SPICE: STRANGE HADRONS AS A PRECISION TOOL FOR STRONGLY INTERACTING SYSTEMS

May 13 – 17, 2024 @ Trento

$\bar{K}N$ interaction and Λ(1405) in a renormalizable framework of Chiral EFT

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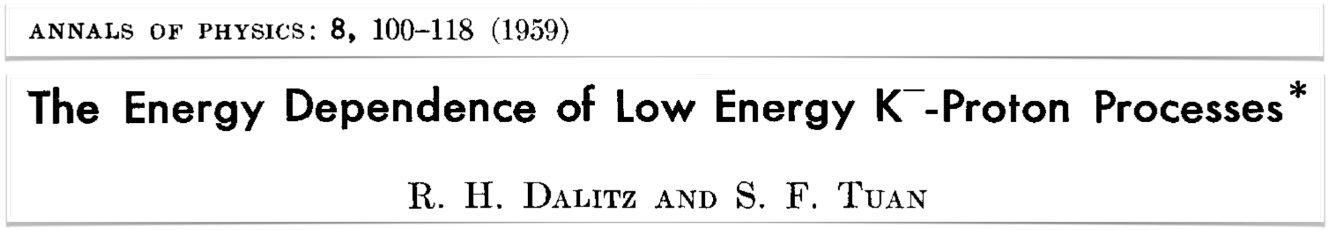
OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary

$\bar{K}N$ interaction

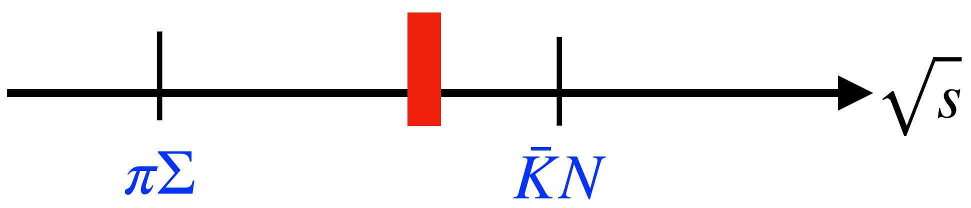
- $\bar{K}N$ interaction is strongly attractive (I=0)
  - **Exotic** $\Lambda(1405)$ resonance → $\bar{K}N$ amplitude in free space



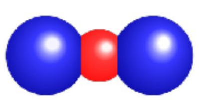
ANNALS OF PHYSICS: 8, 100–118 (1959)

The Energy Dependence of Low Energy K⁻-Proton Processes*

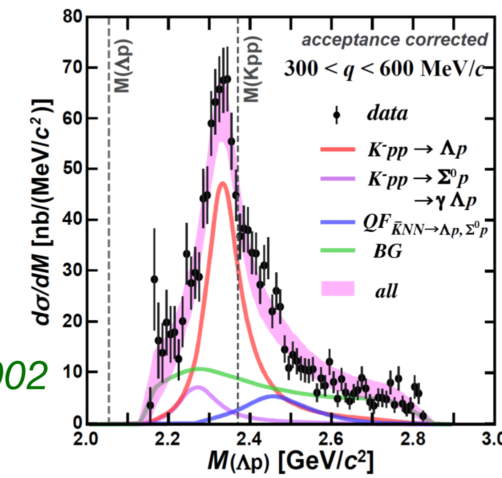
R. H. DALITZ AND S. F. TUAN



  - **New form** of nuclei/atoms: $\bar{K}NN$, $\bar{K}NNN$, multi-$\bar{K}$ N/A J-PARC, DAΦNE, GSI…



E15@J-PARC, PRC102(2020)044002

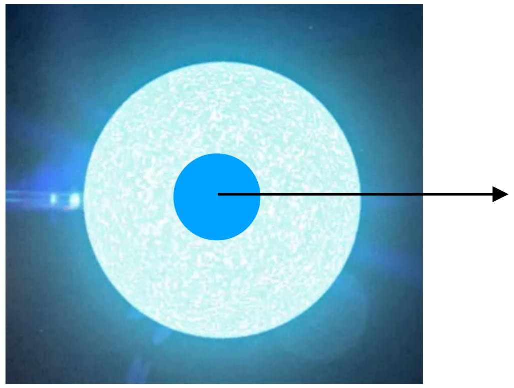


$B_{K^-pp} \sim 40$ MeV

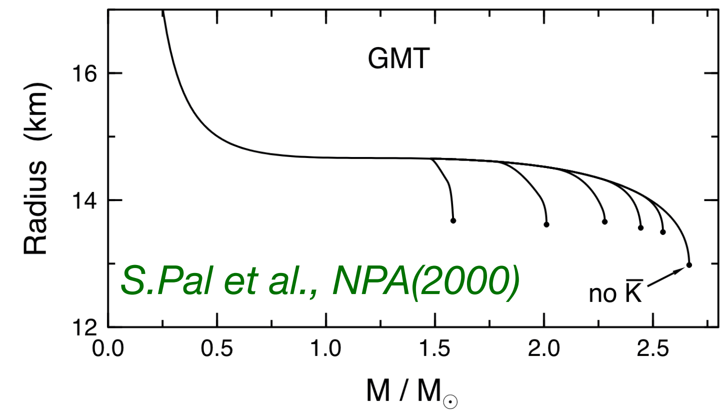
$\Gamma_{K^-pp} \sim 100$ MeV

Details can be seen in today's talks

  - **Kaon-condensate** could change EoS of neutron star

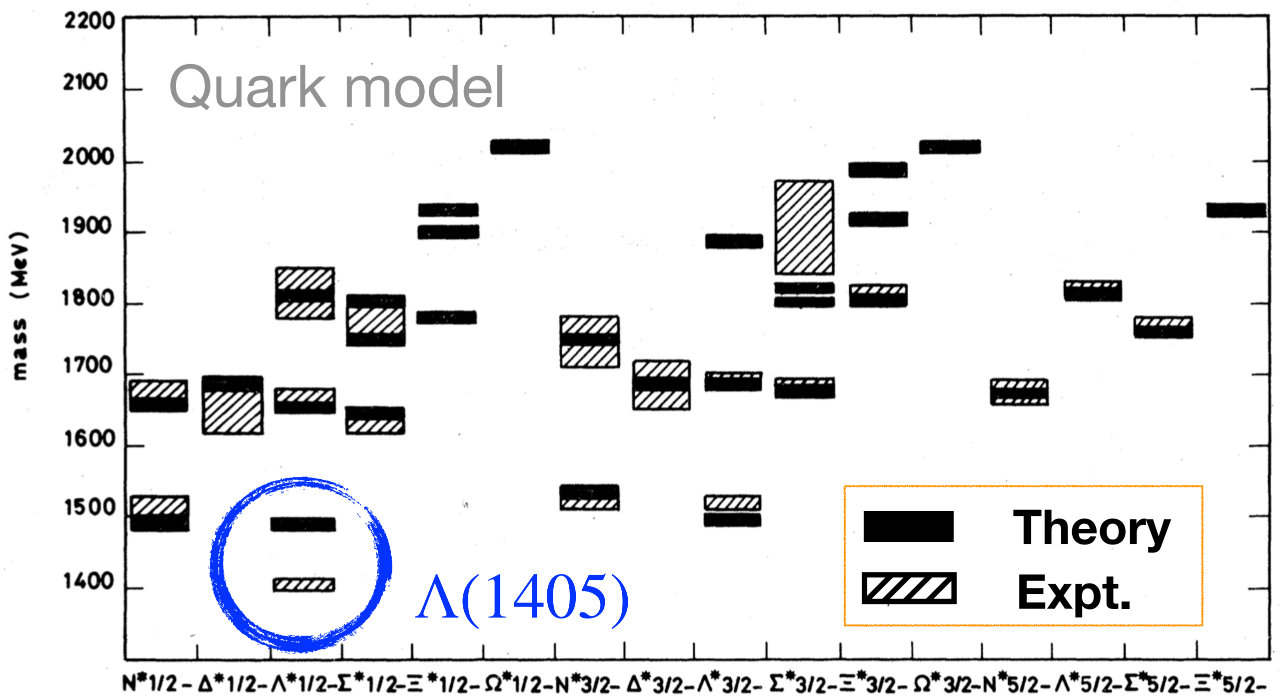


Inner core
- hyperonic matter
- Kaon condensate
- Quark matter



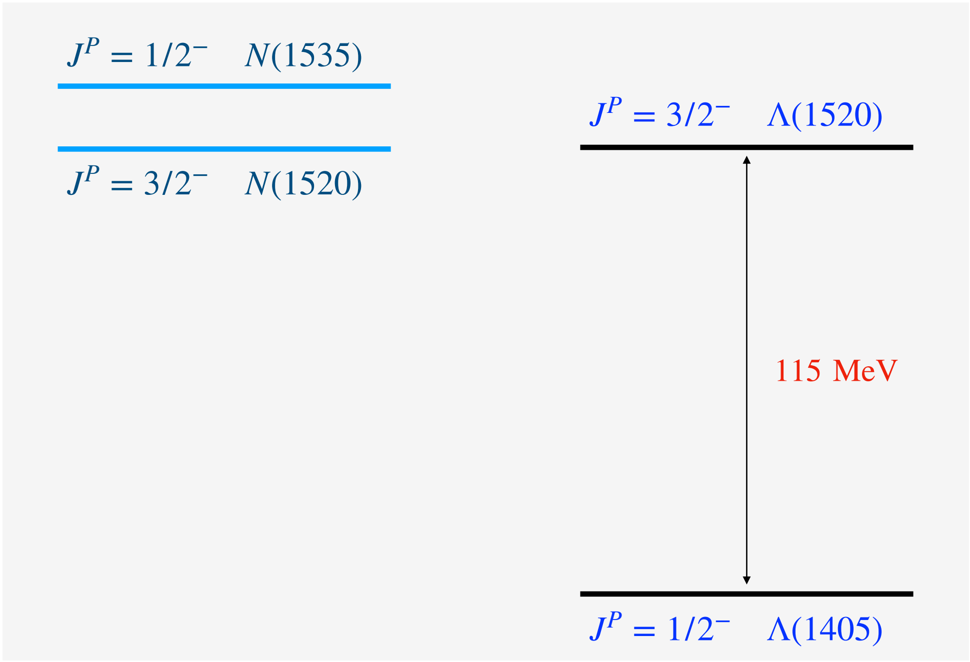
Play an important role in the strangeness nuclear physics

- $\Lambda(1405)$ state is an exotic candidate



N. Isgur and G. Karl, Phys.Rev.D 18 (1978) 4187

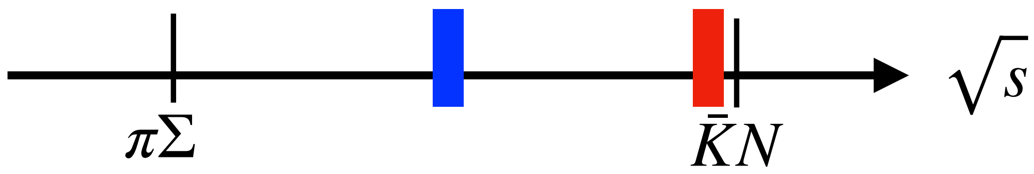
- **Lightest excited baryon with** $J^P = \frac{1}{2}^-$



- Variety of theoretical studies
  - Chiral SU(3) quark model *F. Huang, PRC2007…*
  - QCD sum rules *L.S. Kisslinger, EPJA2011…*
  - Phenomenological potential model *A. Cieplý, NPA2015…*
  - Skyrme model *T. Ezoe, PRD2020…*
  - Hamiltonian effective field theory *Z.-W. Liu, PRD2017…*
  - Chiral unitary approach *N.Kaiser,NPA1995; E.Oset,NPA1998; J.A.Oller&U.-G.Meißner,PLB2001…*

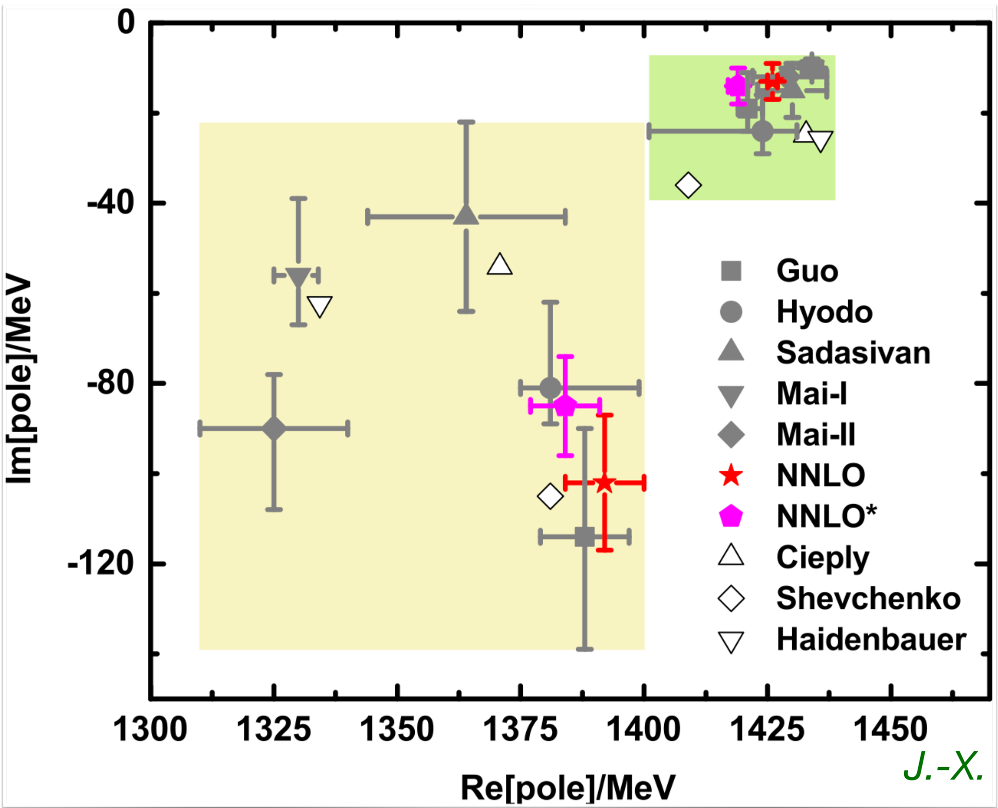
Structure of $\Lambda(1405)$ resonance

- Double-pole predicted by **chiral unitary approach**



$\Lambda(1405)\ 1/2^-$ $\quad I(J^P) = 0(\frac{1}{2}^-)$ Status: ∗∗∗∗

$\Lambda(1380)\ 1/2^-$ $\quad J^P = \frac{1}{2}^-$ Status: ∗∗

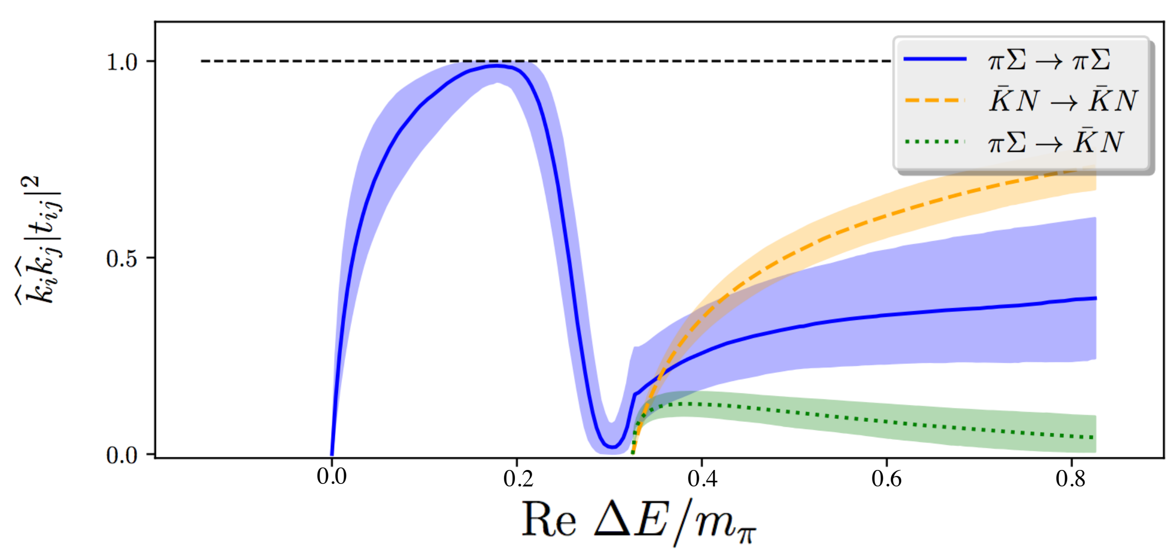


J.-X. Lu, et al., PRL130(2023)071902

✓ Pole 1: Λ(1405) is around 1420 MeV

✓ Pole 2: Λ(1380) needs further studies to fix its position

- Double-pole structure verified by LQCD



$m_\pi \approx 200$ MeV, $m_K \approx 487$ MeV

$$\text{Lower Pole}: \ E_1 = 1392(9)_{\rm stat}(2)_{\rm model}(16)_{\rm a}\ \text{MeV}$$

$$\text{Higher Pole}: \ E_2 = 1455(13)_{\rm stat}(2)_{\rm model}(17)_{\rm a} - i11.5(4.4)_{\rm stat}(4.0)_{\rm model}(0.1)_{\rm a}\ \text{MeV}$$

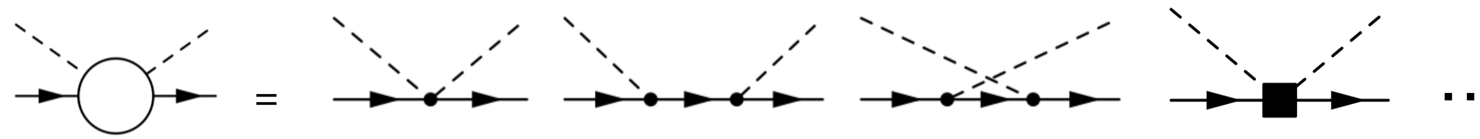
Baryon Scattering Coll., PRL132(2024)051901

Chiral Unitary approach

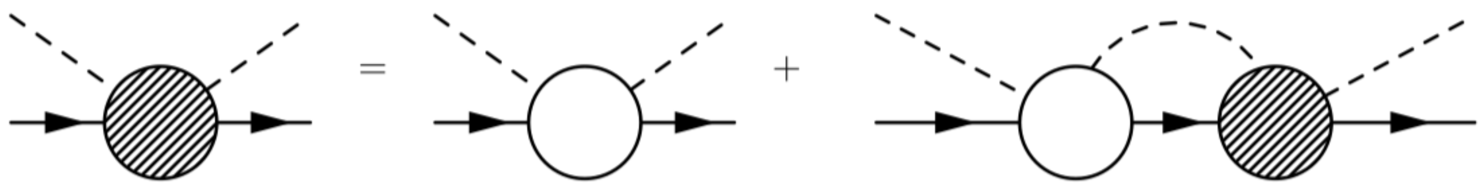
- **Chiral symmetry** of low-energy QCD + **Unitary Relation**

  *J.A.Oller et al.,PPNP45(2000)157-242; T.Hyodo et al.,PPNP*120 (2021)103868 …

- Interaction kernel V: calculate in ChPT order by order
  - Leading, next-to-leading order, …



- Scattering T-matrix: solve scattering equations



  - Lippmann-Schwinge equation or Bethe-Salpeter equation

$$T(p',p) = V(p',p) + i\int \frac{d^4k}{(2\pi)^4} V(p',k)\,G(k)\,T(k,p)$$

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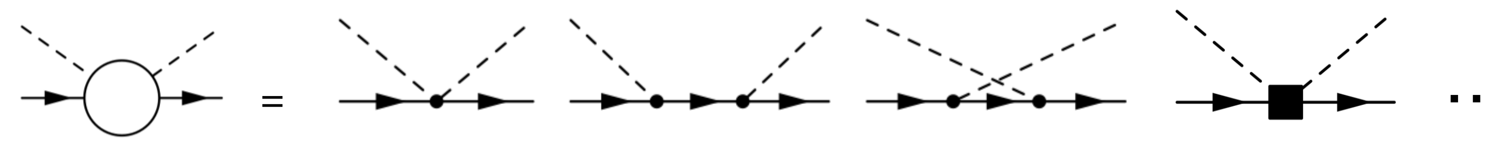

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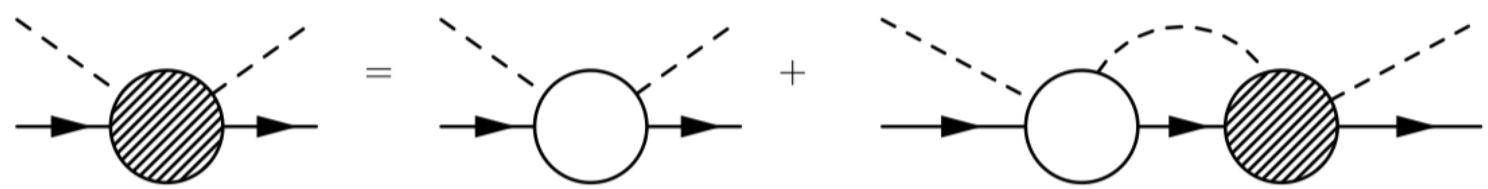

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- On-shell factorization $\rightarrow V(p',p) + V(p',p)\left(i\int \frac{d^4k}{(2\pi)^4} G(k) \right) T(p',p)$

Neglecting off-shell effect

→ cause troubles in the study of three-body interaction?

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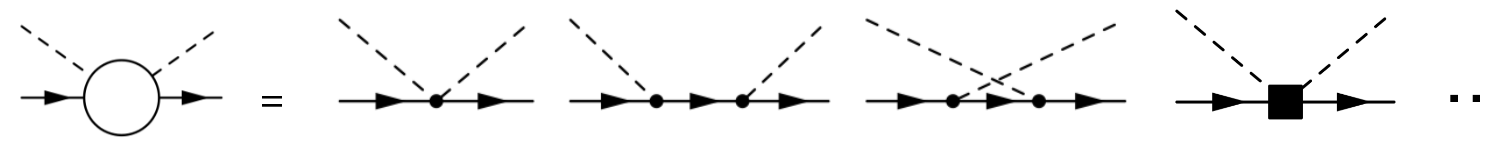

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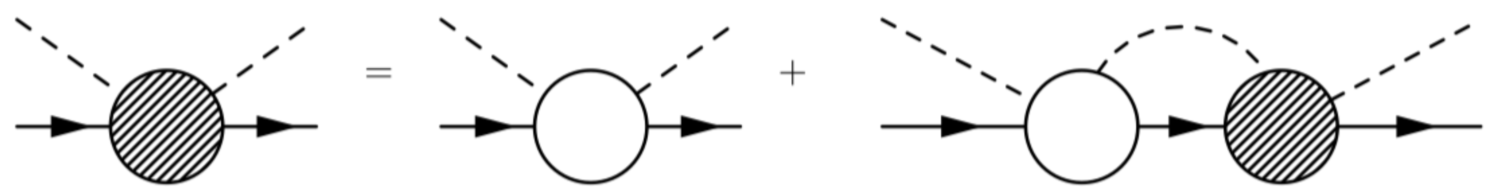

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Neglecting off-shell effect

→ cause troubles in the study of three-body interaction?

- Finite cutoff or subtraction constant to renormalize the loop integral

$$G^R(E,\Lambda) \text{ or } G^R(E,\alpha_i)$$

Cutoff / Model dependence

In this work

- Facing the rapid progress of precision experiments, **a model-independent formalism would be needed** ALICE, AMADEUS, J-PARC, STAR…
- We propose **a renormalizable framework** of Chiral EFT for meson-baryon scattering
  - Apply to the SU(2) sector: pion-nucleon scattering

    XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406
  - Extend to the SU(3) sector: $\bar{K}N$ scattering and $\Lambda(1405)$ state

    XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C81 (2021) 582
  - Investigate the light-quark mass dependence of $\Lambda(1405)$

    XLR, arXiv: 2404.02720 [hep-ph]
  - Next-to-leading order studies

    XLR, et al., In progress

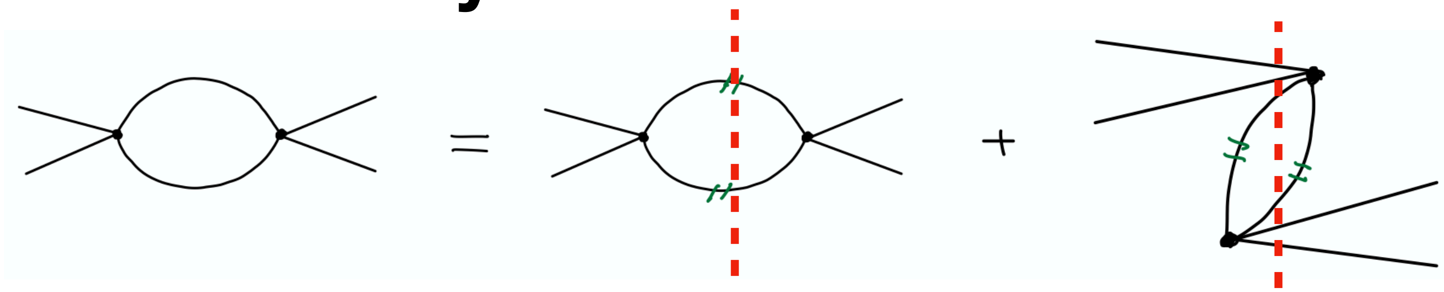
Theoretical framework

Time-ordered perturbation theory

- Definition

S. Weinberg, Phys.Rev.150(1966)1313
G.F. Sterman, "An introduction to quantum field theory", Cambridge (1993)

- Re-express the Feynman integral in a form that makes the connection with on-mass-shell (off-energy shell) state explicit.
  - ✓ Instead the propagators for internal lines as the energy denominators for intermediate states
- **TOPT or old-fashioned perturbation theory**



- Advantages
  - Explicitly show the unitarity
  - Easily to tell the contributions of a particular diagram

- Obtain the rules for time-ordered diagrams
  - Perform Feynman integrations over the zeroth components of the loop momenta
  - Decompose Feynman diagram into sums of time-ordered diagrams
  - Match to the rules of time-ordered diagrams

Diagrammatic rules in TOPT

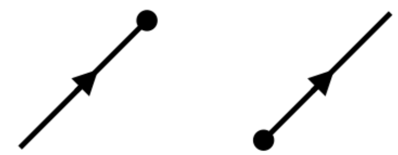
XLR, PoS(CD2021)007

- **External lines**

Spin 0 boson (in, out)

1

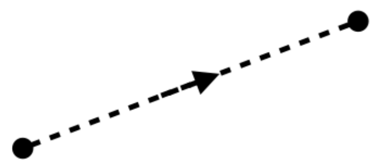
Spin 1/2 fermion (in, out)



$$u(\mathbf{p}), \quad \bar{u}(\mathbf{p}')$$

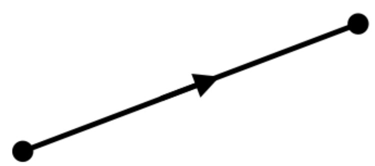
- **Internal lines**

Spin 0 (anti-)boson



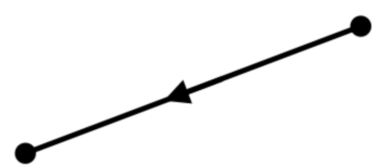
$$\frac{1}{2\,\epsilon_q} \qquad \epsilon_q \equiv \sqrt{\mathbf{q}^2 + M^2}$$

Spin 1/2 fermion



$$\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) \qquad \omega_p \equiv \sqrt{\mathbf{p}^2 + m^2}$$

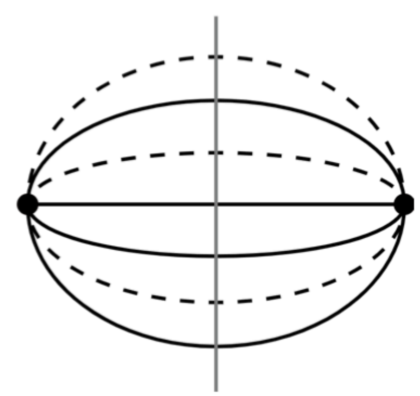
anti-fermion



$$\frac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) - \gamma_0$$

- **Intermediate state**

A set of lines between two vertices



$$\frac{1}{E - \sum_i \omega_{p_i} - \sum_j \epsilon_{q_j} + i\epsilon}$$

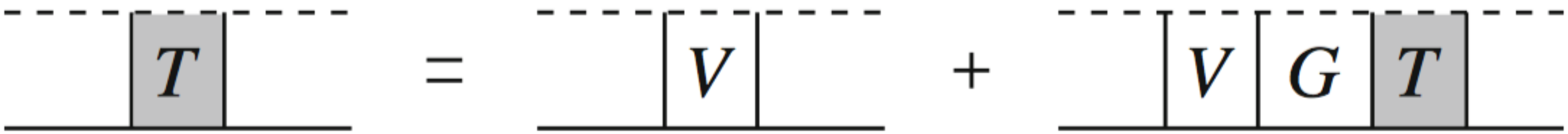
- **Interaction vertices: the standard Feynman rules**
  - Take care of zeroth components of integration momenta

✓ particle $p^0 \to \omega(p, m)$

✓ antiparticle $p^0 \to -\omega(p, m)$

Meson–baryon scattering in TOPT

- Interaction kernel / potential V
  - **Define:** sum up the one-meson and one-baryon **irreducible diagrams**
  - **Power counting:** Q/Λ_χ systematic ordering of all graphs
- Scattering equation



- Coupled-channel integral equation for T-matrix

$$T_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) = V_{M_jB_j,M_iB_i}(\boldsymbol{p}',\boldsymbol{p};E) + \sum_{MB}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} V_{M_jB_j,MB}(\boldsymbol{p}',\boldsymbol{k};E)\, G_{MB}(E)\, T_{MB,M_iB_i}(\boldsymbol{k},\boldsymbol{p};E)$$

- **Meson-baryon Green function in TOPT**

$$G_{MB}(E) = \frac{m}{2\omega(k,M)\,\omega(k,m)} \frac{1}{E-\omega(k,M)-\omega(k,m)+i\epsilon}$$

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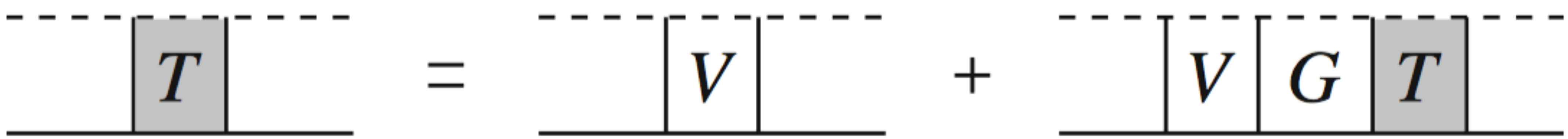

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Potential and scattering equation are obtained on an equal footing!

Leading order studies

XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner,
Eur. Phys. J. C80 (2020) 406; Eur. Phys. J. C81 (2021) 582

Leading order potential

- Chiral effective Lagrangian

$$\mathcal{L}_{\mathrm{LO}} = \frac{F_0^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + \langle \bar{B}\left(i\gamma_\mu\partial^\mu - m\right)B\rangle + \frac{D/F}{2}\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]_\pm\rangle$$
$$-\frac{1}{4}\left\langle V_{\mu\nu}V^{\mu\nu} - 2\mathring{M}_V^2\left(V_\mu - \frac{i}{g}\Gamma_\mu\right)\left(V^\mu - \frac{i}{g}\Gamma^\mu\right)\right\rangle + g\,\langle \bar{B}\gamma_\mu[V^\mu, B]\rangle$$

- Time ordered diagrams

(a) (b) (c) (d)

- **Vector mesons included as explicit degrees of freedom**
  - ✓ One-vector meson exchange potential instead of the Weinberg-Tomozawa term
  - ✓ Improve the ultraviolet behaviour without changing the low-energy physics

- LO potential in TOPT

- Dirac spinor is decomposed as $u_B(p,s) = u_0 + [u(p) - u_0] \equiv (1,0)^\dagger \chi_s$+high order

$$V^{(a+b)}_{M_jB_j,M_iB_i} = -\frac{1}{32F_0^2}\sum_{V=K^*,\rho,\omega,\phi} C^V_{M_jB_j,M_iB_i}\frac{\mathring{M}_V^2}{\omega_V(q_1-q_2)}\left(\omega_{M_i}(q_1)+\omega_{M_j}(q_2)\right)$$
$$\times\left[\frac{1}{E-\omega_{B_i}(p_1)-\omega_V(q_1-q_2)-\omega_{M_j}(q_2)} + \frac{1}{E-\omega_{B_j}(p_2)-\omega_V(q_1-q_2)-\omega_{M_i}(q_1)}\right]$$

$$V^{(c)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} C^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(P)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)}{E-\omega_B(P)}.$$
$$V^{(d)}_{M_jB_j,M_iB_i} = \frac{1}{4F_0^2}\sum_{B=N,\Lambda,\Sigma,\Xi} \tilde{C}^B_{M_jB_j,M_iB_i}\frac{m_B}{\omega_B(K)}\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{q}_1)(\boldsymbol{\sigma}\cdot\boldsymbol{q}_2)}{E-\omega_{M_i}(q_1)-\omega_{M_j}(q_2)-\omega_B(K)}.$$

Ultraviolet Behavior

- One-loop integral $V\,G\,V$

$$I_{VGV} = \int \frac{d^3k}{(2\pi)^3} V(p', k)\, G(k)\, V(k, p) \begin{cases} V = V_{\rm VME}, & I_{VGV} \overset{k\to\infty}{\longrightarrow} \int d^3k\, \frac{1}{k}\, \frac{1}{k^3}\, \frac{1}{k} \\ V = V_{\rm WT}, & I_{VGV} \overset{k\to\infty}{\longrightarrow} \int d^3k\, k\, \frac{1}{k^3}\, k \end{cases}$$

  - Scattering amplitude from the VME potential is **cutoff independent !**

$$T_{\rm VME} = V_{\rm VME} + V_{\rm VME}\, G\, T_{\rm VME}$$

Renormalizable

Ultraviolet Behavior

- One-loop integral $V\,G\,V$

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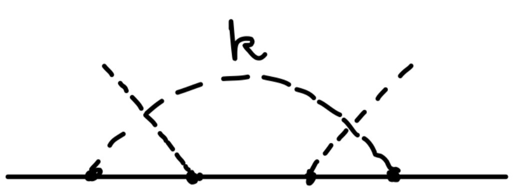
Renormalizable

- Iteration of the crossed-Born term is also renormalizable



$$\to \int d^3k\, \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}'\,\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}}}{k}\, \frac{1}{k^3}\, \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}\,\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}}}{k}$$

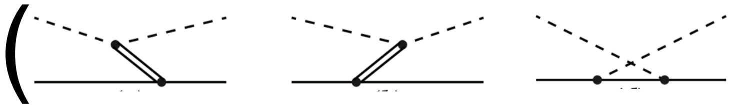
- Only divergence is from the iteration of the Born term

$$\to \int d^3k\, \boldsymbol{\sigma}\cdot\boldsymbol{p}'\,\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}}\, k\, \frac{1}{k^3}\, k\, \boldsymbol{\sigma}\cdot\boldsymbol{p}\,\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}}$$

Quadratical divergence

Subtractive renormalization

- LO potential: **one-baryon irreducible and reducible parts**

$V_{\mathrm{LO}} = V_I$ () $+ V_R$ ()

- LO T-matrix

$$T_{\mathrm{LO}} = V_{\mathrm{LO}} + V_{\mathrm{LO}}\, G\, T_{\mathrm{LO}} \quad \Longrightarrow \quad \begin{cases} T_{\mathrm{LO}} = T_I + (1 + T_I G)\, T_R\, (1 + G\, T_I) \\ T_I = V_I + V_I\, G\, T_I \\ T_R = V_R + V_R\, G\, (1 + T_I G)\, T_R \end{cases}$$

- Irreducible part: $T_I \xrightarrow{\Lambda \sim \infty}$ Finite
- Reducible part: $T_R \xrightarrow{\Lambda \sim \infty}$ Divergent
  - ✓ **Potential can be rewritten as separable form**

$$V_R(p', p; E) = \xi^T(p')\, C(E)\, \xi(p) \qquad \text{C(E): constant} \qquad \xi^T(q) := (1, q)$$

  - ✓ T_R can be rewritten as $T_R(p', p; E) = \xi^T(p')\, \chi(E)\, \xi(p) \qquad \chi(E) = \left[C^{-1} - \xi\, G\, \xi^T - \xi\, G\, T_I^S\, G\, \xi^T\right]^{-1}$

D.B.Kaplan, et al.,NPB478,629(1996); E. Epelbaum, et al.,EPJA51,71(2015)

  - ✓ Using **subtractive renormalization,** replacing Green function $G^{Rn} = G(E) - G(m_B)$

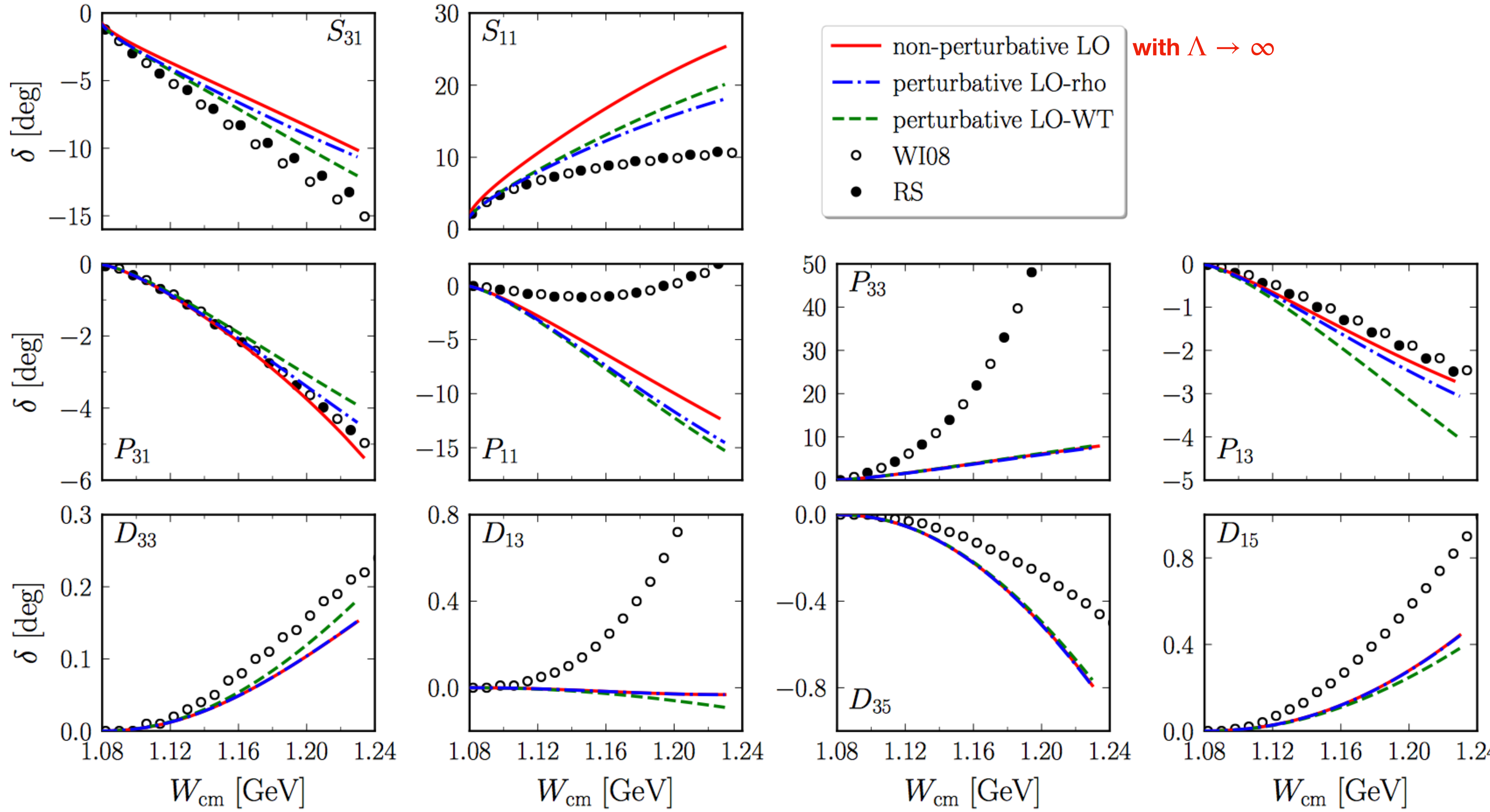
E. Epelbaum, et al., EPJA56(2020)152

Renormalized LO T-matrix

$$T_{\mathrm{LO}}^{Rn} = T_I + \left(\xi^T + T_I\, G^{Rn}\, \xi^T\right)\, \chi^{Rn}(E)\, \left(\xi + \xi\, G^{Rn}\, T_I\right)$$

Pion–Nucleon scattering

- Description phase shifts of pion-nucleon scattering



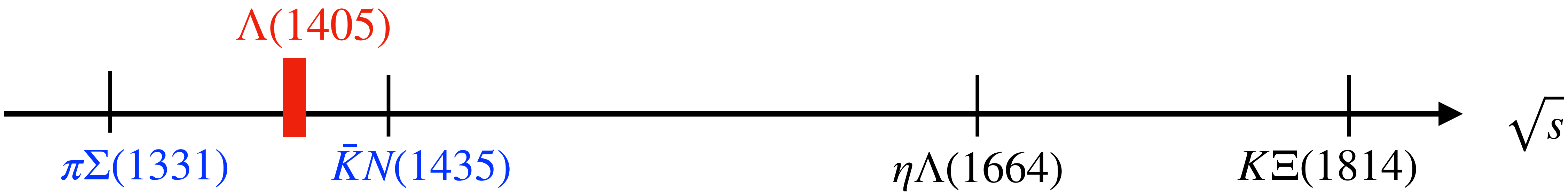
- Rho-meson-exchange contribution is similar as WT term
- Non-perturbative results are only slightly different from the ones of the perturbative approach

✓ **Non-perturbative treatment is valid**, since ChPT has good convergence in SU(2) sector

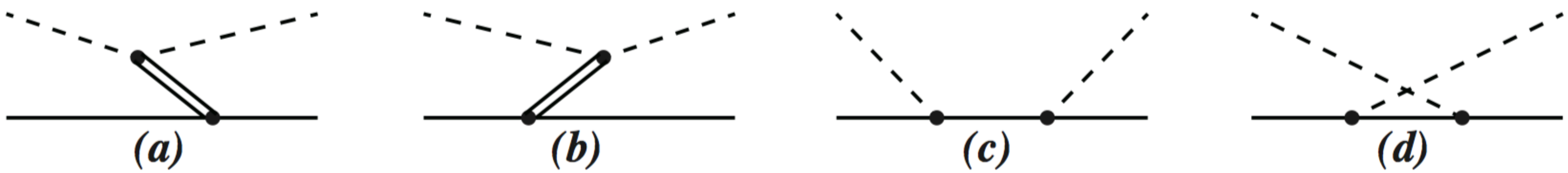
XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, Eur. Phys. J. C80 (2020) 406

S=−1 meson-baryon scattering

- Four coupled channels $\bar{K}N, \pi\Sigma, \eta\Lambda, K\Xi$ in isospin limit



- Focus on the S-wave potential



  - Born term (p-wave) does not contribute
  - Crossed-Born term ∼5% of VME contribution
  - VME potential couplings

C^V	$\pi\Sigma$	$\bar{K}N$	$\eta\Lambda$	$K\Xi$
$\pi\Sigma$	$C^\rho = -16$ attractive	$C^{K^*} = 2\sqrt{6}$	0	$C^{K^*} = -2\sqrt{6}$
$\bar{K}N$		$C^{\{\rho,\omega,\phi\}} = \{-6, -2, -4\}$ attractive	$C^{K^*} = -6\sqrt{2}$	0
$\eta\Lambda$			0	$C^{K^*} = 6\sqrt{2}$
$K\Xi$				$C^{\{\rho,\omega,\phi\}} = \{-6, -2, -4\}$

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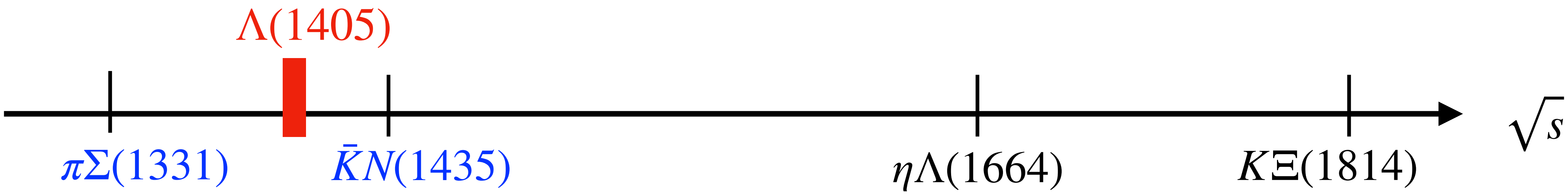

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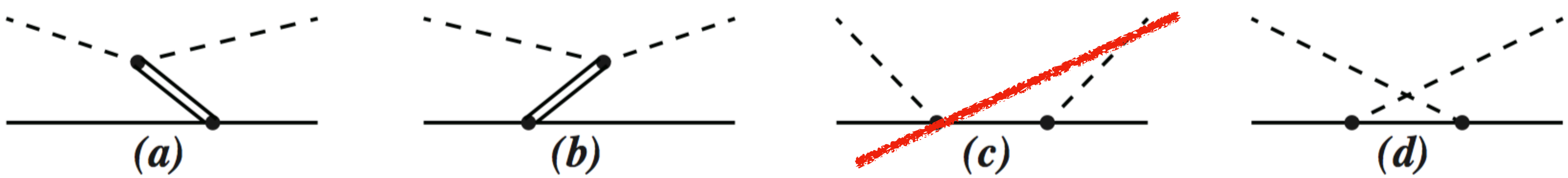

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$\eta\Lambda$		attractive	0	$C^{K^*} = 6\sqrt{2}$
$K\Xi$				$C^{\{\rho,\omega,\phi\}} = \{-6, -2, -4\}$

S=-1 meson-baryon scattering

P. W. scattering equation

$$T^{LJ}_{M_jB_j,M_iB_i}(p',p) = V^{LJ}_{M_jB_j,M_iB_i}(p',p) + \sum_{MB}\int \frac{dkk^2}{(2\pi)^3} V^{LJ}_{M_jB_j,MB}(p',k)\frac{1}{2\,\omega_M\omega_B}\frac{m_B}{E-\omega_M-\omega_B+i\epsilon} T^{LJ}_{MB,M_iB_i}(k,p)$$

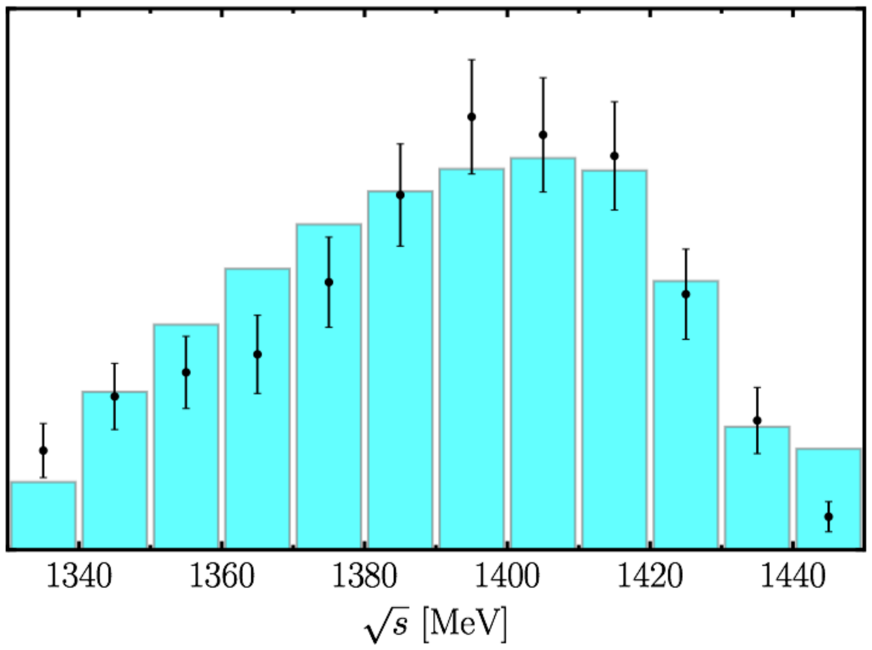
- Take into account **the off-shell effects of potential**
- Use **subtractive reormalization** to obtain the renormalized T-matrix
  - ➡ Cutoff-independent: $\Lambda \to \infty$

No free parameters needed to be fitted!

Two pole positions of $\Lambda(1405)$

		lower pole	higher pole
This work	$F_0 = F_\pi$	$1337.7 - i\,79.1$	$1430.9 - i\,8.0$
(LO)	$F_0 = 103.4$	$1348.2 - i\,120.2$	$1436.3 - i\,0.7$
NLO	*Y. Ikeda,NPA(2012)*	$1381^{+18}_{-6} - i81^{+19}_{-8}$	$1424^{+7}_{-23} - i26^{+3}_{-14}$
	Z.-H.Guo,PRC(2013)-Fit II	$1388^{+9}_{-9} - i114^{+24}_{-25}$	$1421^{+3}_{-2} - i19^{+8}_{-5}$
	M.Mai,EPJA(2015)-sol-2	$1330^{+4}_{-5} - i56^{+17}_{-11}$	$1434^{+2}_{-2} - i10^{+2}_{-1}$
	M.Mai,EPJA(2015)-sol-4	$1325^{+15}_{-15} - i90^{+12}_{-18}$	$1429^{+8}_{-7} - i12^{+2}_{-3}$

πΣ invariant mass spectrum



- Consistent with M. Mai EPJA(2015), in particular for the lower pole

Comes to the unphysical quark mass region —>

Quark mass dependence of $\Lambda(1405)$

XLR, arXiv: 2404.02720 [hep-ph]

$\Lambda(1405)$ from Lattice QCD

- The first lattice study of $\Lambda(1405)$ pole positions

Baryon Scattering Collaboration: PRL 132, 051901 (2024); PRD109,014511(2024)

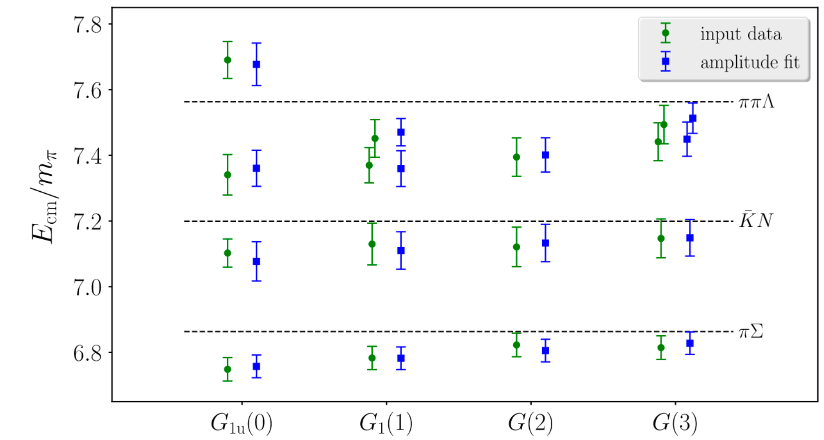
- Focus on the $\pi\Sigma - \bar{K}N$ coupled channels (below $\pi\pi\Lambda$ threshold)
- Pion and Kaon masses: $M_\pi \approx 200$ MeV, $M_K \approx 487$ MeV

Coordinated Lattice Simulations (CLS)

D200 ensemble

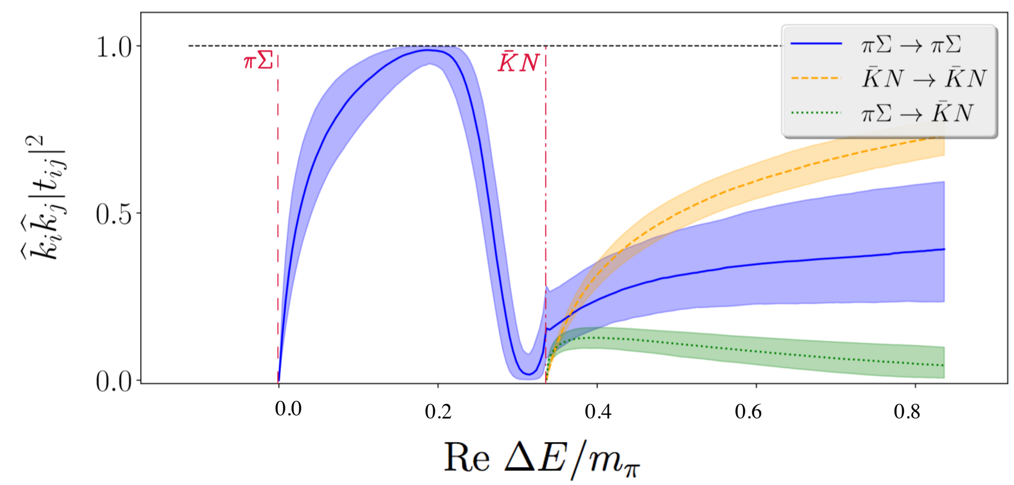
a (fm)	$(L/a)^3 \times T/a$	$m_\pi L$
0.0633(4)(6)	$64^3 \times 128$	4.181(16)

✓ Extract FV energy spectrum



✓ Implement the Lüscher formalism

$$\det[\tilde{K}^{-1}(E_{\rm cm}) - B^P(E_{\rm cm})] = 0.$$



- Two poles of $\Lambda(1405)$

Virtual bound state

$$E_1 = 1392(9)_{\rm stat}(2)_{\rm model}(16)_{\rm a}\,{\rm MeV}$$

$$\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 1.9(4)_{\rm stat}(6)_{\rm model}$$

Resonance

$$E_2 = [1455(13)_{\rm stat}(2)_{\rm model}(17)_{\rm a} - i11.5(4.4)_{\rm stat}(4.0)_{\rm model}(0.1)_{\rm a}]\,{\rm MeV}$$

$$\left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2}_{\bar{K}N}}\right| = 0.53(9)_{\rm stat}(10)_{\rm model}$$

Apply our framework to unphysical world

- BaSc results provide an ideal playground
  - **Check/verify the predictive power** of existing chiral unitary approaches
- We extend the calculation to the unphysical quark mass region
  - Use the same meson and baryon masses as the BaSc study
    - ✓ $M_\pi = 203.7$ MeV, $M_K = 486.4$ MeV, $m_N = 979.8$ MeV, $m_\Sigma = 1193.9$ MeV
    - ✓ $F_0 = F_\pi = 93.2$ MeV
  - Focus on the $\pi\Sigma - \bar{K}N$ coupled channels

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    - ✓ $F_0 = F_\pi = 93.2$ MeV
  - Focus on the $\pi\Sigma - \bar{K}N$ coupled channels
  - **Consistent with the BaSc results**

XLR, arXiv: 2404.02720 [hep-ph]

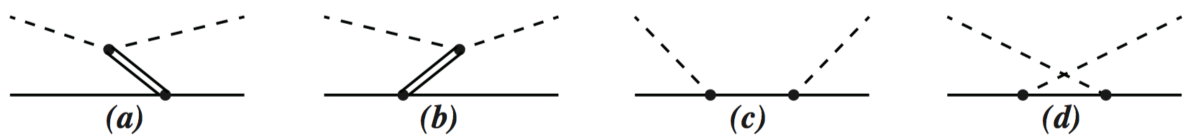
	BaSc [PRL2024]	This work			
$\Lambda(1405)$	z_R [MeV]	z_R [MeV]	$g_{\pi\Sigma}$	$g_{\bar{K}N}$	$\lvert g_{\pi\Sigma}\rvert / \lvert g_{\bar{K}N}\rvert$
Lower pole	**1392(18)**	1387.14	$0.021 + i1.87$	$0.017 + i1.55$	1.21
Higher pole	**1455(21) − i11.5(6.0)**	$1469.86 - i4.71$	$0.038 + i0.98$	$1.51 - i1.22$	0.50

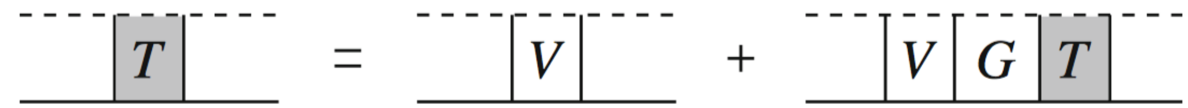
  - If performing a full calculation with $\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi$ channels

Lower pole : $z_R = 1389.05$ MeV   Higher pole : $z_R = 1464.55 - i9.44$ MeV

Quark mass dependence of model variables

- Variables in our LO calculation

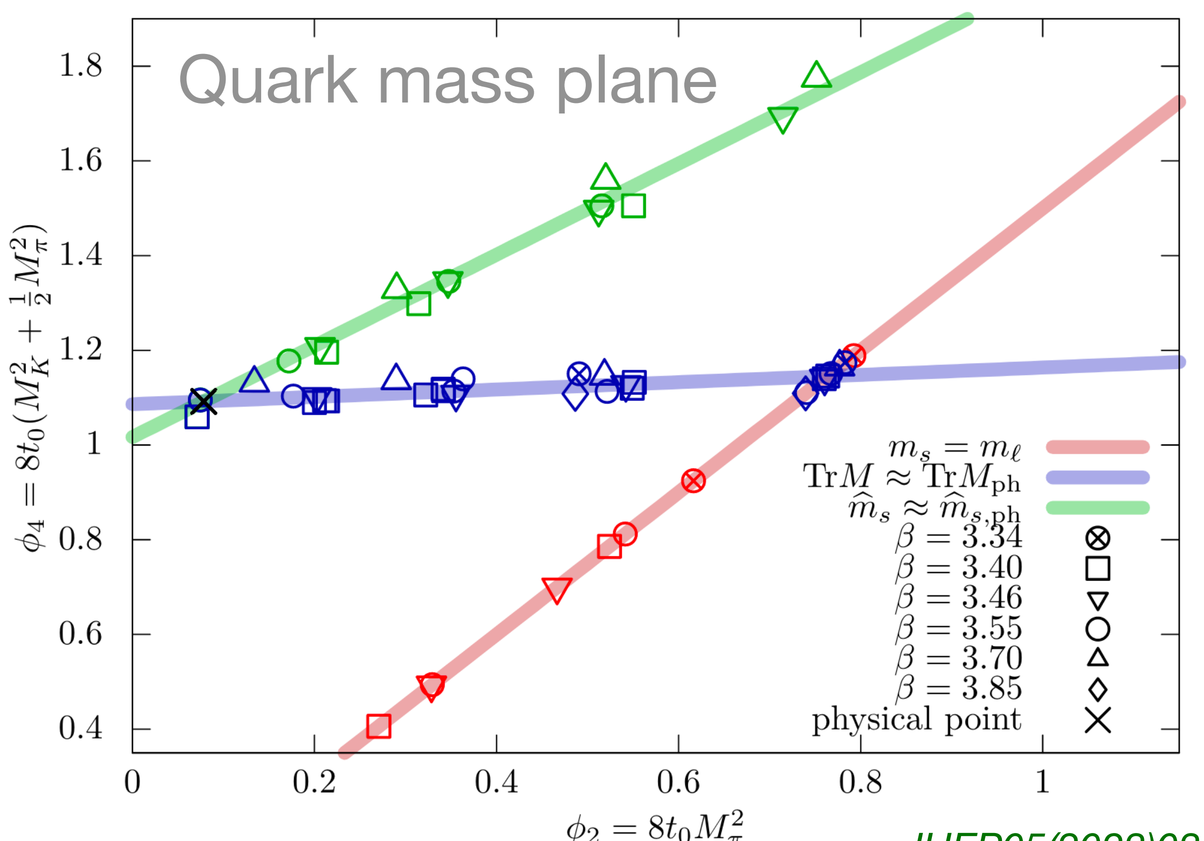




$V_{\mathrm{LO}}(M_{\pi,K,\eta}; m_{N,\Lambda,\Sigma,\Xi}; F_0 \equiv F_\pi; M_{\rho,\omega,K^*,\phi})$ $\qquad$ $G(M_{\pi,K,\eta}; m_{N,\Lambda,\Sigma,\Xi})$

  - How to obtain their quark-mass dependence?
  - Apply **ChPT formulae:** $f(M_\pi, \mathrm{LECs})$
  - Fit **LQCD data** with different quark masses

- Focus on the lattice data based on the CLS configuration



JHEP05(2023)035

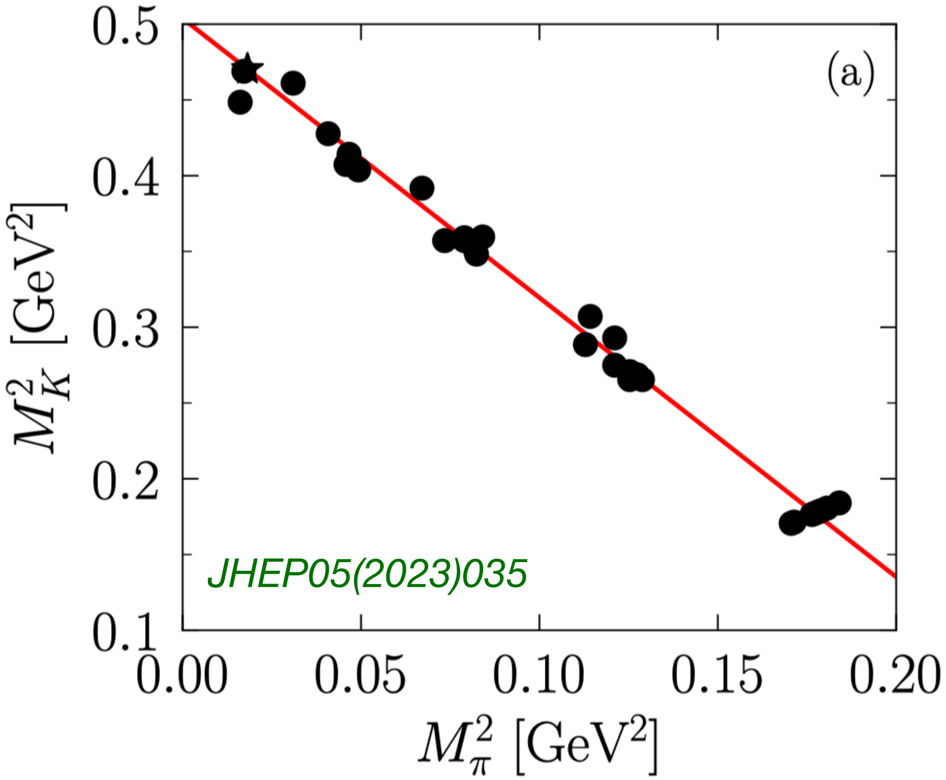
$\mathrm{Tr}(M) = \mathrm{const\ line}$

BaSc: "D200" ensemble

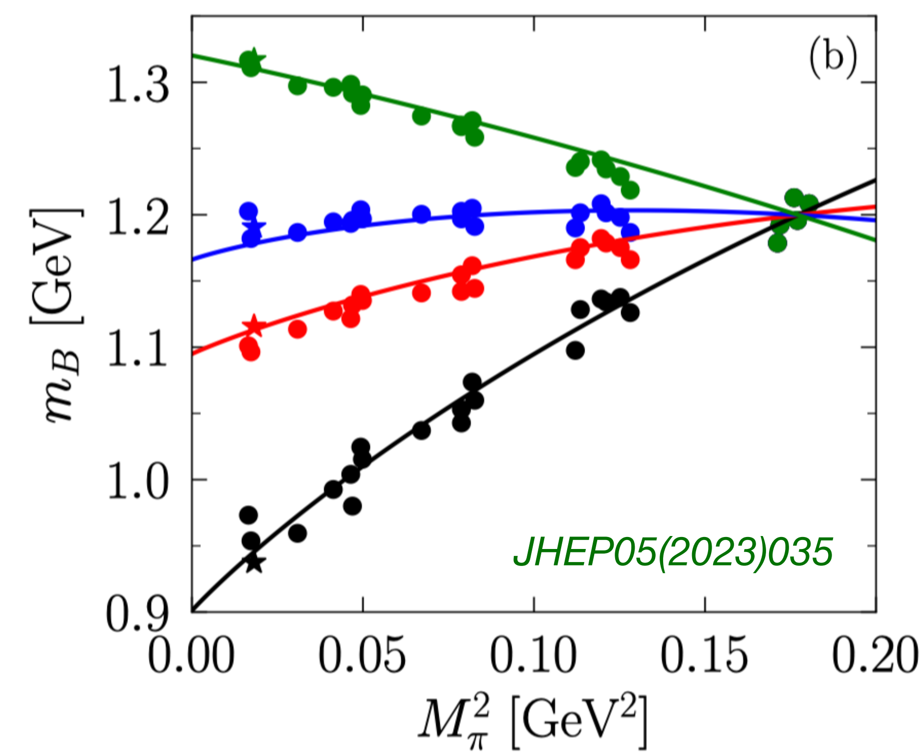
$M_{\pi,K}$	
	JHEP05(2023)035
$m_{N,\Lambda,\Sigma,\Xi}$	
F_π	JHEP 08 (2022) 220
M_ρ	PRD 106(2022)114502

Quark mass dependence of model variables

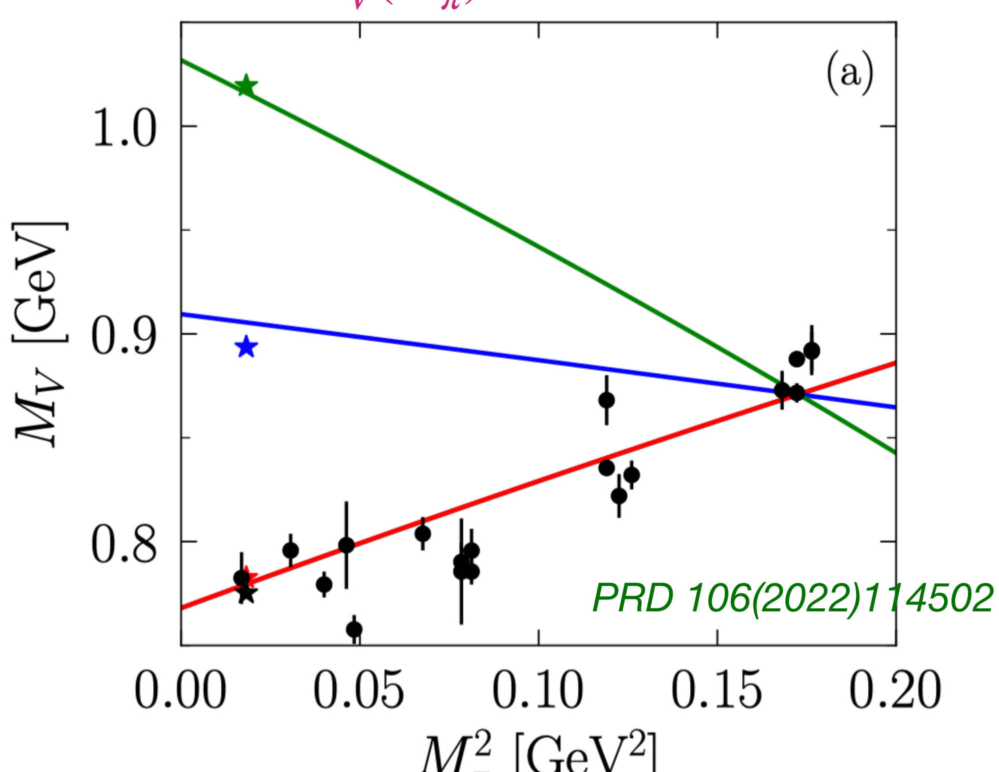
$M_K^2 = a + bM_\pi^2$ $\quad M_\eta^2 = (4M_K^2 - M_\pi^2)/3$



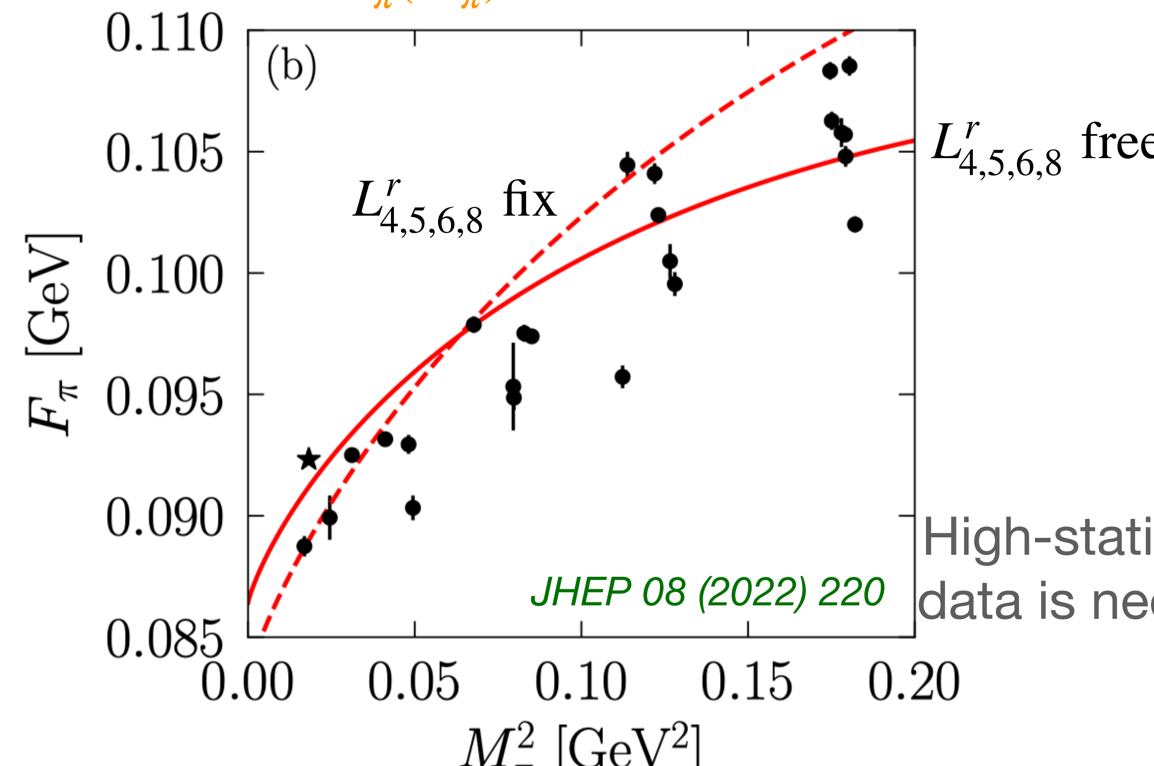
$$m_B = m_0 + \sum_{P=\pi,K} \xi^{(2)}_{PB} M_P^2 + \frac{1}{(4\pi F_0)^2} \sum_{P=\pi,K,\eta} \xi^{(3)}_{PB} H_B\left(\frac{M_P}{m_0}\right)$$



$M_V(M_\pi)$: LO ChPT



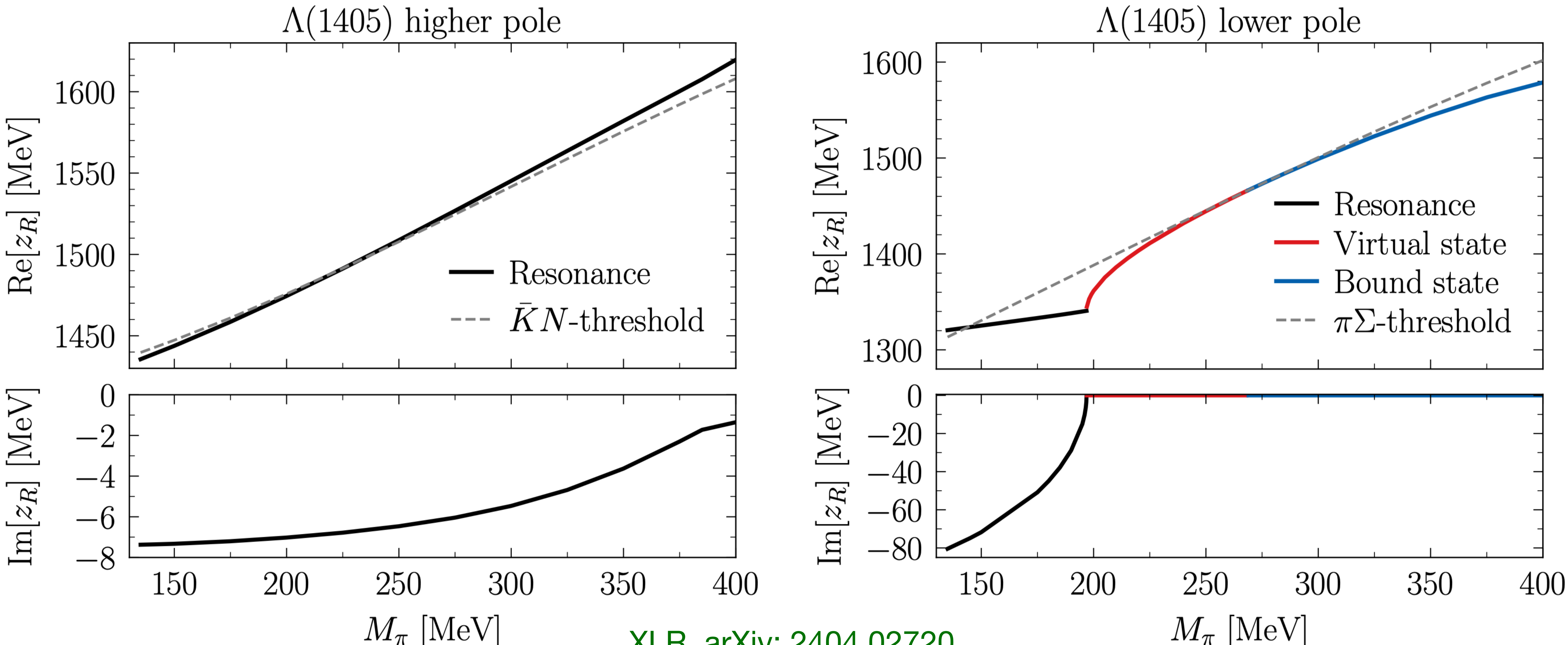
$F_\pi(M_\pi)$: NLO ChPT



High-statistics data is needed

Light-quark mass dependence of $\Lambda(1405)$

- Full calculation with $\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi$ channels



XLR, arXiv: 2404.02720

With the increase of pion mass

- $\bar{K}N$ interaction gradually strengthens
- $\pi\Sigma$ interaction changes rapidly and is enhanced sufficiently

Similar conclusion given by J.M. Xie et al., PRD 108 (2023) L111502

Next-to-leading order studies

XLR, et al., In progress

Beyond leading order

- **Maintain the scattering T-matrix renormalizable**
  - Take LO potential non-perturbatively
  - Higher order corrections are perturbatively included
- Up to NNLO
  - Potential: $V = V_{\rm LO} + V_{\rm NLO} + V_{\rm NNLO}$
  - T-matrix: $T = T_{\rm LO} + T_{\rm NLO} + T_{\rm NNLO}$

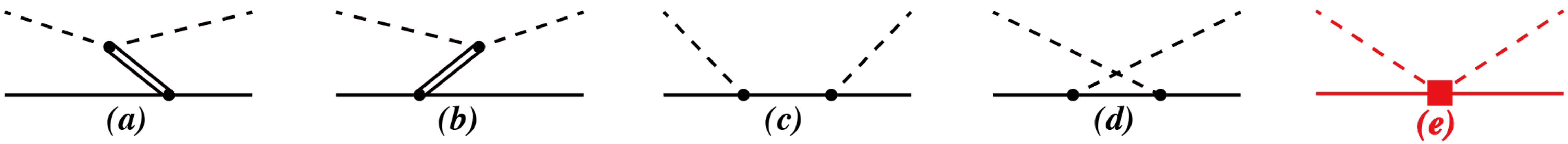
$$T_{\rm LO} = V_{\rm LO} + V_{\rm LO} G T_{\rm LO}$$ (non-perturbative)

$$T_{\rm NLO} = V_{\rm NLO} + V_{\rm LO} G T_{\rm NLO} + V_{\rm NLO} G T_{\rm LO}$$

$$T_{\rm NNLO} = V_{\rm NNLO} + V_{\rm LO} G T_{\rm NNLO} + V_{\rm NLO} G T_{\rm NLO} + V_{\rm NNLO} G T_{\rm LO}$$

  - Use **the subtractive renormalization** to remove divergent terms and power-counting breaking terms

πN scattering at NLO



- Chiral effective Lagrangian

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1 \langle \chi_+ \rangle - \frac{c_2}{4m^2} \langle u^\mu u^\nu \rangle \left(D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[u_\mu, u_\nu \right] \right\} \Psi_N$$

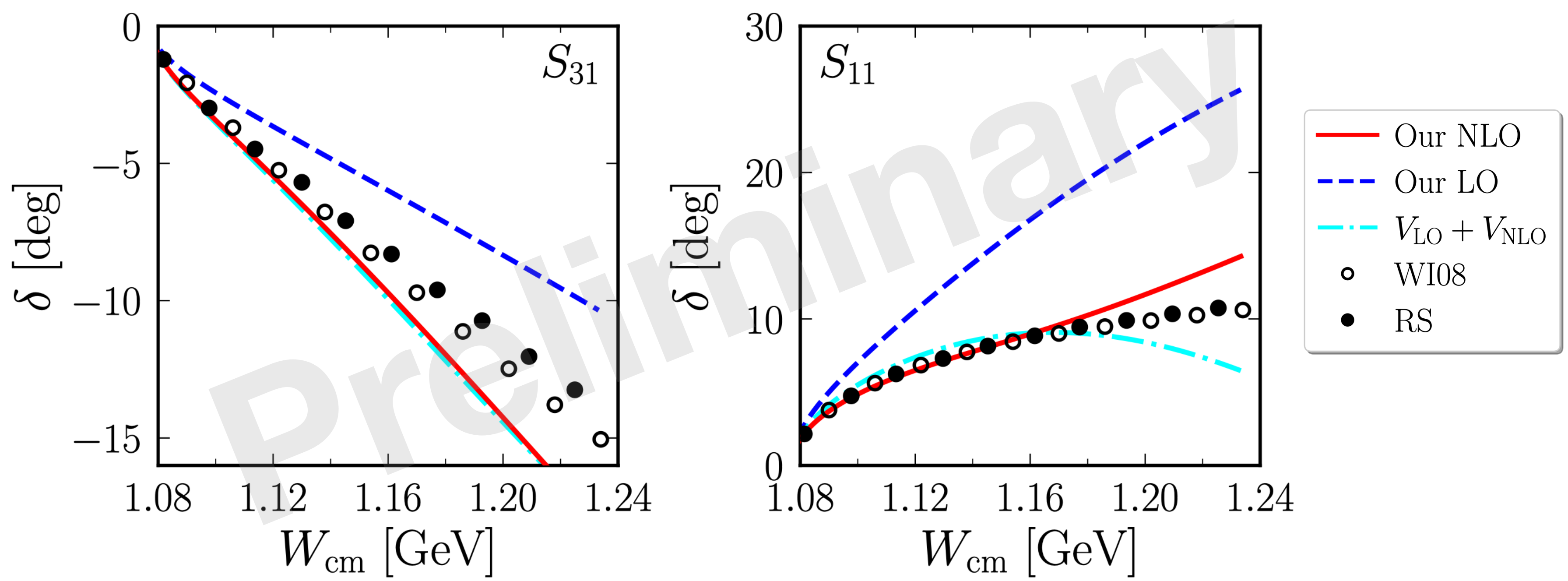
  - Fix $c_1 = -0.74,\ c_2 = 1.81,\ c_3 = -3.61,\ c_4 = 2.17\ \text{GeV}^{-1}$

D. Siemens, et al., PLB770 (2017) 27-34

- NLO potential

$$V = V_{\text{LO}} + V_{\text{NLO}}$$
$$= V^{(a+b+c+d)}|_{u=u_0\sim(1,0)^\dagger} + V^{(a+b+c+d)}|_{u=u_1\sim\mathcal{O}(p)} + V^{(e)}|_{u=u_0\sim(1,0)^\dagger}$$

- Prediction for the πN phase shifts



Summary

- **A renormalized framework** for MB scattering is proposed
  - Time-ordered perturbation theory + Covariant chiral Lagrangians
  - Take into account the off-shell effects of potential
  - Employ the subtractive renormalization
    - ✓ **achieve T-matrix is cutoff-independent**
- Leading order study
  - πN scattering; $\bar{K}N$ scattering with coupled channels
  - Obtain the **two-pole structure** of $\Lambda(1405)$
  - Investigate the light-quark mass dependence of $\Lambda(1405)$
- Next-leading order study
  - NLO correction is perturbatively included
  - πN scattering: improve the description of phase shifts

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HAL QCD

- Next-leading order study
  - NLO correction is perturbatively included
  - πN scattering: improve the description of phase shifts
  - Plan: extend to $\bar{K}N$ scattering, $\Lambda(1405)$, and other resonances

Thank you for your attention!

Back up

Finite cutoff artifact

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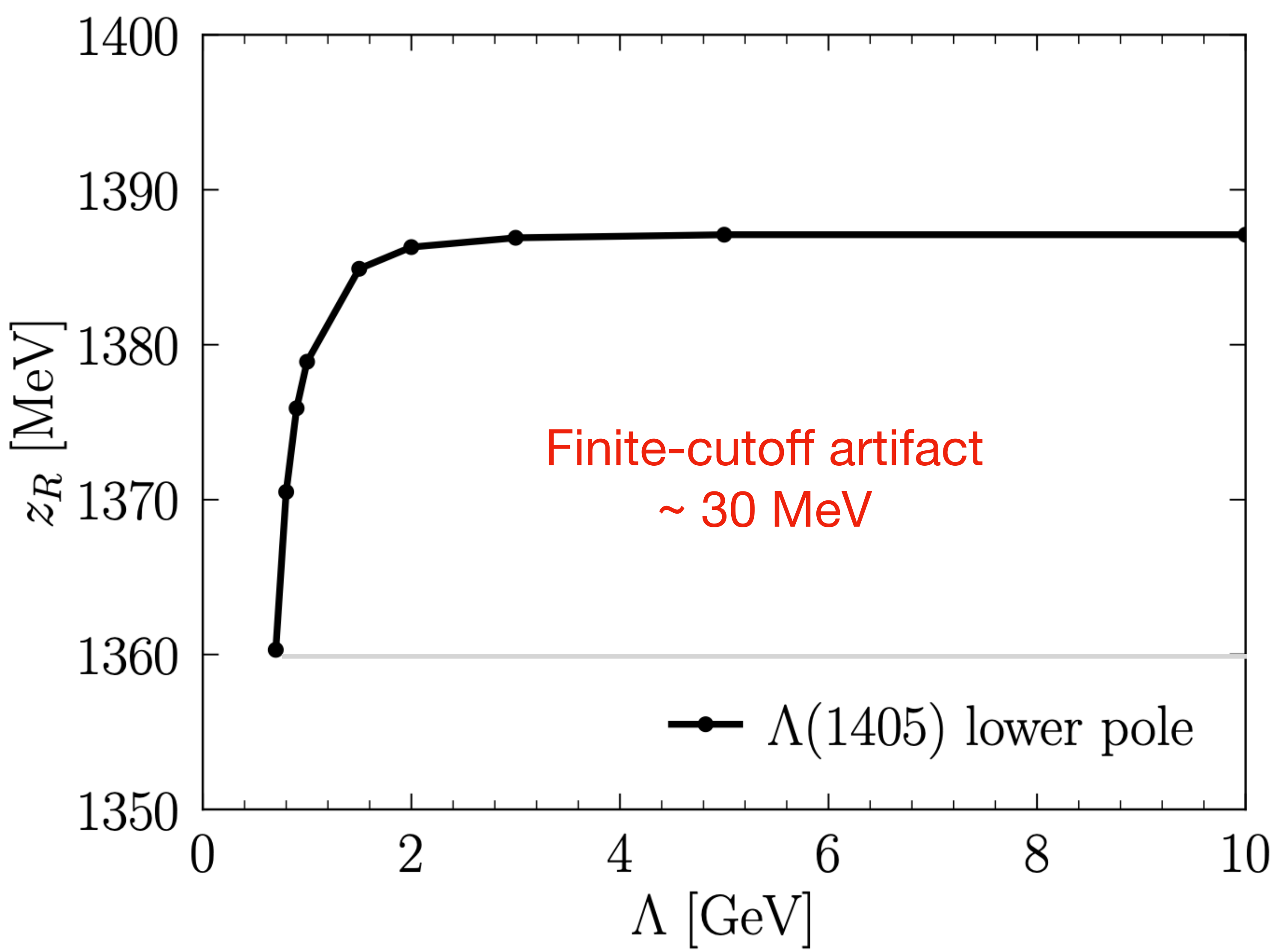

- $\Lambda \geq 3$ GeV, cutoff-independent results achieved!

➡ Renormalized $\pi\Sigma - \bar{K}N$ amplitudes

➡ Fix $\Lambda = 10$ GeV