SU(3)-Symmetric Density Functional Theory

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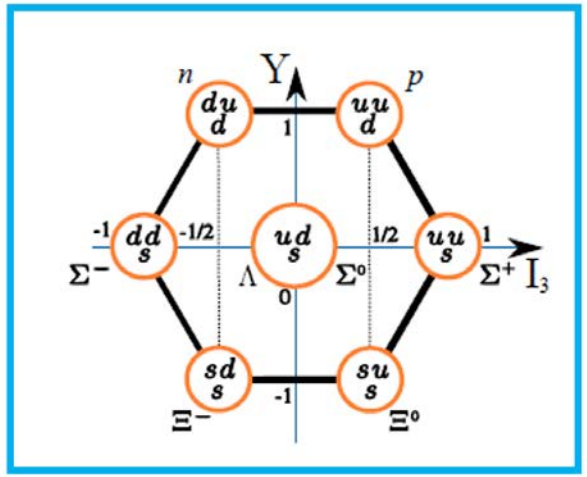
Agenda



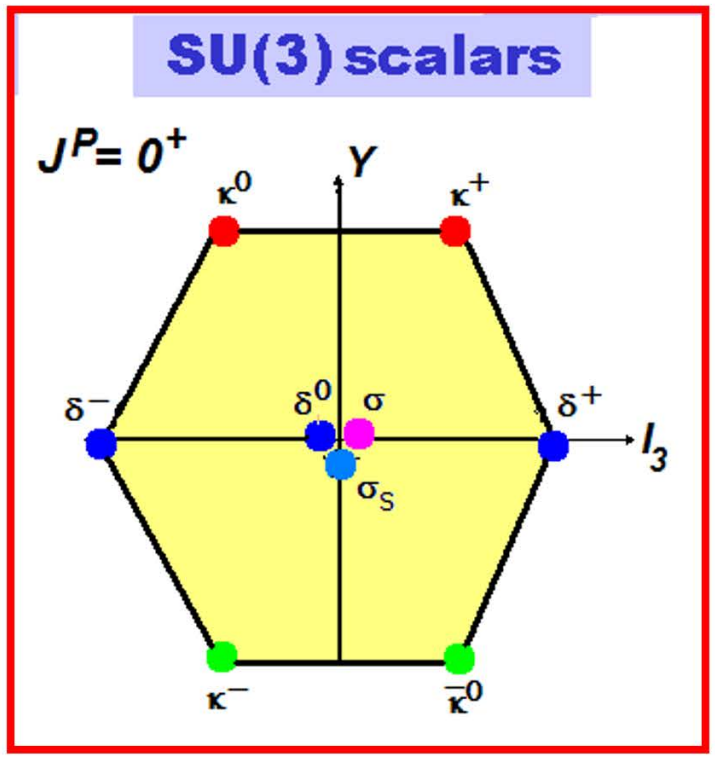
$$B = \frac{1}{\sqrt{2}}\sum_{a=1}^{8}\lambda^a B^a = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

H. Lenske, M. Dhar,
EPJ Web Conf. 271 (2022) 05003 (Proc. Hyp 2022),
2208.04916 [nucl-th]

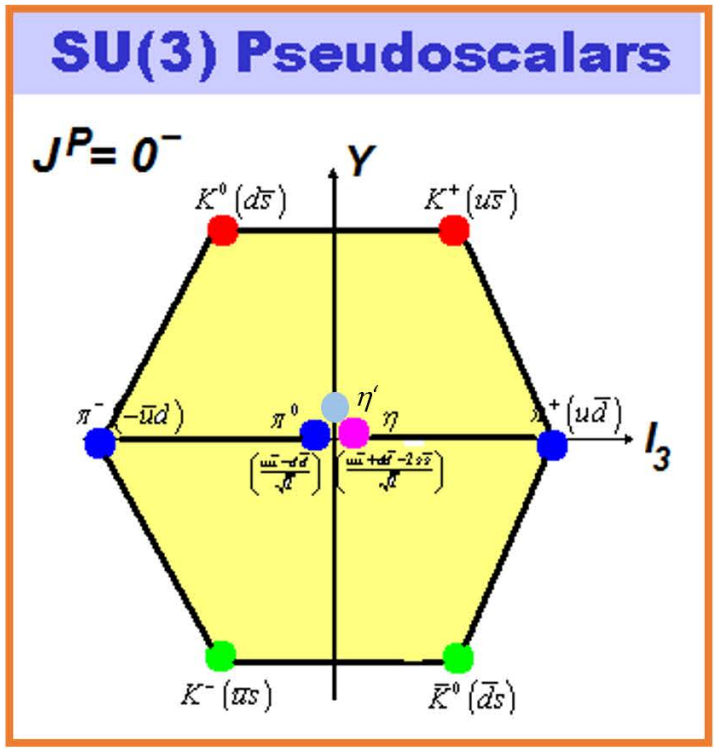
Theoretical Background:

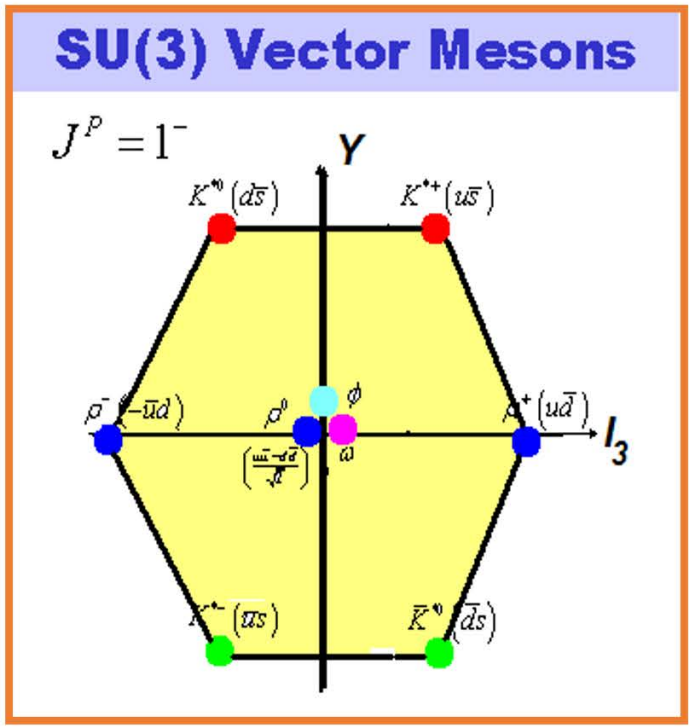
H. Lenske *Lect.Notes Phys.* 641 (2004) 147
H. Lenske, M. Dhar, *Lect.Notes Phys. 948 (2018) 161*
H. Lenske, M. Dhar, Th. Gaitanos, Xu Cao, *Prog.Part.Nucl.Phys.* 98 (2018) 119

SU(3) Meson *8+1* Nonets



Nuclear Mean-Fields





Lagrangian Density of BB-Octet Interactions

$$\mathcal{L}_{int}^{\mathcal{P}} = -\sqrt{2}\left\{g_D\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_8\right]_D + g_F\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_8\right]_F\right\} - g_S\frac{1}{\sqrt{3}}\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_1\right]_S$$

P=Pseudoscalar, Vector, and Scalar Meson Exchange

anti−symmetric $[\bar{B}, B] = \bar{B}B - B\bar{B}$ and symmetric $\{\bar{B}, B\} = \bar{B}B + B\bar{B}$ configurations

$$\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}\right]_D = \mathrm{Tr}\left(\left\{\overline{\mathcal{B}}, \mathcal{B}\right\}\mathcal{P}_8\right) \quad , \quad \left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}\right]_F = \mathrm{Tr}\left(\left[\overline{\mathcal{B}}, \mathcal{B}\right]\mathcal{P}_8\right) \quad , \quad \left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}\right]_S = \mathrm{Tr}(\overline{\mathcal{B}}\mathcal{B})\mathrm{Tr}(\mathcal{P}_1)$$

Octet Baryon Physics:
3 sets of 3 Fundamental = 9 Constants $\{g_D, g_F, g_S\}_{P,V,S}$ fix the in total 48 BB‘M Vertices

Mean-Field relevant
BBM Vector Vertices under Singlet-Octet Meson Mixing

Vertex	Coupling constant
$NN\omega$	$g_N^{\omega}=g_S\cos(\theta)+\sqrt{\frac{3}{2}}g_F\sin(\theta)-\frac{1}{\sqrt{6}}g_D\sin(\theta)$
$NN\phi$	$g_N^{\phi}=g_S\sin(\theta)-\sqrt{\frac{3}{2}}g_F\cos(\theta)+\frac{1}{\sqrt{6}}g_D\cos(\theta)$
$NN\rho$	$g_N^{\rho}=\sqrt{2}(g_F+g_D)$
$\Lambda\Lambda\omega$	$g_\Lambda^{\omega}=g_S\cos(\theta)-\sqrt{\frac{2}{3}}g_D\sin(\theta)$
$\Lambda\Lambda\phi$	$g_\Lambda^{\phi}=g_S\sin(\theta)+\sqrt{\frac{2}{3}}g_D\cos(\theta)$
$\Sigma\Sigma\omega$	$g_\Sigma^{\omega}=g_S\cos(\theta)+\sqrt{\frac{2}{3}}g_D\sin(\theta)$
$\Sigma\Sigma\phi$	$g_\Sigma^{\phi}=g_S\sin(\theta)-\sqrt{\frac{2}{3}}g_D\cos(\theta)$
$\Sigma\Sigma\rho$	$g_\Sigma^{\rho}=\sqrt{2}g_F$
$\Lambda\Sigma\rho$	$g_{\Lambda\Sigma}^{\rho}=\sqrt{\frac{2}{3}}g_D$
$\Xi\Xi\omega$	$g_\Xi^{\omega}=g_S\cos(\theta)-\sqrt{\frac{3}{2}}g_F\sin(\theta)-\frac{1}{\sqrt{6}}g_D\sin(\theta)$
$\Xi\Xi\phi$	$g_\Xi^{\phi}=g_S\sin(\theta)+\sqrt{\frac{3}{2}}g_F\cos(\theta)+\frac{1}{\sqrt{6}}g_D\cos(\theta)$
$\Xi\Xi\rho$	$g_\Xi^{\rho}=\sqrt{2}(g_F-g_D)$

Guiding Principles for a SU(3) DFT Mean-Field Dynamics

- **Interactions inherit SU(3) symmetry from NN (BB) scattering data**
- **Incorporate SU(3)-breaking by use of physical masses and empirical NN coupling constants**
- **Free space, and in-medium Bethe-Salpeter, and vertex equations conserve the fundamental symmetries**
- **Three scalar and vector in-medium NN couplings are needed for in-medium $\{g_D, g_F, g_S\}_{|M=S,V}$**

Covariant SU(3) Density Functional

$$\mathcal{L}_{int}^{DF} = -\sqrt{2} \sum_{\mathcal{M}\in\{\mathcal{P},\mathcal{S},\mathcal{V}\}} \left\{ g_D^{*(\mathcal{M})}(\hat{\rho})\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_8\right]_D + g_F^{*(\mathcal{M})}(\hat{\rho})\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_8\right]_F - g_S^{*(\mathcal{M})}(\hat{\rho})\frac{1}{\sqrt{6}}\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_1\right]_S \right\}$$

$$\hat{\rho} = \mathrm{F}\left(\bar{\Psi}, \Psi\right)$$

Meson and Baryon Field Equations:

$$\left(\partial_\mu \partial^\mu + m_{\mathcal{M}}^2\right) \Phi_{\mathcal{M}}^s = \sum_{BB'} g_{BB'\mathcal{M}}^*(\hat{\rho}) \rho^{BB's}\,, \qquad \left(\partial_\mu \partial^\mu + m_{\mathcal{M}}^2\right) V_{\mathcal{M}}^\lambda = \sum_{BB'} g_{BB'\mathcal{M}}^*(\hat{\rho}) \rho^{BB'\lambda}$$

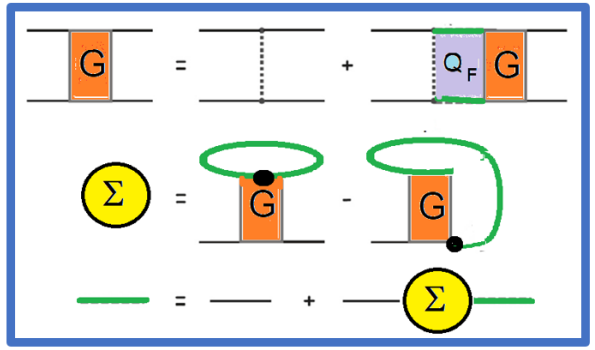
$$\left(\gamma_\mu \left(p^\mu - \Sigma_{\mathcal{B}}^\mu(\hat{\rho})\right) - M_{\mathcal{B}} + \Sigma_{\mathcal{B}}^{(s)}(\hat{\rho})\right) \Psi_{\mathcal{B}} = 0\,.$$

Evaluated in mean-field approximation

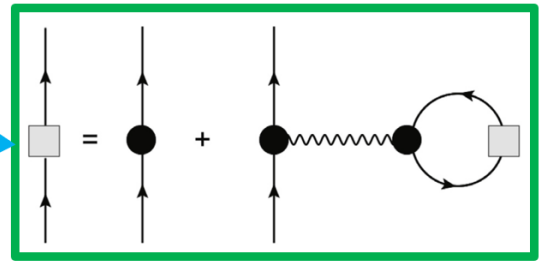
$$\Phi = \langle 0|\Phi|0\rangle + \delta\Phi \;;\; V^\mu = \langle 0|V^\mu|0\rangle + \delta V^\mu \;;\; \hat{\rho} = \langle 0|\hat{\rho}|0\rangle + \delta\rho \;;\; g_{BB'M}(\hat{\rho}) = g_{BB'M}(\rho_B) + \delta\rho \frac{\partial g_{BB'M}(\rho_B)}{\partial \rho_B}$$

$$\phi = \langle 0|\Phi|0\rangle \;;\; V^0 = \langle 0|V^\mu|0\rangle \delta^{\mu 0} \;;\; \rho_B \equiv \langle 0|\hat{\rho}|0\rangle \;;\; \delta\rho = \hat{\rho} - \rho_B$$

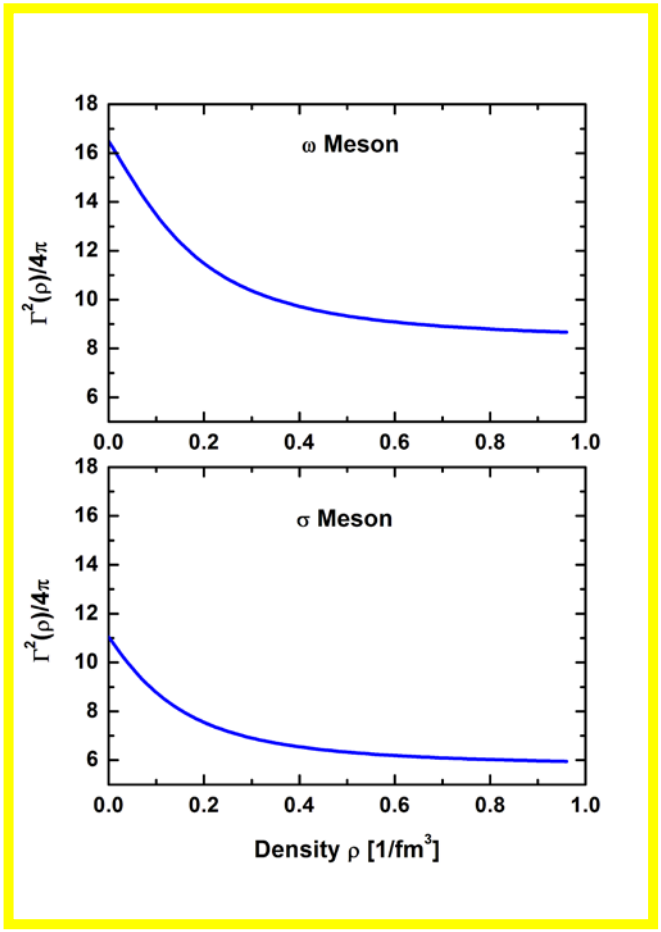
DBHF Approach to SU(3) Mean-Field Dynamics



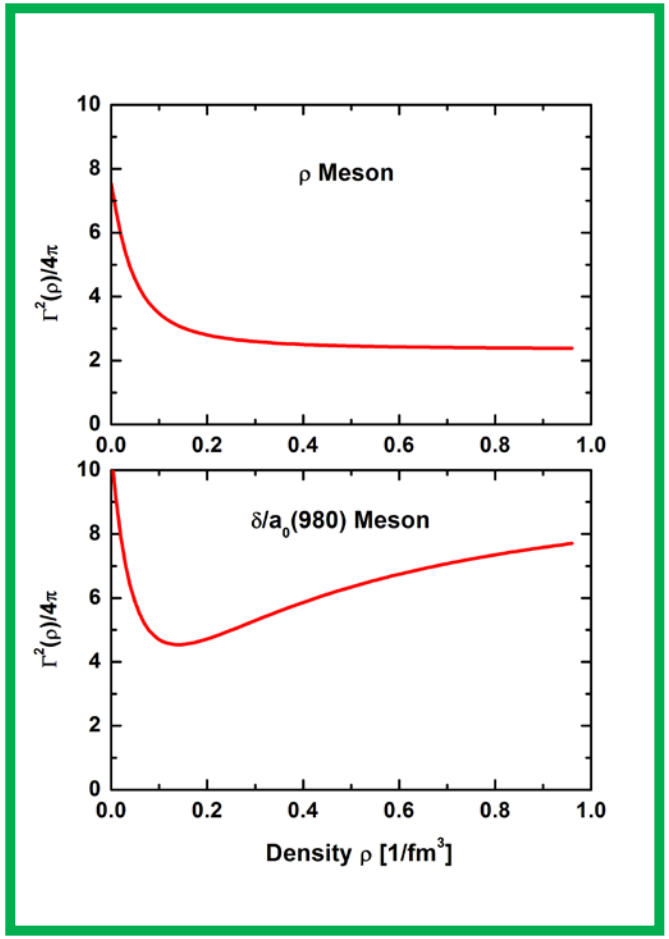
$$\Gamma_{\mathcal{B}\mathcal{B}'a}(q_s, k_F) \simeq \frac{1}{1 - \int dq' V_a \mathcal{G}^* Q_F \,|_{\mathcal{B}\mathcal{B}'}} \, g_{\mathcal{B}\mathcal{B}'a}.$$



Scalar and Vector DBHF NN Vertices as Input for the SU(3) EDF



Isoscalar Vertices

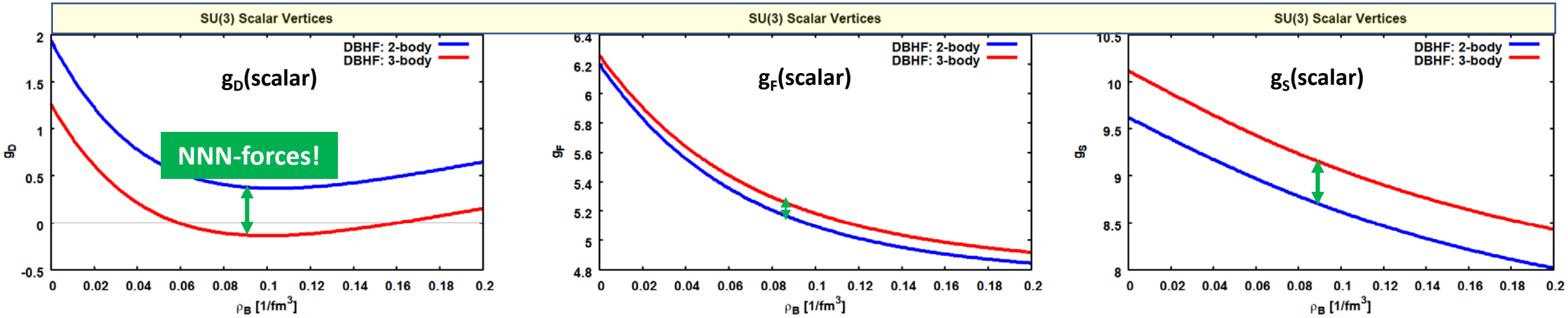


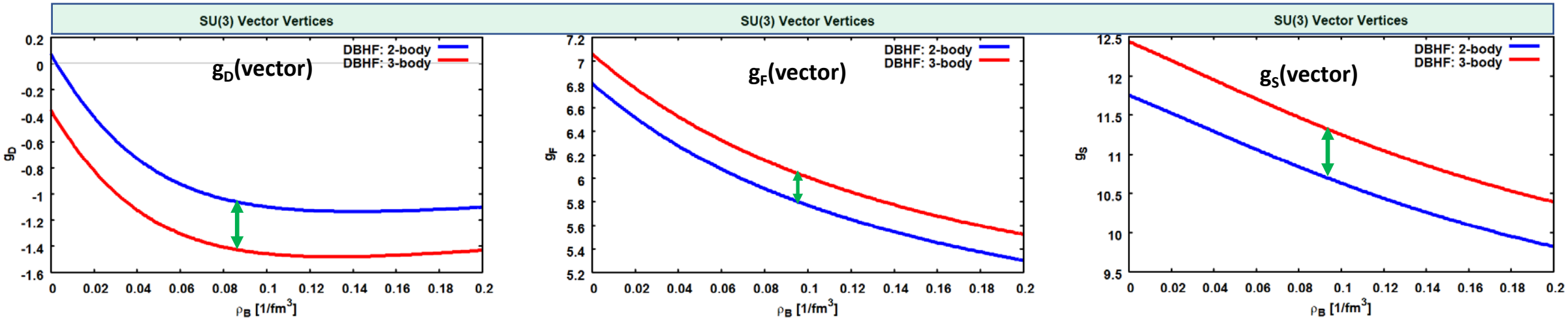
Isovector Vertices

H.L., Lect.Notes Phys. 641 (2004) 147; PPNP 98 (2018) 119

GI-DBHF SU(3) Vertices in Infinite Nuclear Matter

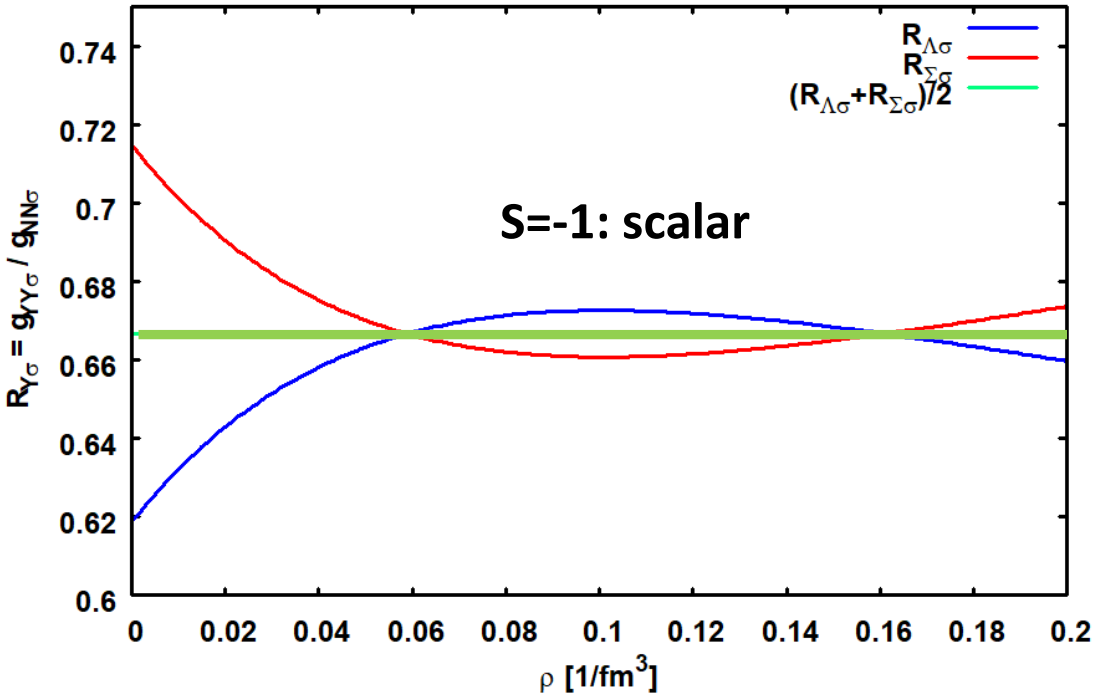
Sìnglet-Octet Meson Mixing: θ_{ideal}=35.26…°

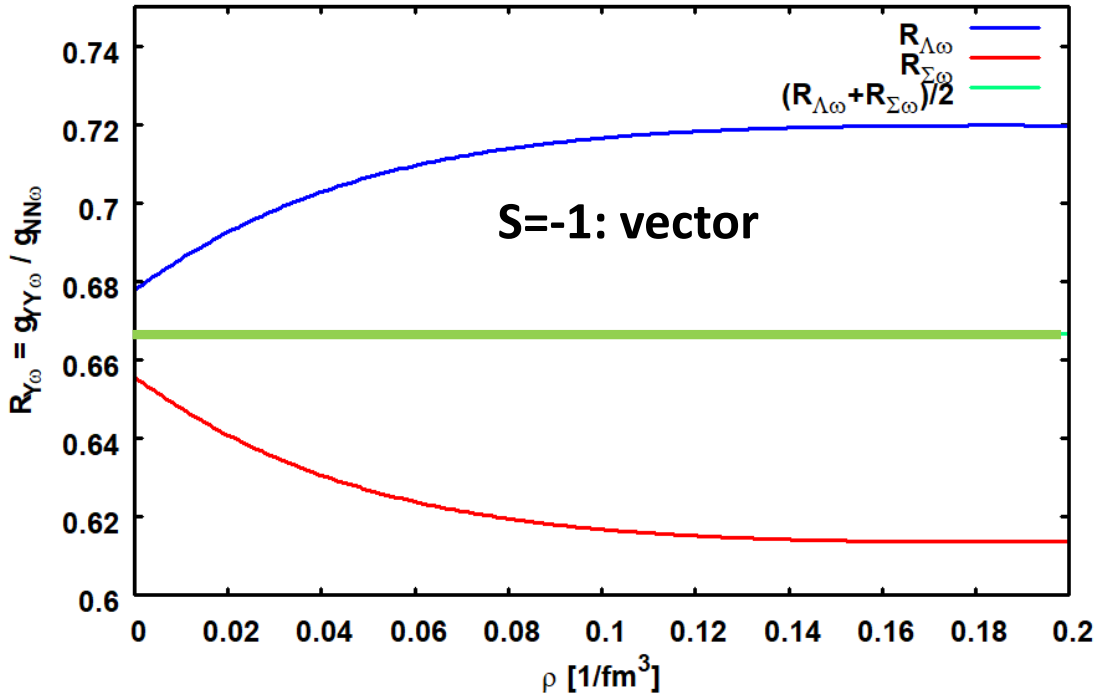




Hyperon Couplings and „Quark Scaling“

$$R_{YM} = \frac{g_{YYm}}{g_{NNm}} \approx \frac{N_Y(u,d-\text{quarks})}{N_Y(\text{quarks})} \underset{S=-1}{=} \frac{2}{3}$$



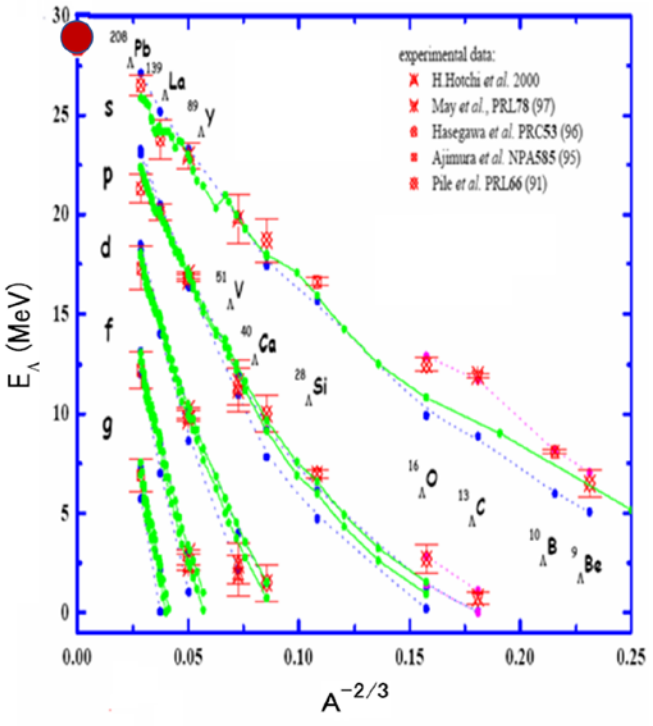


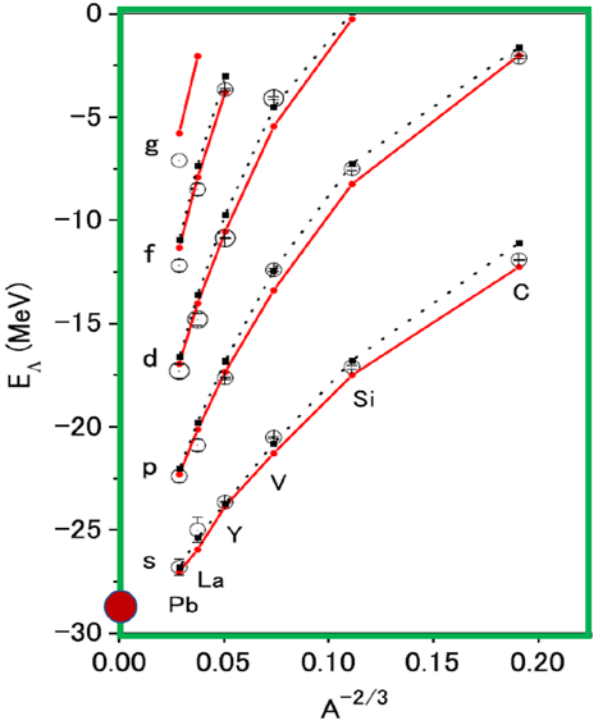
In-Medium Quark Scaling Hypothesis recovered in the average for Y=0,-1 Hypercharge-Multiplets

GI-DHBF Theory and Scaling Approach to Hypernuclei

$R_{\Lambda\sigma}$=0.49±0.3 and $R_{\Lambda\omega}$=0.55±0.4

Covariant GI-GBHF,
H.L. et al.,
PR 61 (2000) 064309
PPNP 98 (2018) 119

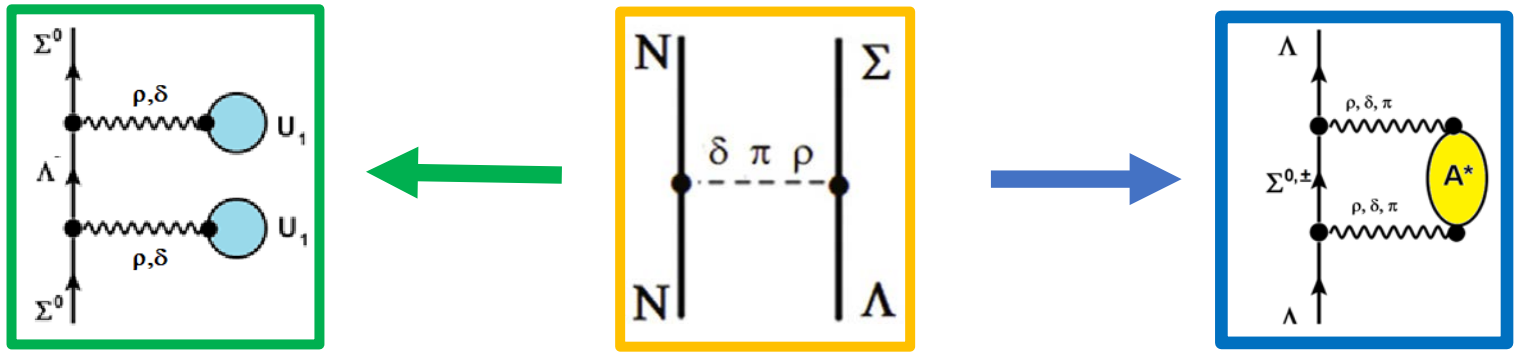




ESC-08 G-matrix,
Yamamoto et al.,
PRC 90 (2014) 045805

- **Single-Λ Hypernuclei:**
  - **Empirical „scaling approach“ for hyperon interactions**
  - **Λ-separation energies are reproduced within exp. error bars**
  - **Hampered by scarce hyperon-nucleon and hyperon-nucleus scattering data**

$\Lambda\Sigma^0$ Mixing by Isovector Interactions

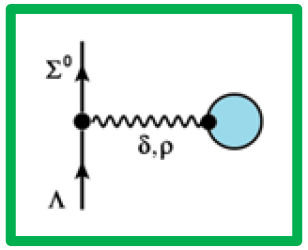


Dover, Feshbach, Gal (PRC 51, 1995)

$$\mathcal{L} = \sum_{b=\Lambda,\Sigma} \bar{\psi}_b \left(\gamma_\mu \left(p^\mu + V_b^\mu \right) - M_b - \phi_b \right) \psi_b + \bar{\psi}_\Lambda U_{\Lambda\Sigma} \psi_\Sigma + h.c.$$

$$U_{\Lambda\Sigma} = \left(g_{\Lambda\Sigma\delta} \boldsymbol{\phi}_\delta + g_{\Lambda\Sigma\rho} \gamma_\mu \mathbf{V}_\rho^\mu \right) \cdot \boldsymbol{\tau}_{\Lambda\Sigma}$$

$\Lambda\Sigma^0$ Mixing in Asymmetric Nuclear Matter by the Static Isovector Mean-Field

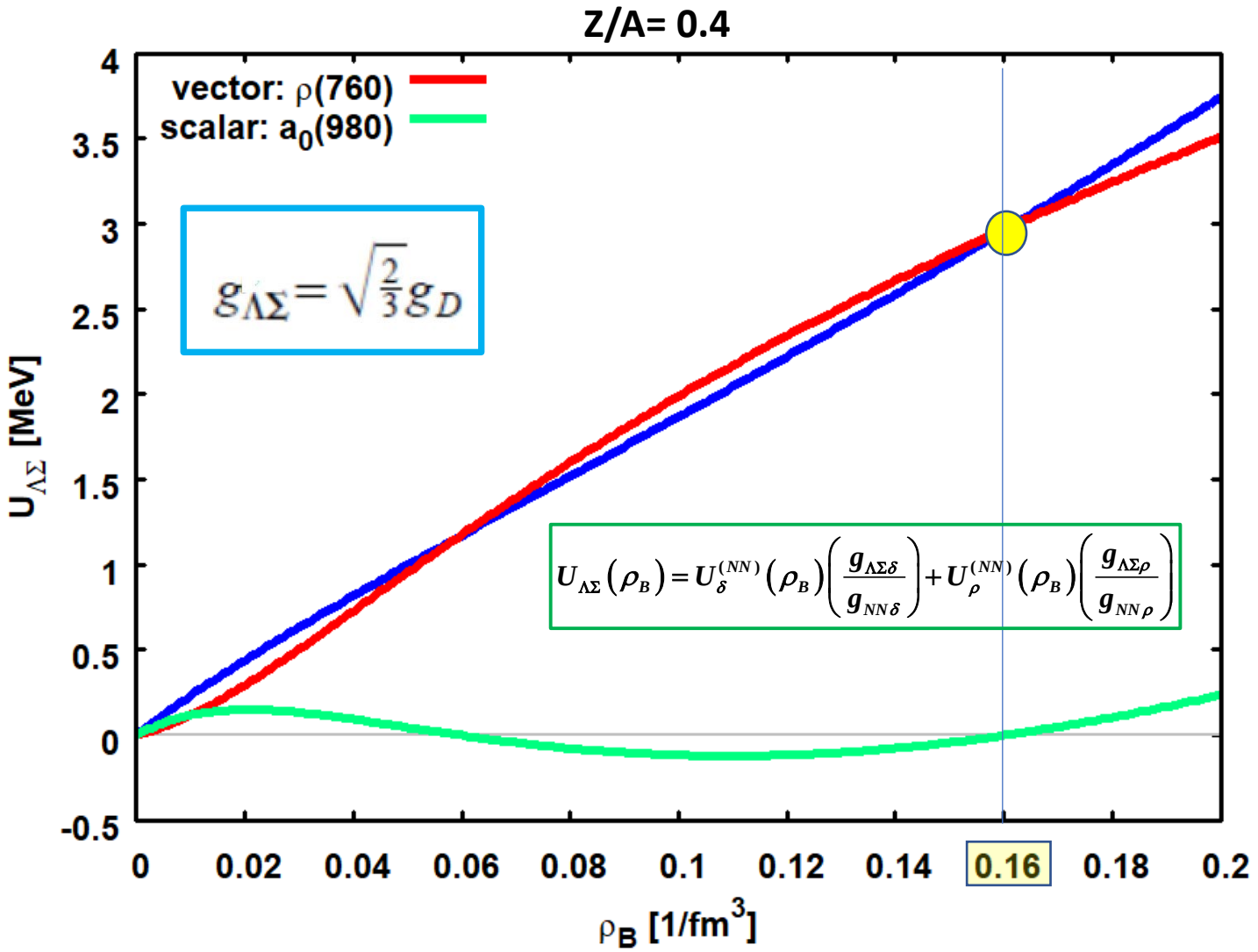


$$U_{\Lambda\Sigma}\left(\rho_B\right) = U_{\delta}^{(NN)}\left(\rho_B\right)\left(\frac{g_{\Lambda\Sigma\delta}}{g_{NN\delta}}\right) + U_{\rho}^{(NN)}\left(\rho_B\right)\left(\frac{g_{\Lambda\Sigma\rho}}{g_{NN\rho}}\right)$$

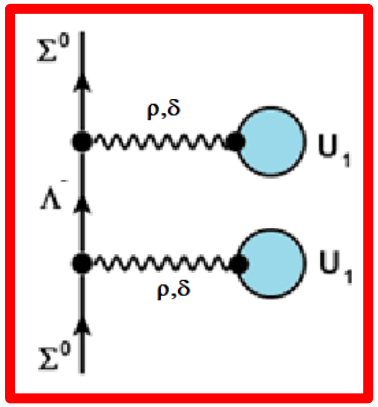
Coupled $\Lambda\Sigma^0$ System induced by Isovector Mean-Field

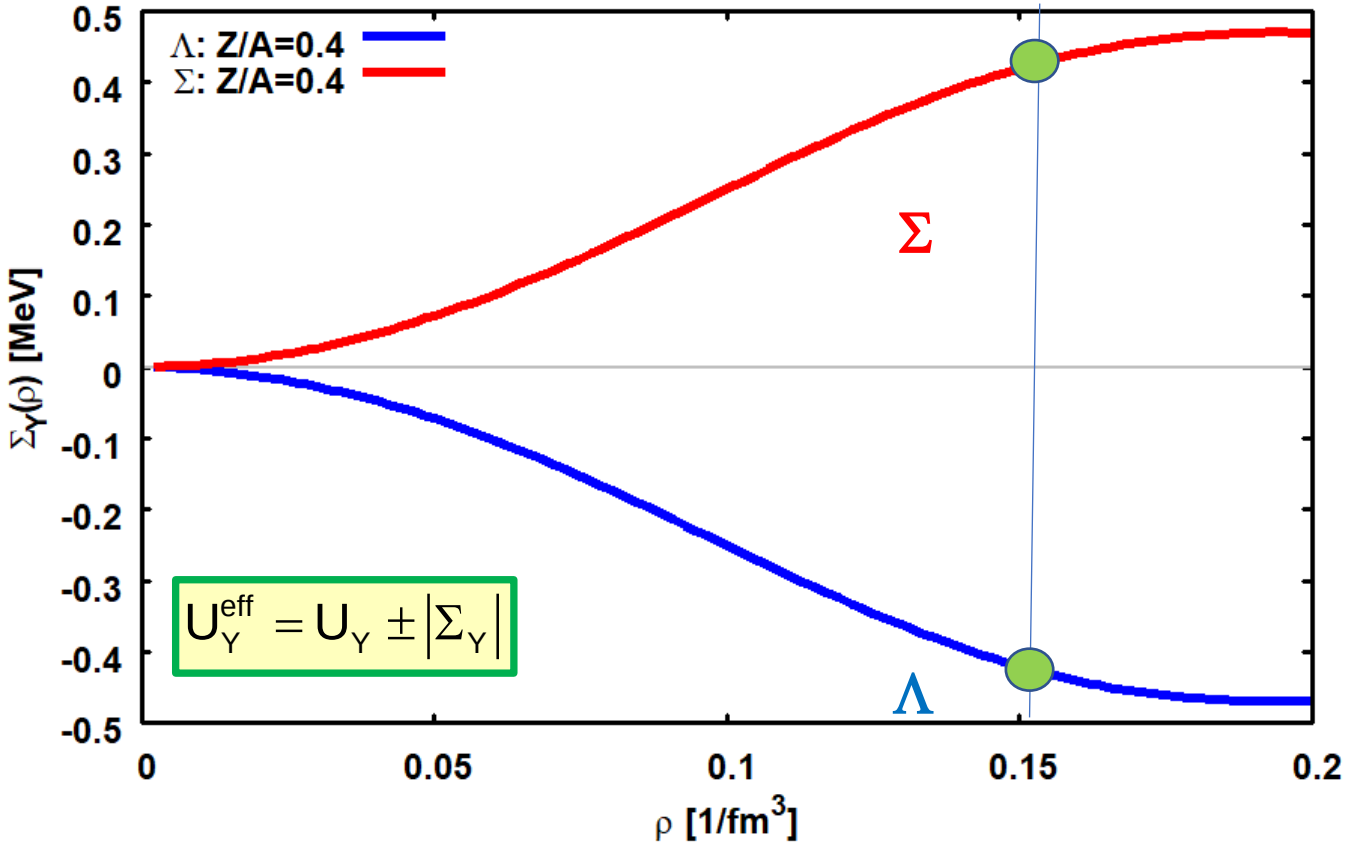
$$\begin{pmatrix} H_{\Lambda A} - E & U_{\Lambda\Sigma} \\ U_{\Lambda\Sigma}^{\dagger} & H_{\Lambda A} + m_{\Sigma\Lambda} - E \end{pmatrix} \begin{pmatrix} \left[\phi_{\Lambda} \otimes \left|A\right\rangle\right]_{I_A N_A} \\ \left[\phi_{\Sigma} \otimes \left|A\right\rangle\right]_{I_A N_A} \end{pmatrix} = 0$$

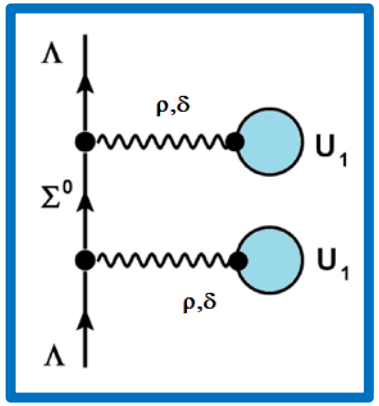
Λ–Σ Mean-Field Mixing SU(3) Potential in Asymmetric Nuclear Matter



Λ and Σ Self-Energies Induced by the Isovector Mean-Field

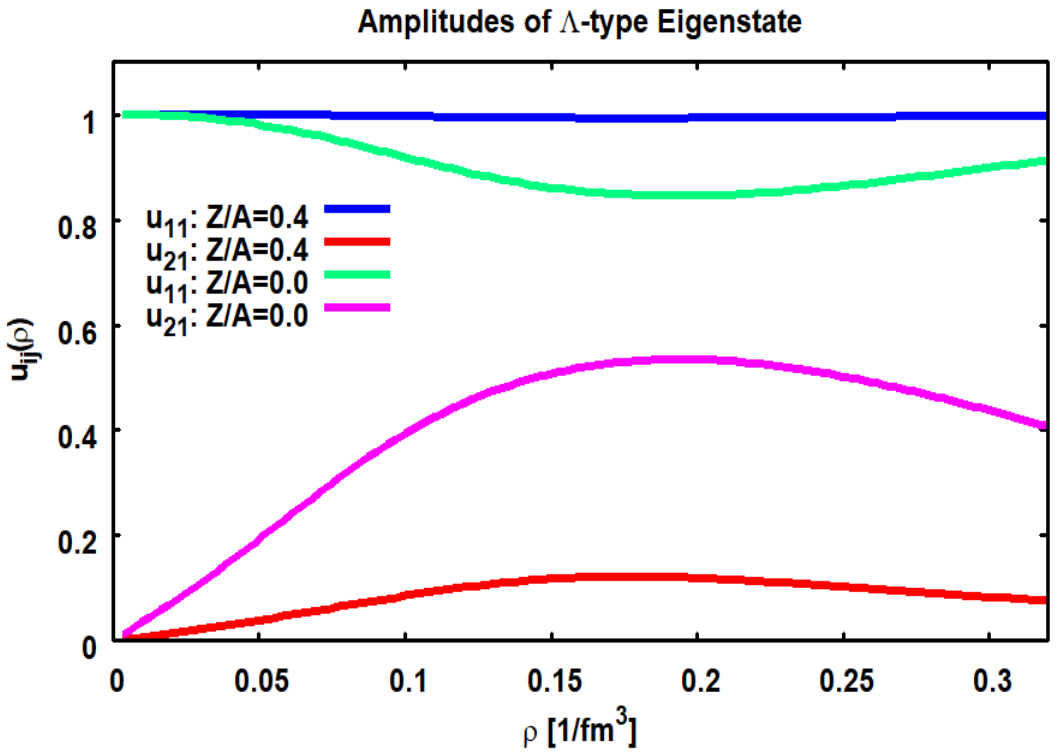


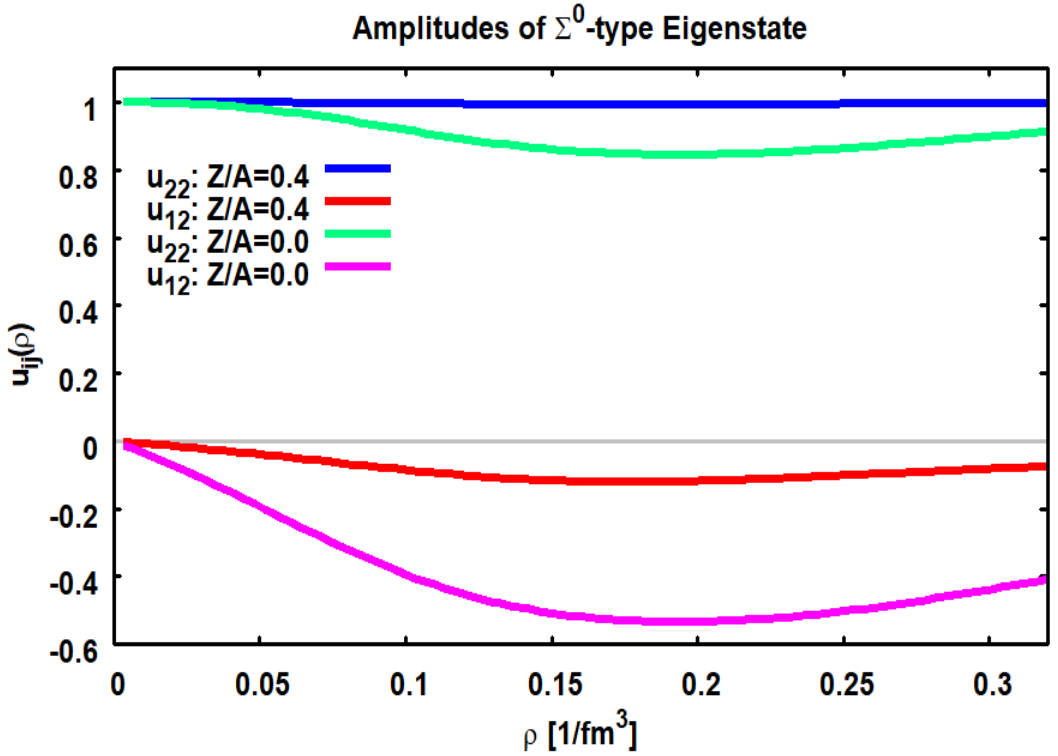




- Depth of effective Λ potential increases
- Depth of effective Σ potential decreases

$\Lambda\Sigma^0$ Mixing in Asymmetric Nuclear Matter



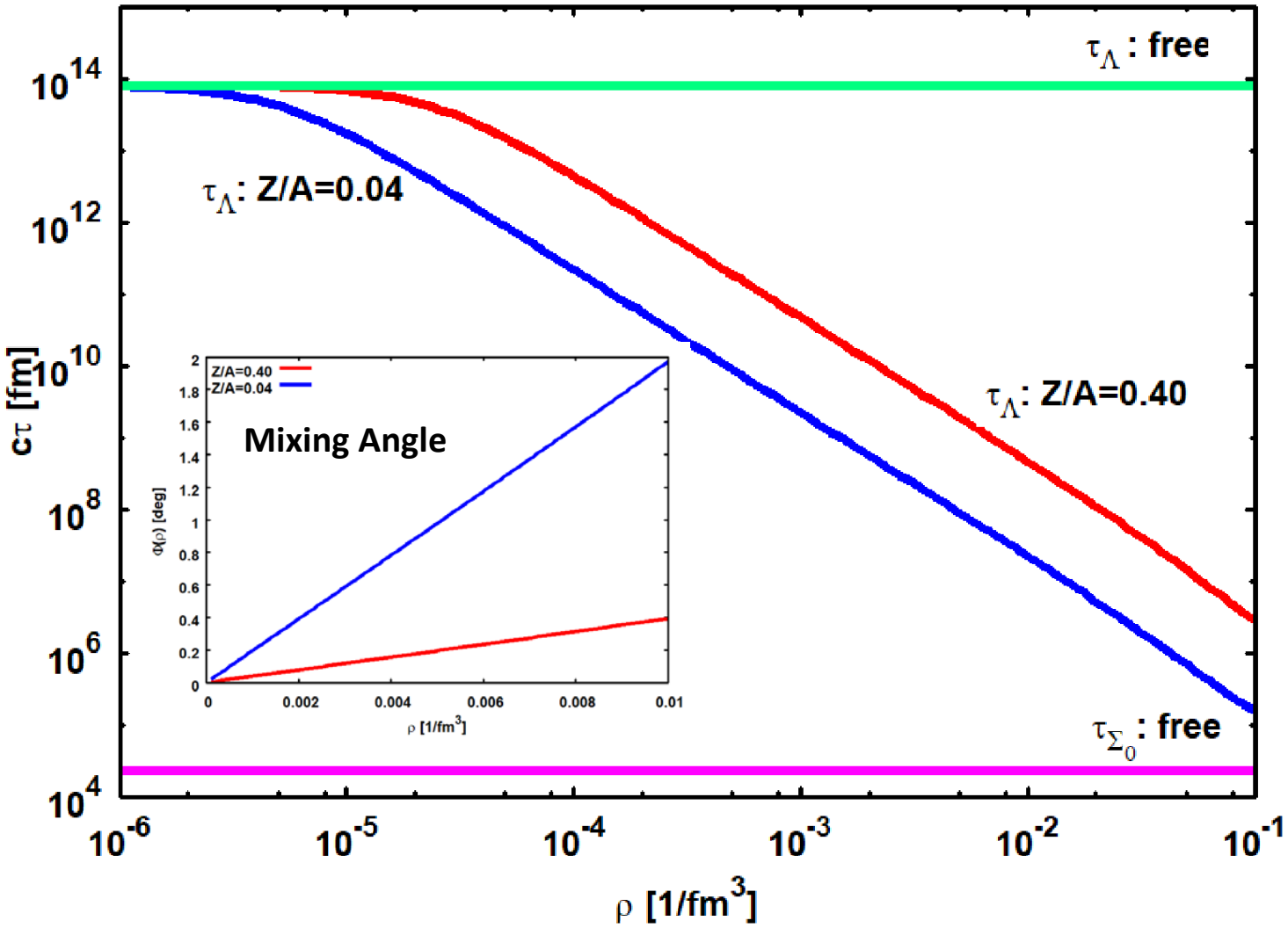


$$\left|\Lambda^*(\rho)\right\rangle = u_{11}(\rho)\left|\Lambda\right\rangle + u_{21}(\rho)\left|\Sigma\right\rangle$$

$$\left|\Sigma^*(\rho)\right\rangle = u_{22}(\rho)\left|\Sigma\right\rangle + u_{12}(\rho)\left|\Lambda\right\rangle$$

In-Medium $\Lambda\Sigma^0$ Mixing and Λ Lifetime

$$\left|\Lambda^*(\rho)\right\rangle = \cos(\Phi)\left|\Lambda\right\rangle + \sin(\Phi)\left|\Sigma\right\rangle \quad \left|\Sigma^*(\rho)\right\rangle = \cos(\Phi)\left|\Sigma\right\rangle - \sin(\Phi)\left|\Lambda\right\rangle$$



Summary and Outlook

- **Octet BB interactions in the SU(3) scheme**
- **SU(3)-symmetric Octet-EDF from DHFB NN-EDF**
- **In-medium Λ–Σ mixing induced by the isovector mean-field**
- **In-medium Λ lifetime**
- **In preparation: hyperon shells (!) in neutron stars, gravitational phase shifts**
- **to come: hypernuclear spectra, magnetic moments…**

H. Lenske, M. Dhar, EPJ Web Conf. 271 (2022) 05003, 2208.04916 [nucl-th]

Supported in part by STRONG 2020 and DFG Le439/16