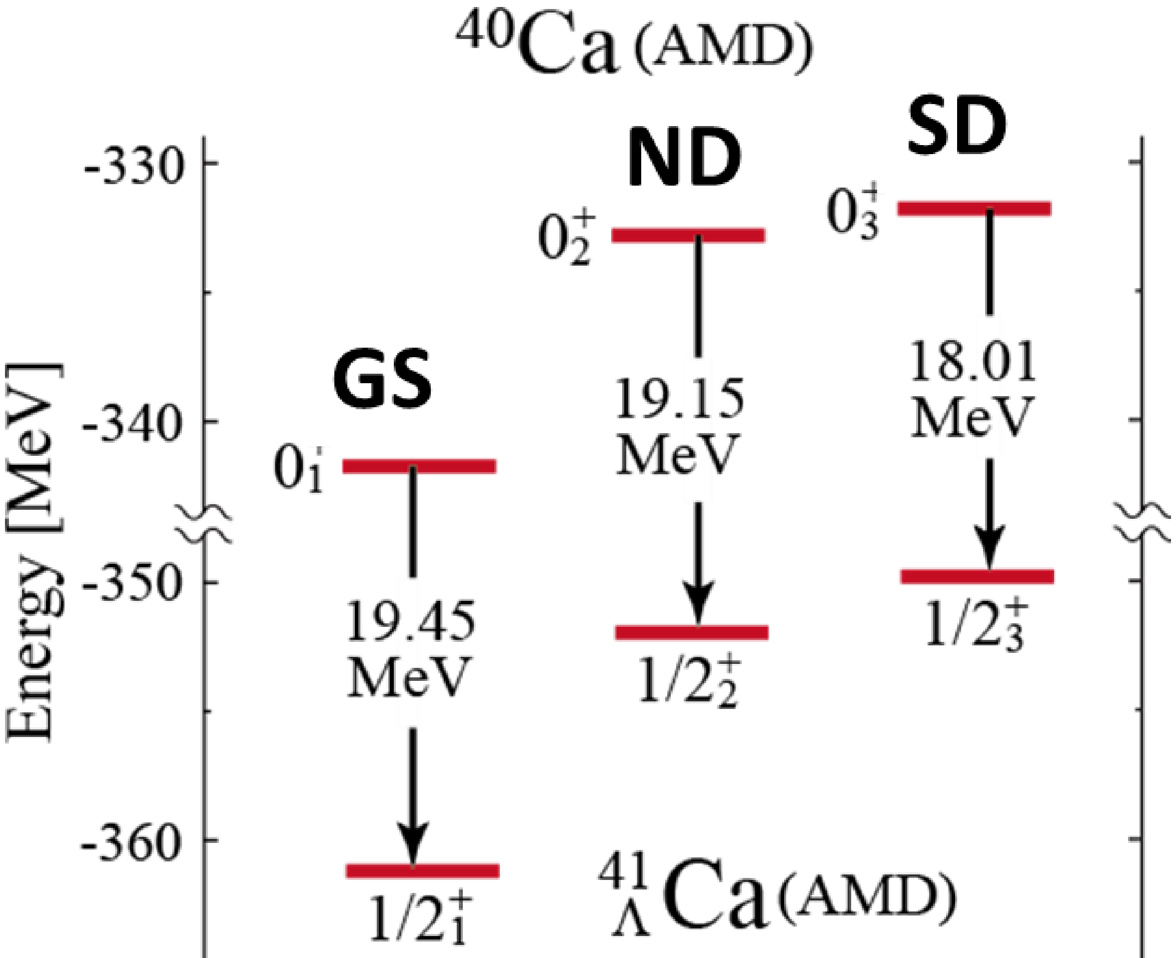


M. I., et al., PRC**89**, 024310(2014)

An extreme cases: superdeformation

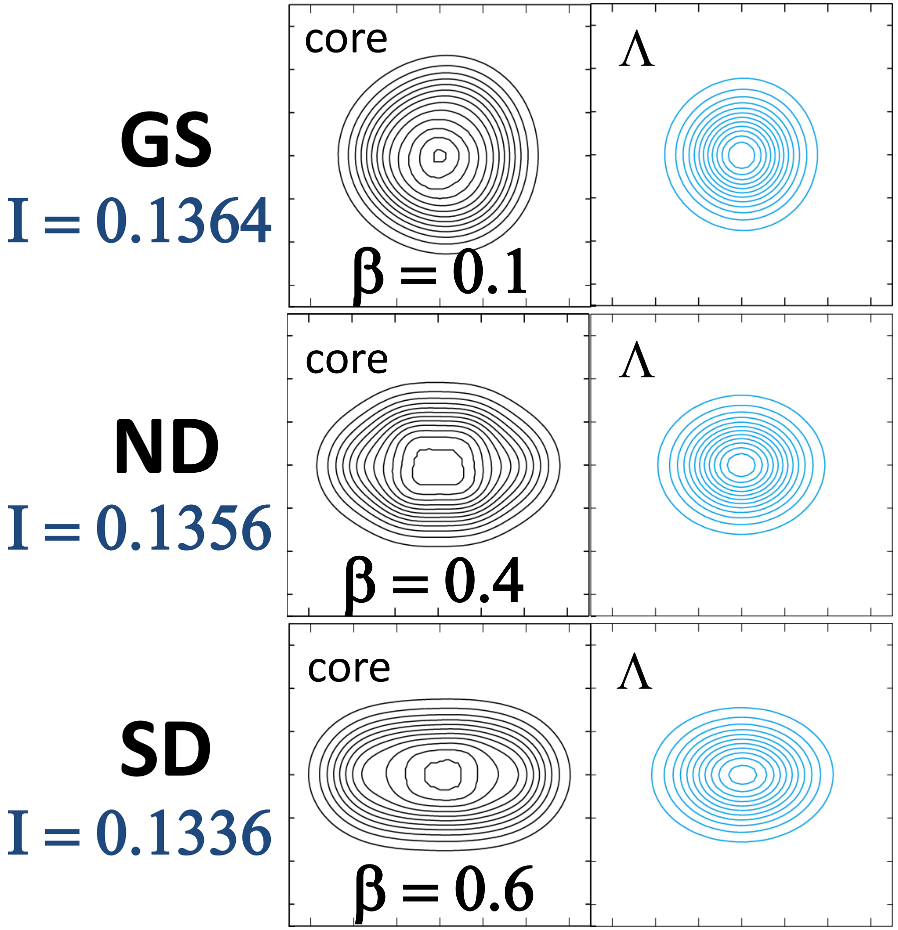
- B_Λ is sensitive to nuclear deformation through overlap b/w Λ and N

M. Isaka, *et al.*, PRC**89**, 024310(2014)



Note: Deformation change is quite small

Overlap: $I = \int d^3r \rho_N(\boldsymbol{r})\rho_\Lambda(\boldsymbol{r})$ [fm^{-3}]



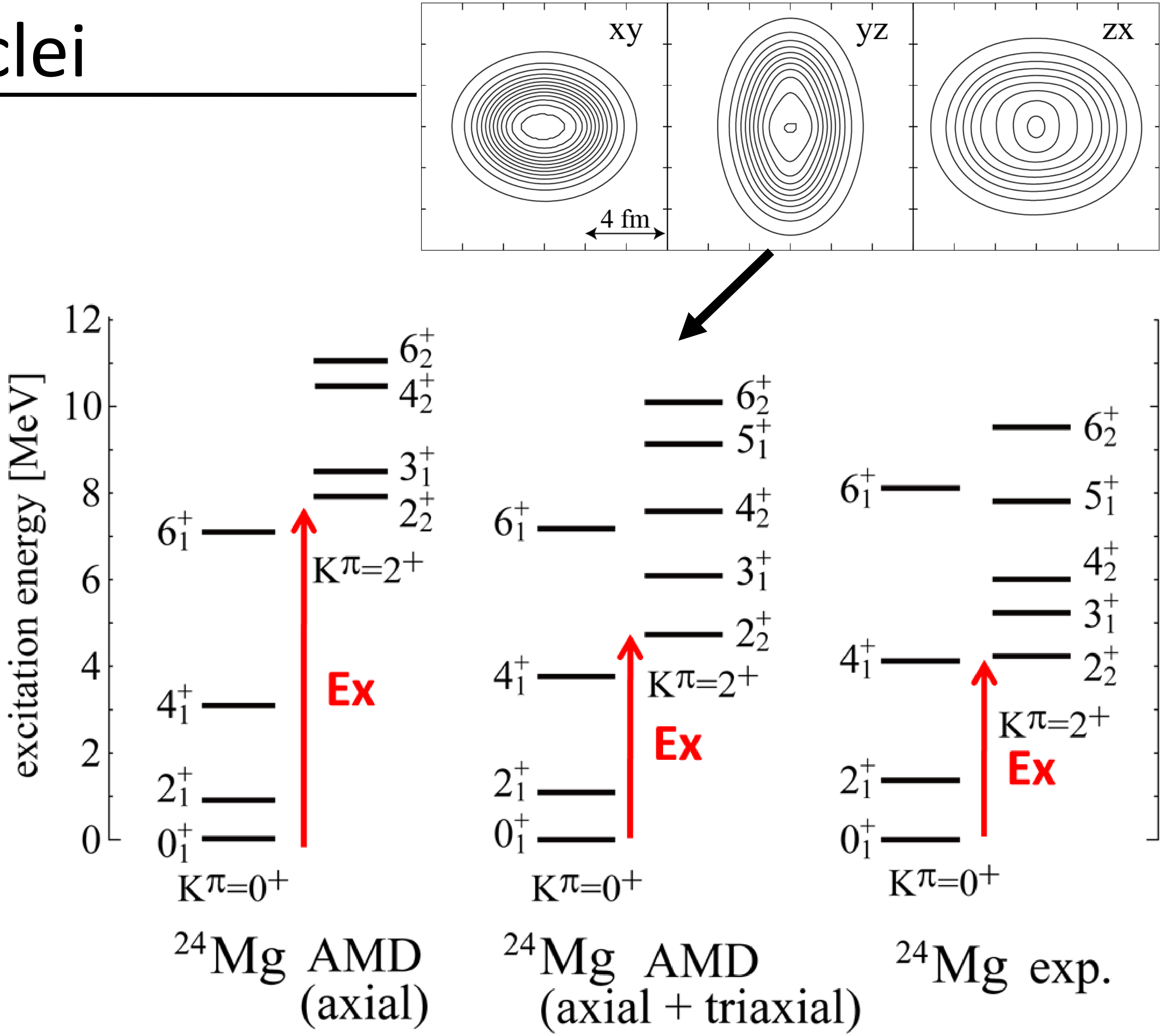
Backup

$^{25}_{\Lambda}Mg$

Deformation of Mg nuclei

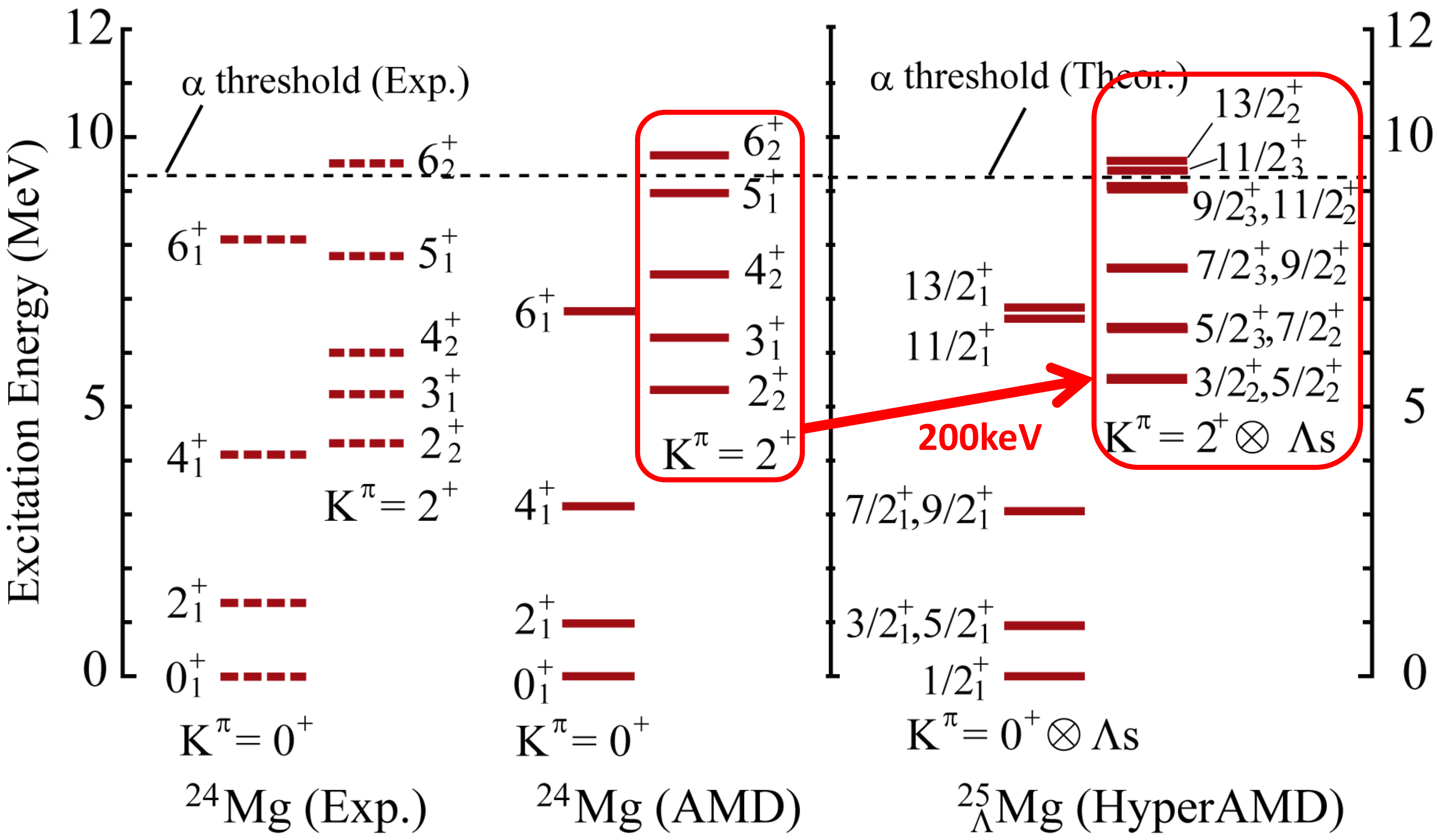
Ex.) ^{24}Mg

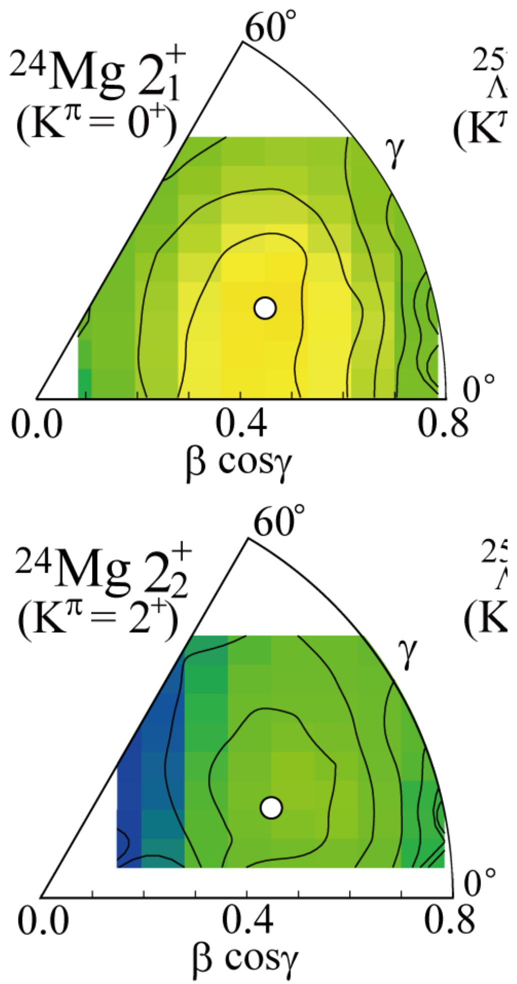
- Largely deformed nuclei far from magic number
- Low-lying 2nd 2^+ band indicates triaxial deformation



Backup: Excitation spectra of $^{25}{}_{\Lambda}$Mg

M. I., M. Kimura, A. Dote and A. Ohnishi, PRC **85** (2012), 034303.





Excitation energy of K$^{\pi}$=2^{+}⊗Λ_s band is shifted up by about 200 keV

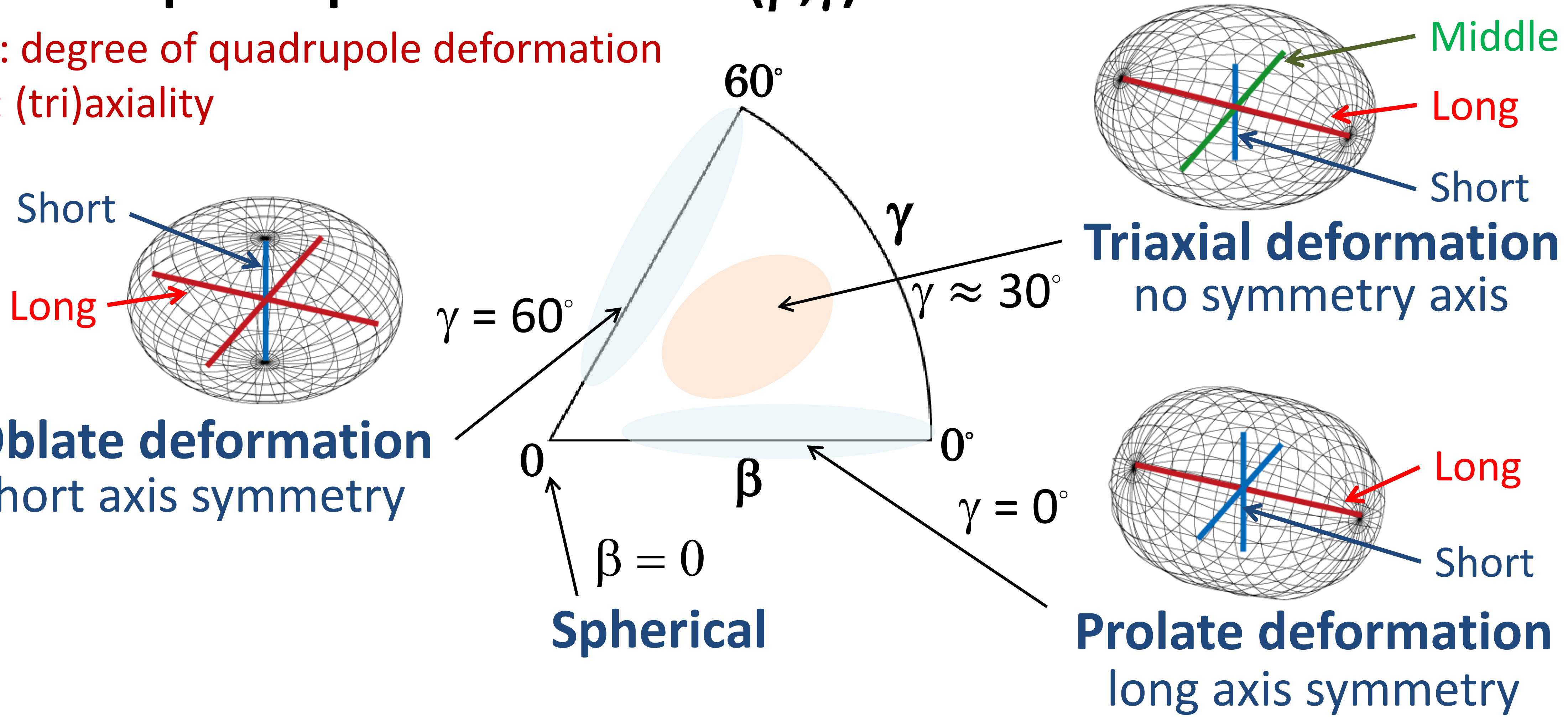
Backup

$^{27}_{\Lambda}$Mg

Deformation of nuclei

◆Most of nuclei are deformed except for magic nuclei

- **Nuclear quadruple deformation (β,γ)**
  - β: degree of quadrupole deformation
  - γ: (tri)axiality



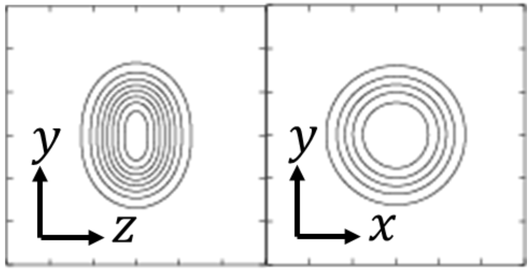
Deformation of nuclei

◆Most of nuclei are deformed except for magic nuclei

●Nuclear quadruple deformation (β,γ)

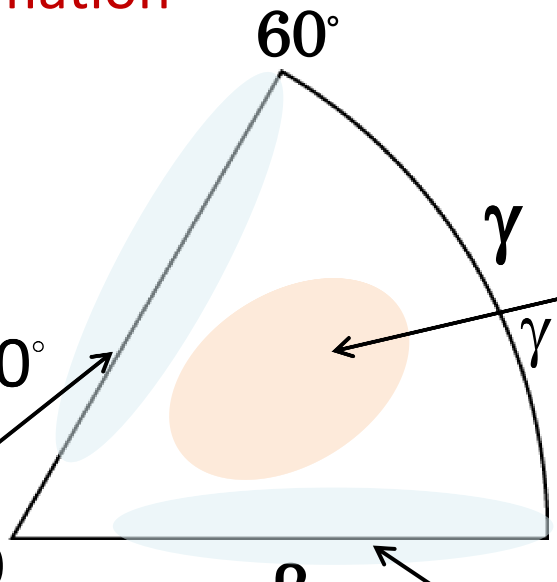
- β: degree of quadrupole deformation
- γ: (tri)axiality

^{12}C



Oblate deformation
short axis symmetry

$\gamma = 60^\circ$



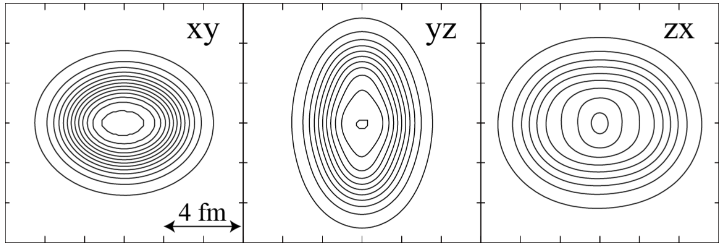
$\gamma \approx 30^\circ$

$\beta = 0$

Spherical

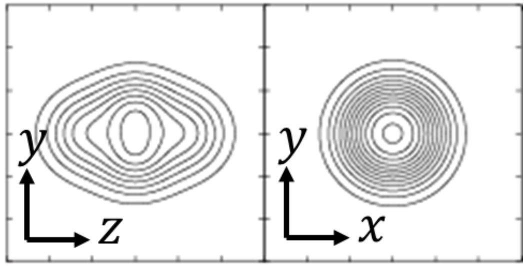
$\gamma = 0^\circ$

^{26}Mg



Triaxial deformation
no symmetry axis

^{20}Ne



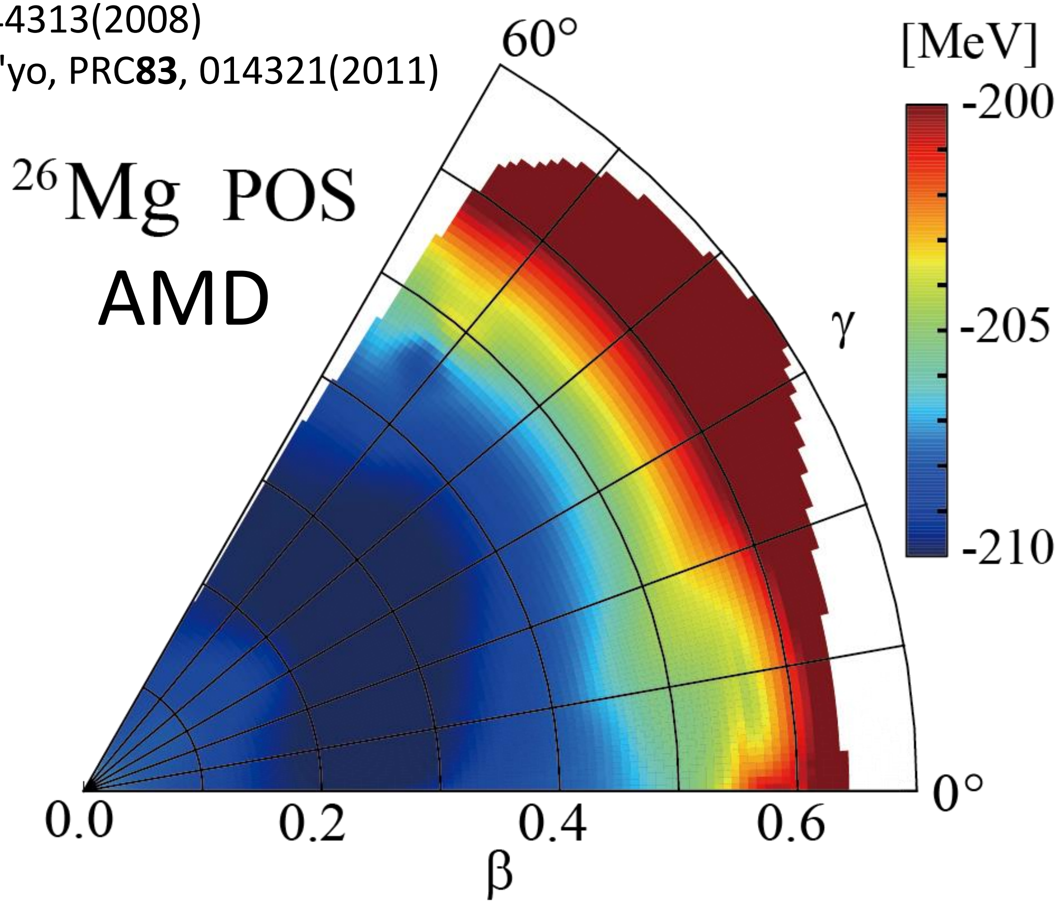
Prolate deformation
long axis symmetry

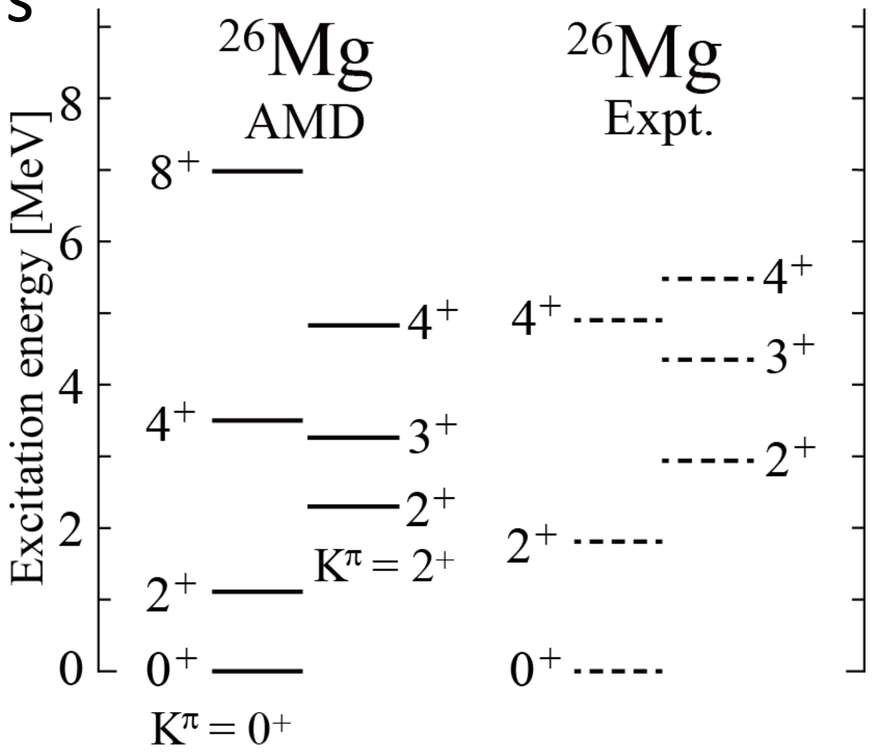
Density distribution with AMD calc

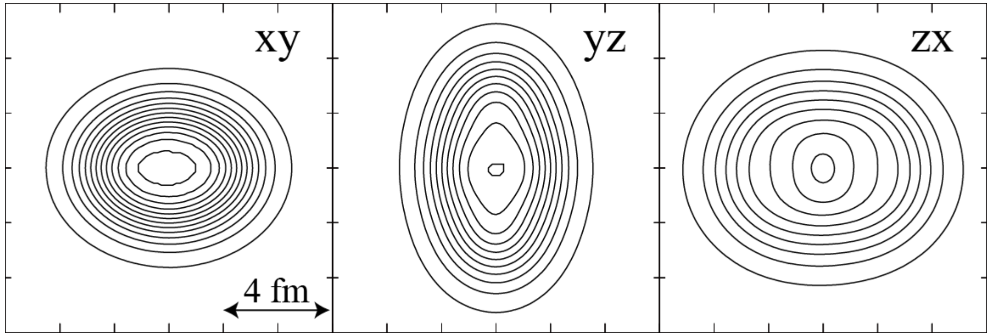
Deformation of Mg nuclei

^{26}Mg
- Shell gap in Nilsson diagram: Z=12 (prolate) vs. N=14 (oblate) → triaxial
- β,γ-soft nature is discussed by several authors

Terasaki et al., NPA**621**, 706(1997)
Rodrıguez-Guzman et al., NPA**709**, 201(2002)
Peru et al., PRC**77**, 044313(2008)
Hinohara, Kanada-En'yo, PRC**83**, 014321(2011)



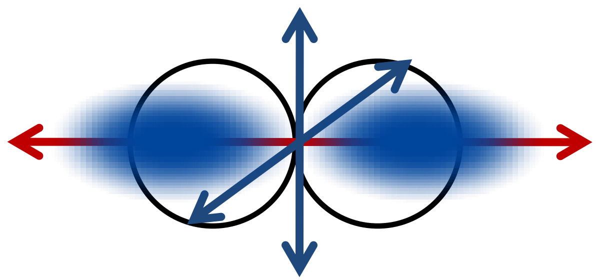




Split of *p*-state in $^{9}_{\Lambda}$Be

◆$^{9}_{\Lambda}$Be with 2α cluster structure

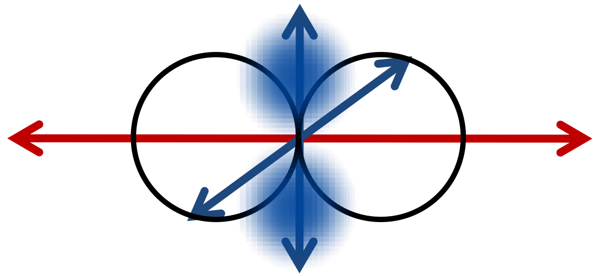
<u>*p* orbit parallel to 2α (long axis)</u>



Large overlap

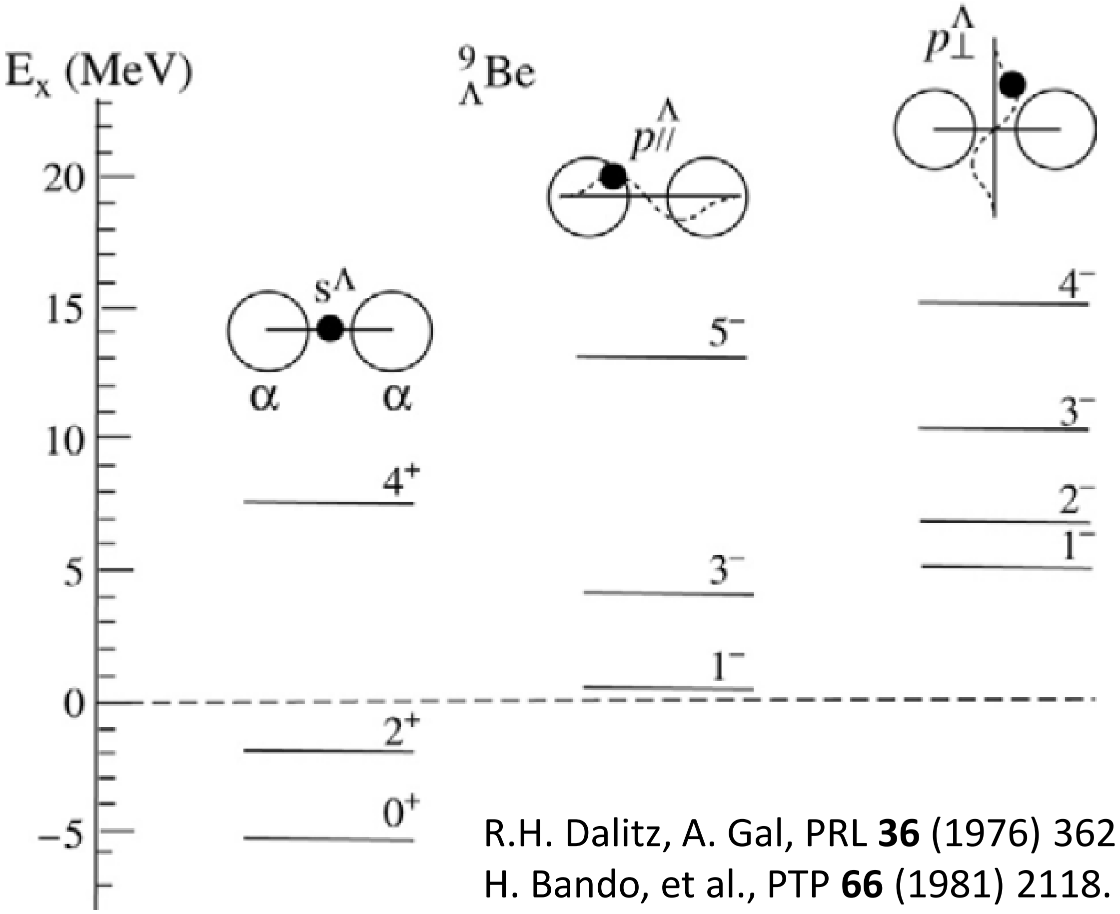
Deeply bound

<u>*p* orbit perpendicular to 2α (short axes)</u>



Small overlap

Shallowly bound



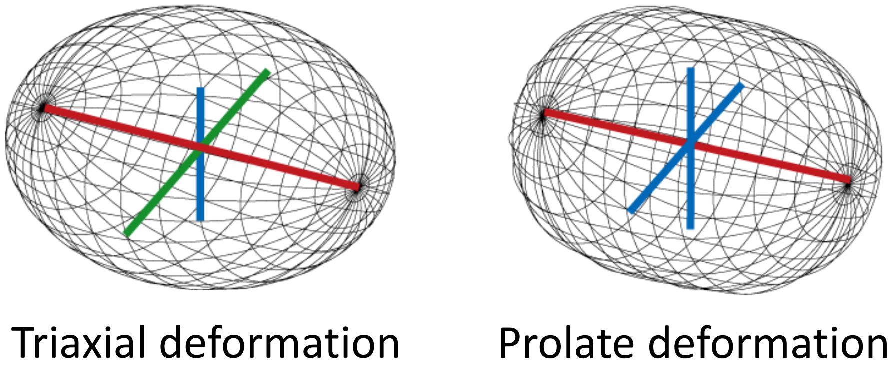
R.H. Dalitz, A. Gal, PRL **36** (1976) 362.
H. Bando, et al., PTP **66** (1981) 2118.
T. Motoba, et al.,PTPS**81**, 42(1985).

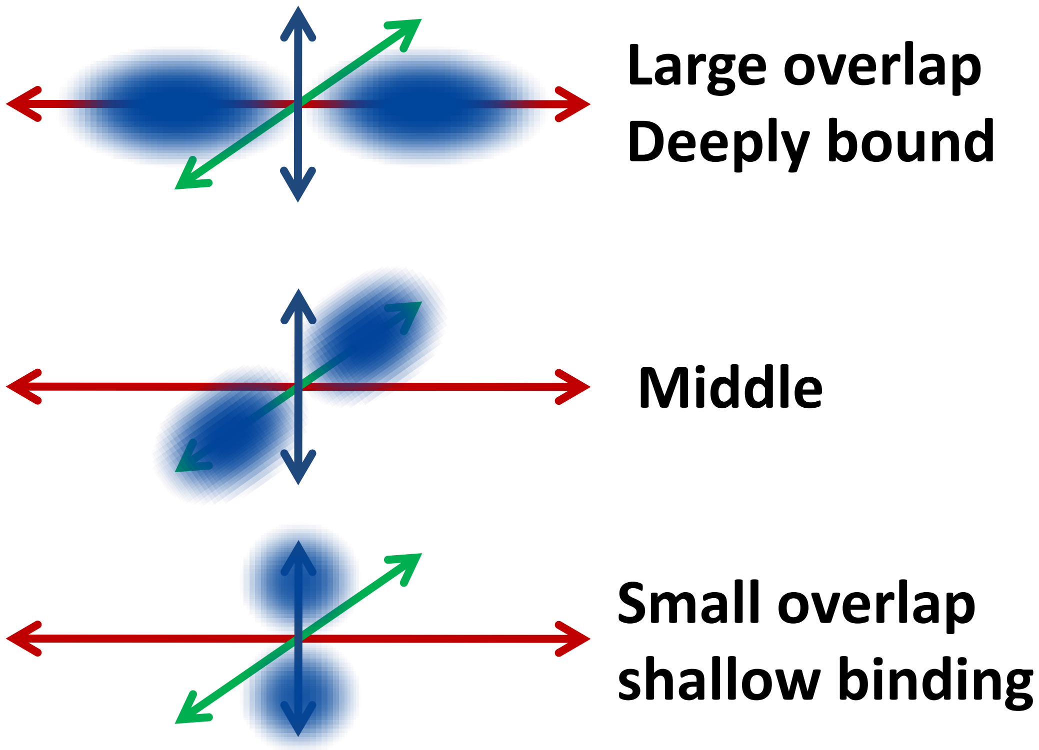
_p_-states splits into 2 bands depending on the direction of _p_-orbits

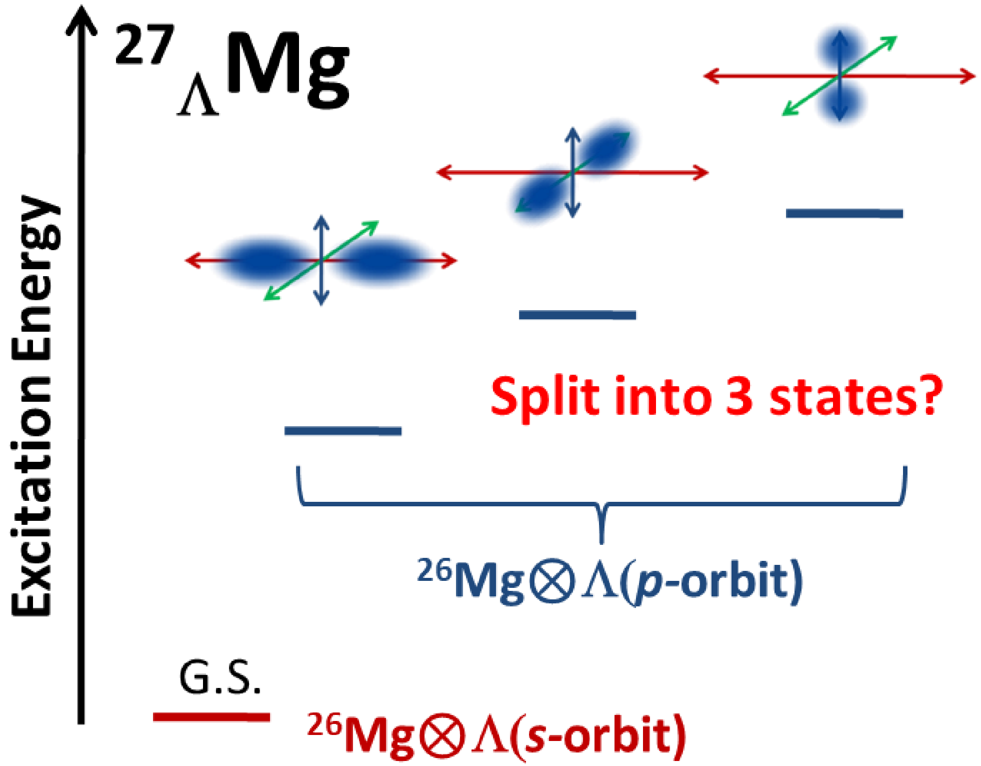
Triaxial deformation

If ^{26}Mg is triaxially deformed nuclei

⟶ *p*-states split into 3 different state





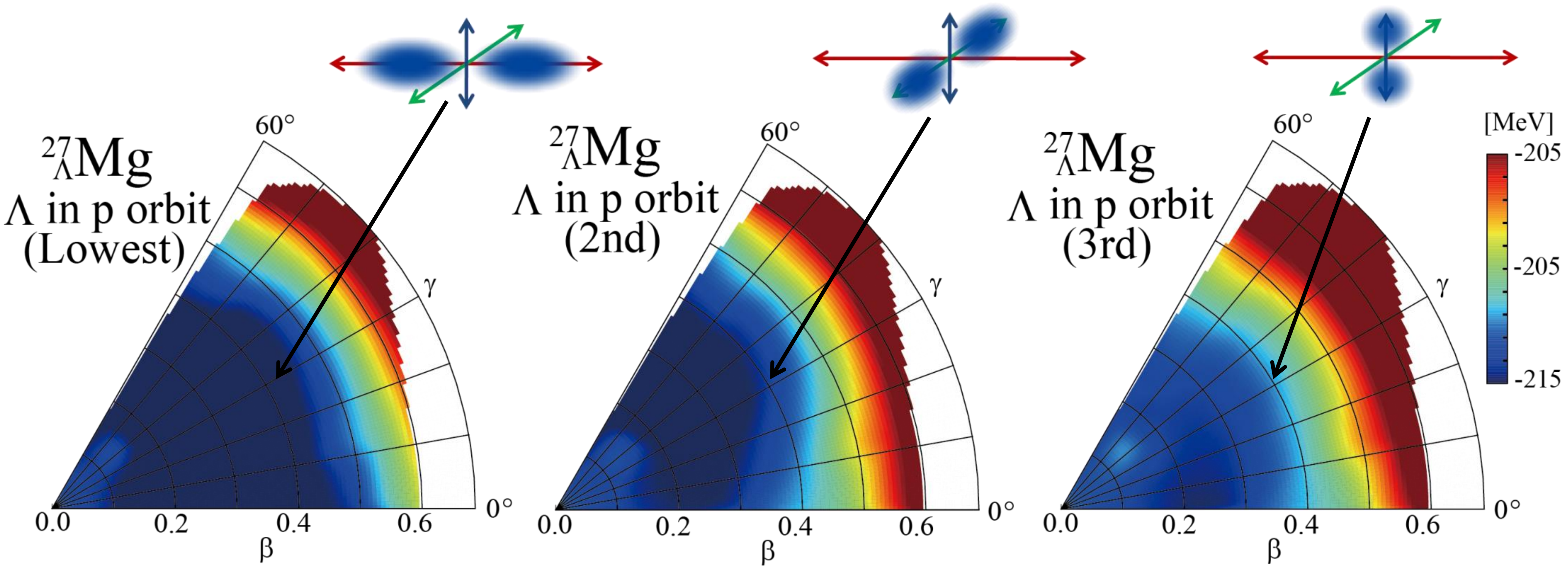


Observing the 3 different *p*-states is strong evidence of triaxial deformation

Energy surface on (β, γ) plane

◆ ***p*-states of $^{27}_{\Lambda}$Mg**

- **3 different *p* states appear by the energy variation with constraints**
- **With different spatial distribution of Λ (in γ ≃ 30 deg. region)**



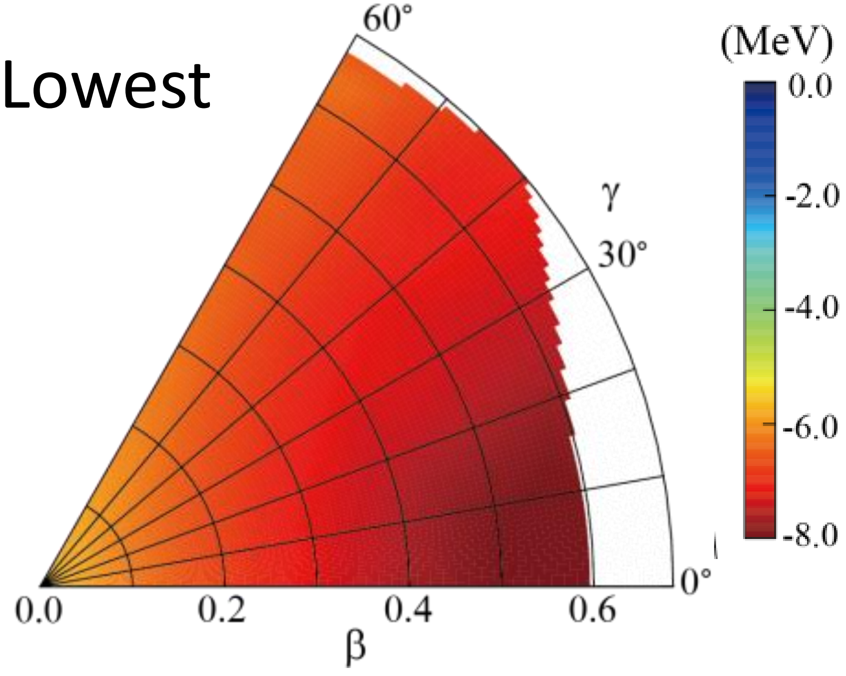
Results： Single particle energy of Λ hyperon

◆Λ single particle energy on (β, γ) plane

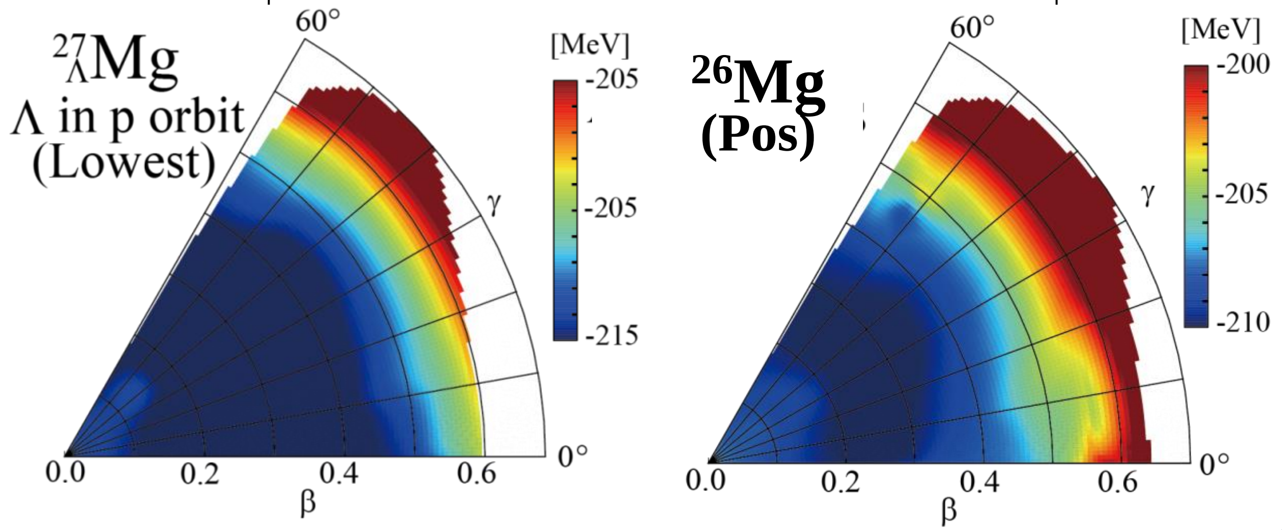
$$\varepsilon_\Lambda(\beta,\gamma) = E_{\Lambda p}(\beta,\gamma) - E_{core}(\beta,\gamma)$$

$^{27}_{\Lambda}$Mg (AMD, Λ in *p* orbit)

Lowest



$\varepsilon_\Lambda(\beta,\gamma)$: energy difference



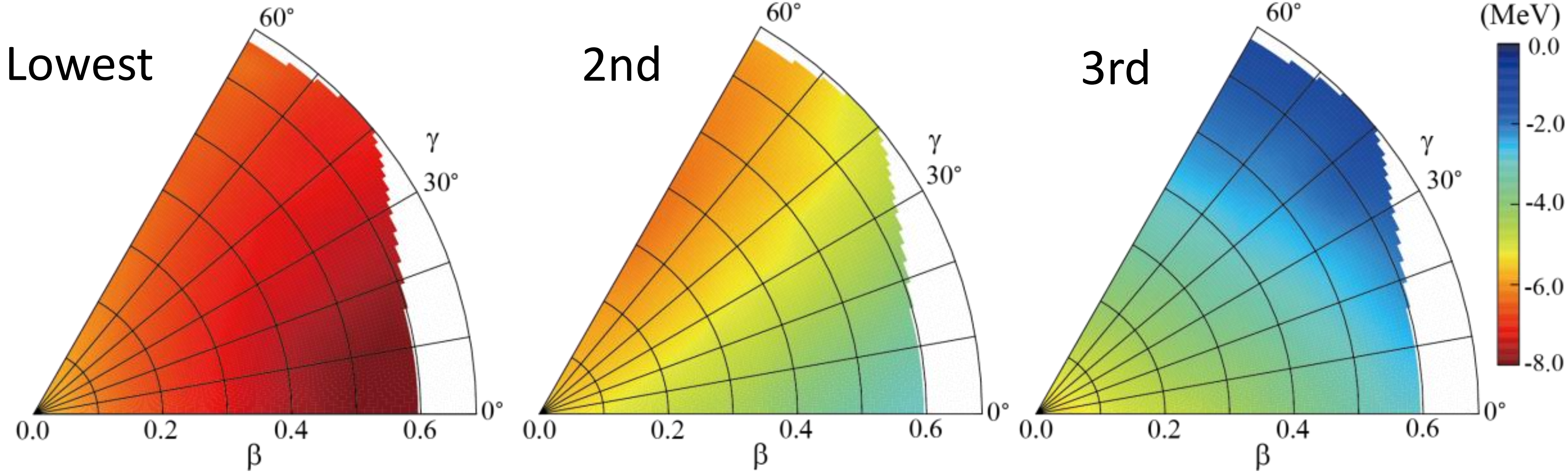
Single particle energy of Λ particle is different in each p state corresponding the difference of overlap between Λ and nucleons

Results： Single particle energy of Λ hyperon

◆Λ single particle energy on (β, γ) plane

$$\varepsilon_\Lambda(\beta,\gamma) = E_{\Lambda p}(\beta,\gamma) - E_{core}(\beta,\gamma)$$

$^{27}_{\Lambda}$Mg (AMD, Λ in *p* orbit)

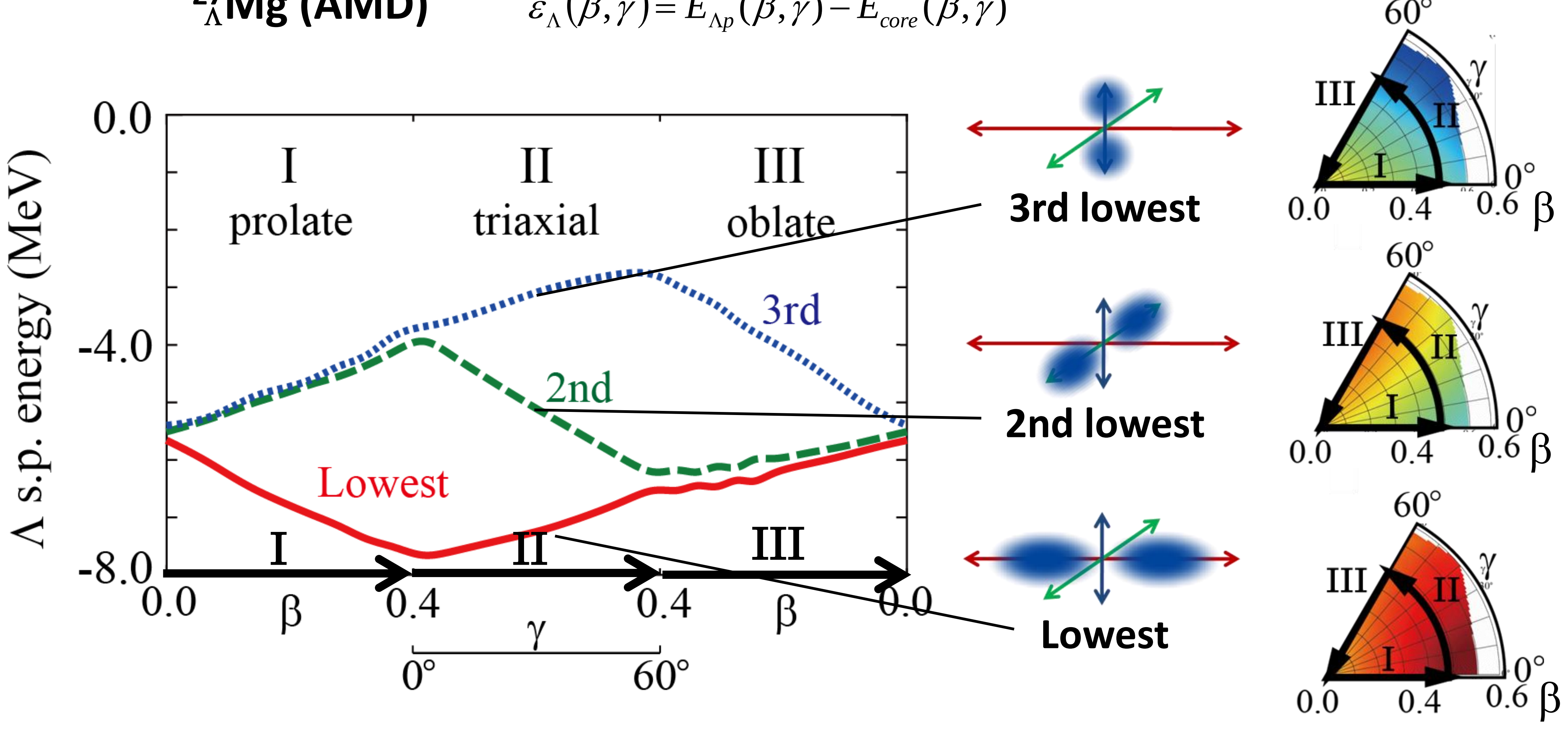


Single particle energy of Λ particle is different in each p state corresponding the difference of overlap between Λ and nucleons

Results : Single particle energy of Λ hyperon ε_Λ

$^{27}_{\Lambda}$Mg (AMD)

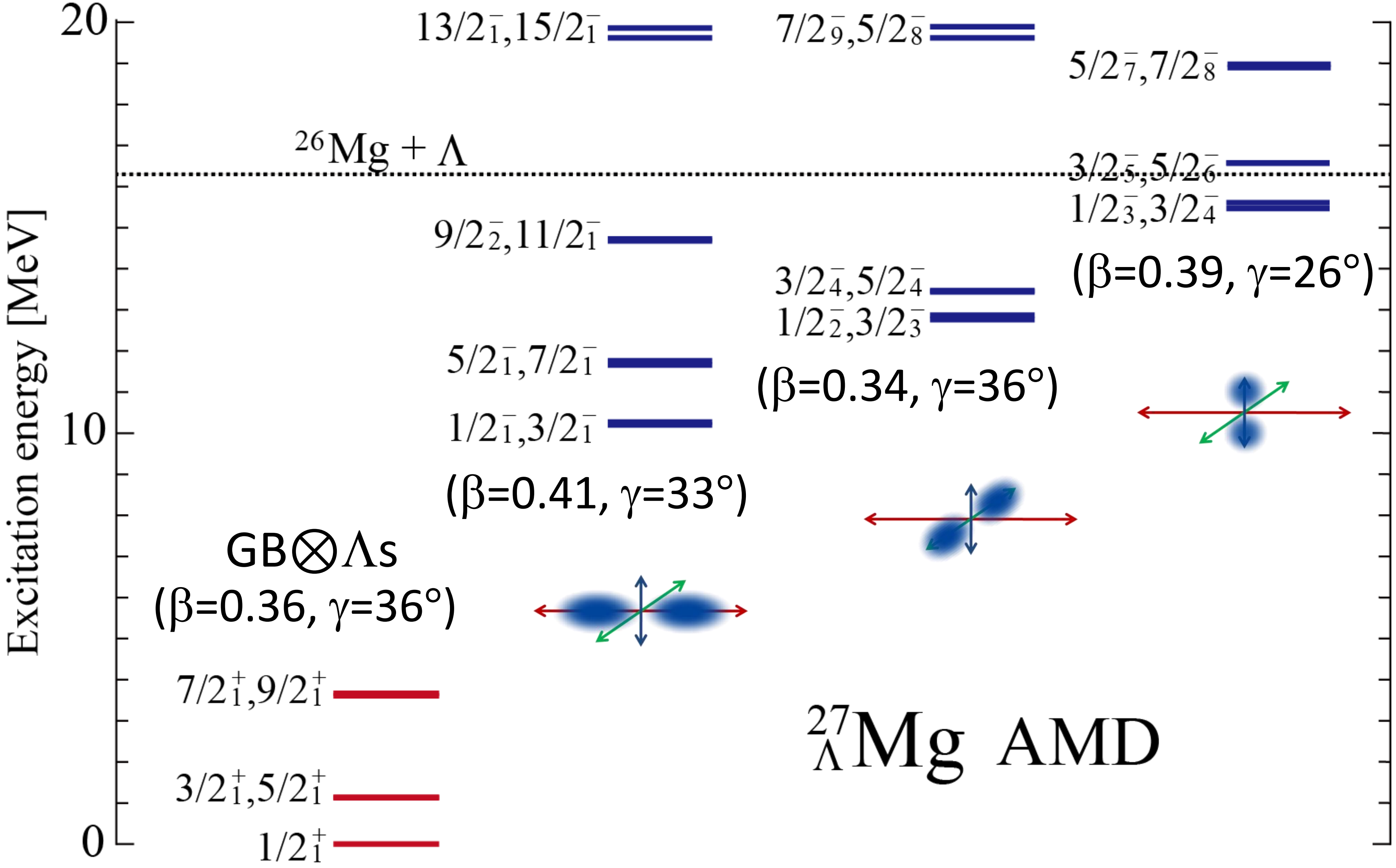
$$\varepsilon_\Lambda(\beta,\gamma) = E_{\Lambda p}(\beta,\gamma) - E_{core}(\beta,\gamma)$$



3 different p-states appear with triaxial deformation

Results: Excitation spectra

- **3 bands are obtained by Λ in *p*-orbit** → **Splitting of the *p* states**

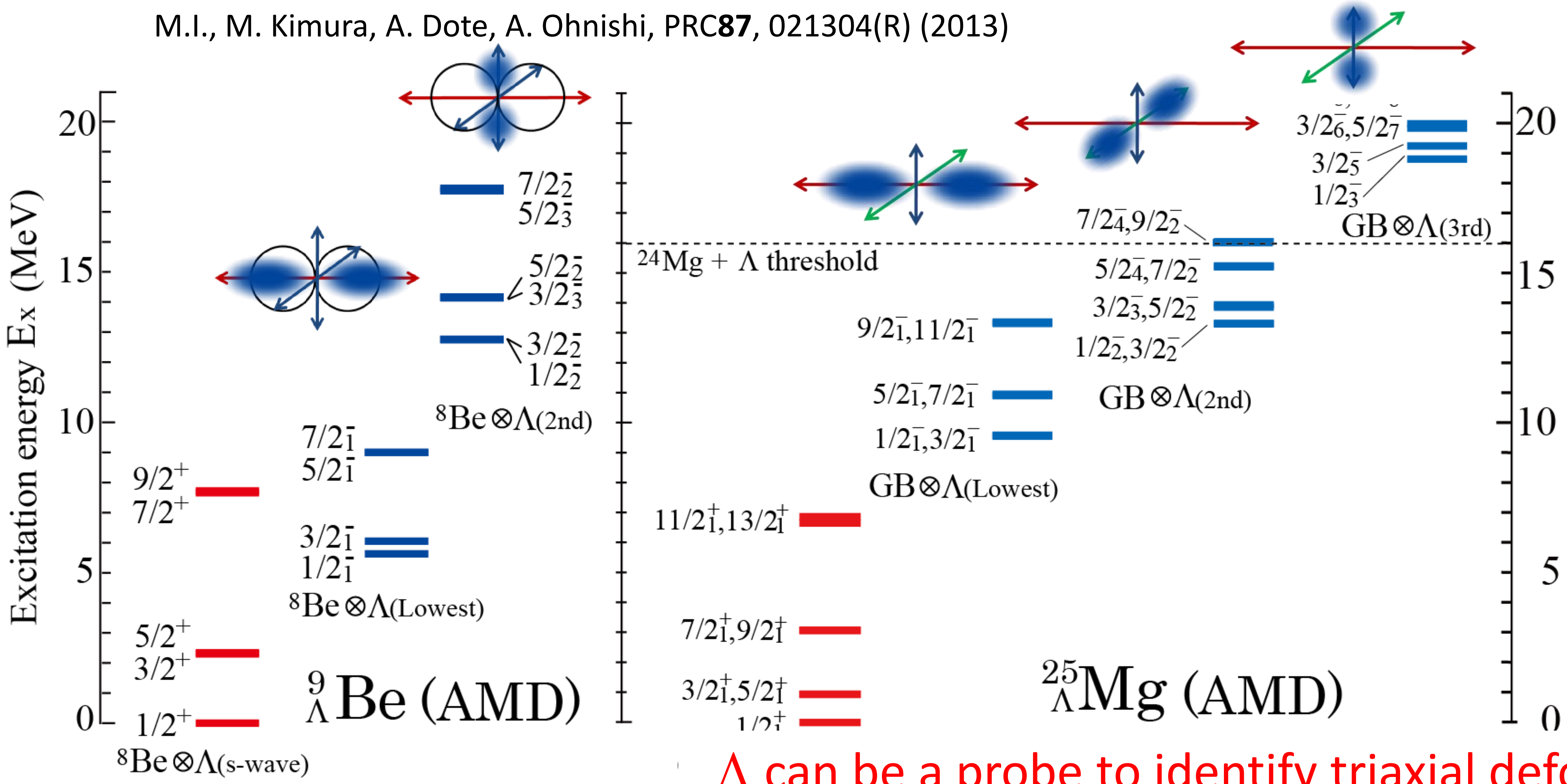


cf. ^{26}Mg ground state:
β=0.41, γ=33°

Λ in *p* orbit coupled to deformed nuclei

Splitting of p orbit with triaxial deformation → 3 different p states

M.I., M. Kimura, A. Dote, A. Ohnishi, PRC**87**, 021304(R) (2013)



Λ can be a probe to identify triaxial deformation

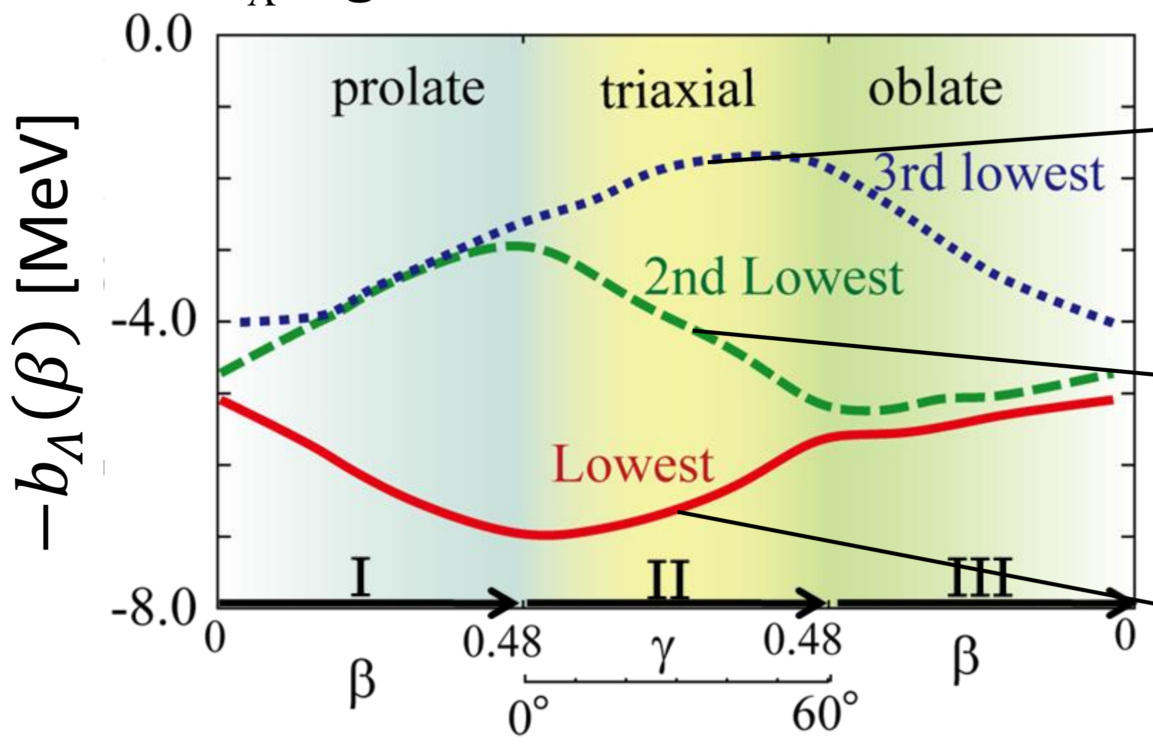
Λ in *p* orbit coupled to deformed nuclei

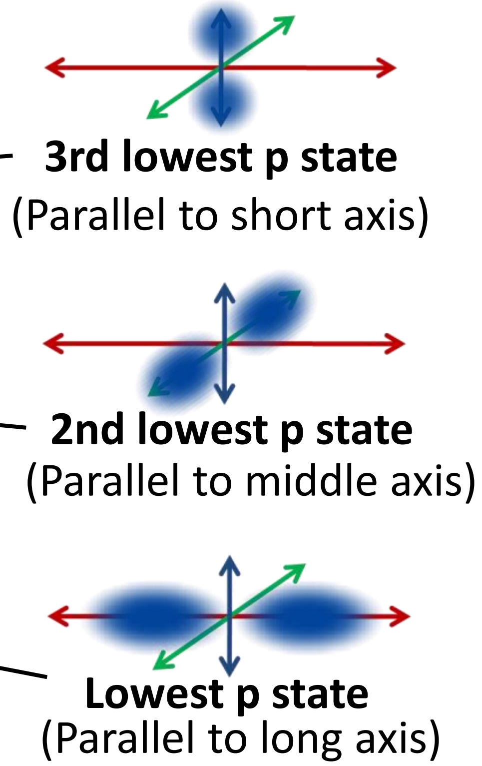
M.I., et al.,
PRC**87**, 021304(R) (2013)

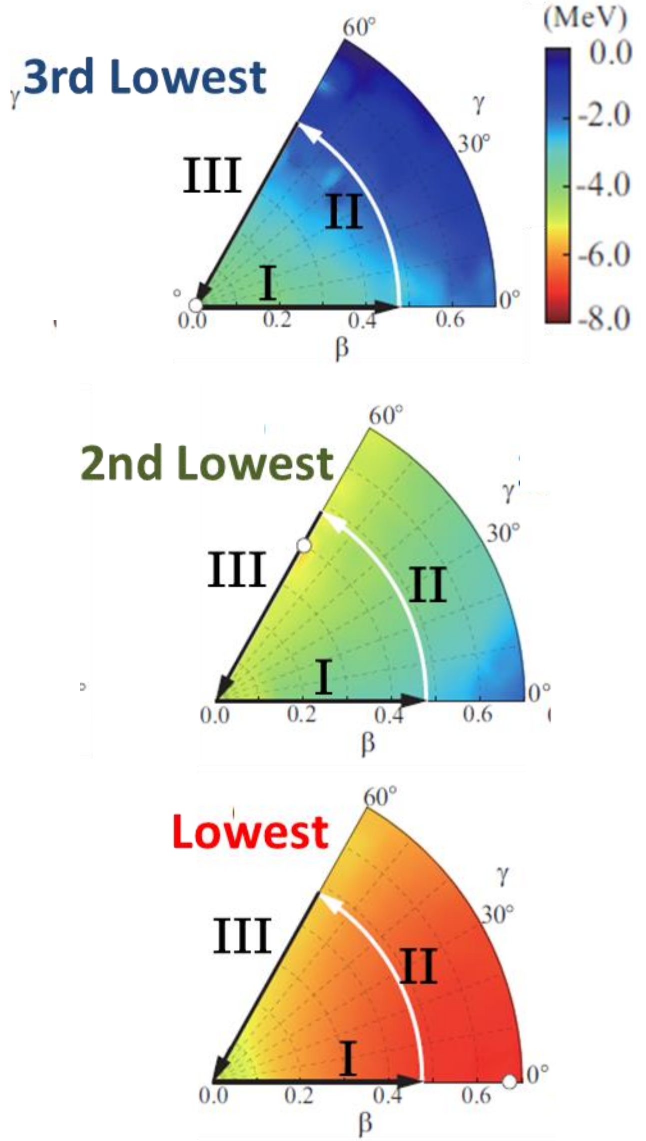
Splitting of p orbit with triaxial deformation → 3 different p states

- 3 *p*-states with different spatial distributions of Λ
- Λ binding energy $b_\Lambda(\beta)$ is different each other due to triaxial deformation

$^{25}_{\Lambda}$Mg $b_\Lambda(\beta) = E_{core}(\beta) - E_\Lambda(\beta)$



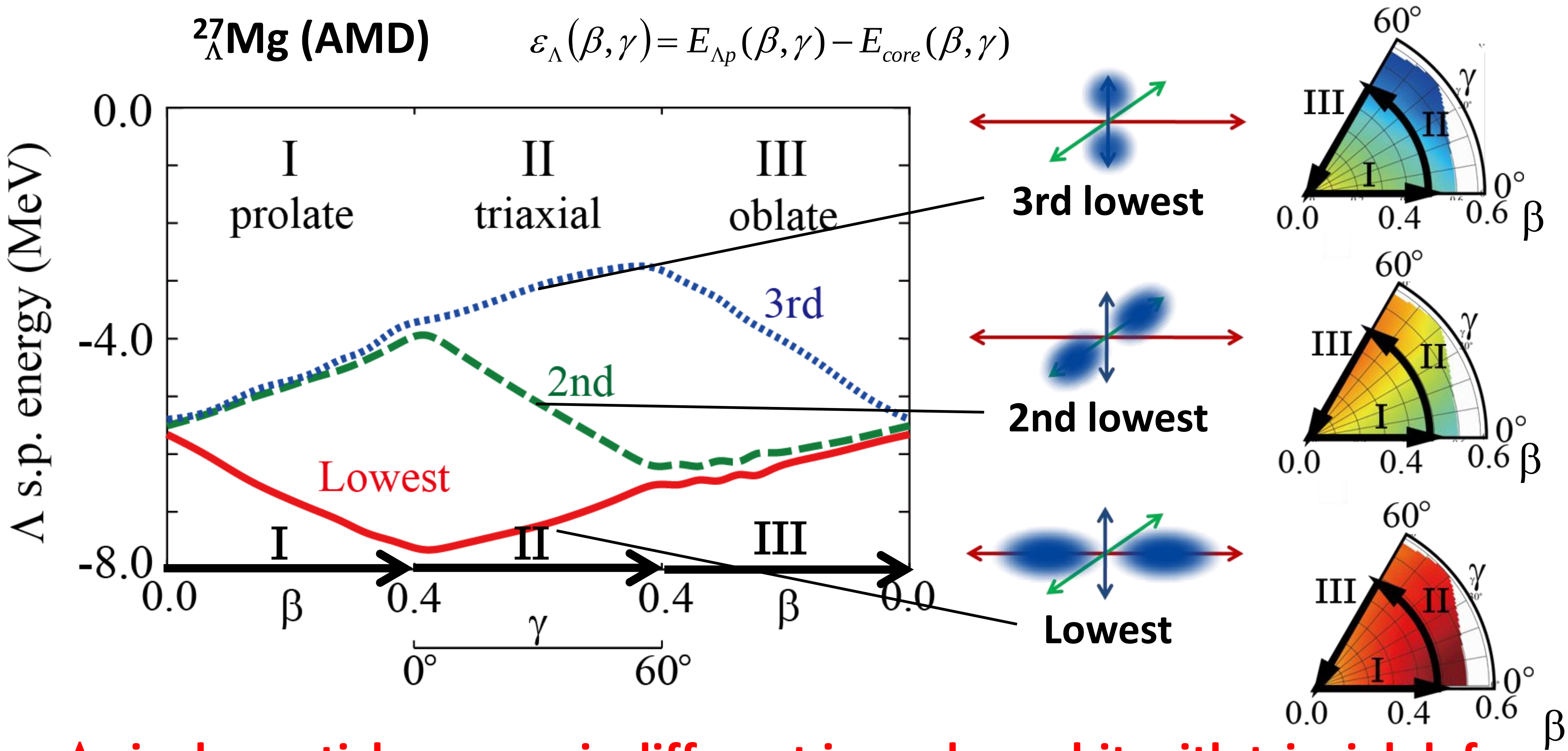




Results : Single particle energy of Λ hyperon ε_Λ

$^{27}_{\Lambda}$Mg (AMD)

$$\varepsilon_\Lambda(\beta,\gamma) = E_{\Lambda p}(\beta,\gamma) - E_{core}(\beta,\gamma)$$

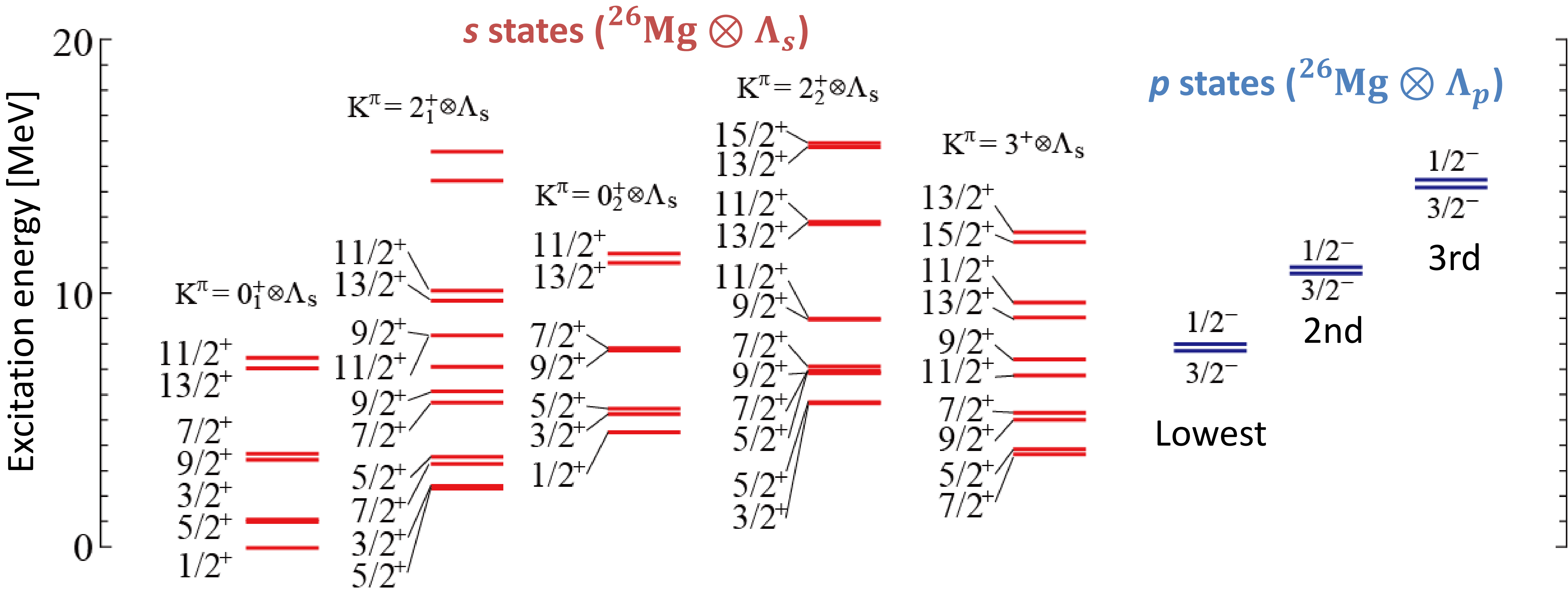


- **Λ single-particle energy is different in each p orbit with triaxial deformation**
- **3 different p-states appear if the core nucleus is triaxially deformed**

Preliminary results: Production cross section of $^{27}_{\Lambda}$Mg

Production cross section of ^{27}Al(γ, K^{+})$^{27}_{\Lambda}$Mg reaction within HyperAMD + PWIA

- *s* states: all rotational bands obtained with AMD calc.
- *p* states: lowest (1/2^{-}, 3/2^{-}) states for each



Backup

Hypernuclear production cross section with AMD

Production cross section with AMD wf

Production cross section of hypernuclei with AMD wave functions to see effects of various structures

- **In future: (γ, K⁺) reaction**

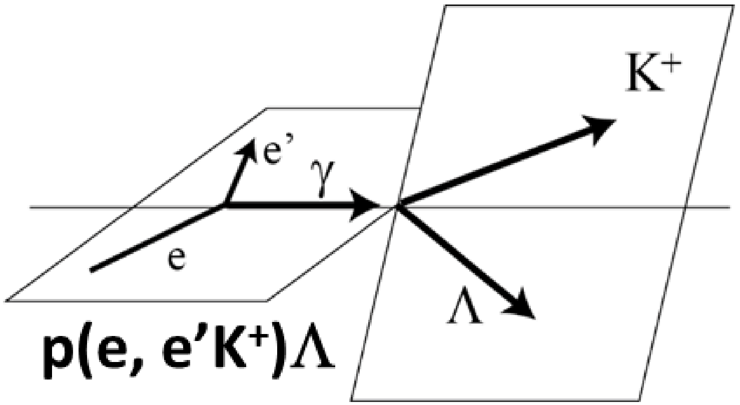
T. Motoba *et al.*, PTP**185**, 224(2010)

$$\frac{d\sigma}{d\Omega}(\theta_K^{\rm Lab}) = \frac{s p_K^2 E_K E_H}{p_K(E_H+E_K) - E_\gamma E_K \cos\theta_K^{\rm Lab}} \sum_{M_f} R(fi;M_f),$$

$$R(fi;M_f) = \frac{1}{2J_i+1} \sum_{M_i} \Psi_{\rm GCM}^{J_f\pi M_f} |\langle \Psi_{\rm GCM}^{J_f\pi M_f} | O | \Psi_{\rm GCM}^{J_i\pi M_i} \rangle|^2$$

AMD + GCM wave functions

Various structure



p(e, e'K⁺)Λ

$$O = \int d^3r \chi_K^{(-)*}(\mathbf{p}, \xi\mathbf{r}) \chi_K^{(+)}(\mathbf{k}, \mathbf{r}) \sum_{j=1}^{A} V_-^{(j)} \delta(\mathbf{r} - \eta\mathbf{r}_j) \langle \mathbf{k}-\mathbf{p}, \mathbf{p} | t | \mathbf{k}, 0 \rangle$$

Plane wave

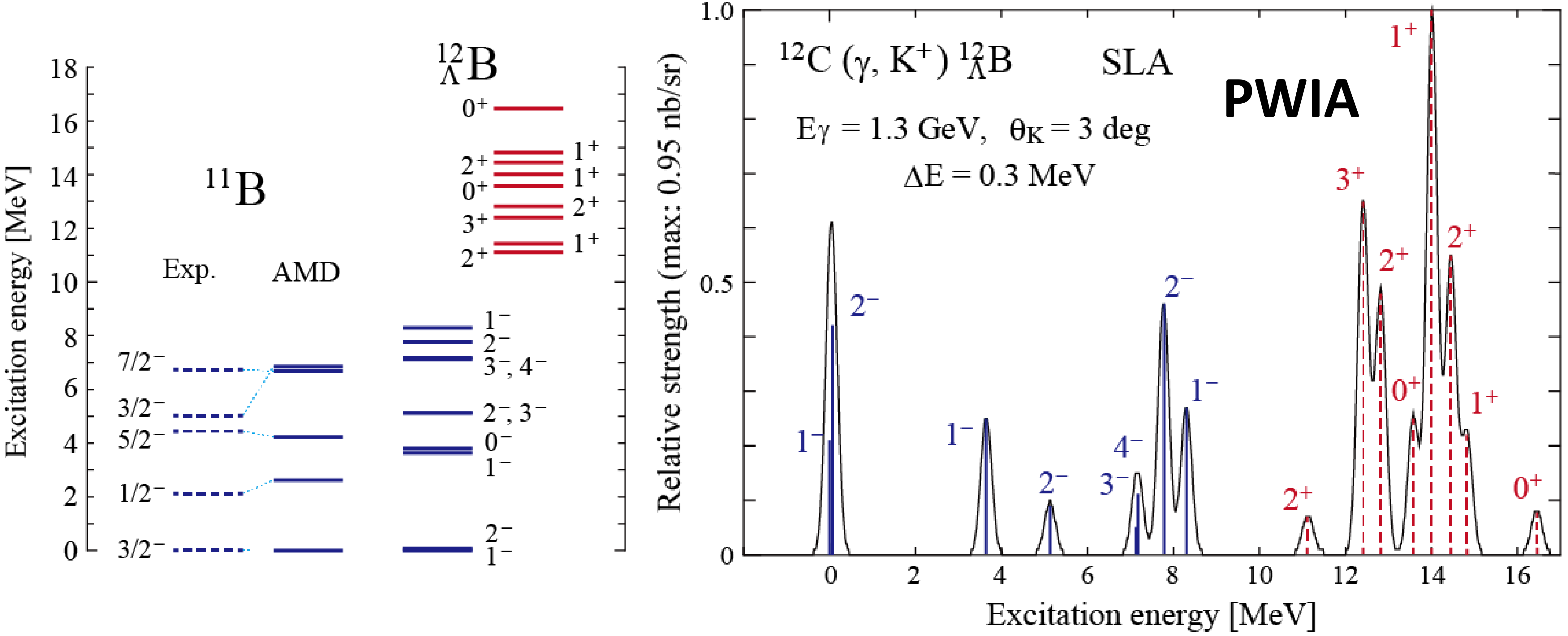
→ Distorted wave

Elementary amplitude

- **Current status: PWIA based on effective nucleon number approach**

Production cross section with AMD wf

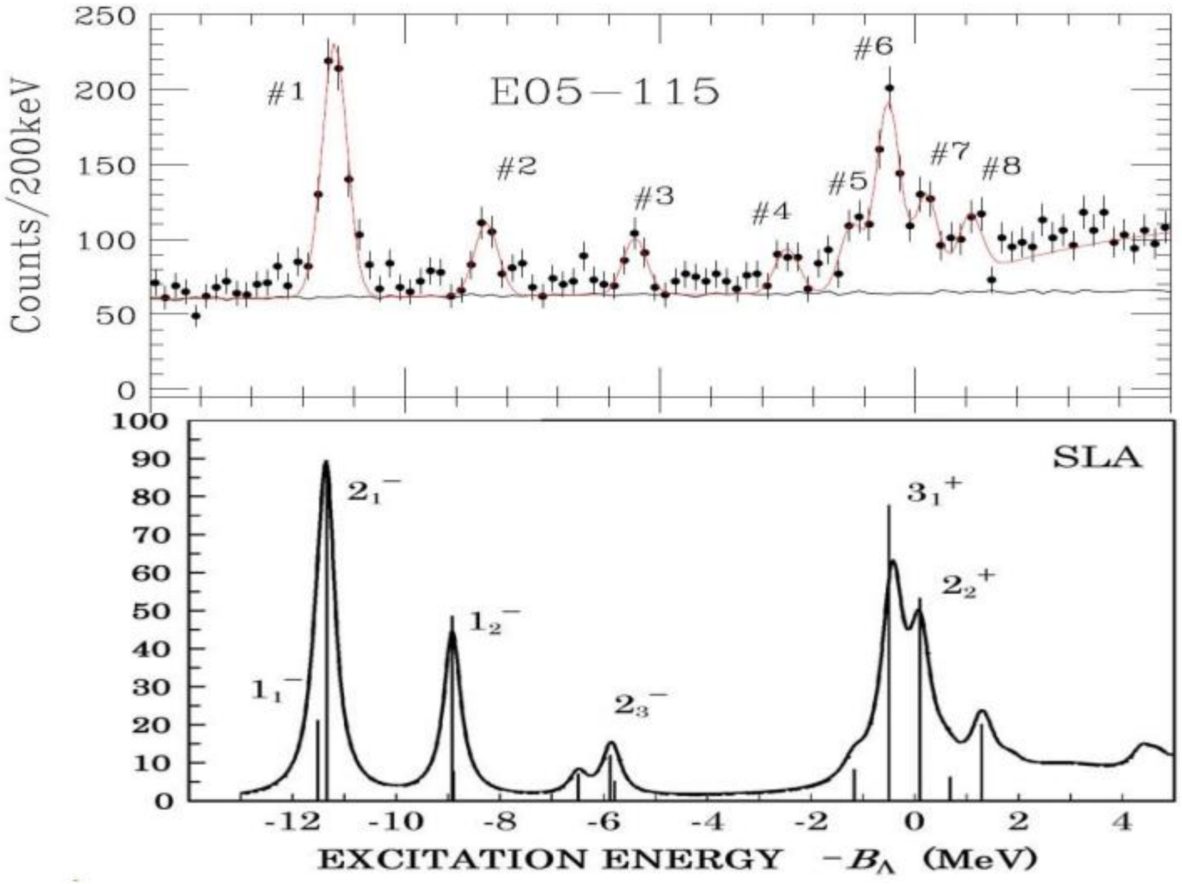
◆Example: $^{12}C(g, K^+)^{12}{}_\Lambda B$



Production cross section with AMD wf

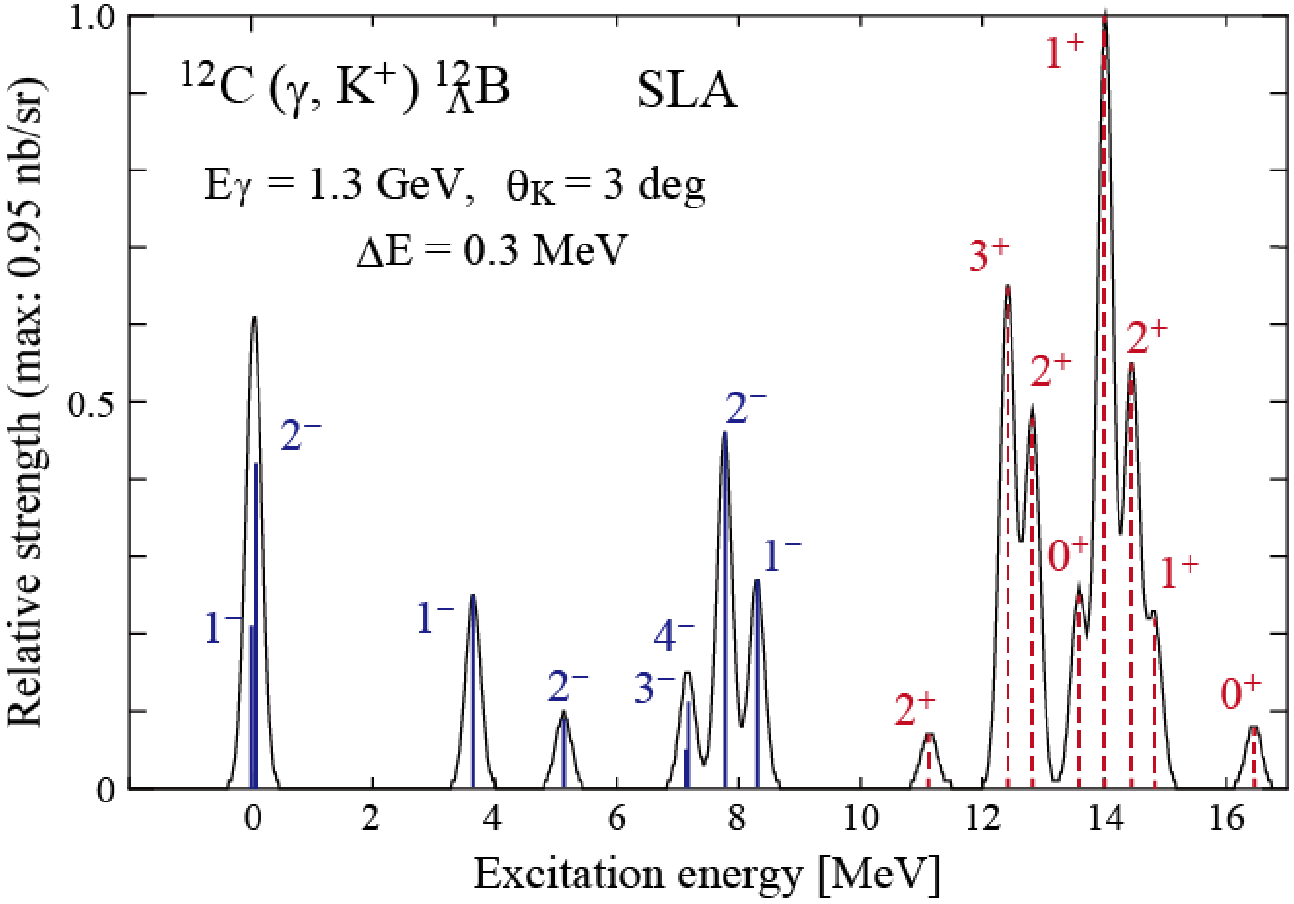
◆Example: $^{12}C(g, K^+)^{12}{}_{\Lambda}B$

L. Tang, et al.,PRC**90**, 034320(2014)



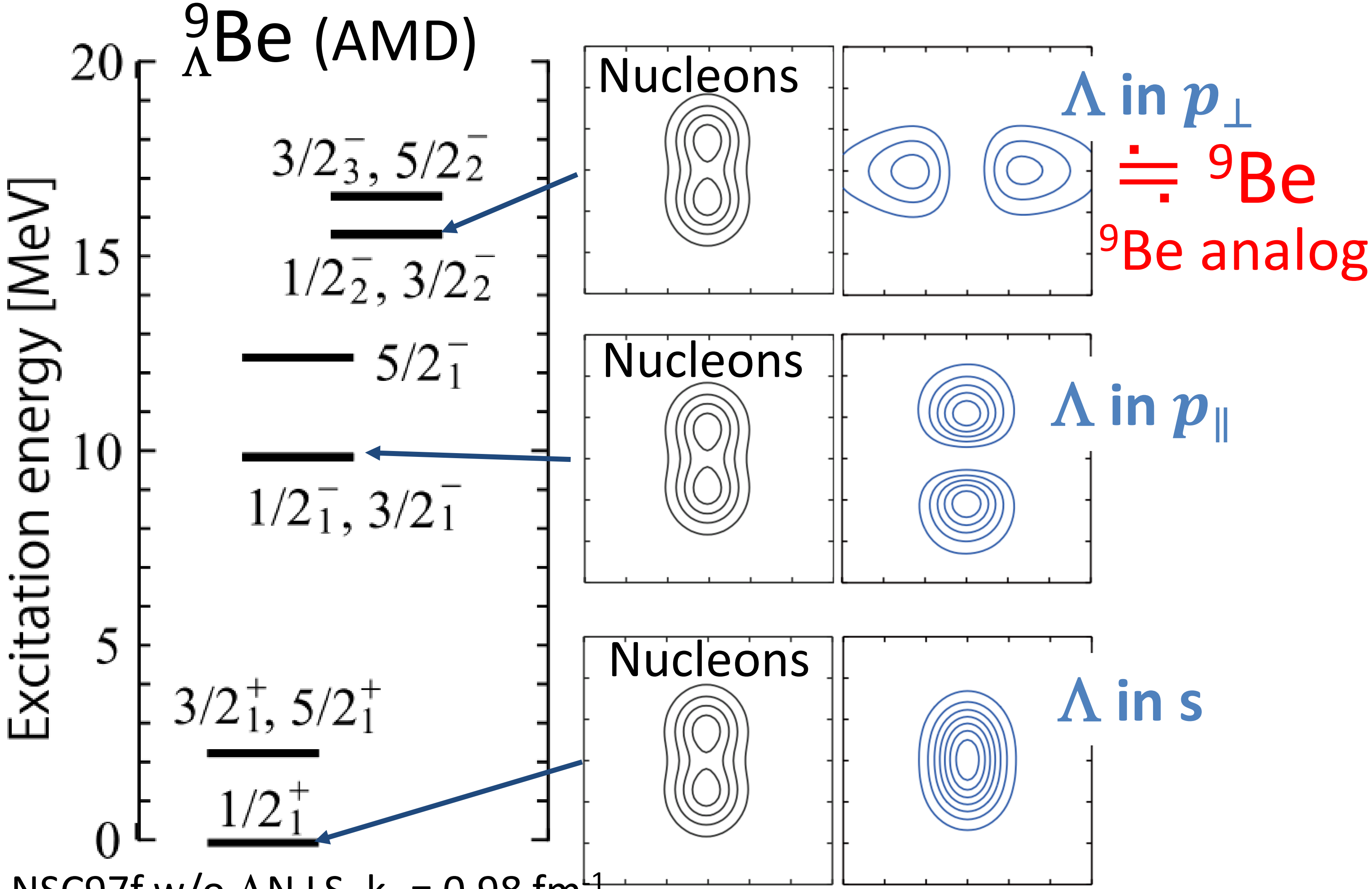
T. Motoba, JPS.Conf.Proc.17,011003(2017)

HyperAMD(present), PWIA



Current Status

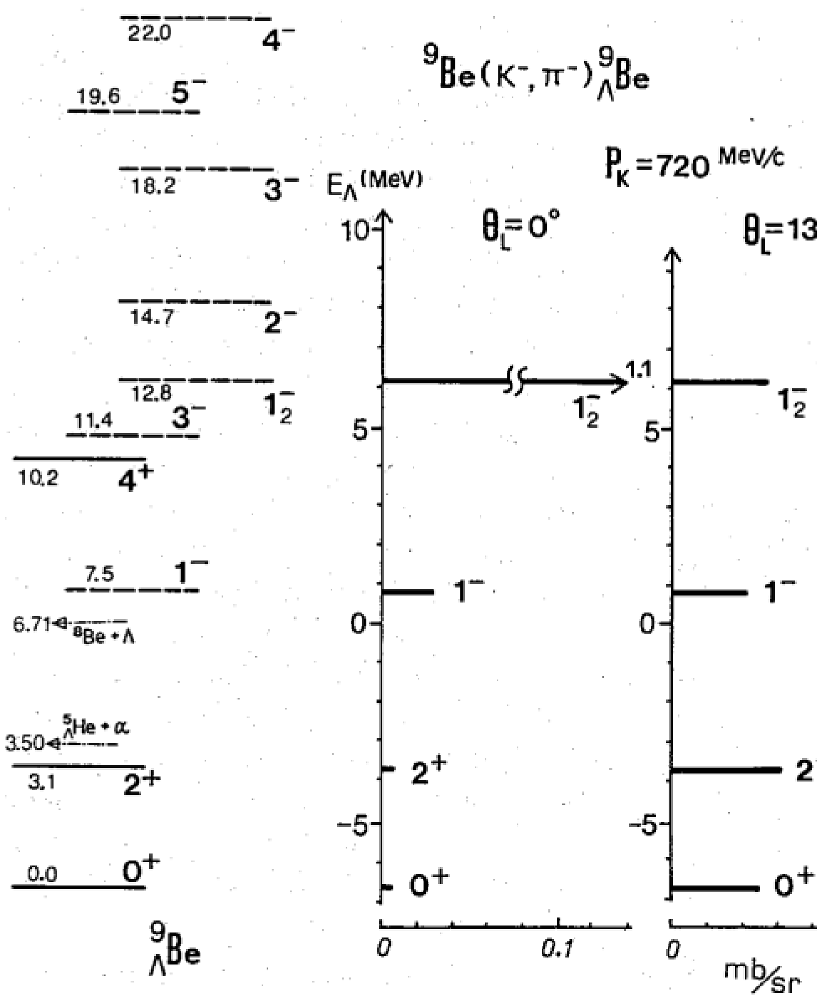
◆Application to ^{9}Be(K^{-}, π^{-})$^{9}_{\Lambda}$Be



NSC97f w/o ΛN LS, k_F = 0.98 fm^{-1}

cf. α + α + Λ cluster model calc

Motoba, et al., PTP71, 222(1984)



Current Status

◆Application to ^{9}Be(K^{-}, π^{-})$^{9}_{\Lambda}$Be

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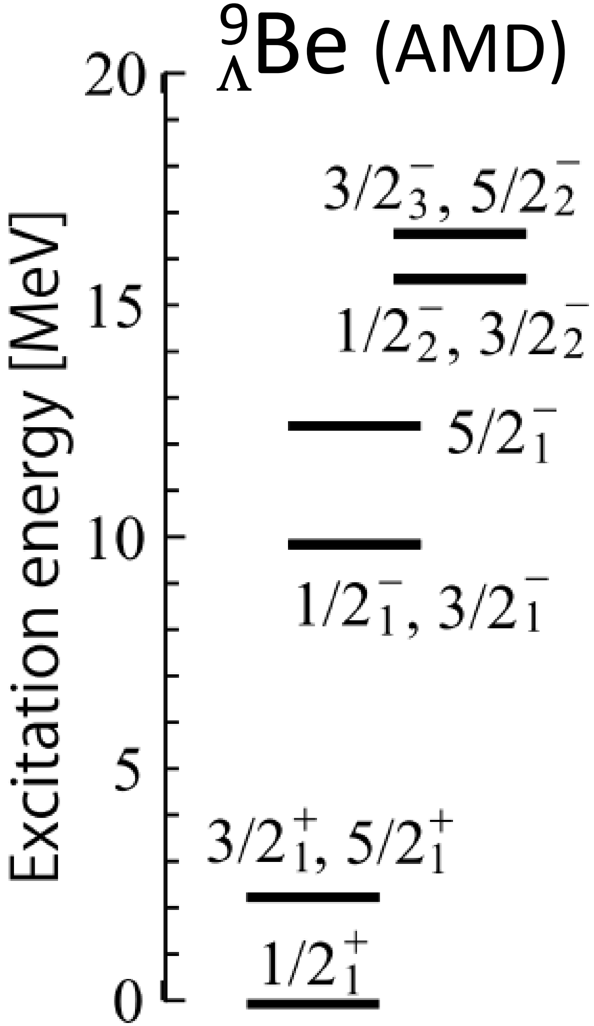
Motoba, et al., PTP71, 222(1984)



NSC97f w/o ΛN LS, k_F = 0.98 fm^{-1}

Λ in $p_\perp$ states