Fine-tuning of the $\bar{K}NN$ and $\bar{K}NNN$ calculations

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Interest to antikaon-nucleon systems: quasi-bound state in the K^-pp system
$\rightarrow$ experimental and theoretical efforts with different results

Experimental evidences (FINUDA, DISTO); J-PARK E15 experiment: clear observation of the K^-pp quasi-bound state with
$BE = 42 \pm 3$ MeV, $\Gamma = 100 \pm 7$ MeV.

The problem: big difference between theoretical and experimental widths

Our K^-pp calculations: Faddeev-type dynamically exact equations with coupled $\bar{K}NN - \pi\Sigma N$ channels, different $\bar{K}N - \pi\Sigma - \pi\Lambda$ and NN interactions

What could change the theoretical results:

- More accurate model of the $\Sigma N - \Lambda N$ interaction ("different models could change the three-body K^-pp pole position quite strongly")
- Inclusion of the πN interaction ("variation of the interaction parameters lead to smaller differences"),
- Direct inclusion of the $\pi\Lambda N$ channel ($\bar{K}NN - \pi\Sigma N - \pi\Lambda N$ calculations, "strong dependence")

$$U_{ij}^{\alpha\beta} = \delta_{\alpha\beta}(1-\delta_{ij})(G_0^{\alpha})^{-1} + \sum_{k=1}^{3}\sum_{\gamma=1}^{5}(1-\delta_{ik})T_k^{\alpha\gamma}G_0{}^{\gamma}U_{kj}^{\gamma\beta}$$

$\bar{K}N$ interaction is stronlgly coupled to $\pi\Sigma$ via $\Lambda(1405)$ resonance $\rightarrow$ $\pi\Sigma$ and $\pi\Lambda$ channel was included directly. Particle channels:

$$\alpha = 1 : |\bar{K}_1 N_2 N_3\rangle, \qquad \alpha = 2 : |\pi_1 \Sigma_2 N_3\rangle \qquad \alpha = 3 : |\pi_1 N_2 \Sigma_3\rangle$$
$$\alpha = 4 : |\pi_1 \Lambda_2 N_3\rangle \qquad \alpha = 5 : |\pi_1 N_2 \Lambda_3\rangle$$

Separable form of the potentials:

$$V_i^{\alpha\beta} = \lambda_i^{\alpha\beta}|g_i^{\alpha}\rangle\langle g_i^{\beta}| \qquad \rightarrow \qquad T_i^{\alpha\beta} = |g_i^{\alpha}\rangle\tau_i^{\alpha\beta}\langle g_i^{\beta}|$$

the three-body equations are rewritten as

$$X_{ij}^{\alpha\beta}(z) = \delta_{\alpha\beta}Z_{ij}^{\alpha} + \sum_{k=1}^{3}\sum_{\gamma=1}^{5}Z_{ik}^{\alpha}\,\tau_k^{\alpha\gamma}\,X_{kj}^{\gamma\beta}$$

Two-body $\bar{K}N - \pi\Sigma - \pi\Lambda$ T-matrices are necessary.

Three antikaon-nucleon interaction models:

- phenomenological $\bar{K}N - \pi\Sigma - \pi\mathbf{\Lambda}$ with one-pole $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma - \pi\mathbf{\Lambda}$ with two-pole $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential, two-pole $\Lambda(1405)$

reproduce (with the same level of accuracy):

- $1s$ level shift and width of kaonic hydrogen *(SIDDHARTA)*
  direct inclusion of Coulomb interaction, no Deser-type formula used
- Cross-sections of $K^-p \to K^-p$ and $K^-p \to MB$ reactions
- Threshold branching ratios γ, R_c and R_n
- ◦ $\Lambda(1405)$ resonance (*one- or two-pole structure*)
  $M^{PDG}_{\Lambda(1405)} = 1405.1^{+1.3}_{-1.0}$ MeV, $\Gamma^{PDG}_{\Lambda(1405)} = 50.5 \pm 2.0$ MeV [*PDG (2023)*]

New $\bar{K}N$ potentials, K^-p scattering

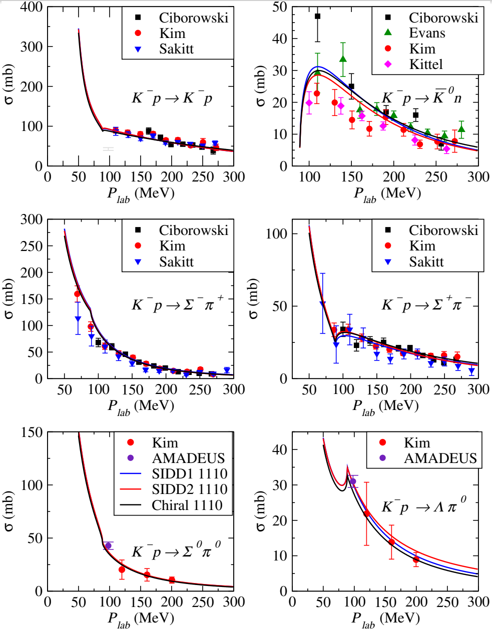


Figure: New $V_{\bar{K}N}$ potentials: one-pole, two-pole phenomenological and chirally motivated

Physical characteristics of the three $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials: $1s$ level shift and width, strong pole(s), γ, R_c, R_n threshold branching ratios, a_{K^-p} scattering length (physical masses in all channels, Coulomb in K^-p).

	$V^{1,\mathrm{SIDD}}_{\bar{K}N-\pi\Sigma-\pi\Lambda}$	$V^{2,\mathrm{SIDD}}_{\bar{K}N-\pi\Sigma-\pi\Lambda}$	$V^{\mathrm{Chiral}}_{\bar{K}N-\pi\Sigma-\pi\Lambda}$	Exp
ΔE_{1s}	-322.6	-323.5	-311.6	$-283 \pm 36 \pm 6$
Γ_{1s}	645.4	633.8	605.8	$541 \pm 89 \pm 22$
E_1	$1429.5 - i\,35.0$	$1430.9 - i\,41.6$	$1429.6 - i\,33.2$	
E_2	–	$1380.4 - i\,79.9$	$1367.8 - i\,66.5$	
γ	2.35	2.36	2.36	2.36 ± 0.04
R_c	0.666	0.664	0.664	0.664 ± 0.011
R_n	0.190	0.189	0.190	0.189 ± 0.015
a_{K^-p}	$-0.77 + i\,0.97$	$-0.78 + i\,0.95$	$-0.75 + i\,0.90$	

Two-term Separable New potential (TSN) of nucleon-nucleon interaction

[N.V.S. Few-body syst 61, 27 (2020)]

$$V_{NN}^{\rm TSN}(k,k') = \sum_{m=1}^{2} g_m(k)\,\lambda_m\; g_m(k')\,,$$

$$g_m(k) = \sum_{n=1}^{3} \frac{\gamma_{mn}}{(\beta_{mn})^2 + k^2}, \quad \text{for } m = 1,2$$

fitted to Argonne $V18$ potential *[R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. C 51, 38 (1995)]* phase shifts

Triplet and singlet scattering lengths a and effective ranges $r_{\rm eff}$

$$a_{np}^{\rm TSN} = -5.400\,{\rm fm}, \qquad r_{{\rm eff},np}^{\rm TSN} = 1.744\,{\rm fm}$$
$$a_{pp}^{\rm TSN} = 16.325\,{\rm fm}, \qquad r_{{\rm eff},pp}^{\rm TSN} = 2.792\,{\rm fm},$$

deuteron binding energy $E_{\rm deu} = 2.2246$ MeV.

New NN potential

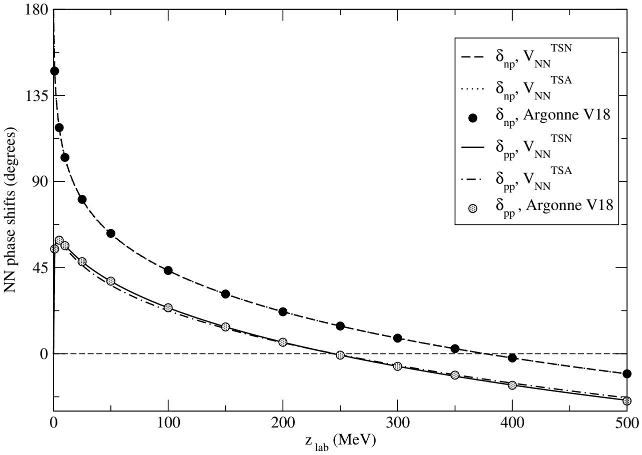


Figure: Phase shifts of *np* and *pp* scattering calculated using the new V_{NN}^{TSN} and $V_{NN}^{\mathrm{TSA-B}}$ potentials plus phase shifts of Argonne $V18$

Spin- and isospin-dependent potential

$$V^{\Sigma N}_{I,S}(k,k') = \lambda^{\Sigma N}_{I,S}\; g^{\Sigma N}_{I,S}(k)\, g^{\Sigma N}_{I,S}(k'), \quad g^{\Sigma N}_{I,S}(k) = \frac{1}{(k^2+\beta^{\Sigma N}_{I,S})^2}$$

- $I = 1/2$: Two-channel $\Sigma N - \Lambda N$ potential, real parameters
- $I = 3/2$: One-channel case, real parameters

Parameters were fitted to experimental cross-sections and scattering lengths from an "advanced" potential

[Haidenbauer et al., Eur. Phys. J. A 59 (2023) 63]

ΣN and ΛN cross-sections: theory vs. experiment

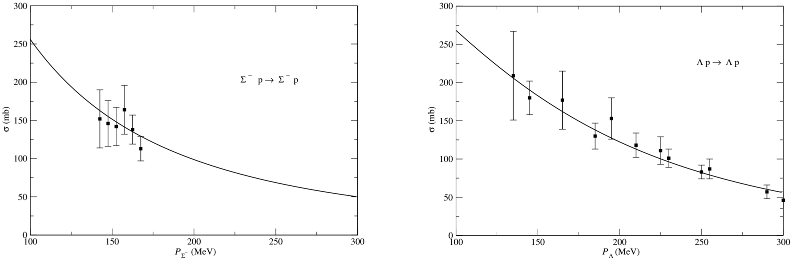


Figure: Comparison with the experimental data on ΣN and ΛN cross-sections

ΣN and ΛN cross-sections: theory vs. experiment

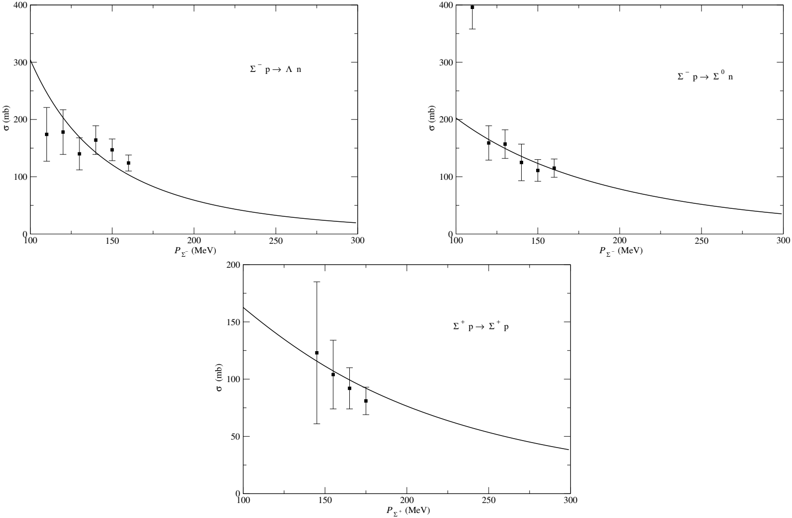


Figure: Comparison with the experimental data on ΣN and ΛN cross-sections

Scattering lengths given by the new $\Sigma N - \Lambda N$ potential (in fm) compared to those of the "advanced" potential (signs are opposite).

	$V^{\Sigma N}_{I,S}$	"Advanced" $V^{\Sigma N}$
$a^{\Sigma N}_{I=1/2,S=0}$	$-1.40 + i\,0.00$	$-1.03 + i\,0.00$
$a^{\Sigma N}_{I=1/2,S=1}$	$-0.03 + i\,5.77$	$-2.60 + i\,2.56$
$a^{\Sigma N}_{I=3/2,S=0}$	2.78	3.47
$a^{\Sigma N}_{I=3/2,S=1}$	-0.37	-0.41
$a^{\Lambda N}_{I=1/2,S=0}$	2.57	2.80
$a^{\Lambda N}_{I=1/2,S=1}$	1.49	1.56

Isospin-dependent potential

$$V_I^{\pi N}(k,k') = \lambda_I^{\pi N}\; g_I^{\pi N}(k)\, g_I^{\pi N}(k'), \quad g_I^{\pi N}(k) = \frac{1}{(k^2 + \beta_I^{\pi N})^2}$$

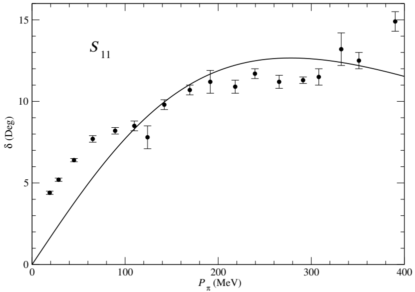
Parameters were fitted to the S-wave phase shifts and scattering lengths:

- $a^{\rm Exp}_{\pi N, I=1/2} = 0.26$ fm
- $a^{\rm Exp}_{\pi N, I=3/2} = -0.11$ fm

Theoretical values:

- $a^{\rm Exp}_{\pi N, I=1/2} = 0.34$ fm
- $a^{\rm Exp}_{\pi N, I=3/2} = -0.34$ fm

πN phase shifts: theory vs. experiment



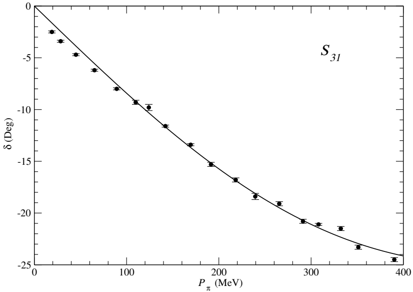


Figure: Comparison with the experimental data on πN phase shifts

K^-pp quasi-bound state: Three-channel three-body $\bar{K}NN - \pi\Sigma N - \pi\Sigma\Lambda$ calculations with new $V_{\Sigma N - \Lambda N}$ and $V_{\pi N}$ potentials compared to the previous results (two-channel three-body $\bar{K}NN - \pi\Sigma N - \pi\Sigma$ calculations with older potentials). Binding energies B_{K^-pp} (MeV) and widths Γ_{K^-pp} (MeV) are shown.

	$V^{1,\mathrm{SIDD}}_{\bar{K}N}$		$V^{2,\mathrm{SIDD}}_{\bar{K}N}$		$V^{\mathrm{Chiral}}_{\bar{K}N}$	
	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}
$V^{\mathrm{2ch}}_{\mathrm{Prev}\Sigma\mathrm{N},\pi\mathrm{N}}$	52.2	67.1	46.6	51.2	29.4	46.4
$V^{\mathrm{3ch}}_{\mathrm{New}\Sigma\mathrm{N},\pi\mathrm{N}}$	33.8	67.6	46.0	65.3	27.8	68.6

The four-body Faddeev-type AGS equations, written for separable potentials [*P. Grassberger, W. Sandhas, Nucl. Phys. B 2, 181-206 (1967)*]

$$\bar{U}^{\sigma\rho}_{\alpha\beta}(z) = (1-\delta_{\sigma\rho})(\bar{G_0}^{-1})_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta}(1-\delta_{\sigma\tau})\bar{T}^{\tau}_{\alpha\gamma}(z)(\bar{G_0})_{\gamma\delta}(z)\bar{U}^{\tau\rho}_{\delta\beta}(z),$$

$$\bar{U}^{\sigma\rho}_{\alpha\beta}(z) = \langle g_\alpha | G_0(z) U^{\sigma\rho}_{\alpha\beta}(z) G_0(z) | g_\beta \rangle,$$

$$\bar{T}^{\tau}_{\alpha\beta}(z) = \langle g_\alpha | G_0(z) U^{\tau}_{\alpha\beta}(z) G_0(z) | g_\beta \rangle, \quad (\bar{G_0})_{\alpha\beta}(z) = \delta_{\alpha\beta}\tau_\alpha(z).$$

Operators $\bar{U}^{\sigma\rho}_{\alpha\beta}$ and $\bar{T}^{\tau}_{\alpha\beta}$ contain four-body $U^{\sigma\rho}_{\alpha\beta}(z)$ and three-body $U^{\tau}_{\alpha\beta}(z)$ transition operators of the general form, correspondingly. Separable form of the "effective three-body potentials" :

$$\bar{T}^{\tau}_{\alpha\beta}(z) = |\bar{g}^{\tau}_{\alpha}\rangle \bar{\tau}^{\tau}_{\alpha\beta}(z) \langle \bar{g}^{\tau}_{\beta}|$$

→ the four-body equations can be rewritten as
[A. Casel, H. Haberzettl, W. Sandhas, Phys. Rev. C 25, 1738 (1982)]

$$\bar{X}^{\sigma\rho}_{\alpha\beta}(z) = \bar{Z}^{\sigma\rho}_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta} \bar{Z}^{\sigma\tau}_{\alpha\gamma}(z)\bar{\tau}^{\tau}_{\gamma\delta}(z)\bar{X}^{\tau\rho}_{\delta\beta}(z)$$

with new four-body transition $\bar{X}^{\sigma\rho}$ and kernel $\bar{Z}^{\sigma\rho}$ operators.

Partitions of the $\bar{K}NNN$ system

two partitions of $3+1$ type: $|\bar{K}+(NNN)\rangle$, $|N+(\bar{K}NN)\rangle$,
one of the $2+2$ type: $|(\bar{K}N)+(NN)\rangle$

$K^-ppn - \bar{K}^0nnp$ system: $\bar{K}NNN$ with $I^{(4)}=0, S^{(4)}=1/2,\ L^{(4)}=0$

- $\bar{K}NN\,(I^{(3)}=1/2, S^{(3)}=0$ or $1)$
- $NNN\,(I^{(3)}=1/2, S^{(3)}=1/2)$
- $\bar{K}N+NN\,(I^{(4)}=0, S^{(4)}=1/2)$ – a special system with two non--interacting pairs of particles; 3-body system of equations to be solved

Full system of equations was solved with:

- 1-term separable $\bar{K}N-\pi\Sigma-\pi\Lambda$ potential, 2-term separable NN potential (input)
- 1-term separabelized 3-body NNN, $\bar{K}NN$ "T-matrices" and "3-body" $\bar{K}N+NN$ "T-matrices"

EDPE/EDPA separabelization

Separabelization of $3+1$ and 2+2 amplitudes: Energy Dependent Pole Expansion/Approximation (EDPE/ EDPA) *[S. Sofianos, N.J. McGurk, H. Fiedeldeldey, Nucl. Phys. A 318, 295 (1979)]*

Three-body Faddeev-type AGS equations written in momentum basis:

$$X_{\alpha\beta}(p,p';z) = Z_{\alpha\beta}(p,p';z)+\sum_{\gamma=1}^{3}\int_0^\infty Z_{\alpha\gamma}(p,p'';z)\,\tau_\gamma(p'';z)\,X_{\gamma\beta}(p'',p';z)p''^2dp'',$$

eigenvalues λ_n and eigenfunctions $g_{n\alpha}(p;z)$ of the system – from

$$g_{n\alpha}(p;z) = \frac{1}{\lambda_n}\sum_{\gamma=1}^{3}\int_0^\infty Z_{\alpha\gamma}(p,p';z)\,\tau_\gamma(p';z)\,g_{n\gamma}(p';z)p'^2dp'$$

EDPE/EDPA method: solution of the eigenequations for a fixed energy z, usually $z = E_B$. After that energy dependent form-factors $g_{n\alpha}(p;z)$ and propagators $(\Theta(z))^{-1}_{mn}$ are calculated. The separable version of a three-body amplitude:

$$X_{\alpha\beta}(p,p';z) = \sum_{m,n=1}^{\infty} g_{m\alpha}(p;z)\,\Theta_{mn}(z)\,g_{n\beta}(p';z).$$

Dependence of the binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the $K^- ppn - \bar{K}^0 nnp$ system on three $\bar{K}N$ interaction models.

	Two-channel $\bar{K}N(-\pi\Sigma)$ potential		Three-channel $\bar{K}N(-\pi\Sigma - \pi\Lambda)$ potential	
	B	Γ	B	Γ
$V^{1,\mathrm{SIDD}}_{\bar{K}N}$	51.2	50.8	41.5	41.8
$V^{2,\mathrm{SIDD}}_{\bar{K}N}$	46.4	39.9	39.6	37.5
$V^{\mathrm{Chiral}}_{\bar{K}N}$	30.5	42.8	32.1	54.8

- Fine tuning of the $K^- pp$ quasi-bound state: calculations with the three-channel $\bar{K}N - \pi\Sigma\ - \pi\Lambda$ potentials, new version of the $\Sigma N - \Lambda N$ and πN potentials: Larger widths then before calculated using all three $\bar{K}NN$ potentials; the width are comparable
- Fine-tuning of the $\bar{K}NNN$ system: calculations with exact optical version of the three-channel $\bar{K}N - \pi\Sigma\ - \pi\Lambda$ potentials lead to strong changes in binding energies and/or widths

To do list:

- limitations on the potentials from femtoscopy measurements;
- influence of the new potentials on the $K^- np$ system (quasi-bound state remains?);
- renew the prediction for the kaonic deuterium