Hypernuclei from the Lattice

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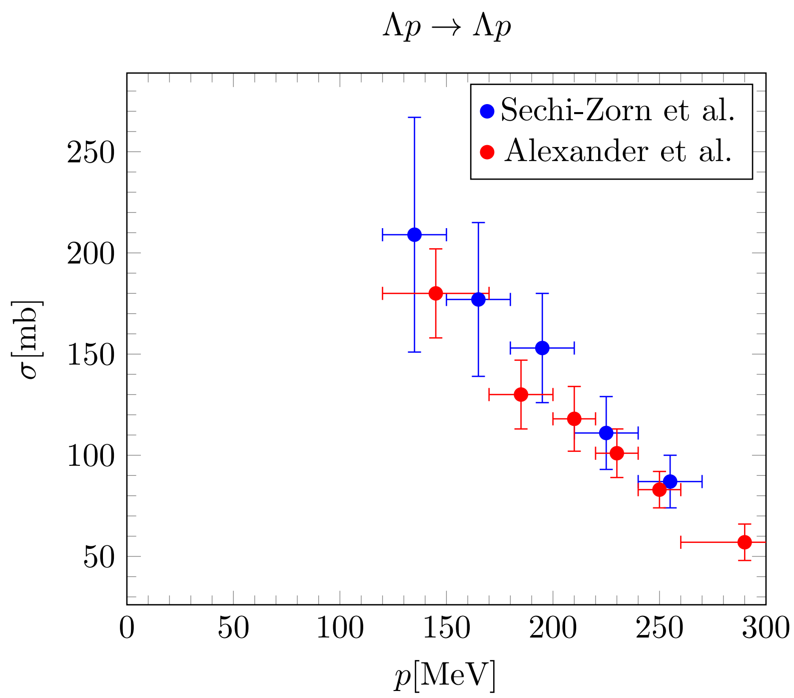
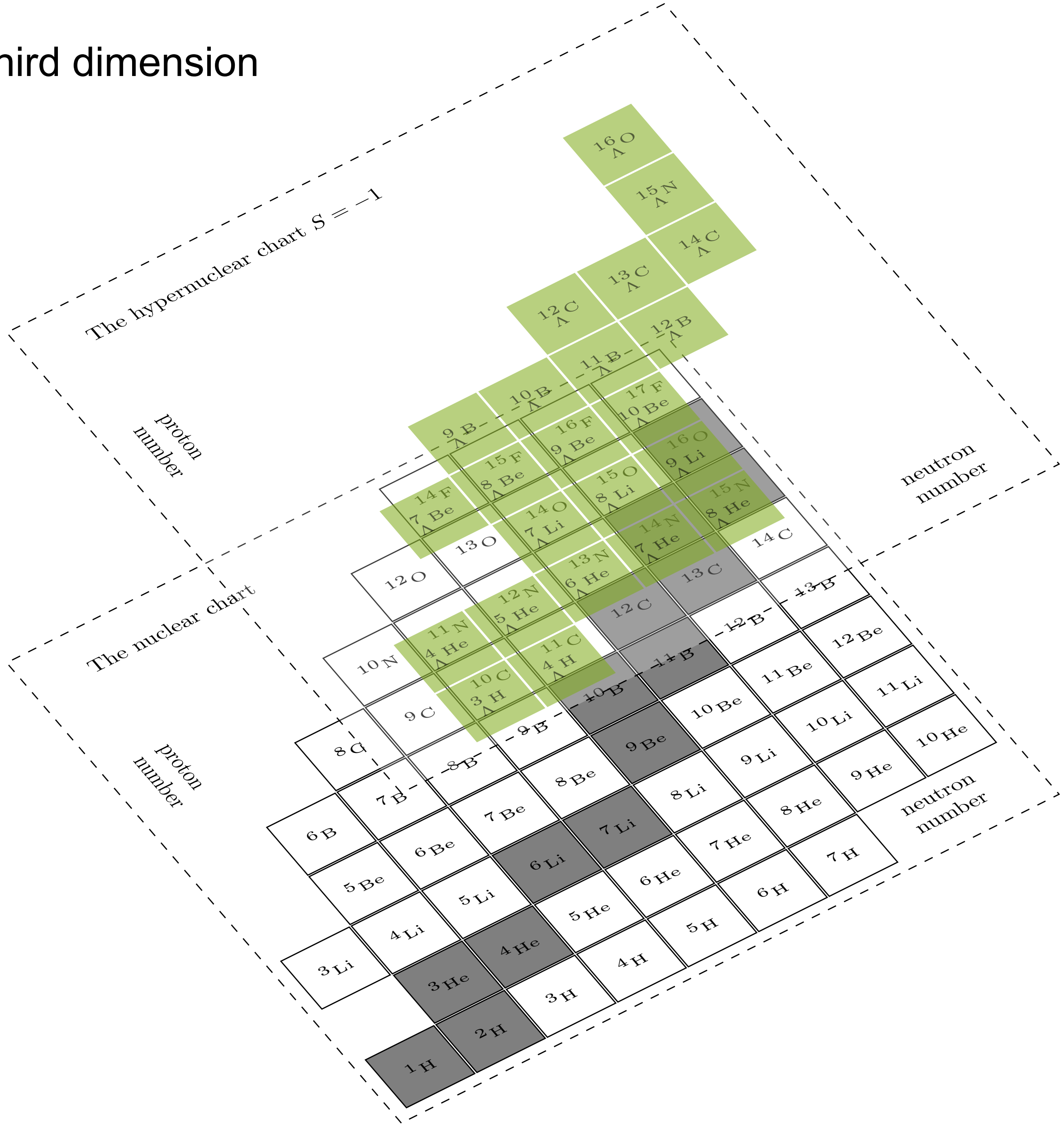
Outline

- Motivation
  - From NLEFT to (Hyper) NLEFT
    - Lattice Interaction
    - Results for light nuclei
    - Results for medium mass nuclei
- Summary and Outlook

Hypernuclear physics in a nutshell

- Strangeness extents the nuclear chart to a third dimension
- Unique opportunity to study the strong force
  Without the Pauli principle
- Typical approach from nuclear physics
  does not work since two-body data is sparse

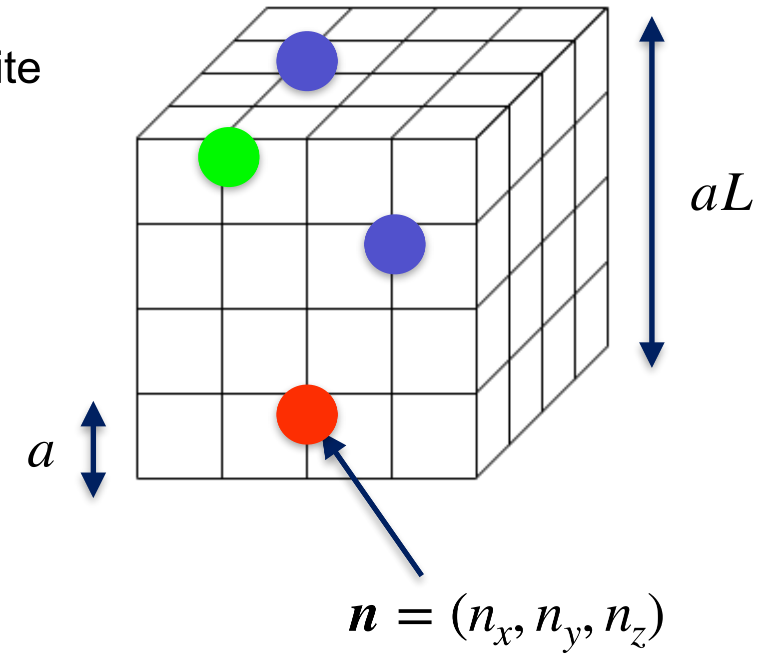




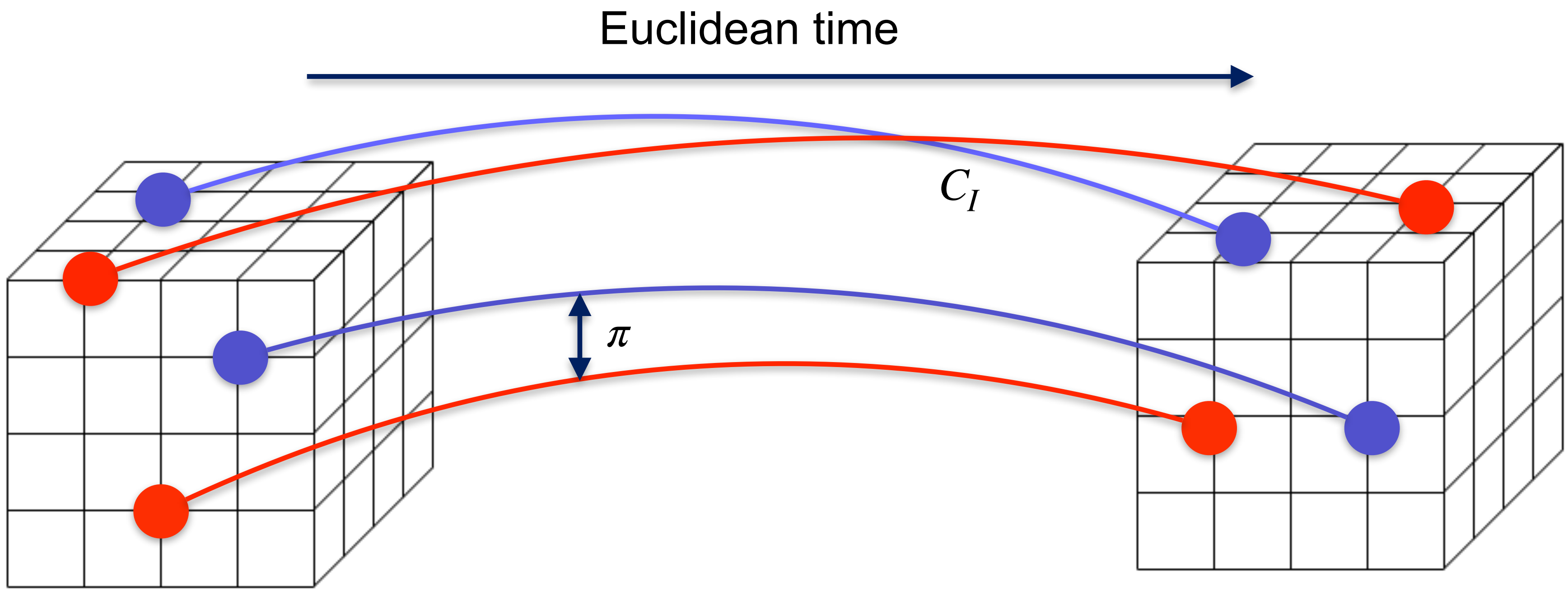
- Gateway : Three-Body Systems

Method: Lattice Monte Carlo

- Lattice ⇒ Cubic Volume of size $(La)^3$ with discrete lattice site
- Discretized chiral potentials ,contact interactions one-pion exchange, coulomb (Epelbaum et al.)
- Do euclidean time evolution and extract i.e. energies as transient energy $E = -\frac{d}{d\tau}\ln(Z(\tau))$



Method: Lattice Monte Carlo



- Auxiliary Fields to handle many particles efficiently:
- Idea: Replace Interactions between nucleons with Interaction of a nucleon with an auxiliary field

$$\exp(-\frac{C}{2}(N^\dagger N)^2) = \sqrt{\frac{1}{2}}\int \mathrm{d}A \exp\left[-\frac{A^2}{2} + \sqrt{C}A(N^\dagger N)\right]$$

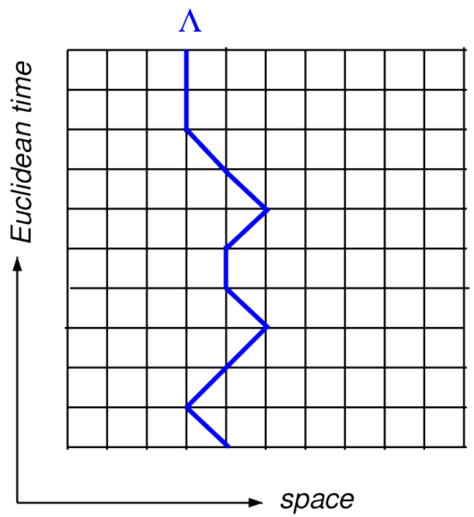
Since Nucleons only interact with an auxiliary field⇒ Perfect for parallel computing

Starting point for (Hyper) Nuclear Lattice EFT

Very Successful Nuclear Program:

Using AFMC and shuttle algorithm

Wave function matching to obtain precise results for nuclei and charge radii



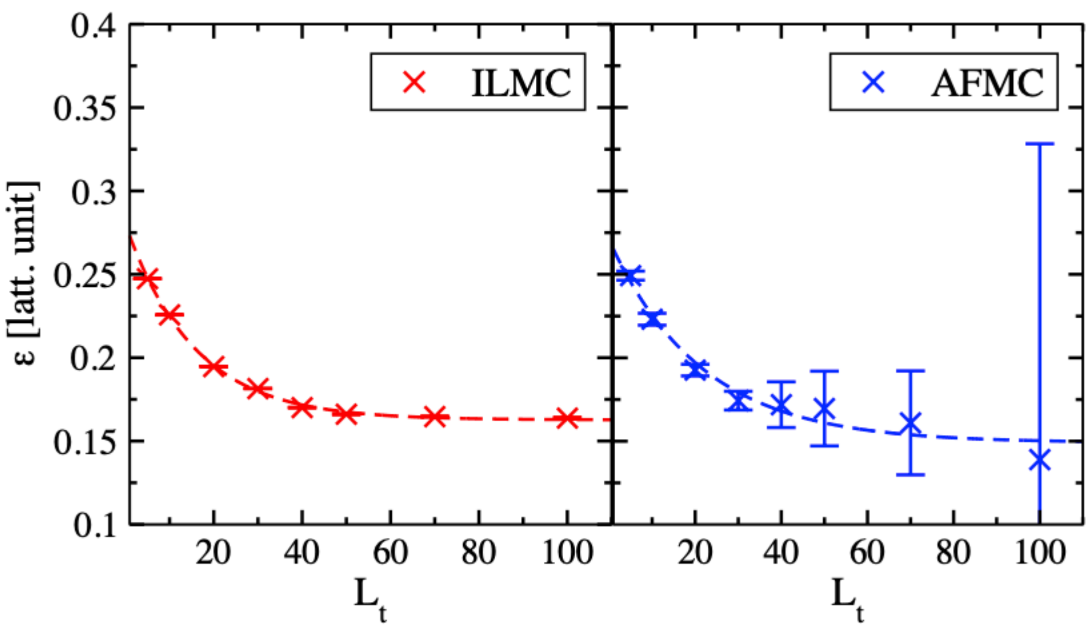
(D. Frame, T. A. Lähde, D. Lee, U.-G. Meißner)

AFMC does not converge as good as in a pure nuclear matter simulation

Need to develop a method that threats this impurities more efficient

Treat Impurity as worldline:

(S.Bour, D.Lee, H.-W. Hammer, U.-G. Meißner)



Starting point for (Hyper) Nuclear Lattice EFT

- Challenge with IFMC, need to collect millions of worldlines

→ Can we still do hypernuclear calculations with AFMC ?

→ Important for possible applications with many Hyperons

- Taylor interaction to work non-perturbative with our best NN interaction

→ Evolve together with NN counterparts

Constraints smearing parameters to the NN ones

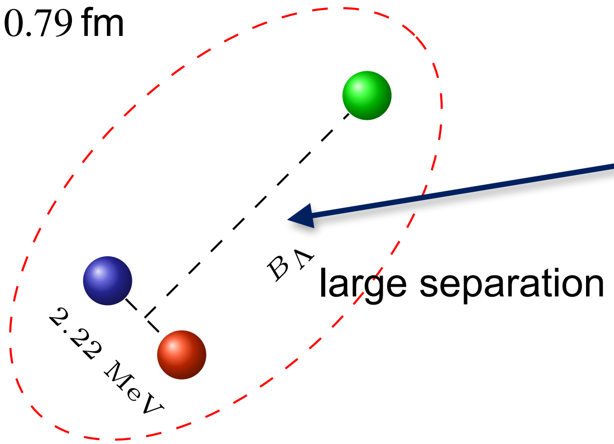
	Phase
$A = 3$	0.97
$A = 4$	0.89
$A = 5$	1 ← α – core
$A = 7$	0.92
$A = 13$	0.97

$L = 12 \quad Lt = 500$

This is very promising, for larger hypernuclei

Construction of a first Lattice ΛN interaction

$$\sqrt{\langle r_{\Lambda d}^2 \rangle} \approx 10.79\,\text{fm}$$



Emulsion:

$B_\Lambda = 0.130 \pm 0.050$ MeV Juric 1973

Heavy Ion:

$B_\Lambda = 0.406 \pm 0.120$ MeV Star 2020

$B_\Lambda = 0.102 \pm 0.063$ MeV Alice 2023

World Average:

$B_\Lambda = 0.164 \pm 0.043$ MeV Mainz 2024

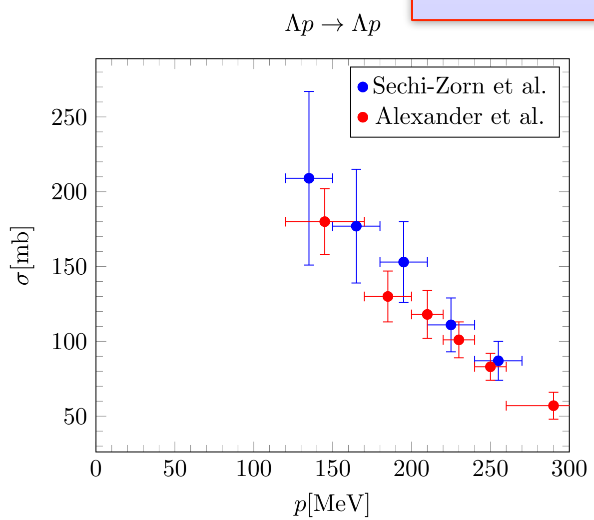
Shallow S-Wave State

$$J^P = \frac{1}{2}^+$$

Distinguishable

$$I = 0 \Rightarrow \frac{1}{\sqrt{2}}(pn - np)\Lambda$$

Combine 2-Body data with hypertriton in exact calculation



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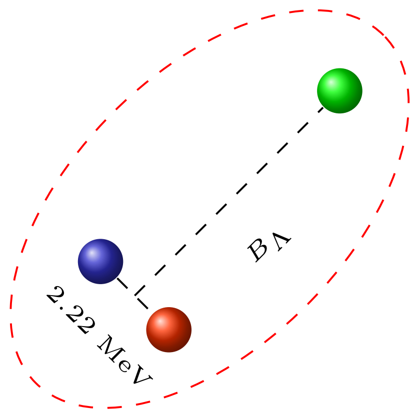
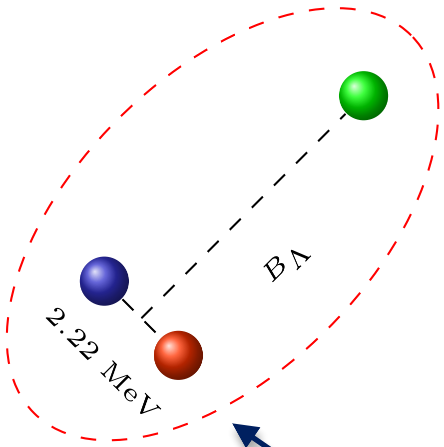


Construction of a first Lattice ΛN interaction



large separation → Large box sizes needed

GPU Lanczos Code to fix 2-Body forces

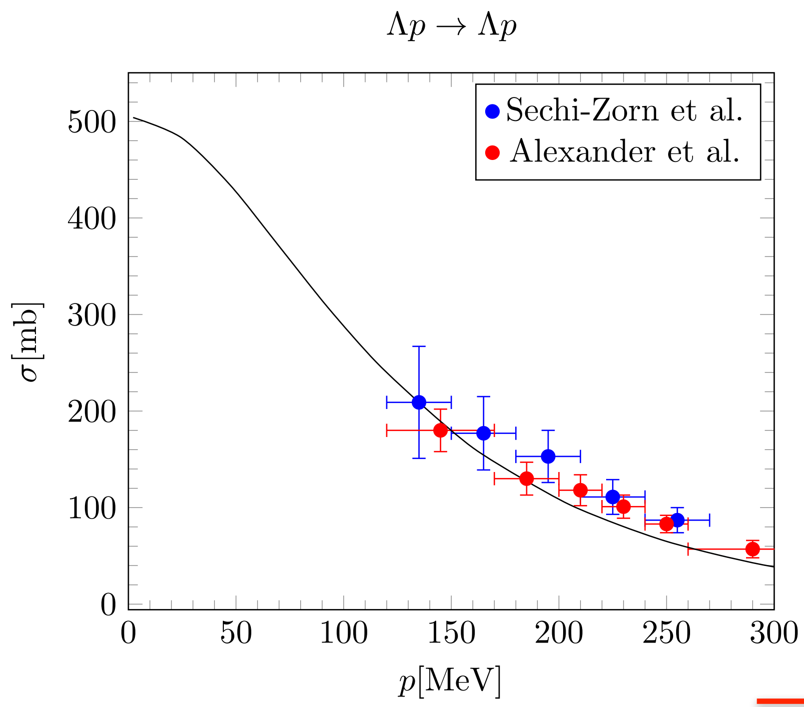
→ Allows L = 23.76 fm

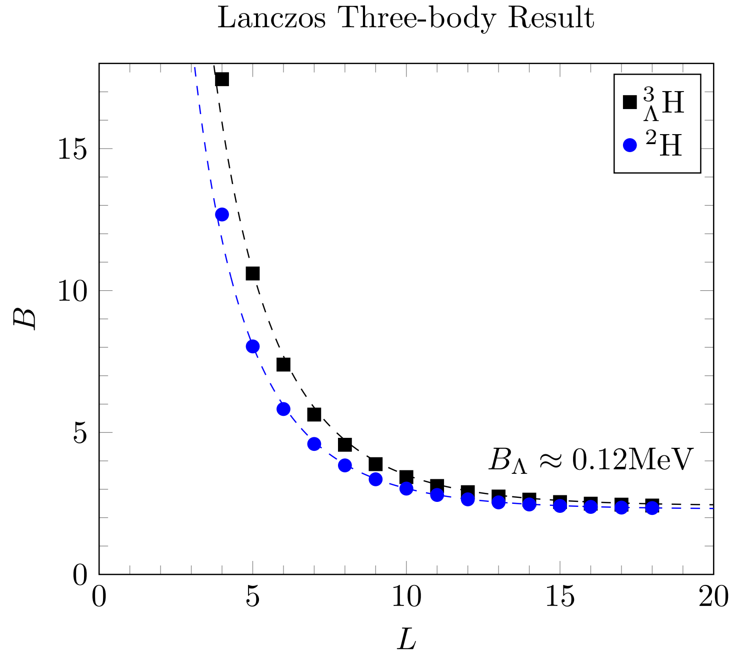
Tightly bound compared to Λ separation

Use simpler nuclear Interaction

Non-pertubative LO interaction → $^3S_1$ / $^1S_0$ interaction

Construction of a first Lattice ΛN interaction





Best SMS N^2LO interaction

(Haidenbauer et al.)

$a_s = -2.80$ fm $\quad r_s = 2.89$ fm

$a_t = -1.58$ fm $\quad r_t = 3.09$ fm

This interaction

$a_s = -2.89$ fm $\quad r_s = 3.28$ fm

$a_t = -1.60$ fm $\quad r_t = 3.94$ fm

Phase shift similar to $p \sim 60$ MeV

$$E(L) = E_{L\to\infty} + \frac{A}{L} e^{-\frac{L}{L_0}}$$

$\approx$ Emulsion

$$B^{\Lambda}_{L\to\infty} = (90 + 30)\ \text{keV} \approx 120\ \text{keV}$$

2-Body

GIR corrections

Results: Two Body interaction (L=12 l.u.) (light nuclei)

During Evolution:

Spin-averaged Interaction:

$$C = \frac{3\,{}^3S_1 + {}^1S_0}{4}$$

Perturbative part:

Spin-dependent Interaction:

$$C_S = \frac{{}^3S_1 - {}^1S_0}{4}$$

Nuclear Interaction:

N^3LO interaction, same as for WFM results

Results: Two Body	Experiment
$B_\Lambda({}^3\mathrm{H}_\Lambda) = 0.38 \pm 0.08$ MeV	0.164 ± 0.43 MeV

→ Box effect, consistent with exact L=12 result

Results: Two Body	Experiment
$B_\Lambda({}^4\mathrm{H}_\Lambda^{0^+}) = 2.08 \pm 0.16$ MeV	2.169 ± 0.042 MeV
$B_\Lambda({}^4\mathrm{H}_\Lambda^{1^+}) = 1.20 \pm 0.16$ MeV	1.081 ± 0.042 MeV

→ Splitting quite good, missing 0.2 MeV

Results: Two Body	Experiment
$B_\Lambda({}^5\mathrm{He}_\Lambda) = 3.39 \pm 0.06$ MeV	3.102 ± 0.03 MeV

→ Smaller overbinding compared to other LO Calculations

Results: Two Body	Experiment
$B_\Lambda({}^7\mathrm{Li}_\Lambda) = 5.07 \pm 0.50$ MeV	5.619 ± 0.06 MeV

→ Typically overbound by ~1 MeV in LO calculations

Results: Two Body interaction, further analysis

Missing ~ 0.6 MeV in A=7 systems

A=5 system only slightly overbound

More Attraction → Modify C

→ Fixes $^{7}\text{Li}_{\Lambda}$

→ Strong overbinding of $^{5}\text{He}_{\Lambda}$

Strong overbinding of $^{4}\text{H}_{\Lambda}$

$B_{\Lambda}(^{3}\text{H}_{\Lambda}) \sim 0.2 - 0.3$

→ Can three-body forces help us here?

Structure of contact three-body forces

(Petschauer et al.)

$$V_{ct}^{\Lambda NN} = C_1(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)(3+\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)$$ ← $I=1$

$$+C_2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_3)(1-\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)$$

$$+C_3(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)(1-\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)$$

(C_2, C_3 terms) ← $I=0$ → Can Influence Hypertriton

C_2 only interaction that depends on Λ spin → ${}^4\mathrm{H}_\Lambda^{0^+} - {}^4\mathrm{H}_\Lambda^{1^+}$ Splitting

C_1, C_3 are iso/spin interchanged to each other → Might not split for small Hypernuclei

→ Effectively N3LO χEFT(NN) + LO $\not\pi$EFT(YN)

Results: Fitting 3-Body forces

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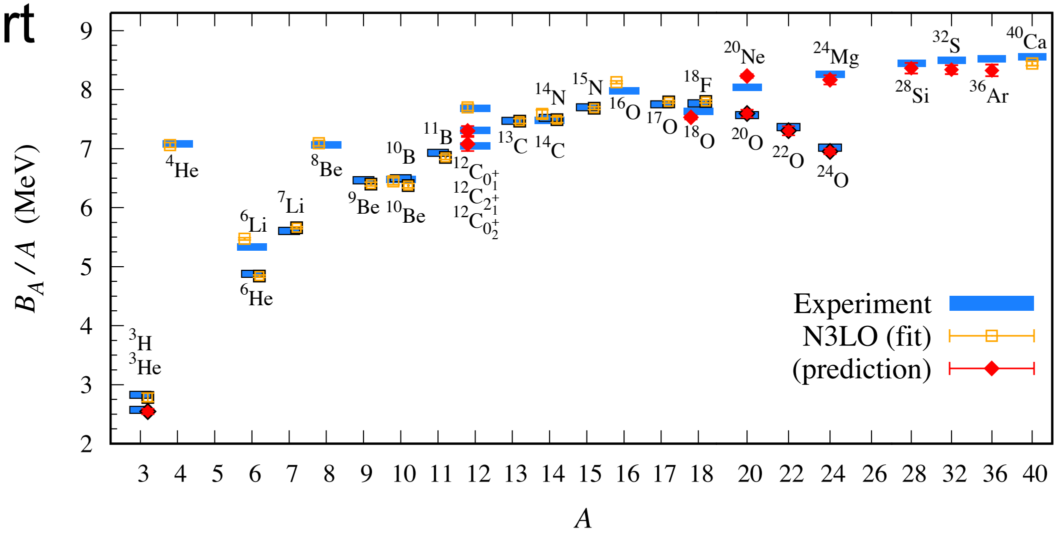
Nuclear 3-Body Forces are fitted as part of the WFM interaction



(Elhatisari et al.)

→ Use similar classes of non-local as well as local smeared YNN Forces

→ Leads to a total of 343 (7 each) combination of YNN forces

→ Only 25 combinations have a $\chi_R^2 > 1.5$

→ Freedom in higher nuclei

→ Constrain e.g $^9B_\Lambda$, $^{13}C_\Lambda$

Three-Body Results (Light nuclei), contrained by $^{9}B_{\Lambda}$

$$V_{ct}^{\Lambda NN} = C_1(1 - \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3)(3 + \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)$$ → locally smeared

$$+C_2\boldsymbol{\sigma}_1 \cdot (\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3)(1 - \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)$$ → not smeared

$$+C_3(3 + \boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_3)(1 - \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)$$ → locally smeared

Results: Three-body	Experiment	
$B_{\Lambda}(^{3}\mathrm{H}_{\Lambda}) = 0.43 \pm 0.08$ MeV	0.164 ± 0.43 MeV	← Finite Volume effect
$B_{\Lambda}(^{4}\mathrm{H}_{\Lambda}^{0^+}) = 2.25 \pm 0.18$ MeV	2.258 ± 0.042 MeV	← Use average of A=4 systems
$B_{\Lambda}(^{4}\mathrm{H}_{\Lambda}^{1^+}) = 1.01 \pm 0.18$ MeV	1.011 ± 0.042 MeV	←
$B_{\Lambda}(^{5}\mathrm{He}_{\Lambda}) = 3.09 \pm 0.12$ MeV	3.10 ± 0.03 MeV	
$B_{\Lambda}(^{7}\mathrm{Li}_{\Lambda}^{\frac{1}{2}^+}) = 5.61 \pm 0.96$ MeV	5.62 ± 0.06 MeV	Missing L=2 Contributions
$B_{\Lambda}(^{7}\mathrm{Li}_{\Lambda}^{\frac{3}{2}^+}) = 5.36 \pm 0.96$ MeV	4.93 ± 0.06 MeV	←

Three-Body Results (towards medium mass nuclei)

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$$V_{ct}^{\Lambda NN} = C_1(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)(3+\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)$$ → locally smeared

$$+C_2\boldsymbol{\sigma}_1\cdot(\boldsymbol{\sigma}_2+\boldsymbol{\sigma}_3)(1-\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)$$ → not smeared

$$+C_3(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)(1-\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)$$ → locally smeared

Results: Three-body	Experiment
$B_\Lambda(^9\mathrm{Be}_\Lambda) = 6.63 \pm 0.55$ MeV	6.614 ± 0.072 MeV
$B_\Lambda(^{13}\mathrm{C}_\Lambda) = 11.47 \pm 0.84$ MeV	11.79 ± 0.16 MeV
$B_\Lambda(^{16}\mathrm{O}_\Lambda) = 11.72 \pm 1.58$ MeV	13.00 ± 0.09 MeV

→ Same set but restricted by $^{16}O_\Lambda$

$B_\Lambda(^9\mathrm{Be}_\Lambda) = 6.74 \pm 0.56$ MeV $B_\Lambda(^{13}\mathrm{C}_\Lambda) = 11.51 \pm 0.84$ MeV $B_\Lambda(^{16}\mathrm{O}_\Lambda) = 12.8 \pm 1.6$ MeV

→ Uncertainty dominated by the modified nuclear part

→ All hypernuclei can be described within uncertainties

How does the Λ glue the core together? Example: TBF

→ Only consider contributions from changes in the NNN force

Naiv expectation: The further the hyperon is away the less it is modifying the nuclear core

→ A=3 no effect

A=4 ≈ 30 keV

A=5 ≈ 120 keV

A=7 ≈ 700 keV

A=9 ≈ 500 keV

A=12 ≈ 3.3 MeV

→ Contributes to nuclear uncertainty

Possible Paths to improvement

- Go to higher orders in the two-body interaction

  → Typical LO problems go away in other methods

- Include two-Pion exchange/Pion exchange 3B Forces

  → Long-Range behaviour of the interaction

- fit two-body forces with better nuclear interaction

  → Removes any dependence of the NN Force on the YN Force

- Improve statistics in the NN part of the hypernuclei

  → Main uncertainty from sampling of the NN part of the nucleus

Summary and Outlook

Good Results for light hypernuclei nuclei A=3-16
with $N^3LO(NN)$ and $LO(YN)$ interaction

Method scales with A, straightforward application to the whole hypernuclear chart

Many possible path ways to improve the results

Calculate the hypernuclear chart

Many excited states in A=7/9 hypernuclei