What neutron stars tell about the hadron-quark phase transition

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Overview

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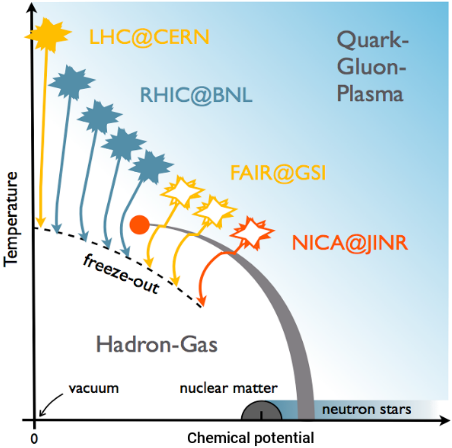
   Motivation

2. Quark-meson model
   Parametrization at $T = 0$
   Extremum equations for $\phi_{N/S}$ and $\Phi, \bar{\Phi}$
   Results
   Critical endpoint

3. Results for Neutron Stars
   Bayesian inference
   Data and constraints
   Analysis

4. Summary

Dense strongly interacting matter



What is the phase diagram and EOS for dense strongly interacting matter?

At $\mu = 0$: lattice and experiments (STAR/PHENIX and ALICE).

For $\mu >> 0$ no precise theory and no heavy ion experiment.

Dense matter at T=0

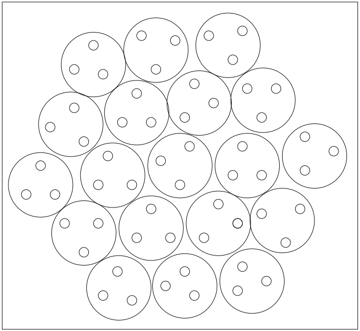
Is there a phase transition at T=0? If yes, at which density? What are the phases?

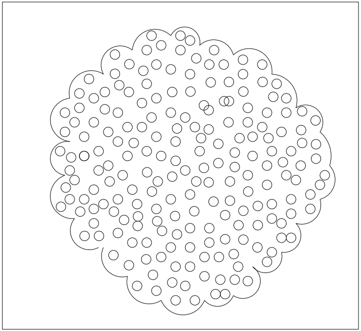
$V_{proton} = 2.48 fm^3$ (with R_{em}), densest packing with spheres: 74%

$\rightarrow \rho_{max} = 0.3 fm^{-3} \approx 1.8\rho_0$ by maximal packing

Reid hard core potential: $R_{hc} \approx 0.5 R_{em} \rightarrow \rho_{max} \approx 15\rho_0$ at hard core overlap

heavy ion collisions: no sharp transition until 2-3 ρ_0



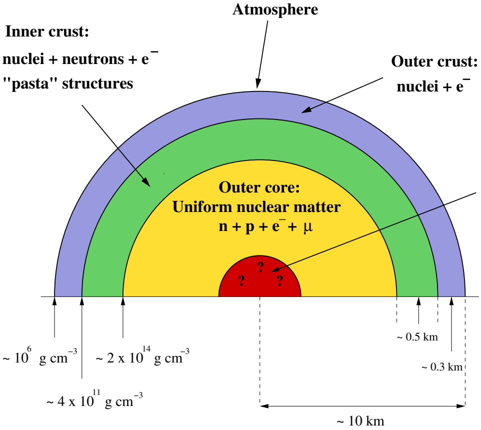


Neutron Stars a challenge and a possibility

Neutron stars are contain the densest matter in the Universe (Laboratory for strong interaction)

What is the structure of neutron stars (what are the constituents), hybrid stars? Superfluids?

Strange matter is unlikely, three-body repulsion for Λ, Σ (Weise)



Vidana, arxiv:1805.00837

Tolman-Oppenheimer-Volkoff (TOV) equation

Solving the Einstein's equation for spherically symmetric case and homogeneous matter $\longrightarrow$ TOV eqs.:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{[p(r)+\varepsilon(r)]\,[M(r)+4\pi r^3 p(r)]}{r[r-2M(r)]} \tag{1}$$

with

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \varepsilon(r)$$

These are integrated numerically for a specific $p(\varepsilon)$

- For a fixed ε_c central energy density Eq. (1) is integrated until $p = 0$
- Varying ε_c a series of compact stars is obtained (with given M and R)
- Once the maximal mass is reached, the stable series of compact stars ends

Modelling the strongly interacting matter

Plan is to have an interaction with the right global symmetry pattern describing the hadronic (with binding) and the quark phase as well.

1. Starting point is an SU(3) linear sigma model with (pseudo)scalar and (axial)vector nonets. We obtained a very good description for the meson masses and decay widths.

   D. Parganlija, P. Kovács, Gy. Wolf, F. Giacosa, D.H. Rischke, Phys. Rev. D87 (2013) 014011
2. We added the baryon octet and decuplet, baryon masses are from spontanous breaking of chiral symmetry

   P. Kovács, Á. Lukács, J. Váróczy, Gy. Wolf, M. Zétényi, Phys. Rev. D89 (2014) 054004
3. We added te isospin breaking

   P. Kovács, Gy. Wolf, N. Weickgenannt and D.H. Rischke, Phys. Rev. D (2024)
4. Nonzero temperature, chemical potential: We added Polyakov-loops, quarks: Quark-meson model, very good agreement with lattice at $\mu = 0$.

   P. Kovács, Zs. Szép and Gy. Wolf, Phys. Rev. D93 (2016) 114014
5. Future: I plan to add baryons in chirally symmetric way, but they should have finite mass in the restored phase: probably the mirror baryon model.

EOS

hadronic matter - soft: SFHo

(Steiner, A. W., Hempel, M., Fischer, T. Astrophys. J. 774 (2013) 17) and Hempel, M., Schaffner-Bielich, J. Nucl. Phys. A837 (2010) 210)

relativistic mean-field model (nucleons, σ, ω, ρ with quartic copulings), with K=245 MeV, L=47.1 MeV, m^*/m_n=0.76.

hadronic matter - stiff: DD2

(Typel, S., Ropke, G., Klahn, T., Blaschke, D., Wolter, H. Phys. Rev. C81, (2010) 015803

relativistic mean-field with light clusters, K = 243 MeV, L=58 MeV m^*/m_n=0.63.

Concatenation

Quark matter: Quark-meson model

It seems that a strong first order phase transition is ruled out by astrophysical constraints: J.-E. Christian and J. Schaffner-Bielich, Phys. Rev. D 103, 063042 (2021), and Weise's talk yesterday.

Hadron-quark crossover with polynomial interpolation ($\rho = \rho_B$):

$$
\begin{aligned}
\varepsilon(\rho_B) &= \varepsilon_{hadronic}(\rho_B) \qquad \rho_B < \rho_{BL}, \\
\varepsilon(\rho_B) &= \sum_{k=0}^{5} C_k \rho_B^k \qquad \rho_{BL} \leq \rho_B \leq \rho_{BU} \\
\varepsilon(\rho_B) &= \varepsilon_{qm}(\rho_B) \qquad \rho_{BU} < \rho_B.
\end{aligned}
$$

C_k is determined by the requirement that the energy density, ε and its first two derivatives with respect to ρ_B, pressure and sound velocity is continuous at the boundaries.
2 parameters: $\Gamma = 0.5 * (\rho_{BU} - \rho_{BL})$ and $\overline{\rho_B} = 0.5 * (\rho_{BL} + \rho_{BU})$

Lagrangian (2/1)

$$\mathcal{L}_{\mathrm{Tot}} = \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2$$
$$- \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)]$$
$$+ c_1(\det\Phi + \det\Phi^\dagger) + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\})$$
$$+ \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger).$$
$$+\bar{\Psi}i\partial\!\!\!/\Psi - g_F\bar{\Psi}(\Phi_S + i\gamma_5\Phi_{PS})\Psi + g_V\bar{\Psi}\gamma^\mu\left(V_\mu + \frac{g_A}{g_V}\gamma_5 A_\mu\right)\Psi$$

+Polyakov loops

P. Kovács, Zs. Szép, Gy. Wolf, Phys. Rev. D93 (2016) 114014

Lagrangian (2/2)

where

$$D^\mu \Phi = \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu [T_3, \Phi]$$

$$\Phi = \sum_{i=0}^{8} (\sigma_i + i\pi_i) T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8} (\rho_i^\mu - b_i^\mu) T_i, \quad L^\mu = \sum_{i=0}^{8} (\rho_i^\mu + b_i^\mu) T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu [T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu]\}$$

$$\bar{\Psi} = (\bar{u}, \bar{d}, \bar{s})$$

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv < \sigma_{N/S} >$$

Determination of the parameters of the Lagrangian

16 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A) \longrightarrow$ Determined by the min. of χ^2:

$$\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[\frac{Q_i(x_1, \ldots, x_N) - Q_i^{\mathrm{exp}}}{\delta Q_i} \right]^2 ,$$

where $(x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1, \ldots, x_N)$ calculated from the model, while Q_i^{exp} taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Tree-level masses $\rightarrow$ 15 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:
  $\Gamma_{\rho\rightarrow\pi\pi}, \Gamma_{\Phi\rightarrow KK}, \Gamma_{K^\star\rightarrow K\pi}, \Gamma_{a_1\rightarrow\pi\gamma}, \Gamma_{a_1\rightarrow\rho\pi}, \Gamma_{f_1\rightarrow KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\rightarrow K\pi},$
  $\Gamma_{f_0^L\rightarrow\pi\pi}, \Gamma_{f_0^L\rightarrow KK}, \Gamma_{f_0^H\rightarrow\pi\pi}, \Gamma_{f_0^H\rightarrow KK}$
- $T_c = 155$ MeV from lattice

Features of our approach

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with U^{glue}
- u,d,s constituent quarks, $(m_u = m_d)$
- mesonic fluctuations included in the grand canonical potential:

$$\Omega(T, \mu_q) = -\frac{1}{\beta V} ln(Z)$$

- Fermion vacuum and thermal fluctuations
- Five order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five T/μ-dependent equations

T/μ_B dependence of the condensates

Ω: grand canonical potential

$$\frac{\partial\Omega}{\partial\Phi} = \left.\frac{\partial\Omega}{\partial\bar{\Phi}}\right|_{\varphi_N=\phi_N,\varphi_S=\phi_S} = 0$$

$$\frac{\partial\Omega}{\partial\phi_N} = \left.\frac{\partial\Omega}{\partial\phi_S}\right|_{\Phi,\bar{\Phi}} = 0, \qquad \text{(after the SSB)}$$

$$\frac{\partial\Omega}{\partial v_0} = 0 \qquad \text{(only contribute at } \mu > 0)$$

five order parameters:

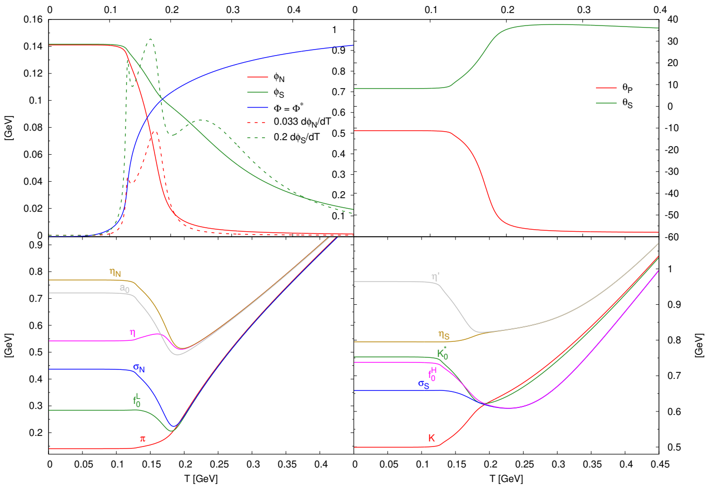
$(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five T/μ-dependent equations

ϕ_N, ϕ_S: expectation values for the scalar field

$\Phi, \bar{\Phi}$: expectation values for the Polyakov loops

v_0 expectation value for the vector field

With low mass scalars, $m_{f_0^L} = 300$ MeV

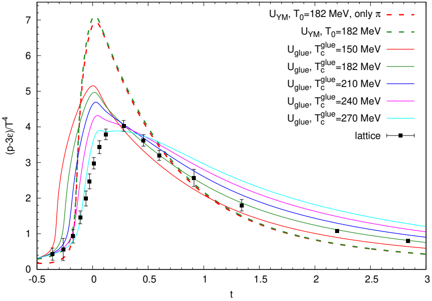


chiral symmetry is restored at high T as the chiral partners (π, f_0^L), (η, a_0) and $(K, K_0^\star)$, (η', f_0^H) become degenerate

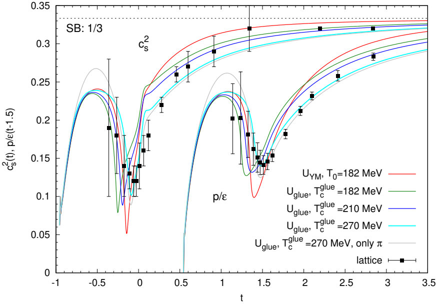
$U(1)_A$ symmetry is not restored, as the axial partners (π, a_0) and (η, f_0^L) do not become degenerate

Observables

interaction measure



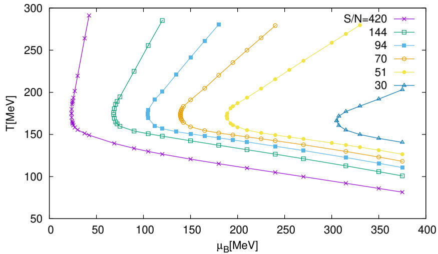
speed of sound

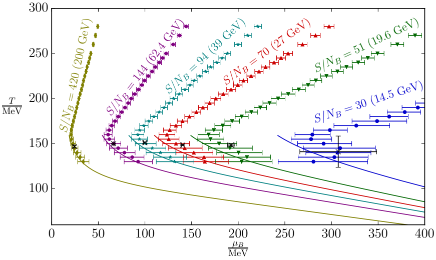


Isentropic trajectories in the $T - \mu_{\rm B}$ plane

our model, where $\mu_B^{\rm CEP} > 850{\rm MeV}$

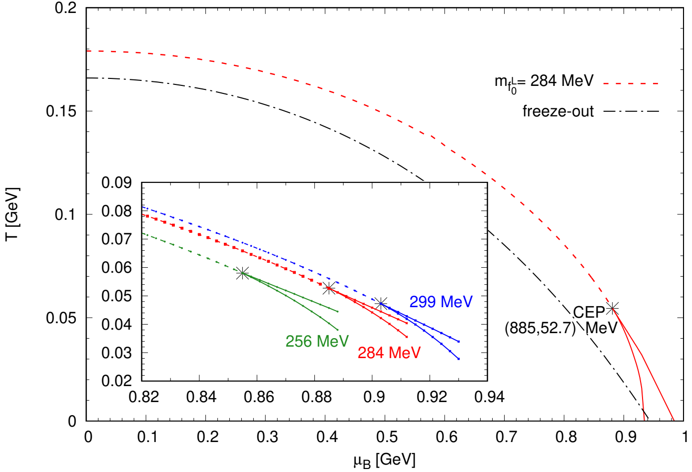
lattice (analytic continuation)
Günther *et al.*, arXiv:1607.02493





same qualitative behavior of the isentropic trajectories for $\mu_B \leq 400$ MeV
$\Longrightarrow$ indication that in the lattice result there is no CEP in this region of μ_B

$T - \mu_B$ phase diagram



- we use U^{glue} with $T_c^{glue} = 210$ MeV
- freeze-out curve from Cleymans et al., J.Phys.G32, S165
- Curvature at $\mu_B = 0$ $\kappa = 0.0193$, close to the lattice value $\kappa = 0.020(4)$ (Cea *et*

Bayesian inference

Unsetted parameters: m_σ, g_v, $\overline{\rho_B} \equiv 0.5(\rho_{BL} + \rho_{BU})$, $\Gamma \equiv 0.5(\rho_{BU} - \rho_{BL})$

290 MeV $\leq m_\sigma \leq$ 700 MeV
$0 \leq g_v \leq 10$
$2\rho_0 \leq \overline{\rho_B} \leq 5\rho_0$
$\rho_0 \leq \Gamma \leq 4\rho_0$ with the constraint: $\rho_{BL} = \overline{\rho_B} - \Gamma > \rho_0$
We created $\sim$ 18000 EOSs to be used in the Bayesian analysis
Bayes theorem:
θ is a parameter set, $p(\theta)$ is the prior probability for θ, $p(data|\theta)$ is the probability that for given θ, the data is measured. Then

$$p(\theta|data) = \frac{p(data|\theta)p(\theta)}{p(data)}$$

$p(data)$ is a normalization constant. We assume $p(\theta)$ is uniform in the allowed hypersurface. For independent observations:

$$p(data|\theta) = p(M_{max}|\theta)p(NICER|\theta)p(\overline{\Lambda}|\theta)$$

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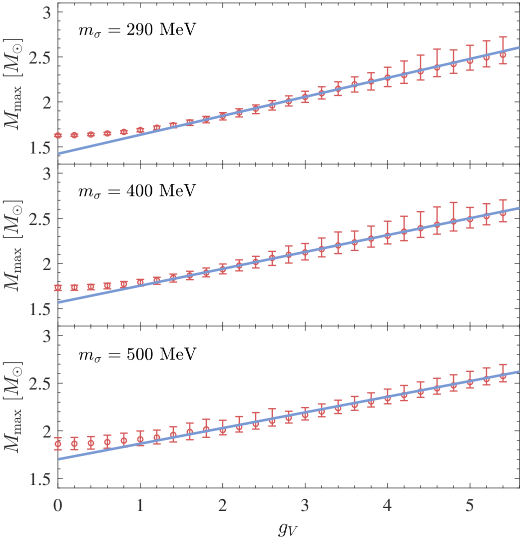
Data

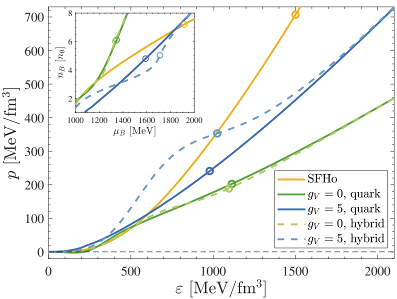
- $2M_\odot$: PSR J0348+0432 with a mass $2.01 \pm 0.04 M_\odot$,
  PSR J1614-2230 with a mass $1.908 \pm 0.016 M_\odot$
- perturbative QCD EOS should converge to it keeping $c_s < 1$,
  $\mu_{QCD} = 2.6 GeV, n_{QCD} = 6.471/fm^3, p_{QCD} = 3823 MeV/fm^3$
- NICER: (M,R) values for PSR J0030+0451, PSR J0740+6620 (Miller)
- tidal deformability GW170817: : $70 < \Lambda(1.4M_\odot) < 720$ Abbot (2019)
- hypermassive NS GW170817: no prompt black hole formation, (Rezzola):
  $2.01 \geq M_{TOV}/M_\odot \geq 2.16$
- massgap neutron star: $2.59 \pm 0.09 M_\odot$
- Hess J1731-347 neutron star: mass= $0.77 \pm 0.19 M_\odot$, $R = 10.4 \pm 0.8 km$

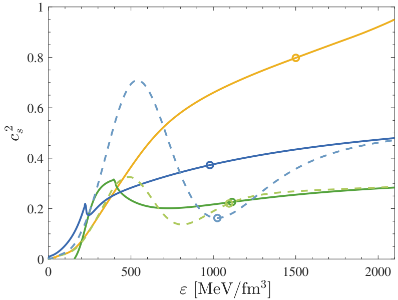
g_v dependence of the M_{max}



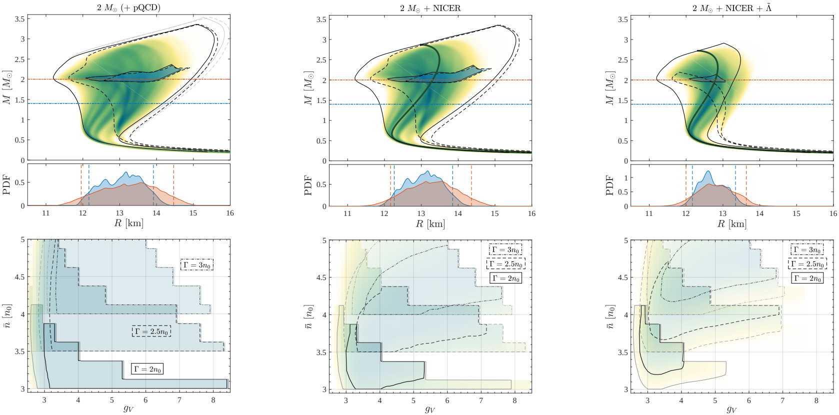
The errorbars are obtained by varying $\overline{\rho}$ and Γ.

EOS and sound velocity



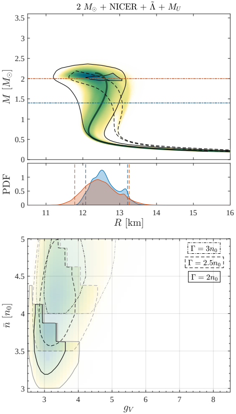


Bayesian analysis

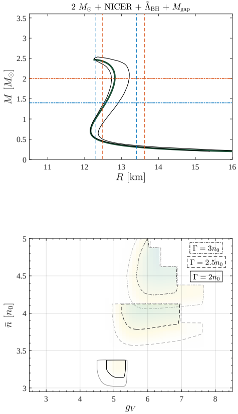


prior (M_{max} + p_{QCD}) | prior + NICER | prior + NICER + GW

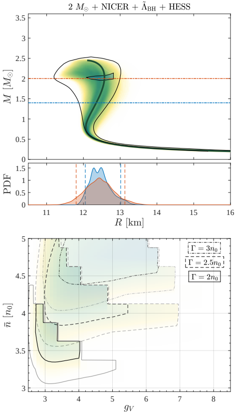
Bayesian analysis



prior + NICER + GW + HMNS

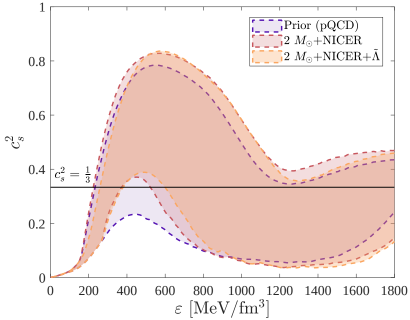


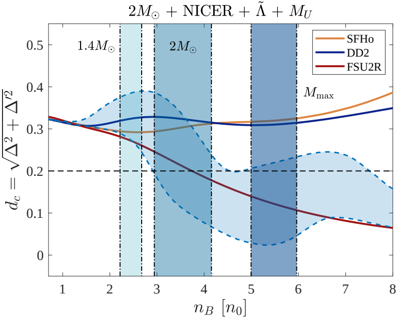
prior + NICER + GW + Mgap



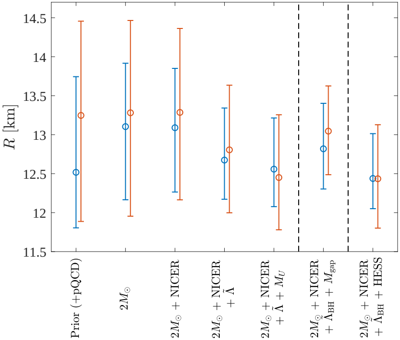
prior + NICER + GW + HESS

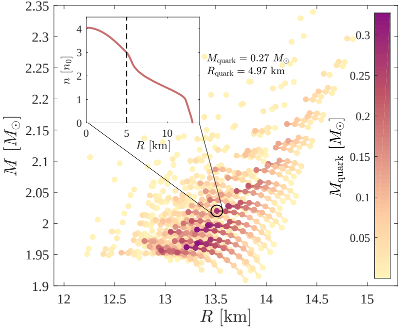
Sound velocity and Bayesian analysis





Bayesian analysis





Summary and Conclusions

- Our model can reproduce the lattice calculations at $\mu = 0$
- With our model we can fulfill the present astronomical constraints
- The central density do not go above $6\rho_0$.
- The radius of the neutron stars are 12.8 ± 0.8 km.
- hadronic and quark phase ought to be handled with the same model to drop ad-hoc parameters

Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_S = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$
Scalars: $a_0(980$ or $1450), K_0^\star(800$ or $1430)$,
(σ_N, σ_S) : 2 of $f_0(500, 980, 1370, 1500, 1710)$

Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star-} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv < \sigma_{N/S} >$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like Tr$[(D_\mu\Phi)^\dagger(D_\mu\Phi)]$:

$$\begin{aligned}
\pi_N - a^\mu_{1N} &:\quad -g_1\phi_N a^\mu_{1N}\partial_\mu\pi_N, \\
\pi - a^\mu_1 &:\quad -g_1\phi_N(a^{\mu+}_1\partial_\mu\pi^- + a^{\mu 0}_1\partial_\mu\pi^0) + \text{h.c.}, \\
\pi_S - a^\mu_{1S} &:\quad -\sqrt{2}g_1\phi_S a^\mu_{1S}\partial_\mu\pi_S, \\
K_S - K^\star_\mu &:\quad \frac{ig_1}{2}(\sqrt{2}\phi_S - \phi_N)(\bar{K}^{\star 0}_\mu\partial^\mu K^0_S + K^{\star -}_\mu\partial^\mu K^+_S) + \text{h.c.}, \\
K - K^\mu_1 &:\quad -\frac{g_1}{2}(\phi_N + \sqrt{2}\phi_S)(K^{\mu 0}_1\partial_\mu\bar{K}^0 + K^{\mu +}_1\partial_\mu K^-) + \text{h.c.}.
\end{aligned}$$

Diagonalization→ Wave function renormalization

Thermodynamical Observables

We include mesonic thermal contribution to p for (π, K, f_0^l)

$$\Delta p(T) = -nT \int \frac{d^3q}{(2\pi)^3} ln(1 - e^{-\beta E(q)}), \quad E(q) = \sqrt{q^2 + m^2}$$

- pressure: $p(T, \mu_q) = \Omega_H(T = 0, \mu_q) - \Omega_H(T, \mu_q)$
- entropy density: $s = \frac{\partial p}{\partial T}$
- quark number density: $\rho_q = \frac{\partial p}{\partial \mu_q}$
- energy density: $\epsilon = -p + Ts + \mu_q \rho_q$
- scaled interaction measure: $\frac{\Delta}{T^4} = \frac{\epsilon - 3p}{T^4}$
- speed of sound at $\mu_q = 0$: $c_s^2 = \frac{\partial p}{\partial \epsilon}$

Inclusion of the vector meson- quark interaction

$$\mathcal{L}_{\rm Vq} = -g_V \sqrt{6}\bar{\Psi}\gamma_\mu V_0^\mu \Psi$$
$$V_0^\mu = \frac{1}{\sqrt{6}}\mathrm{diag}(v_0 + \frac{v_8}{\sqrt{2}}, v_0 + \frac{v_8}{\sqrt{2}}, v_0 - \sqrt{2}v_8) \quad (2)$$

vector fields: like Walecka model, nonzero expectation values are built up at nonzero chemical potential. For simplicity

$$< v_0^\mu >= v_0\delta^{0\mu}, \quad < v_8^\mu >= 0$$

Modification of the grand canonical potential:

$$\Omega(T=0, \mu_q, g_v) = \Omega(T=0, \tilde{\mu}_q, gv=0) - \frac{1}{2}m_v^2 v_0^2,$$

with $\tilde{\mu}_Q = \mu_q - g_v v_0$

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P}\exp\left[i\int_0^\beta d\tau A_4(\vec{x},\tau)\right]$$

$\longrightarrow$ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement

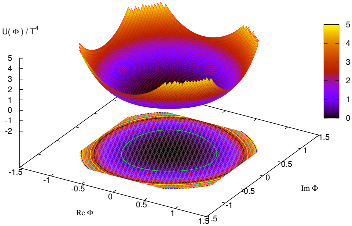
low T: confined phase, $\langle\Phi(\vec{x})\rangle, \langle\bar{\Phi}(\vec{x})\rangle = 0$
high T: deconfined phase, $\langle\Phi(\vec{x})\rangle, \langle\bar{\Phi}(\vec{x})\rangle \neq 0$

Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space
Effects of the gauge fields:

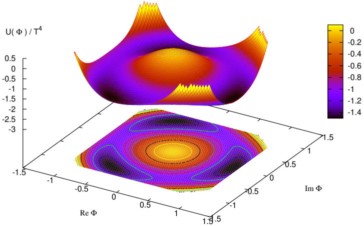
- In this gauge the effect of the gauge field on the quarks acts like an imaginary chemical potential
  $\longrightarrow$ modified quark distribution function.
- Polyakov potential: $\mathcal{U}\left(\Phi,\bar{\Phi}\right)$ models the free energy of a pure gauge theory, parameters are fitted to the pure gauge lattice data

Polyakov loop potential

"Color confinement"
$\langle\Phi\rangle = 0 \longrightarrow$ no breaking of $\mathbb{Z}_3$
one minimum



"Color deconfinement"
$\langle\Phi\rangle \neq 0 \longrightarrow$ spontaneous breaking of $\mathbb{Z}_3$
minima at $0, 2\pi/3, -2\pi/3$
one of them spontaneously selected



from H. Hansen et al., PRD75, 065004 (2007)

Effects of Polyakov loops on FD statistics

Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p - \mu_q) \longrightarrow f_\Phi^+(E_p) = \frac{\left(\bar{\Phi} + 2\Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}{1 + 3\left(\bar{\Phi} + \Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}$$

$$f(E_p + \mu_q) \longrightarrow f_\Phi^-(E_p) = \frac{\left(\Phi + 2\bar{\Phi} e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}{1 + 3\left(\Phi + \bar{\Phi} e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}$$

$\Phi, \bar{\Phi} \to 0 \Longrightarrow f_\Phi^\pm(E_p) \to f(3(E_p \pm \mu_q))$ $\Phi, \bar{\Phi} \to 1 \Longrightarrow f_\Phi^\pm(E_p) \to f(E_p \pm \mu_q)$

three-particle state appears: mimics confinement of quarks within baryons

at $T = 0$ there is no difference between models with and without Polyakov loop

Thank you for your attention!