Dense QCD equation of state: duality and implications from lattice QCD

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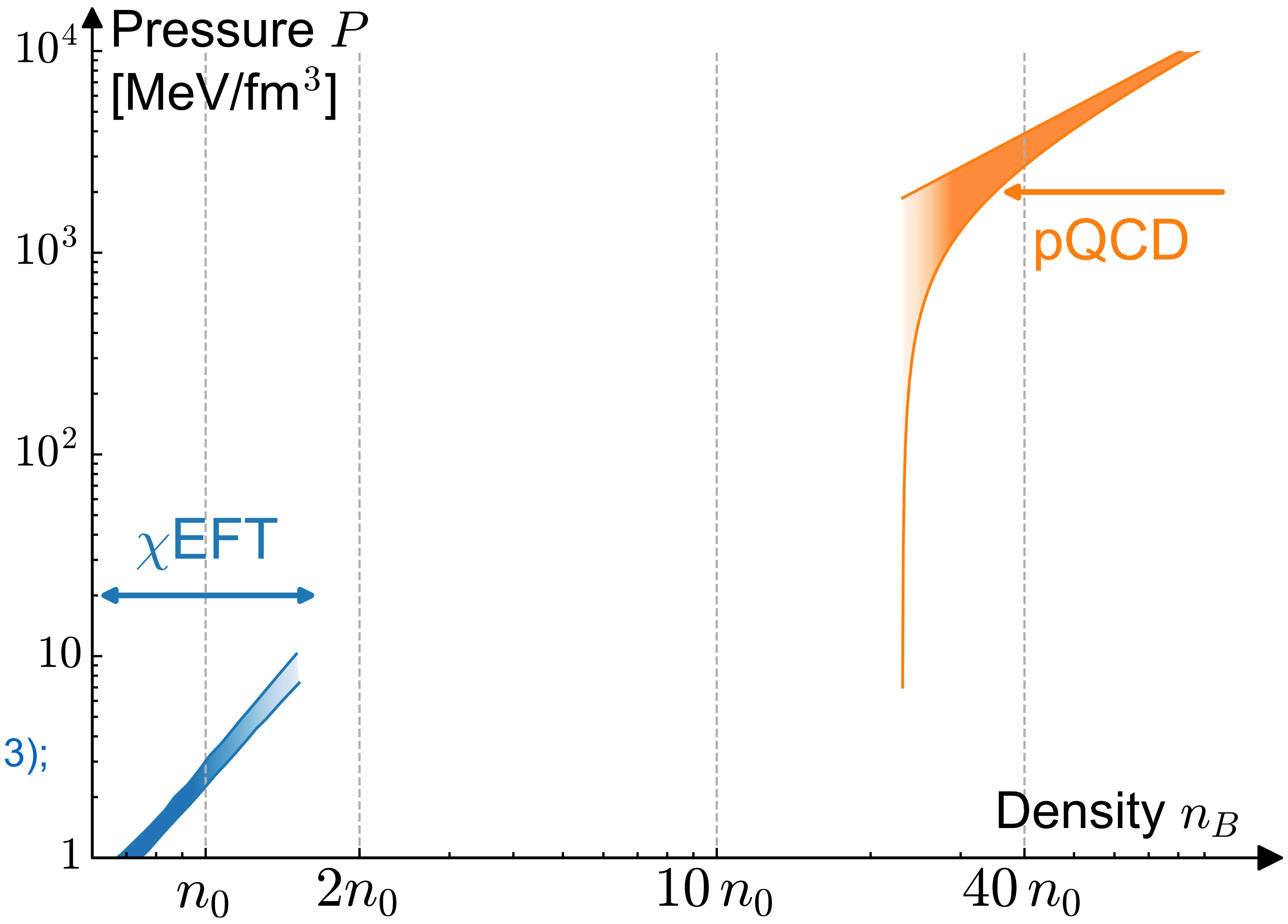
References:
[1] Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL129 (2022) [2207.06753]
[2] Y. Fujimoto, T. Kojo, L. McLerran, PRL132 (2024) [2306.04304]
[3] Y. Fujimoto, S. Reddy, PRD 109 (2024) (Selected for Editors' suggestion) [2310.09427]
[4] Y. Fujimoto, PRD109 (2024) [2312.11443]; to appear [2405.?????]

26 April, 2024 - ECT* Workshop: *The physics of strongly interacting matter*

QCD equation of state (EoS)

Freedman,McLerran(1978);
Baluni(1979);
Kurkela,Romatschke,Vuorinen,
Gorda,Säppi,
Paatelainen,Seppänen+(2009-)



Tews,Krüger,Hebeler,Schwenk(2013);
Drischler,Furnstahl,
Melendez,Philips(2020);
Keller,Hebeler,Schwenk(2022);
& many others

Nuclear density: $n_0 = 0.16$ fm^{-3}

“Uncertainty” in pQCD

Freedman,McLerran(1978);Fraga,Pisarski,Schaffner-Bielich(2001)

$$P_{\rm pQCD}(\mu;\bar\Lambda) = \frac{3\mu^2}{4\pi^2}\left[1 - 2\frac{\alpha_s(\bar\Lambda)}{\pi} - \left(2\ln\frac{\alpha_s(\bar\Lambda)}{\pi} + \frac{29}{6}\ln\frac{\bar\Lambda^2}{(2\mu)^2} + 17.39\right)\left(\frac{\alpha_s(\bar\Lambda)}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3)\right]$$

- **$\bar\Lambda$: renormalization scale**

  … only ambiguity in pQCD from perturbative series truncation

- Canonical choice: $\bar\Lambda = 2\mu$ (typical hard interaction scale)

- “Uncertainty” quantified by varying by factor 2

  i.e. $X \in [1/2,\ 2]$ with $X \equiv \bar\Lambda/(2\mu)$

  … ad hoc procedure, purely based on historical practice

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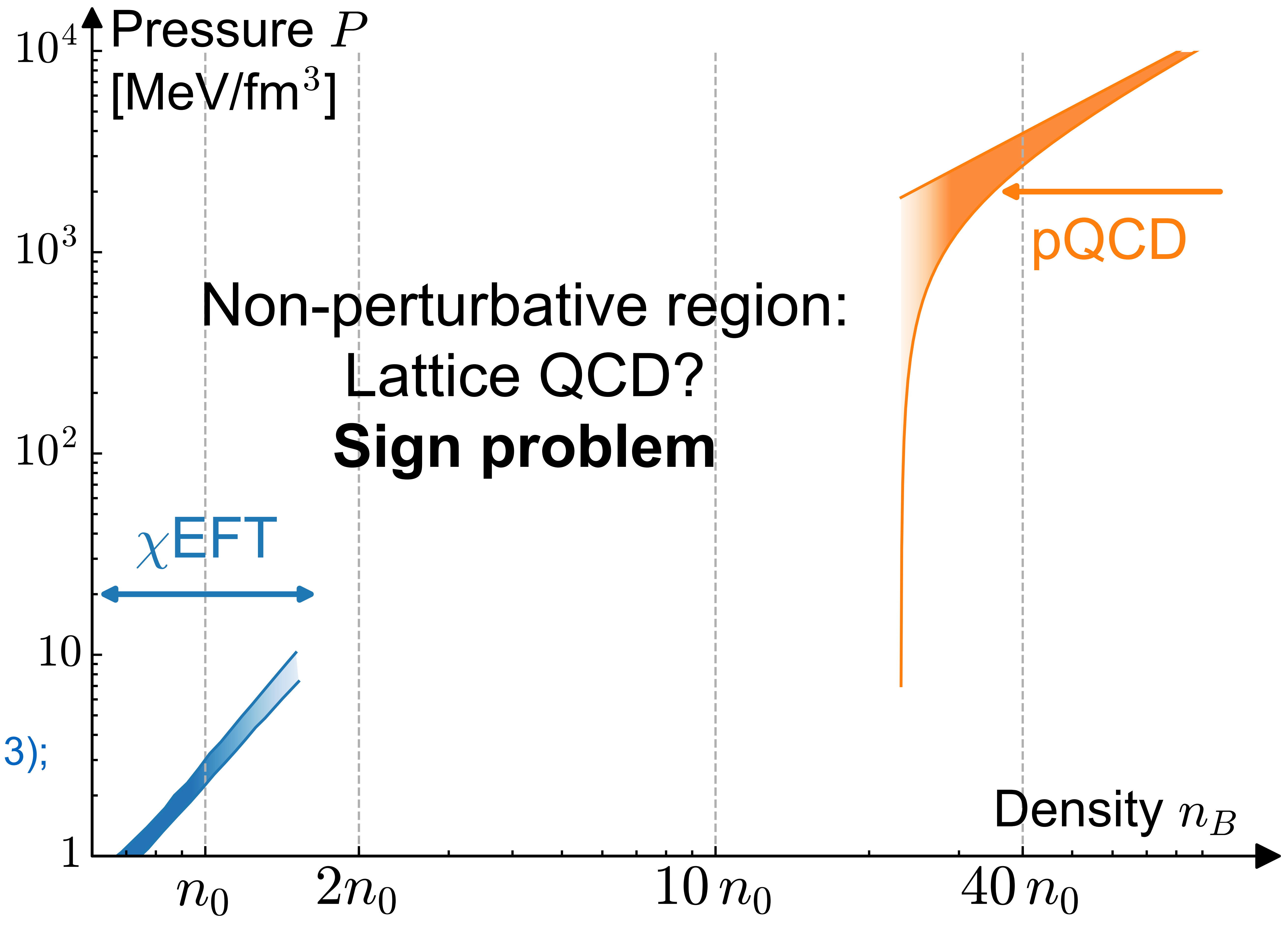

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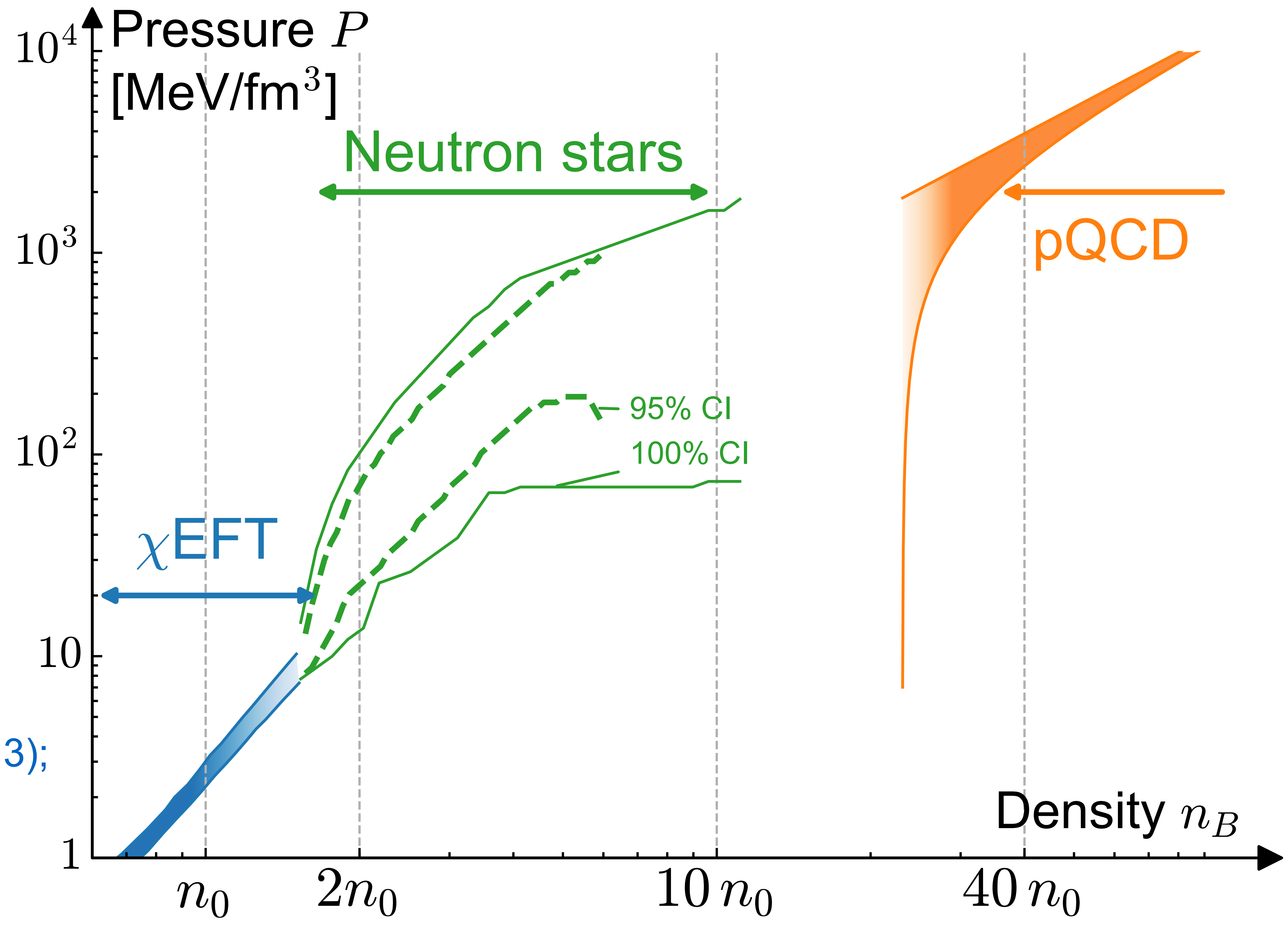

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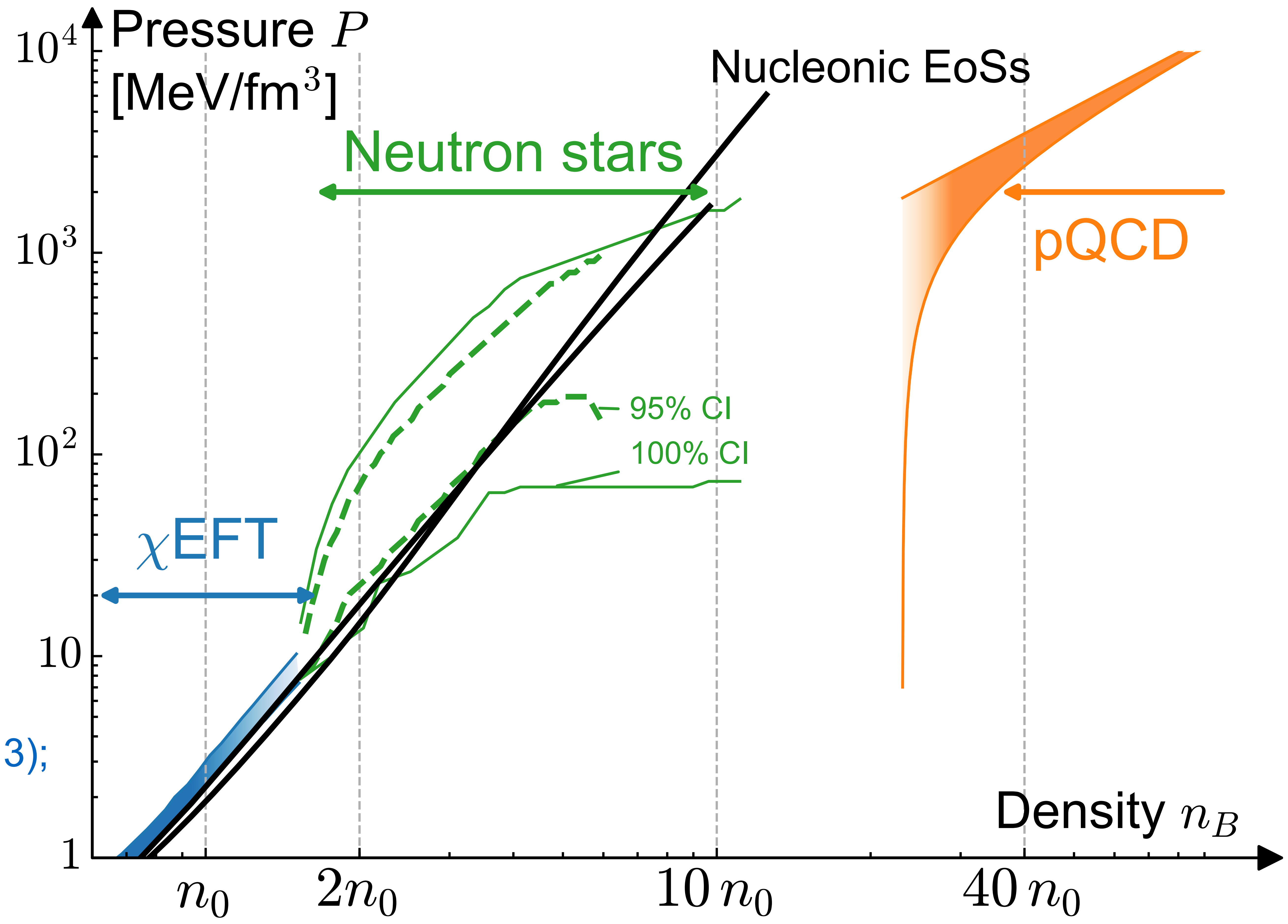

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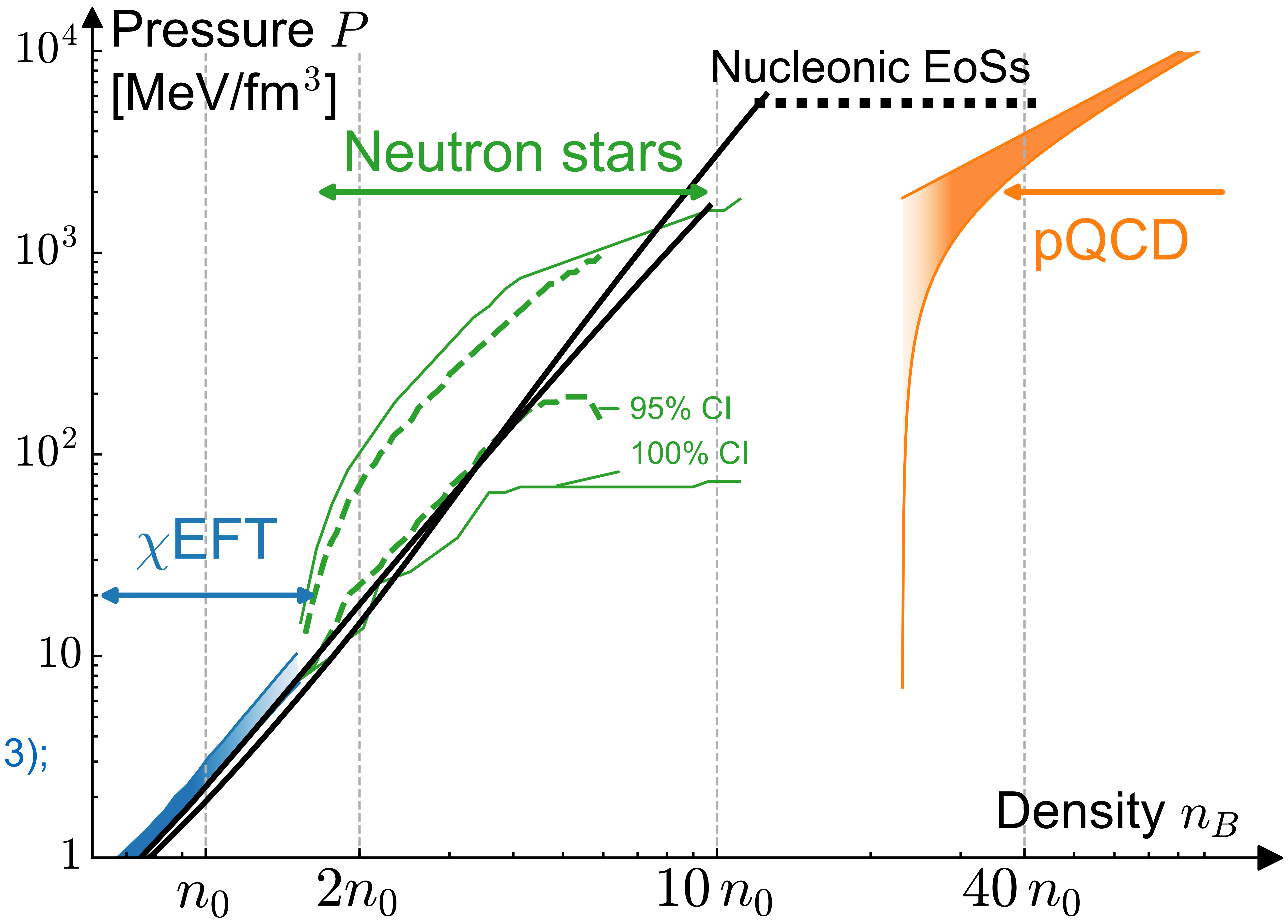

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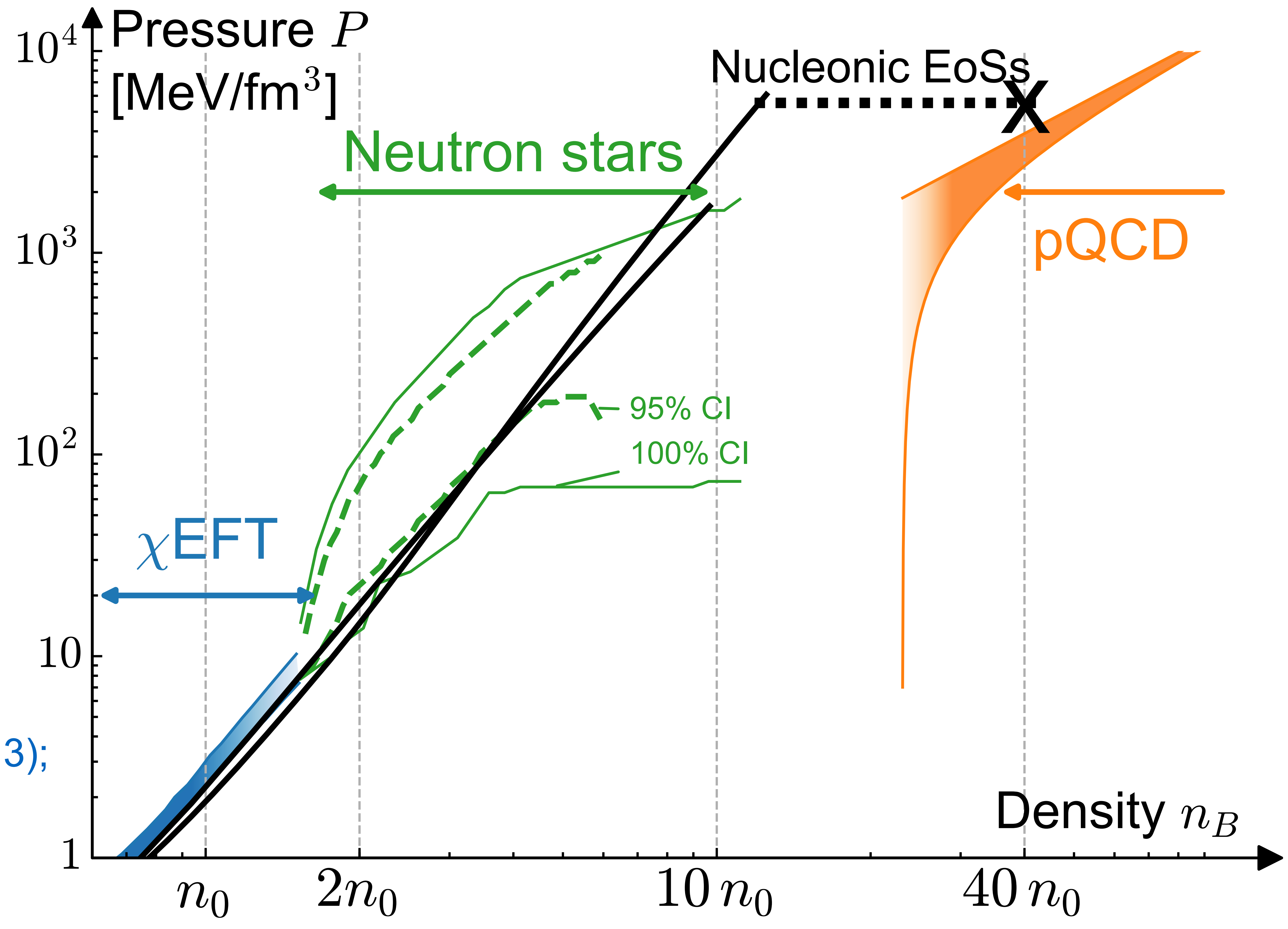

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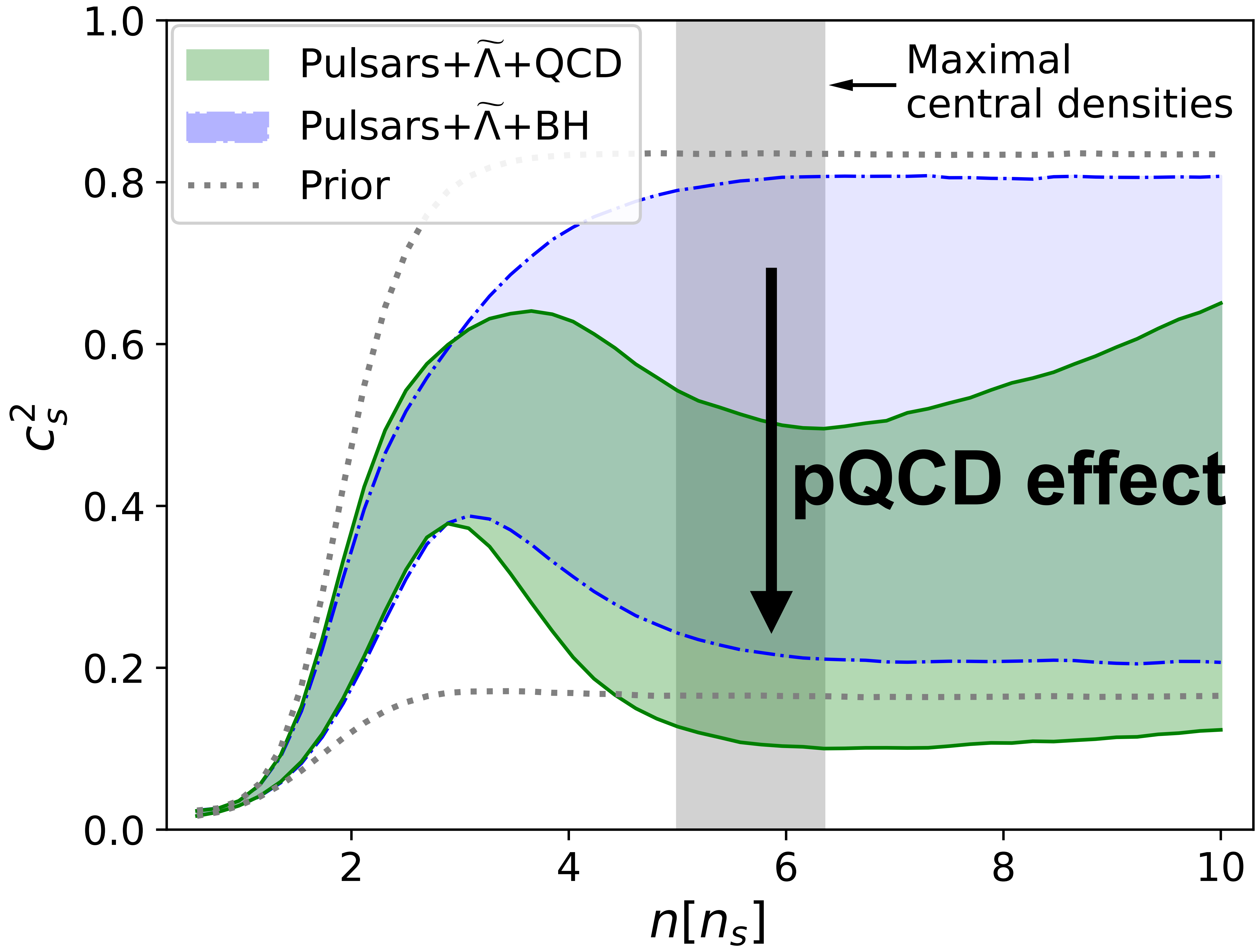
The effect of pQCD on the EoS



Softening at high density

Annala et al.(2020); Gorda,Komoltsev,Kurkela(2022); Altiparmak,Ecker,Rezzola(2022); Fujimoto,Fukushima,McLerran,Praszalowicz(2022); Marczenko,McLerran,Redlich,Sasaki(2022)

cf. no softening

Somasundaram,Margueron,Tews(2022); Brandes,Weise,Kaiser(2023); …

Disagreement?
→ No. But, depends on the density up to which the EoS is modeled

Komoltsev,Somasundaram,Gorda,Kurkela,Margueron,Tews(2023)

QCD at finite isospin density

Alford,Kapustin,Wilczek (1999); Kogut,Sinclair (2002-);
Beane,Detmold,Savage et al. (NPLQCD) (2007-);
Endrodi et al. (2014-)…

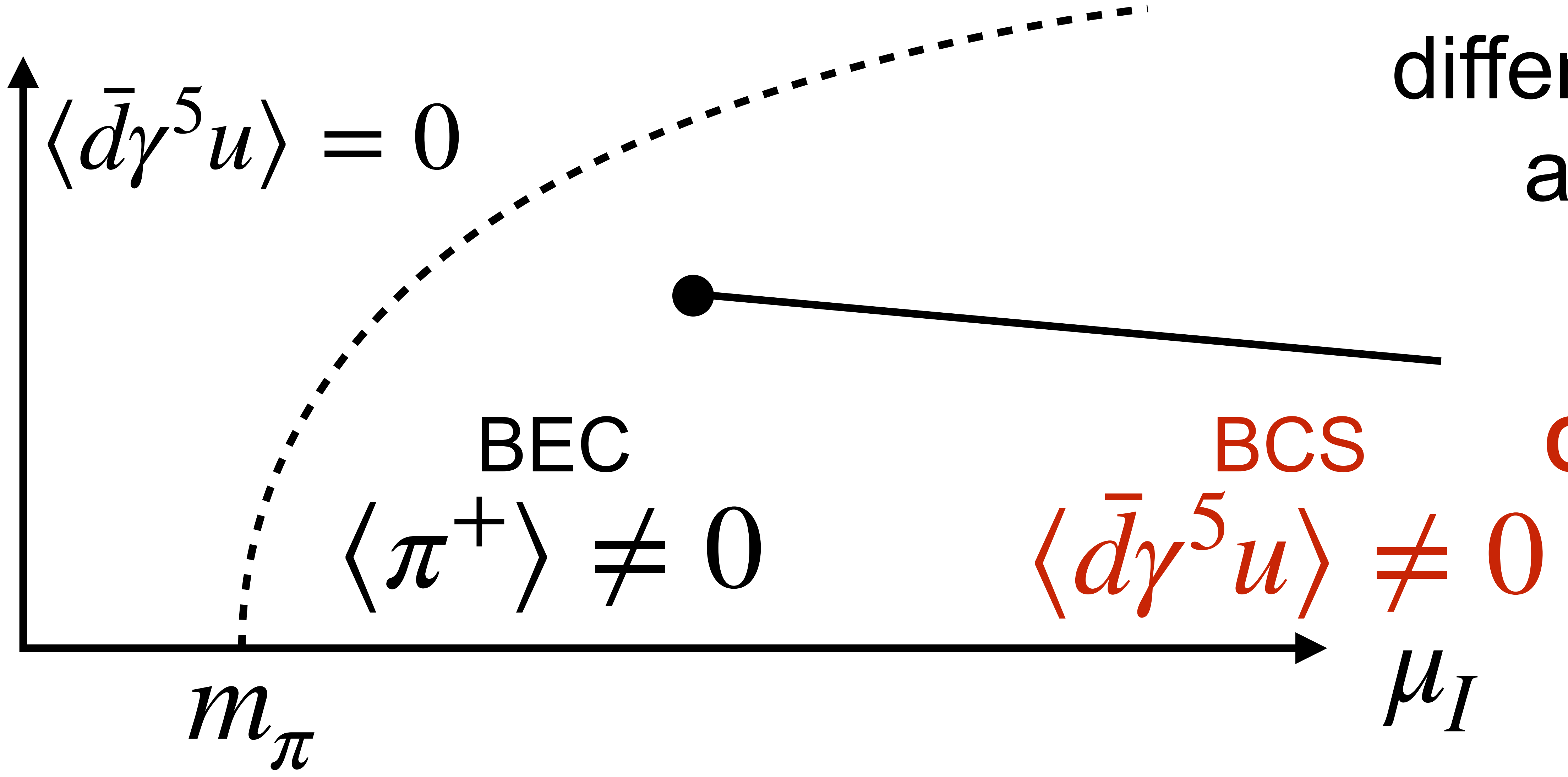
- **No sign problem** → can be simulated on the lattice!

- Isospin chemical potential (conjugate to isospin density I_3):

$$\mu_u = \frac{\mu_I}{2}, \quad \mu_d = -\frac{\mu_I}{2}$$ … Fermi surface of u & $\bar{d}$

- Phase structure:

Son,Stephanov (2000)



different from phase structure at finite baryon density

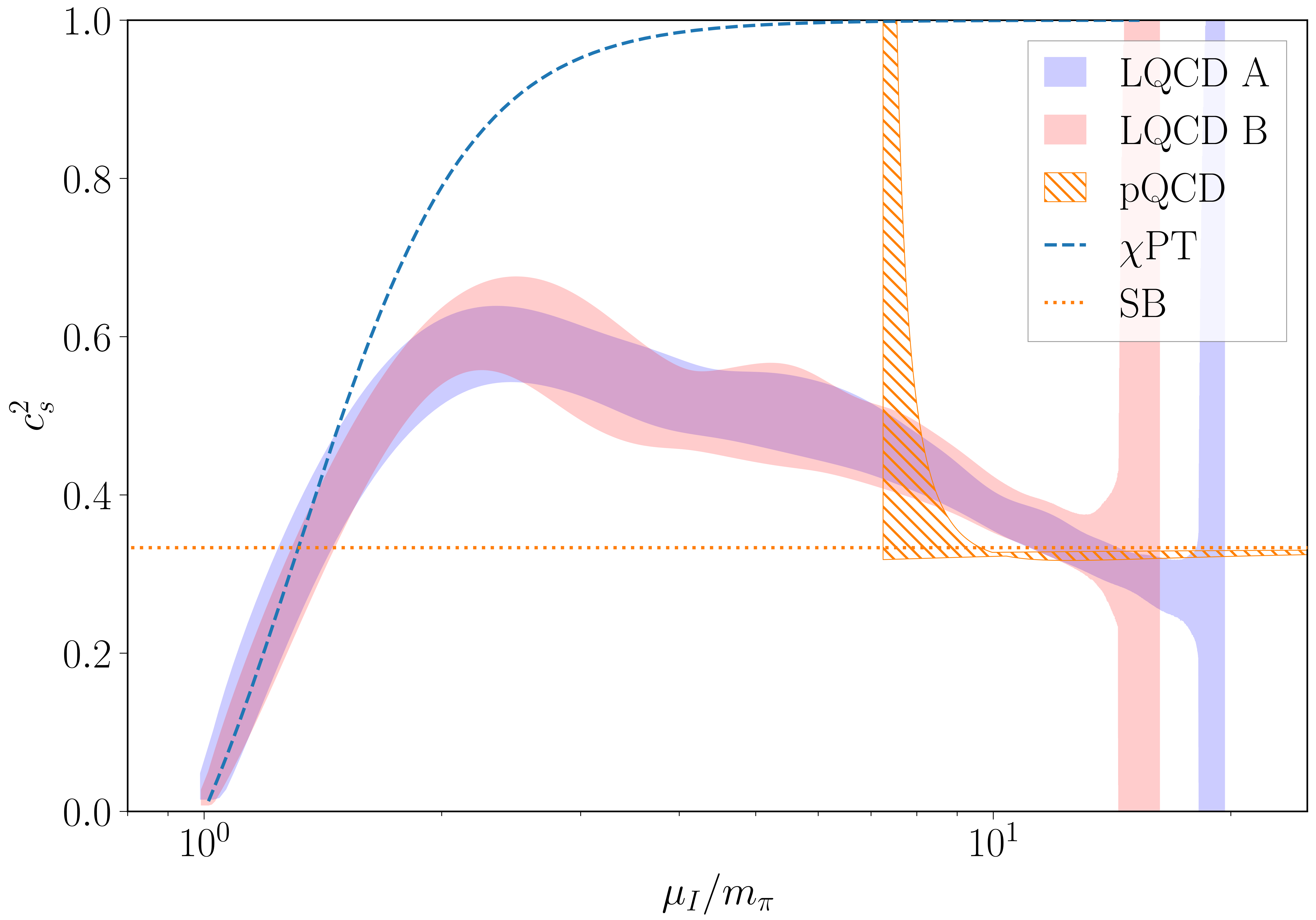
Cooper pairing

QCD EoS at finite isospin density

Recent impact:

Abbott et al. (NPLQCD collaboration) (2023)

EoS is calculated up to $n_I \sim 180\, n_{\text{sat}}$ by lattice QCD



Outline

1. **Implications from lattice QCD at finite isospin density**

    a) Bounds on isospin symmetric EoS from QCD inequality

    Y. Fujimoto, S. Reddy, PRD109 (2024)

    b) Comparison with weak-coupling results

    Y. Fujimoto, PRD109 (2024);
    Y. Fujimoto, in preparation

2. **Duality and conformality in dense QCD**

    Trace anomaly, Quarkyonic matter

    Y. Fujimoto, T. Kojo, L. McLerran, PRL132 (2024);
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Notation

- QCD_I: QCD at finite μ_I and zero μ_B
- QCD_B: QCD at finite μ_B and zero μ_I

QCD inequality

Cohen (2003); Fujimoto,Reddy (2023);
see also: Moore,Gorda (2023)

- **Dirac operator**: $\mathscr{D}(\mu) \equiv \gamma^\mu D_\mu + m - \mu\gamma^0$, **property**: $\det \mathscr{D}(-\mu) = [\det \mathscr{D}(\mu)]^*$

- **QCD$_I$**: $Z_I(\mu_I) = \int [dA] \det \mathscr{D}(\frac{\mu_I}{2}) \det \mathscr{D}(-\frac{\mu_I}{2}) e^{-S_G} = \int [dA] \left| \det \mathscr{D}(\frac{\mu_I}{2}) \right|^2 e^{-S_G}$

charge conjugation symmetry $\mu_B \to -\mu_B$

- **QCD$_B$**: $Z_B(\mu_B) = \int [dA] \det \mathscr{D}(\frac{\mu_B}{N_c}) \det \mathscr{D}(\frac{\mu_B}{N_c}) e^{-S_G} = \int [dA] \mathrm{Re} \left[\det \mathscr{D}(\frac{\mu_B}{N_c}) \right]^2 e^{-S_G}$

Note: this is **isospin symmetric** because there is no isospin imbalance

- From the relation $\mathrm{Re}\, z^2 \le |z^2| = |z|^2$:

$$Z_B(\mu_B) \le \int [dA] \left| \det \mathscr{D}(\frac{\mu_B}{N_c}) \right|^2 e^{-S_G} = Z_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

QCD inequality

Cohen (2003); Fujimoto,Reddy (2023);
see also: Moore,Gorda (2023)

- Take the log of the following inequality:

$$Z_B(\mu_B) \leq Z_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

- QCD inequality for pressure $P \propto \log Z$:

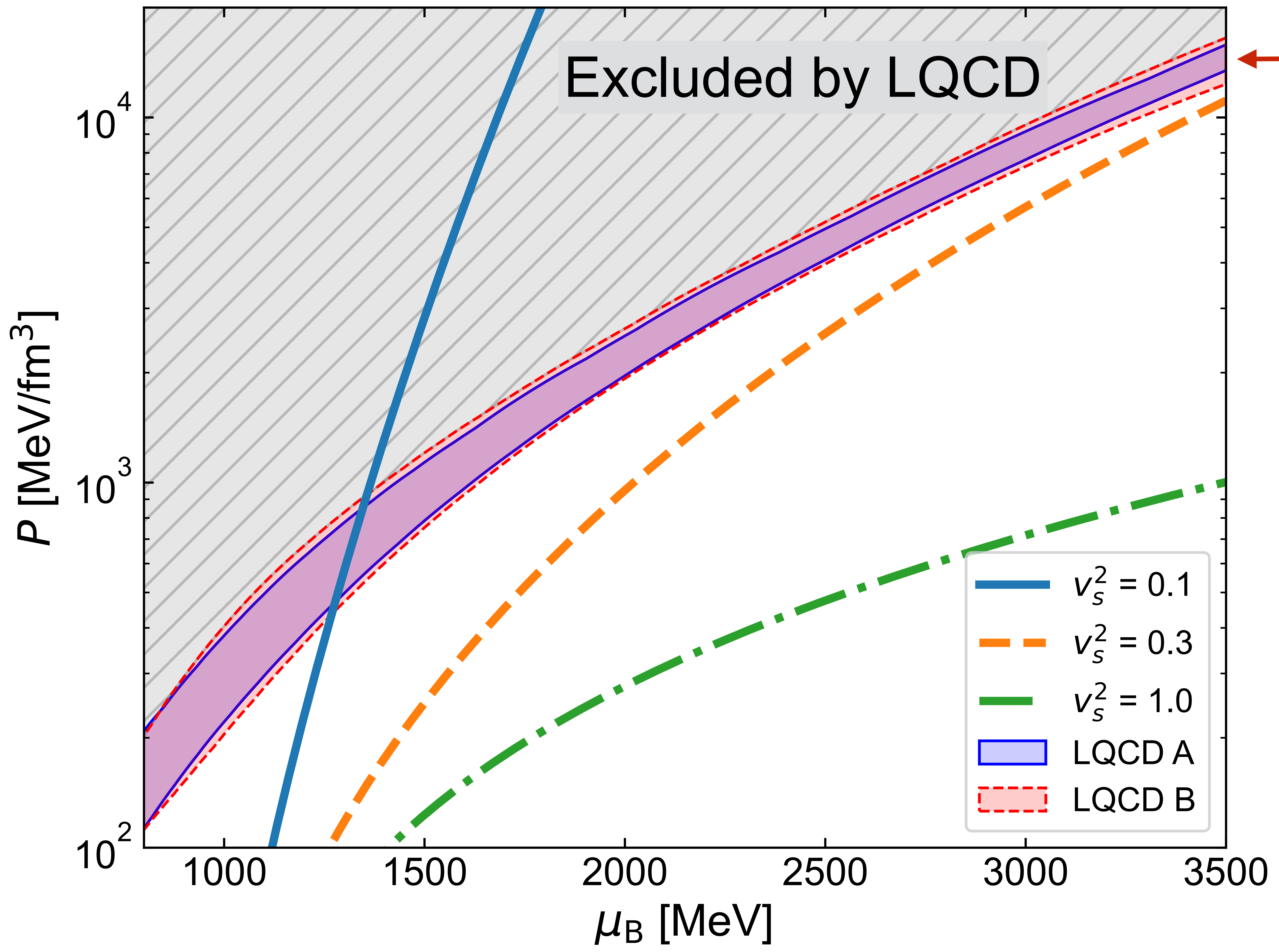
$$P_B(\mu_B) \leq P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Pressure of dense QCD$_B$ matter
(what we want to know)

Pressure of dense QCD$_I$ matter
(what we already know
from lattice QCD)

Direct use of QCD inequality

Lattice data: Abbott et al. (2023); Fujimoto,Reddy (2023)



Lattice data: upper bound

$$P_B(\mu_B) \leq P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

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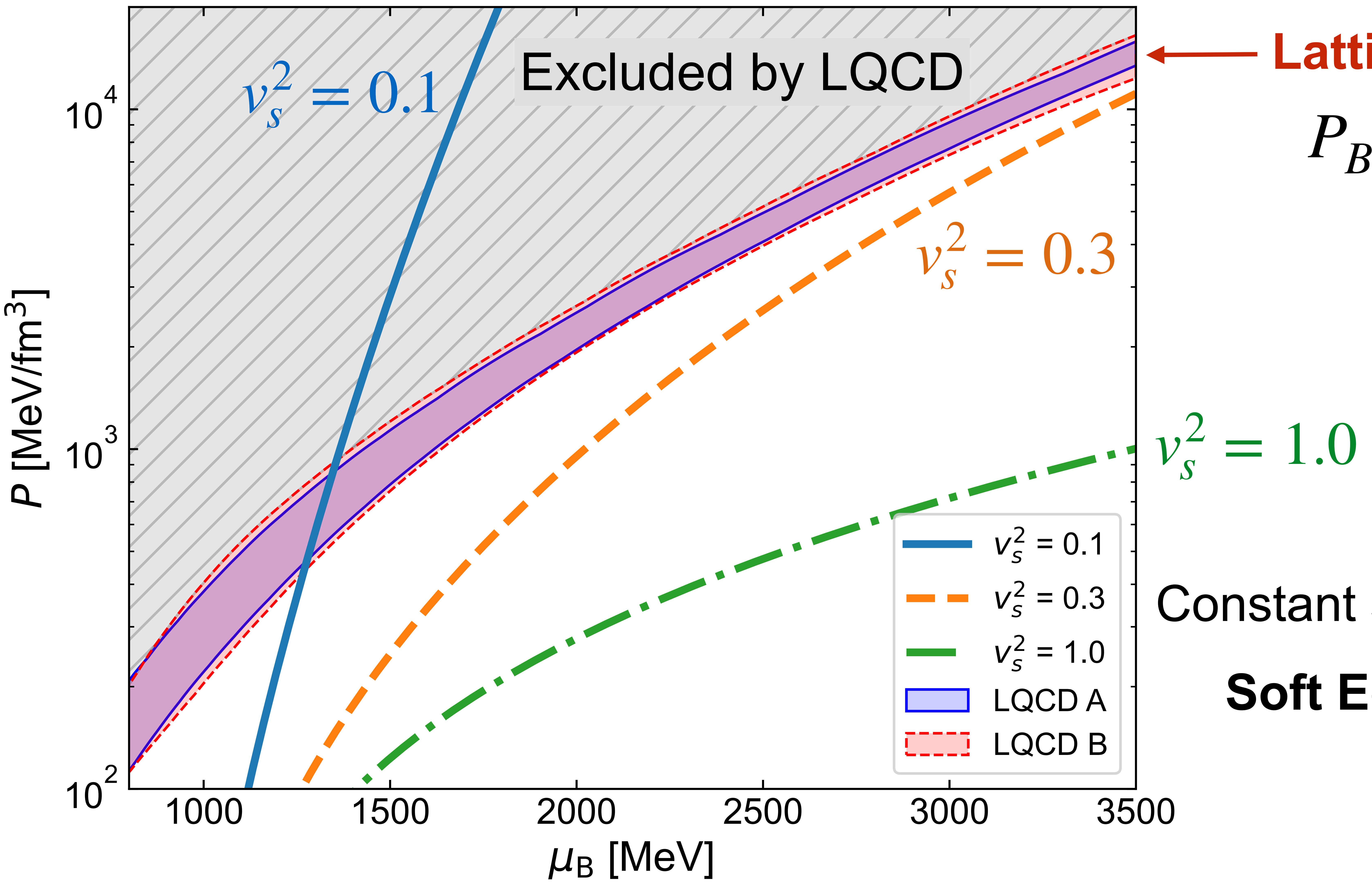


Lattice data: upper bound

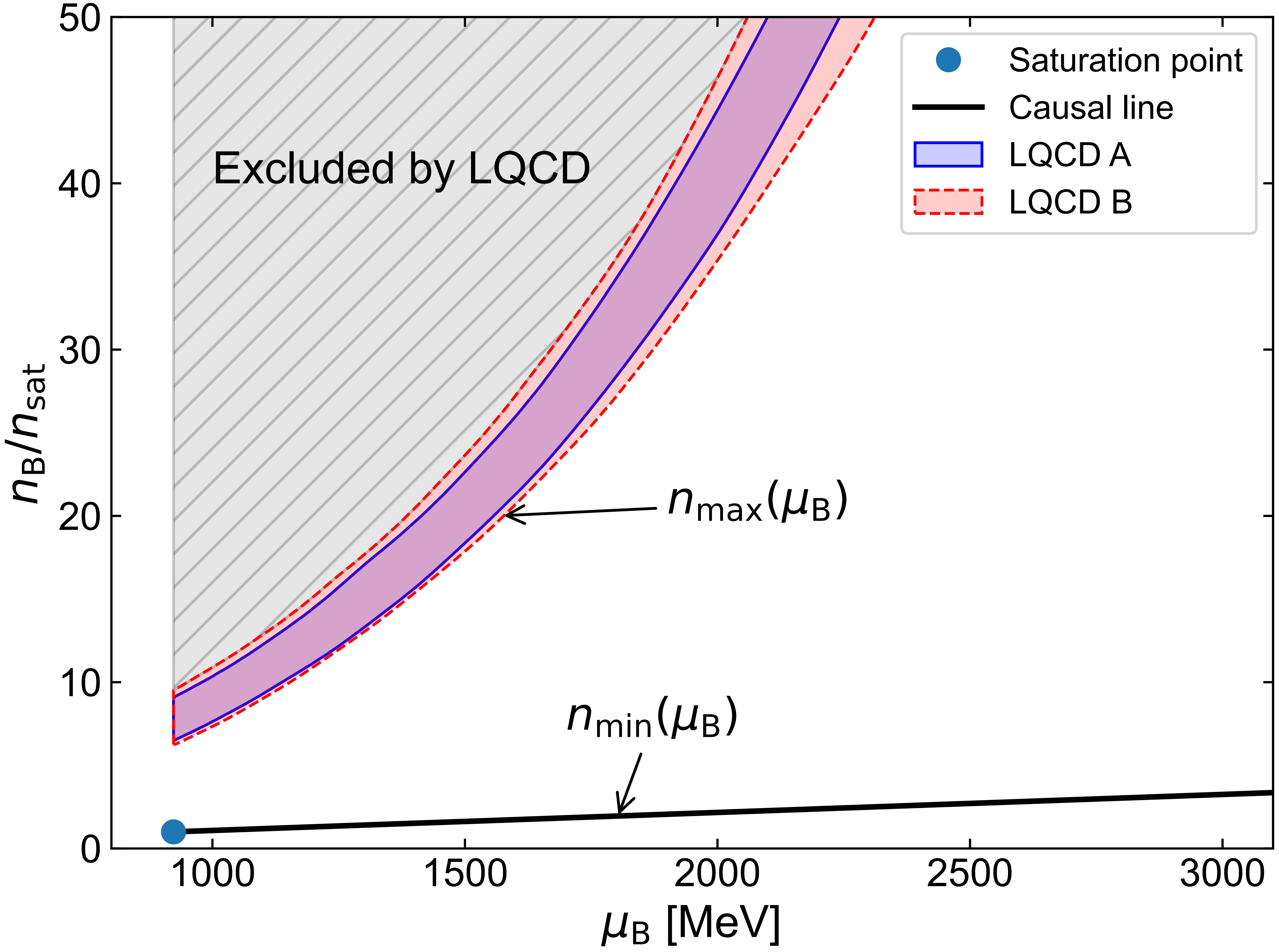
$$P_B(\mu_B) \le P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

Constant sound speed EoS: $P(\varepsilon) \propto v_s^2 \varepsilon$

Soft EoS (smaller P at a given ε) is excluded

Bounds on $n_B(\mu_B)$

Komoltsev,Kurkela (2021); Fujimoto,Reddy (2023)



Properties $n_B(\mu_B)$ must satisfy:

① Stability:

$$\frac{d^2P}{d\mu_B^2} \geq 0 \;\Rightarrow\; \frac{dn_B}{d\mu_B} \geq 0$$

② Causality $v_s^2 \leq 1$:

$$v_s^2 = \frac{n_B}{\mu_B}\frac{d\mu_B}{dn_B} \leq 1 \;\Rightarrow\; \frac{dn_B}{d\mu_B} \geq \frac{n_B}{\mu_B}$$

③ QCD inequality on the integral:

$$\int_{\boxed{\mu_{\rm sat}}}^{\mu_B} d\mu'\, n_B(\mu') \;\leq P_I\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$

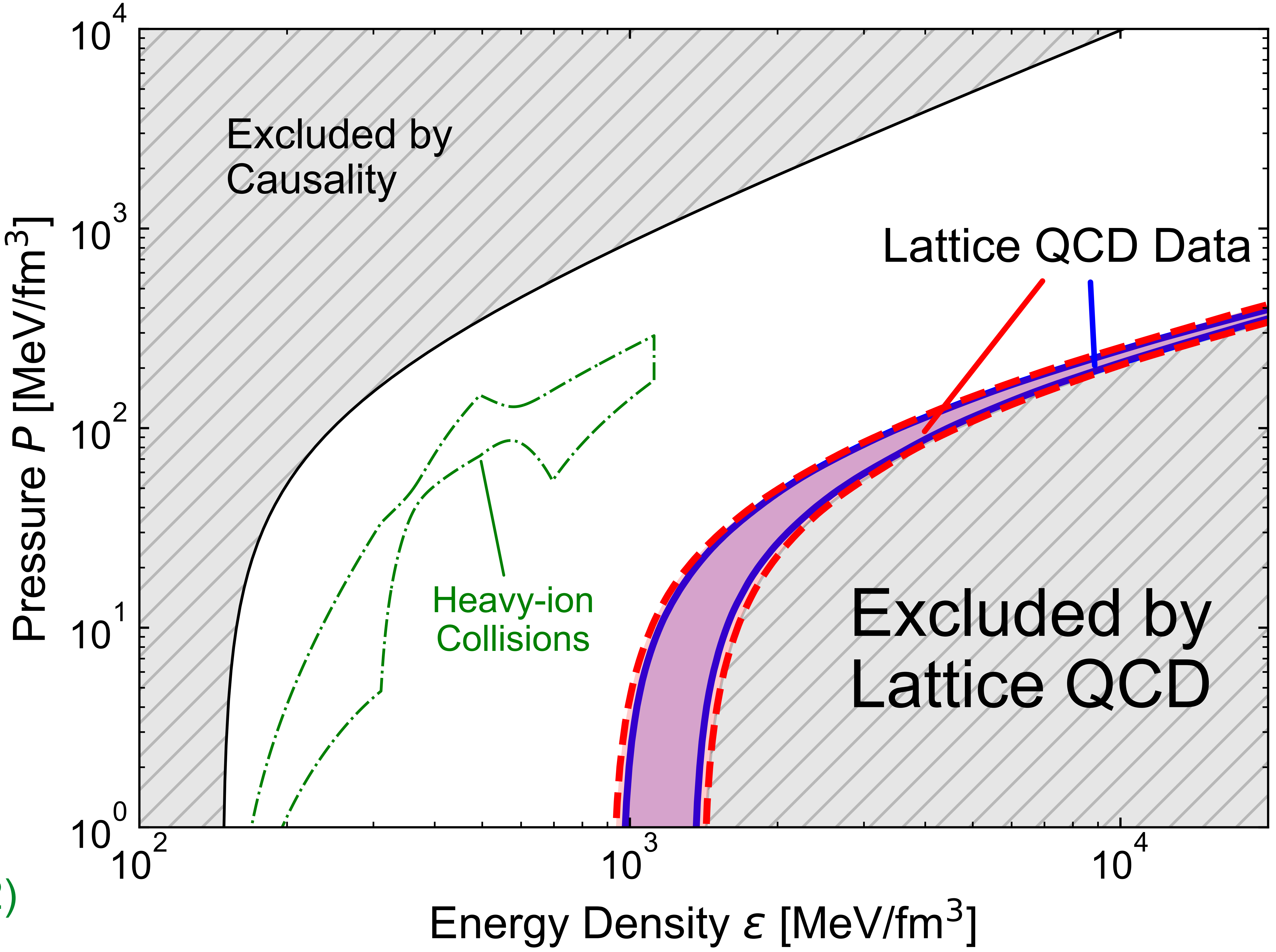
Lower bound of the integral must be specified

fix it to the **empirical saturation property**

Robust bounds on $P(\varepsilon)$

Fujimoto, Reddy (2023)

From the relation $\varepsilon = -P + \mu_B n_B$:



Soft EoS at large ε is excluded

Heavy-ion:
Oliinychenko et al. (2022)

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Applicability of weak-coupling results?

Freedman,McLerran (1978); Baluni (1979);
Kurkela,Romatschke,Vuorinen,Gorda,Säppi,
Paatelainen,Seppänen+ (2009-)

Bulk pQCD thermodynamics in weak-coupling α_s expansion:

$$P_{\mathrm{pQCD}}(\mu) = \frac{3\mu^2}{4\pi^2}\left[1 - 2\frac{\alpha_s}{\pi} - \left(2\ln\frac{\alpha_s}{\pi} + \frac{29}{6}\ln\frac{\bar{\Lambda}^2}{(2\mu)^2} + 17.39\right)\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3)\right]$$

$(N_c = 3,\ N_f = 3)$

- Convergence seems to be good up to $\mathcal{O}(\alpha_s^2)$
- Valid down to $\mu \sim 10^3$ MeV?

- **Universal for QCD$_B$ and QCD$_I$ up to $\mathcal{O}(\alpha_s^2)$**

  $\rightarrow$ Lattice QCD$_I$ can be used as benchmark

Moore,Gorda (2023);
Navarette,Paatelainen,Seppänen (2024)

Condensation energy

e.g. Alford,Rajagopal,Schafer,Schmitt (2008); Fujimoto (2023)

Contribution of the Cooper pairing gap to bulk thermodynamics (pressure)

Difference in pressure w/ and w/o the gap formation:

$$\delta P \equiv P(\Delta \neq 0) - P(\Delta = 0)$$

Weak-coupling expression up to next-to-leading order:

$$\delta P = \frac{3\mu^2}{2\pi^2}\Delta^2 \left[1 + \frac{\pi}{3}\left(\frac{\alpha_s}{\pi}\right)^{1/2} \right]$$ … condensation energy

D.o.S

Pairing gap, weak-coupling formula

Cooper pairing gap in weak coupling

Son (1998); Schäfer,Wilczek (1999); Pisarski,Rischke (1999);
Brown,Liu,Ren (1999); Wang,Rischke (2001); …

- Color-superconducting gap up to next to leading order:

$$\ln\left(\frac{\Delta}{\mu}\right) = -\frac{\sqrt{3}\pi}{2\sqrt{c_R}}\left(\frac{\alpha_s}{\pi}\right)^{-\frac{1}{2}} - \frac{5}{2}\ln\left(N_f\frac{\alpha_s}{\pi}\right) + \ln\frac{2^{\frac{13}{2}}}{\pi} - \frac{\pi^2+4}{12c_R} - \zeta + \mathcal{O}(\alpha_s^{\frac{1}{2}})$$

$c_R = 2/3$ for $\bar{\mathbf{3}}$, $c_R = 4/3$ for $\mathbf{1}$ channel, $\zeta = \frac{1}{3}\ln 2$ for CFL, $\zeta = 0$ otherwise

… this formula is also universal for QCD_B (color superconductivity)

and QCD_I (pion condensation-like Cooper pairing) Fujimoto (2023)

→ Lattice QCD_I can also be used as benchmark

Cooper pairing gap in weak coupling

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Pros and cons for the applicability:

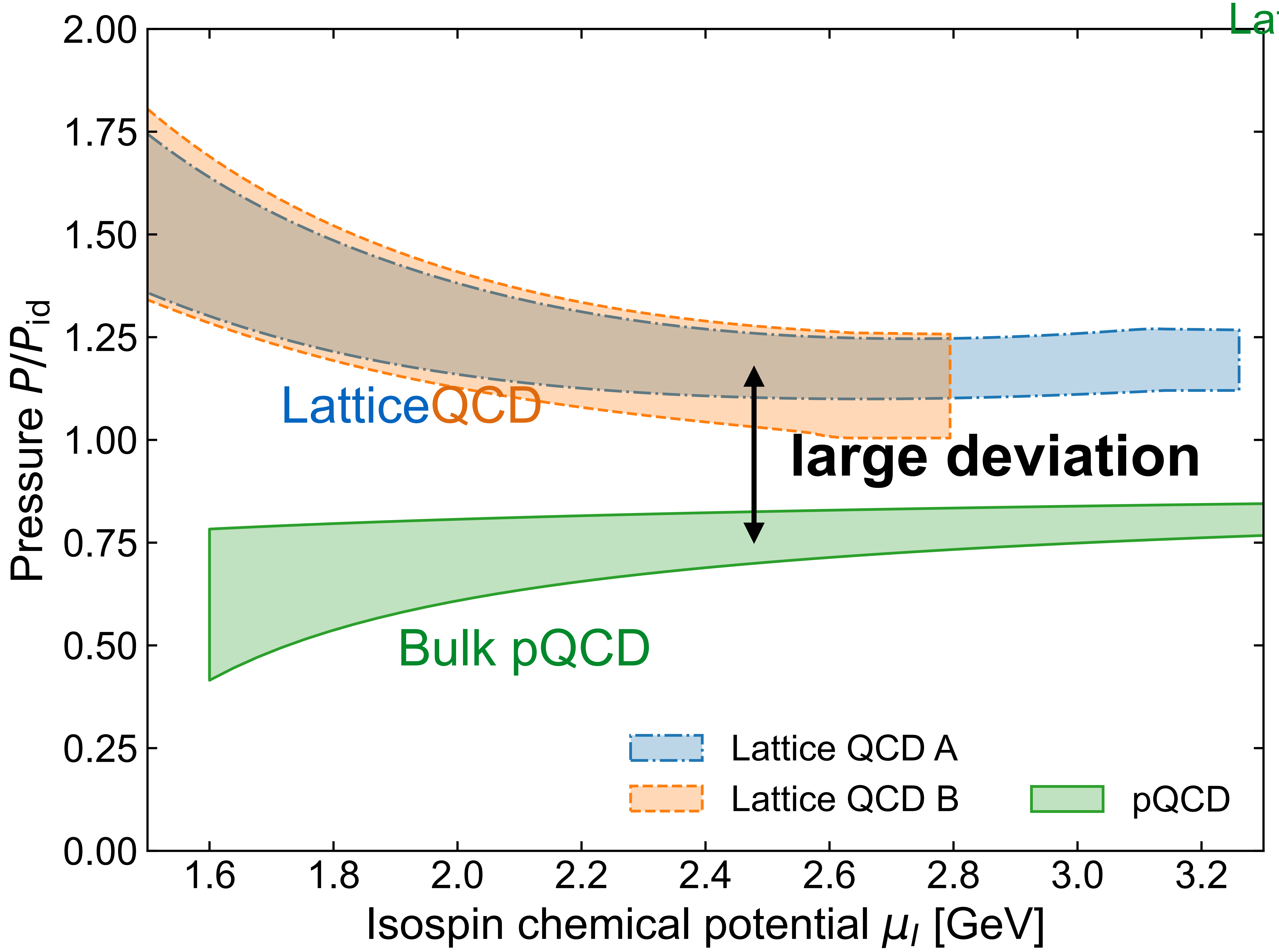
Con: - Folklore — only applicable at very large μ

e.g. $\mu \sim 10^8$ MeV
[Rajagopal,Shuster (2000)]

Pros: - Standard pQCD (e.g. collider pheno) is valid down to $\mu \sim 10^3$ MeV
- Derivation of Δ is valid as long as $\Delta \ll m_\mathrm{D} \ll \mu$ (scale separation)

Weak-coupling results vs lattice data

Lattice data: Abbott et al. (2023); Fujimoto (2023)



Bulk pQCD pressure: $P_{\rm pQCD}(\mu) = \frac{3\mu^2}{4\pi^2}\left[1 - 2\frac{\alpha_s}{\pi} - \left(2\ln\frac{\alpha_s}{\pi} + \frac{29}{6}\ln\frac{\bar{\Lambda}^2}{(2\mu)^2} + 17.39\right)\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3)\right]$

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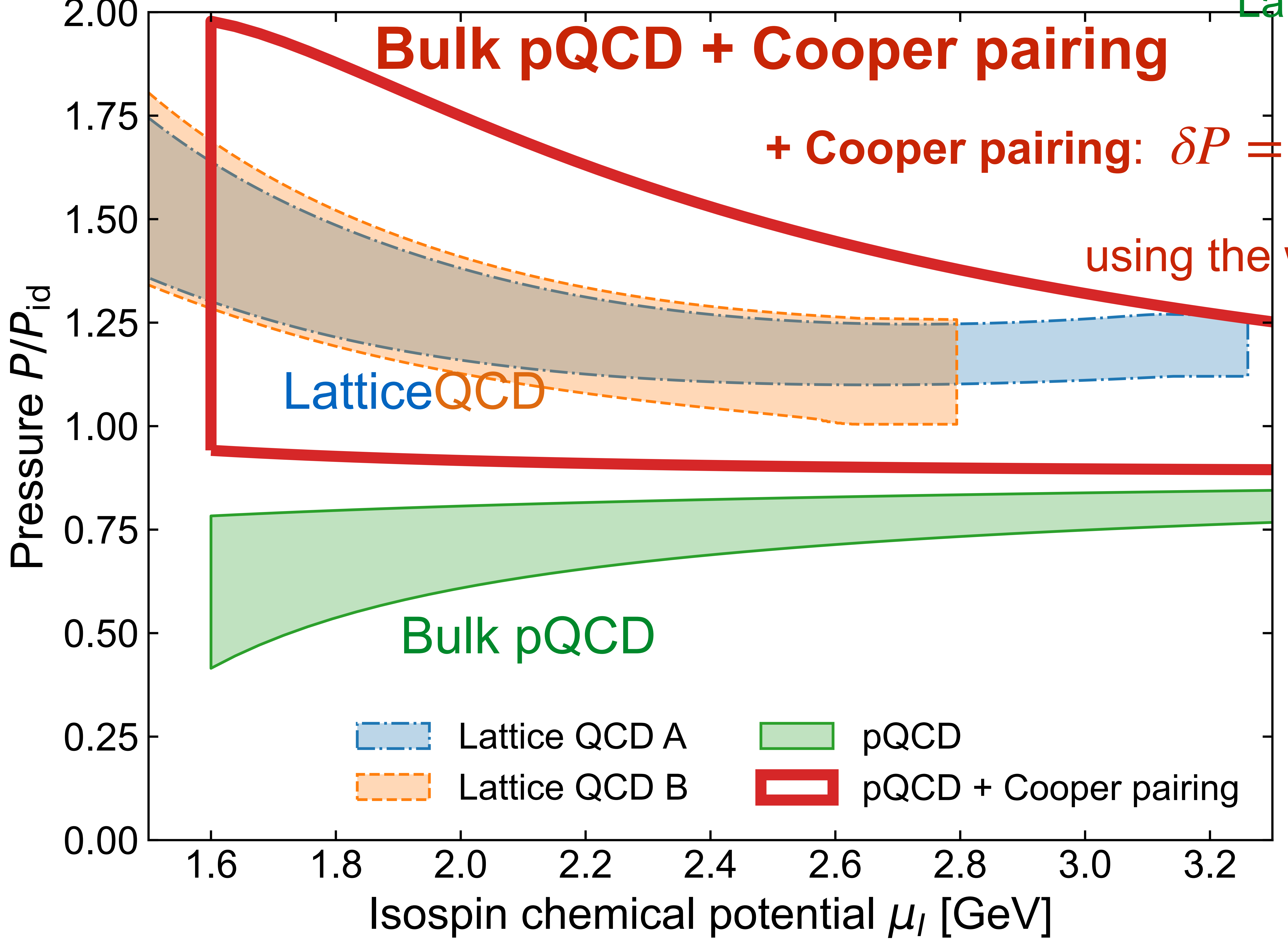

+ Cooper pairing: $\delta P = \frac{3\mu_I^2}{8\pi^2}\Delta^2\left[1+\frac{\pi}{3}\left(\frac{\alpha_s}{\pi}\right)^{1/2}\right]$

using the weak-coupling formula for Δ

Bulk pQCD pressure: $P_{\rm pQCD}(\mu) = \frac{3\mu^2}{4\pi^2}\left[1 - 2\frac{\alpha_s}{\pi} - \left(2\ln\frac{\alpha_s}{\pi} + \frac{29}{6}\ln\frac{\bar{\Lambda}^2}{(2\mu)^2} + 17.39\right)\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3)\right]$

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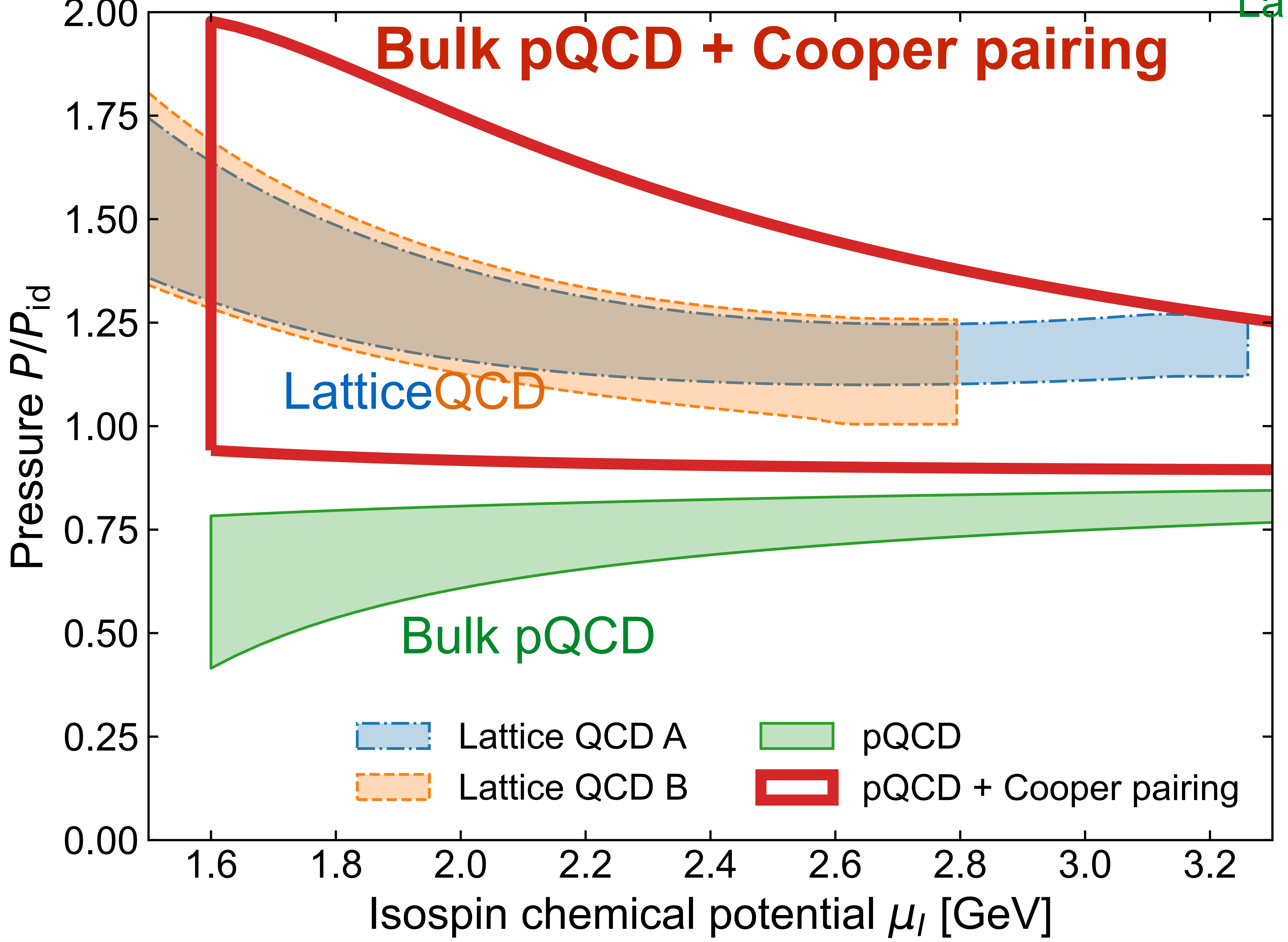


Empirical evidence for the dense-QCD weak-coupling results to be applicable down to $\mu \sim 800$ MeV

At least the magnitude is correct

Is the gap Δ the only correction?

Alford,Braby,Paris,Reddy (2004)

$$P = a_4\mu^4 + a_2\mu^2 - B$$

- a_4: Ideal gas behavior + pQCD correction (Dominant)

- a_2: **Gap correction $a_2 \propto \Delta^2$ (large, ~20-200%),**
  Quark mass $a_2 \propto -m_f^2$ (small, ~1%)
  Temperature $a_2 \propto T^2$ (small, ~1%)

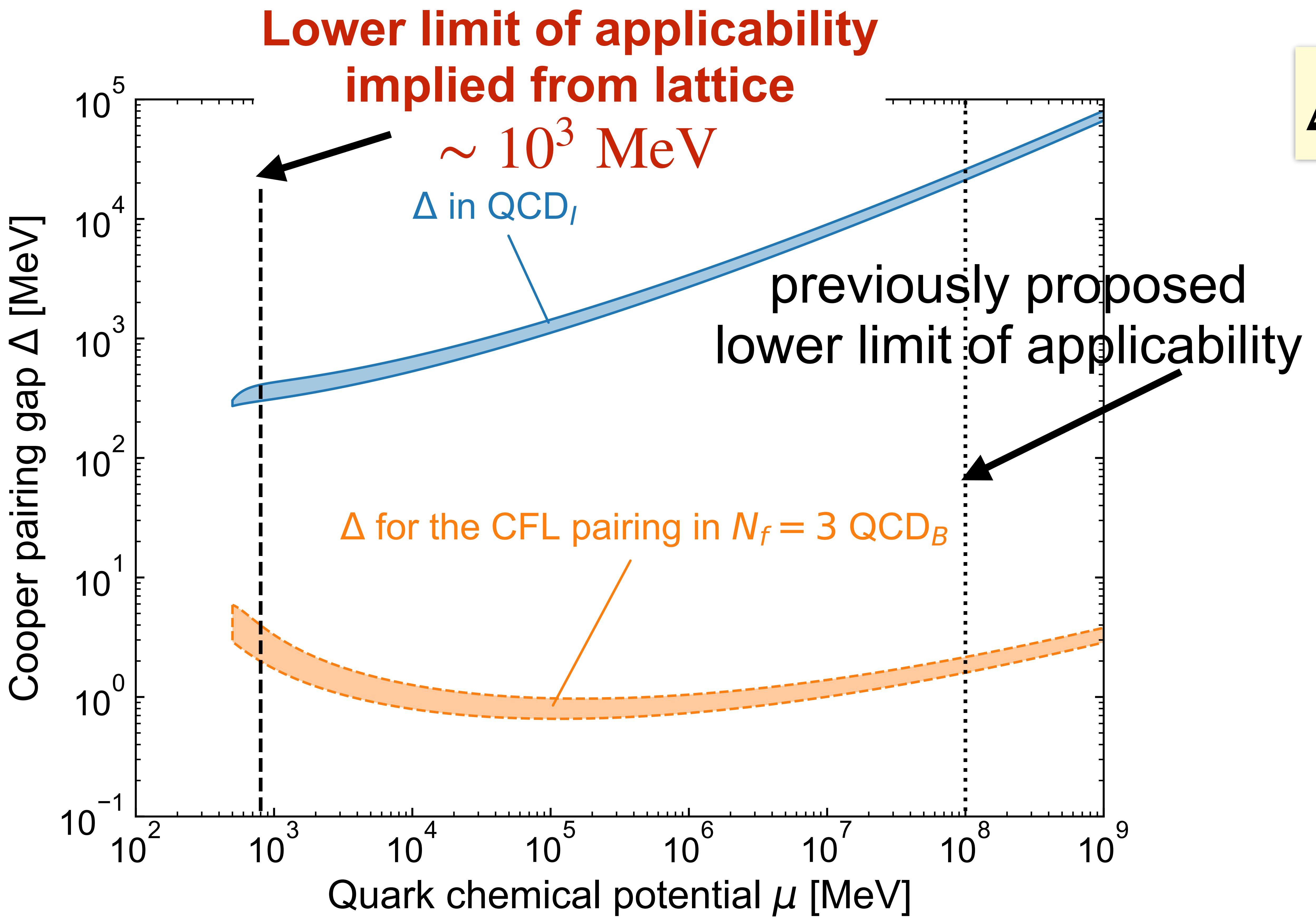
- B: Bag constant, typically $B^{1/4} \simeq 200\ \mathrm{MeV}$ (small, ~0.5%)
  Instantons, suppressed by $\dfrac{m_f}{\Lambda_{\mathrm{QCD}}} \sim 10^{-3}$

Shuryak; Kallman; Abrikosov; de Carvalho; Chemtob; Baluni (late 70s - early 80s)

Impact on QCD_B: color superconductivity

Fujimoto, *in prep.* (2024)

Weak-coupling Cooper pairing gap formula is reliable down to $\mu \sim 10^3$ MeV



$\Delta_{\mathrm{CFL}} \sim 2-3$ MeV at $\mu = 800$ MeV

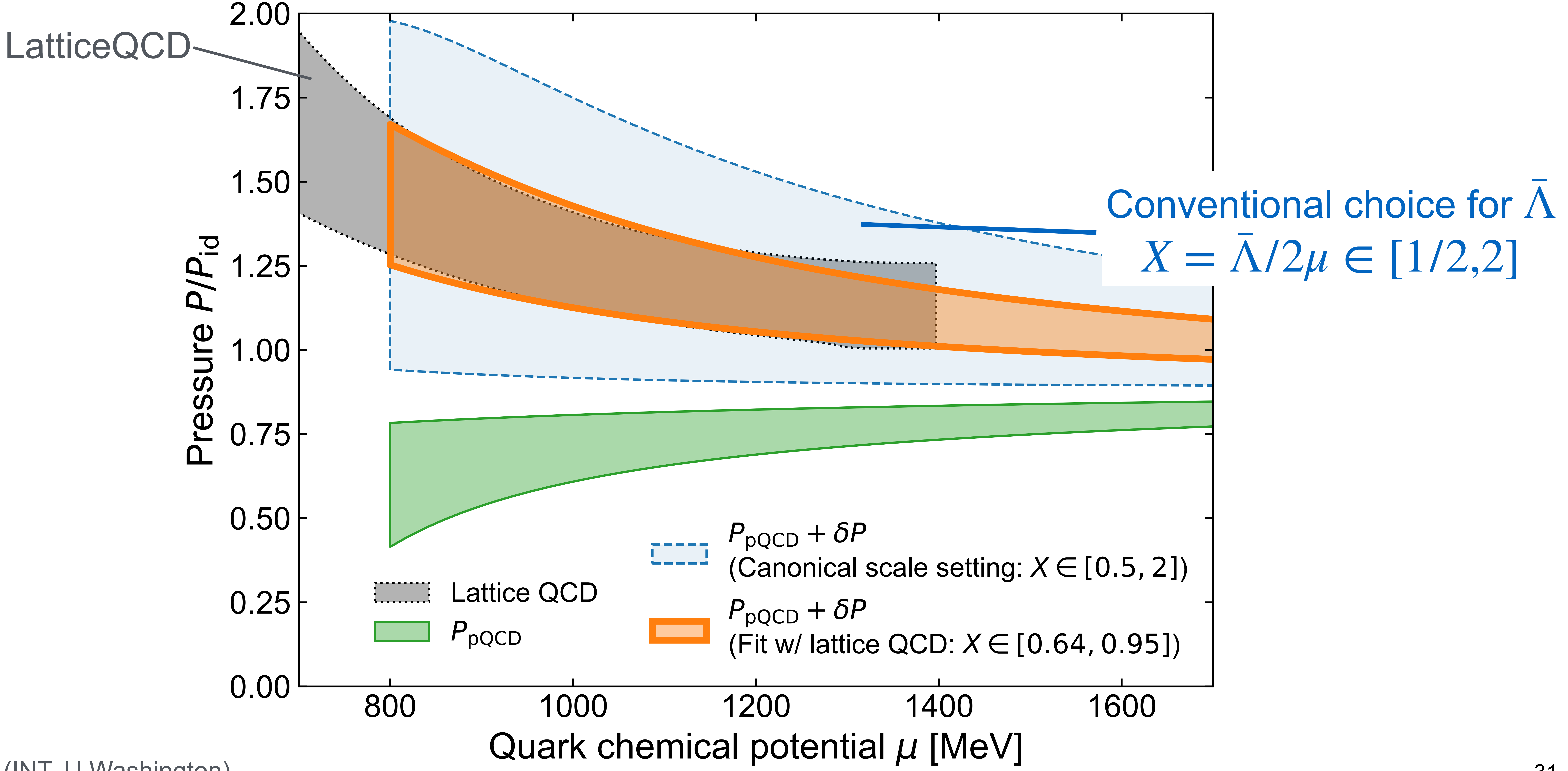
- A negligibly **small** contribution to bulk thermodynamics

- Comparable to the stress by strange quark mass: $\Delta_{\mathrm{CFL}} \sim m_s^2/4\mu$

$\rightarrow$ CFL may not be the ground state?

Prescription for $\bar{\Lambda}$ determination

Fujimoto, *in prep.* (2024)



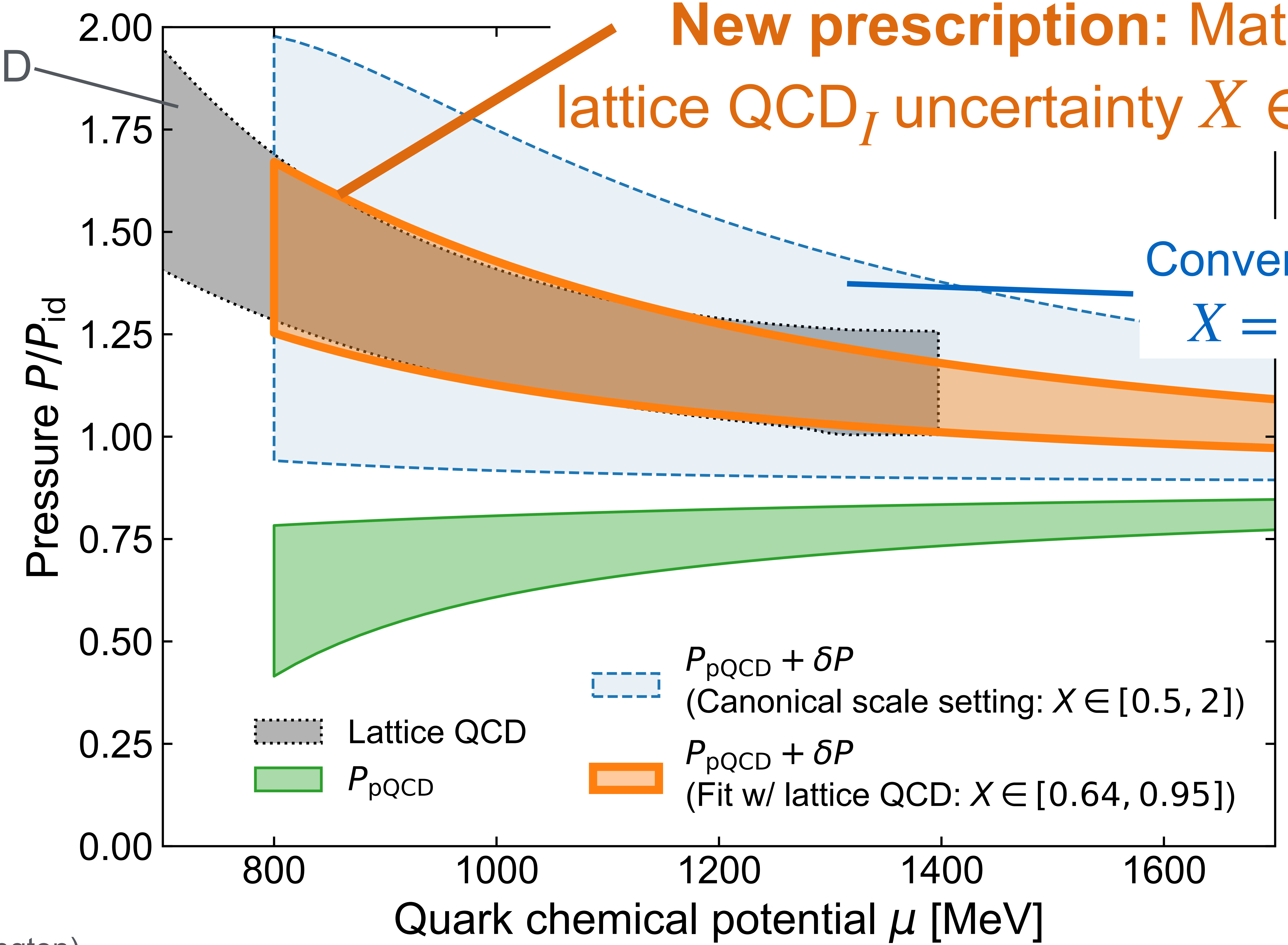
Prescription for $\bar{\Lambda}$ determination

Fujimoto, *in prep.* (2024)

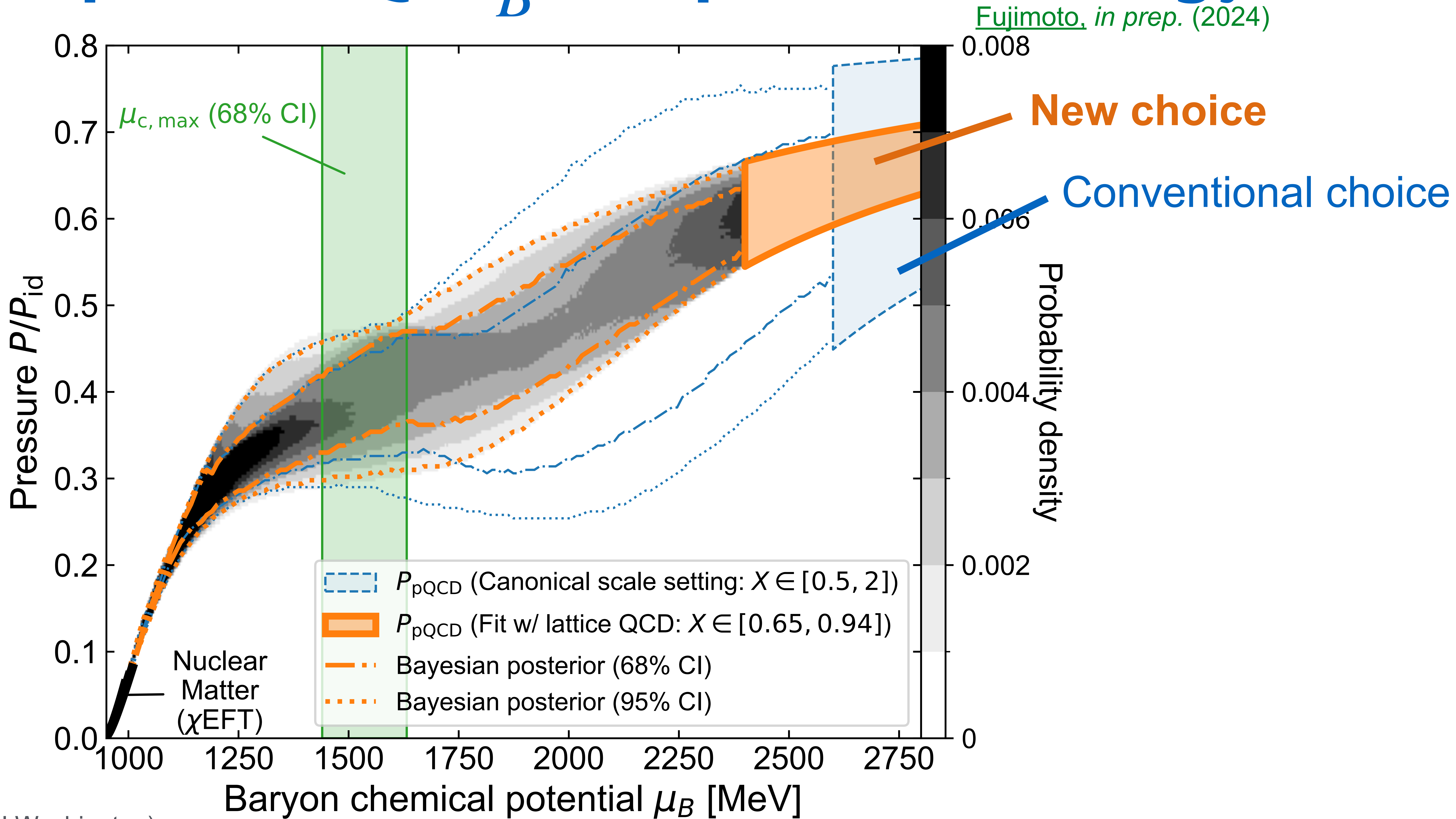
New prescription: Matching $\bar{\Lambda}$ to lattice QCD_I uncertainty $X \in [0.64,\ 0.95]$

Conventional choice for $\bar{\Lambda}$
$X = \bar{\Lambda}/2\mu \in [1/2,2]$

LatticeQCD



Impact on QCD_B: NS phenomenology



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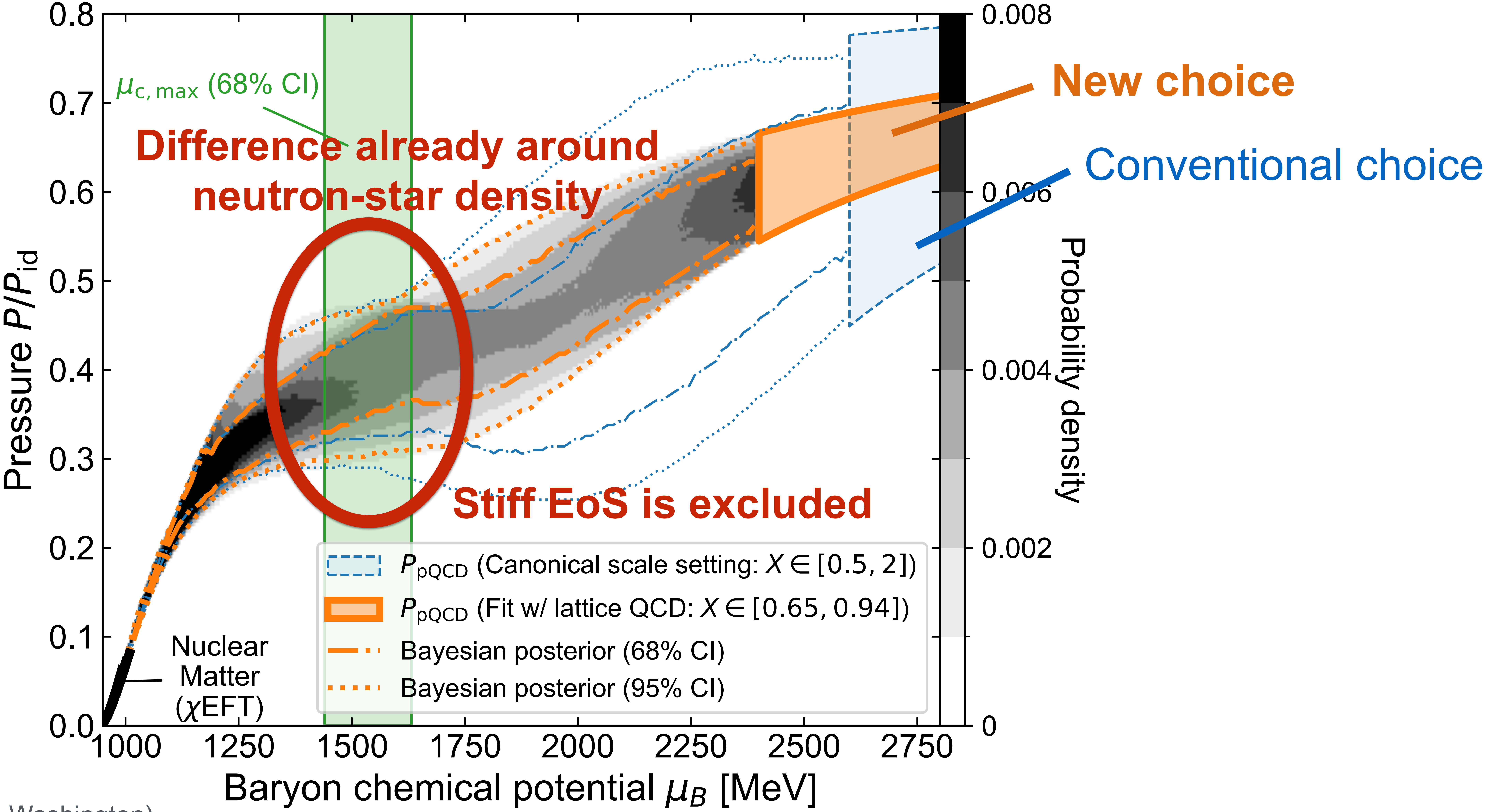
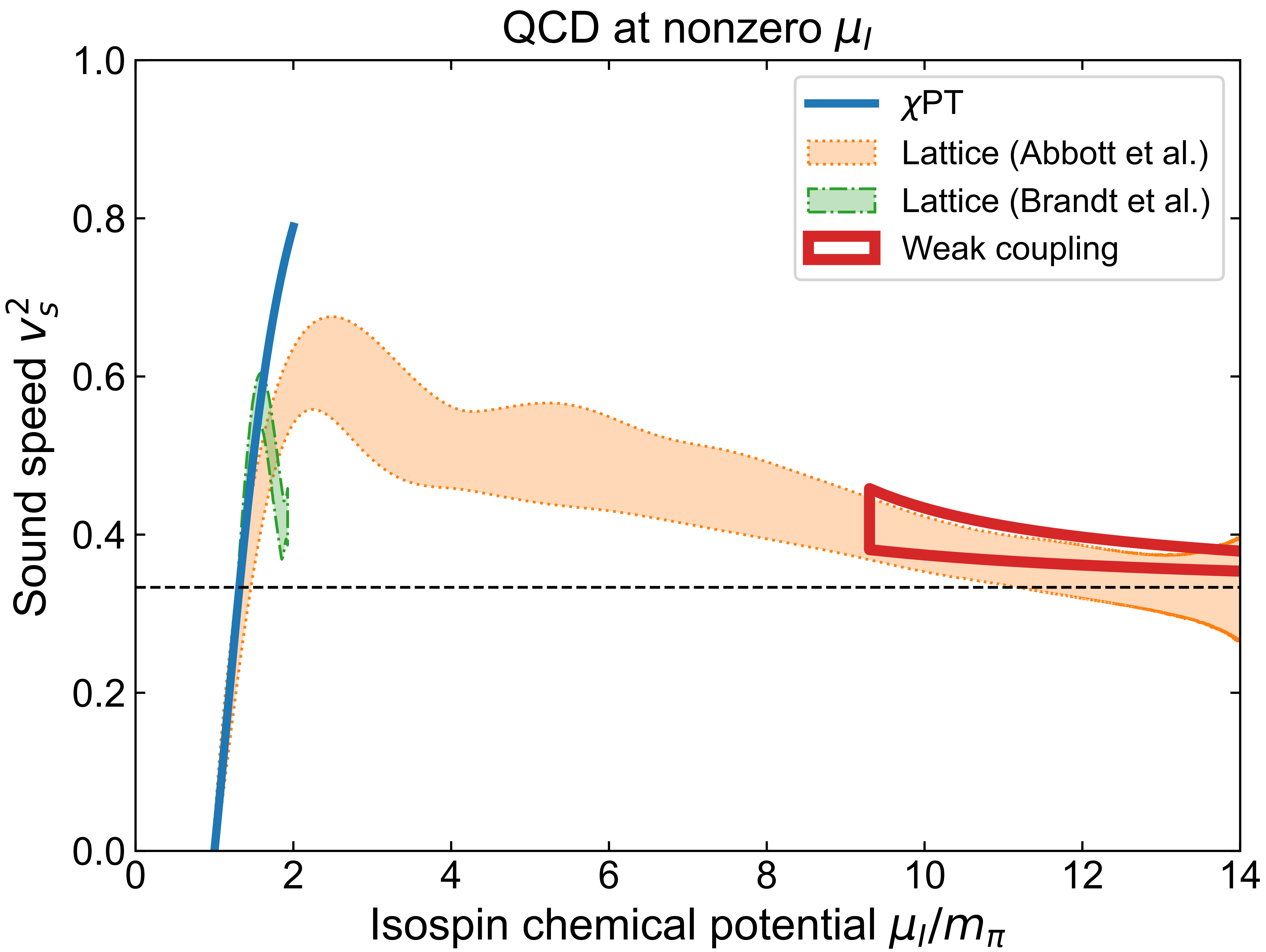


Outlook: Finite-μ lattice simulations

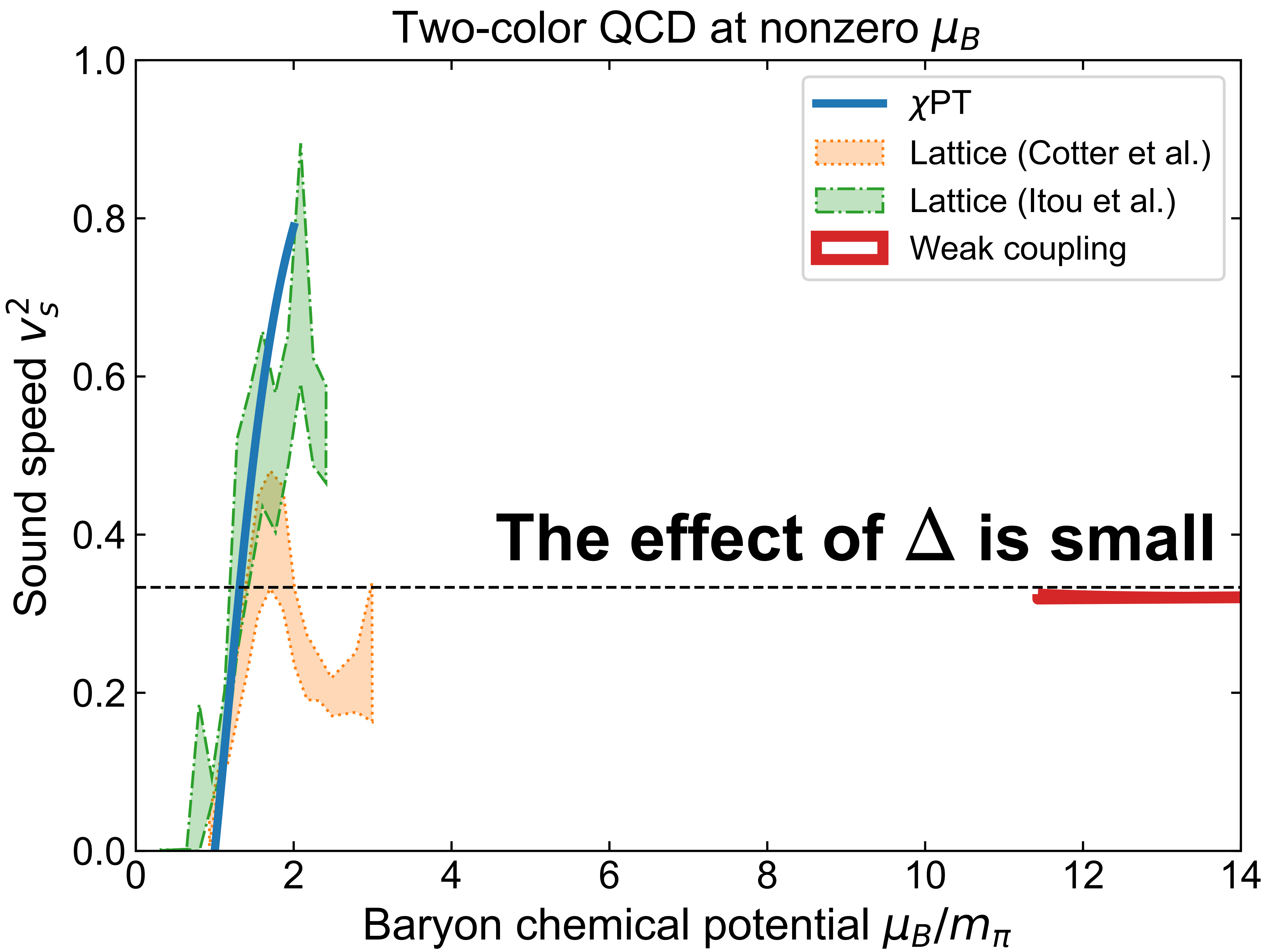
Fujimoto, *in prep.* (2024)

Already calculated



QCD_I

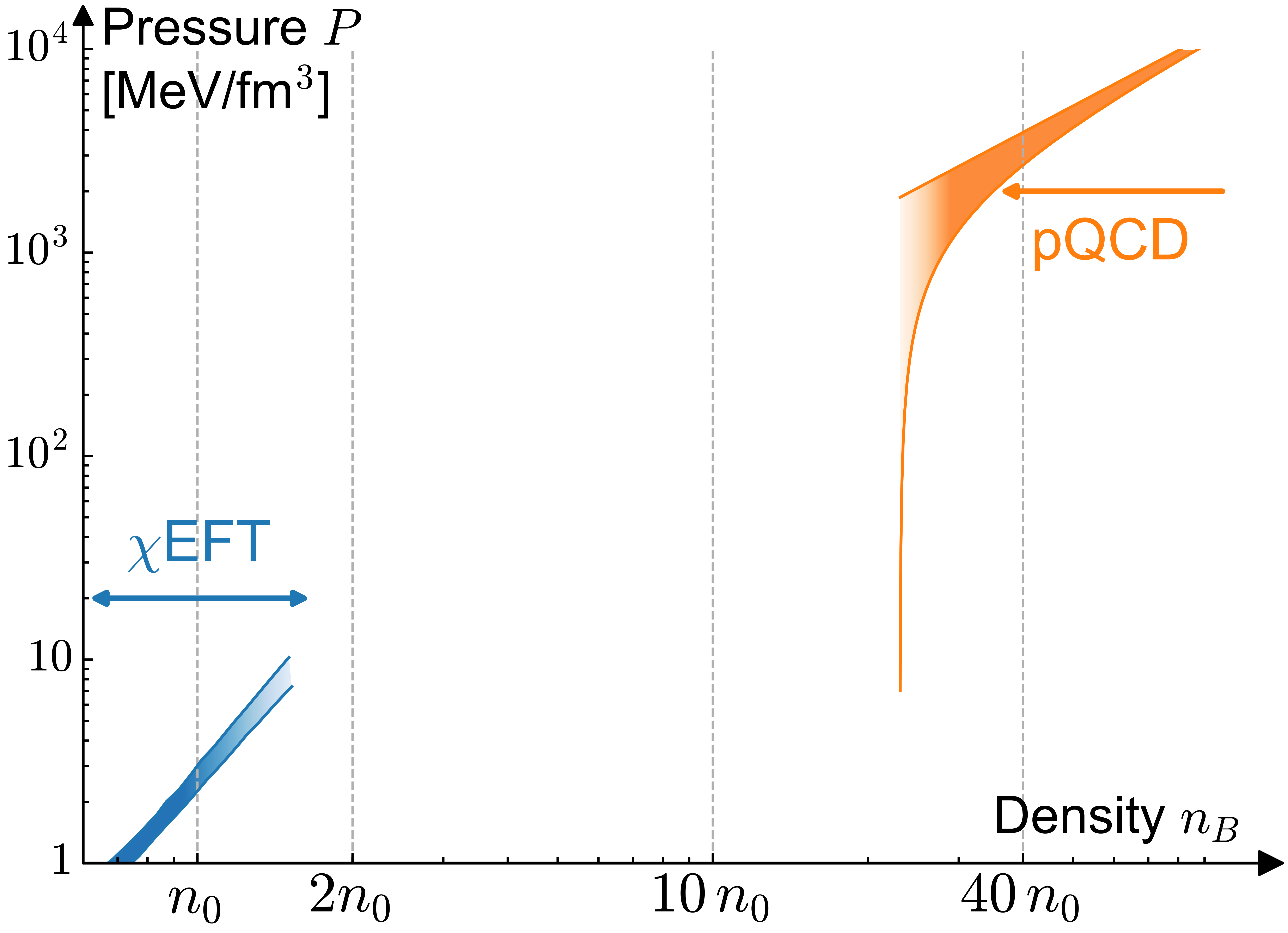
Calculable w/o the sign problem



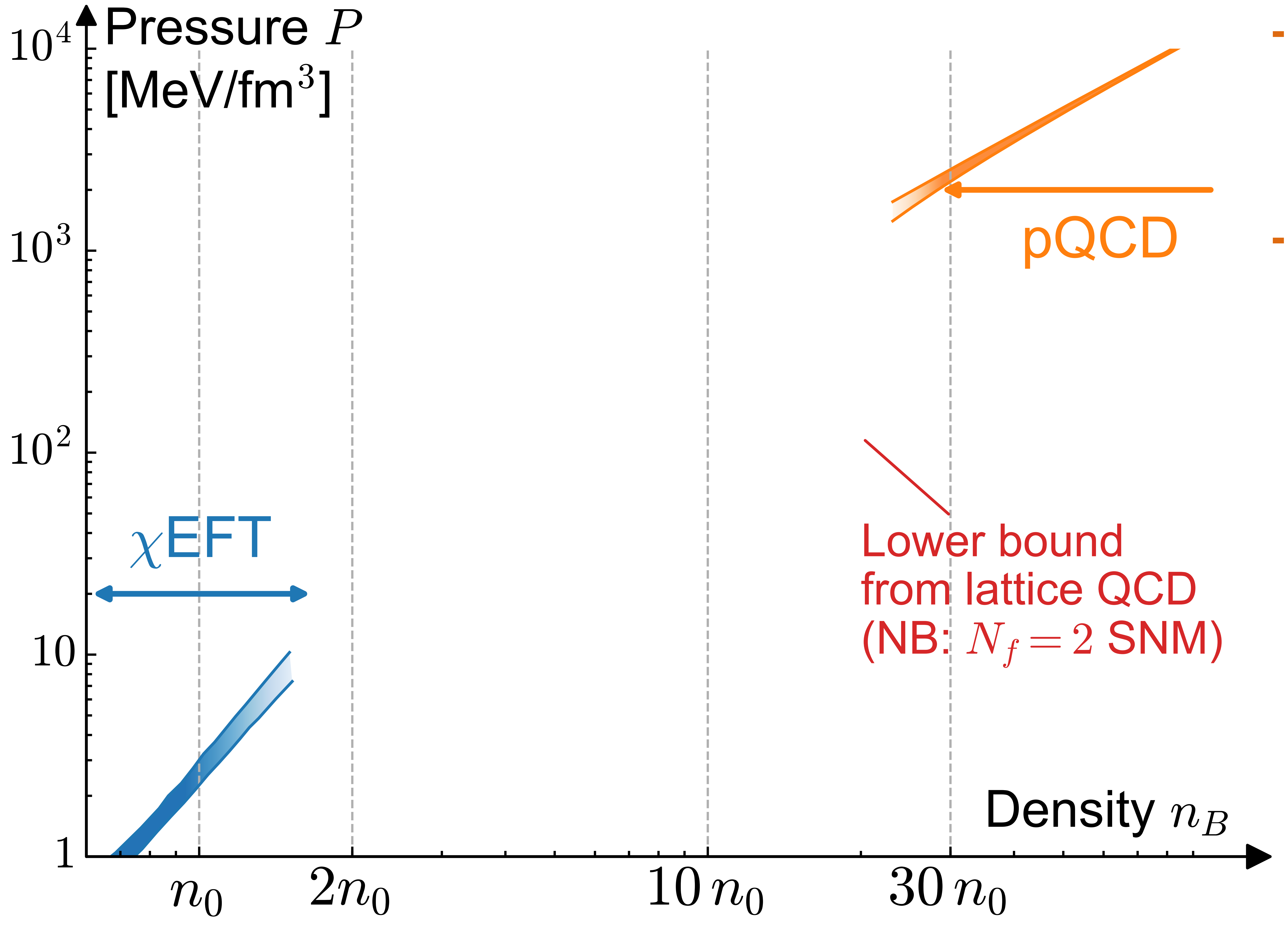
$N_c = 2$ QCD

at $\mu_B > 0$

Summary of Part 1



Summary of Part 1



- **Matched uncertainty w/ lattice QCD$_I$ data**
- **Color-superconducting gap Δ negligible**

Outline

1. **Implications from lattice QCD at finite isospin density**

   a) Bounds on isospin symmetric EoS from QCD inequality

   Y. Fujimoto, S. Reddy, PRD109 (2024)

   b) Comparison with weak-coupling results

   Y. Fujimoto, PRD109 (2024);
   Y. Fujimoto, in preparation

2. **Duality and conformality in dense QCD**

   Trace anomaly, Quarkyonic matter

   Y. Fujimoto, T. Kojo, L. McLerran, PRL132 (2024);
   Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL129 (2022)

Conformal limit

Weak coupling limit $\alpha_s \to 0$ is achieved when $\varepsilon \to \infty$.

The pQCD EoS has properties in this **conformal limit** as:

Trace anomaly: $\varepsilon - 3P \sim \beta_0 \mu^4 \left(\frac{\alpha_s}{\pi}\right)^2 \to 0$

Sound speed: $v_s^2 = \frac{dP}{d\varepsilon} \sim \frac{1}{3} \frac{1}{1 + \beta_0 \left(\frac{\alpha_s}{\pi}\right)^2} \to \frac{1}{3}$

At the intermediate density, $\varepsilon - 3P = 0$ and $v_s^2 = 1/3$ are different conditions

Trace anomaly and effective d.o.f.

Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL129 (2022)

- Trace anomaly:
  related to the changes in the effective degrees of freedom ν

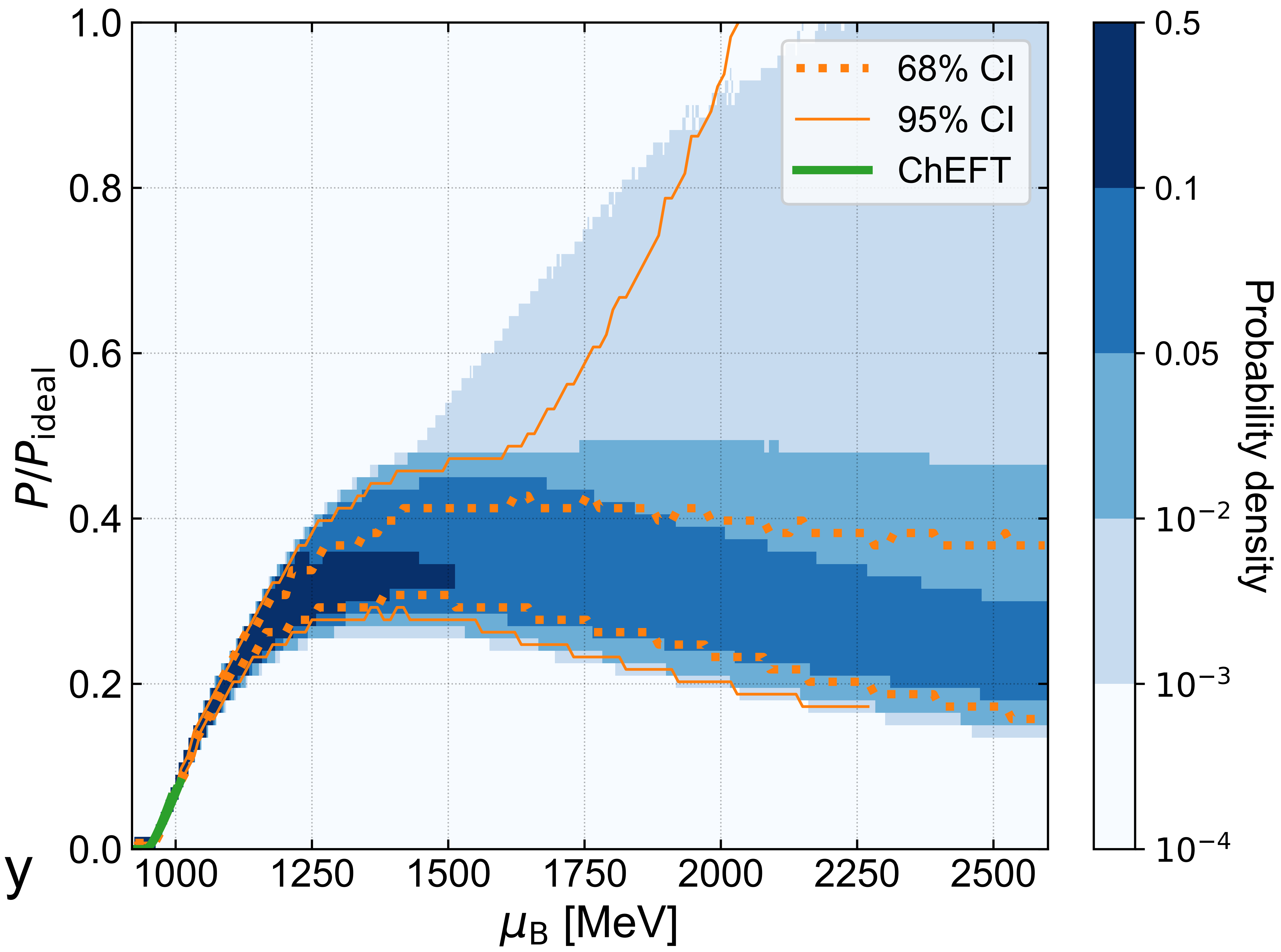
$$\frac{\varepsilon - 3P}{P_{\text{ideal}}} = \frac{d\nu}{d\ln\mu}$$

$$(P = \nu P_{\text{ideal}})$$

- $\nu \sim 1$ in quark matter regime

- If ν increases: positive trace anomaly
  if ν decreases: negative trace anomaly

w/o using pQCD information:



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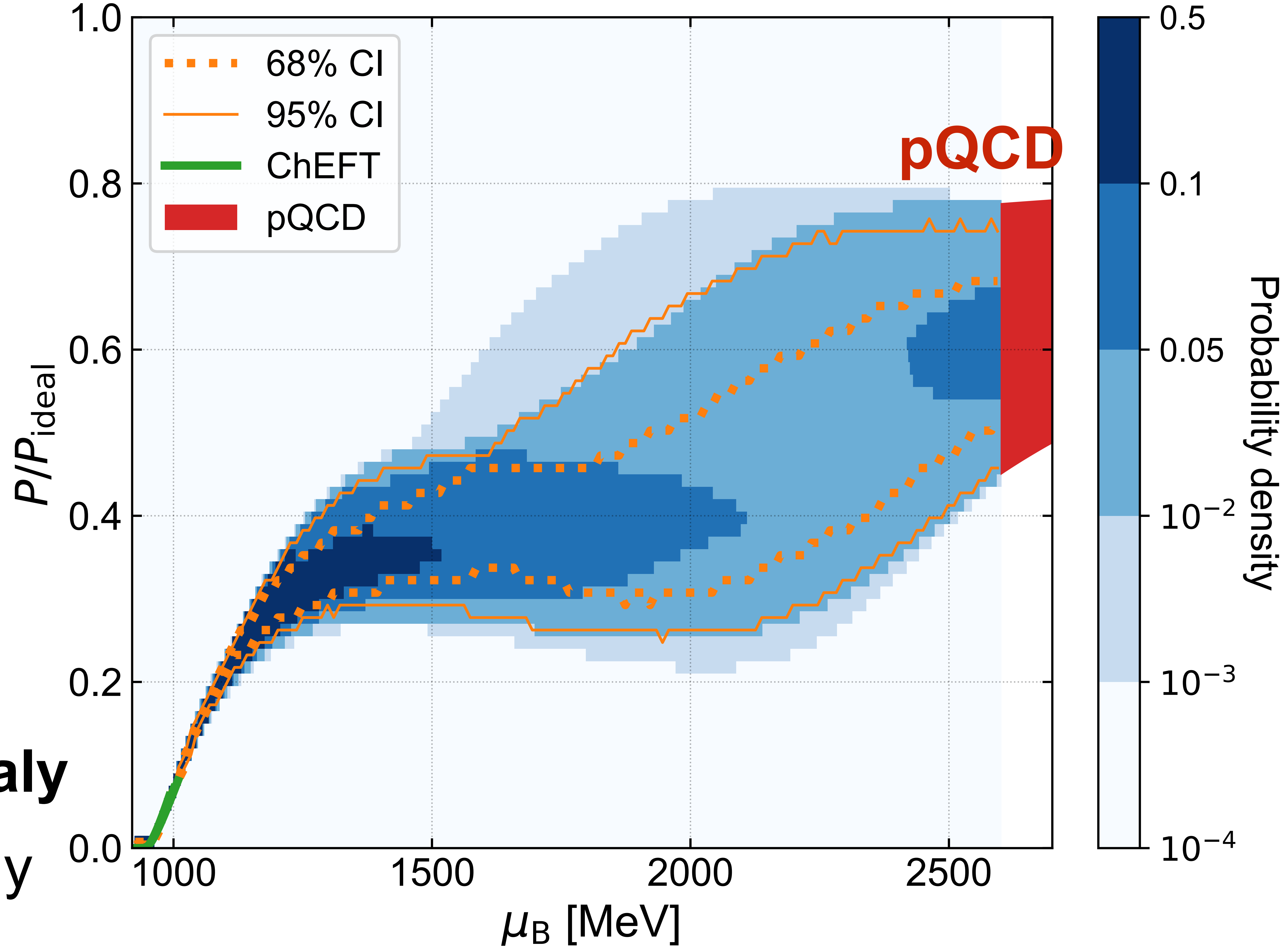
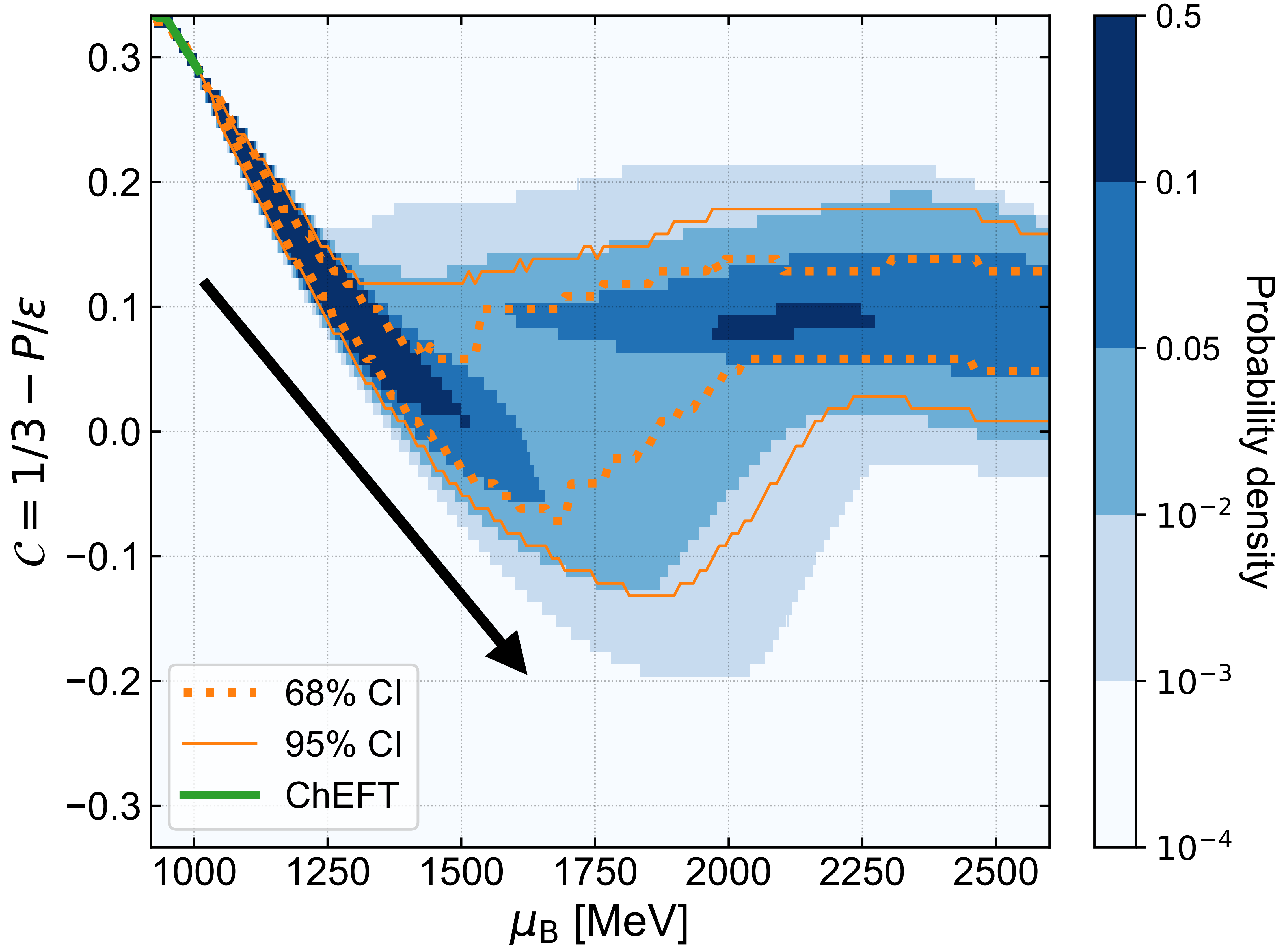


Positive trace anomaly favored by QCD effect

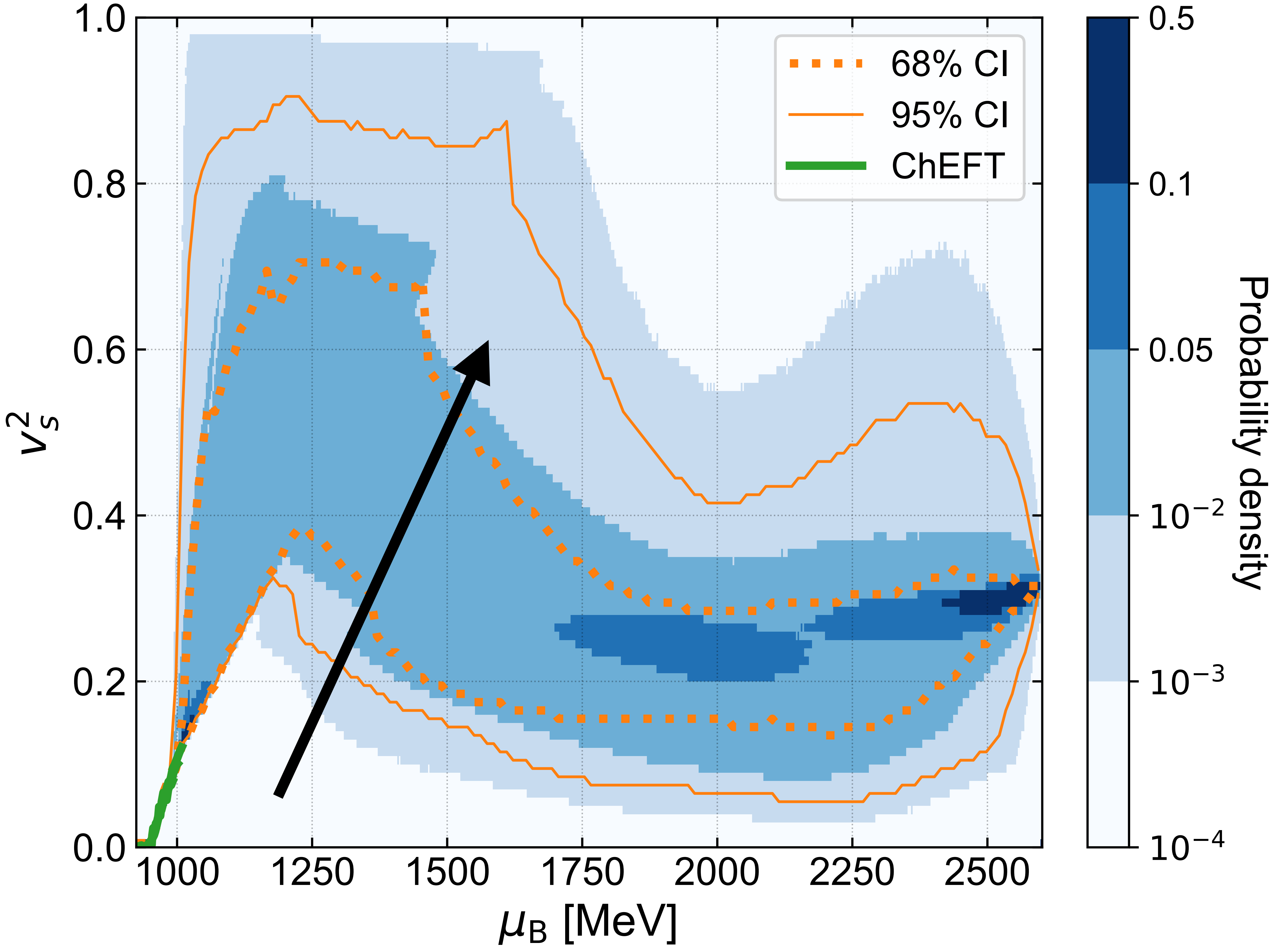
Trace anomaly and peak in sound speed

Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL129 (2022)

Normalized trace anomaly:
$(\varepsilon - 3P)/3\varepsilon$



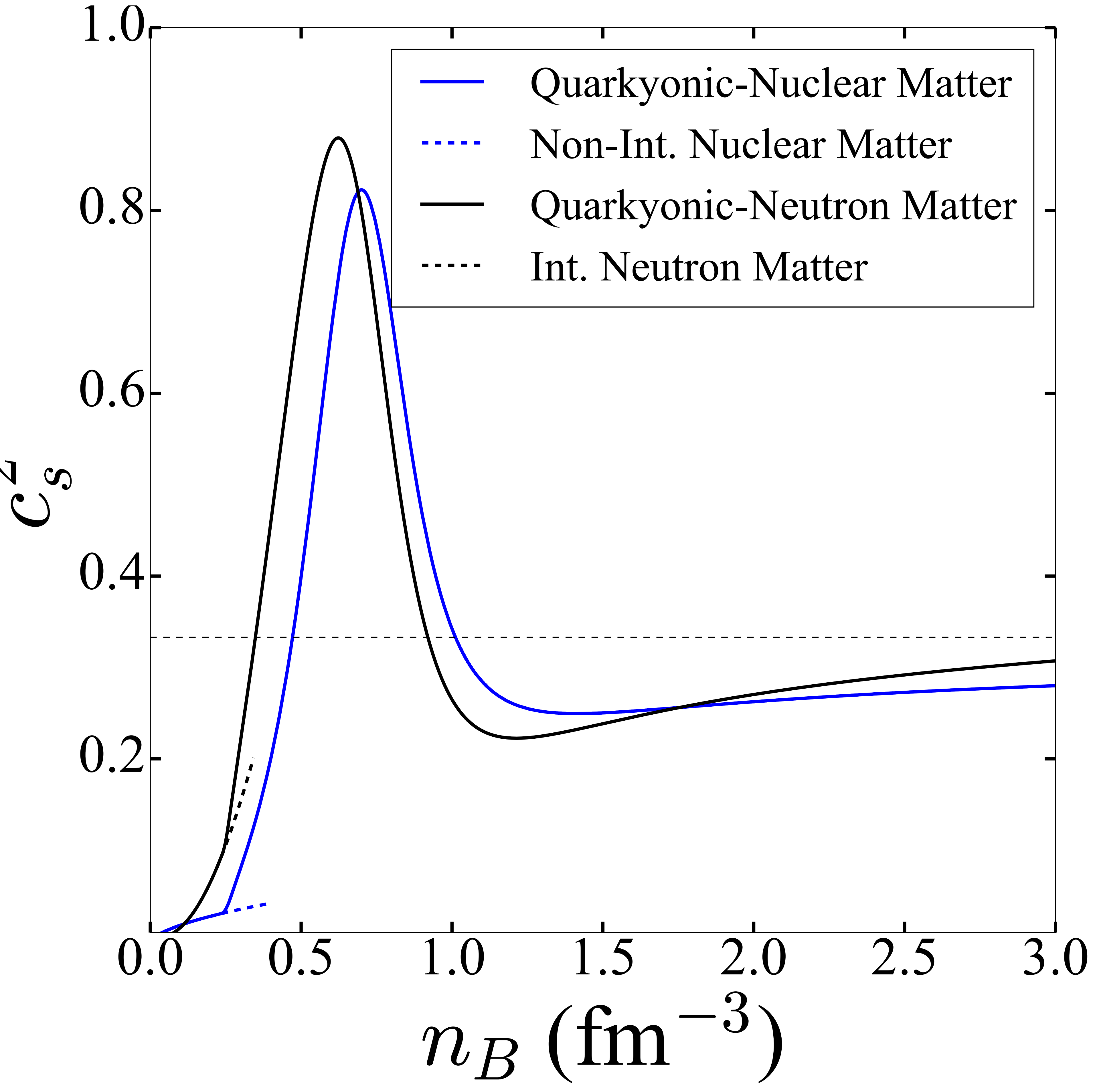
Sound speed:
$v_s^2 = dP/d\varepsilon$



Rapid approach to $\varepsilon - 3P \to 0$ drives the peak in v_s^2

Quarkyonic matter: EoS model for neutron star

McLerran,Reddy (2018):

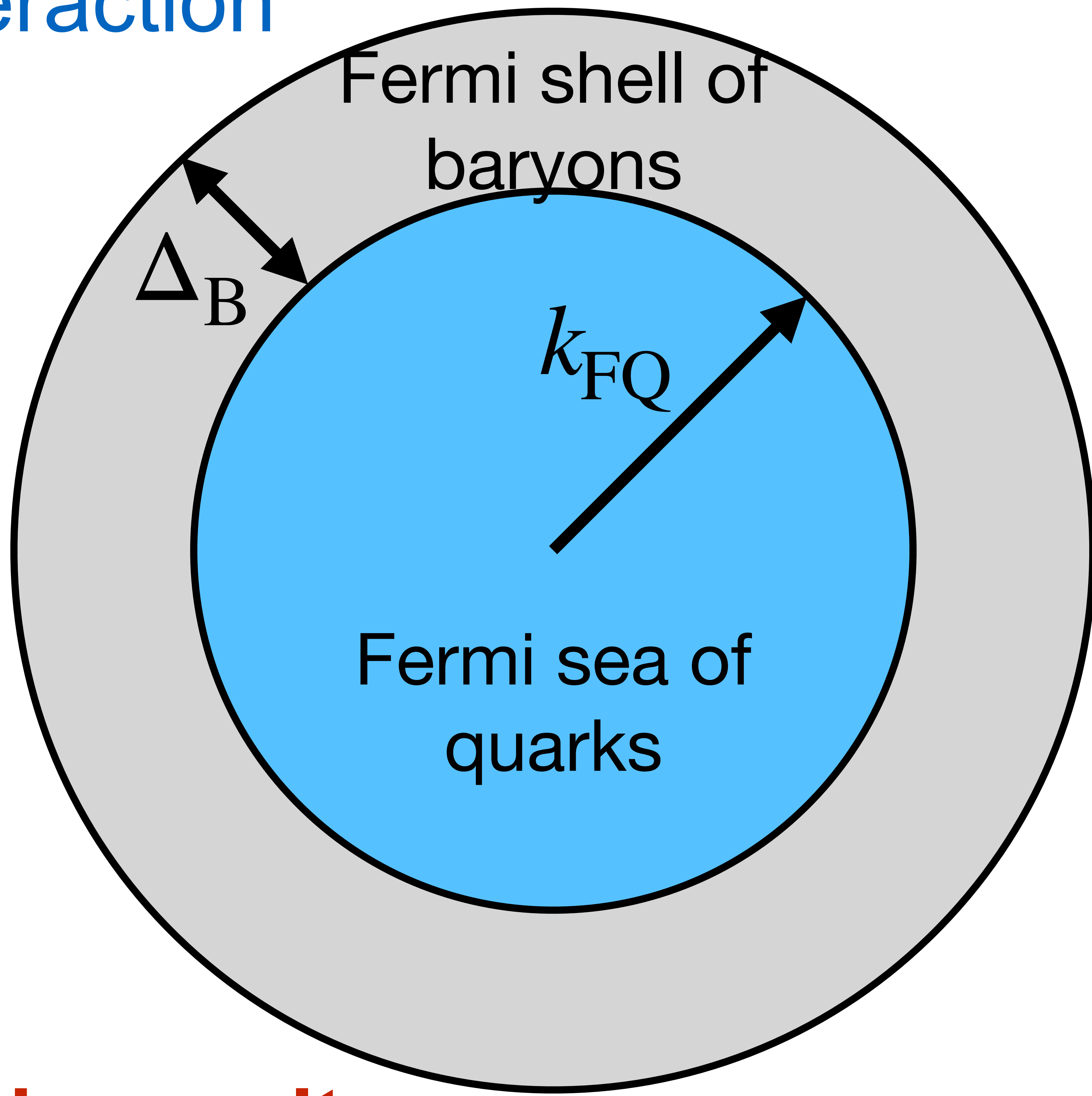


This EoS model was derived by assuming the following picture of Fermi baryon "shell":

Fermi sea: dominated by interaction that is less sensitive to IR
→ quarks

Fermi shell: interaction sensitive to IR d.o.f.
→ baryons, mesons, glues…

McLerran,Pisarski (2007)



This talk: reinterpretation of this result

Central tenet of Quarkyonic matter

- **Naive picture of deconfinement at high density:** Collins,Perry (1974)
  In weak-coupling regime, quarks are deconfined
  … Led by Debye screening of the confinement potential

- **Quarkyonic matter**: Large-N_c QCD implies… McLerran,Pisarski (2007)
  Dense QCD matter at high density can be described either as
  - Confined baryons (because confining interaction is less screened)
  - (weakly-coupled) Quarks

→ implies duality between quark and confined baryonic matter

Duality in Fermi gas model

Fujimoto, Kojo, McLerran (2023)

Implement duality in Fermi gas model
(= simultaneous description in terms of baryons & quarks)

Fermi gas model w/ an explicit duality:

$$\varepsilon = \int_{\boldsymbol{k}} E_{\mathrm{B}}(k) f_{\mathrm{B}}(k) = \int_{\boldsymbol{k}} E_{\mathrm{Q}}(q) f_{\mathrm{Q}}(q)$$

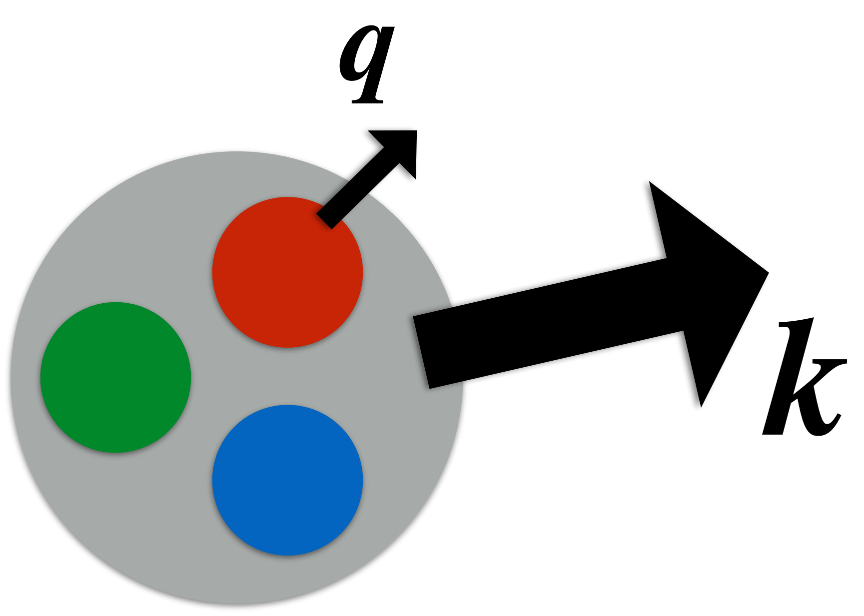
$$n_{\mathrm{B}} = \int_{\boldsymbol{k}} f_{\mathrm{B}}(k) = \int_{\boldsymbol{q}} f_{\mathrm{Q}}(q)$$

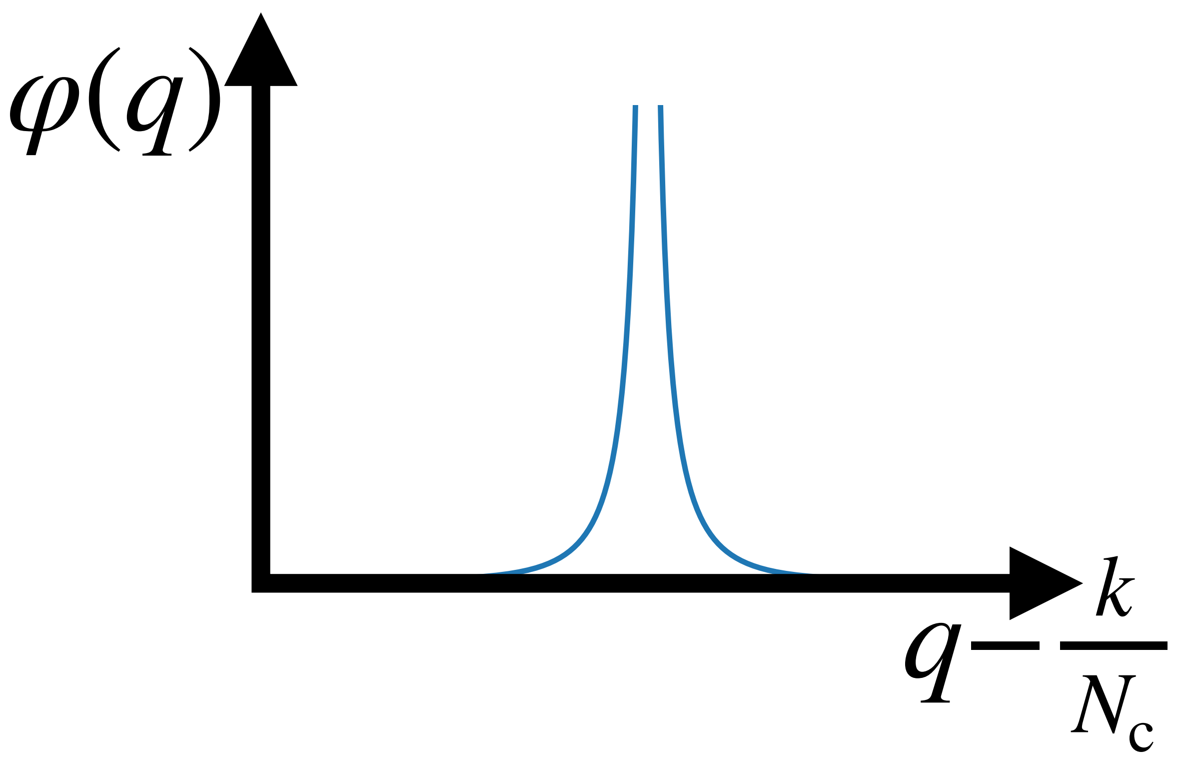
$0 \leq f_{\mathrm{B,Q}} \leq 1$: Pauli exclusion

$E_{\mathrm{B}}(k) = \sqrt{k^2 + M_N^2}$: ideal baryon dispersion relation

Modeling of confinement:

$$f_{\mathrm{Q}}(q) = \int_{\boldsymbol{k}} \varphi\left(\boldsymbol{q} - \frac{\boldsymbol{k}}{N_{\mathrm{c}}}\right) f_{\mathrm{B}}(k)$$



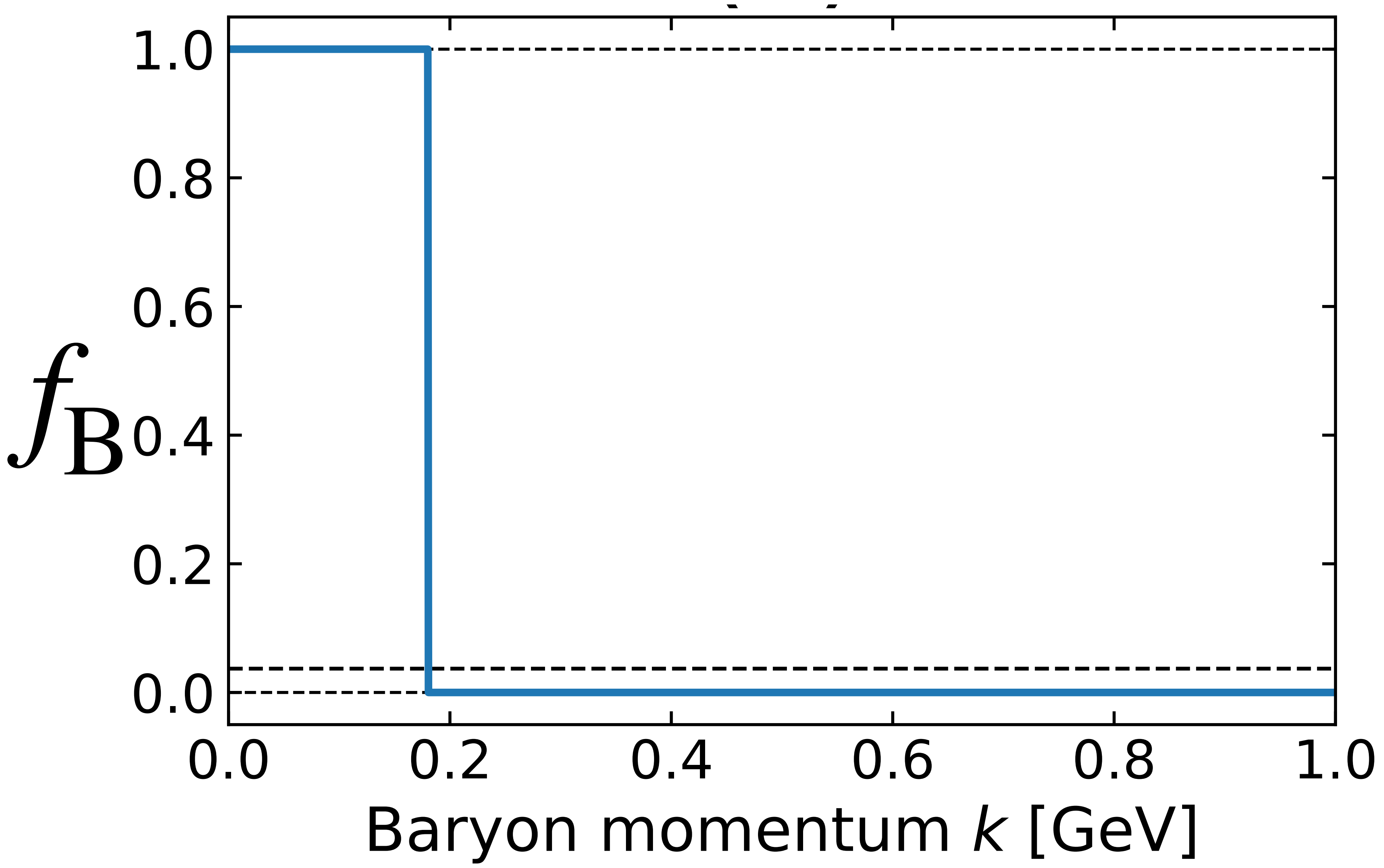


Duality in Fermi gas model

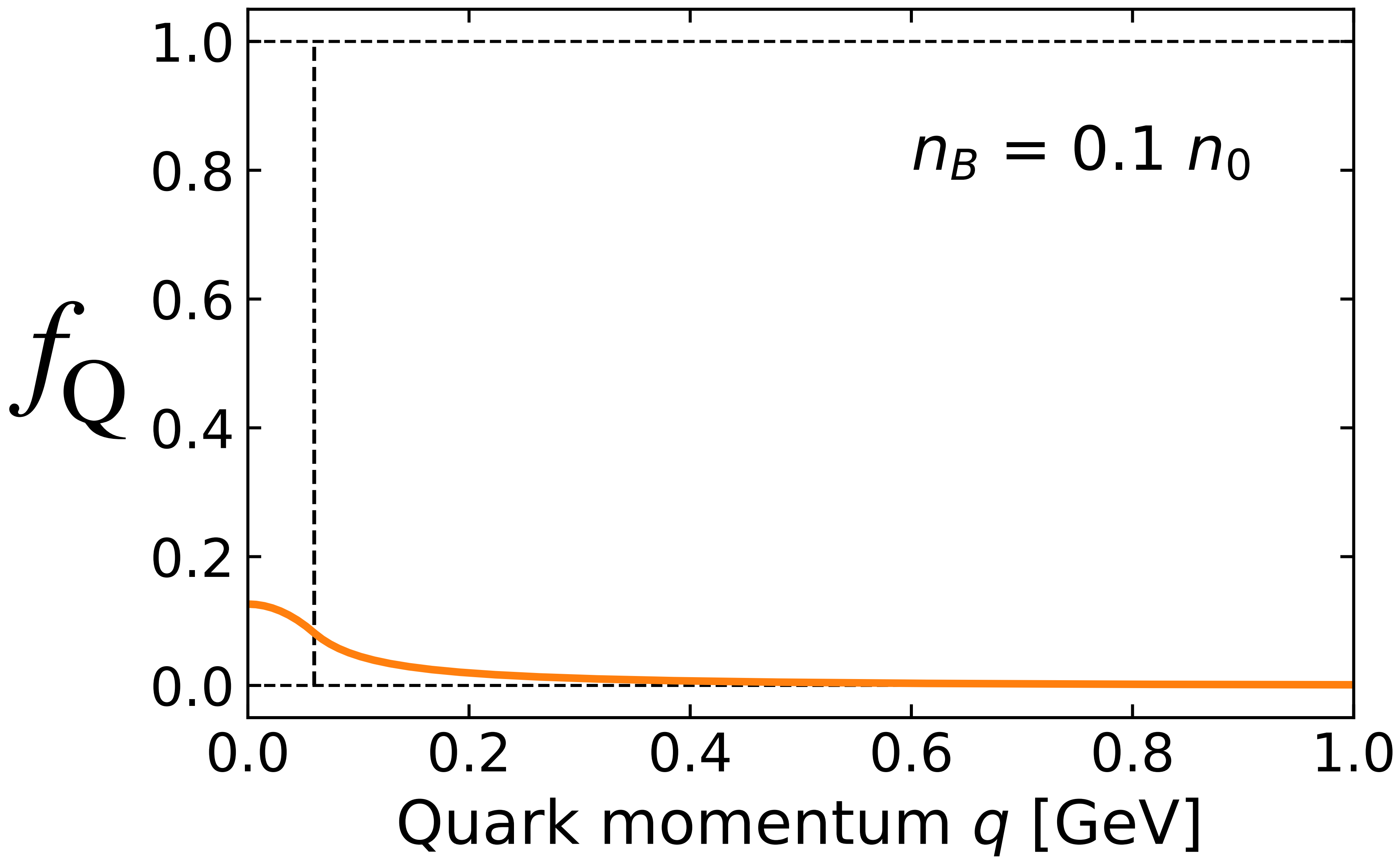
Fujimoto,Kojo,McLerran (2023)

At low density…

Fermi-Dirac distribution for baryons



Quarks do not fill up the Fermi sea yet

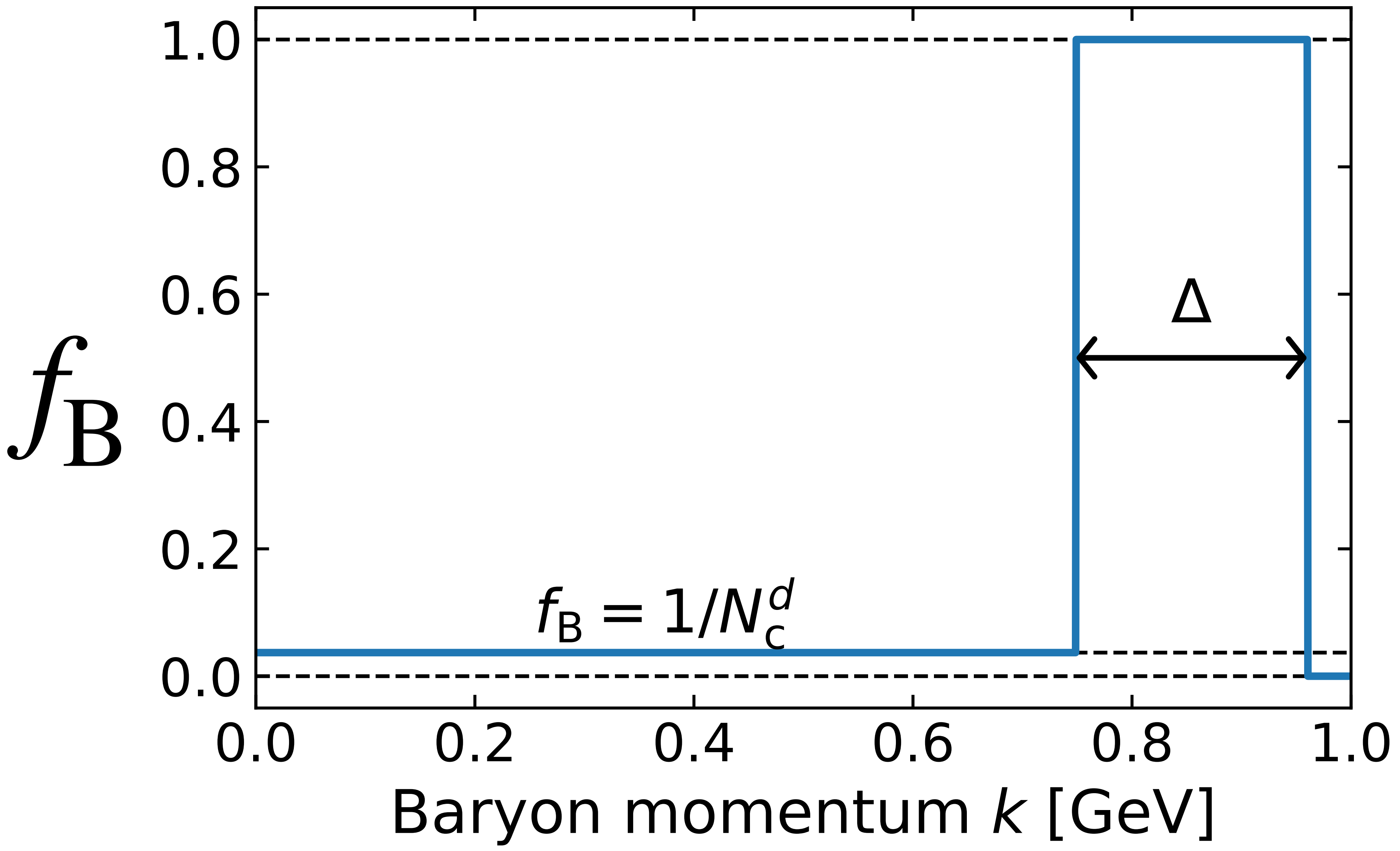


Duality in Fermi gas model

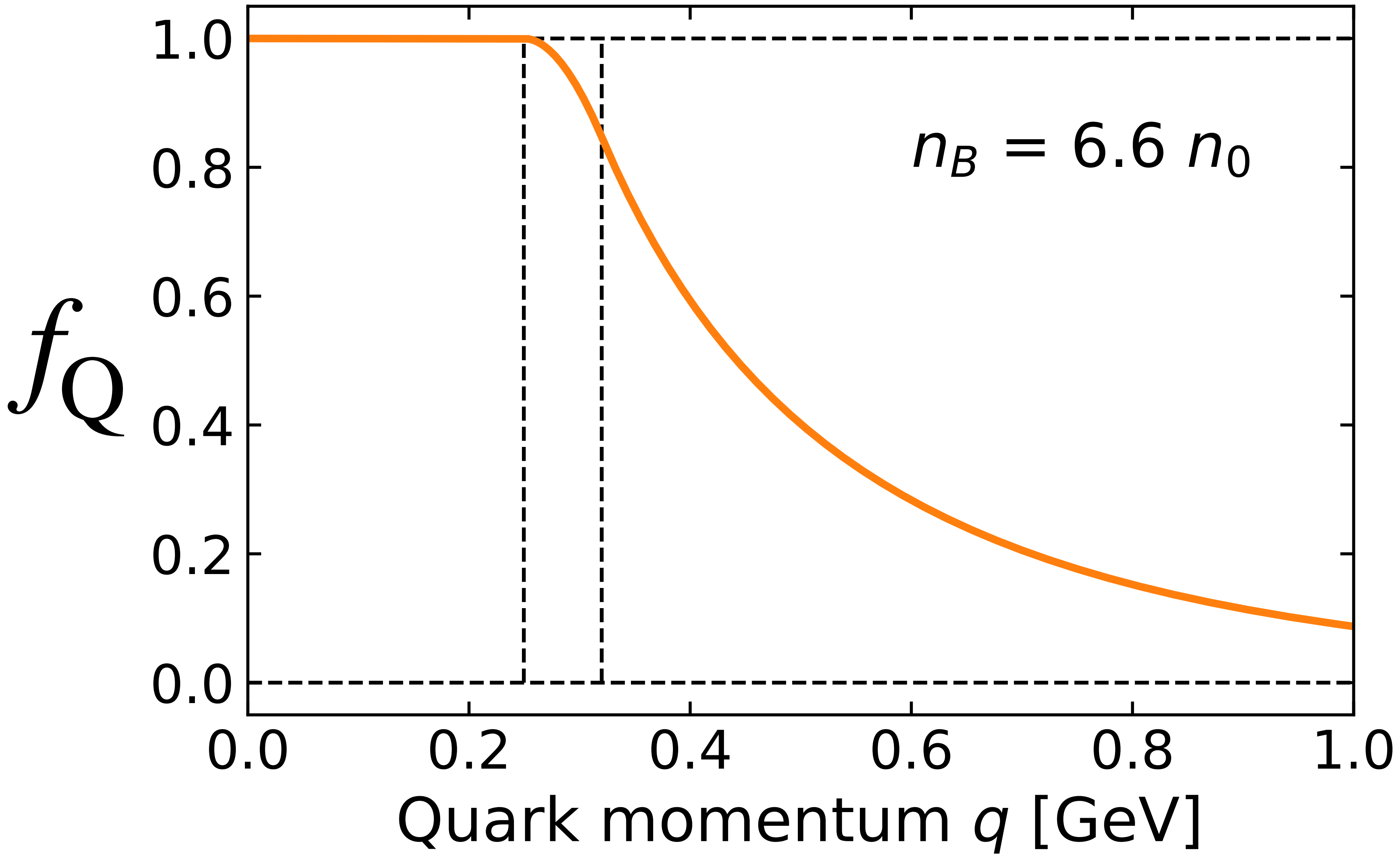
Fujimoto,Kojo,McLerran (2023)

At sufficiently high density…

Fermi-Dirac distribution for baryons is modified



Quark obeys the FD distribution (with a tail from confinement)

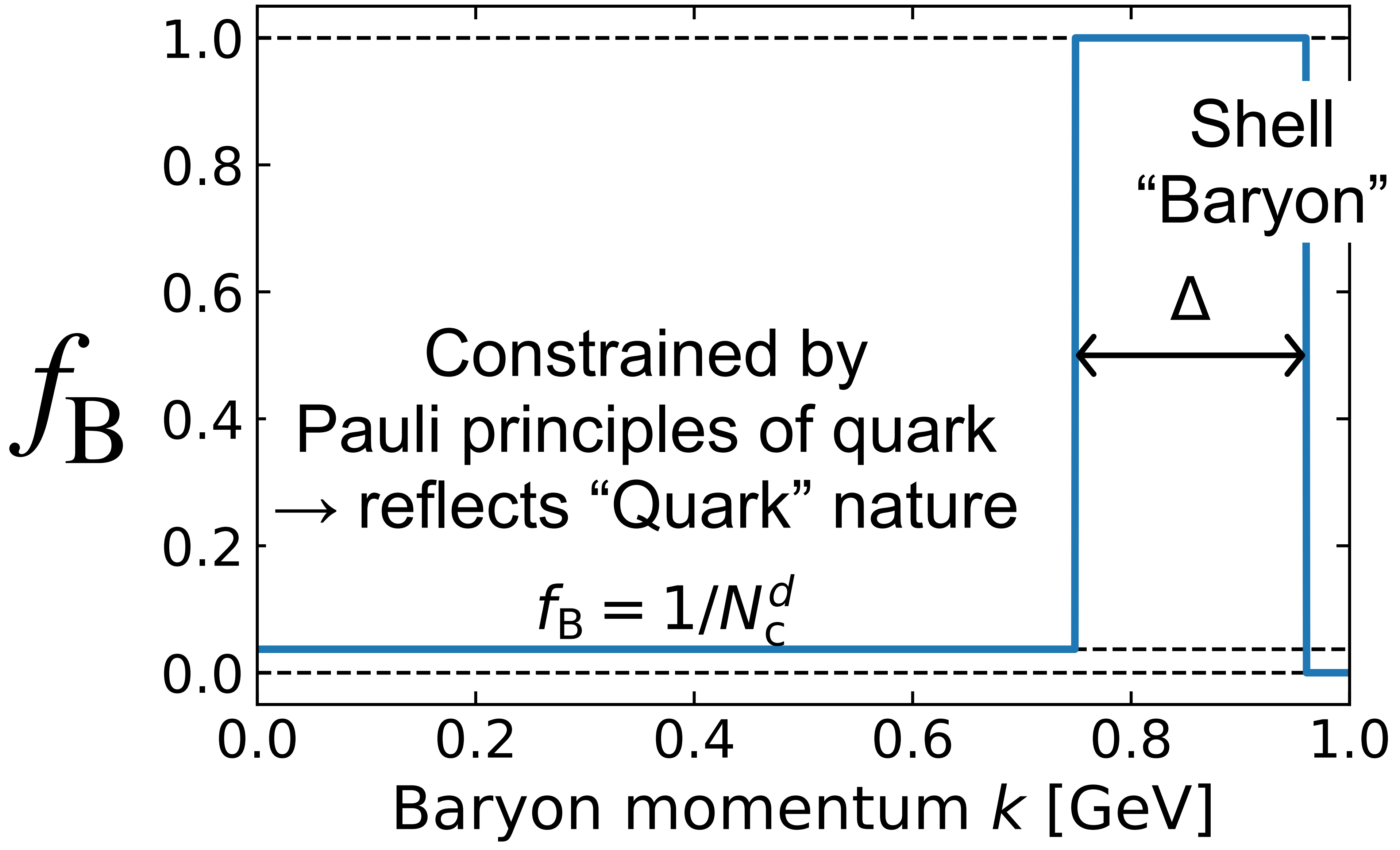


Equivalence to Quarkyonic model

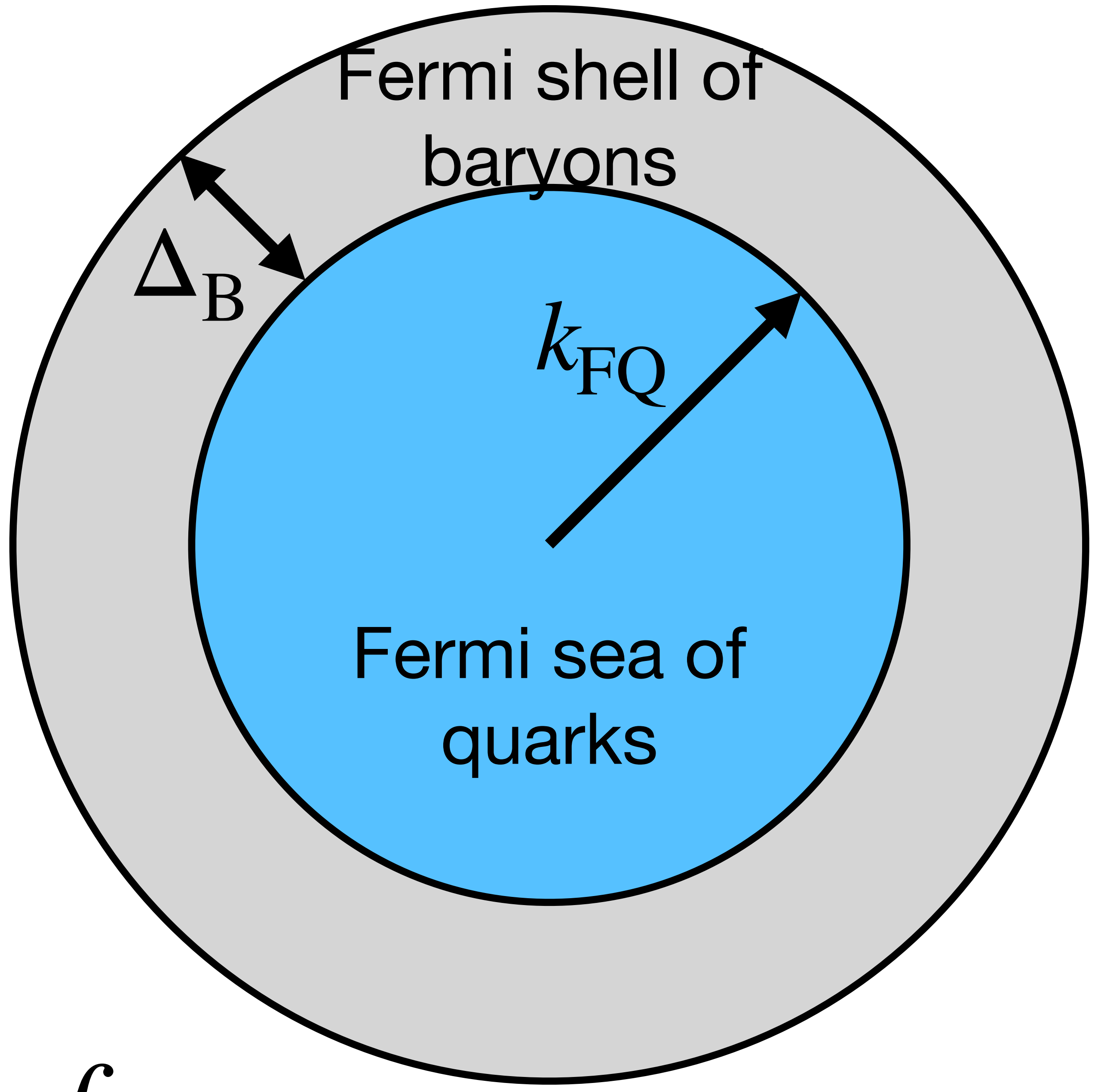
Fujimoto,Kojo,McLerran (2023)

At sufficiently high density…

Fermi-Dirac distribution for baryons is modified



McLerran,Pisarski (2007)
McLerran,Reddy (2018)



Fermi shell structure arises in f_B
(Note: this is still **pure baryonic description**)

This picture is equivalent to
McLerran-Reddy model of the EoS
based on the McLerran-Pisarski shell picture

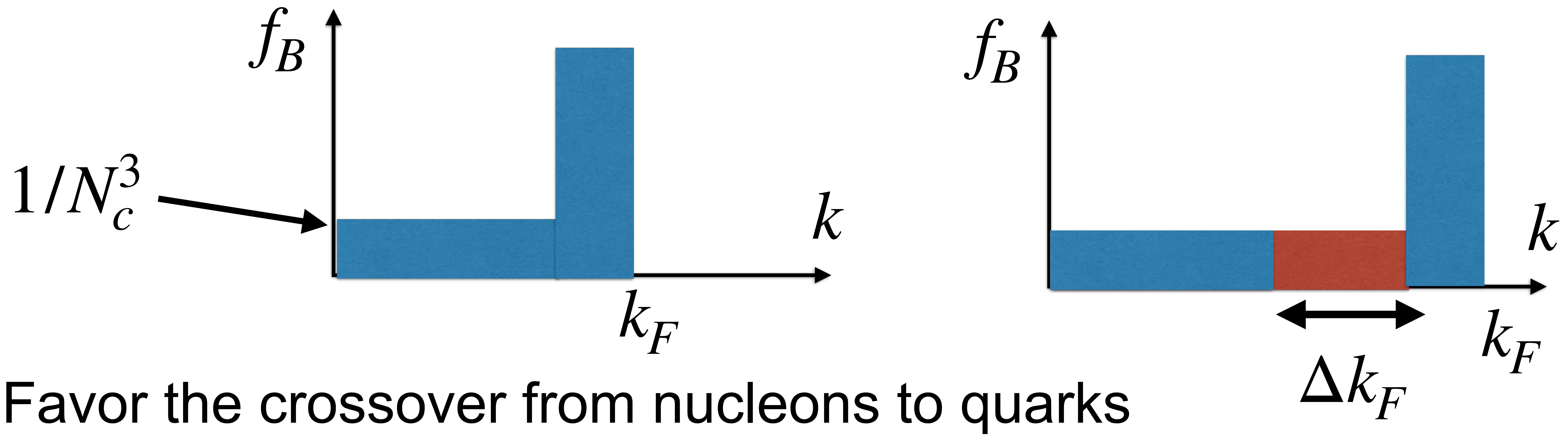
Rapid stiffening in the EoS

Fujimoto,Kojo,McLerran (2023)

A partial occupation of available baryon phase space leads to **large sound speed:**

$$v_s^2 = \frac{n_\mathrm{B}}{\mu_\mathrm{B} dn_\mathrm{B}/d\mu_\mathrm{B}} \to \frac{\delta\mu_\mathrm{B}}{\mu_\mathrm{B}} \sim v_s^2 \frac{\delta n_\mathrm{B}}{n_\mathrm{B}}$$

If baryons have underoccupied state, the change in density is small while the change in Fermi energy ($\sim k_F$) is large



→ Favor the crossover from nucleons to quarks

Summary

- **QCD$_I$:** a testing ground for QCD$_B$. Lattice simulation feasible

- **QCD inequality:** Robust constraints on the symmetric nuclear matter EoS from lattice QCD & saturation property

- **Weak-coupling results:** Matches well with lattice QCD$_I$.
  Empirical evidence for the validity down to $\mu \sim 10^3\ \mathrm{MeV}$.
  Color-superconducting gap negligible at $\mu \sim 800$ MeV,
  Crosscheck with lattice-QCD can be provided in $N_c = 2$.

- **Quarkyonic matter:** reinterpretation as a hadron-quark duality