Lambda(1405) in the flavor SU(3) limit from lattice QCD

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(HAL QCD Collaboration)

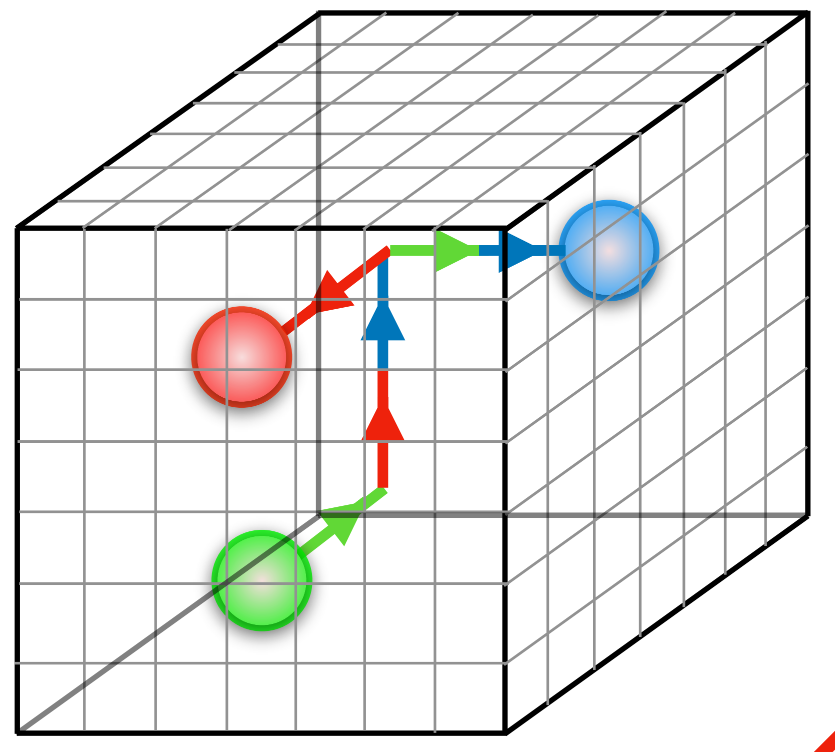
based on
K. M. and S. Aoki, "Study on Lambda(1405) in the flavor SU(3) limit in the HAL QCD method,"
PoS **LATTICE2023**, 063 (2024) [arXiv:2311.17421[hep-lat]]

SPICE: Strange hadrons as a Precision tool for strongly InteraCting systEms
@ECT*, May13, 2024

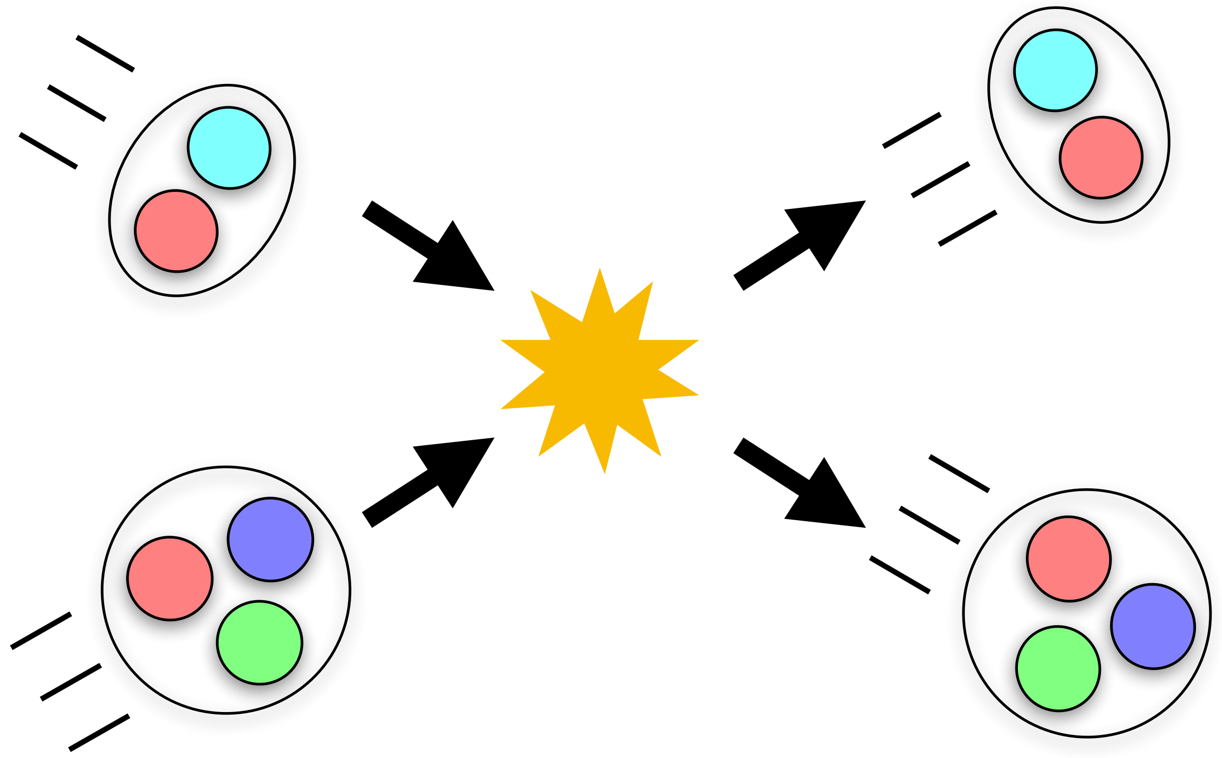
Introduction

- ultimate goal: understand the exotic hadrons from lattice QCD
- key: hadron scatterings (interactions)

Lattice QCD

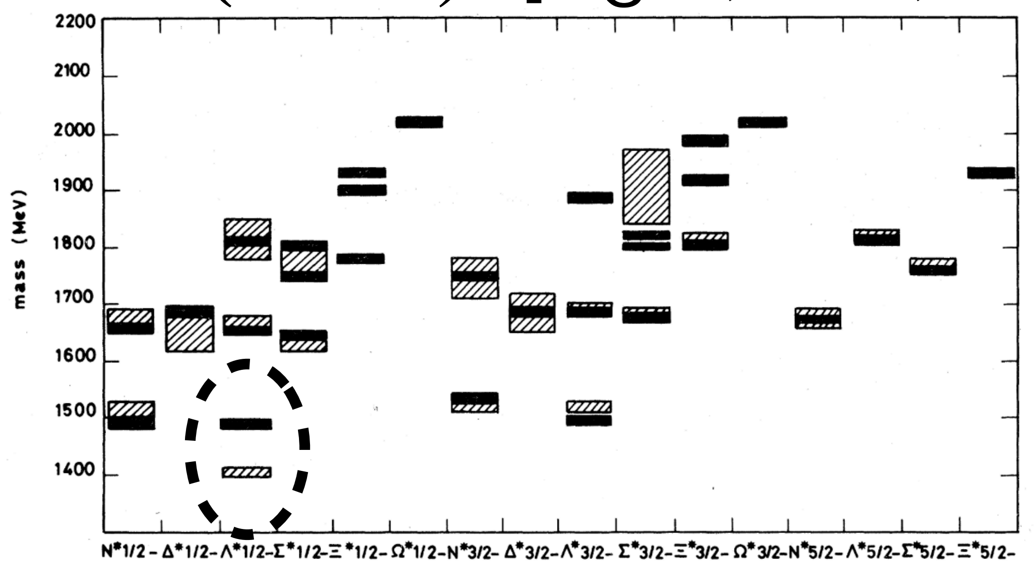


hadron scatterings

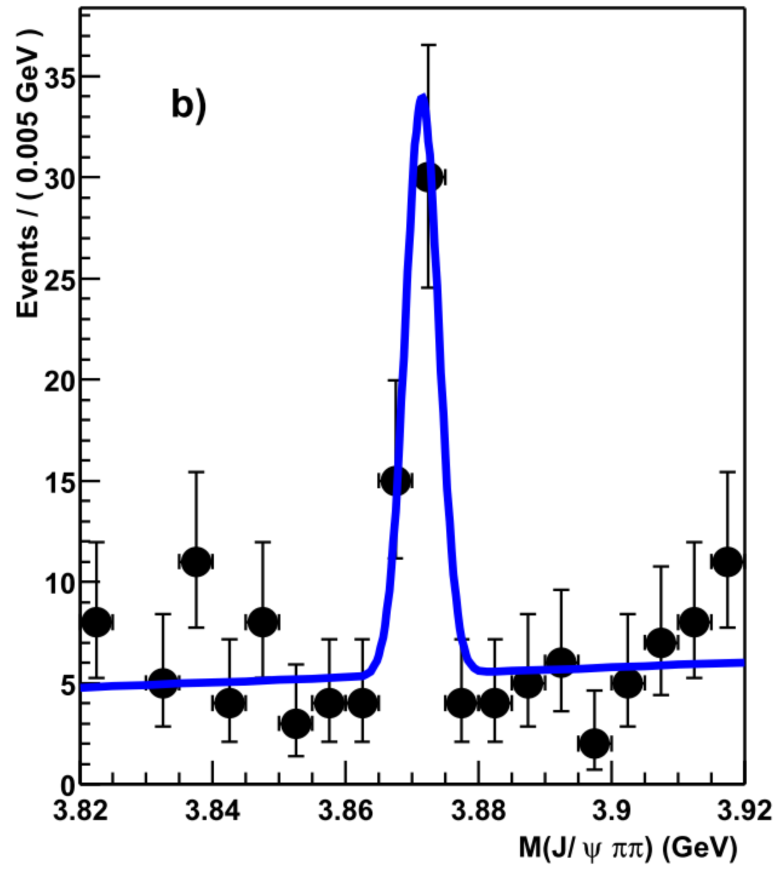


exotic hadrons

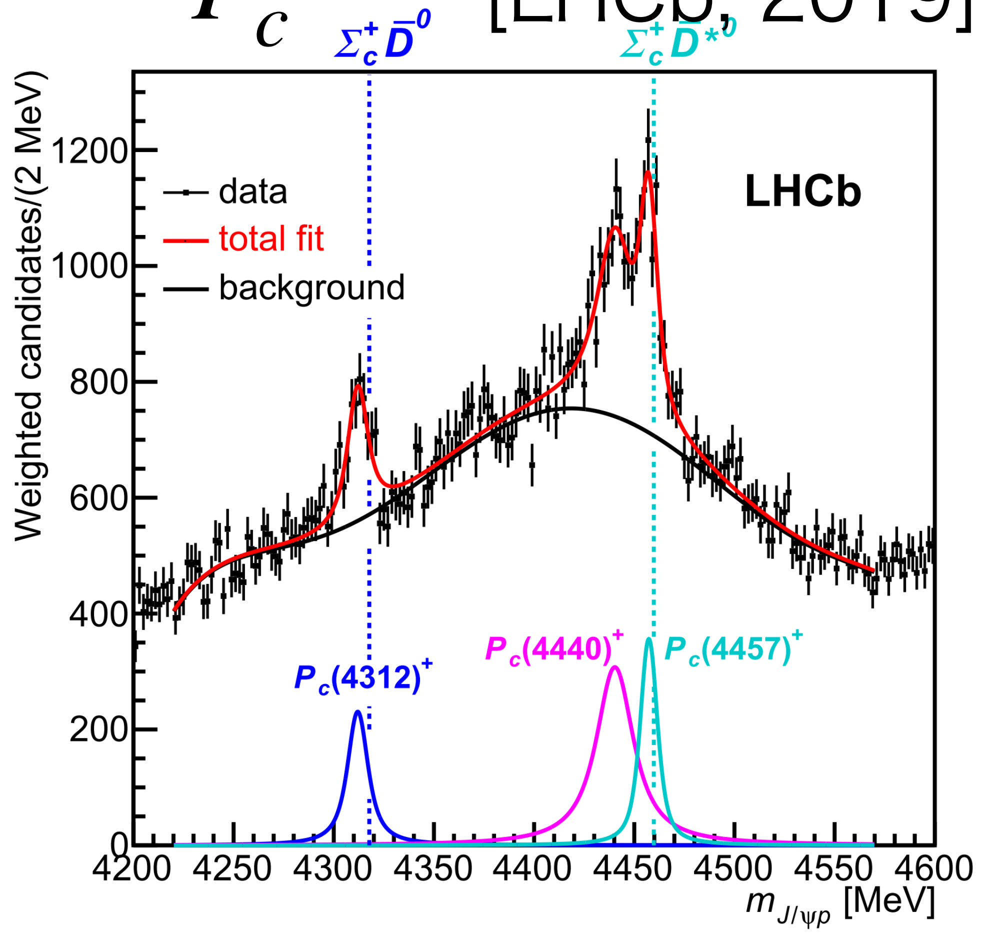
$\Lambda(1405)$ [Isgur, Karl, 1978]



$X(3872)$ [Belle, 2003]



P_c [LHCb, 2019]



- Finite-volume method [Lüscher, 1991]
  : temporal info of correlation functions ⟶ energy ⟶ phase shift
- HAL QCD method [Ishii, Aoki, Hatsuda 2007]
  : temporal & **spatial** info of correlation functions ⟶ **potential** ⟶ phase shift

(Time-dependent) HAL QCD method

[Ishii, Aoki, Hatsuda 2007]
[Ishii et al. 2011]

- R-correlator:

$$R(\mathbf{r},t) = \frac{\langle O_1(\mathbf{r},t)O_2(\mathbf{0},t)\bar{J}(0)\rangle}{\langle O_1(t)\bar{O}_1(0)\rangle\langle O_2(t)\bar{O}_2(0)\rangle} \approx \sum_n C_{\bar{J},n}\underline{\Psi^{W_n}(\mathbf{r})}e^{-(W_n-m_1+m_2)t}$$

Nambu-Bethe-Salpeter (NBS) wave function

- time-dependent equation

(μ: reduced mass)

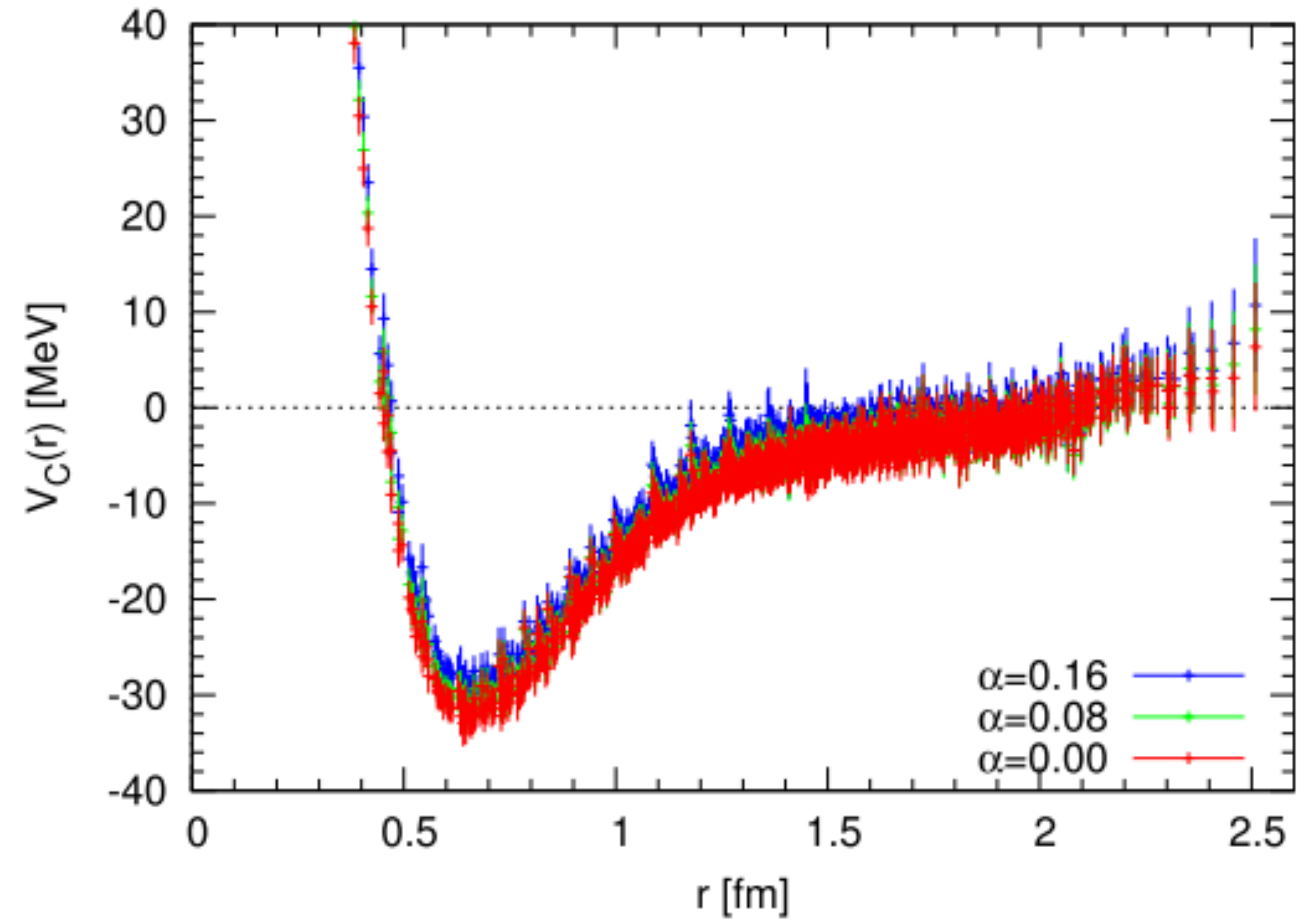
$$\int d^3r'\ U(\mathbf{r},\mathbf{r}')R(\mathbf{r}',t) \simeq \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right)R(\mathbf{r},t)$$

$$\approx V(r)\delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

(leading-order (LO) approximation)

$$\rightarrow\quad V(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r},t)}\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right)R(\mathbf{r},t)$$

e.g. NN potential



[Ishii et al. 2011]

HAL QCD Collaborations

- **H**adrons to **A**tomic nuclei from **L**attice QCD (**HAL** QCD) Collaboration

- Members

S. Aoki, E. Itou (YITP, Kyoto Univ., Japan)

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T. Sugiura (Rissho Univ., Japan)

K. Murase (Tokyo Metropolitan University, Japan)

T. Aoyama (ISSP, Tokyo Univ., Japan)

T. M. Doi (Kyoto Univ., Japan)

K. Murakami (TITech, Japan)

F. Etminan (Univ. of Birjand, Iran)

H. Tong (Univ. of Bonn, Germany)

L. Zhang (UCAS)

Quark pair annihilations

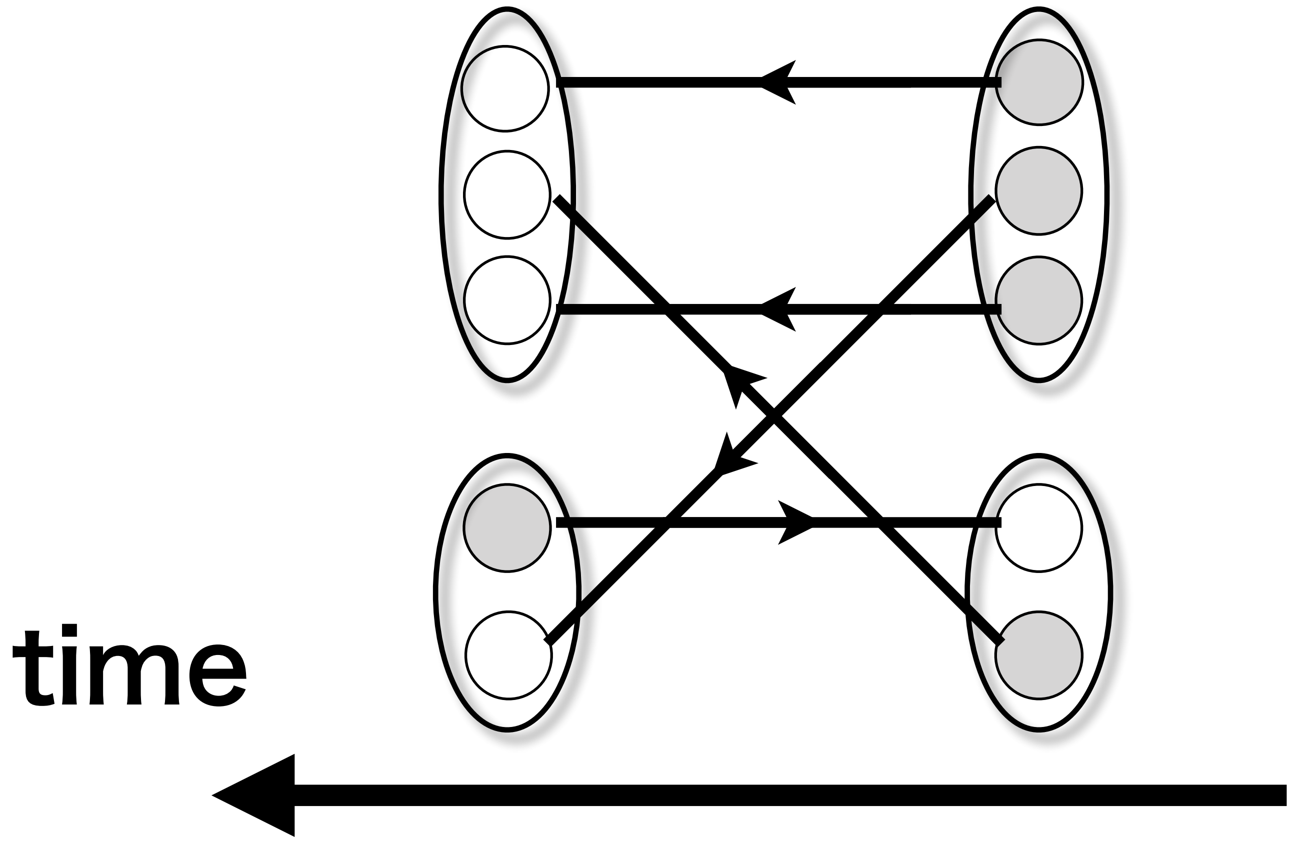
- two types of exotic hadrons:
  with and without **quark pair annihilations**

* neglect $Q\bar{Q}$ annihilation

- **w/o** quark pair annihilations:

$QQ\bar{q}\bar{q}$, $Q\bar{Q}q\bar{q}'$ *, $Q\bar{Q}qqq$ *, $q\bar{q}'qqq$

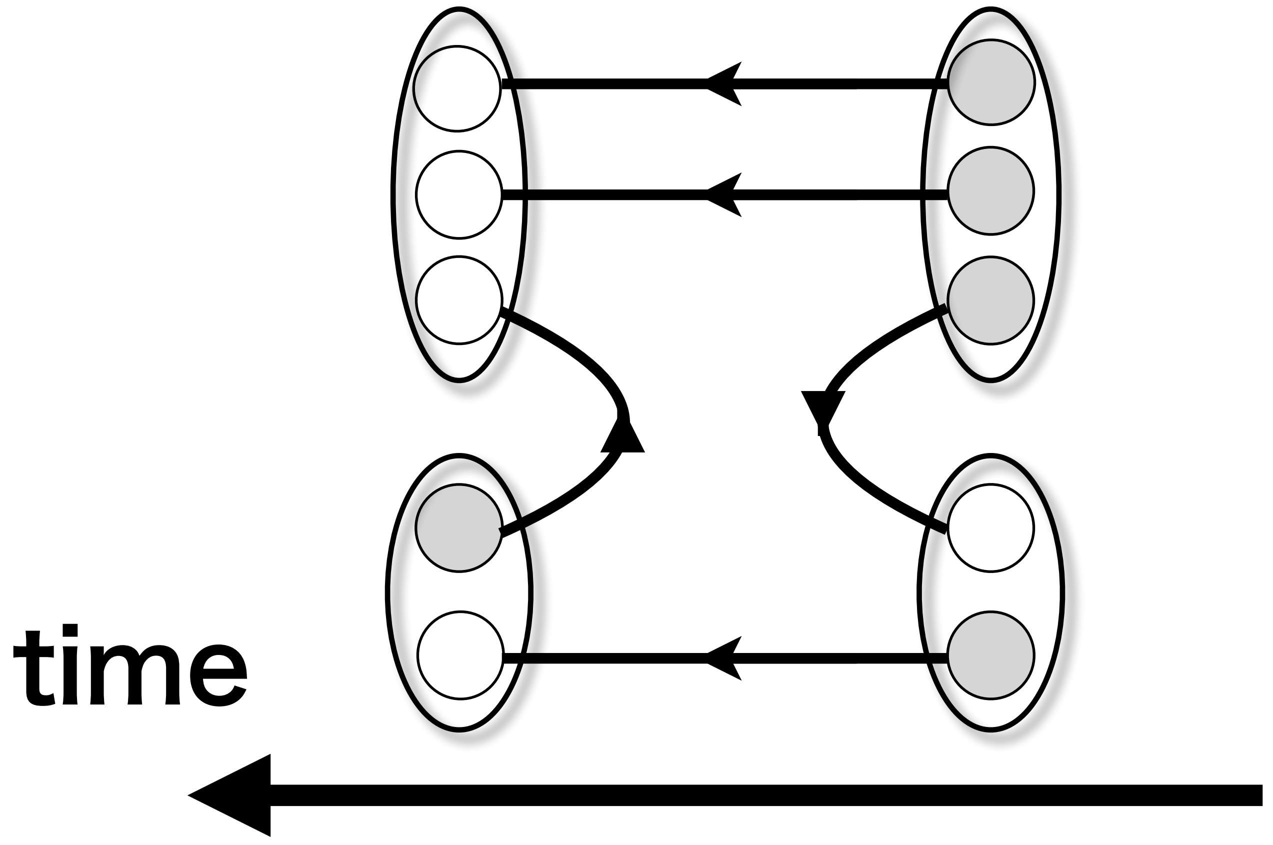
T_{cc} Z P_c Θ^+



- **w/** quark pair annihilations:

resonances, $Q\bar{Q}q\bar{q}$, $q\bar{q}q\bar{q}$, $q\bar{q}qqq$

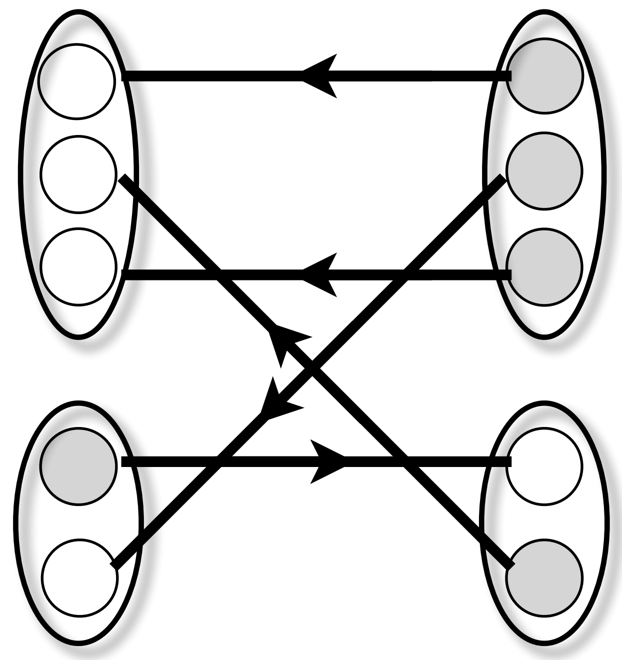
X f_0/σ $\Lambda(1405)$



- w/ quark pair annihilations: much more computational cost in lattice QCD

➡ **situation is much different**

Exotic hadrons w/o quark pair annihilations



- The HAL QCD studies have been done **in almost realistic setups** recently

m_π

$\gtrsim$ 400 MeV

- $Z_c(3900)$ [Ikeda et al., 2016]
- P_c pentaquark [Sugiura et al., 2018]
- $\Theta^+(1540)$ [Ikeda, 2011 (PoS)] [**KM**, Akahoshi, Aoki, 2020]
- T_{bb} tetraquark [T. Aoki, S. Aoki, Inoue, 2023]

146 MeV near-physical point

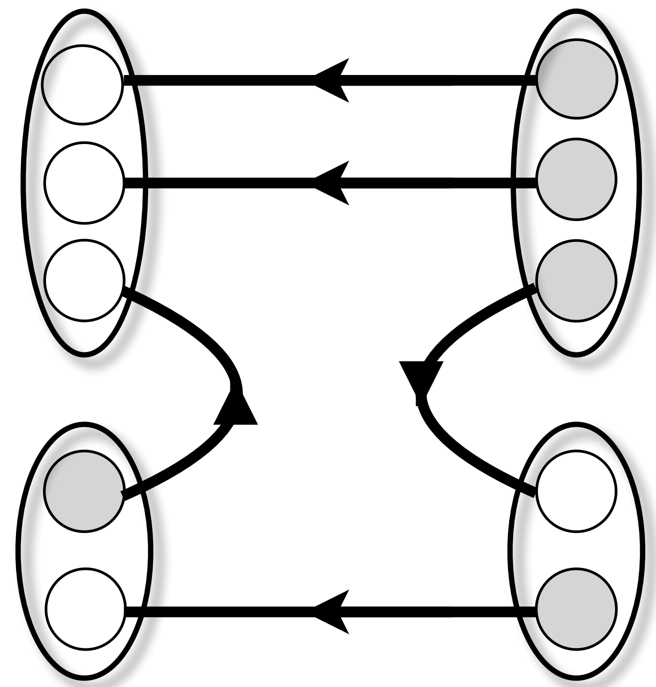
- $\Omega\Omega$ dibaryon [Gongyo et al., 2018]
- $N\Omega$ dibaryon [Iritani et al., 2019]
- H-dibaryon [Sasaki et al., 2020]
- $\Omega_{ccc}\Omega_{ccc}$ dibaryon [Lyu et al., 2021]
- T^+_{cc} tetraquark [Lyu et al., 2023]
- $N\phi$ + femtoscopy [Lyu et al., 2022] [Chizzali et al., 2022]

138 MeV physical point

configuration generated using Supercomputer Fugaku (Japan) → (hadron interactions: on-going)

[Itou et al. (HAL QCD), in preparation]

Exotic hadrons w/ quark pair annihilations

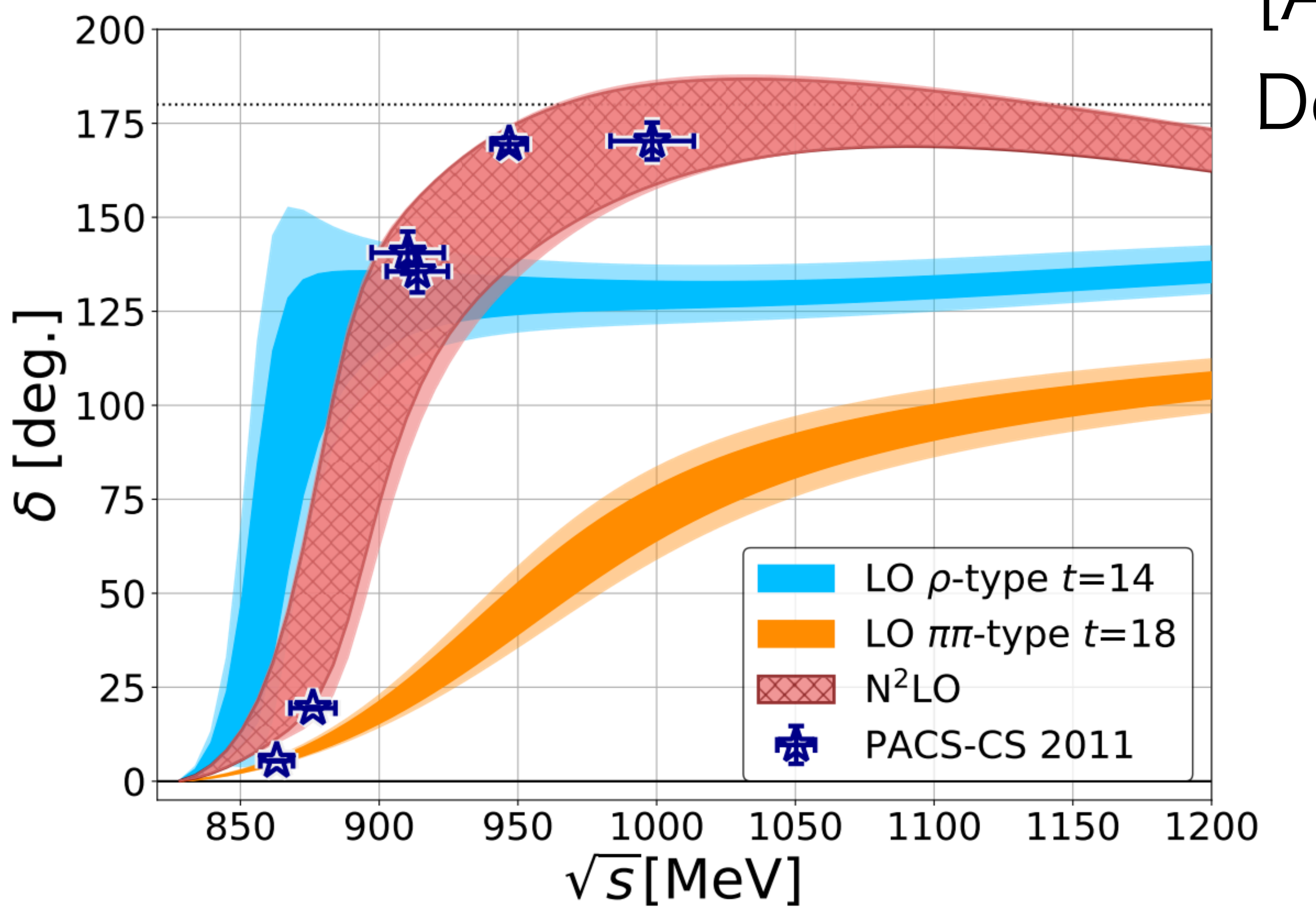


- hadron resonances/most of exotic hadrons:
  quark-pair annihilation diagrams appear

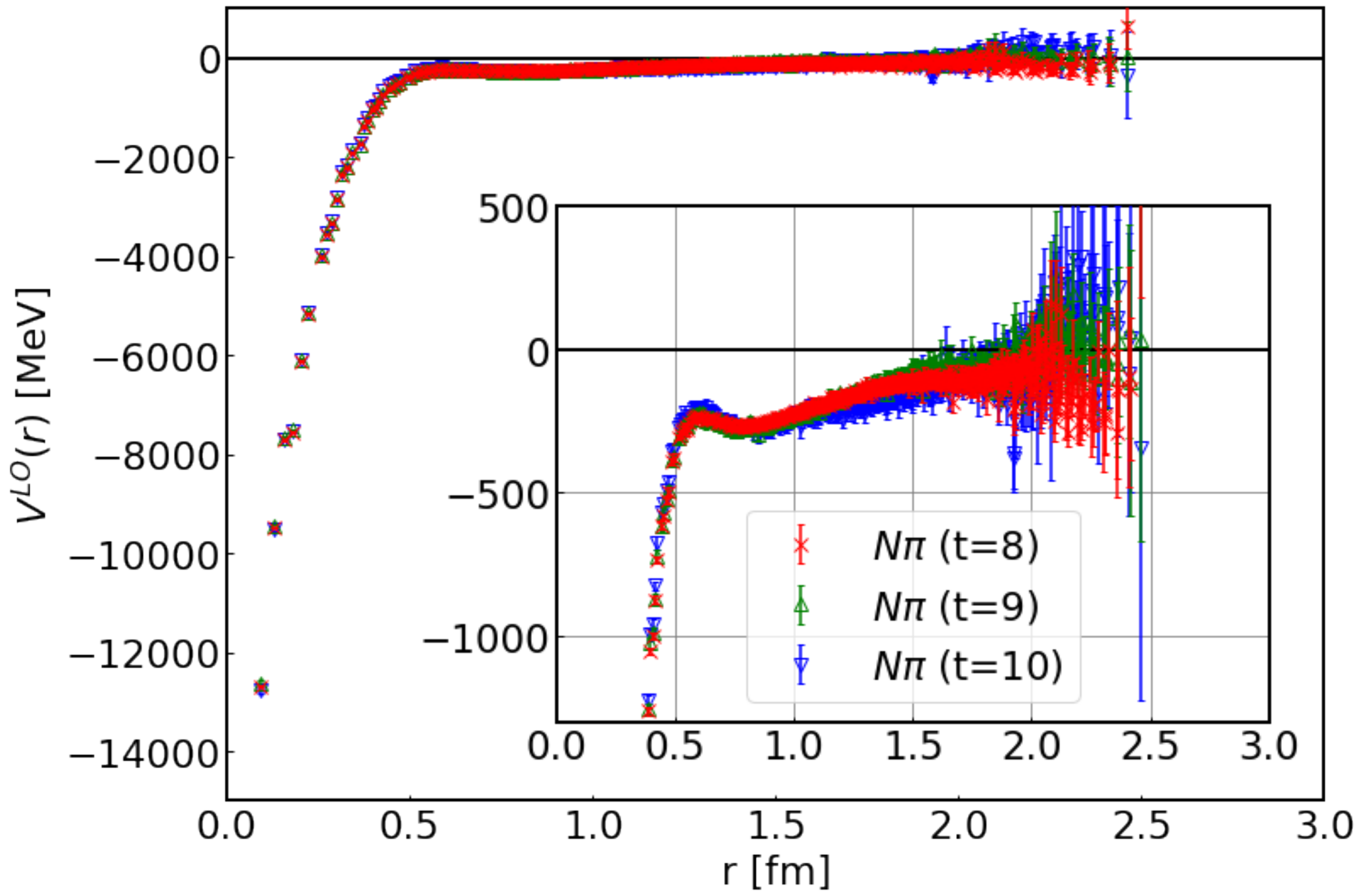
  ➡ computational cost is very high ← $\times O(L^4)$ larger

- **new technique to suppress the cost**
  allowed such calculation in HAL QCD method [Akahoshi, Aoki, Doi, 2021]

  - $\pi\pi \to \rho$ (resonance) [Akahoshi, Aoki, Doi, 2021]



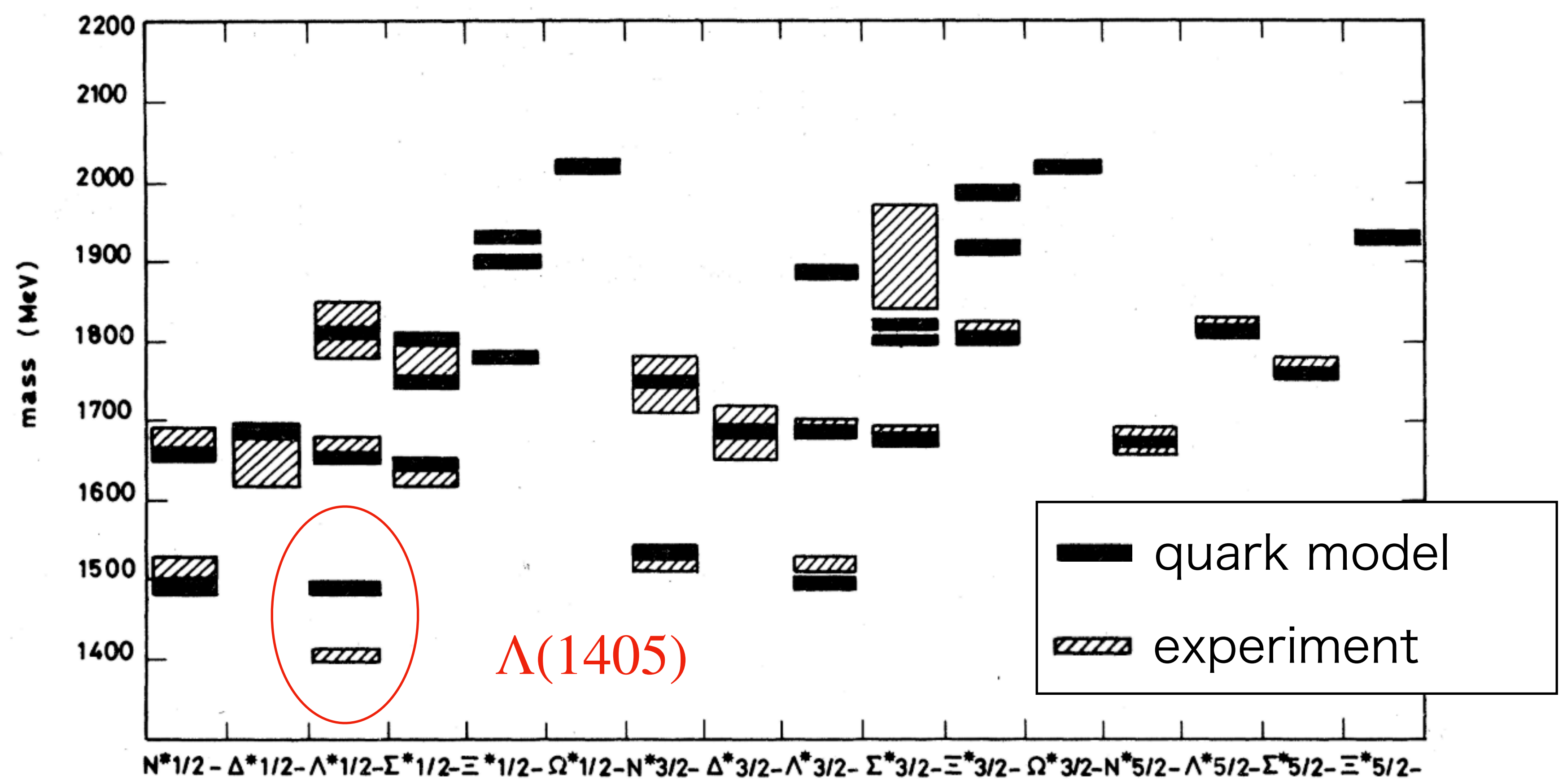
  - P-wave $N\pi$ $(\Xi\bar{K})$ ➡ Δ (Ω) (stable) [KM, Akahoshi, Aoki, Doi, Sasaki, 2023]



- next step: **exotic hadrons** $(\Lambda(1405)$ etc.)

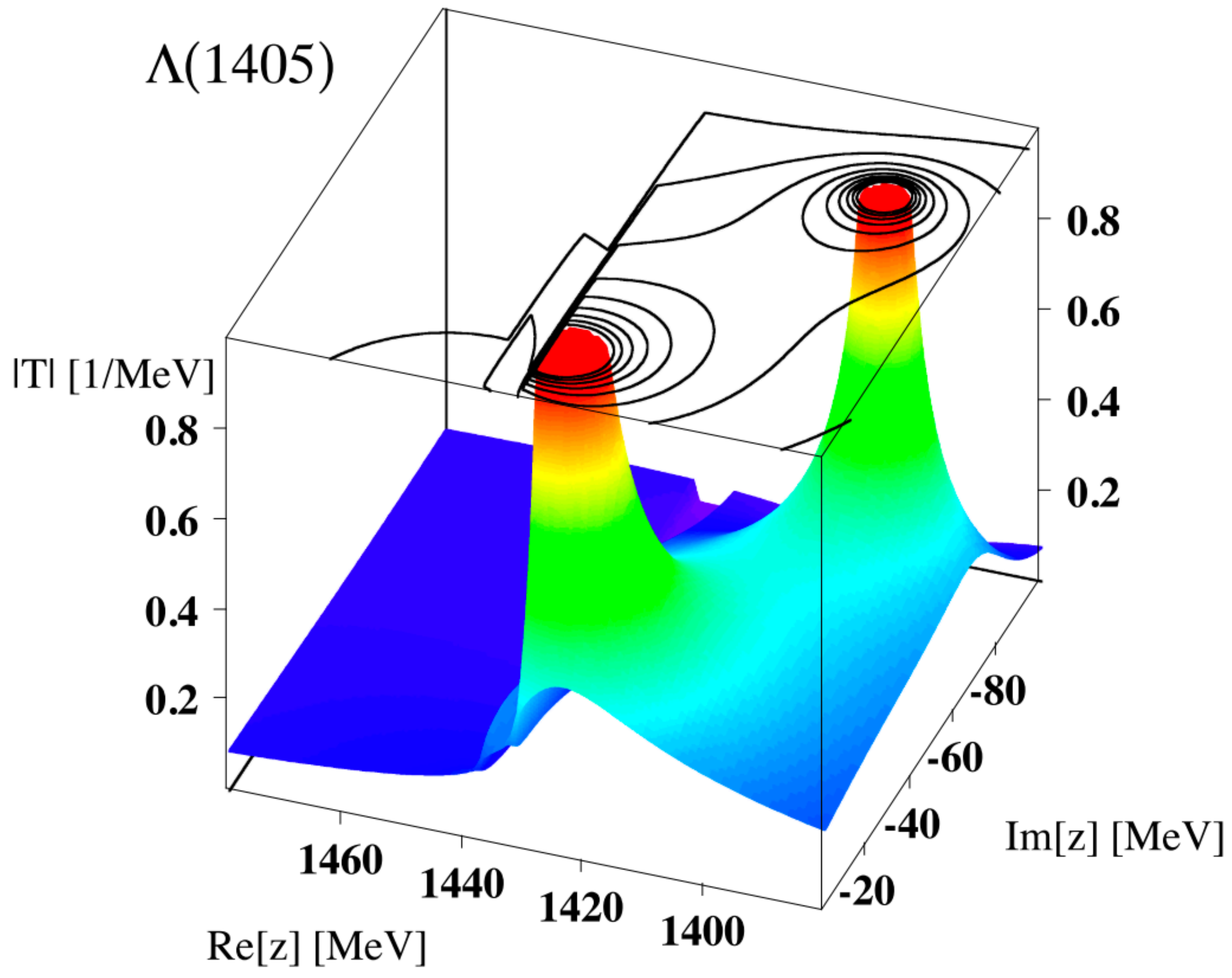
$\Lambda(1405)$

- $\Lambda(1405)$: not a simple Λ baryon



- one pole? **two poles?**
    - chiral unitary model [Oller and Meissner, 2001] [Jido, Oller, Oset, Ramos, Meissner, 2003]
    - lattice QCD using finite-volume method at $m_\pi \approx 200$ MeV [Bulava et al. (BaSc Collab.), 2024]

→ virtual state below $\pi\Sigma$ + resonance below $\bar{K}N$

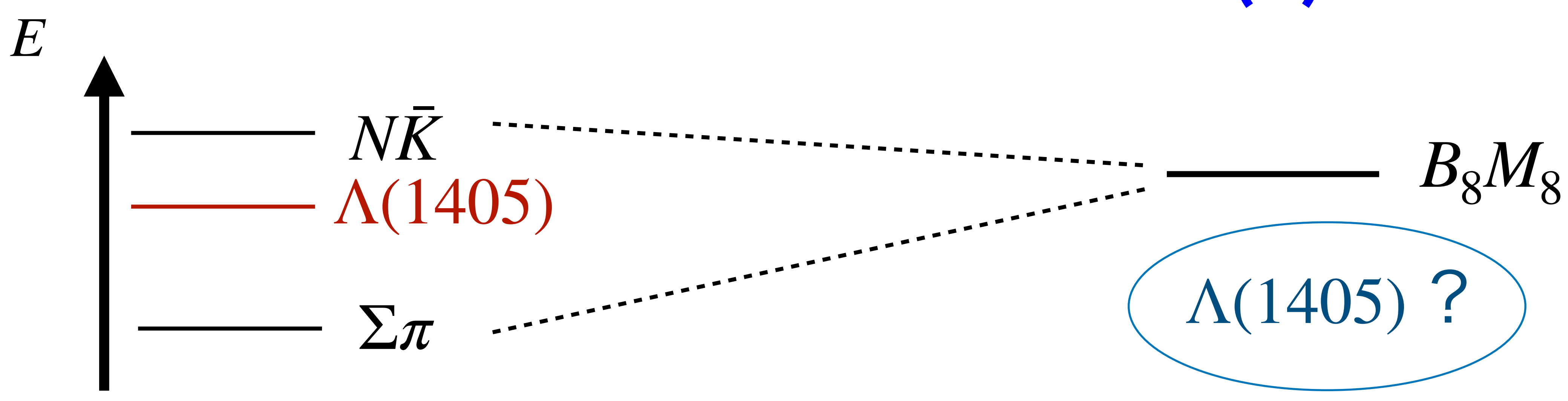


(Hyodo and Jido, Prog. Part. Nucl. Phys. **67** (2012), 55-98)

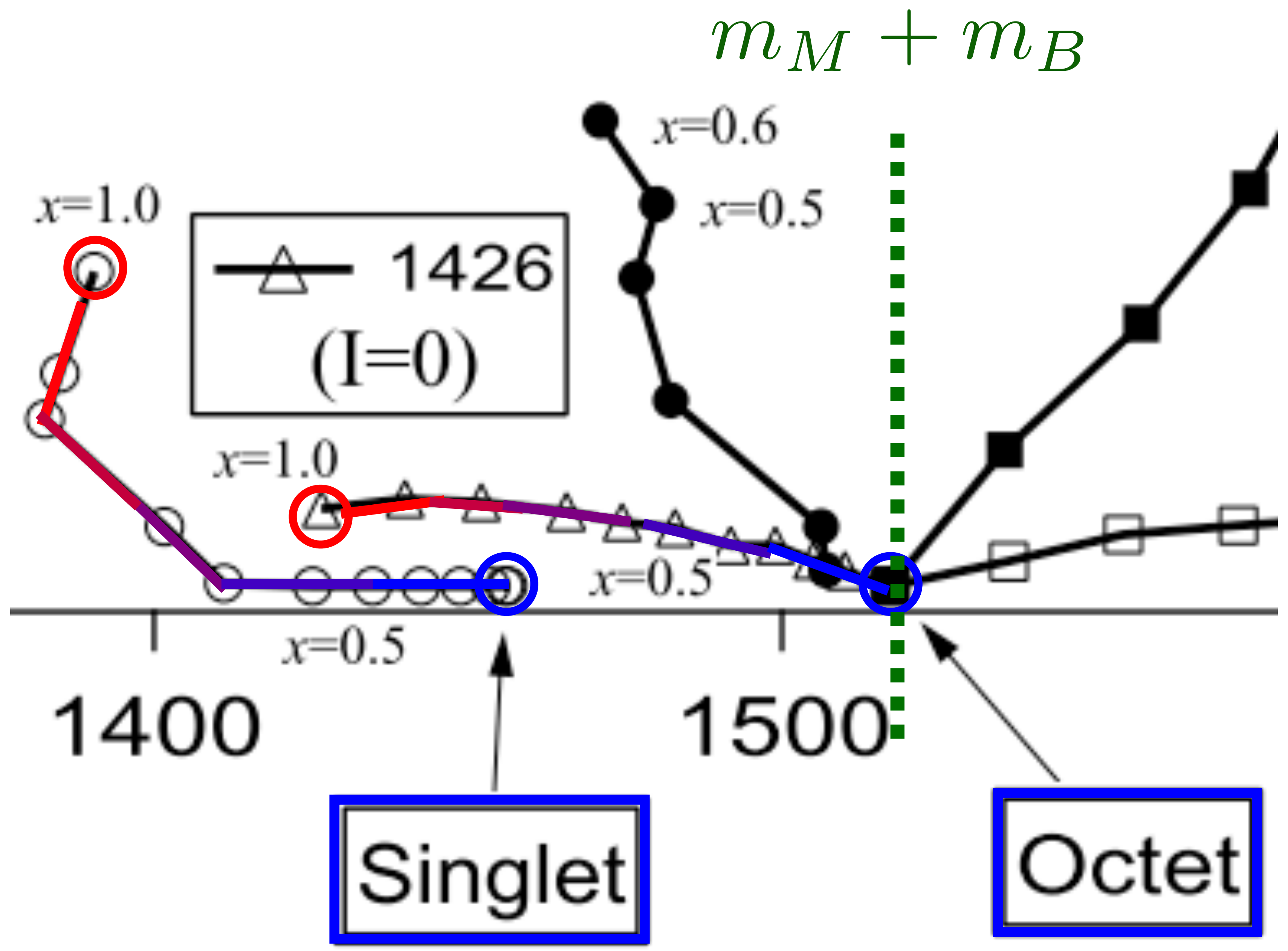
- this talk: **study from HAL QCD approach**

$\Lambda(1405)$ in flavor SU(3) limit

- $\Lambda(1405)$ in **flavor SU(3) limit** $m_u = m_d = m_s$



(almost) single-channel analysis



(Jido et al., Nucl. Phys. A **725** (2003), 181-200)

- previous study in the chiral unitary model

Physical point

two poles constituting $\Lambda(1405)$

SU(3) limit

one pole in singlet and the other in octet channels

$\boxed{\text{goal in this work}}$

understand the mechanism to generate these poles via the HAL QCD potential

Setups

- channels: $\underset{\text{meson}}{\underline{8}} \otimes \underset{\text{baryon}}{\underline{8}} = 27 \oplus 10 \oplus 10^* \oplus \underline{8_s \oplus 8_a \oplus 1}$

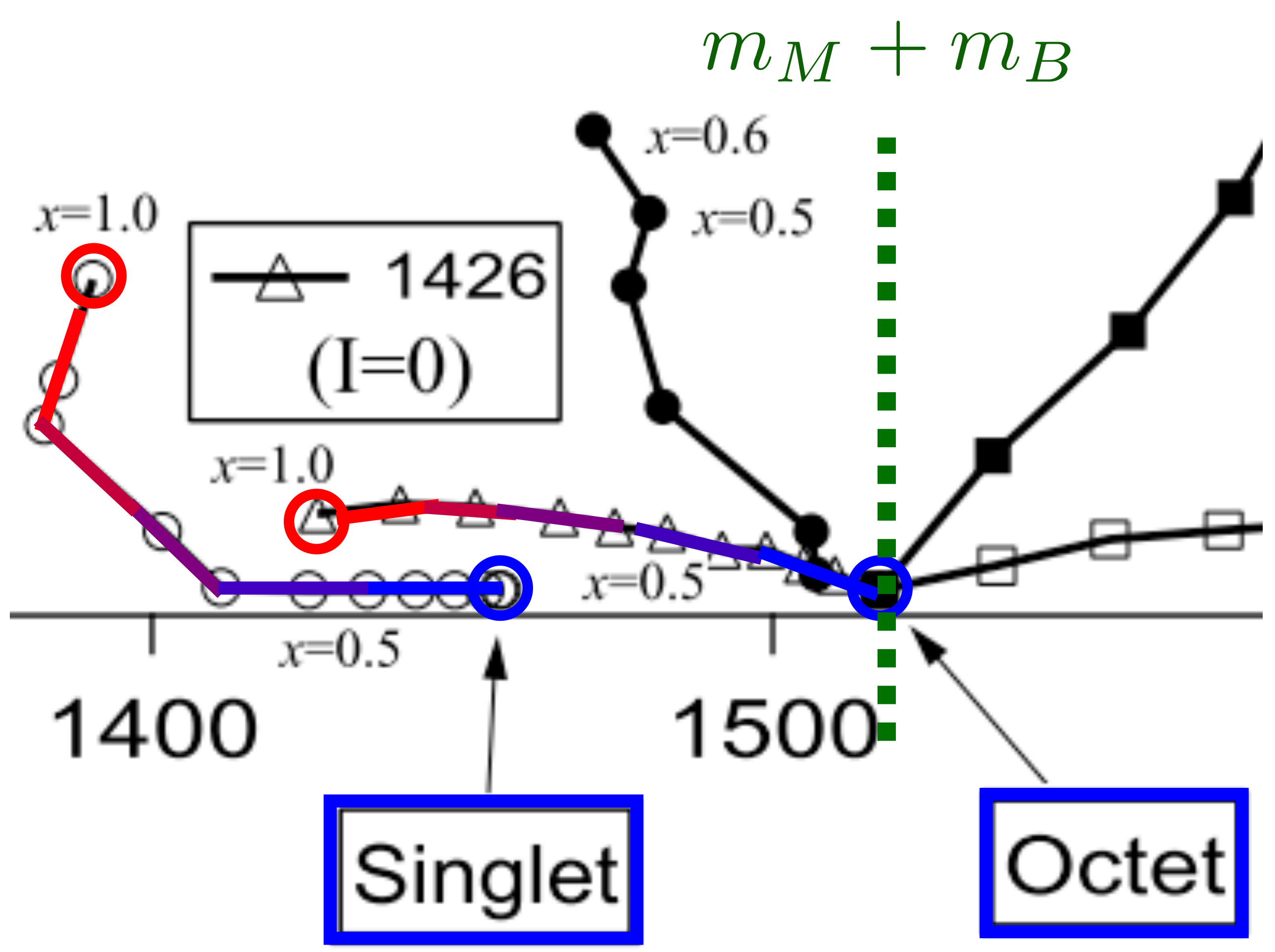
- S-wave analysis

- LO approximation in the HAL QCD potential

$$U(\mathbf{r}, \mathbf{r}') \approx V(\mathbf{r})\delta^{(3)}(\mathbf{r} - \mathbf{r}')$$

- **neglect** 8_s **and** 8_a **coupling** in this work

$$\begin{pmatrix} V_{8_s 8_s}(r) & V_{8_s 8_a}(r) \\ V_{8_a 8_s}(r) & V_{8_a 8_a}(r) \end{pmatrix} \approx \begin{pmatrix} V_{8_s 8_s}(r) & 0 \\ 0 & V_{8_a 8_a}(r) \end{pmatrix}$$



(Jido et al., Nucl. Phys. A **725** (2003), 181-200)

cf. chiral perturbation theory w/ WT interaction:

- **no coupling between** 8_s **and** 8_a
- interactions for 8_s and 8_a are the same

Lattice setups

- $a \approx 0.12$ fm, 32^4 lattices, $m_M \approx 670$ MeV
  $m_B \approx 1489$ MeV (cf. $m_M = 368$ MeV, $m_B = 1151$ MeV in chiral unitary)

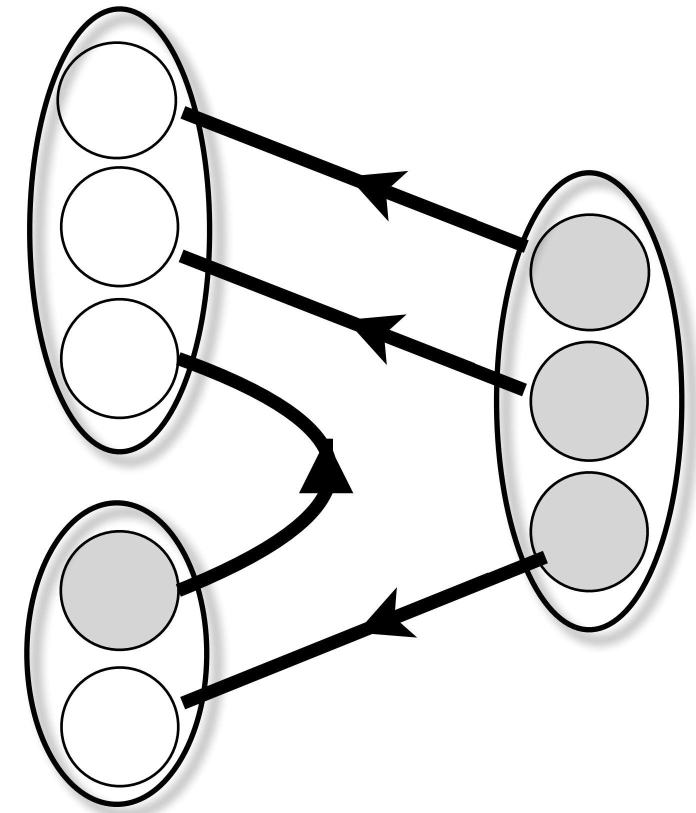
- R-correlators

$(\mathrm{rep} = 1, 8_s, 8_a)$

$$R^{(\mathrm{rep})}(\mathbf{r}, t) = \frac{\langle (M(\mathbf{r},t)B(\mathbf{0},t))_{(\mathrm{rep})}\underline{\bar{\Lambda}(0)}\rangle}{\langle M(t)\bar{M}(0)\rangle\langle B(t)\bar{B}(0)\rangle}$$

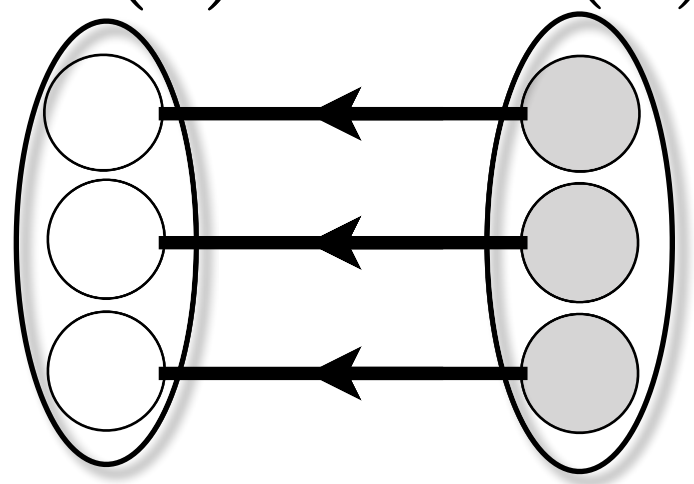
$\sim \sum_{\mathbf{z}} \bar{u}(\mathbf{z})\bar{d}(\mathbf{z})\bar{s}(\mathbf{z})$

: 3-quark type
(octet, singlet)

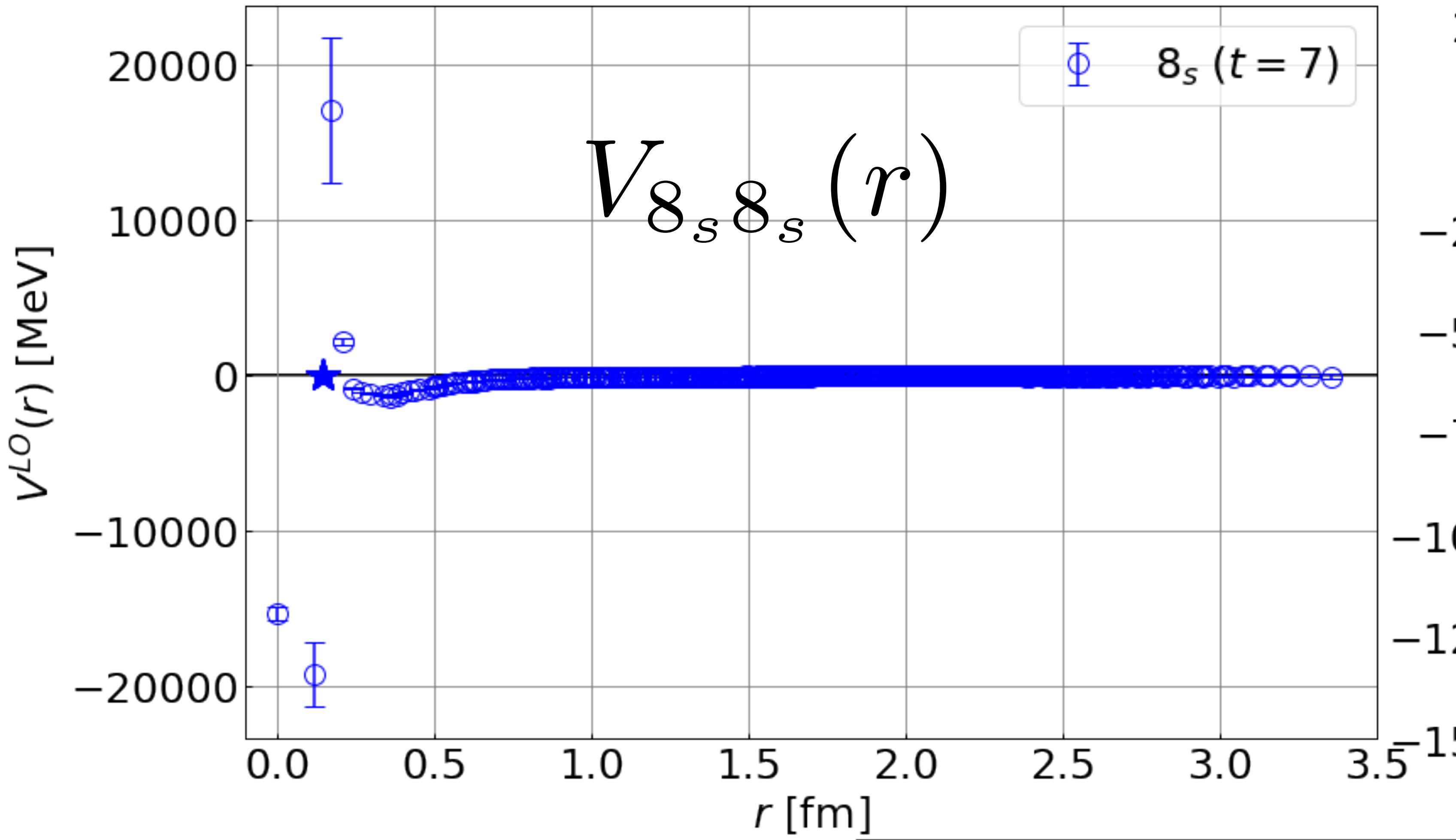


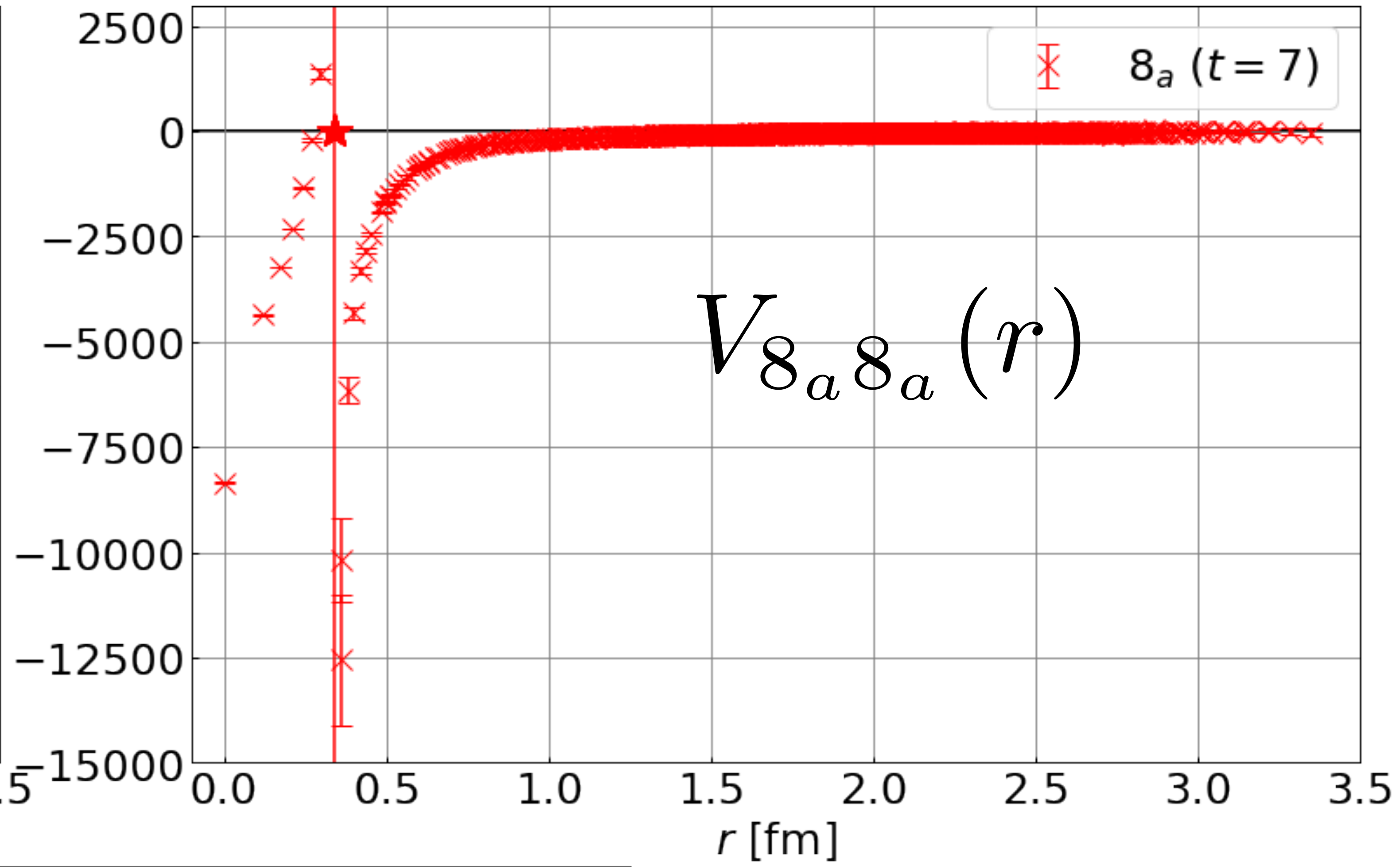
- **one bound state in each channel from $\langle \Lambda(t)\bar{\Lambda}(0)\rangle$:**

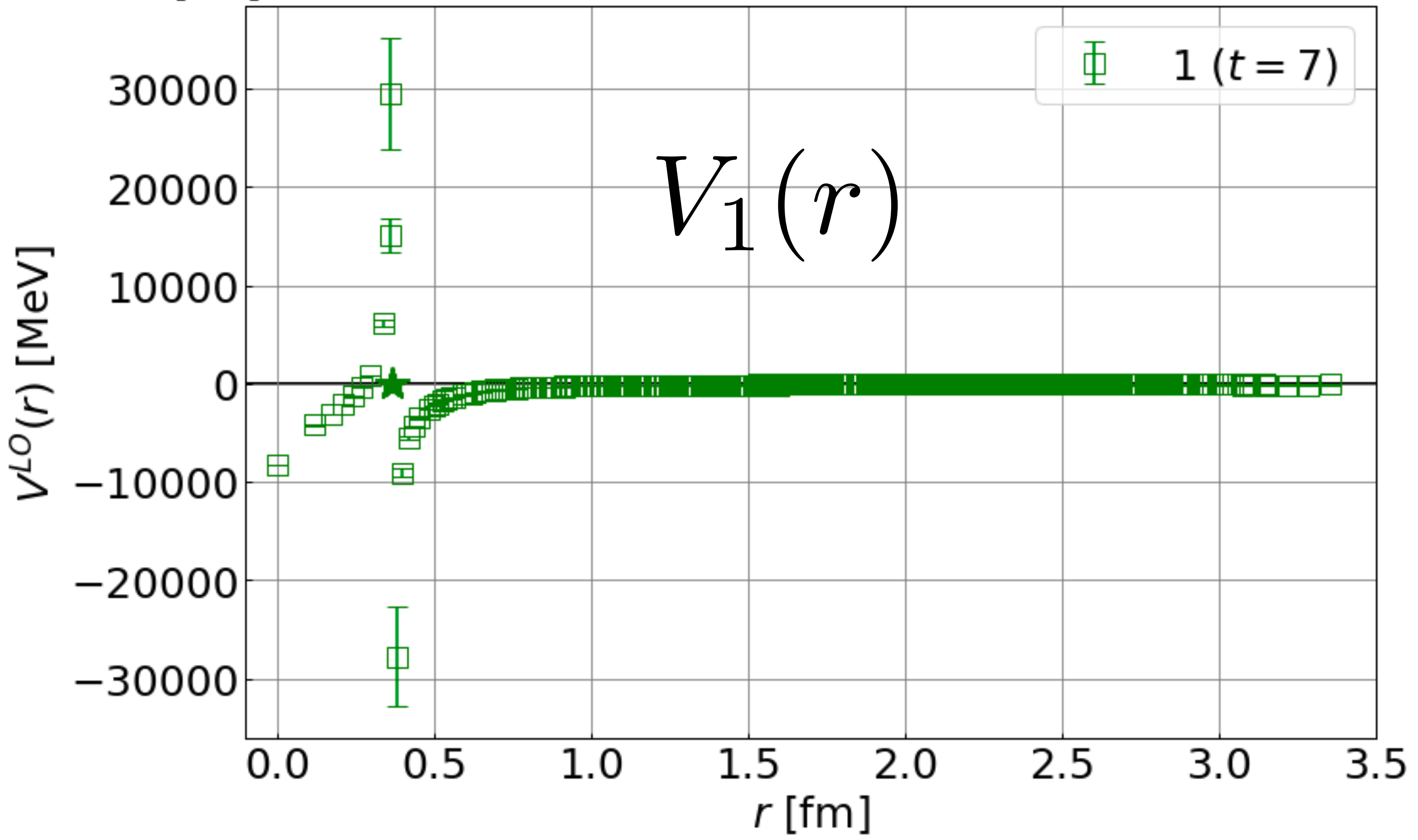
  - $m_M + m_B - m_{\mathrm{bound}}^{(\mathrm{octet})} = 156(8)_{\mathrm{stat}}$ MeV
  - $m_M + m_B - m_{\mathrm{bound}}^{(\mathrm{singlet})} = 227(5)_{\mathrm{stat}}$ MeV



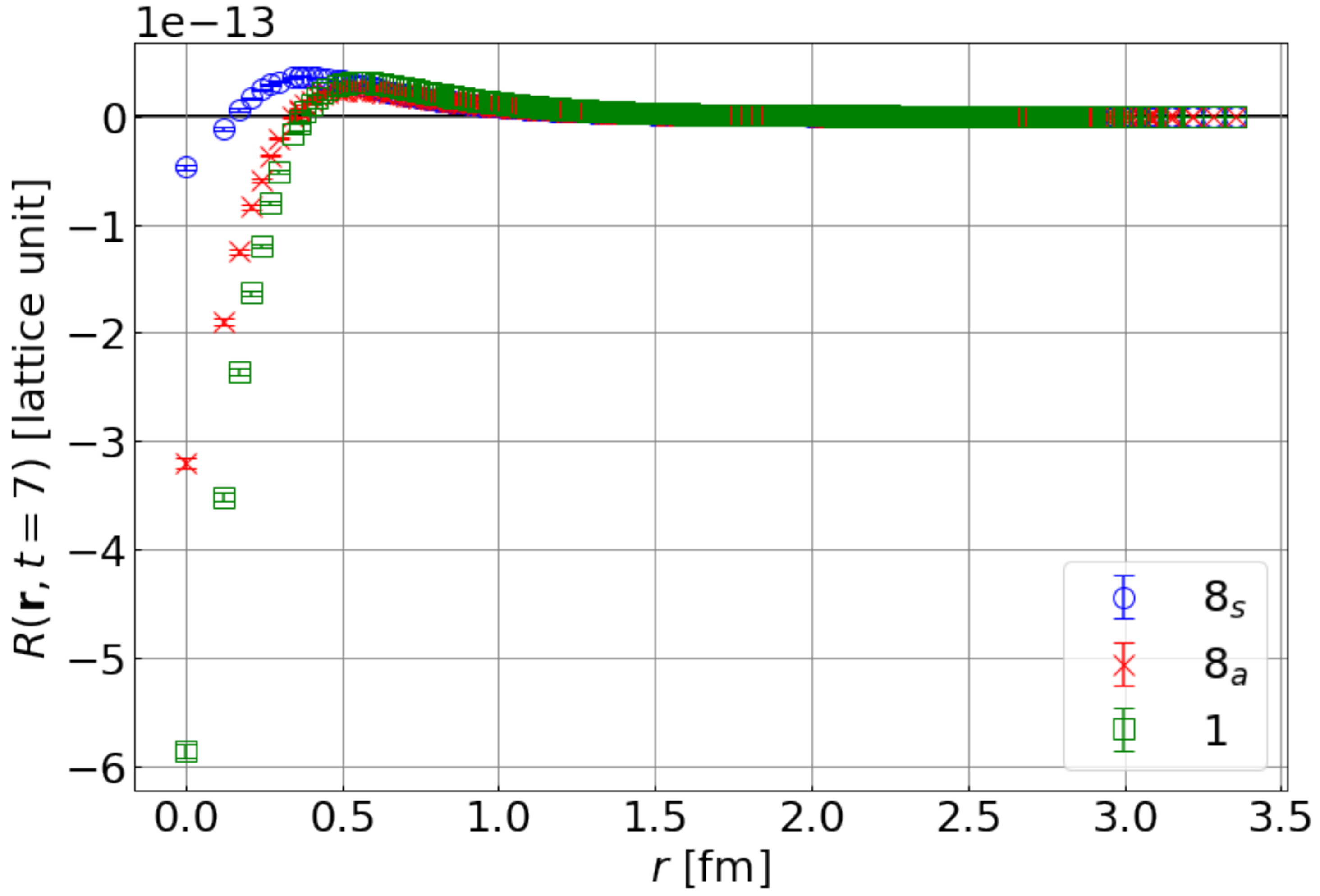
LO potentials







R-correlators $R(\mathbf{r}, t)$



$$V(r) \approx \frac{1}{R(\mathbf{r},t)} \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1 + 3\delta^2}{8\mu} \frac{\partial^2}{\partial t^2} \right) R(\mathbf{r},t)$$

- singular behavior because of the R-correlators crossing zero

no problem in principle,
but difficult to obtain reliable results⋯

Utilizing the two octet R-correlators

- **assume 8_s and 8_a are degenerated** in this work

→ $R^{(8_s)}(\mathbf{r},t)$, $R^{(8_a)}(\mathbf{r},t)$: **different potentials, but produce the same scattering amplitude**

cf. chiral perturbation theory w/ WT interaction:

- no coupling between 8_s and 8_a
- **interactions for 8_s and 8_a are the same**

→ same situation for $R^{(8_{\rm mix})}(\mathbf{r},t) = R^{(8_s)}(\mathbf{r},t) - cR^{(8_a)}(\mathbf{r},t)$ at any c

$$R^{(8_s)}(\mathbf{r},t) \rightarrow V(r)$$

different

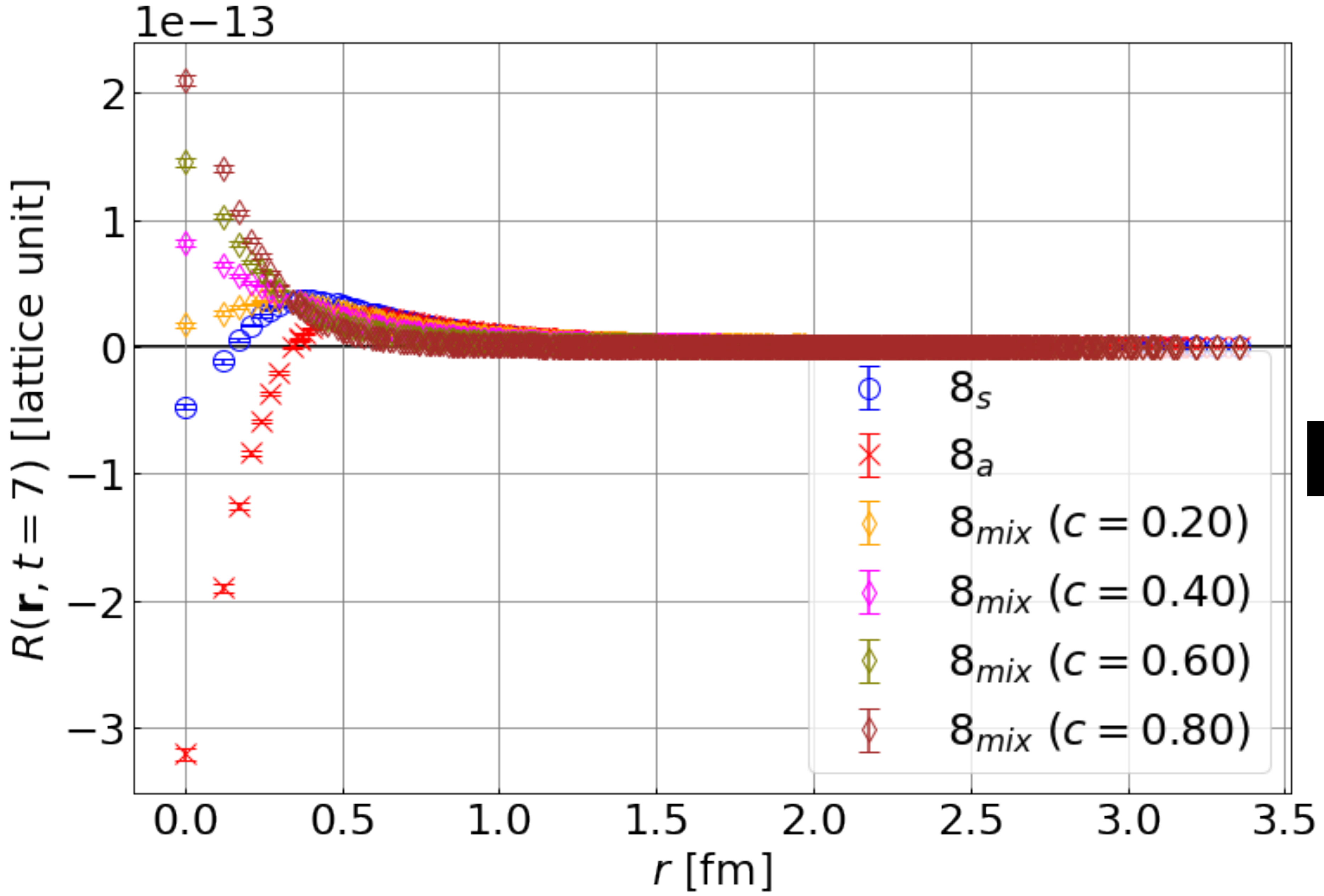
$$R^{(8_a)}(\mathbf{r},t) \rightarrow V'(r) \rightarrow \mathcal{M}(s)$$

$$R^{(8_{\rm mix})}(\mathbf{r},t) \equiv R^{(8_s)}(\mathbf{r},t) - cR^{(8_a)}(\mathbf{r},t) \rightarrow V^{(c)}(r)$$

- c is set such that $R^{(8_{\rm mix})}(\mathbf{r},t)$ **does not cross zero**

LO potentials from mixed R-correlators

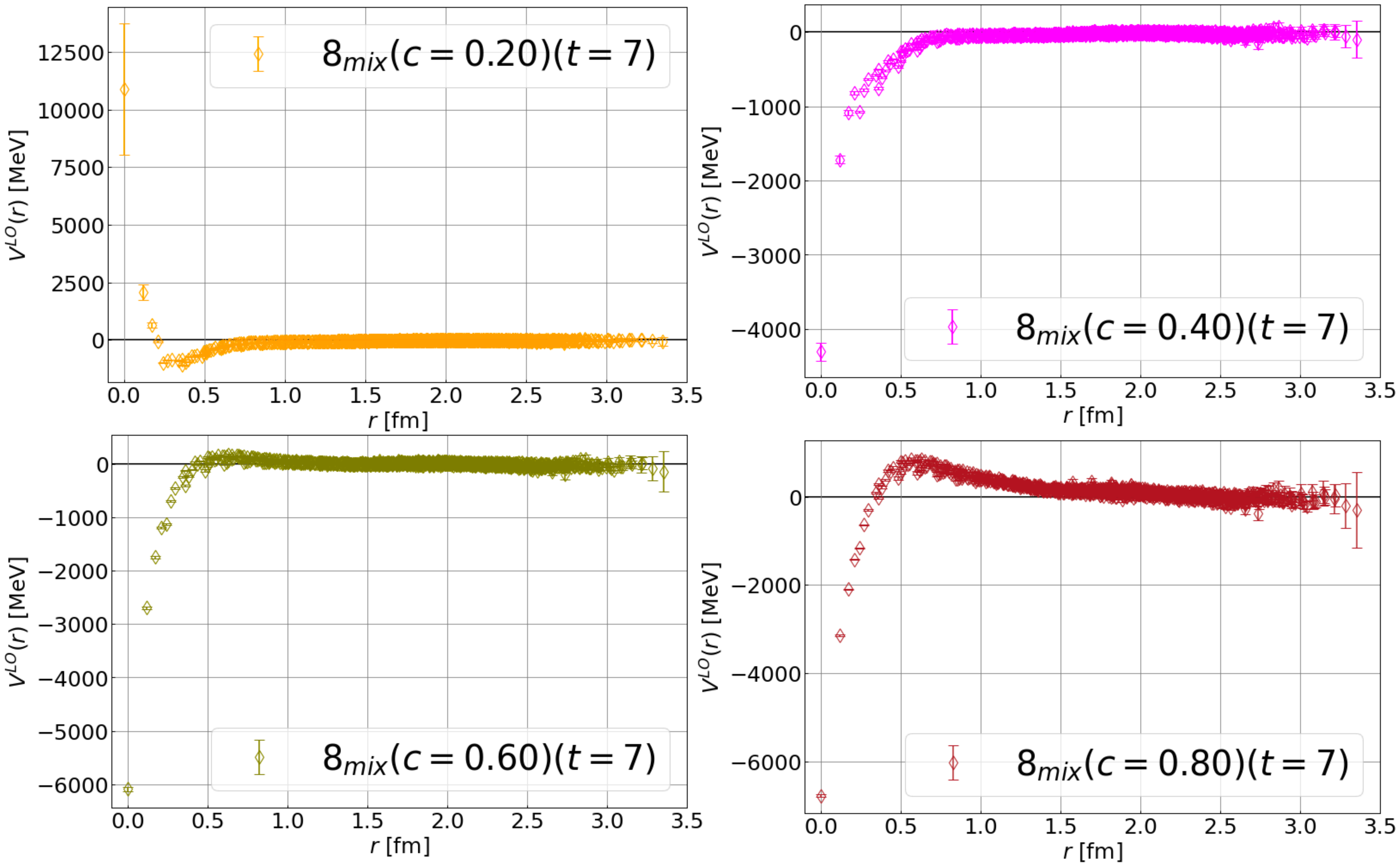
$$R^{(8_{\rm mix})}(\mathbf{r},t) = R^{(8_s)}(\mathbf{r},t) - cR^{(8_a)}(\mathbf{r},t)$$



- the zero points disappear

in $0.2 \lesssim c \lesssim 0.8$

$V^{(c)}(r)$



- **attractive for all** c

- the shape drastically changes for different c

➡ **physical observables?**

Binding energy in octet channel

- solve Schrödinger equation

➡ binding energy for each c

c	0.2	0.25	0.3	0.4	0.6	0.8
$E_{\mathrm{bind}}^{(\mathrm{octet})}$ [MeV]	179(4)	177(5)	177(5)	163(7)	132(13)	99(15)

$$\Rightarrow \quad E_{\mathrm{bind}}^{(\mathrm{octet})} = 163(7)_{\mathrm{stat}} \begin{pmatrix} +16 \\ -64 \end{pmatrix}_{\mathrm{sys}} \ \mathrm{MeV}$$

- consistent with the value from $\langle \Lambda_{\mathrm{octet}}(t) \bar{\Lambda}_{\mathrm{octet}}(0) \rangle$ ($156(8)_{\mathrm{stat}}$ MeV)

➡ **our analysis (and assumption)** is more or less **reliable**

- systematic error possibly comes from:
  - effect of the coupling 8_s, 8_a
  - difference between 8_s, 8_a
  - non-locality effect

Summary

- we study $\Lambda(1405)$ in flavor SU(3) limit from the meson-baryon scatterings using the HAL QCD method

- R-correlator in each irrep. have zero point, producing potential with singular point

- we utilize the mixed R-correlators in the **octet channel** to obtain the non-singular potential

- the potentials from different mixed R-correlators change the shapes, but give similar binding energies

Future work

- the singular behavior is due to the zeros of the 3-point functions (wave functions)

- such behavior does not happen in the usual QM

→ the singular behavior: effects beyond QM (QFT)

non-locality in the HAL QCD method

- Future work: use **separable potential** instead of the local one to avoid the singular behavior

$$U(\mathbf{r}, \mathbf{r}') \simeq g v(\mathbf{r}) v(\mathbf{r}')$$