

Theoretical investigation of the reaction mechanisms for the $\bar{K}N$ and $\bar{K}NN$ systems with K^- beam at J-PARC

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- [1] T. S. , E. Oset, and A. Ramos, PTEP 2016 123D03.
 - [2] T. S. , E. Oset, and A. Ramos, JPS Conf. Proc. 26 (2019) 023009.
 - [3] J. Yamagata-Sekihara, T. S. , and D. Jido, under discussion.
 - [4] T. S. , E. Oset, and A. Ramos, under discussion.

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1. Introduction
2. The $\bar{K}N$ system in the $K^- d \rightarrow \pi \Sigma n$ reaction: J-PARC E31
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4. Summary

1. Introduction

1. Introduction

++ Strange hadrons as a precision tool ++

HIGGSTAN.

- **Hadrons = Particles composed of quarks, interact via strong interactions.**







- Underlying theory for strong interactions: **Quantum Chromodynamics (QCD).**

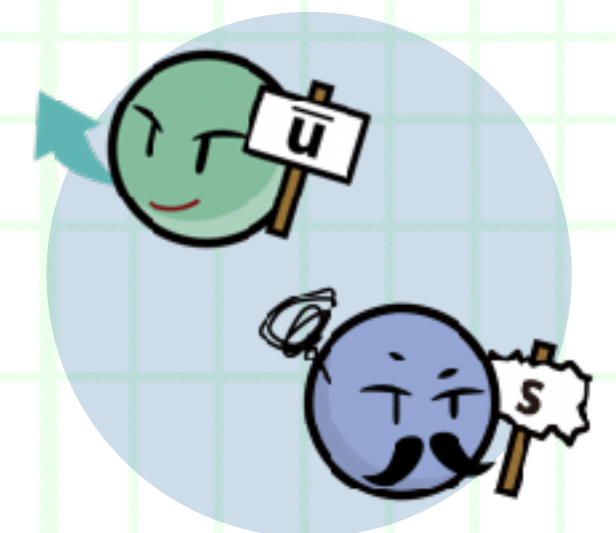
$$\mathcal{L}_{\text{QCD}} = \sum_{f=\text{flavor}} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

- We want to understand strong interactions from QCD.

- Recently, much attention has paid to the hadronic systems **with strange quarks = non-zero strangeness.**

- **The experiments and numerical simulations are now available !**
- A different perspective on hadron physics.

クォーク Quarks	電荷 Charge	世代 Generation		
	スピン Spin	I	II	III
	+2/3	 up	 charm	 top
	1/2			
	-1/3	 down	 strange	 bottom
	1/2			



K- meson.

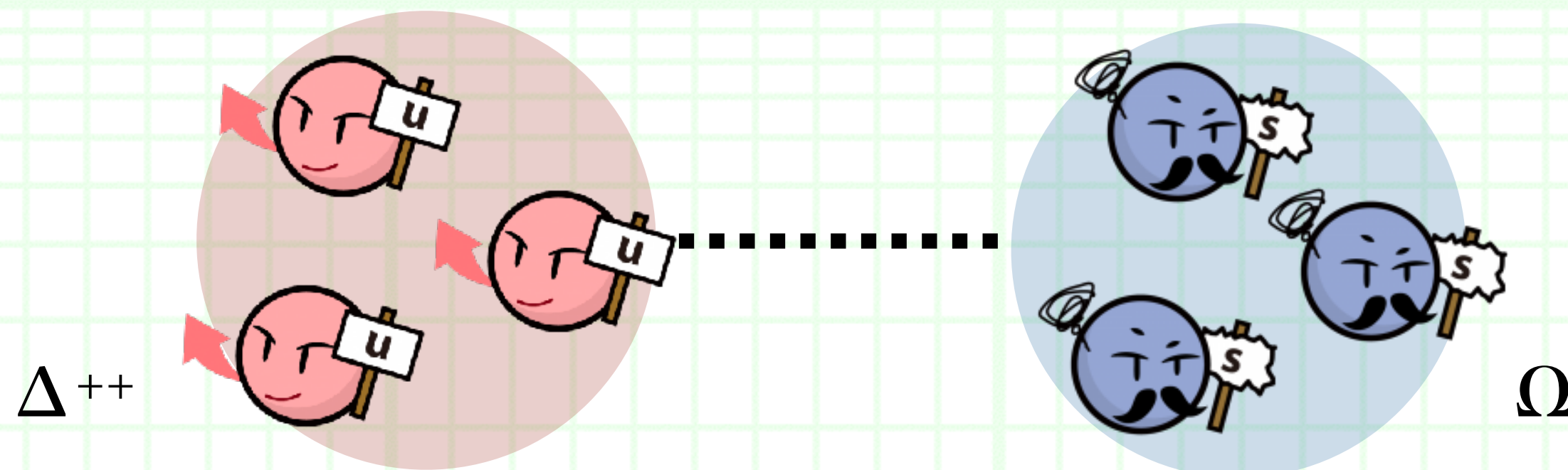
1. Introduction

++ An example: Baryon-baryon interaction ++ **Attractive**



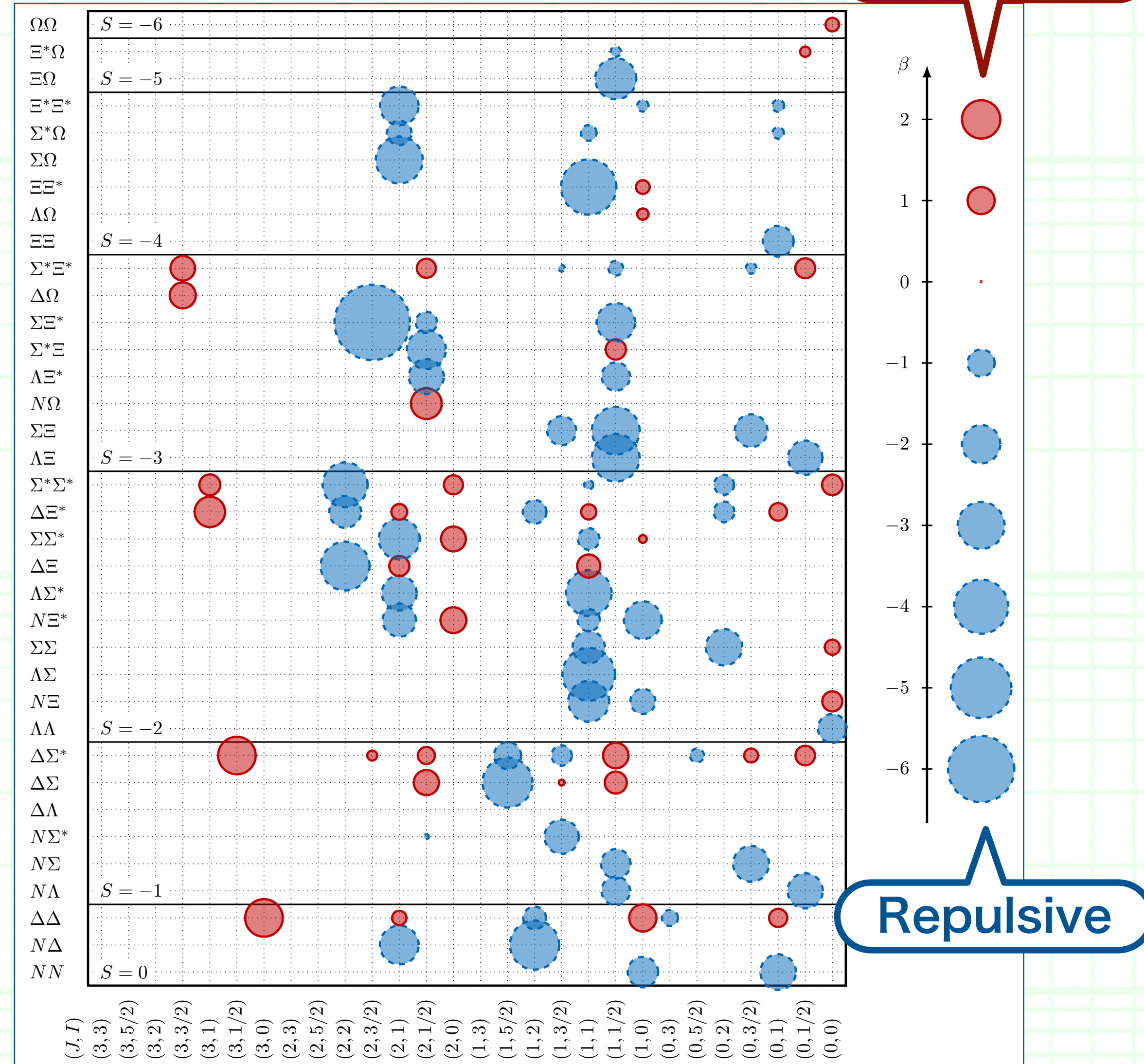
■ An example: Baryon-baryon interaction.

- Do the properties of the nuclear force persist even in the baryon-baryon interactions with strangeness ?



- Baryon-baryon Int. in a quark model.

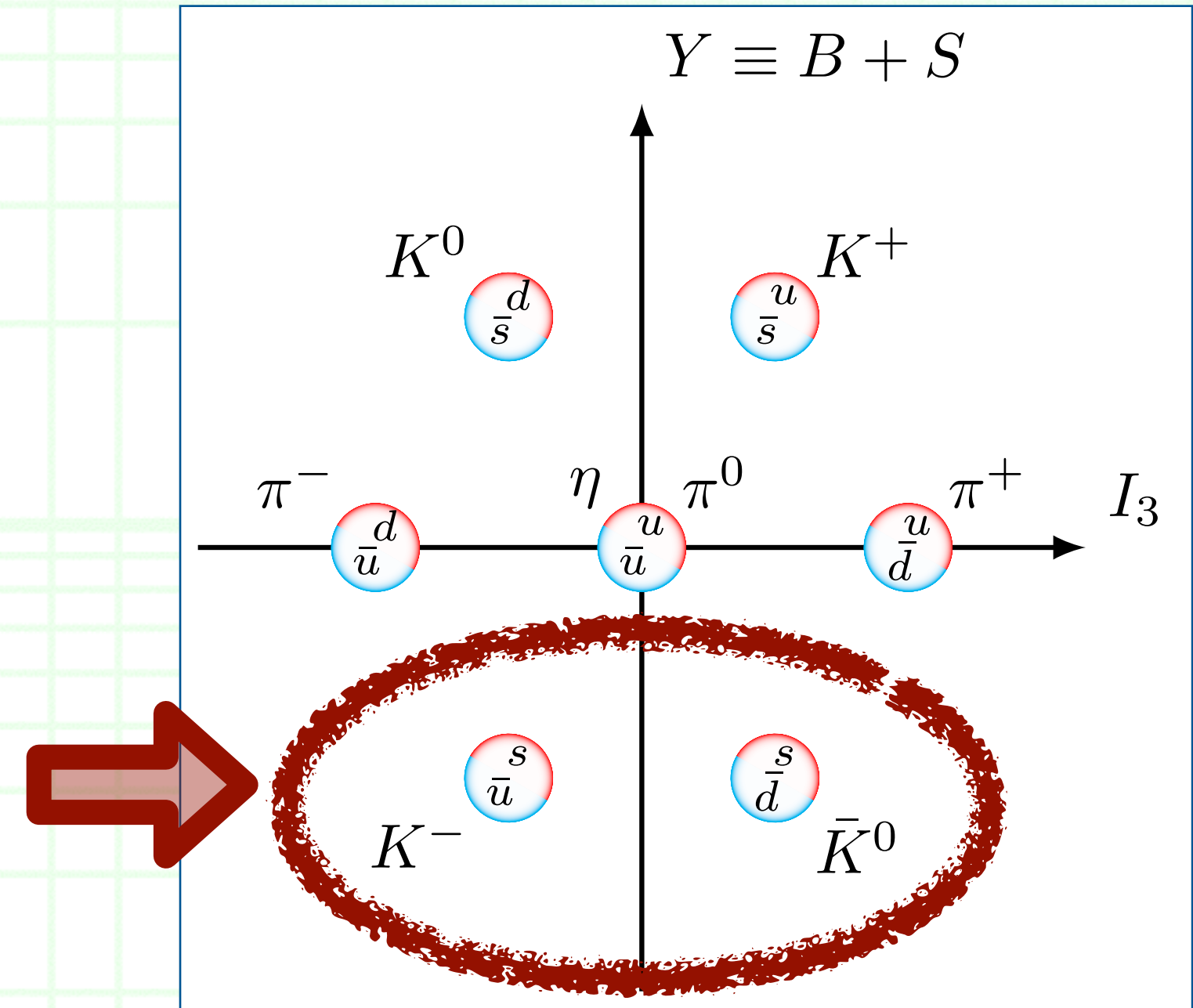
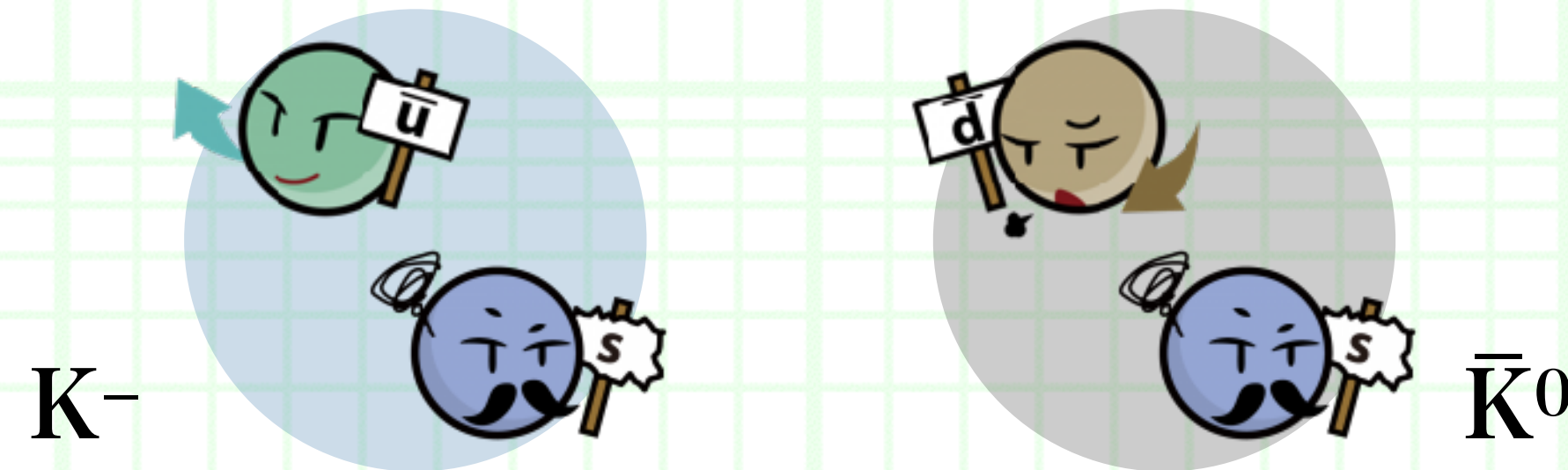
T. S. and T. Hashiguchi, Phys. Rev. C108 (2023) 065202.



1. Introduction

++ Anti-kaon \bar{K} ++

- We focus on the anti-kaon $\bar{K} = (K^-, \bar{K}^0)$.



Octet mesons

- \bar{K} is **lighter** as a Nambu-Goldstone boson.
→ The dynamics is constrained by **the spontaneous breaking of chiral symmetry.**

$$\mathcal{L}_{\text{QCD}} \rightarrow \mathcal{L}_{\text{ChPT}} = \sum_n \mathcal{L}_n$$

– Chiral perturbation theory (ChPT).

- \bar{K} is **heavier** than the pion $\pi = (\pi^+, \pi^0, \pi^-)$.
→ Easier to make **a bound state**, when the interaction is attractive enough.

1. Introduction

++ The $\bar{K}N$ and $\bar{K}NN$ systems ++

■ Indeed, the $\bar{K}N$ interaction in chiral dynamics is **attractive enough to generate a bound state as $\Lambda(1405)$.**

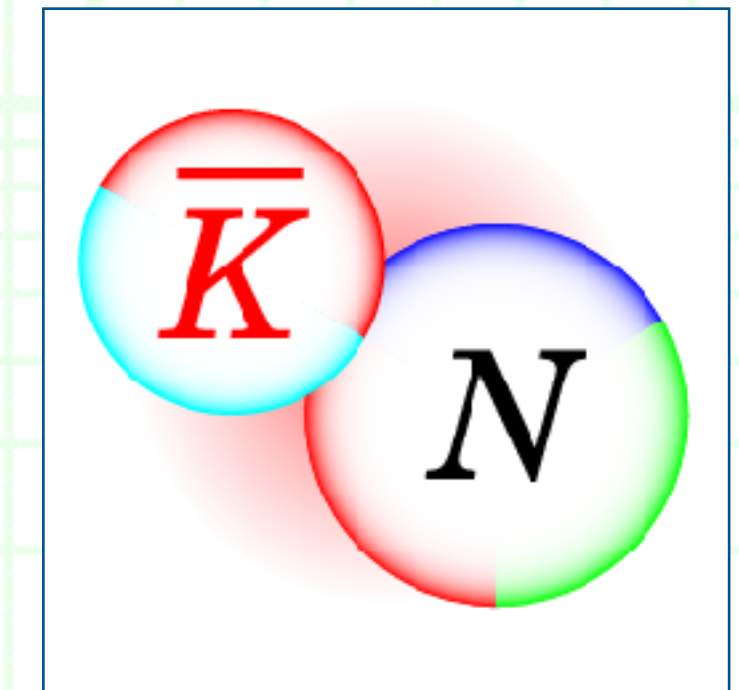
□ Theoretical studies support the $\bar{K}N$ molecular nature of the $\Lambda(1405)$ resonance.

• **Compositeness** = the norm of the two-body wave function.

T. S. , Hyodo and Jido, PTEP [2015](#) 063D04; Kamiya and Hyodo, PTEP [2017](#) 023D02; ...

• **Dominant $\bar{K}N$ component in lattice QCD simulations.**

Hall et al., Phys. Rev. Lett. [114](#) (2015) 132002.



Bound state as $\Lambda(1405)$!

■ We can extend the discussion from the $\bar{K}N$ to the $\bar{K}NN$ bound state.

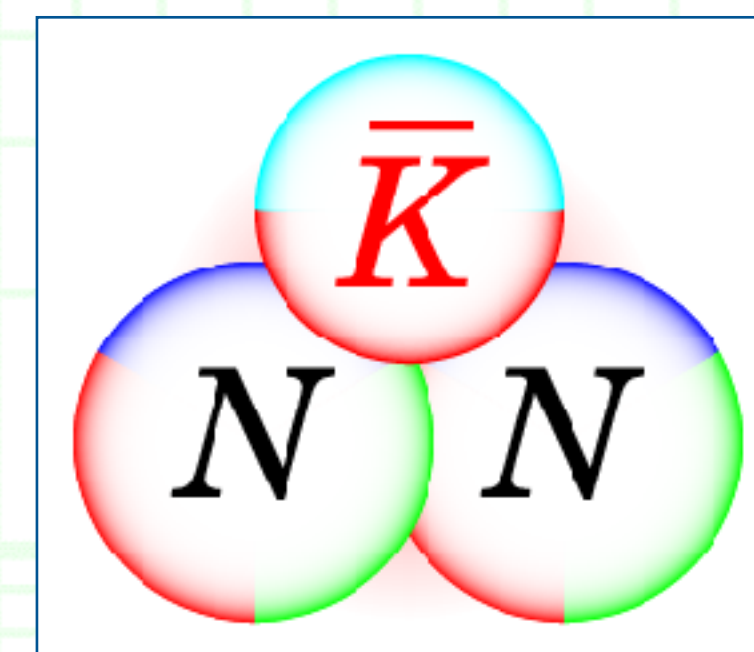
— The simplest kaonic nucleus.

□ Theoretically, **the attractive $\bar{K}N$ interaction indicates that the $\bar{K}NN$ system is bound.**

Expect to be bound !

$\bar{K}N$ Int.: attractive

NN Int.: attractive

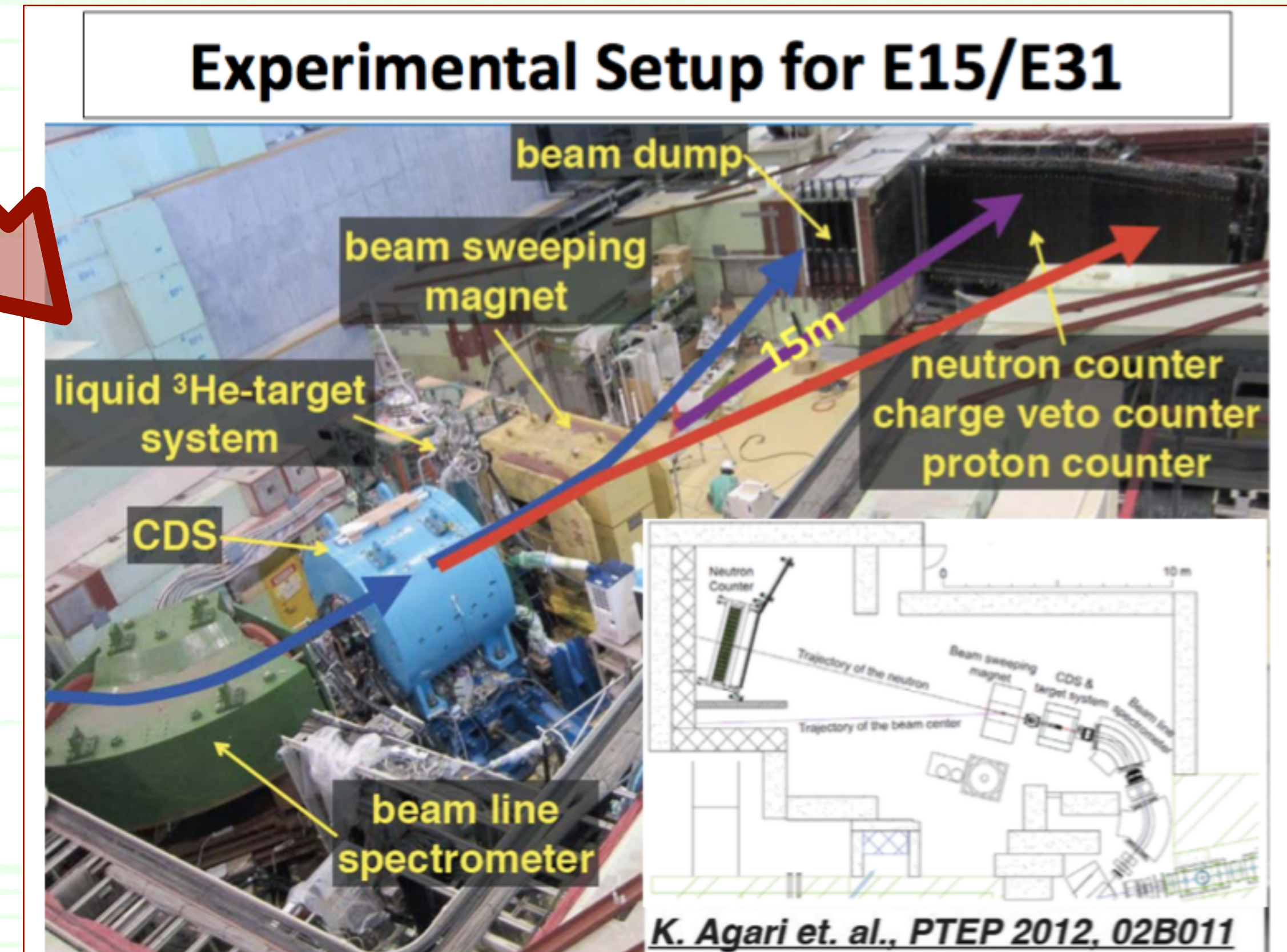
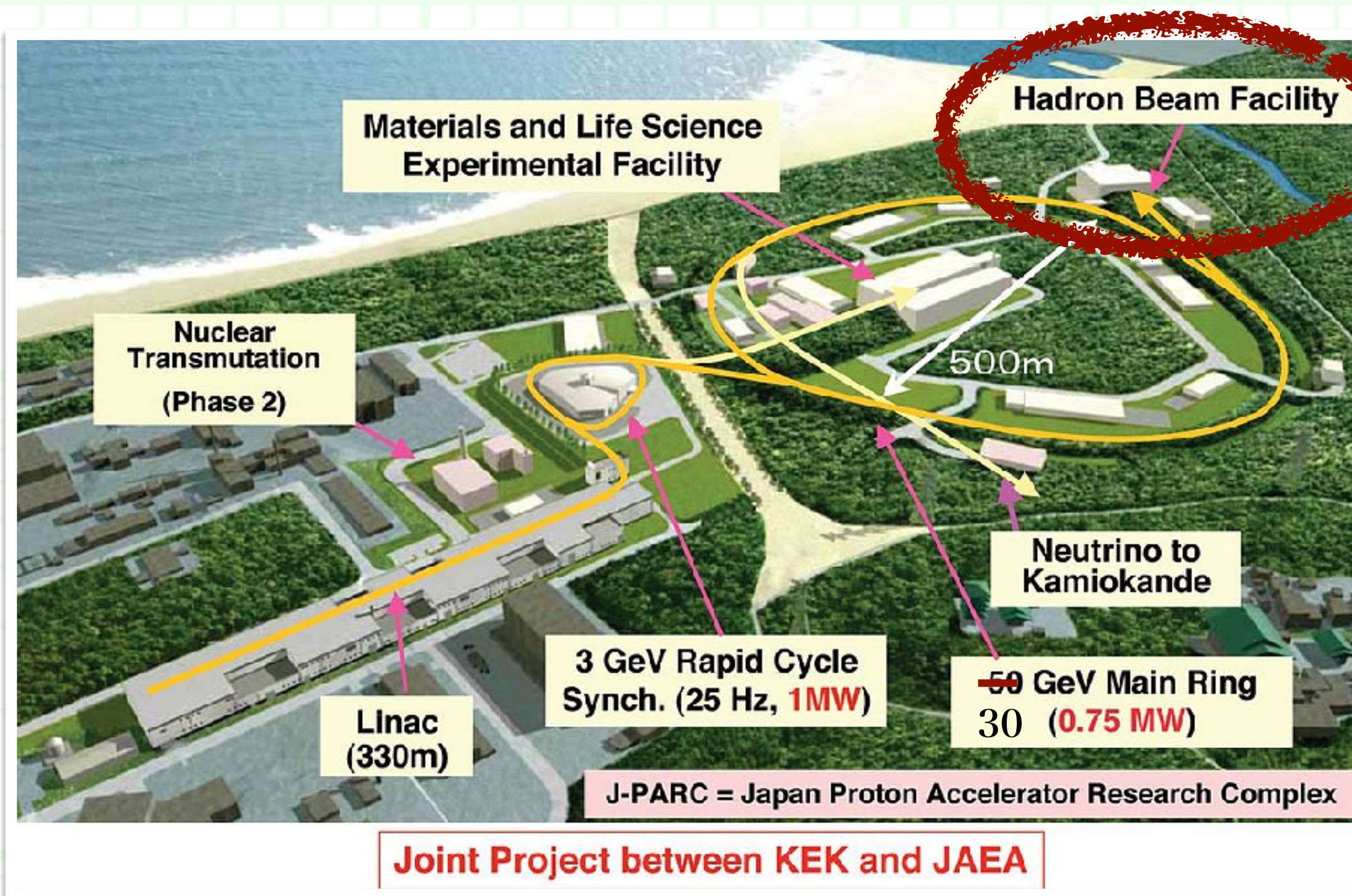


1. Introduction

++ \bar{K} experiments at J-PARC ++

- Such $\bar{K}N$ and $\bar{K}NN$ systems can be studied at J-PARC.
- Today **the world's highest-intensity kaon beam** is available at J-PARC !

Sakuma-san, talk at ECT* (2017).

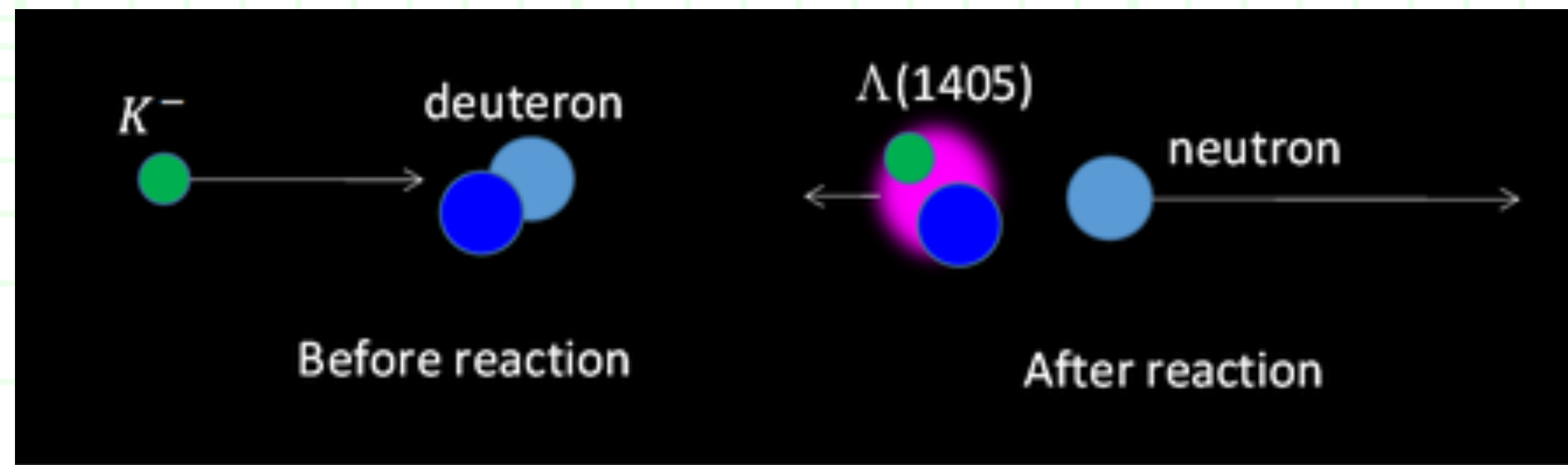


1. Introduction

++ J-PARC E31 Exp. for the $\bar{K}N$ system ++

- The J-PARC E31 Exp. was performed to produce the $\Lambda(1405)$ resonance from the $\bar{K}N$ channel in the $K^- d \rightarrow \pi \Sigma n$ reaction.

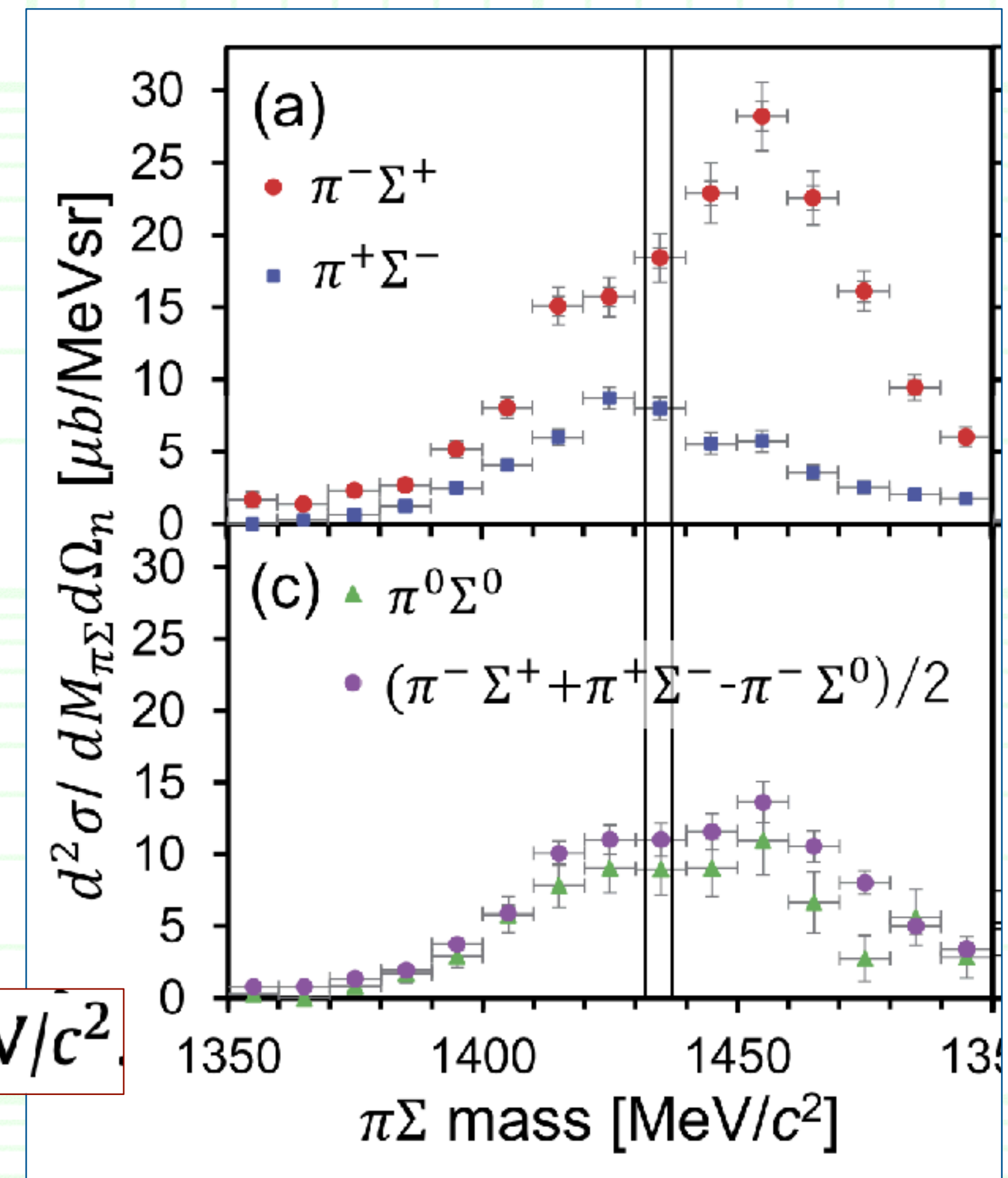
J-PARC
press release.



- If the $\Lambda(1405)$ is indeed a $\bar{K}N$ bound state, **peak position** and **pole position** would provide good information on **the binding energy**.
- J-PARC E31 Exp. analysis suggests:

$$1417.7_{-7.4}^{+6.0}(\text{fit})_{-1.0}^{+1.1}(\text{syst.}) + [-26.1_{-7.9}^{+6.0}(\text{fit})_{-2.0}^{+1.7}(\text{syst.})]i \text{ MeV}/c^2$$

J-PARC E31 Collab., Phys. Lett. **B837** (2023) 137637.

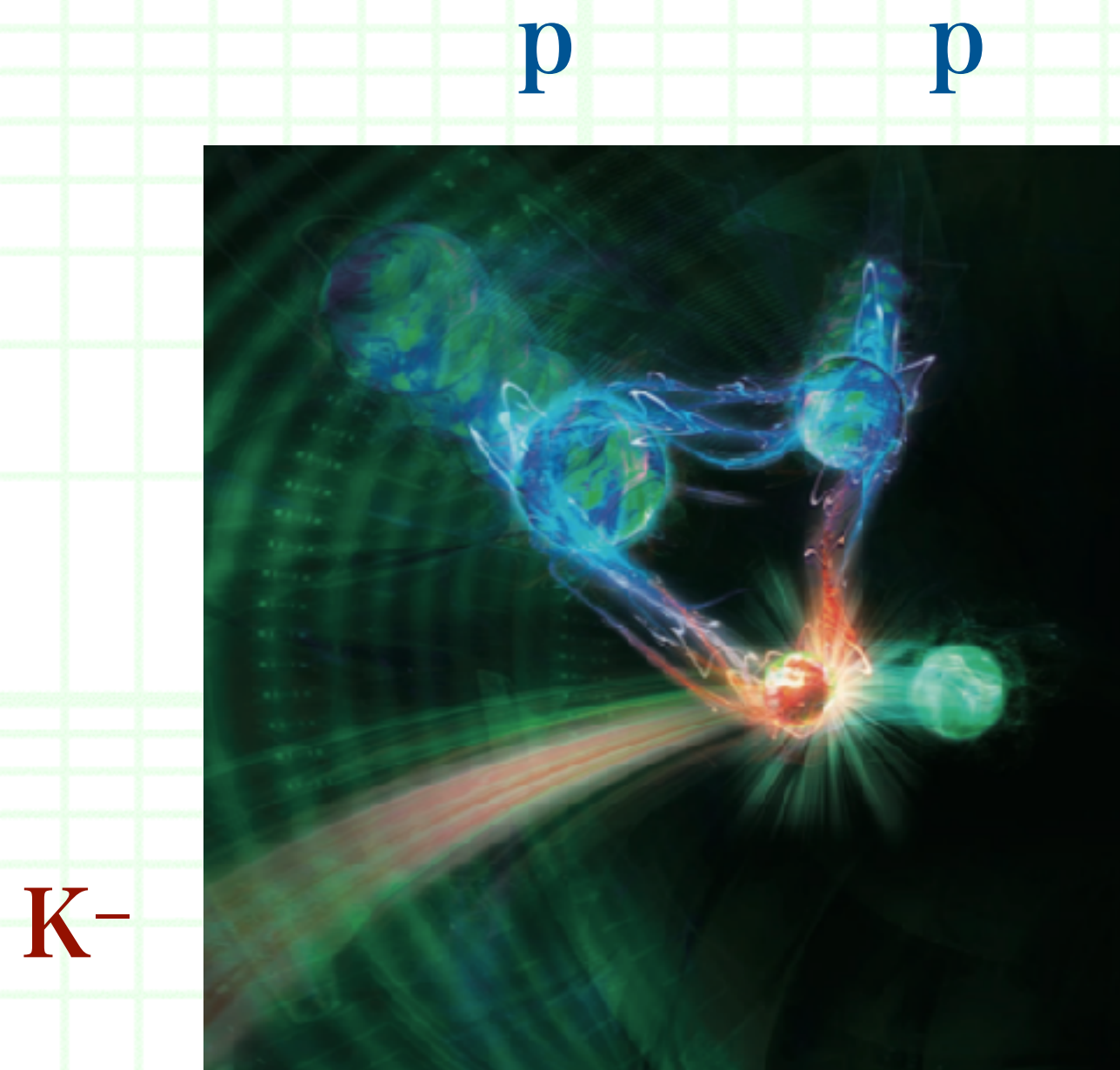


1. Introduction

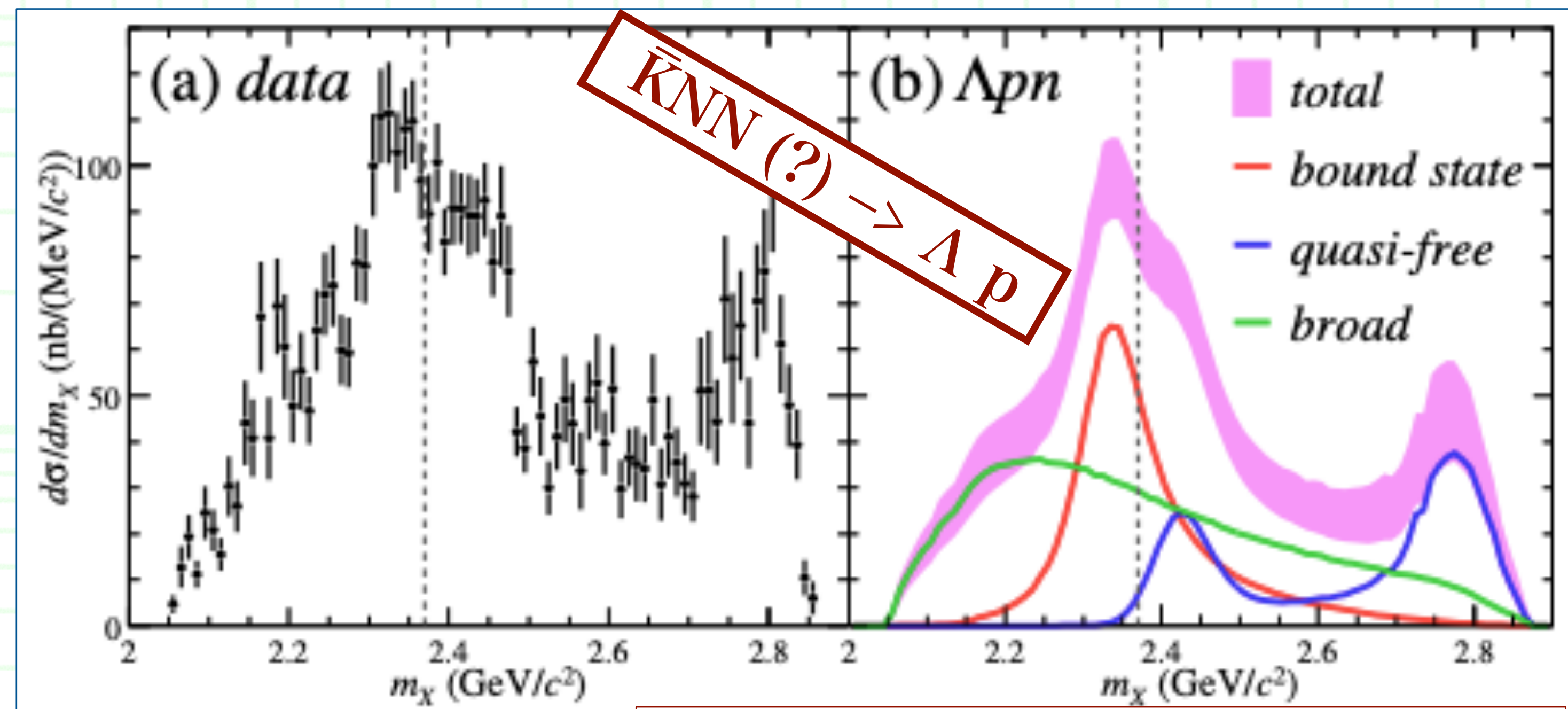
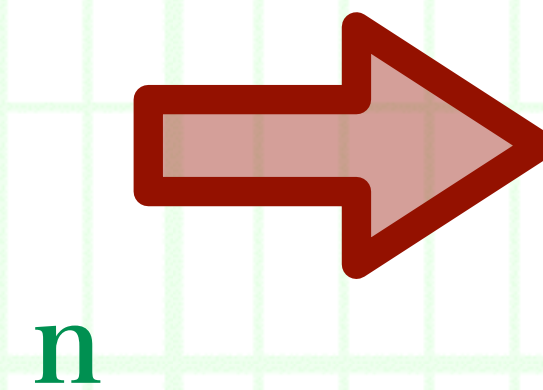
++ J-PARC E15 Exp. for the $\bar{K}NN$ system ++

- The J-PARC E15 Exp. was performed to search for the $\bar{K}NN$ bound state in the $K^- ^3\text{He} \rightarrow \Delta pn$ reaction.

Yamaga et al. [J-PARC E15], Phys. Rev. C102 (2020) 044002.



The Austrian Academy of Sciences, Harald Ritsch.



$$B_K = 42 \pm 3(\text{stat.})_{-4}^{+3}(\text{syst.}) \text{ MeV}$$

$$\Gamma_K = 100 \pm 7(\text{stat.})_{-9}^{+19}(\text{syst.}) \text{ MeV}$$

□ Is this really the signal of the $\bar{K}NN$ bound state ?

1. Introduction

++ Motivation ++

■ Are these peaks really the signals of the $\bar{K}N$ and $\bar{K}NN$ systems ?

– Does the \bar{K} **survive** the reaction ?

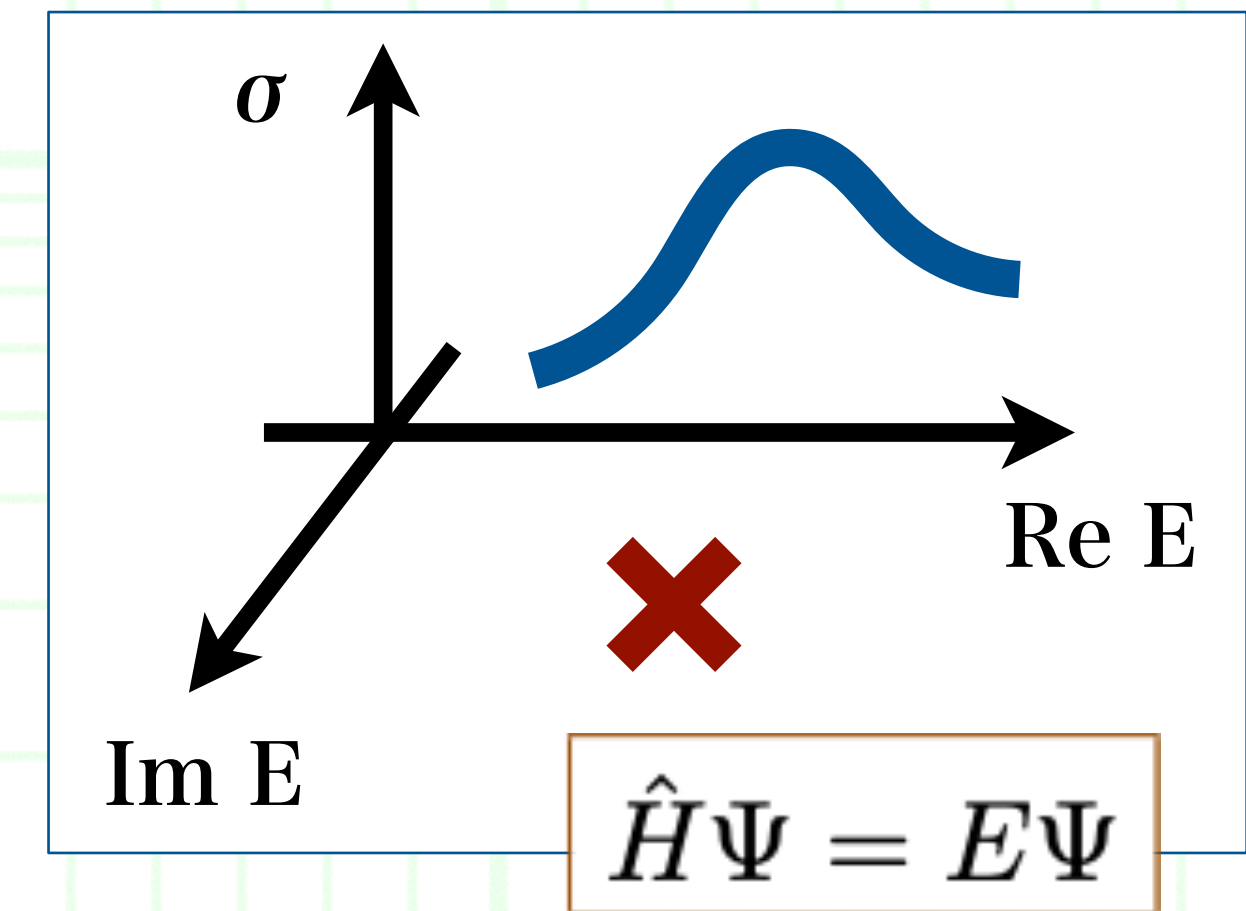
■ Resonance poles vs. Exp. peaks.

Resonance poles correspond to **the eigenstates** of the Schrödinger Eq.

Exp. peak position, which is **on the real energy axis**, may differ from the real part of the pole position due to **interference between the pole and background**.

■ We aim to **connect the resonance pole and Exp. observables** by reaction calculations.

– Determine **the analytic form of the scattering Amp.** on the complex energy plane.



2. The $\bar{K}N$ system in the $K^- d \rightarrow \pi \Sigma n$ reaction: J-PARC E31

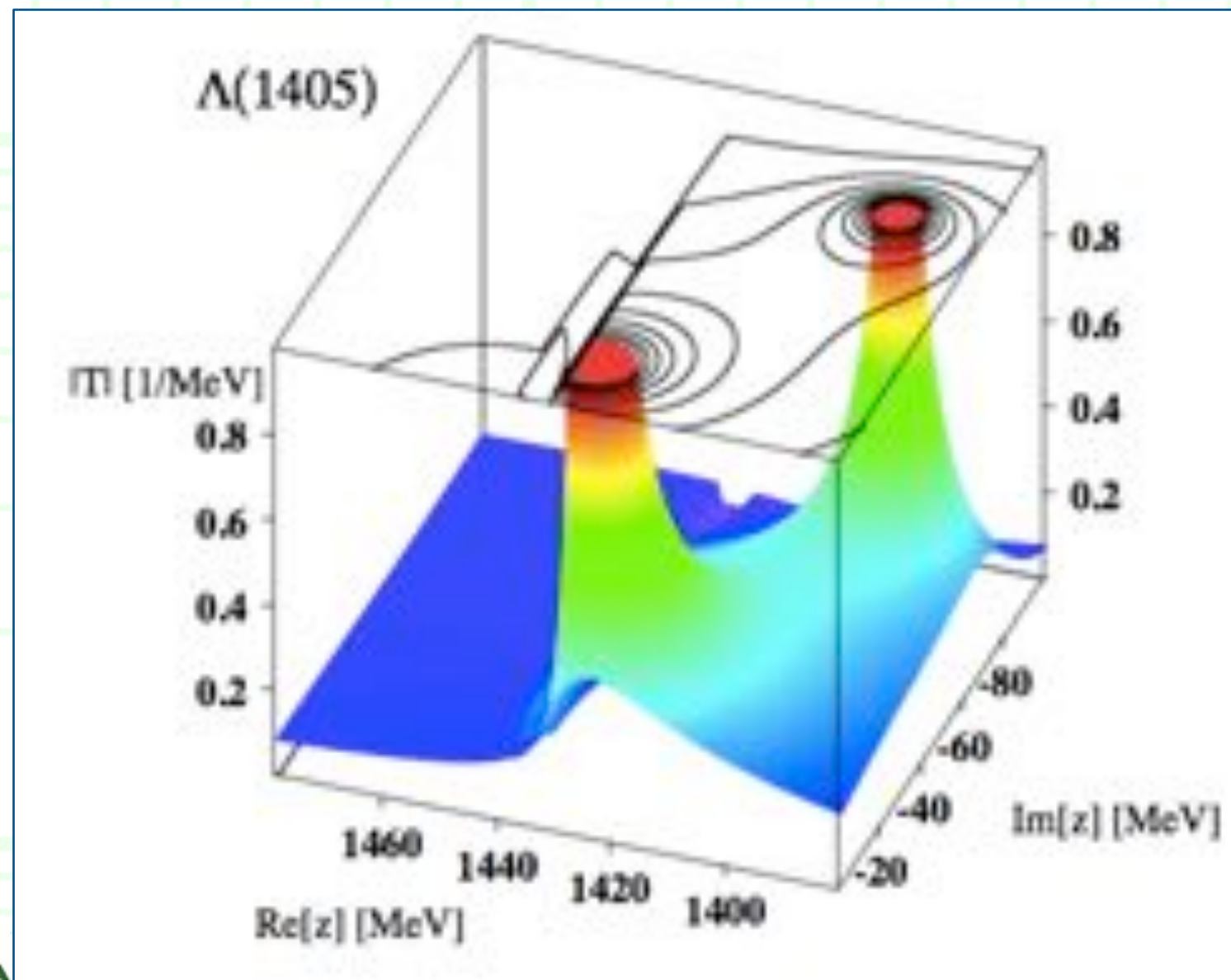
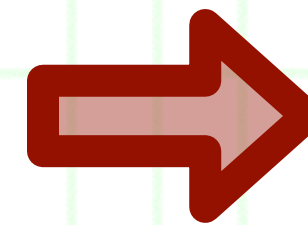
2. The $\bar{K}N$ system in $K^- d \rightarrow \pi \Sigma n$

++ Resonance pole(s) of the $\Lambda(1405)$ ++

PDG Live.

■ The pole structure of the $\Lambda(1405)$ resonance has been a hot topic in the $\bar{K}N$ physics.

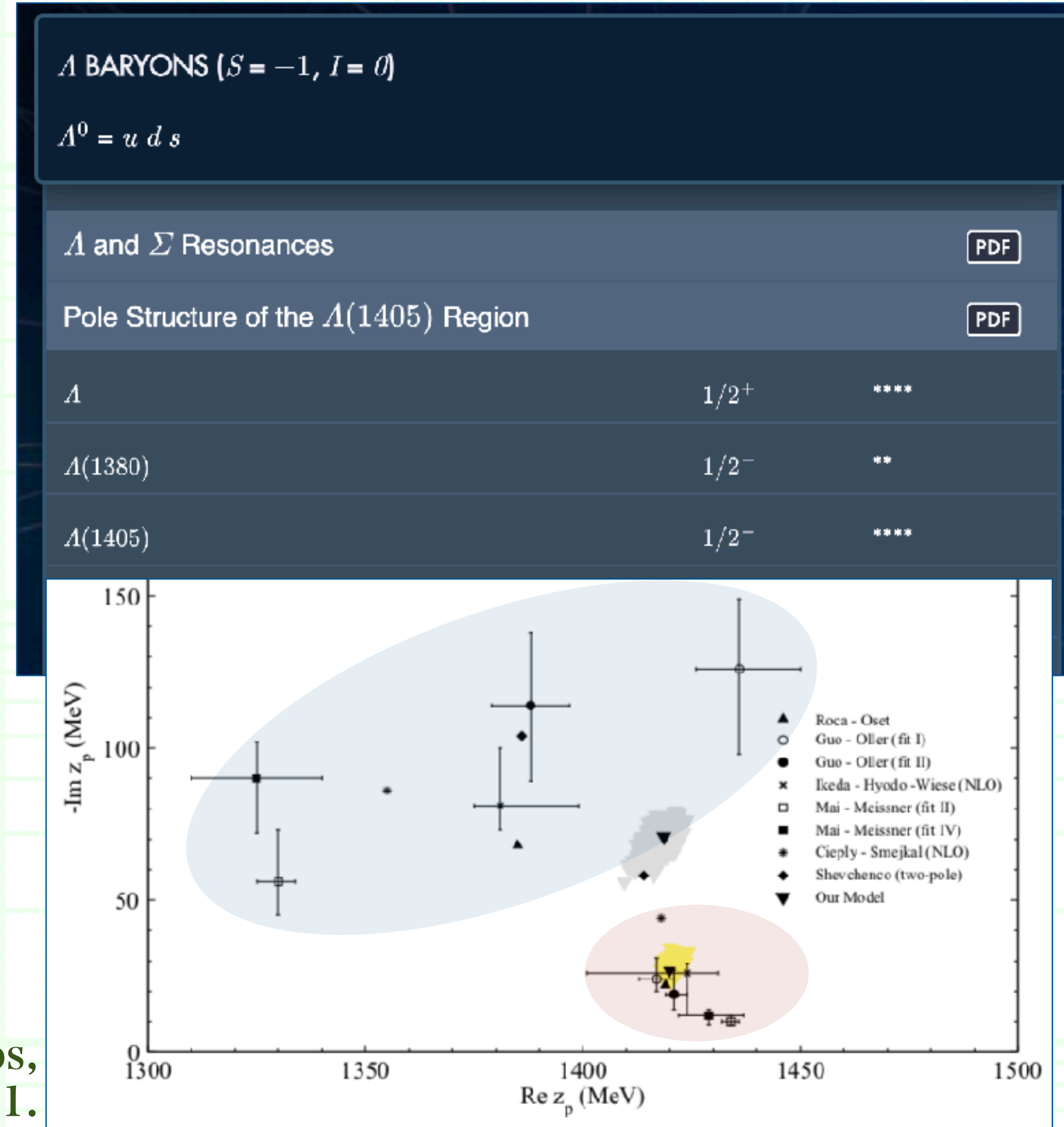
- Two poles ?
- The higher pole strongly couples to $\bar{K}N$?
- Where is the lower pole ?



Hyodo and Jido,
Prog. Part. Nucl. Phys.
67 (2012) 55.

**Resonance pole
in the scattering Amp.**

Feijoo, Magas and Ramos,
Phys. Rev. C99 (2019) 035211.

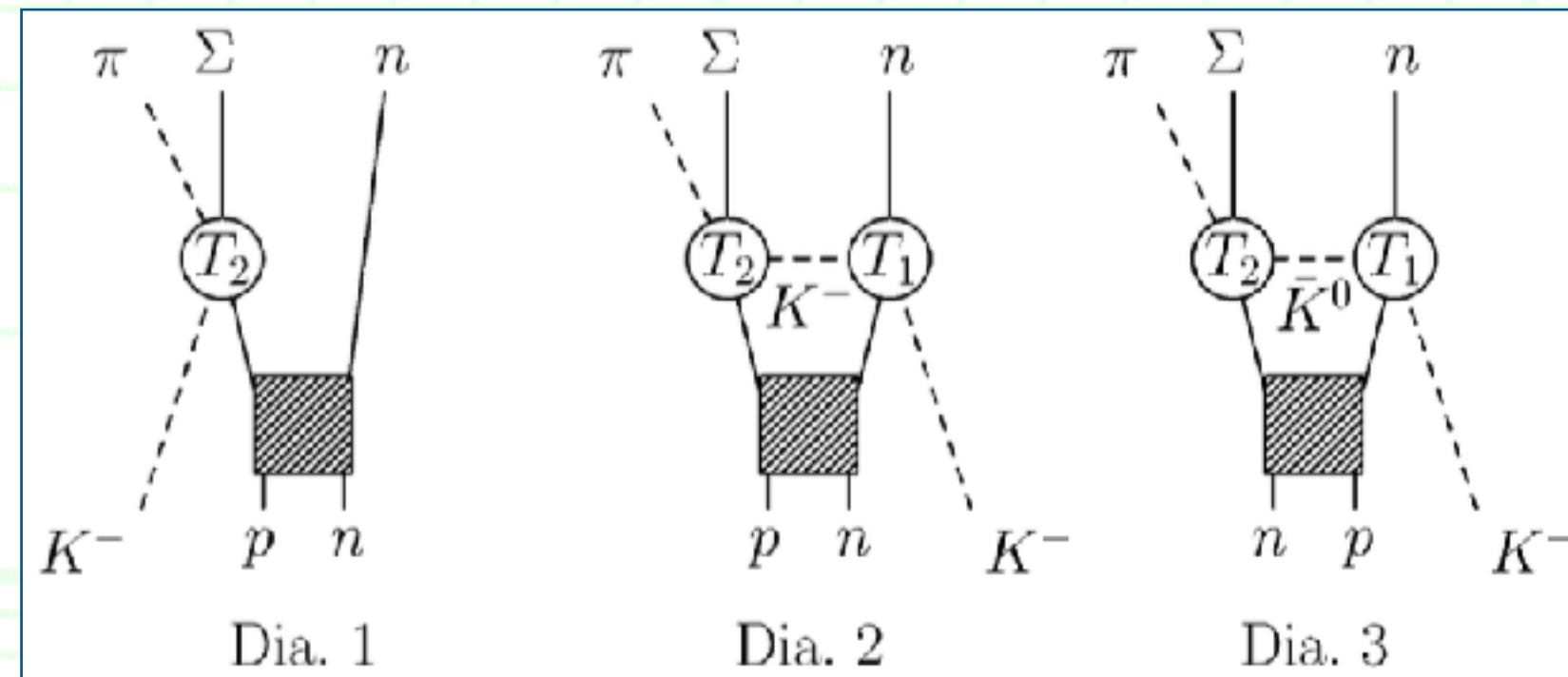
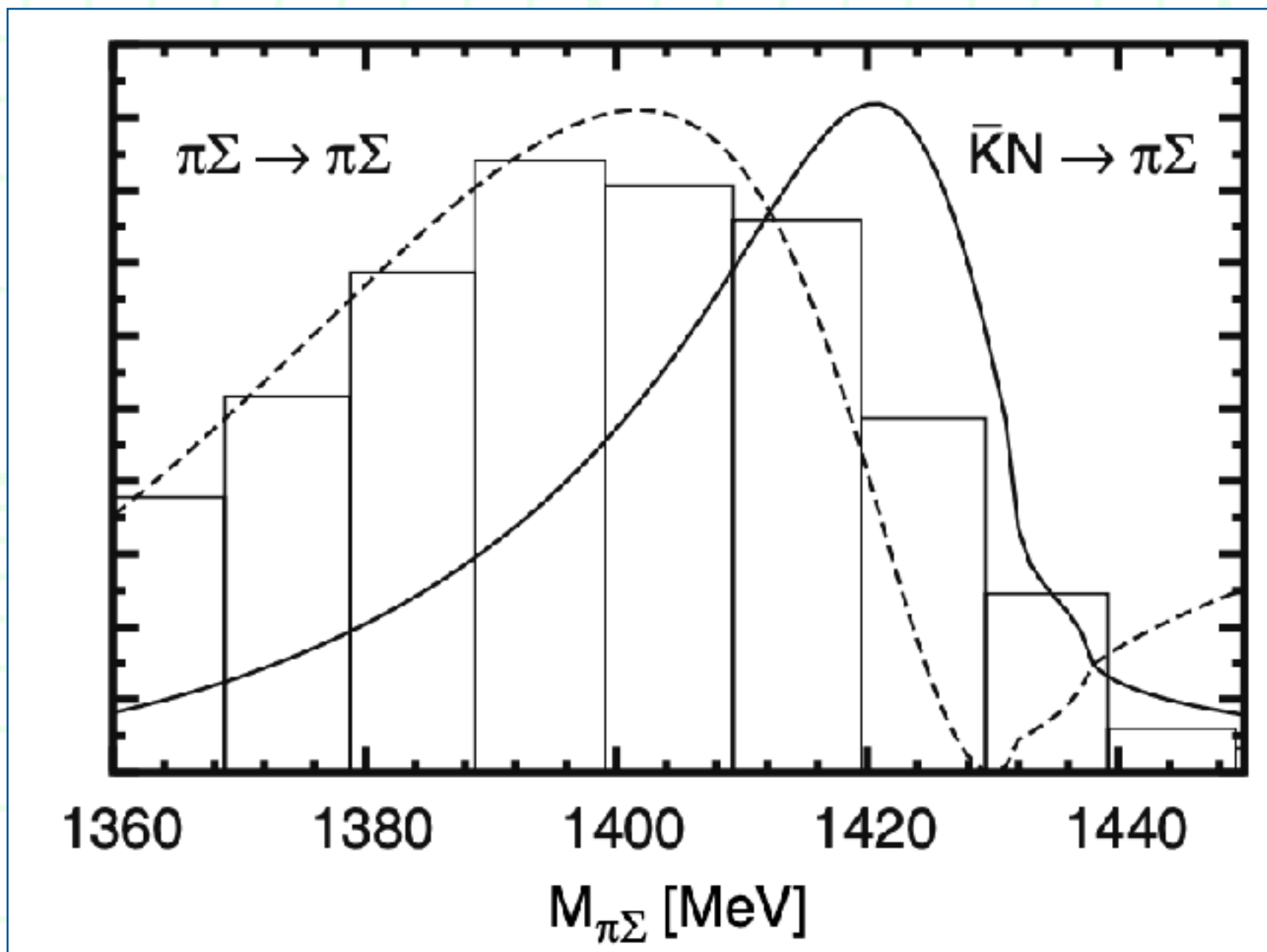


2. The $\bar{K}N$ system in $K^- d \rightarrow \pi \Sigma n$

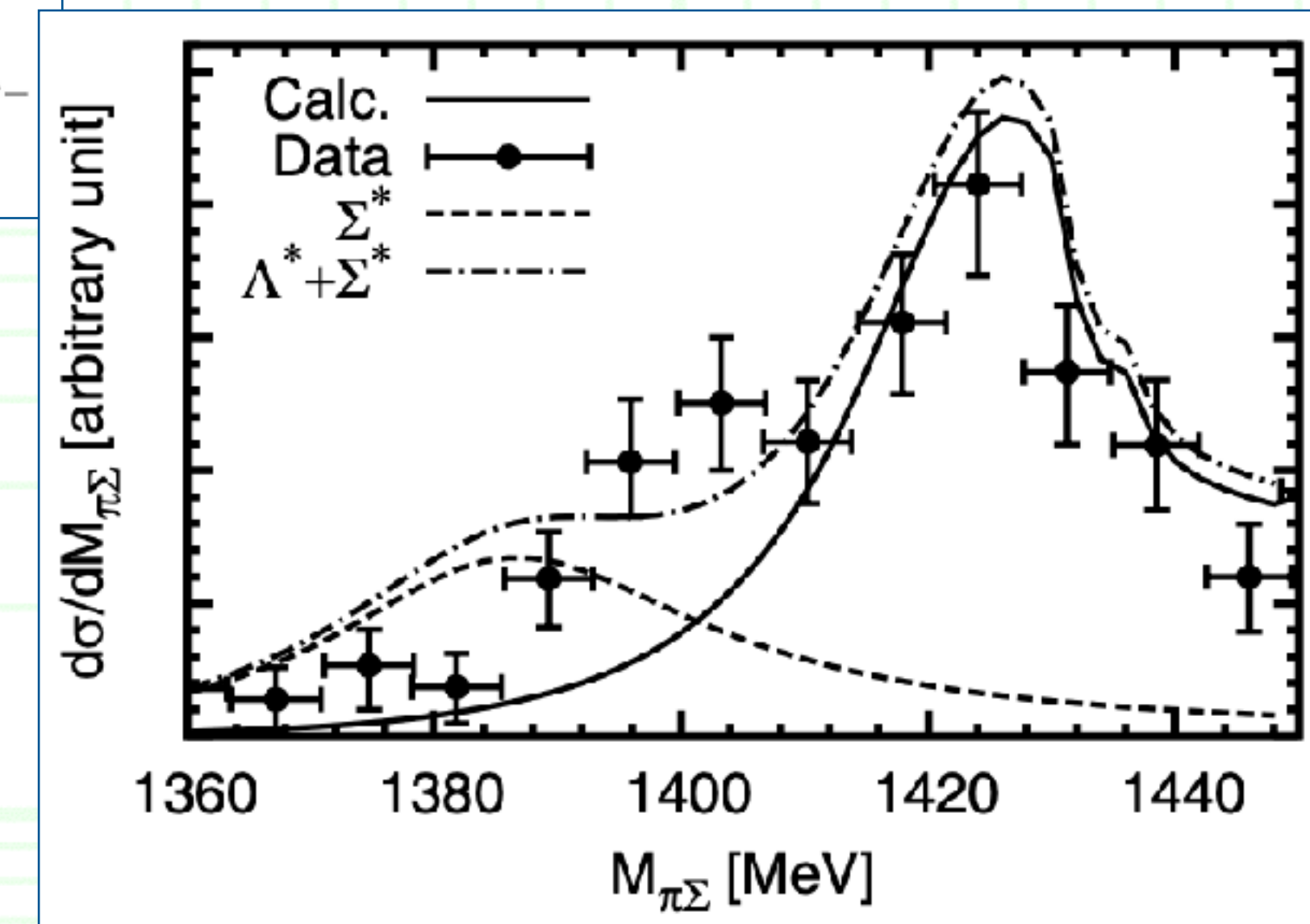
++ Resonance pole(s) of the $\Lambda(1405)$ ++

■ To elucidate the $\Lambda(1405)$ pole structure, the $\Lambda(1405)$ production directly from the $\bar{K}N$ channel was proposed.

→ **The $K^- d \rightarrow \pi \Sigma n$ reaction is the best way.** D. Jido, E. Oset, and T. S., Eur. Phys. J. A42 (2009) 257.



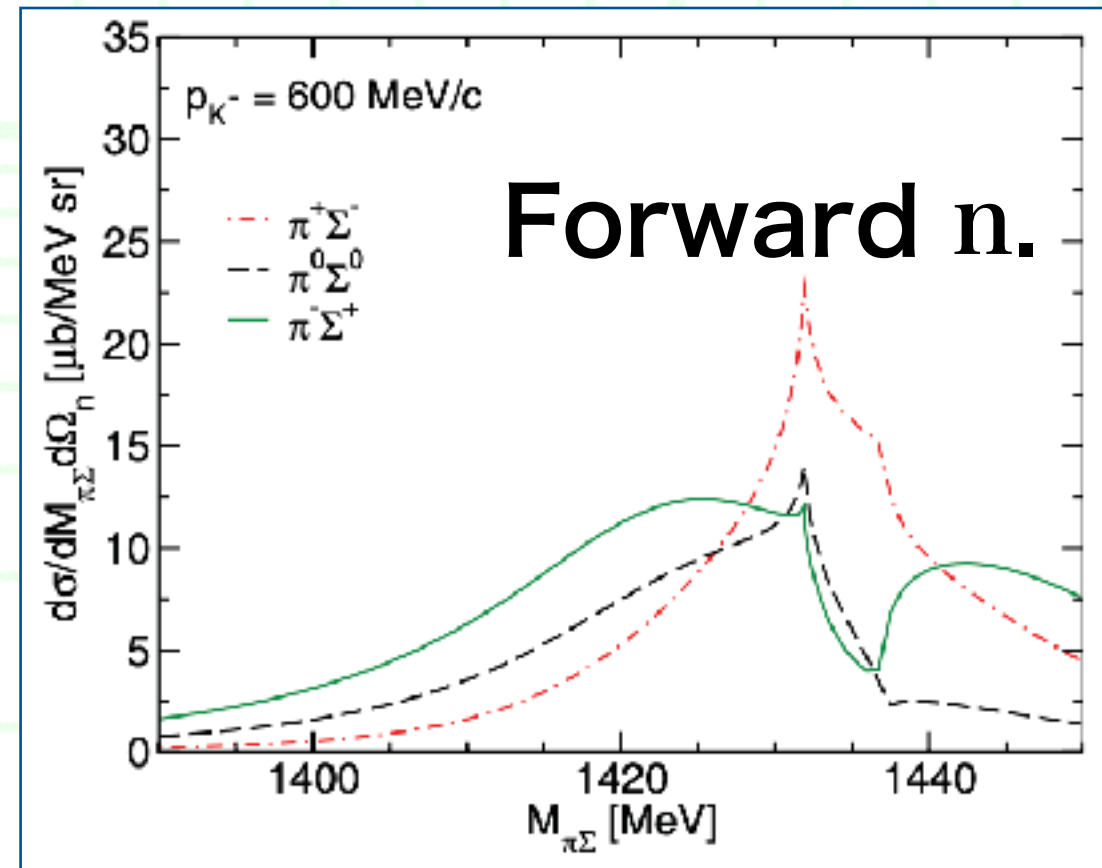
$p_K = 800 \text{ MeV}/c$
Whole angles.



2. The $\bar{K}N$ system in $K^- d \rightarrow \pi \Sigma n$

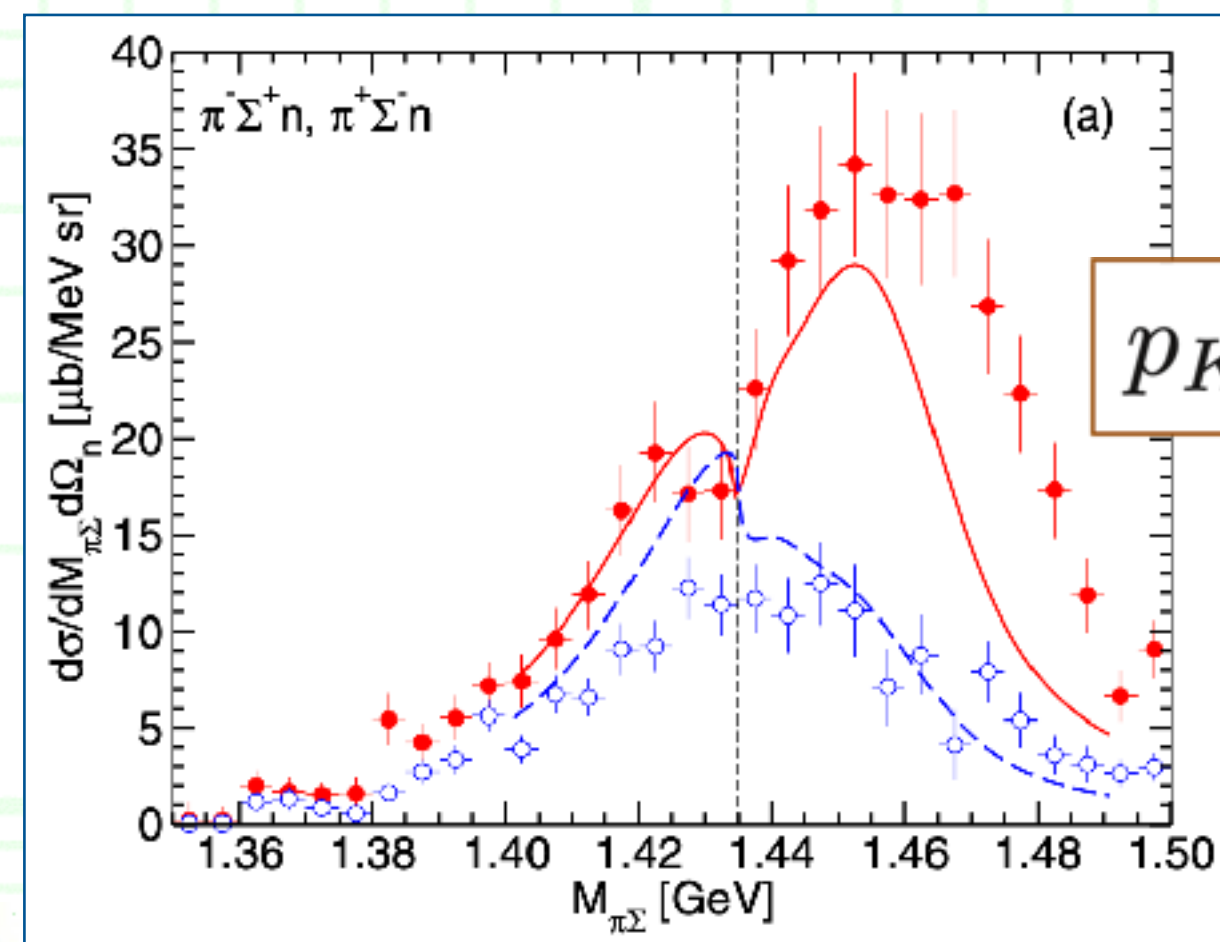
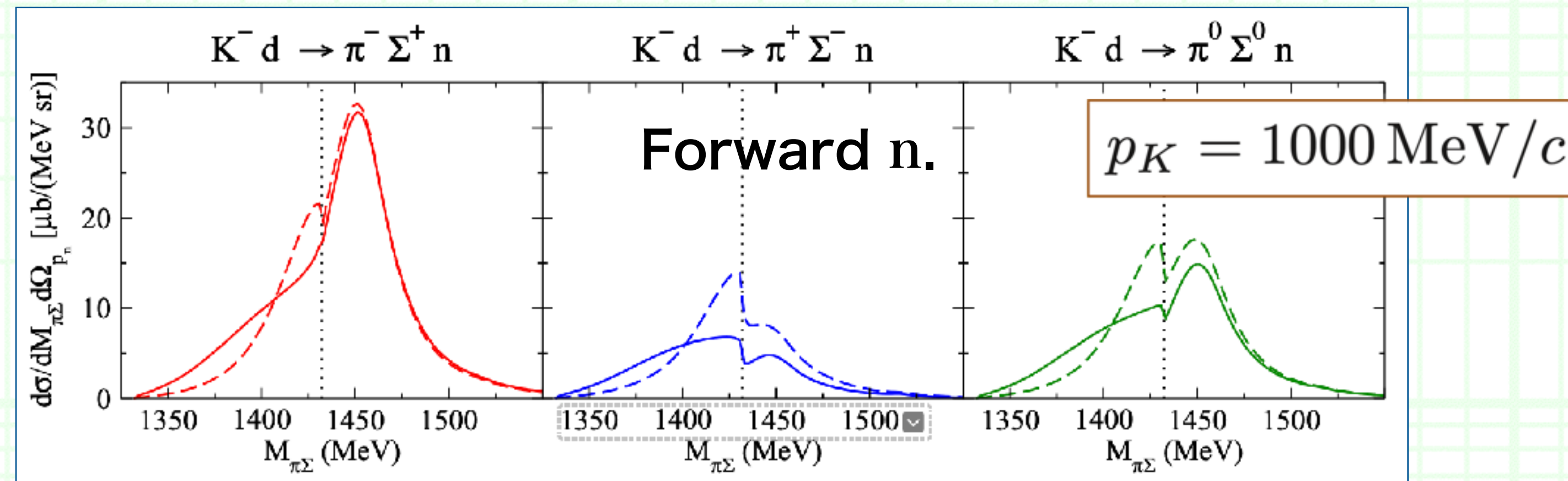
++ The $K^- d \rightarrow \pi \Sigma n$ reaction ++

■ Preceding theoretical studies on the $K^- d \rightarrow \pi \Sigma n$ reaction.



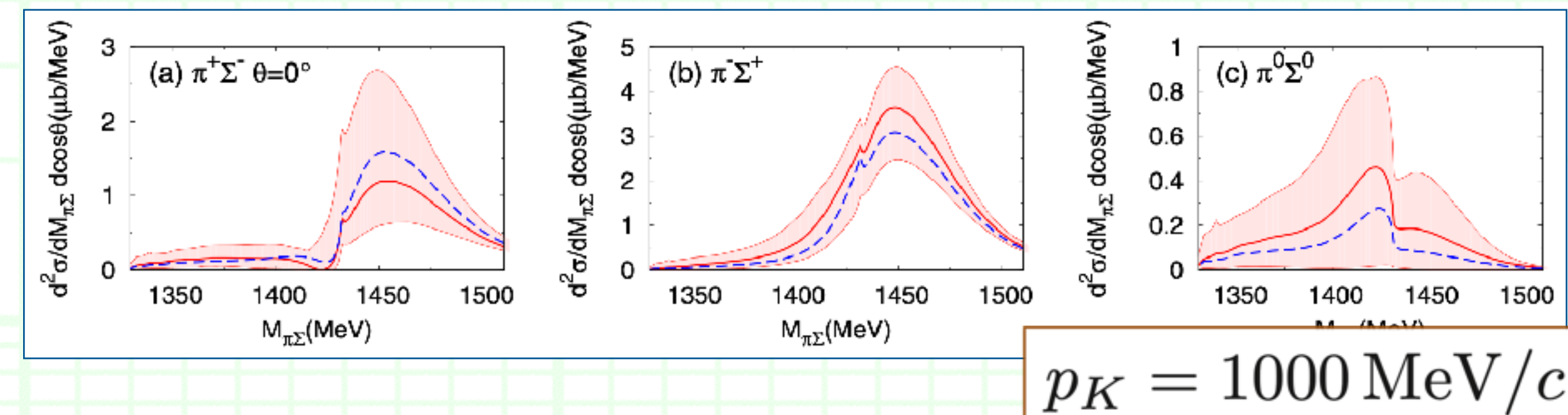
Miyagawa and Haidenbauer (2012).

Kamano and Lee (2016).



Miyagawa, Haidenbauer, and Kamada (2018).

Ohnishi, Ikeda, Hyodo, and Weise (2016).

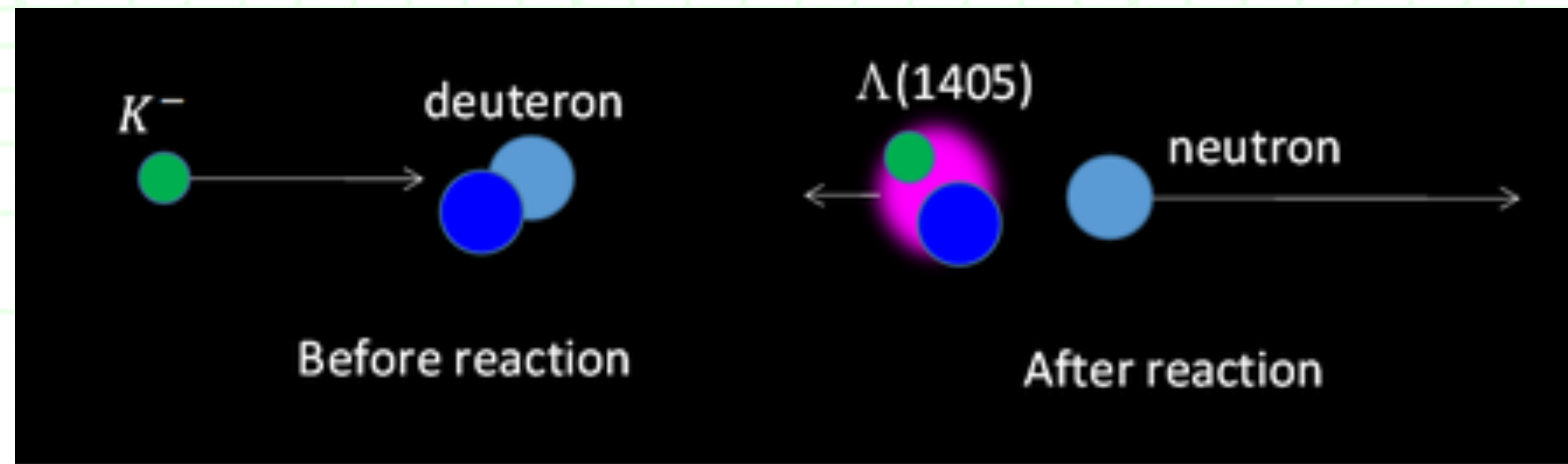


2. The $\bar{K}N$ system in $K^- d \rightarrow \pi \Sigma n$

++ J-PARC E31 Exp. for the $\bar{K}N$ system ++

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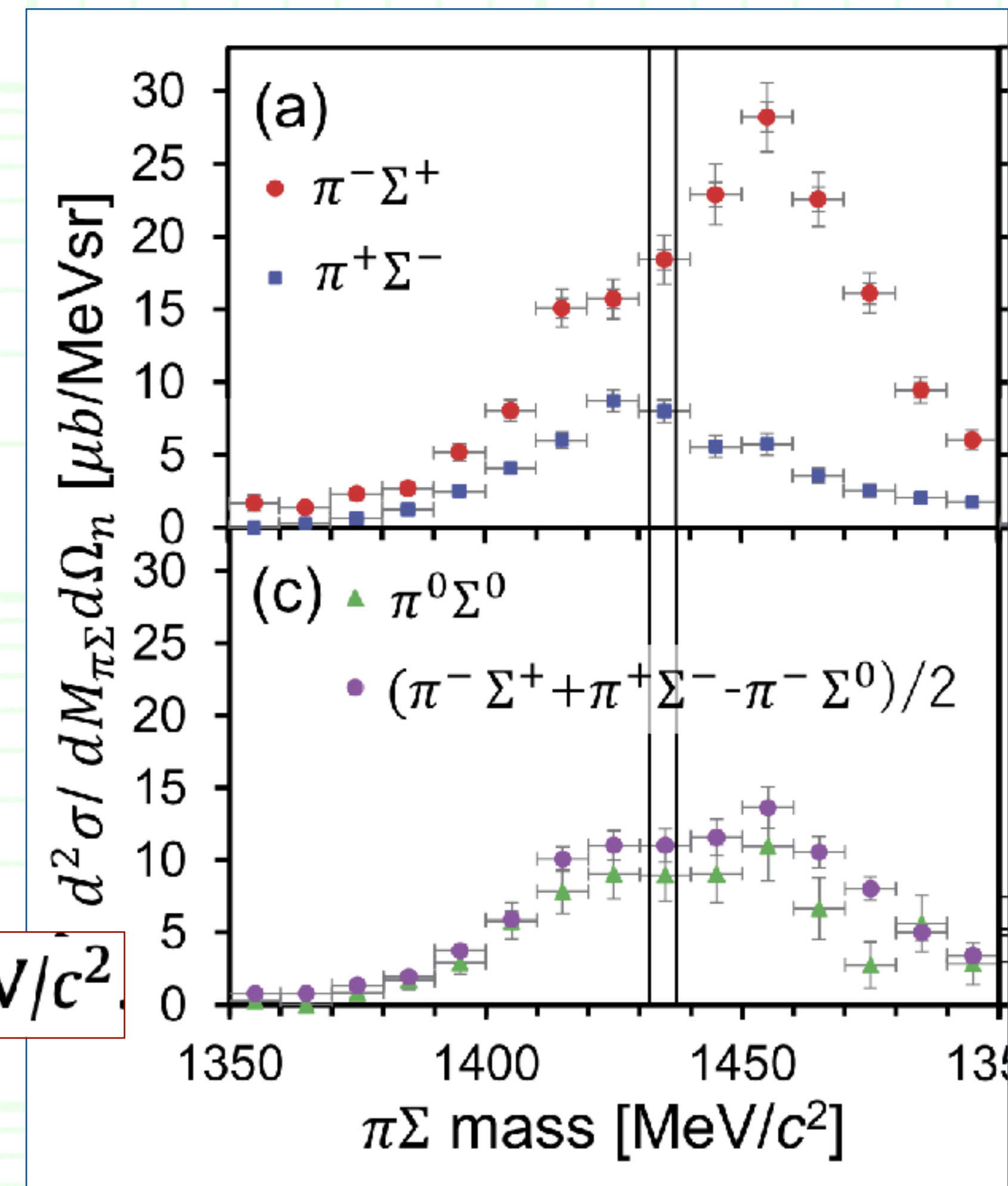
J-PARC
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J-PARC E31 Collab., Phys. Lett. B837 (2023) 137637.



2. The $\bar{K}N$ system in $K^- d \rightarrow \pi \Sigma n$

++ Our calculation ++

- We calculate the cross section of the $K^- d \rightarrow \pi \Sigma n$ reaction.

J. Yamagata-Sekihara, T. S., and D. Jido, under discussion.

- **Angular** (= momentum transfer q_{trans}) dependence ? $q_{\text{trans}} = p_K - p_n$ at Lab. frame

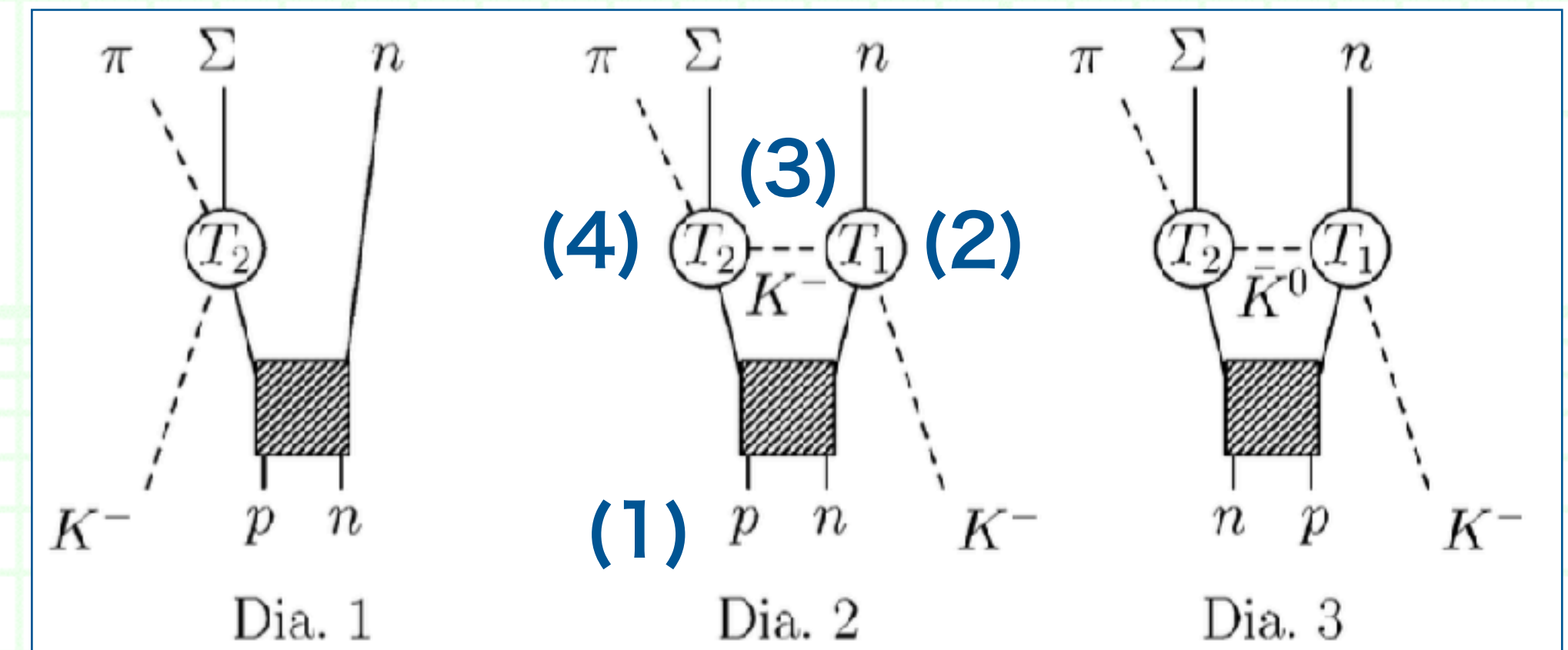
- Contribution from **each component** of the reaction diagram ?

(1) Deuteron wave function.

(2) 1st step T_1 ($\bar{K}N \rightarrow \bar{K}N$, $P_K = 1 \text{ GeV}/c$).

(3) \bar{K} propagator.

(4) 2nd step T_2 ($\bar{K}N \rightarrow \pi \Sigma$).



→ We aim to construct a precise model to **determine the pole position of the $\Lambda(1405)$** in the complex energy plane.

2. The $\bar{K}N$ system in $K^- d \rightarrow \pi \Sigma n$

++ Deuteron wave function & \bar{K} propagator ++

■ Deuteron wave function.

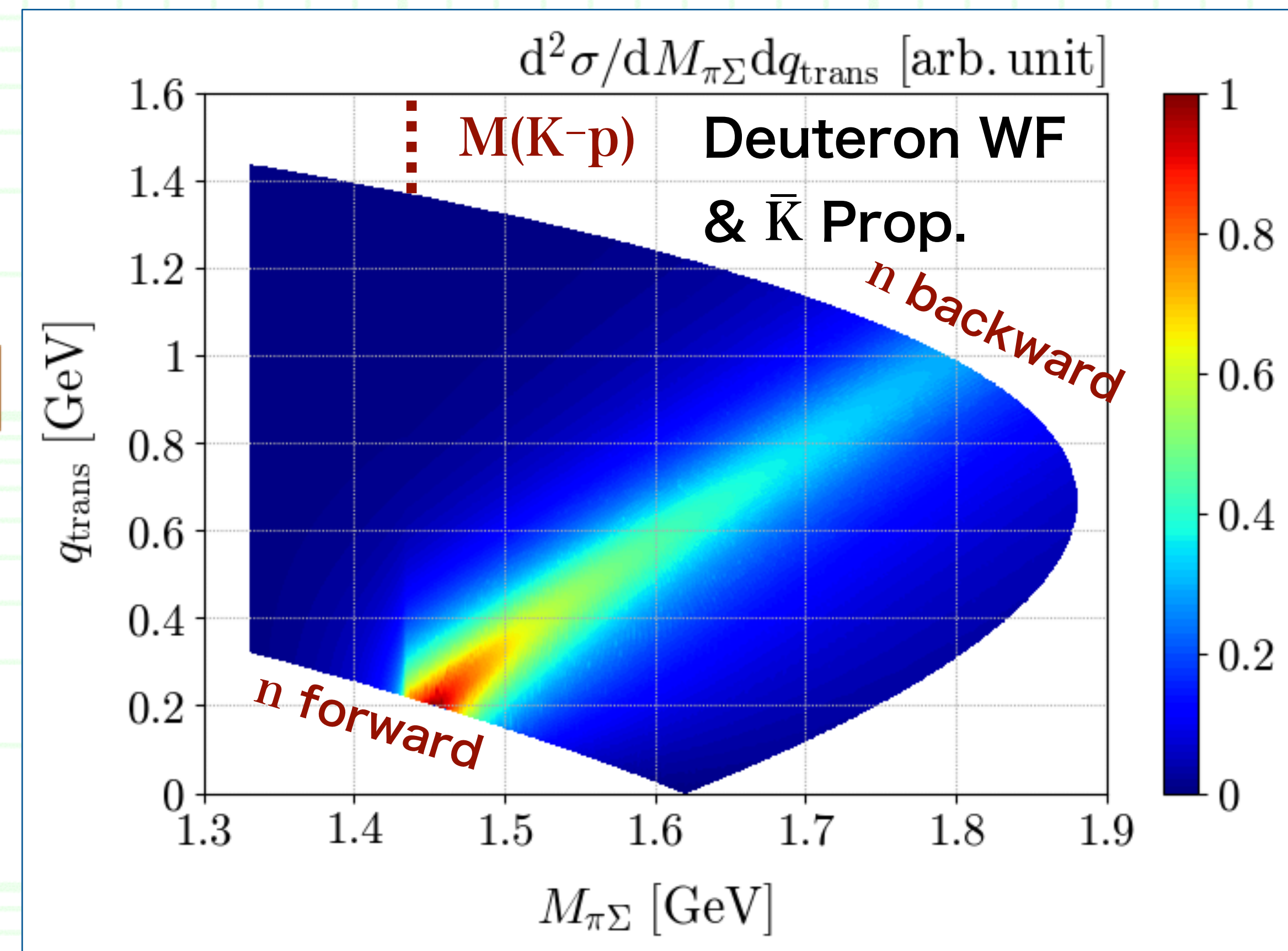
- Taken from **the CD-Bonn potential** (s wave only, $\sim 95\%$): Small uncertainty.

Machleidt, Phys. Rev. C63 (2011) 024001.

■ Together with the \bar{K} propagator, the scattering Amp. becomes:

$$\text{Amp.} = \int \frac{d^3 q_{\text{ex}}}{(2\pi)^3} \frac{\varphi_{\text{deut}}(|\mathbf{q}_{\text{ex}} - \mathbf{q}_{\text{trans}}|)}{q_{\text{ex}}^2 - m_K^2 + i0} \quad \mathbf{q}_{\text{trans}} = \mathbf{p}_K - \mathbf{p}_n$$

- **Band width** owing to the deuteron WF.
– Off-shell N inside the deuteron.
- On this band, we may treat **the propagating \bar{K} as (almost) on-shell particle.**



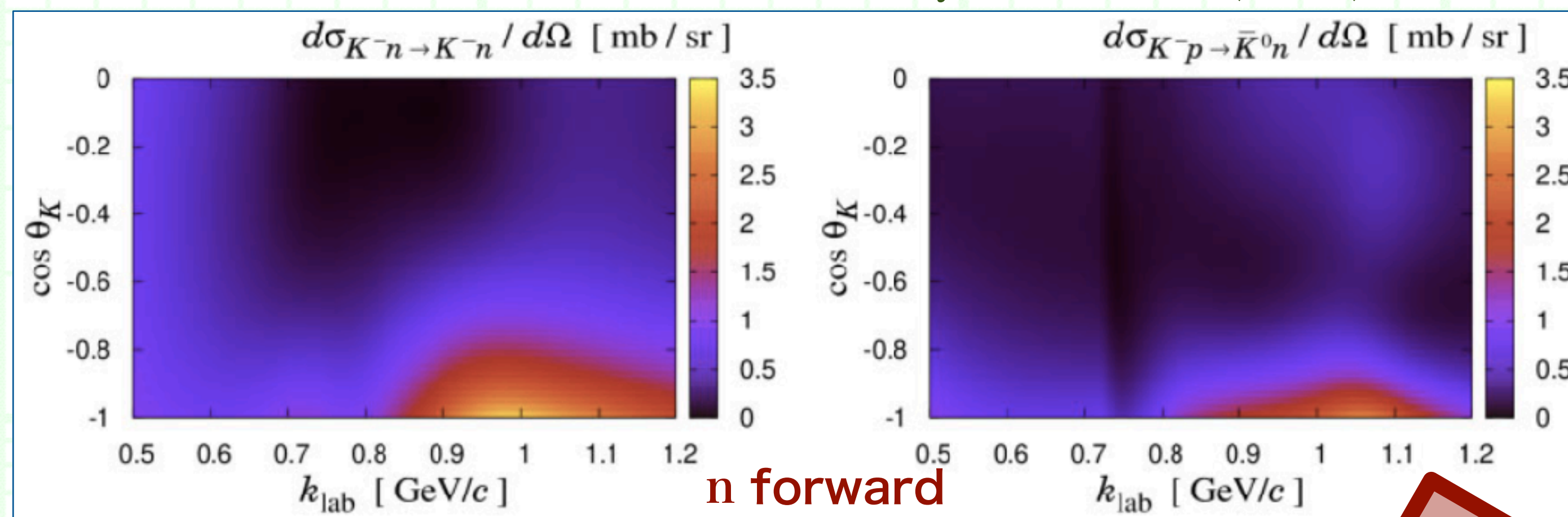
2. The $\bar{K}N$ system in $K^- d \rightarrow \pi \Sigma n$

++ & the 1st step $\bar{K}N \rightarrow \bar{K}N ++$

■ Inclusion of the 1st step T_1 ($\bar{K}N \rightarrow \bar{K}N$, $P_K = 1 \text{ GeV}/c$).

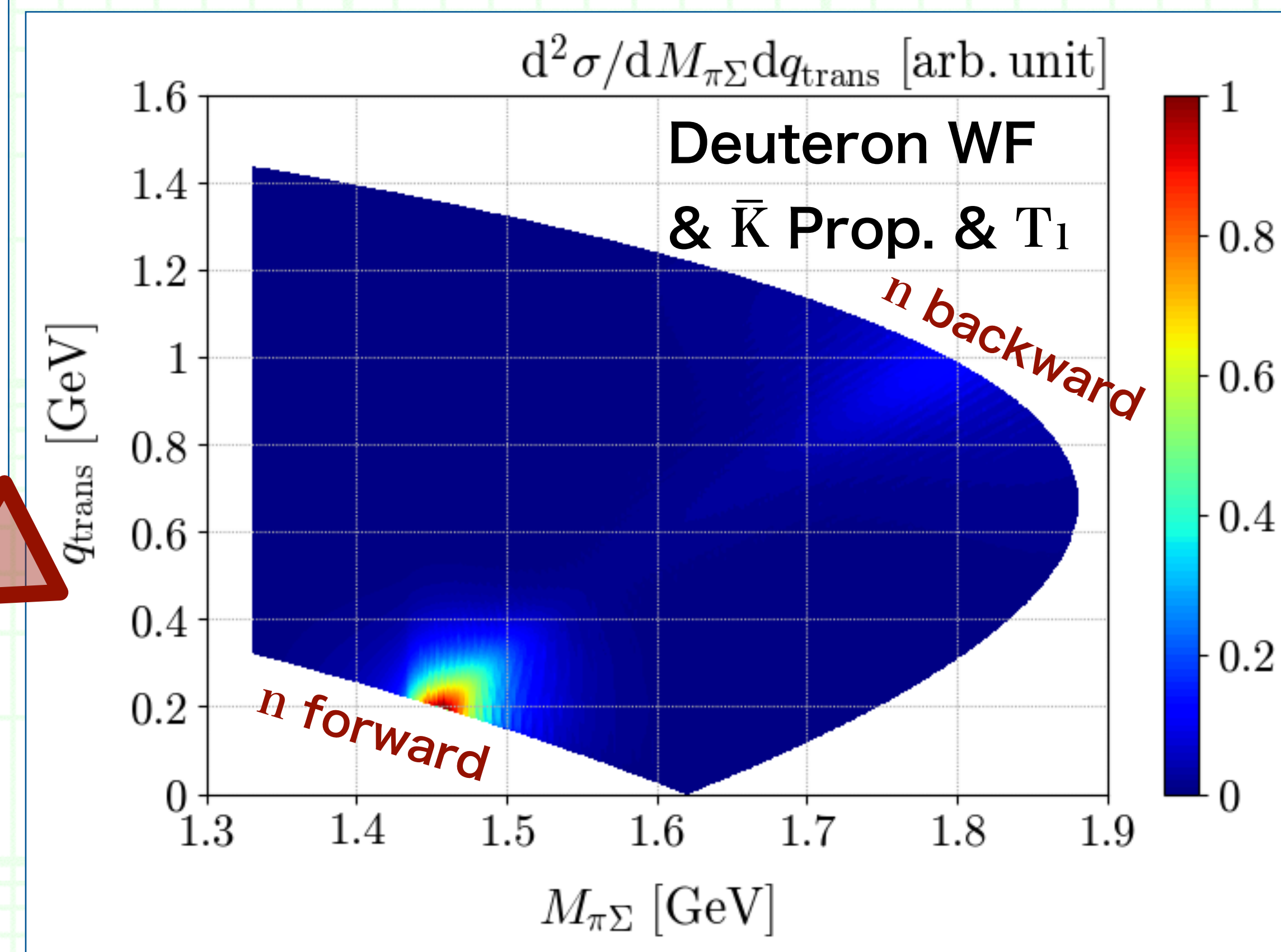
→ Employ the Kamano et al. on-shell amplitude.

Kamano, Nakamura, Lee, and Sato, Phys. Rev. C90 (2014) 065204.



$$\text{Amp.} = \int \frac{d^3 q_{\text{ex}}}{(2\pi)^3} \frac{\varphi_{\text{deut}}(|\mathbf{q}_{\text{ex}} - \mathbf{q}_{\text{trans}}|)}{q_{\text{ex}}^2 - m_K^2 + i0} T_1^{(\text{Kamano})}$$

□ Dominated by the **small momentum transfer region** $q_{\text{trans}} < 0.4 \text{ GeV}$.



2. The $\bar{K}N$ system in $K^- d \rightarrow \pi \Sigma n$

++ & the 2nd step $\bar{K}N \rightarrow \pi \Sigma ++$

■ Inclusion of the 2nd step T_2 ($\bar{K}N \rightarrow \pi \Sigma$).

→ Employ **the Ikeda-Hyodo-Weise amplitude**, which contains the $\Lambda(1405)$.

Ikeda, Hyodo, and Weise, Nucl. Phys. A881 (2012) 98.

$$\text{Amp.} = \int \frac{d^3 q_{\text{ex}}}{(2\pi)^3} \varphi_{\text{deut}}(|\mathbf{q}_{\text{ex}} - \mathbf{q}_{\text{trans}}|) \frac{1}{q_{\text{ex}}^2 - m_K^2 + i0} T_1^{(\text{Kamano})} T_2^{(\text{IHW})}$$

→ Full calculation.

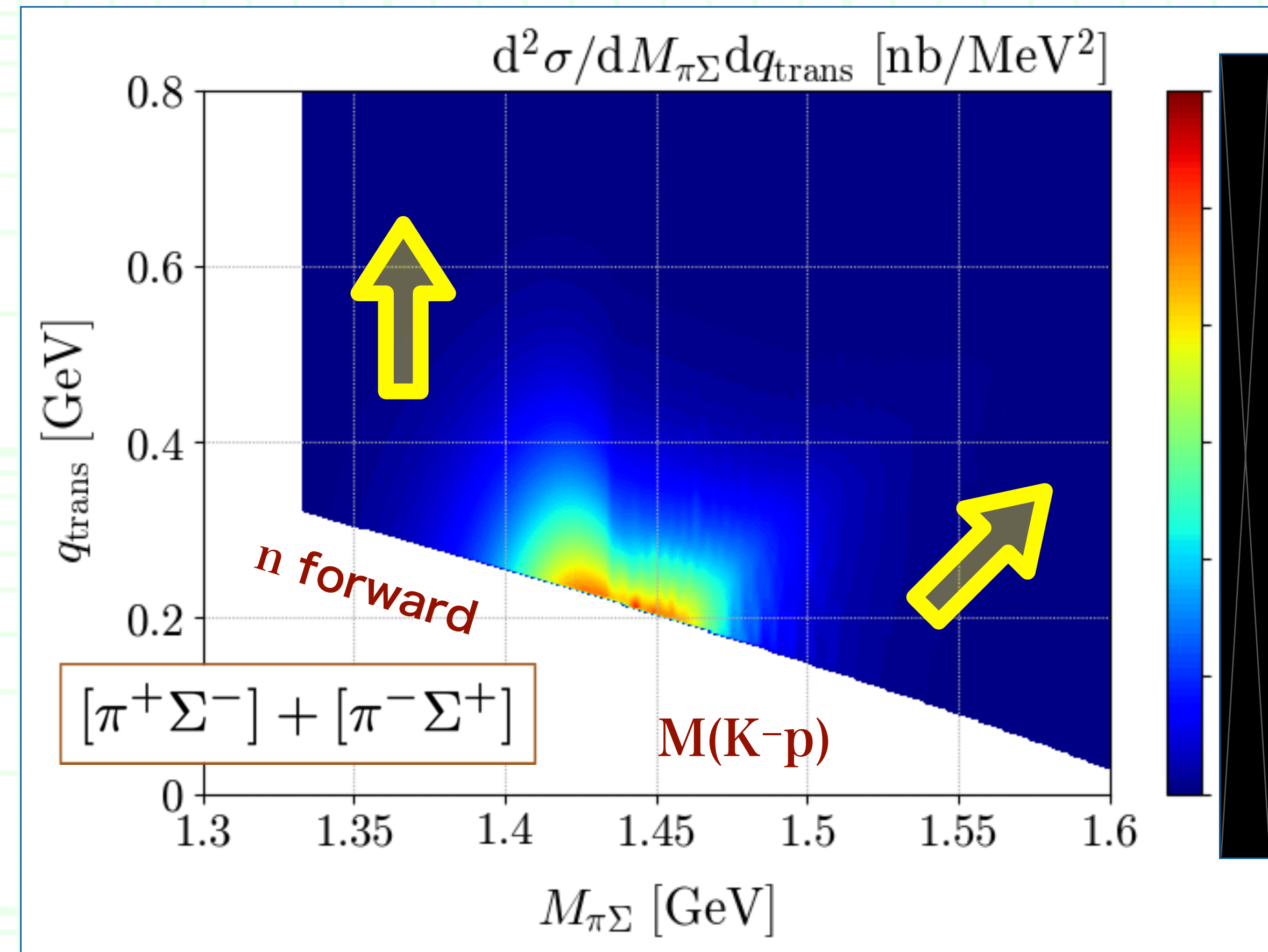
□ We have two trends.

• Below the $\bar{K}N$ threshold:

The $\Lambda(1405)$ signal.

• Above the $\bar{K}N$ threshold:

The quasi-free \bar{K} propagation.

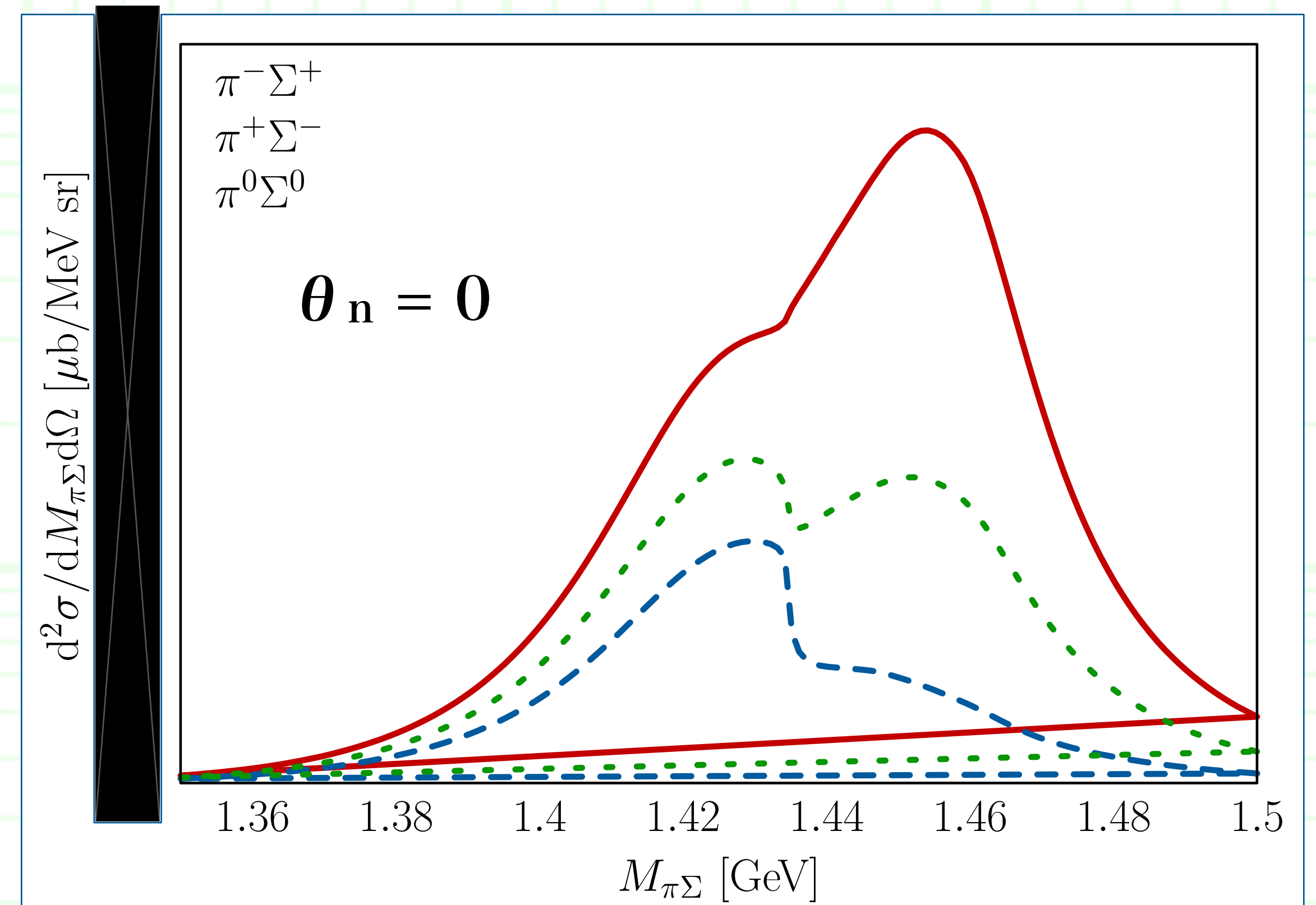
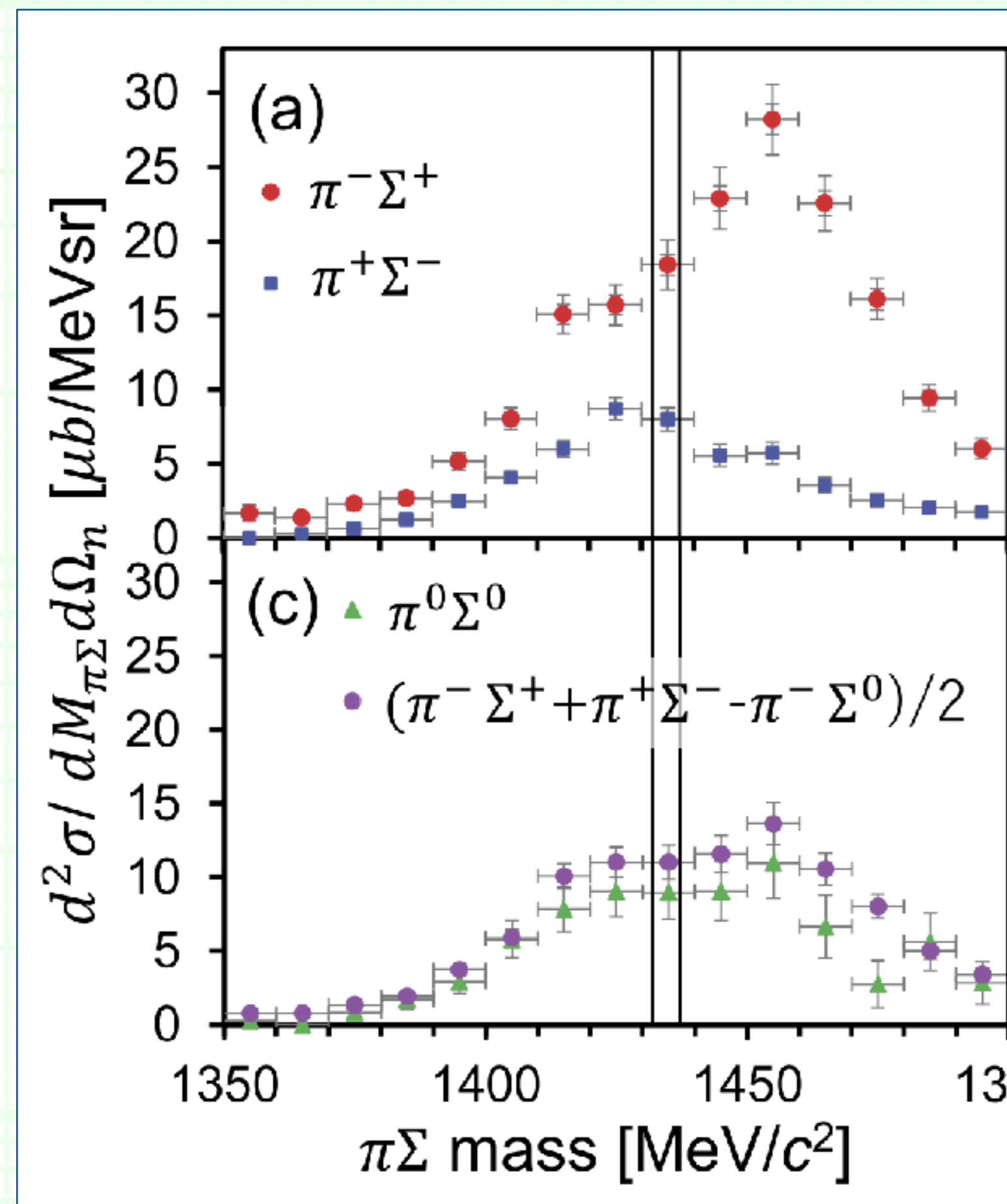


2. The $\bar{K}N$ system in $K^- d \rightarrow \pi \Sigma n$

++ Spectrum at the forward n ++

■ We can compare the $\pi \Sigma$ spectrum at the forward n condition with the Exp. data.

J-PARC E31 Collab., Phys. Lett. B837 (2023) 137637.



□ Quite similar shapes, although the peak heights are quantitatively different.

2. The $\bar{K}N$ system in $K^- d \rightarrow \pi \Sigma n$

++ Summary and outlook ++

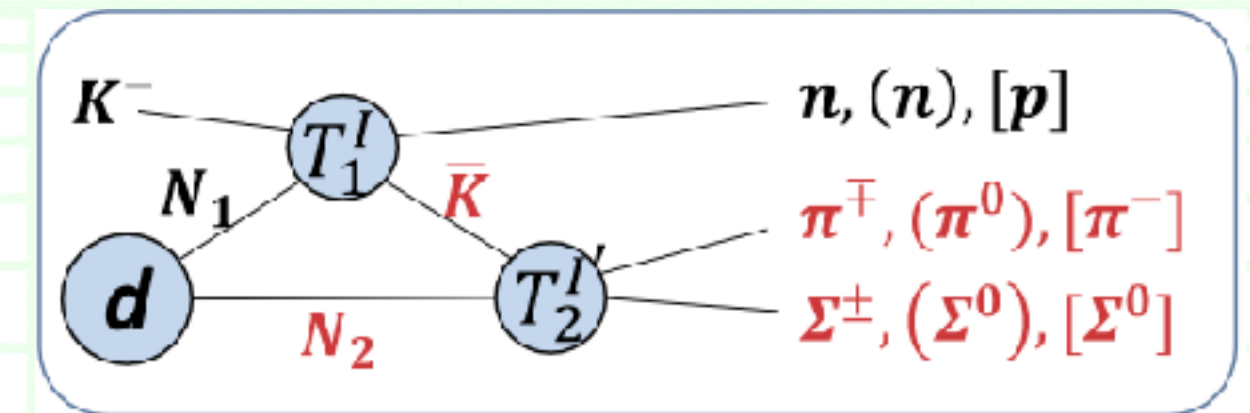
■ After all, what makes the structure in the $K^- d \rightarrow \pi \Sigma n$ reaction ?

Deuteron wave function: Robust.

J-PARC E31 Collab., Phys. Lett. B837 (2023) 137637.

\bar{K} propagator \times 1st step $T_1^{(\text{Kamano})}$ (on-shell) : Fairly robust.

2nd step $T_2^{(\text{IHW})}$ (contains $\Lambda(1405)$) : Fairly robust.



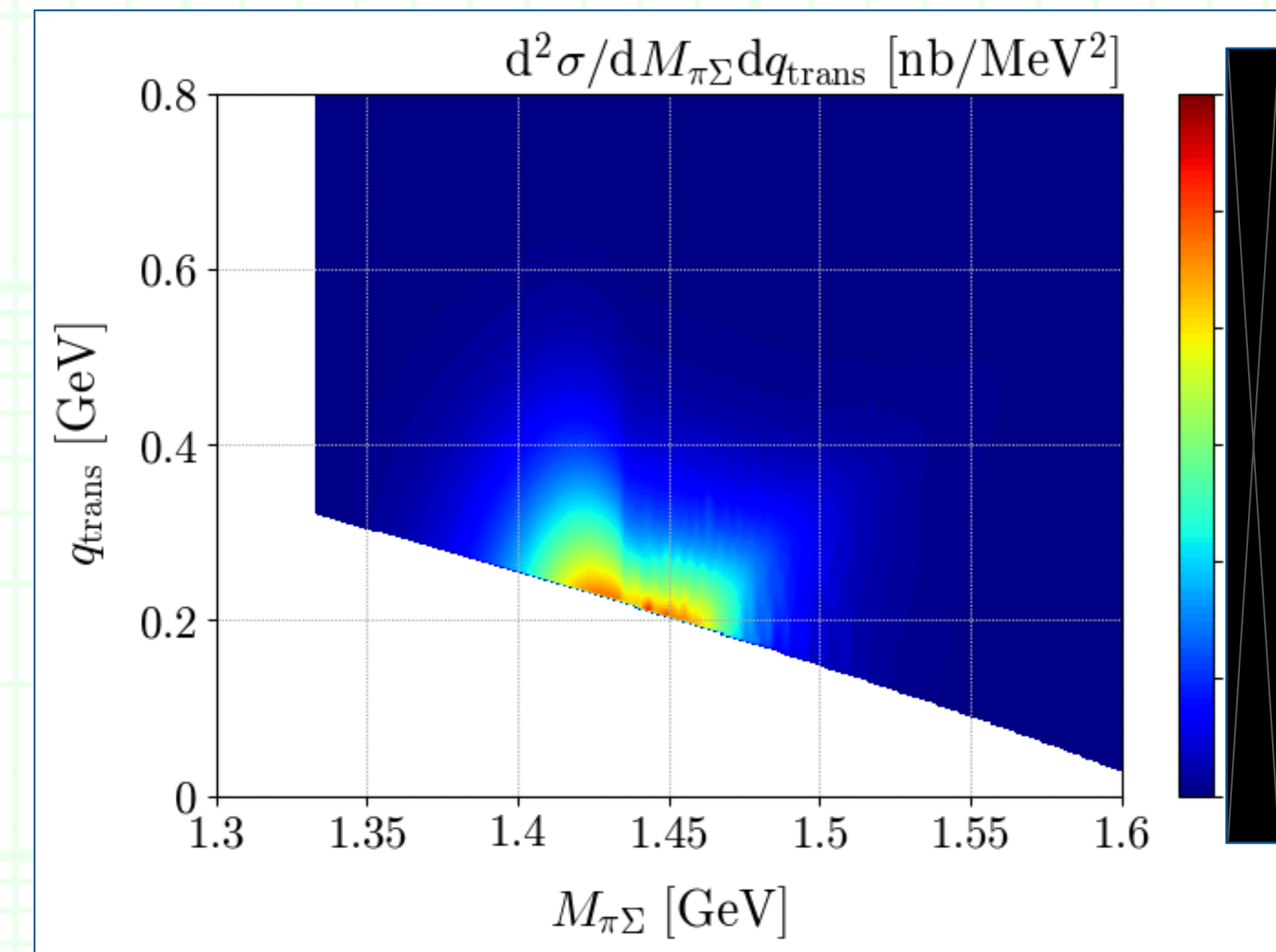
→ Double-step process makes the structure !
 \bar{K} survives the reaction.

■ Then, we can upgrade the reaction calculation.

Final-state interaction ?

Difference from the $\Sigma(1385)$ / $\Lambda(1520)$?

→ More precise properties of the $\Lambda(1405)$.



3. The $\bar{K}NN$ system in the $K^- \text{ } ^3\text{He} \rightarrow \Lambda \text{ p n}$ reaction: J-PARC E15

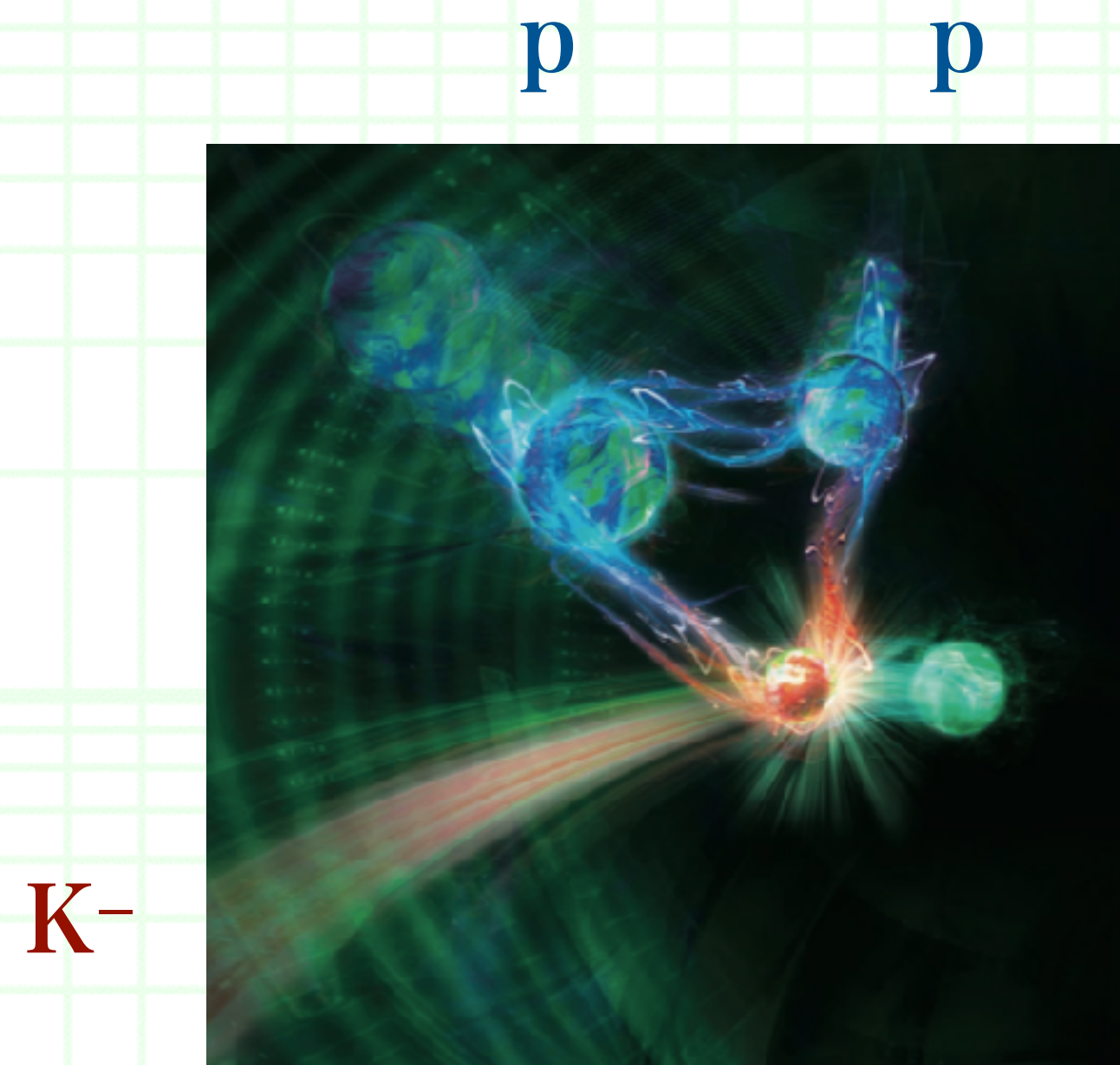
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in the $K^- \ ^3\text{He} \rightarrow \Lambda pn$ reaction.

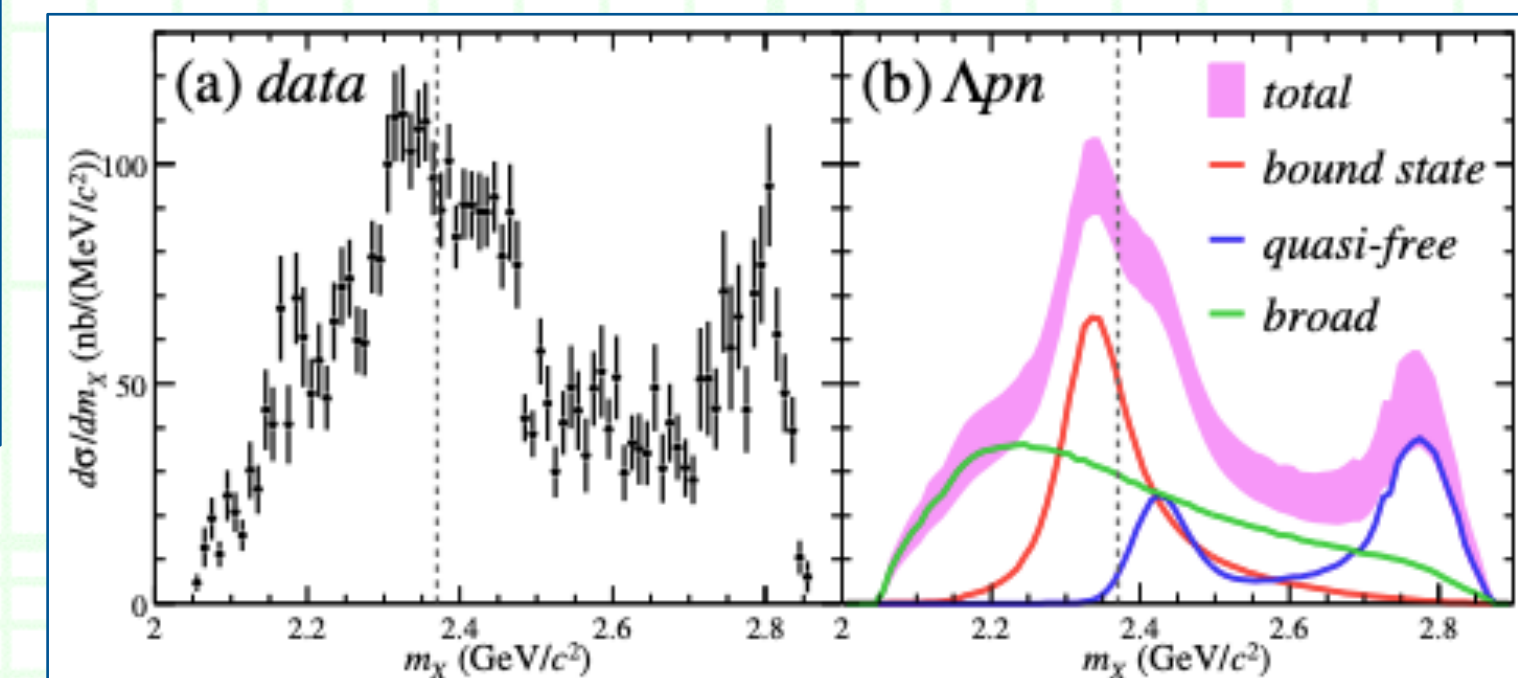
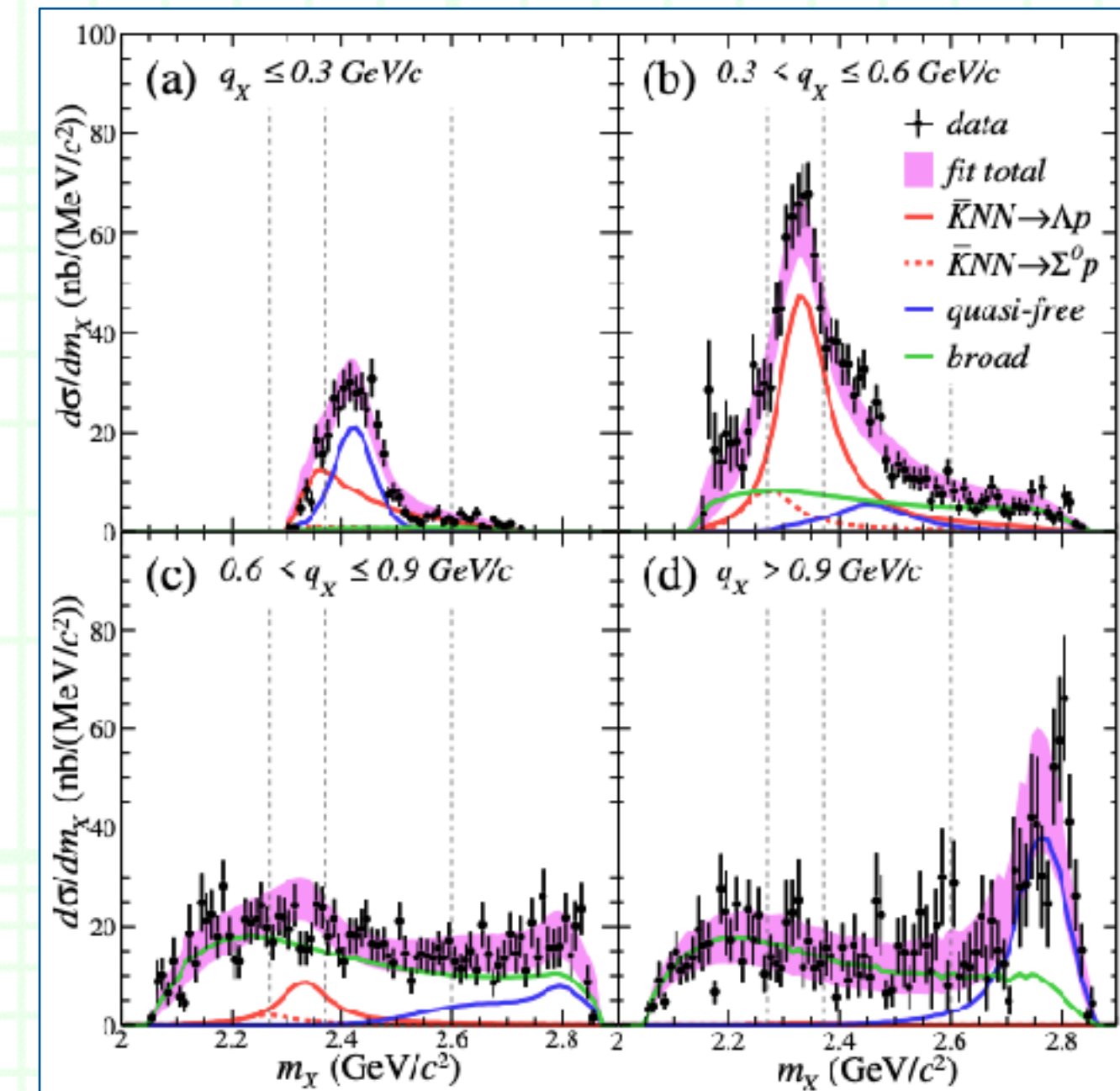
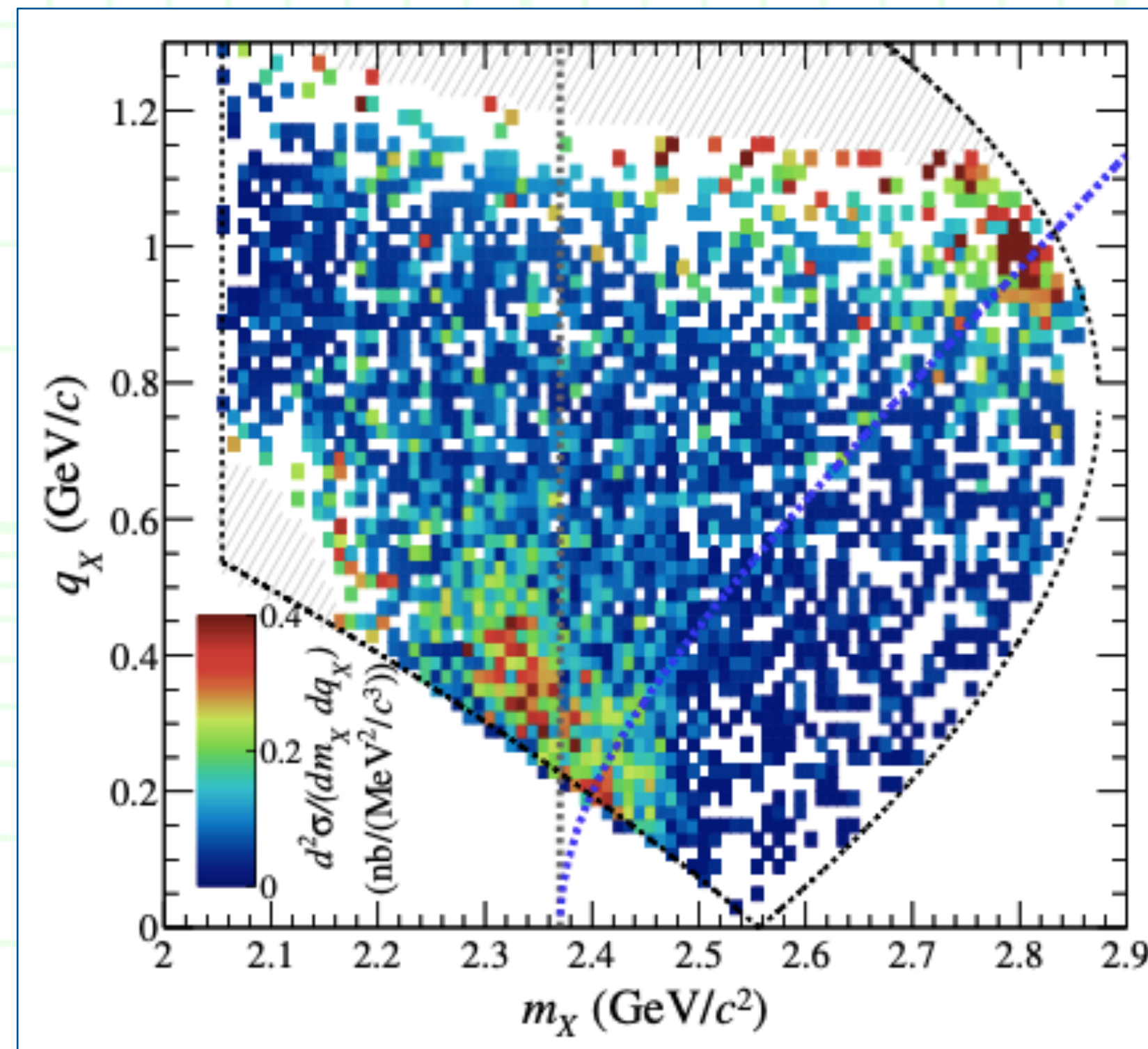
Yamaga et al. [J-PARC E15],
Phys. Rev. C102 (2020) 044002.



K^-

The Austrian Academy of Sciences, Harald Ritsch.

n



$$B_K = 42 \pm 3(\text{stat.})_{-4}^{+3}(\text{syst.}) \text{ MeV} \quad \Gamma_K = 100 \pm 7(\text{stat.})_{-9}^{+19}(\text{syst.}) \text{ MeV}$$

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- We calculate the cross section of the $K^- \text{ } ^3\text{He} \rightarrow \Lambda pn$ reaction.

T. S., E. Oset, and A. Ramos, PTEP 2016 123D03; under discussion.

- Momentum transfer q_{trans} dependence ?

$$q_{\text{trans}} = \mathbf{p}_K - \mathbf{p}_n \text{ at Lab. frame}$$

- Contribution from each component of the reaction diagram ?

(1) ^3He wave function.

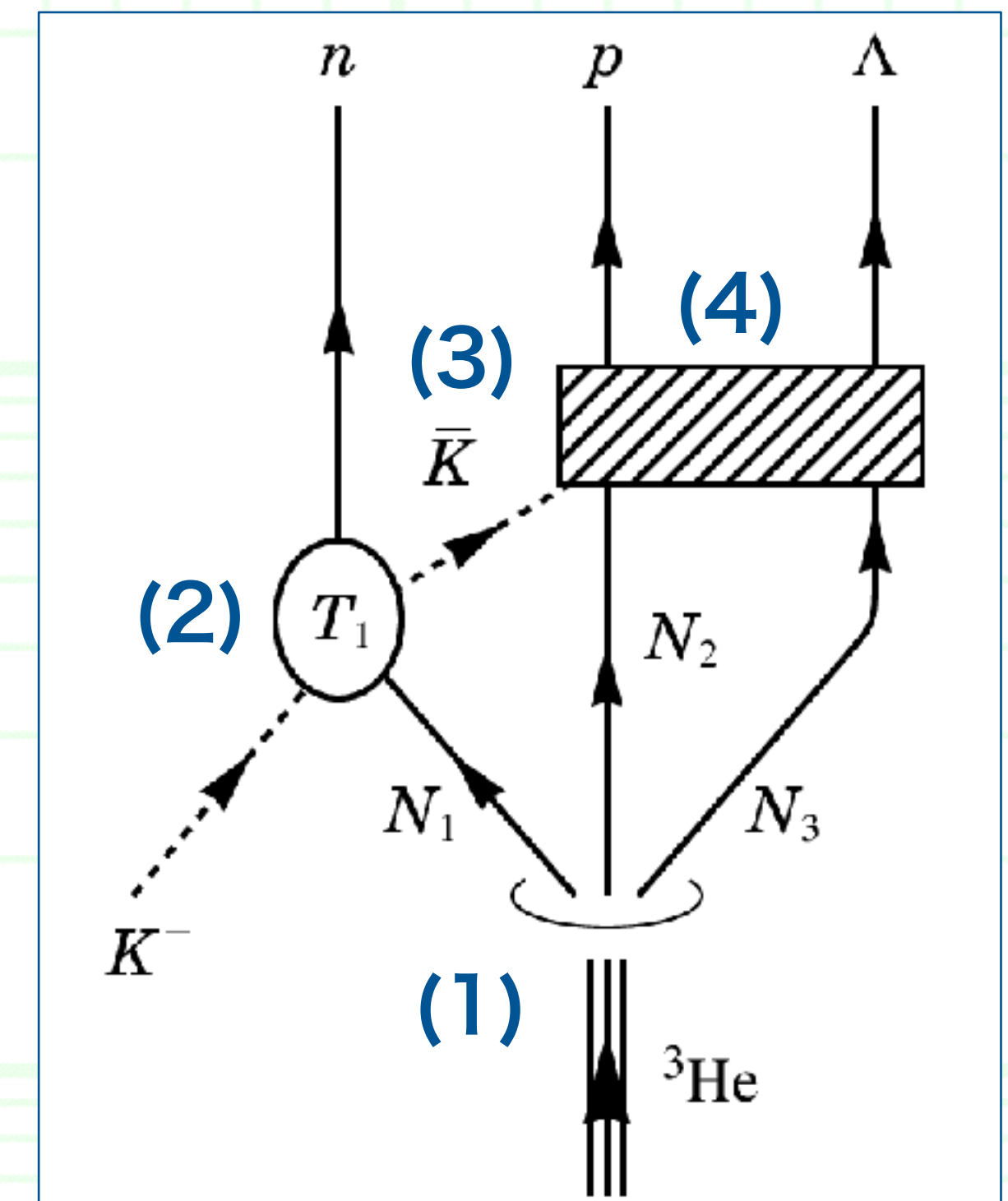
(2) 1st step T_1 ($\bar{K}N \rightarrow \bar{K}N$, $P_K = 1 \text{ GeV}/c$).

(3) \bar{K} propagator.

(4) Faddeev & \bar{K} absorption $T(\bar{K}NN \rightarrow \Lambda p)$.

→ We aim to construct a precise model

to search for the $\bar{K}NN$ pole in the complex energy plane.



3. The $\bar{K}NN$ system in $K^- \text{}^3\text{He} \rightarrow \Lambda pn$

++ ${}^3\text{He}$ wave function ++

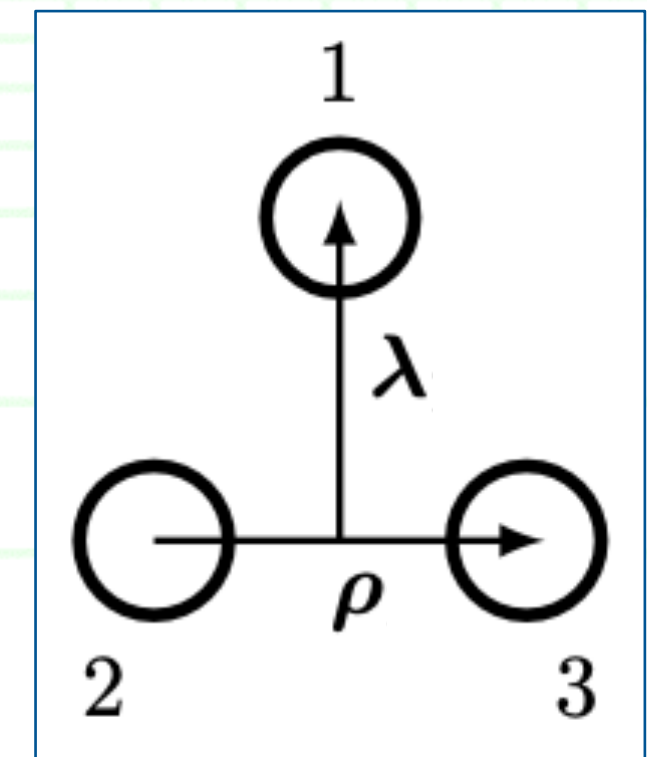
■ ${}^3\text{He}$ wave function.

- **A separable parameterization** fit for the NNN wave function with the CD-Bonn potential (s wave only, $\sim 90\%$).

Baru, Haidenbauer, Hanhart, and Niskanen,
Eur. Phys. J. A16 (2003) 437.

1S_0

1S_0



$$|{}^3\text{He}_\uparrow\rangle = v_{{}^1S_0}(p_\rho)w_{{}^1S_0}(p_\lambda) \left| -\frac{1}{\sqrt{3}}n_\uparrow(p_\uparrow p_\downarrow - p_\downarrow p_\uparrow) + \frac{1}{2\sqrt{3}}p_\uparrow(p_\uparrow n_\downarrow + n_\uparrow p_\downarrow - p_\downarrow n_\uparrow - n_\downarrow p_\uparrow) \right\rangle$$

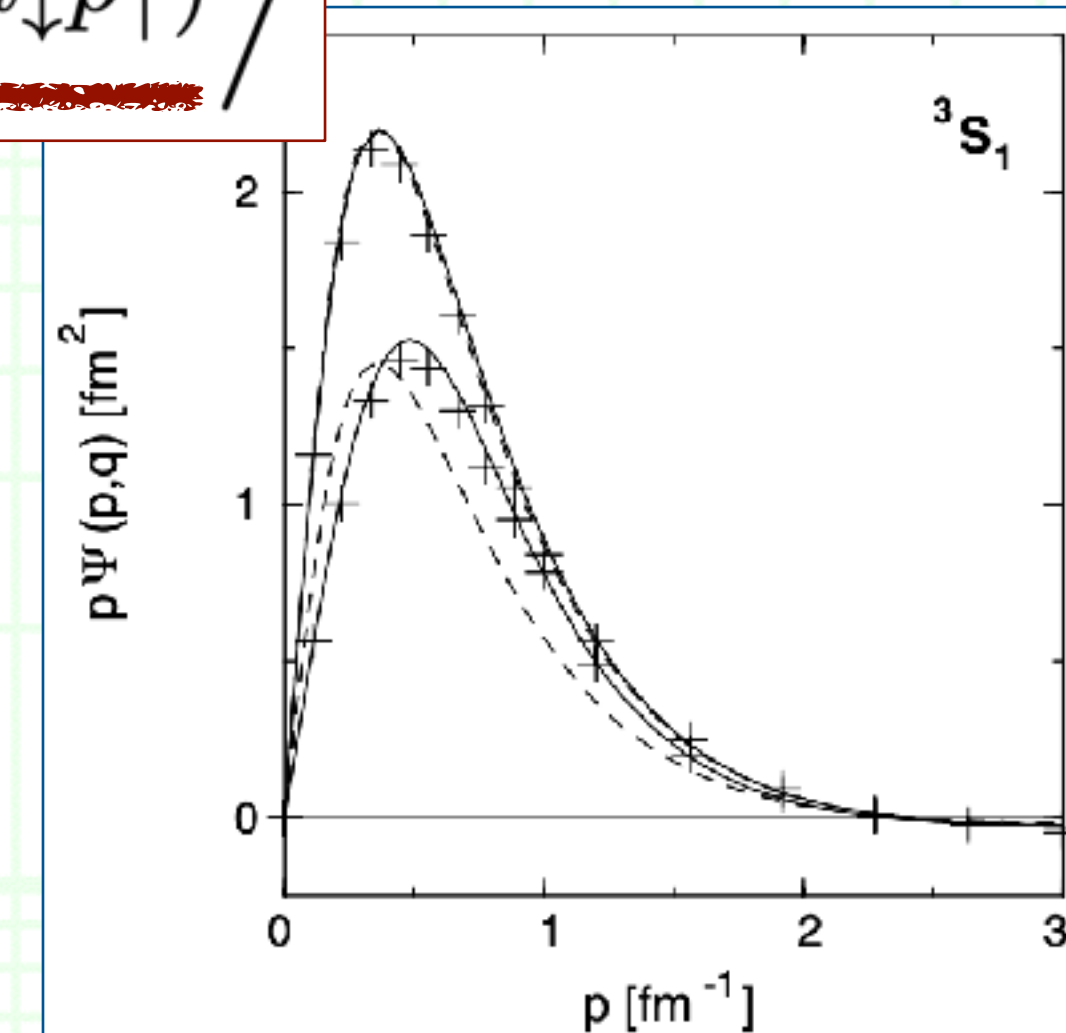
$$+ v_{{}^3S_1}(p_\rho)w_{{}^3S_1}(p_\lambda) \left| -\frac{1}{\sqrt{3}}p_\downarrow(p_\uparrow n_\uparrow - n_\uparrow p_\uparrow) + \frac{1}{2\sqrt{3}}p_\uparrow(p_\uparrow n_\downarrow - n_\uparrow p_\downarrow + p_\downarrow n_\uparrow - n_\downarrow p_\uparrow) \right\rangle$$

3S_1

3S_1

— Functions v and w are given by a five-term expansion:

$$v_\nu(p) = \sum_{n=1}^5 \frac{a_n^\nu}{p^2 + (m_n^\nu)^2}, \quad w_\nu(p) = \sum_{n=1}^5 \frac{b_n^\nu}{p^2 + (M_n^\nu)^2} \quad (\nu = {}^1S_0, {}^3S_1)$$



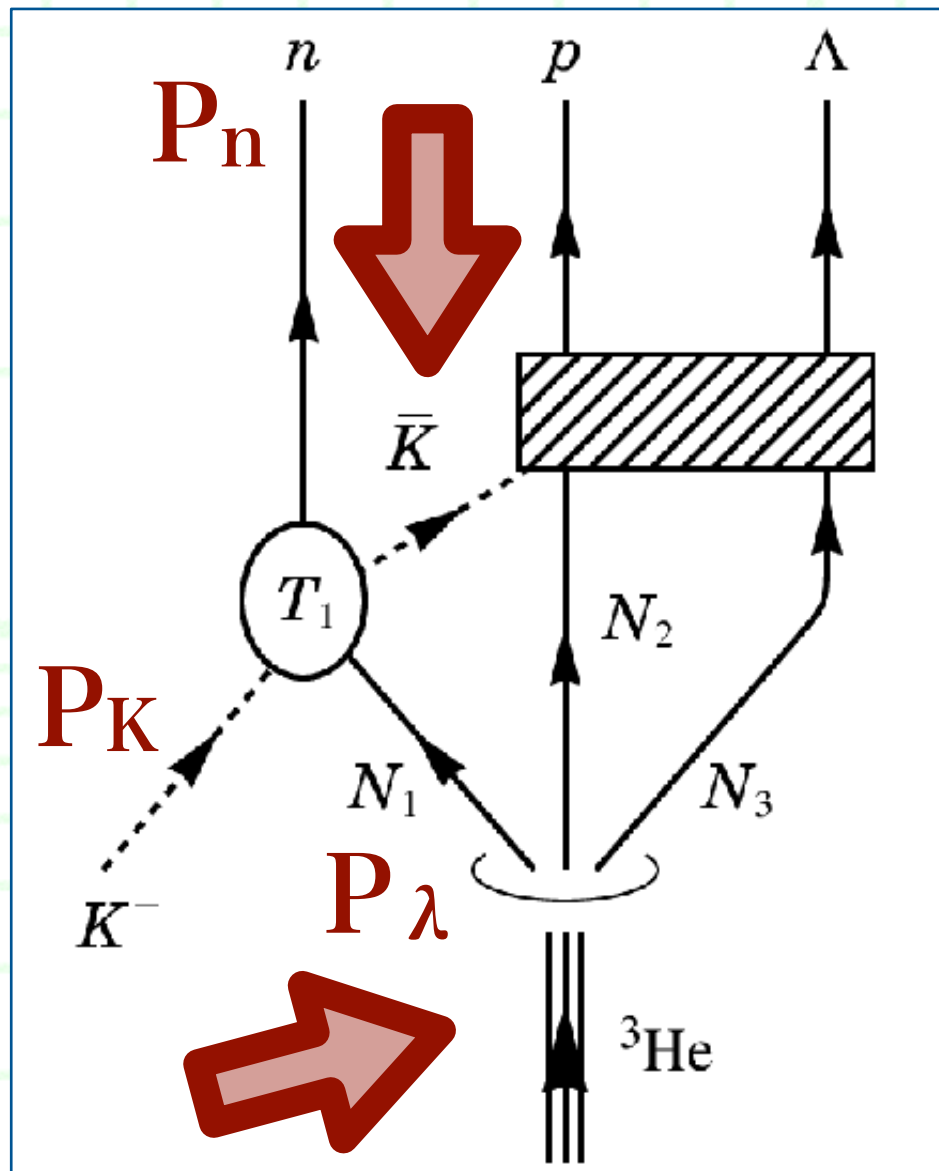
3. The $\bar{K}NN$ system in $K^- \ ^3\text{He} \rightarrow \Lambda pn$

++ & \bar{K} propagator ++

■ Together with the \bar{K} propagator, the scattering Amp. becomes:

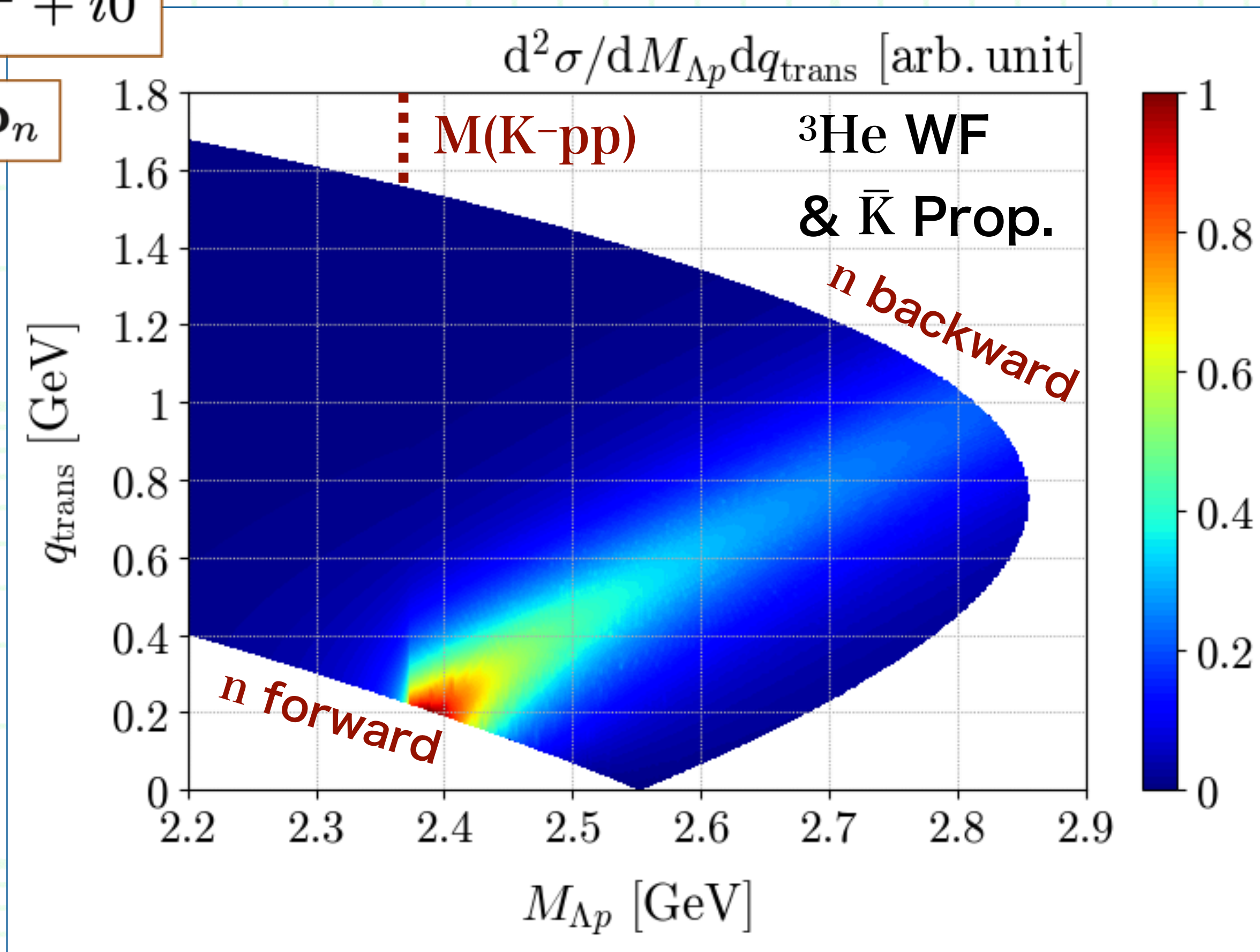
$$\text{Amp.} = \int \frac{d^3 p_\lambda}{(2\pi)^3} \frac{w_\nu(p_\lambda)}{(q_{\text{ex}}^0)^2 - |\mathbf{p}_\lambda + \mathbf{p}_K - \mathbf{p}_n|^2 - m_K^2 + i0}$$

$$q_{\text{trans}} = \mathbf{p}_K - \mathbf{p}_n$$



- Again we have a band structure.
- Band width owing to the ^3He WF.
 - Off-shell N inside ^3He .

- On this band, we may treat the propagating \bar{K} as (almost) on-shell particle.



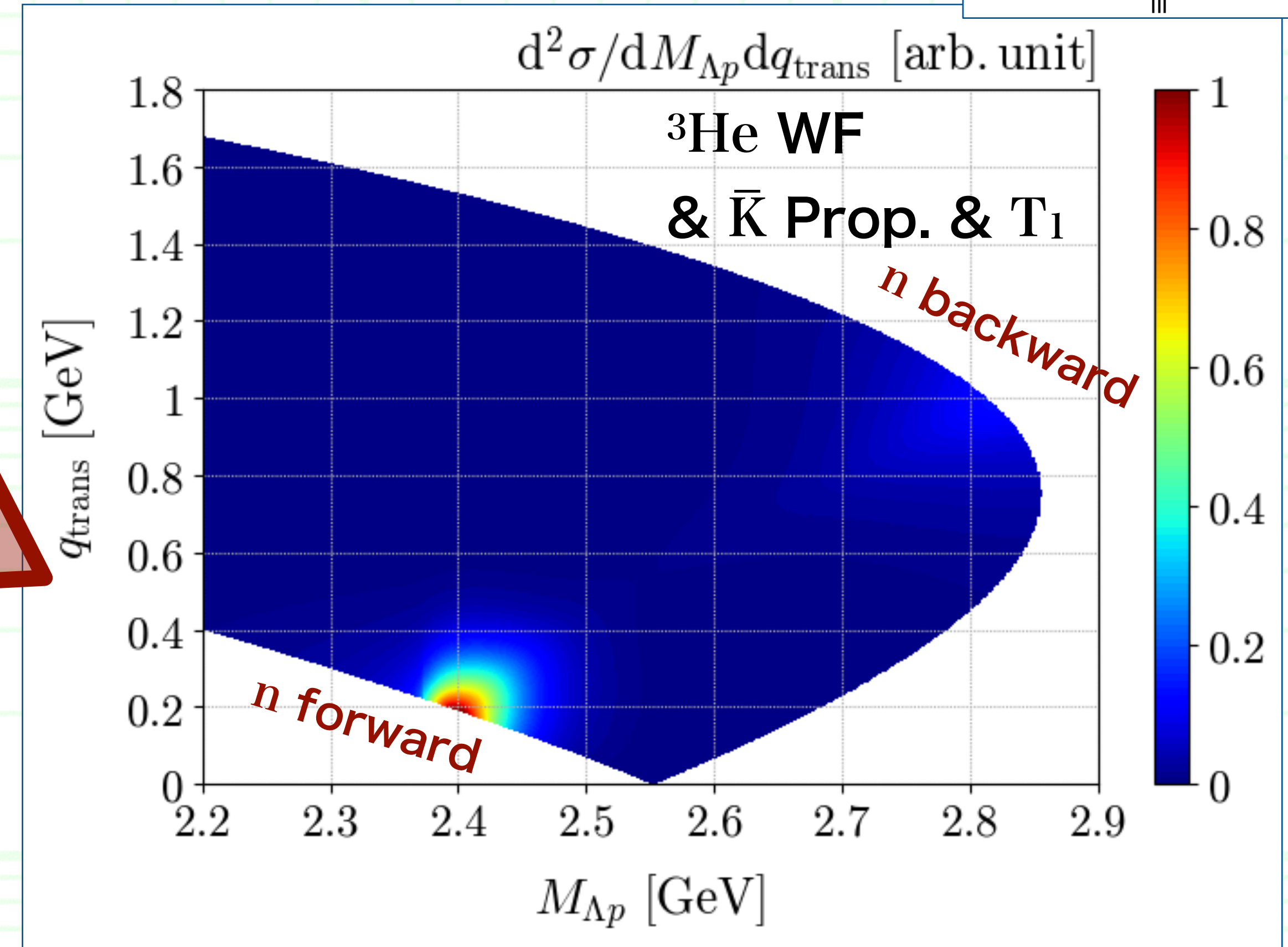
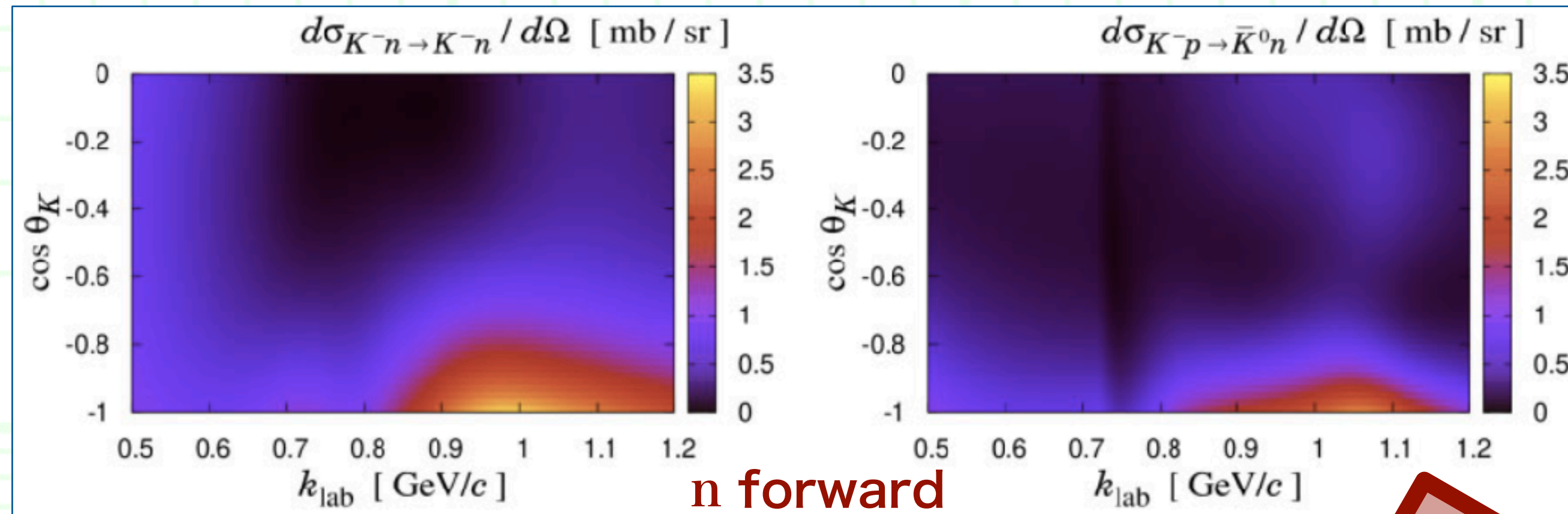
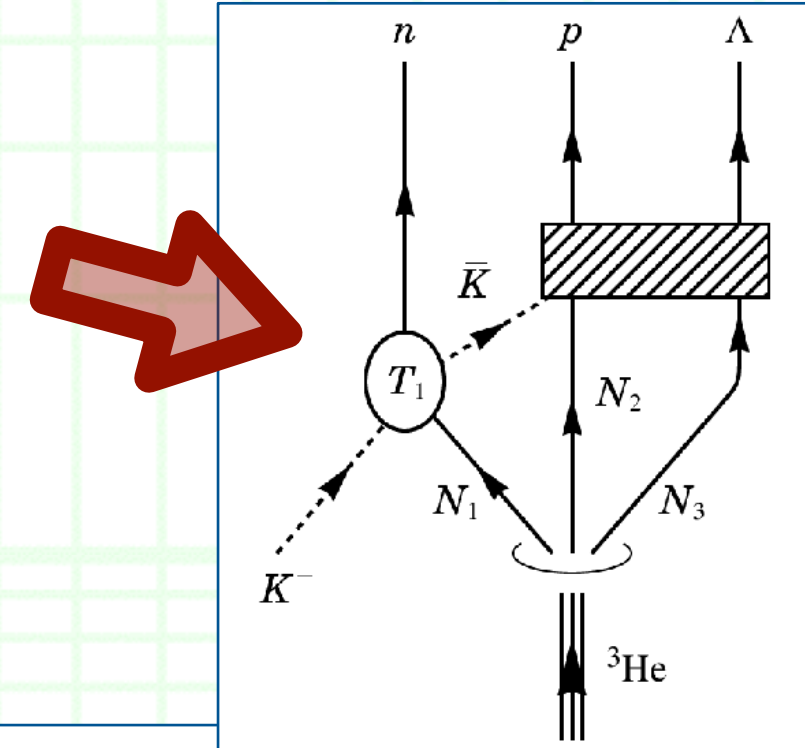
3. The $\bar{K}NN$ system in $K^- 3\text{He} \rightarrow \Lambda pn$

++ & the 1st step $\bar{K}N \rightarrow \bar{K}N ++$

■ Inclusion of the 1st step T_1 ($\bar{K}N \rightarrow \bar{K}N$, $P_K = 1 \text{ GeV}/c$).

→ Again employ **Kamano et al. on-shell amplitude.**

Kamano, Nakamura, Lee, and Sato, Phys. Rev. C90 (2014) 065204.



$$\text{Amp.} = \int \frac{d^3 p_\lambda}{(2\pi)^3} \frac{w_\nu(p_\lambda) T_1^{(\text{Kamano})}}{(q_{\text{ex}}^0)^2 - |\mathbf{p}_\lambda + \mathbf{p}_K - \mathbf{p}_n|^2 - m_K^2 + i0}$$

□ Dominated by the **small momentum transfer region** $q_{\text{trans}} < 0.4 \text{ GeV}$, again.

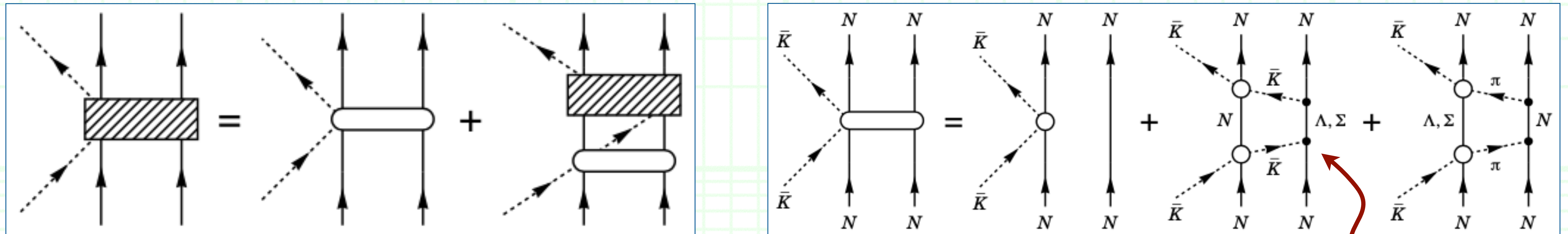
3. The $\bar{K}NN$ system in $K^- \text{ } ^3\text{He} \rightarrow \Lambda p n$

++ The AGS equation and $\bar{K}NN \rightarrow \Lambda p$ ++

■ Solve the Faddeev Eq. with the explicit NN interaction as well as $\bar{K}N$.

□ NN interaction: Separable form which reproduces the NN(1S_0 , 3S_1) phase shift.

□ $\bar{K}N$ interaction: $\bar{K}N \rightarrow \bar{K}N$ in chiral dynamics & Two-nucleon absorption.



→ Solve the Alt-Grassberger-Sandhas (AGS) integral equation.

$\bar{K}N\Lambda$ vertex

$$X_{i,j}(E, p_i, p_j) = Z_{i,j}(E, p_i, p_j) + \sum_n \int_0^\infty \frac{dk}{2\pi^2} k^2 Z_{i,n}(E, p_i, k) T_n(E, k) X_{n,j}(E, k, p_j)$$

– $\bar{K}NN$ bound state is generated !

3. The $\bar{K}NN$ system in $K^- \text{}^3\text{He} \rightarrow \Lambda pn$

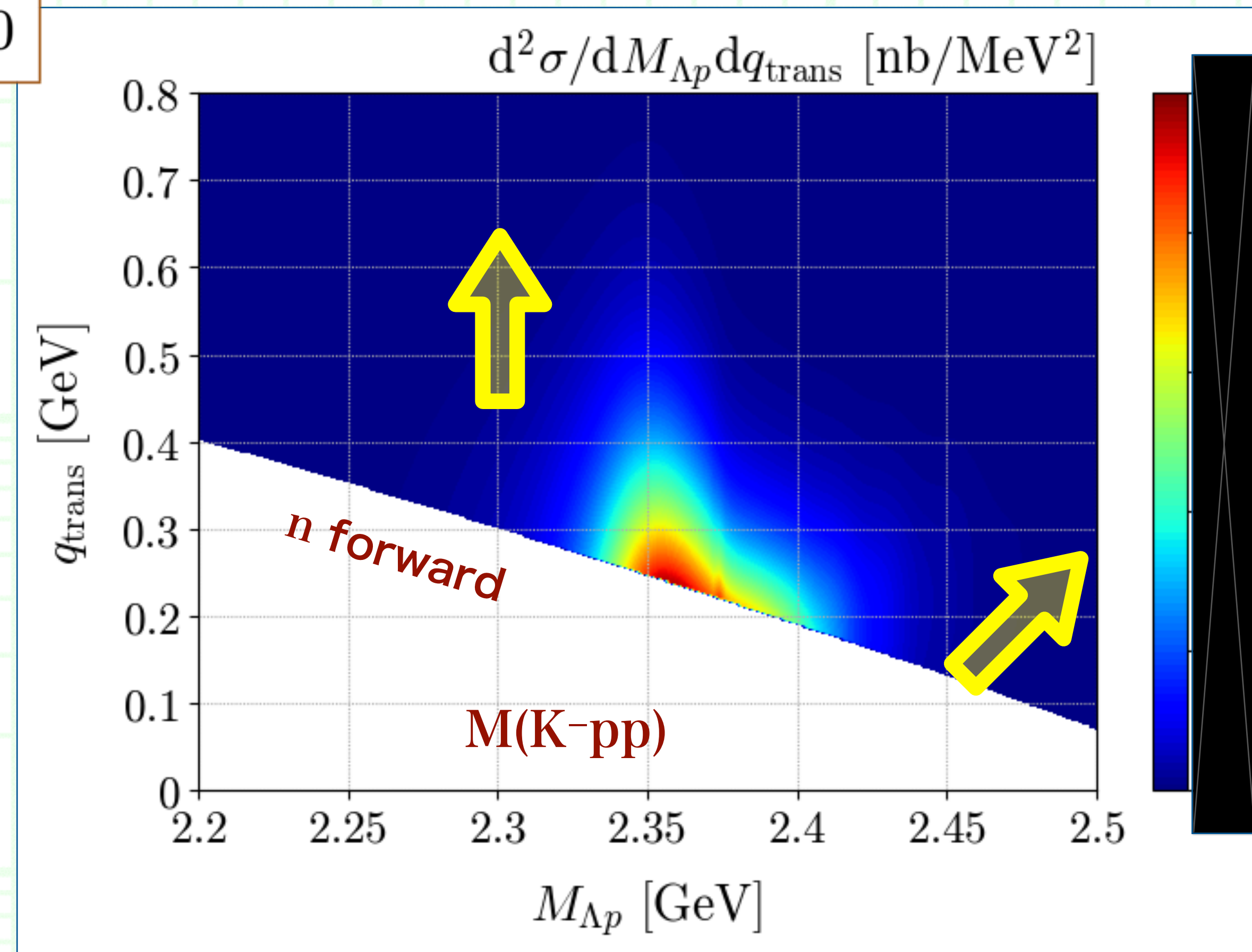
++ Reaction cross section ++

- Inclusion of the $\bar{K}NN \rightarrow \Lambda p$ part. \rightarrow Full calculation.

$$\text{Amp.} = \int \frac{d^3 p_\lambda}{(2\pi)^3} \frac{w_\nu(p_\lambda) T_1^{(\text{Kamano})} T_{\bar{K}NN \rightarrow \Lambda p}}{(q_{\text{ex}}^0)^2 - |\mathbf{p}_\lambda + \mathbf{p}_K - \mathbf{p}_n|^2 - m_K^2 + i0}$$

$$T_{\bar{K}NN \rightarrow \Lambda p} = \int \frac{d^3 p_\rho}{(2\pi)^3} \frac{v_\nu(p_\rho) V_{\bar{K}N\Lambda} X_{\text{AGS}}}{(q'_{\text{ex}})^2 - m_K^2 + i0}$$

- We have two trends.
 - Below the $\bar{K}NN$ threshold:
The $\bar{K}NN$ bound state signal.
 - Above the $\bar{K}NN$ threshold:
The quasi-free \bar{K} propagation.
- These two trends are consistent with the Exp. data.



3. The $\bar{K}NN$ system in $K^- ^3\text{He} \rightarrow \Lambda pn$

++ Summary and outlook ++

■ Consistency of the $K^- ^3\text{He} \rightarrow \Lambda pn$ reaction cross section.

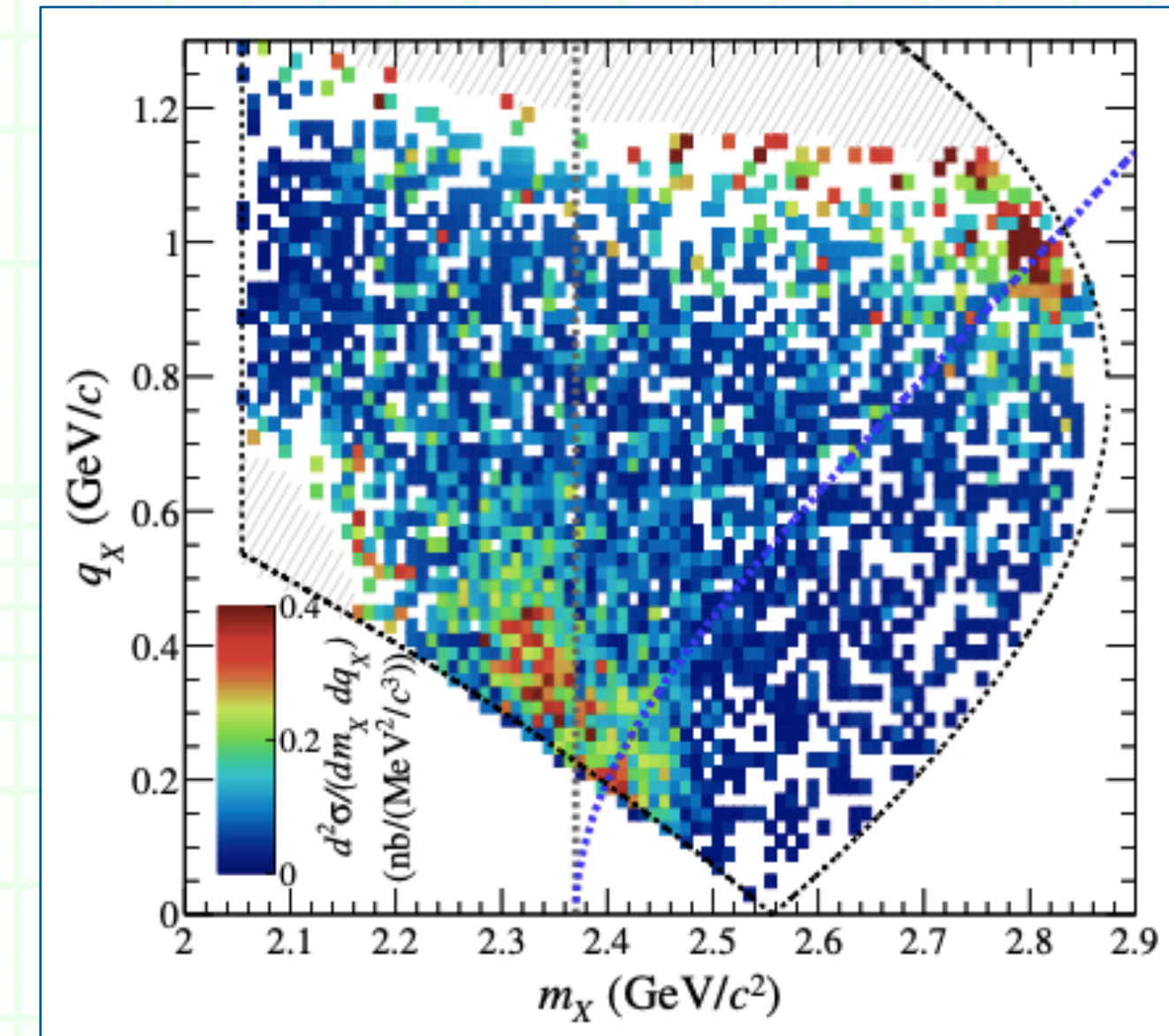
1. Appearance of the quasi-free kaon line.

→ \bar{K} is indeed mediated.

2. The q independent signal below the $\bar{K}NN$ threshold.

→ Strongly support the existence of the $\bar{K}NN$ bound state.

Yamaga et al. [J-PARC E15],
Phys. Rev. C102 (2020) 044002.

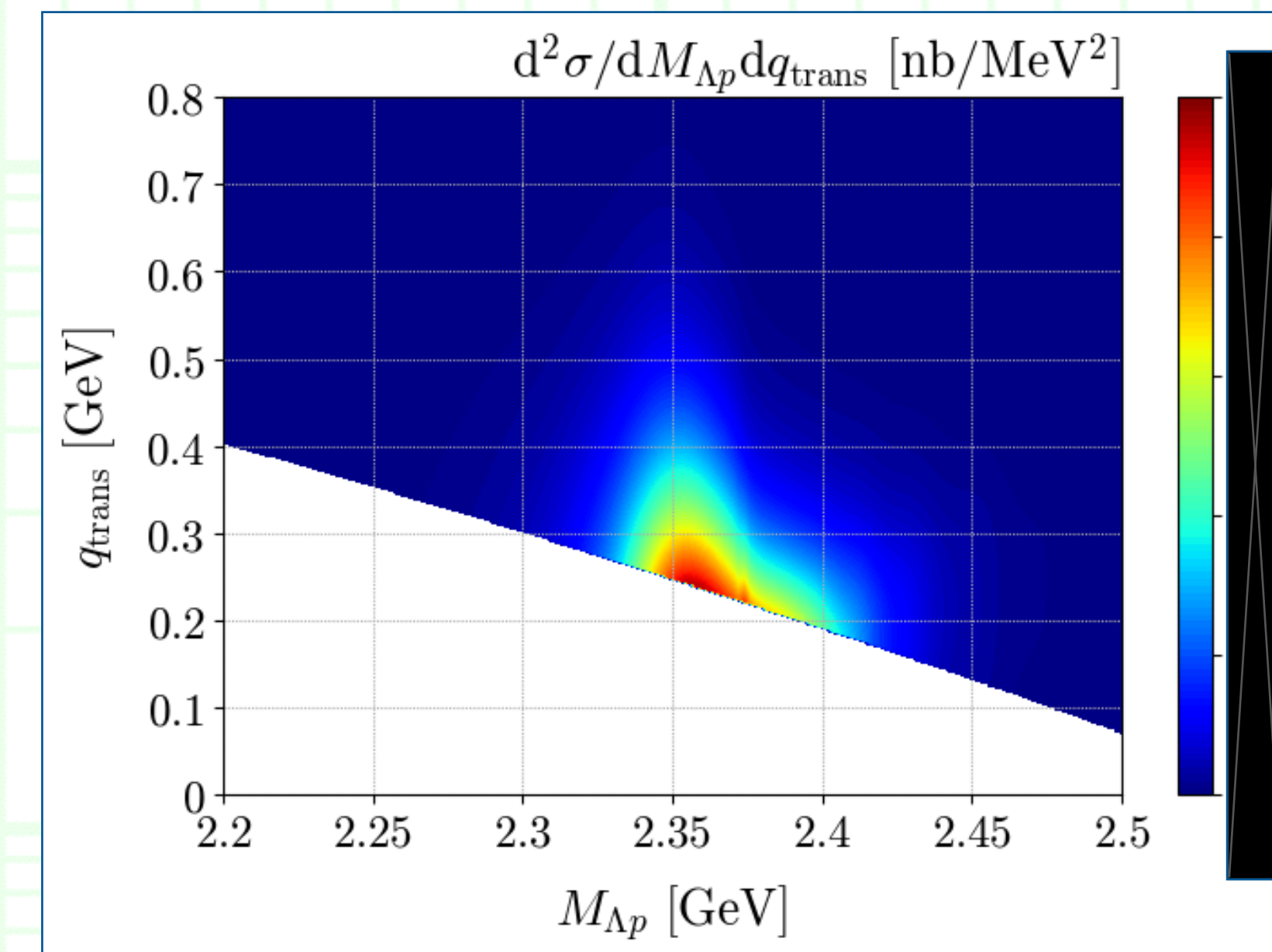


■ Then, we can investigate the scattering amplitude.

Pole position of the $\bar{K}NN$ bound state ?

Spin/parity of the bound state ?

...



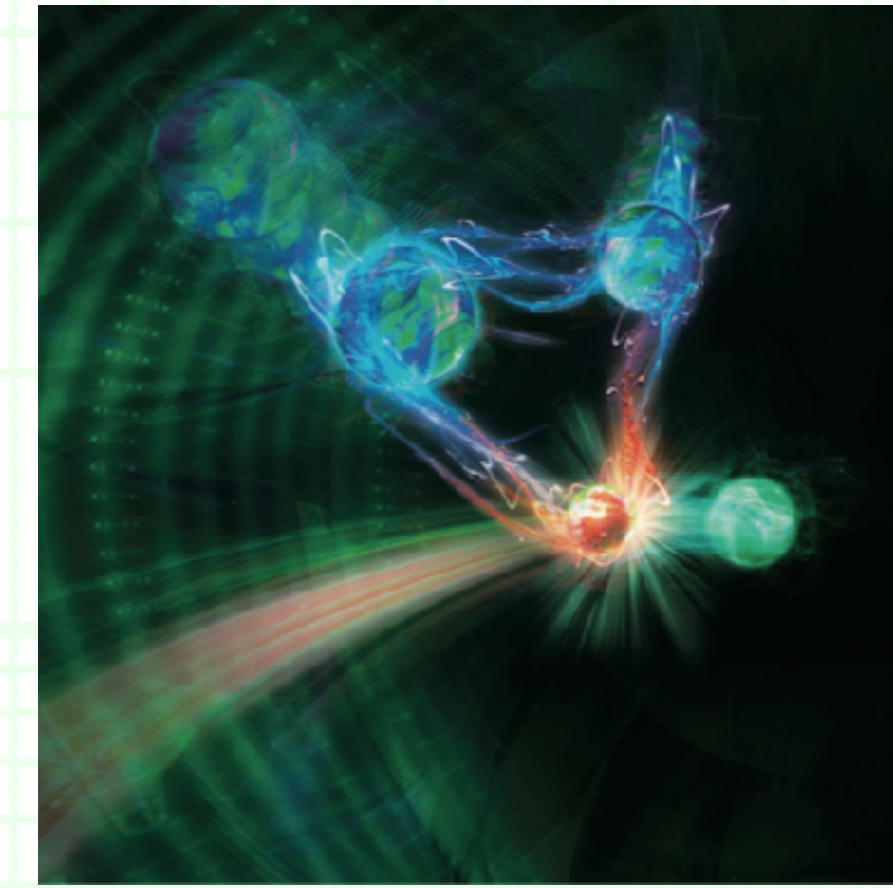
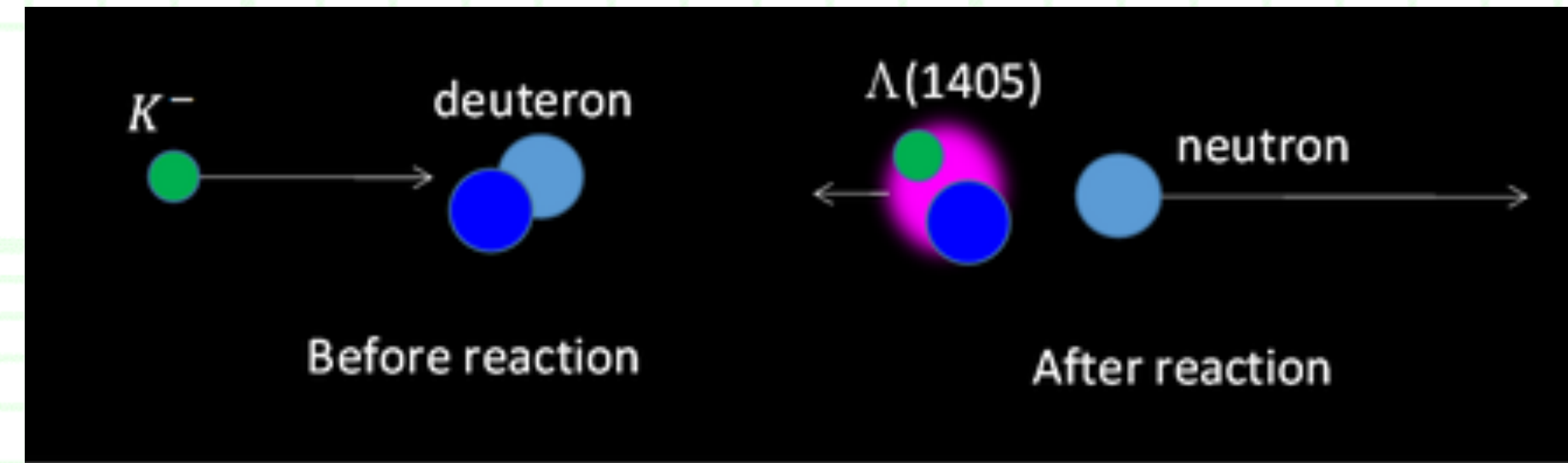
4. Summary

4. Summary

■ Are the peaks of the J-PARC Exps. the signals of the $\bar{K}N$ and $\bar{K}NN$ systems ?

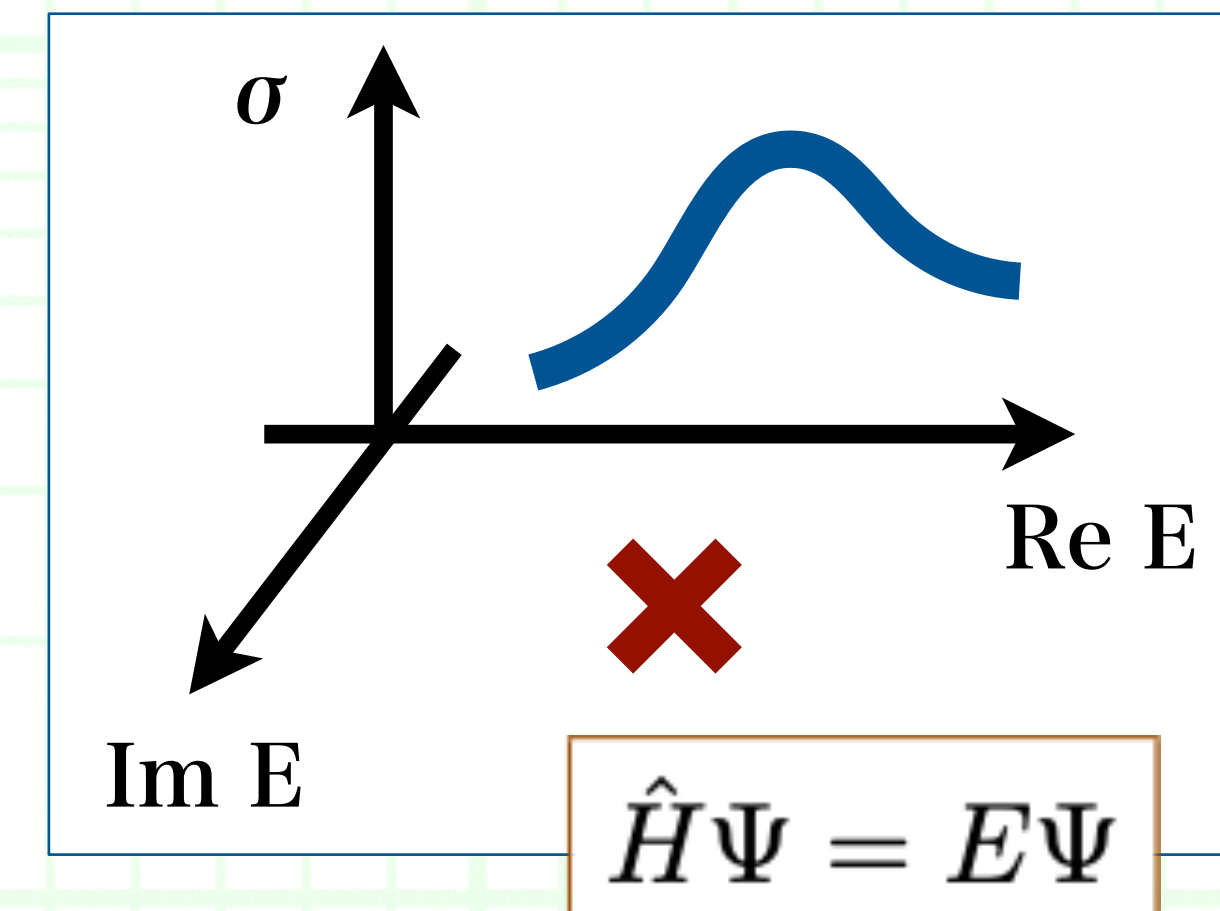
— Yes. \bar{K} is indeed mediated !

The peak structure is consistent with our calculations.



■ We have constructed scattering amplitudes of the reaction $K^- d \rightarrow \pi \Sigma n$ and $K^- {}^3\text{He} \rightarrow \Lambda p n$ to connect the resonance pole and Exp. observables.

□ We want to extract the information on the resonance pole.
The research is on going ...



**Thank you very much
for your kind attention !**