Bayesian inference of thermal effects in dense matter

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Beznogov & Raduta, Phys. Rev. C 107, 045803 (2023)
Raduta, Beznogov & Oertel, arXiv: 2402.14593

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Overview

- EOS in the era of multimessenger astrophysics
- C(ovariant) D(ensity) F(unctional) EOS for cold and dense matter: a Bayesian approach
  - model dependence of the results
  - correlations with parameters of N(uclear) M(atter)
- Thermal effects on thermal energy density and pressure, entropy per baryon, heat capacities at constant volume and pressure, adiabatic and thermal index, speed of sound in CDF EOS
  - correlations with effective Dirac mass
  - pseudo-Sommerfeld expansion
  - test of alternative Sommerfeld expansion [Constantinou+, Ann. Phys. (2015)]
- Conclusions

Phenomenological EOS in the era of multimessenger astrophysics (I)

- EOS - an essential ingredient for atomic nuclei, NM and NS
- EOS - built from effective interactions designed to comply with the symmetry properties of the space; with parameters tunned such as to describe nuclear data (masses, charge r.m.s. radii, neutron skin thicknesses, charge radii diff. in mirror nuclei, energy of giant monopole/dipole/quadrupole resonances, dipole polarizability, flow and particle production in heavy ion collisions, etc.)
  e.g.: Skyrme, Gogny, MDI, CDF
- well known around $(n_{\rm sat}, \delta=0)$ with $n_{\rm sat} \sim 2.7 \cdot 10^{14} {\rm g/cm^3} \sim 0.16~{\rm fm^{-3}}$ and $\delta = (n_n - n_p)/n$
- uncertain at high densities and isospin asymmetries

Phenomenological EOS in the era of multimessenger astrophysics (II)

- availability of NS data, e.g., $M_G/M_\odot \gtrsim 2$ [1], tidal deformabilities [2], radii [3], resulted in a renaissance; better constraints at (n,δ) not accesible so far
- situation is still difficult: few data; large error bars; dependence on NS mass and the unknown EOS, different density domains are explored; every global param. depends in a specific and convoluted way on the EOS behavior over different density domains
- complexity & uncertainties best handled in statistical approaches, e.g., Bayesian

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[1] Antoniadis+, Science (2013); Arzoumanian+, ApJSS (2018); Cromartie+, Nature (2020); Fonseca+, ApJL (2021) [2] Abbott+, PRL (2017); Abbott+, PRX (2019) [3] Miller+, ApJL (2019); Riley+, ApJL (2019); Miller+, ApJL (2021); Riley+, ApJL (2021); Vinciguerra+, ApJ (2024)

Bayesian inferences of EOSs for cold and dense matter

- Phenomenological models successfully used in nuclear physics are employed:
  - non-relativistic models with Skyrme effective interactions [1],
  - CDF with D(ensity) D(ependent) and non-linear couplings [2]

- Constraints: nuclear empirical parameters (NEP), χEFT calculations of P(pure) N(eutron) (M)atter, astro. measurements of NSs; various combinations

- Conclusions: EOS, properties of NSs, global parameters of NSs, correlations depend on priors, sets of constraints, structure of the density functional

... In the following, CDF with DD couplings will be considered

[1] Zhou, Xu & Papakonstantinou, PRC (2023); Beznogov & Raduta, ApJ (2024); Beznogov & Raduta, arXiv:2403.19325
[2] Malik+, ApJ (2022); Malik+, PRD (2023); Beznogov & Raduta, PRC (2023); Char+, PRD (2023)

Covariant Density Functional model with Density Dependent couplings

Lagrangian density [1,2]: $\mathscr{L}(\sigma,\omega,\rho;\Gamma_\sigma,\Gamma_\omega,\Gamma_\rho)$

Density-dependent couplings [3]: $\Gamma_M(n)=\Gamma_{M,0}h_M(x)$, $\quad x=n/n_{\rm sat}$, with

$$h_M(x)=\exp[-(x^{a_M}-1)],\ M=\sigma,\ \omega;\quad h_\varrho(x)=\exp[-a_\varrho(x-1)]$$

6 parameters: $\Gamma_{\sigma,0}$, $\Gamma_{\omega,0}$, $\Gamma_{\rho,0}$, a_σ, a_ω, a_ρ

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[1] Walecka, Ann. Phys. (1974); [2] Typel & Wolter, Nucl. Phys. A (1999); [3] Malik, Ferreira, Agrawal & Providencia, Ap.J. (2022)

Constraints

Quantity	Units	Value	Std. deviation	Ref.
$n_{\rm sat}$	$\rm fm^{-3}$	0.153	0.005	[1]
$E_{\rm sat}$	MeV	-16.1	0.2	[2]
$K_{\rm sat}$	MeV	230	40	[3,4]
$J_{\rm sym}$	MeV	32.5	1.8	[5]
P_1	MeV/fm^3	0.509	2×0.093	[6]
P_2	MeV/fm^3	1.238	2×0.302	[6]
P_3	MeV/fm^3	2.482	2×0.687	[6]
$M_{\rm G}^*$	$M_\odot$	>2.0	—	[7]

NM parameters;

density behavior of PNM as predicted by χEFT; 1, 2, 3: $n_B = 0.08,\ 0.12\ , 0.16\ \rm fm^{-3}$

astro. constraint

[1] Typel & Wolter, NuclPhysA (1999); [2] Dutra+, PhysRevC (2014); [3] Todd-Rutel & Piekarezicz, PhysRevLett (2005); [4] Shlomo+, EPJA (2006); [5] Essik+, PhysRevC (2021); [6] Hebeler+, ApJ (2013); [7] Fonseca+, ApJL (2021);

Priors, Likelihood

Priors: uniform (uninformative) distributions [1]

Likelihood: $\log \mathscr{L}_q \propto -\chi_q^2 = -\sum_{n=1}^{N} \chi_n^2 - \chi_{\mathrm{thr.}}^{\prime 2}$

1) Uncorrelated obs., e.g., n_{sat}, E_{sat}, K_{sat}, L_{sym}, $(E/A)_i$:

$$-\chi_n^2 = -\frac{1}{2}\left(\frac{d_i - \xi_i(\Theta)}{\mathcal{Z}_i}\right)^2$$

2) "Hard-wall" (threshold), M_{G}^*: $-\chi_{\mathrm{thr.}}^{\prime 2} = -10^{10}$, if $M_{\mathrm{G}}^*/M_\odot < 2$

[1] Malik, Ferreira, Agrawal & Providencia, ApJ (2022)

Marginalized posterior distributions (I): NM parameters

Quantity	Units	Med.	68% CI	Data	Ref.
n_{sat}	fm^{-3}	0.153	$^{+0.0049}_{-0.0049}$	0.153 ± 0.005	[1]
E_{sat}	MeV	-16.1	$^{+0.2}_{-0.2}$	-16.1 ± 0.2	[2]
K_{sat}	MeV	247	$^{+33}_{-28}$	230 ± 40	[3,4]
Q_{sat}	MeV	-39.9	$^{+160}_{-130}$	–	
Z_{sat}	MeV	1360	$^{+410}_{-830}$	–	
J_{sym}	MeV	32.1	$^{+1.8}_{-1.8}$	32.5 ± 1.8	[5]
L_{sym}	MeV	42.3	$^{+15}_{-13}$	–	
K_{sym}	MeV	-105	$^{+27}_{-24}$	–	
Q_{sym}	MeV	932	$^{+360}_{-420}$	–	
Z_{sym}	MeV	-6440	$^{+3100}_{-3800}$	–	
m_{eff}	m_N	0.657	$^{+0.041}_{-0.045}$	0.55 ± 0.05	[6]
P_1	$\mathrm{MeV/fm}^3$	0.537	$^{+0.1}_{-0.12}$	0.509 ± 0.186	[7]
P_2	$\mathrm{MeV/fm}^3$	1.2	$^{+0.39}_{-0.39}$	1.238 ± 0.604	[7]
P_3	$\mathrm{MeV/fm}^3$	2.67	$^{+0.82}_{-0.72}$	2.482 ± 1.374	[7]

[1] Typel & Wolter, NuclPhysA (1999); [2] Dutra+, PhysRevC (2014); [3] Todd-Rutel & Piekarezicz, PhysRevLett (2005); [4] Shlomo+, EPJA (2006); [5] Essik+, PhysRevC (2021); [6] Typel, PhysRevC (2005); [7] Hebeler+, ApJ (2013);

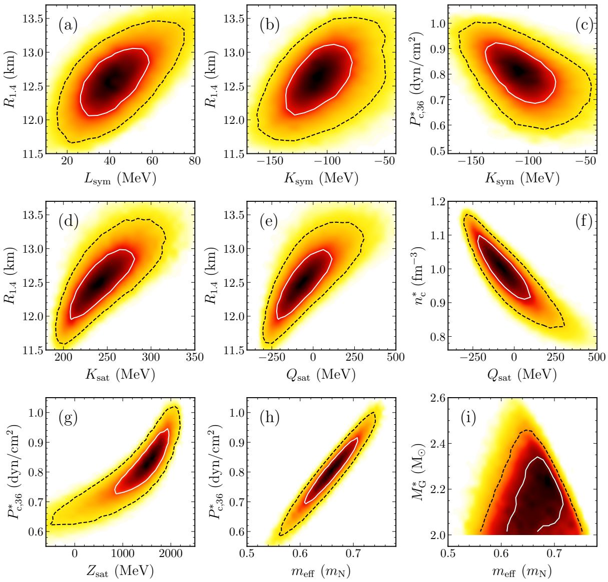
Marginalized posterior distributions (II): NS parameters

Quantity	Units	Med.	68% CI	Obs.	Ref.
M_G^*	$M_\odot$	2.16	$^{+0.15}_{-0.11}$	$\gtrsim 2$	[1]
M_B^*	$M_\odot$	2.57	$^{+0.2}_{-0.15}$	–	–
n_c^*	fm^{-3}	0.98	$^{+0.079}_{-0.083}$	–	–
$\rho_{c,15}^*$	$10^{15}\mathrm{g/cm^3}$	2.21	$^{+0.19}_{-0.18}$	–	–
$P_{c,36}^*$	$10^{36}\mathrm{dyn/cm^2}$	0.798	$^{+0.091}_{-0.097}$	–	–
$R_{1.4}$	km	12.6	$^{+0.42}_{-0.42}$	12.45 ± 0.65	[2]
$\Lambda_{1.4}$	–	519	$^{+120}_{-100}$	190^{+390}_{-120} *	[3]
$R_{2.0}$	km	12.2	$^{+0.62}_{-0.64}$	12.35 ± 0.75	[2]
M_{DU}	$M_\odot$	–	–	–	–

[1] Fonseca+, ApJL (2021); [2] Miller+, ApJLett (2021); [3] Abbott+, PhysRevLett (2018)

* at 90% CI

Correlations between NM and NS params



- $R_{1.4} - L_{sym}$ (very weak)
- $R_{1.4}$ - K_{sat}, Q_{sat} (weak)
- P_c^* - Z_{sat}; n_c^* - Q_{sat} (weak)
- P_c^* - m_{eff} (strong)

Finite temperature EOSs

- simulations of core-collapse supernovae, proto-NS evolution, BNS mergers, stellar BH formation require 3D EOSs; values of state variables and microscopic quantities are provided for wide domains of baryonic density [$10^{-10} \le n_B \le 1-10$ fm^{-3}], temperature [$0 \le T \le 100$ MeV] and charge fraction [$0 \le Y_Q \le 0.6$]
- many such EOSs are available*; based on ab initio/relativistic/non-relativistic approaches; account for different properties of NM ($K_{\rm sat}$, $E_{\rm sym}$, $L_{\rm sym}$, etc.); various compositions (nucleonic, admixtures of hyperons, Δs, π, K, quarks)

Question: which properties of NM govern the finite-T behavior?

In this talk:
- CDFs with DD couplings;
- correlations between $e_{\rm th}$, $p_{\rm th}$, S/A, $\Gamma_{\rm th}$, Γ_S, C_V, C_P, c_S^2 and Dirac mass;
- the 10^5 models of run 1 in Ref. [1] are considered;
- $n = 0.15,\ 0.6\ \mathrm{fm}^{-3}$, $T = 20, 50$ MeV and $Y_Q = 0.5$

* https://compose.obspm.fr/

[1] Beznogov & Raduta, Phys.Rev.C (2023)

CDF in short

Energy density and pressure:

$$e = e_{\mathrm{kin}} + e_{\mathrm{int}},$$
$$p = p_{\mathrm{kin}} + p_{\mathrm{int}} + p_{\mathrm{rearrang}}.$$

Kinetic terms:

$$e_{\mathrm{kin}} = \sum_{i=\mathrm{n},p} \frac{1}{\pi^2} \int_0^\infty dk k^2 E_i(k) f_{\mathrm{FD}}\left(E_i(k) - \mu_i^*\right),$$
$$p_{\mathrm{kin}} = \frac{1}{3} \sum_{i=\mathrm{n},p} \frac{1}{\pi^2} \int_0^\infty \frac{dk k^4}{E_i(k)} f_{\mathrm{FD}}\left(E_i(k) - \mu_i^*\right).$$

Interaction terms:

$$e_{\mathrm{int}} = \frac{m_\sigma^2}{2}\bar{\sigma}^2 + \frac{m_\omega^2}{2}\bar{\omega}^2 + \frac{m_\rho^2}{2}\bar{\rho}^2,$$
$$p_{\mathrm{int}} = -\frac{m_\sigma^2}{2}\bar{\sigma}^2 + \frac{m_\omega^2}{2}\bar{\omega}^2 + \frac{m_\rho^2}{2}\bar{\rho}^2.$$

Mean field expectations:

$$m_\sigma^2 \bar{\sigma} = \sum_{i=\mathrm{n},p} \Gamma_\sigma n_i^s,$$
$$m_\omega^2 \bar{\omega} = \sum_{i=\mathrm{n},p} \Gamma_\omega n_i,$$
$$m_\rho^2 \bar{\rho} = \sum_{i=\mathrm{n},p} \Gamma_\rho t_{3i} n_i,$$

where

$$n_i^{\mathrm{s}} = \frac{1}{\pi^2} \int_o^\infty \frac{dk k^2 m_i^*}{E_i(k)} f_{\mathrm{FD}}\left(E_i(k) - \mu_i^*\right)$$
$$n_i = \frac{1}{\pi^2} \int_o^\infty dk k^2 f_{\mathrm{FD}}\left(E_i(k) - \mu_i^*\right).$$

with $E_i(k) = \sqrt{k^2 + m_i^{*2}}$.

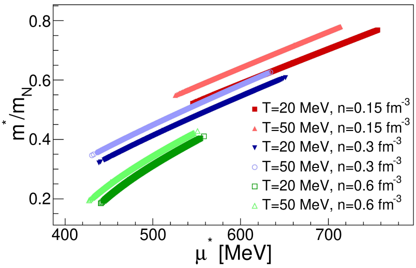
CDF at finite temperature

effective chemical potential
$\mu_i^* = \mu_i - \Gamma_\omega \bar{\omega} - \Gamma_\rho t_{3i} \bar{\rho} - \Sigma_R,$

and effective mass $m_i^* = m_{\rm N} - \Gamma_\sigma \bar{\sigma}$

are linked via

$$n_i = \frac{1}{\pi^2} \int_o^\infty dk k^2 f_{\rm FD}\left(E_i(k) - \mu_i^*\right).$$



$\Rightarrow e_{\rm th}, p_{\rm th}$, etc. depend on m^*

μ^* and m^* are n and T-dependent

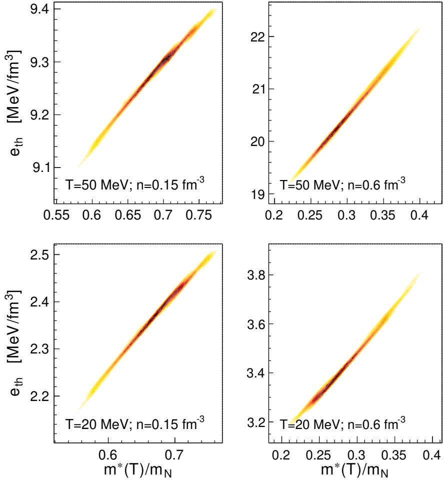
T-modifications are comparable

In non-rel. models, thermal eff. are determined by the Landau mass [1]

[1] Constantinou+, PhysRevC (2014); ibid. (2015)

Thermal energy

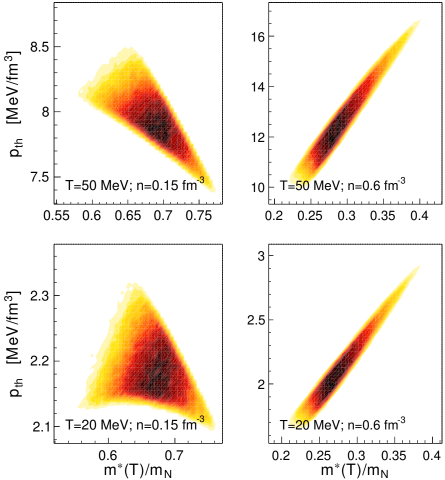
$$e_{\rm th} = e(n, Y_p, T) - e(n, Y_p, T=0)$$



- excellent, almost linear scaling with the Dirac effective mass;
- can be understood within the low-T approximation, but persists beyond it

Thermal pressure

$p_{\rm th} = p(n, Y_p, T) - p(n, Y_p, T=0)$



"high" densities

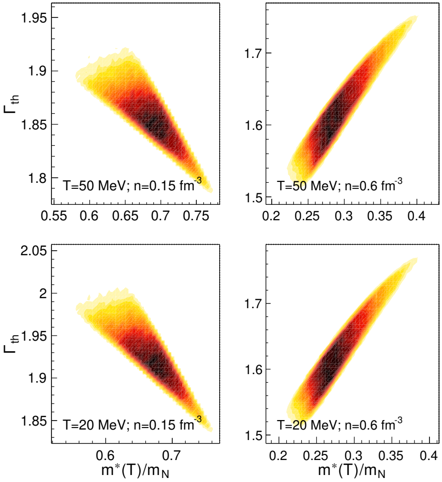
- $p_{\rm th}$ is positively correlated with the Dirac effective mass
- correlations weakening is due to the rearrangement term
- similar quality at low and at high temperatures

"low" densities

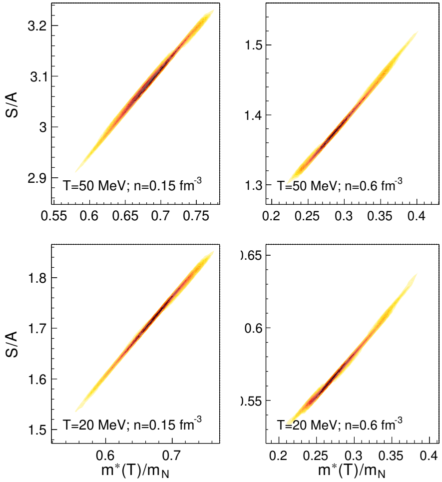
- $p_{\rm int;\ th} > p_{\rm kin;\ th}$, $p_{\rm th}$ is negatively correlated with the Dirac effective mass
- the correlation is stronger at high T

Thermal index

$\Gamma_{\rm th} = 1 + p_{\rm th}/e_{\rm th}$



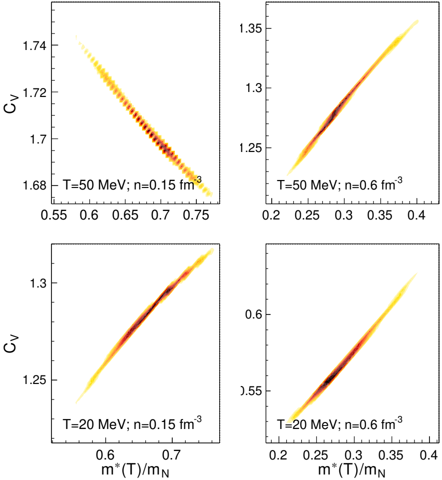
- strong correlations between $\Gamma_{\rm th}$ and the Dirac effective mass
- two regimes: positive (negative) correlations at "high" ("low") densities
  similar to the case of $p_{\rm th}$



- excellent, almost linear scaling with Dirac mass
- similar quality at all (n, T)

Heat capacity at constant volume

$$C_V = T\left(\partial (S/A)/\partial T\right)_{V,\{N_i\}}$$

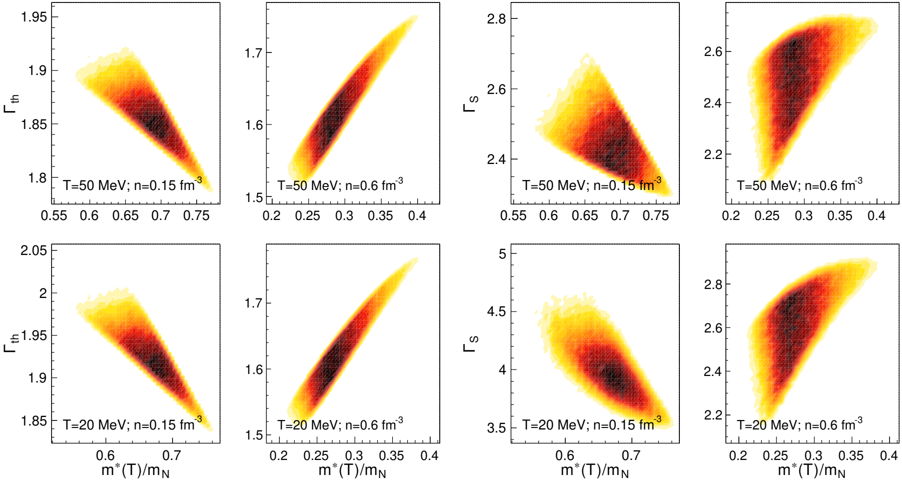


- excellent, almost linear scaling with Dirac mass
- change in sign at low densities and high temperatures

Adiabatic index & speed of sound

$\Gamma_S = (\partial \ln P / \partial \ln n)_S$

$c_S^2 = (dP/de)_{S,A,Y_p} = \Gamma_S P / (e+P)$



Low temperature approx. (I)

Pseudo-Sommerfeld expansion:

$$e_{\rm kin}(T) \approx \frac{\mu^*}{8\pi^2}\left(2\mu^{*2} - m^{*2}\right)\sqrt{\mu^{*2} - m^{*2}} - \frac{m^{*4}}{8\pi^2}\ln\left(\frac{\sqrt{\mu^{*2} - m^{*2}}}{m^*} + \frac{\mu^*}{m^*}\right)$$

$$+ \frac{1}{6}T^2 \frac{\mu^*\left(3\mu^{*2} - 2m^{*2}\right)}{\left(\mu^{*2} - m^{*2}\right)^{1/2}}$$

$$p_{\rm kin}(T) \approx \frac{\mu^*}{24\pi^2}\left(2\mu^{*2} - 5m^{*2}\right)\sqrt{\mu^{*2} - m^{*2}} + \frac{1}{8\pi^2} m^{*4}\ln\left(\frac{\sqrt{\mu^{*2} - m^{*2}}}{m^*} + \frac{\mu^*}{m^*}\right)$$

$$+ \frac{1}{6}T^2 \mu^*\left(\mu^{*2} - m^{*2}\right)^{1/2}$$

$\Rightarrow$ In this limit, $e_{\rm th}$, $p_{\rm th}$, etc. depend effectively only on $m^*(T)$

Numerical results: linear correlations of S/A and $e_{\rm th}$ with $m^*(T)$ survive also beyond this limit; evolution of astro. simulations are expected to depend on $m^*(T)$

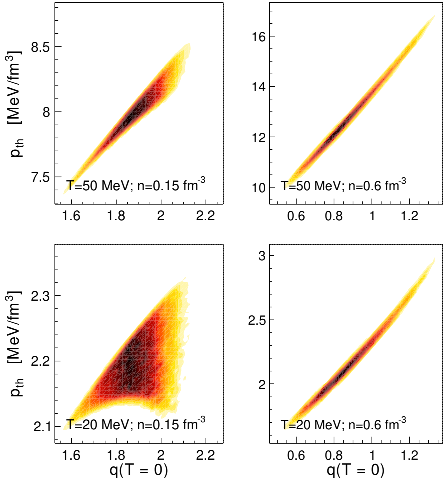
Low temperature approx. (II)

Assuming that $\delta m^*(T) \approx 0$, Constantinou et al. [1] showed that p_{th}, μ_{th} depend on $q = m^{*2}/E_F^{*2}\left(1 - 3d\ln m^*/d\ln n\right)$:

$$p_{th} = \frac{1}{3} a n T^2 \left(1 + q\right) + \mathcal{O}(T^4); \tag{1}$$

[1] Constantinou, Muccioli, Prakash & Lattimer, Ann. Phys. (2015)

Thermal pressure p_{th}



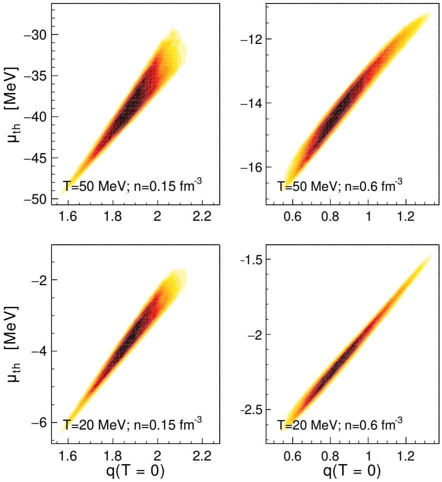
verified in certain cases

Assuming that $\delta m^*(T) \approx 0$, Constantinou et al. [1] showed that $p_{\rm th}$, $\mu_{\rm th}$ depend on $q = m^{*2}/E_F^{*2}\left(1 - 3d\ln m^*/d\ln n\right)$:

$$\mu_{\rm th} = -\frac{2}{3} a T^2 \left(1 - \frac{q}{2}\right) + \mathcal{O}(T^4)$$

[1] Constantinou, Muccioli, Prakash & Lattimer, Ann. Phys. (2015)

Thermal chemical potential $\mu_{\rm th}$



verified in all these cases, incl. high T

Low temperature approx. (II)

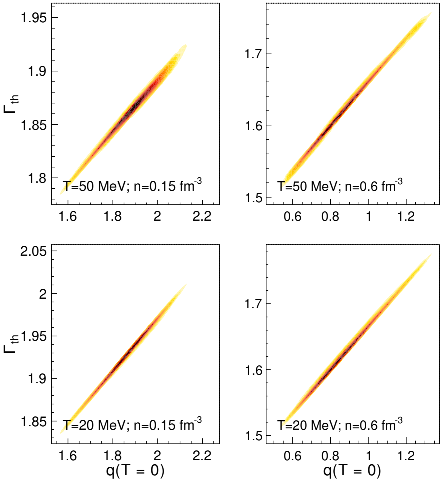
Assuming that $\delta m^*(T) \approx 0$, Constantinou et al. [1] showed that $p_{\rm th}$, $\mu_{\rm th}$ depend on $q = m^{*2}/E_F^{*2}\left(1 - 3d\ln m^*/d\ln n\right)$:

$$p_{\rm th} = \frac{1}{3} a n T^2 \left(1+q\right) + \mathscr{O}(T^4);$$

$$\Gamma_{\rm th} = 1 + \frac{p_{\rm th}}{e_{\rm th}}$$

[1] Constantinou, Muccioli, Prakash & Lattimer, Ann. Phys. (2015)

Thermal index $\Gamma_{\rm th}$



excellent scaling, including at high T

Conclusions

- 10^5 Covariant Density Functional EOS models built within a Bayesian approach are used to study finite-T effects in dense matter
- thermal quantities depend on the Dirac effective mass, with $e_{\rm th}$, S/A being the most sensitive
  - pseudo-Sommerfeld expansion explains the effect, which persists also at high T
- $p_{\rm th}$, $\mu_{\rm th}$, $\Gamma_{\rm th}$ are strongly correlated with $q = m^{*2}/E_F^{*2}\left(1-3d\ln m^*/d\ln n\right)$ [Constantinou+, Ann.Phys. (2015)]
  - the effect persists also at high T and when $\delta m^*(T) \neq 0$
- numerical simulations of CCSN, BNS are expected to show m^*-dependent evolutions
  - similarly to the case of Landau effective mass in non-relativistic models [Yasin+, PRL (2020); Schneider+, ApJ (2020)]]