

Fine-tuning of the $\bar{K}NN$ and $\bar{K}NNN$ calculations

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Interest to antikaon-nucleon systems: quasi-bound state in the K^-pp system
→ experimental and theoretical efforts with different results

Experimental evidences (FINUDA, DISTO); J-PARK E15 experiment:
clear observation of the K^-pp quasi-bound state with
 $BE = 42 \pm 3$ MeV, $\Gamma = 100 \pm 7$ MeV.

The problem: big difference between theoretical and experimental widths

Our K^-pp calculations: Faddeev-type dynamically exact equations with
coupled $\bar{K}NN - \pi\Sigma N$ channels, different $\bar{K}N - \pi\Sigma - \pi\Lambda$ and NN interactions

What could change the theoretical results:

- More accurate model of the $\Sigma N - \Lambda N$ interaction (“different models could change the three-body K^-pp pole position quite strongly”)
- Inclusion of the πN interaction (“variation of the interaction parameters lead to smaller differences”),
- Direct inclusion of the $\pi\Lambda N$ channel ($\bar{K}NN - \pi\Sigma N - \pi\Lambda N$ calculations, “strong dependence”)

Faddeev-type three-body AGS equations, three channels

$$U_{ij}^{\alpha\beta} = \delta_{\alpha\beta}(1 - \delta_{ij})(G_0^\alpha)^{-1} + \sum_{k=1}^3 \sum_{\gamma=1}^5 (1 - \delta_{ik}) T_k^{\alpha\gamma} G_0^\gamma U_{kj}^{\gamma\beta}$$

$\bar{K}N$ interaction is strongly coupled to $\pi\Sigma$ via $\Lambda(1405)$ resonance \rightarrow $\pi\Sigma$ and $\pi\Lambda$ channel was included directly. Particle channels:

$$\begin{aligned} \alpha = 1 : |\bar{K}_1 N_2 N_3\rangle, & \quad \alpha = 2 : |\pi_1 \Sigma_2 N_3\rangle & \quad \alpha = 3 : |\pi_1 N_2 \Sigma_3\rangle \\ & \quad \alpha = 4 : |\pi_1 \Lambda_2 N_3\rangle & \quad \alpha = 5 : |\pi_1 N_2 \Lambda_3\rangle \end{aligned}$$

Separable form of the potentials:

$$V_i^{\alpha\beta} = \lambda_i^{\alpha\beta} |g_i^\alpha\rangle \langle g_i^\beta| \quad \rightarrow \quad T_i^{\alpha\beta} = |g_i^\alpha\rangle \tau_i^{\alpha\beta} \langle g_i^\beta|$$

the three-body equations are rewritten as

$$X_{ij}^{\alpha\beta}(z) = \delta_{\alpha\beta} Z_{ij}^\alpha + \sum_{k=1}^3 \sum_{\gamma=1}^5 Z_{ik}^\alpha \tau_k^{\alpha\gamma} X_{kj}^{\gamma\beta}$$

Two-body $\bar{K}N - \pi\Sigma - \pi\Lambda$ T -matrices are necessary.

Three antikaon-nucleon interaction models:

- phenomenological $\bar{K}N - \pi\Sigma - \pi\Lambda$ with **one-pole** $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma - \pi\Lambda$ with **two-pole** $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential, **two-pole** $\Lambda(1405)$

reproduce (with the same level of accuracy):

- 1s level shift and width of kaonic hydrogen (*SIDDHARTA*)
direct inclusion of Coulomb interaction, no Deser-type formula used
- Cross-sections of $K^-p \rightarrow K^-p$ and $K^-p \rightarrow MB$ reactions
- Threshold branching ratios γ , R_c and R_n
- $\Lambda(1405)$ resonance (*one- or two-pole structure*)
 $M_{\Lambda(1405)}^{PDG} = 1405.1_{-1.0}^{+1.3}$ MeV, $\Gamma_{\Lambda(1405)}^{PDG} = 50.5 \pm 2.0$ MeV [*PDG (2023)*]

New $\bar{K}N$ potentials, K^-p scattering

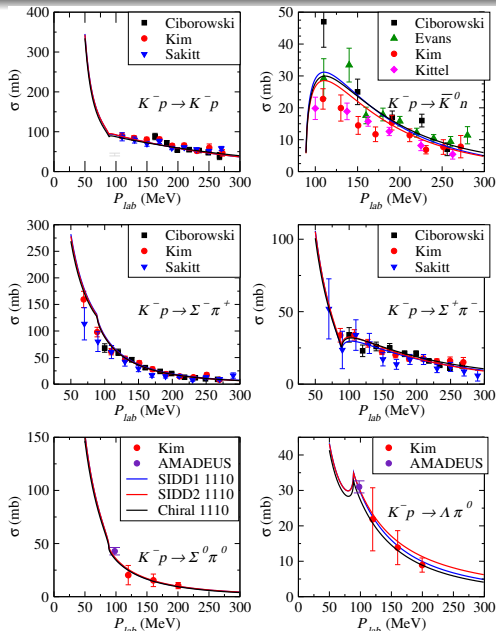


Figure: New $V_{\bar{K}N}$ potentials: one-pole, two-pole phenomenological and chirally motivated

Physical characteristics of the three new $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials

Physical characteristics of the three $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials: $1s$ level shift and width, strong pole(s), γ , R_c , R_n threshold branching ratios, a_{K-p} scattering length (physical masses in all channels, Coulomb in K^-p).

	$V_{\bar{K}N-\pi\Sigma-\pi\Lambda}^{1,\text{SIDD}}$	$V_{\bar{K}N-\pi\Sigma-\pi\Lambda}^{2,\text{SIDD}}$	$V_{\bar{K}N-\pi\Sigma-\pi\Lambda}^{\text{Chiral}}$	Exp
ΔE_{1s}	-322.6	-323.5	-311.6	$-283 \pm 36 \pm 6$
Γ_{1s}	645.4	633.8	605.8	$541 \pm 89 \pm 22$
E_1	$1429.5 - i 35.0$	$1430.9 - i 41.6$	$1429.6 - i 33.2$	
E_2	-	$1380.4 - i 79.9$	$1367.8 - i 66.5$	
γ	2.35	2.36	2.36	2.36 ± 0.04
R_c	0.666	0.664	0.664	0.664 ± 0.011
R_n	0.190	0.189	0.190	0.189 ± 0.015
a_{K-p}	$-0.77 + i 0.97$	$-0.78 + i 0.95$	$-0.75 + i 0.90$	

Two-term Separable New potential (TSN) of nucleon-nucleon interaction

[N. V.S. *Few-body syst* 61, 27 (2020)]

$$V_{NN}^{\text{TSN}}(k, k') = \sum_{m=1}^2 g_m(k) \lambda_m g_m(k'),$$

$$g_m(k) = \sum_{n=1}^3 \frac{\gamma_{mn}}{(\beta_{mn})^2 + k^2}, \quad \text{for } m = 1, 2$$

fitted to Argonne V18 potential [R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, *Phys. Rev. C* 51, 38 (1995)] phase shifts

Triplet and singlet scattering lengths a and effective ranges r_{eff}

$$a_{np}^{\text{TSN}} = -5.400 \text{ fm}, \quad r_{\text{eff}, np}^{\text{TSN}} = 1.744 \text{ fm}$$

$$a_{pp}^{\text{TSN}} = 16.325 \text{ fm}, \quad r_{\text{eff}, pp}^{\text{TSN}} = 2.792 \text{ fm},$$

deuteron binding energy $E_{\text{deu}} = 2.2246 \text{ MeV}$.

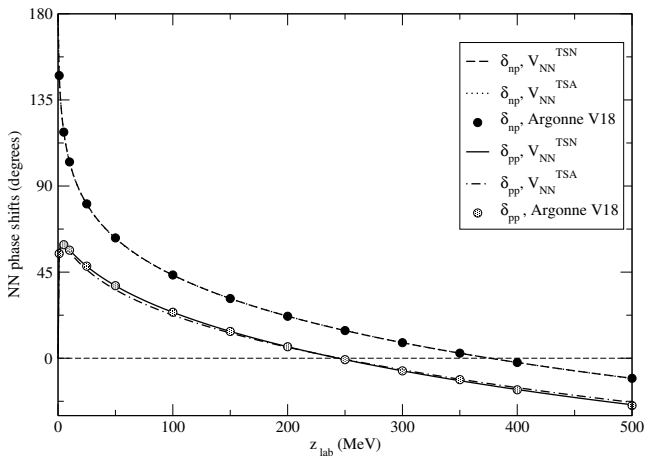


Figure: Phase shifts of np and pp scattering calculated using the new V_{NN}^{TSN} and V_{NN}^{TSA-B} potentials plus phase shifts of Argonne V18

Spin- and isospin-dependent potential

$$V_{I,S}^{\Sigma N}(k, k') = \lambda_{I,S}^{\Sigma N} g_{I,S}^{\Sigma N}(k) g_{I,S}^{\Sigma N}(k'), \quad g_{I,S}^{\Sigma N}(k) = \frac{1}{(k^2 + \beta_{I,S}^{\Sigma N})^2}$$

- $I = 1/2$: Two-channel $\Sigma N - \Lambda N$ potential, real parameters
- $I = 3/2$: One-channel case, real parameters

Parameters were fitted to **experimental cross-sections** and **scattering lengths** from an "advanced" potential

[Haidenbauer et al., *Eur. Phys. J. A* 59 (2023) 63]

ΣN and ΛN cross-sections: theory vs. experiment

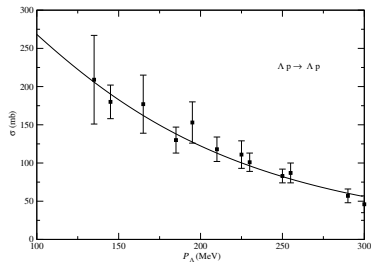
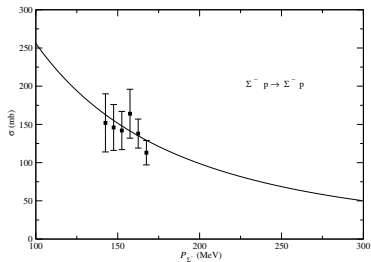


Figure: Comparison with the experimental data on ΣN and ΛN cross-sections

ΣN and ΛN cross-sections: theory vs. experiment

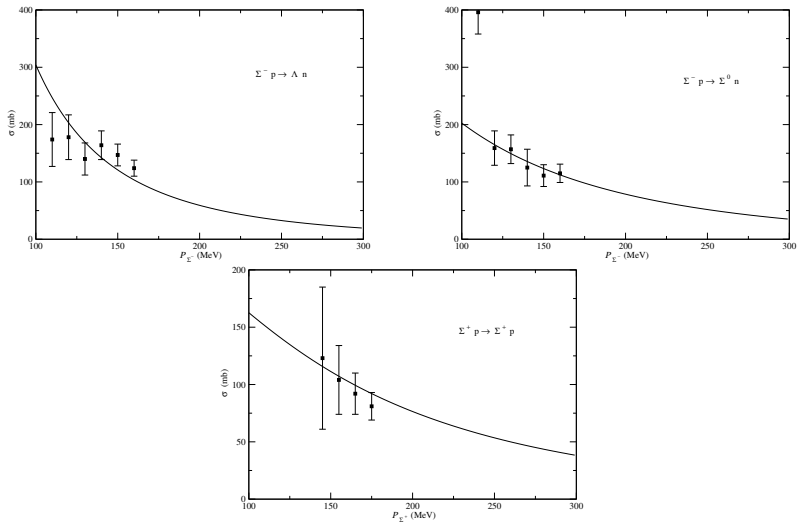


Figure: Comparison with the experimental data on ΣN and ΛN cross-sections

Scattering lengths given by the new $\Sigma N - \Lambda N$ potential

[Scattering lengths](#) given by the new $\Sigma N - \Lambda N$ potential (in fm) compared to those of the "advanced" potential (signs are opposite).

	$V_{I,S}^{\Sigma N}$	"Advanced" $V^{\Sigma N}$
$a_{I=1/2,S=0}^{\Sigma N}$	$-1.40 + i 0.00$	$-1.03 + i 0.00$
$a_{I=1/2,S=1}^{\Sigma N}$	$-0.03 + i 5.77$	$-2.60 + i 2.56$
$a_{I=3/2,S=0}^{\Sigma N}$	2.78	3.47
$a_{I=3/2,S=1}^{\Sigma N}$	-0.37	-0.41
$a_{I=1/2,S=0}^{\Lambda N}$	2.57	2.80
$a_{I=1/2,S=1}^{\Lambda N}$	1.49	1.56

Isospin-dependent potential

$$V_I^{\pi N}(k, k') = \lambda_I^{\pi N} g_I^{\pi N}(k) g_I^{\pi N}(k'), \quad g_I^{\pi N}(k) = \frac{1}{(k^2 + \beta_I^{\pi N})^2}$$

Parameters were fitted to the S -wave phase shifts and scattering lengths:

- $a_{\pi N, I=1/2}^{\text{Exp}} = 0.26 \text{ fm}$
- $a_{\pi N, I=3/2}^{\text{Exp}} = -0.11 \text{ fm}$

Theoretical values:

- $a_{\pi N, I=1/2}^{\text{Exp}} = 0.34 \text{ fm}$
- $a_{\pi N, I=3/2}^{\text{Exp}} = -0.34 \text{ fm}$

πN phase shifts: theory vs. experiment

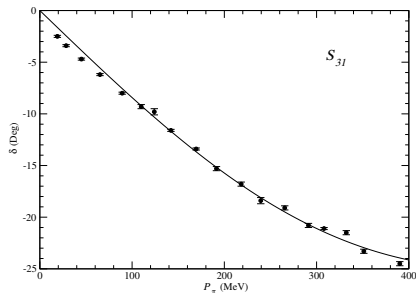
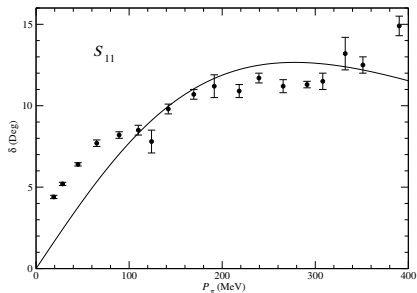


Figure: Comparison with the experimental data on πN phase shifts

K^-pp quasi-bound state: Three-channel three-body $\bar{K}NN - \pi\Sigma N - \pi\Sigma\Lambda$ calculations with new $V_{\Sigma N-\Lambda N}$ and $V_{\pi N}$ potentials compared to the previous results (two-channel three-body $\bar{K}NN - \pi\Sigma N - \pi\Sigma$ calculations with older potentials). Binding energies B_{K^-pp} (MeV) and widths Γ_{K^-pp} (MeV) are shown.

	$V_{\bar{K}N}^{1,\text{SIDD}}$		$V_{\bar{K}N}^{2,\text{SIDD}}$		$V_{\bar{K}N}^{\text{Chiral}}$	
	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}	B_{K^-pp}	Γ_{K^-pp}
$V_{\text{Prev}\Sigma N, \pi N}^{2\text{ch}}$	52.2	67.1	46.6	51.2	29.4	46.4
$V_{\text{New}\Sigma N, \pi N}^{3\text{ch}}$	33.8	67.6	46.0	65.3	27.8	68.6

Four-body equations

The four-body Faddeev-type AGS equations, written for separable potentials [P. Grassberger, W. Sandhas, Nucl. Phys. B 2, 181-206 (1967)]

$$\begin{aligned}\bar{U}_{\alpha\beta}^{\sigma\rho}(z) &= (1 - \delta_{\sigma\rho})(\bar{G}_0^{-1})_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta} (1 - \delta_{\sigma\tau})\bar{T}_{\alpha\gamma}^{\tau}(z)(\bar{G}_0)_{\gamma\delta}(z)\bar{U}_{\delta\beta}^{\tau\rho}(z), \\ \bar{U}_{\alpha\beta}^{\sigma\rho}(z) &= \langle g_\alpha | G_0(z) U_{\alpha\beta}^{\sigma\rho}(z) G_0(z) | g_\beta \rangle, \\ \bar{T}_{\alpha\beta}^{\tau}(z) &= \langle g_\alpha | G_0(z) U_{\alpha\beta}^{\tau}(z) G_0(z) | g_\beta \rangle, \quad (\bar{G}_0)_{\alpha\beta}(z) = \delta_{\alpha\beta} \tau_\alpha(z).\end{aligned}$$

Operators $\bar{U}_{\alpha\beta}^{\sigma\rho}$ and $\bar{T}_{\alpha\beta}^{\tau}$ contain four-body $U_{\alpha\beta}^{\sigma\rho}(z)$ and three-body $U_{\alpha\beta}^{\tau}(z)$ transition operators of the general form, correspondingly. Separable form of the "effective three-body potentials" :

$$\bar{T}_{\alpha\beta}^{\tau}(z) = |\bar{g}_\alpha^\tau\rangle \bar{\tau}_{\alpha\beta}^{\tau}(z) \langle \bar{g}_\beta^\tau|$$

→ the four-body equations can be rewritten as

[A. Casel, H. Haberzettl, W. Sandhas, Phys. Rev. C 25, 1738 (1982)]

$$\bar{X}_{\alpha\beta}^{\sigma\rho}(z) = \bar{Z}_{\alpha\beta}^{\sigma\rho}(z) + \sum_{\tau,\gamma,\delta} \bar{Z}_{\alpha\gamma}^{\sigma\tau}(z) \bar{\tau}_{\gamma\delta}^{\tau}(z) \bar{X}_{\delta\beta}^{\tau\rho}(z)$$

with new four-body transition $\bar{X}^{\sigma\rho}$ and kernel $\bar{Z}^{\sigma\rho}$ operators.

Partitions of the $\bar{K}NNN$ system

two partitions of 3 + 1 type: $|\bar{K} + (NNN)\rangle, |N + (\bar{K}NN)\rangle,$

one of the 2 + 2 type: $|(\bar{K}N) + (NN)\rangle$

$K^-ppn - \bar{K}^0nnp$ system: $\bar{K}NNN$ with $I^{(4)} = 0, S^{(4)} = 1/2, L^{(4)} = 0$

- $\bar{K}NN$ ($I^{(3)} = 1/2, S^{(3)} = 0$ or 1)
- NNN ($I^{(3)} = 1/2, S^{(3)} = 1/2$)
- $\bar{K}N + NN$ ($I^{(4)} = 0, S^{(4)} = 1/2$) – a special system with two non-interacting pairs of particles; 3-body system of equations to be solved

Full system of equations was solved with:

- 1-term separable $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential, 2-term separable NN potential (input)
- 1-term separabilized 3-body $NNN, \bar{K}NN$ "T-matrices" and "3-body" $\bar{K}N + NN$ "T-matrices"

Separabelization of 3 + 1 and 2+2 amplitudes: **Energy Dependent Pole Expansion/Approximation (EDPE/ EDPA)** [*S. Sofianos, N.J. McGurk, H. Fiedeldeldey, Nucl. Phys. A 318, 295 (1979)*]

Three-body Faddeev-type AGS equations written in momentum basis:

$$X_{\alpha\beta}(p, p'; z) = Z_{\alpha\beta}(p, p'; z) + \sum_{\gamma=1}^3 \int_0^{\infty} Z_{\alpha\gamma}(p, p''; z) \tau_{\gamma}(p''; z) X_{\gamma\beta}(p'', p'; z) p''^2 dp'',$$

eigenvalues λ_n and eigenfunctions $g_{n\alpha}(p; z)$ of the system – from

$$g_{n\alpha}(p; z) = \frac{1}{\lambda_n} \sum_{\gamma=1}^3 \int_0^{\infty} Z_{\alpha\gamma}(p, p'; z) \tau_{\gamma}(p'; z) g_{n\gamma}(p'; z) p'^2 dp'$$

EDPE/EDPA method: solution of the eigenequations for a fixed energy z , usually $z = E_B$. After that energy dependent form-factors $g_{n\alpha}(p; z)$ and propagators $(\Theta(z))_{mn}^{-1}$ are calculated. The separable version of a three-body amplitude:

$$X_{\alpha\beta}(p, p'; z) = \sum_{m,n=1}^{\infty} g_{m\alpha}(p; z) \Theta_{mn}(z) g_{n\beta}(p'; z).$$

Results: $\bar{K}NNN$ quasi-bound state

Dependence of the binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the $K^-ppn - \bar{K}^0nnp$ system on three $\bar{K}N$ interaction models.

	Two-channel		Three-channel	
	$\bar{K}N(-\pi\Sigma)$	potential	$\bar{K}N(-\pi\Sigma - \pi\Lambda)$	potential
	B	Γ	B	Γ
$V_{\bar{K}N}^{1,\text{SIDD}}$	51.2	50.8	41.5	41.8
$V_{\bar{K}N}^{2,\text{SIDD}}$	46.4	39.9	39.6	37.5
$V_{\bar{K}N}^{\text{Chiral}}$	30.5	42.8	32.1	54.8

- Fine tuning of the K^-pp quasi-bound state: calculations with the three-channel $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials, new version of the $\Sigma N - \Lambda N$ and πN potentials: **Larger widths** then before calculated using all three $\bar{K}NN$ potentials; the width **are comparable**
- Fine-tuning of the $\bar{K}NNN$ system: calculations with exact optical version of the three-channel $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials lead to strong changes in binding energies and/or widths

To do list:

- limitations on the potentials from femtoscopy measurements;
- influence of the new potentials on the K^-np system (quasi-bound state remains?);
- renew the prediction for the kaonic deuterium