Dynamics in the inner crust: heavy impurity in Fermi superfluid

Daniel Pęcak

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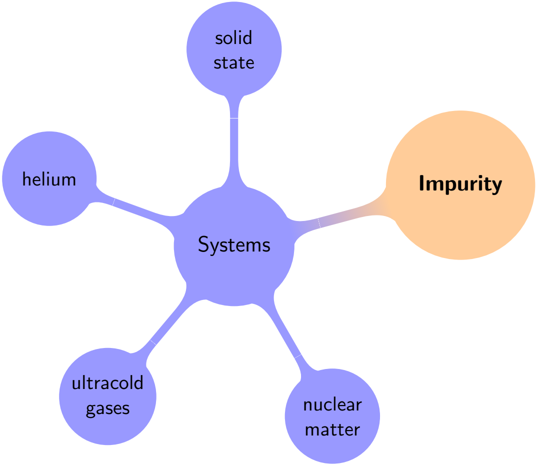
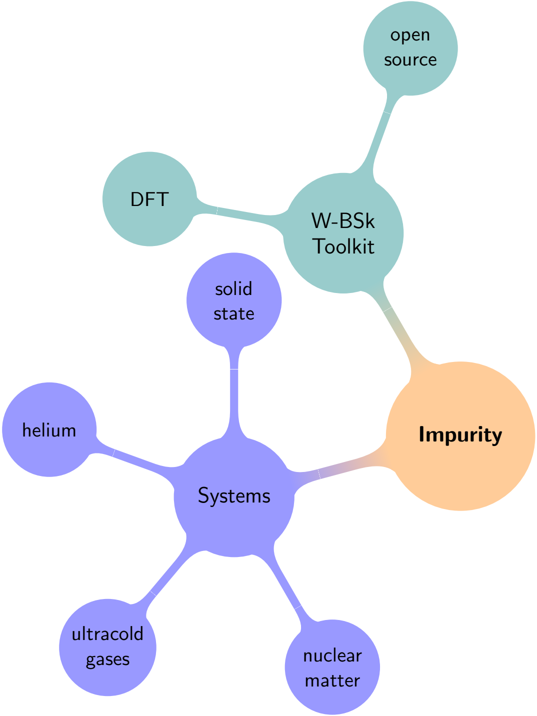
PL GRID

NARODOWE CENTRUM NAUKI

25^{th} April 2024, ECT*, Trento


solid
state
helium
Impurity
Systems
ultracold
gases
nuclear
matter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DFT
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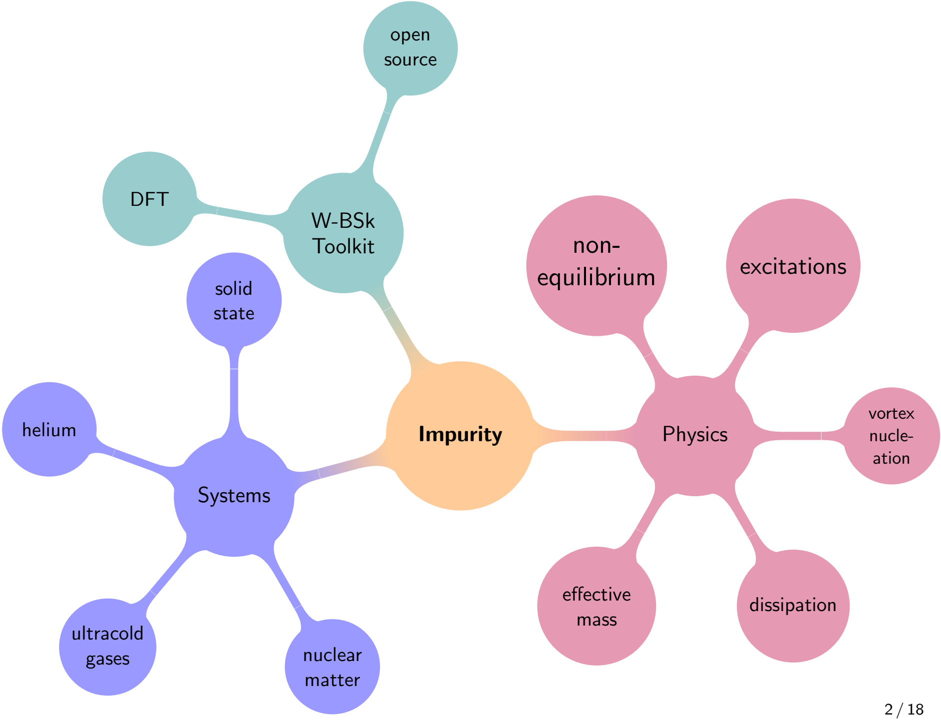
helium
Impurity
Systems
ultracold gases
nuclear matter


open source
DFT
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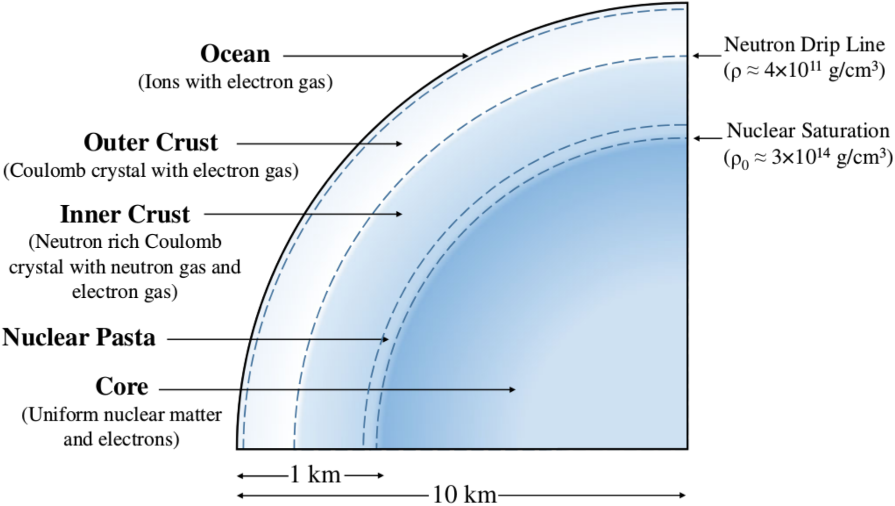
solid state
helium
Systems
ultracold gases
nuclear matter
Impurity
non-equilibrium
excitations
Physics
vortex nucle-ation
effective mass
dissipation



Caplan, M. E., and C. J. Horowitz, Reviews of Modern Physics 89, 041002 (2017)

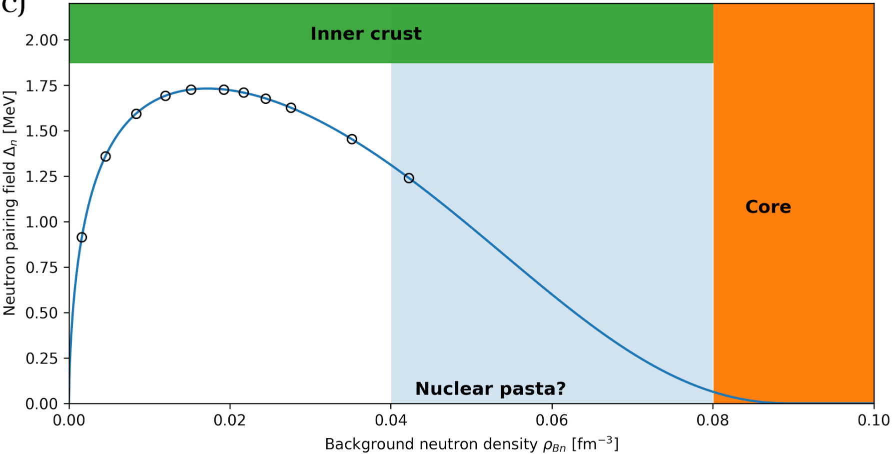
Crust composition

ρ_{max} [g cm^{-3}]	Element	Z	N	R_{cell} [fm]
8.02×10^6	^{56}Fe	26	30	1404.05
2.71×10^8	^{62}Ni	28	34	449.48
1.33×10^9	^{64}Ni	28	36	266.97
1.50×10^9	^{66}Ni	28	38	259.26
3.09×10^9	^{86}Kr	36	50	222.66
1.06×10^{10}	^{84}Se	34	50	146.56
2.79×10^{10}	^{82}Ge	32	50	105.23
6.07×10^{10}	^{80}Zn	30	50	80.58
8.46×10^{10}	^{82}Zn	30	52	72.77
9.67×10^{10}	^{128}Pd	46	82	80.77
1.47×10^{11}	^{126}Ru	44	82	69.81
2.11×10^{11}	^{124}Mo	42	82	61.71
2.89×10^{11}	^{122}Zr	40	82	55.22
3.97×10^{11}	^{120}Sr	38	82	49.37
4.27×10^{11}	^{118}Kr	36	82	47.92

ρ [g cm^{-3}]	Element	Z	N	R_{cell} [fm]
4.67×10^{11}	^{180}Zr	40	140	53.60
6.69×10^{11}	^{200}Zr	40	160	49.24
1.00×10^{12}	^{250}Zr	40	210	46.33
1.47×10^{12}	^{320}Zr	40	280	44.30
2.66×10^{12}	^{500}Zr	40	460	42.16
6.24×10^{12}	^{950}Sn	50	900	39.32
9.65×10^{12}	^{1100}Sn	50	1050	35.70
1.49×10^{13}	^{1350}Sn	50	1300	33.07
3.41×10^{13}	^{1800}Sn	50	1750	27.61
7.94×10^{13}	^{1500}Zr	40	1460	19.61
1.32×10^{14}	^{982}Ge	32	950	14.38

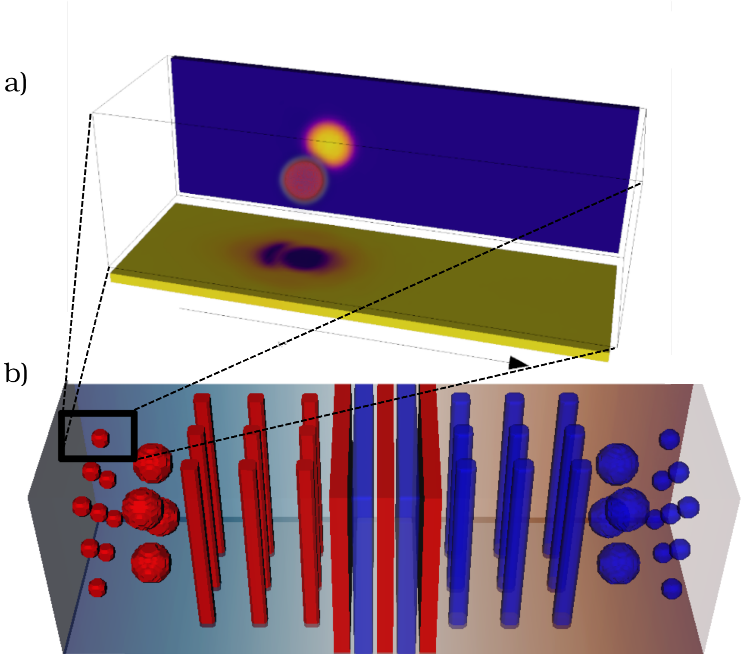
Chamel, Nicolas, and Pawel Haensel. "Physics of neutron star crusts." Living Reviews in relativity 11.1 (2008): 1-182.

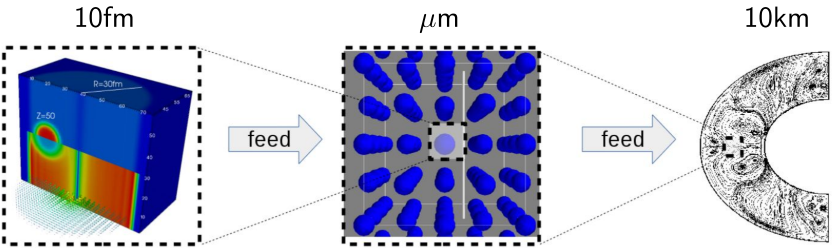
c)



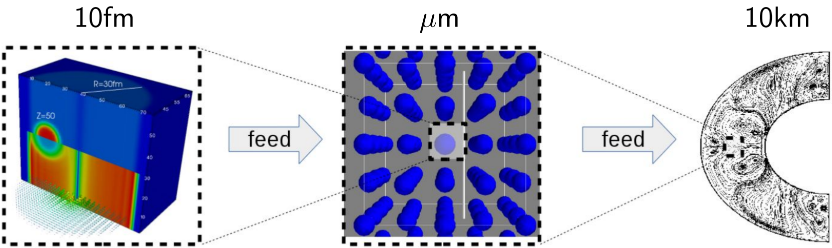
Inner crust
Core
Nuclear pasta?
Neutron pairing field Δ_n [MeV]
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.00
0.02
0.04
0.06
0.08
0.10
Background neutron density ρ_{Bn} [fm^{-3}]

Nonequilibrium dynamics



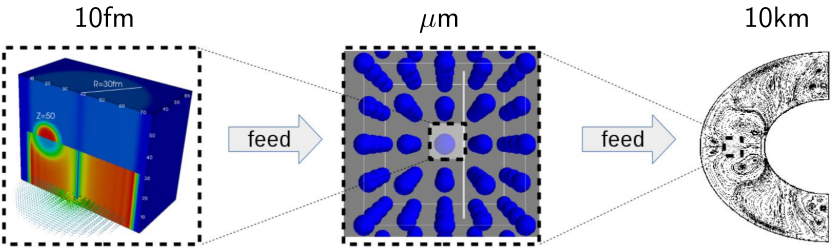


10fm
μm
10km
R=30fm
Z=50
feed
feed



LUMI, Finland (#5 Top 500)

Piz Daint, Switzerland (#37 Top 500)



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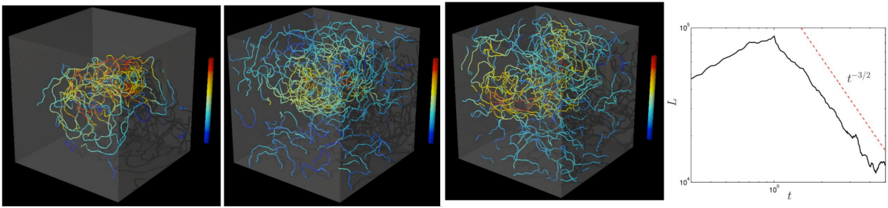

R. Hänninen, A. W. Baggaley, Proc. Nat. Acad. Sci. U.S.A. 111, 4667 (2014)

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W-SLDA Toolkit

Self-consistent solver of mathematical problems which have structure formally equivalent to Bogoliubov-de Gennes equations.

static problems: st-wslda

$$\begin{pmatrix} h_a(\boldsymbol{r}) - \mu_a & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_b^*(\boldsymbol{r}) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix}$$

time-dependent problems: td-wslda

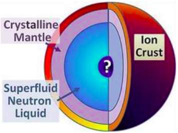
$$i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t) - \mu_a & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -h_b^*(\boldsymbol{r},t) + \mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix}$$

Extension to nuclear matter in neutron stars

Unified solvers for static and time-dependent problems

Dimensionalities of problems: 3D, 2D and 1D

Extension to nuclear matter in neutron stars



The W-SLDA Toolkit has been expanded to encompa
available as the W-BSk Toolkit.

ALL FUNCTIONALITIES →

Integration with VisIt: visualization, animation and analysis tool

ce

W-BSk Toolkit

Getting the code

DOWNLOAD

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Changelog

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \boldsymbol{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \boldsymbol{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \boldsymbol{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \boldsymbol{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

$$\rho(r) = \sum_k |v_k(r)|^2$$

$$\tau(r) = \sum_k |\nabla v_k(r)|^2$$

$$\nu(r) = \sum_k u_k(r) v_k^*(r)$$

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \boldsymbol{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \boldsymbol{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

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$$h(r) = \frac{\delta\varepsilon}{\delta\rho} - \nabla\frac{\delta\varepsilon}{\delta\tau}\nabla - \frac{i}{2}\left\{\frac{\delta\varepsilon}{\delta\boldsymbol{j}}, \nabla\right\}$$

$$\Delta(r) = \frac{\delta\varepsilon}{\delta\nu}$$

Superfluid **L**ocal **D**ensity **A**pproximation

A. Bulgac, Physical Review A **76**, 040502 (2007)

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Hartree-Fock-Bogoliubov equations

$$\begin{pmatrix} h(r) & \Delta(r) \\ \Delta^*(r) & -h^*(r) \end{pmatrix} \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = \epsilon_k \begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix}$$

$$\varepsilon(\rho, \vec{\nabla}\rho, \nu, \tau, \boldsymbol{j}) = \frac{\hbar^2}{2M}\tau + \varepsilon_\rho(\rho) + \varepsilon_\tau(\rho, \tau, \boldsymbol{j}) + \varepsilon_{\Delta\rho}(\rho, \vec{\nabla}\rho) + \varepsilon_\pi(\rho, \vec{\nabla}\rho, \nu)$$

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Quality of results highly depends on the quality of density functional!

1. Energy density

Under the assumption of invariance under time reversal, the HFB energy is written as the integral of a purely local energy-density functional

$$E_{\rm HFB} = \int \mathcal{E}_{\rm HFB}(\boldsymbol{r})\, {\rm d}^3\boldsymbol{r}, \tag{A1}$$

where

$$\mathcal{E}_{\rm HFB}(\boldsymbol{r}) = \mathcal{E}_{\rm Sky}[\rho_n(\boldsymbol{r}), \nabla\rho_n(\boldsymbol{r}), \tau_n(\boldsymbol{r}), \mathbf{J}_n(\boldsymbol{r}), \rho_p(\boldsymbol{r}), \nabla\rho_p(\boldsymbol{r}), \tau_p(\boldsymbol{r}), \mathbf{J}_p(\boldsymbol{r})] + \mathcal{E}_{\rm Coul}[\rho_p(\boldsymbol{r})] + \mathcal{E}_{\rm pair}[\rho_n(\boldsymbol{r}), \tilde{\rho}_n(\boldsymbol{r}), \rho_p(\boldsymbol{r}), \tilde{\rho}_p(\boldsymbol{r})]. \tag{A2}$$

The first term here, the energy density for the Skyrme force of this paper, is given by

$$\begin{aligned}
\mathcal{E}_{\rm Sky} = {} & \sum_{q=n,p} \frac{\hbar^2}{2M_q}\tau_q + \frac{1}{2}t_0\left[\left(1+\frac{1}{2}x_0\right)\rho^2 - \left(\frac{1}{2}+x_0\right)\sum_{q=n,p}\rho_q^2\right] + \frac{1}{4}t_1\left\{\left(1+\frac{1}{2}x_1\right)\left[\rho\tau + \frac{3}{4}(\nabla\rho)^2\right]\right. \\
& \left. - \left(\frac{1}{2}+x_1\right)\sum_{q=n,p}\left[\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2\right]\right\} + \frac{1}{4}t_2\left\{\left(1+\frac{1}{2}x_2\right)\left[\rho\tau - \frac{1}{4}(\nabla\rho)^2\right] + \left(\frac{1}{2}+x_2\right)\right. \\
& \left. \times \sum_{q=n,p}\left[\rho_q\tau_q - \frac{1}{4}(\nabla\rho_q)^2\right]\right\} + \frac{1}{12}t_3\rho^\alpha\left[\left(1+\frac{1}{2}x_3\right)\rho^2 - \left(\frac{1}{2}+x_3\right)\sum_{q=n,p}\rho_q^2\right] \\
& + \frac{1}{4}t_4\left\{\left(1+\frac{1}{2}x_4\right)\left[\rho\tau + \frac{3}{4}(\nabla\rho)^2\right] - \left(\frac{1}{2}+x_4\right)\sum_{q=n,p}\left[\rho_q\tau_q + \frac{3}{4}(\nabla\rho_q)^2\right]\right\}\rho^\beta \\
& + \frac{\beta}{8}t_4\left[\left(1+\frac{1}{2}x_4\right)\rho(\boldsymbol{\nabla}\rho)^2 - \left(\frac{1}{2}+x_4\right)\boldsymbol{\nabla}\rho\cdot\sum_{q=n,p}\rho_q\boldsymbol{\nabla}\rho_q\right]\rho^{\beta-1} + \frac{1}{4}t_5\left\{\left(1+\frac{1}{2}x_5\right)\left[\rho\tau - \frac{1}{4}(\nabla\rho)^2\right]\right. \\
& \left. + \left(\frac{1}{2}+x_5\right)\sum_{q=n,p}\left[\rho_q\tau_q - \frac{1}{4}(\nabla\rho_q)^2\right]\right\}\rho^\gamma - \frac{1}{16}(t_1x_1 + t_2x_2)\,J^2 + \frac{1}{16}(t_1 - t_2)\sum_{q=n,p}J_q^2 \\
& - \frac{1}{16}(t_4x_4\rho^\beta + t_5x_5\rho^\gamma)J^2 + \frac{1}{16}(t_4\rho^\beta - t_5\rho^\gamma)\sum_{q=n,p}J_q^2 + \frac{1}{2}W_0\left(\boldsymbol{J}\cdot\boldsymbol{\nabla}\rho + \sum_{q=n,p}\boldsymbol{J}_q\cdot\boldsymbol{\nabla}\rho_q\right).
\end{aligned} \tag{A3}$$

Brussels-Montreal (BSk) functional

Experimental data

- atomic masses
- nuclear charge radii
- symmetry energy
- incompressibility

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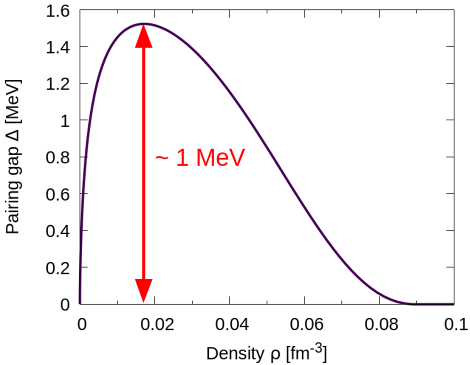
N-body calculations

- EoS of pure neutron matter
- $^1S_0$ pairing gaps in nuclear matter
- effective masses in nuclear matter

Chamel et al., Phys. Rev. C **80**, 065804 (2009)
Goriely et al., Phys. Rev. Lett. **102**, 152503 (2009)
Goriely et al., Phys. Rev. C **93**, 034337 (2016)



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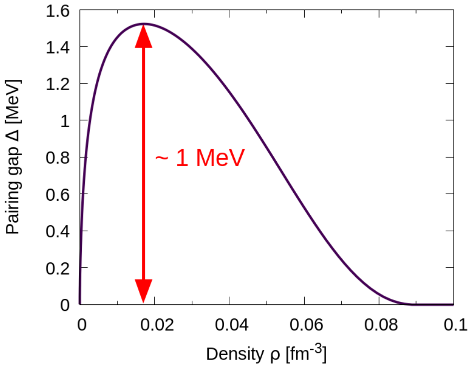
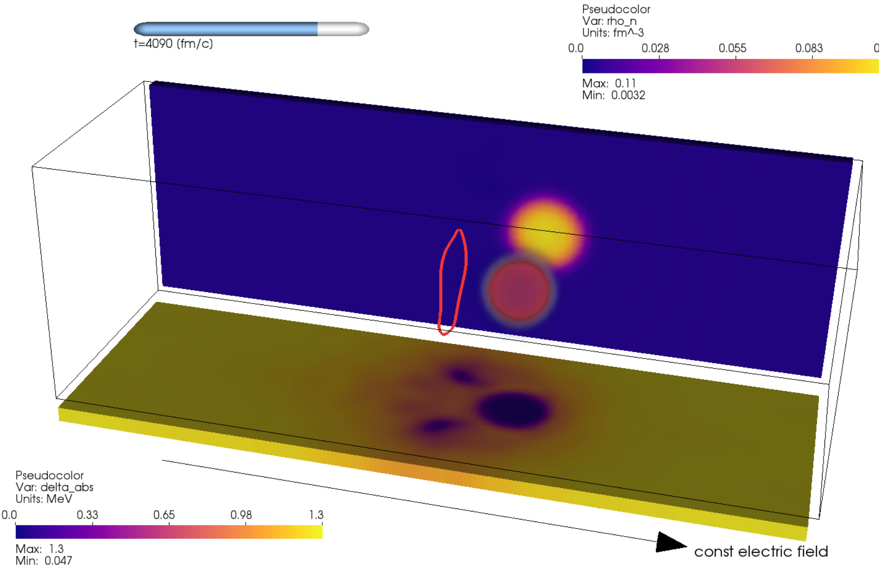
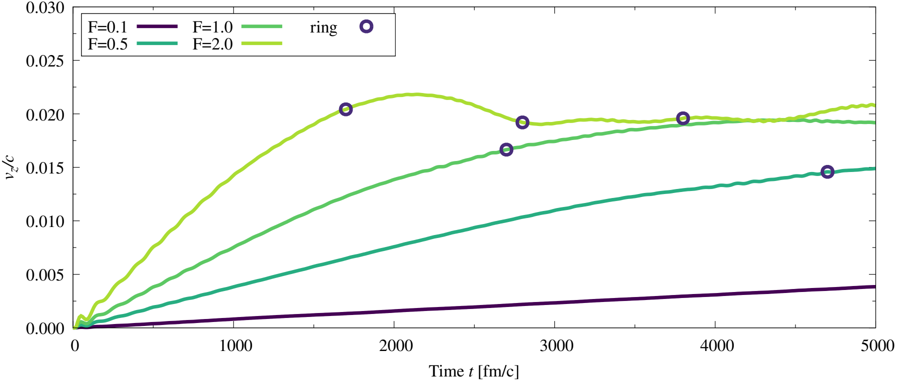
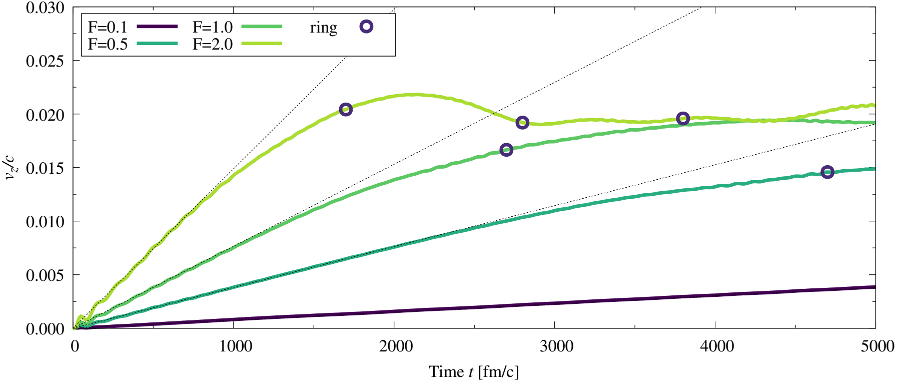


$$\varepsilon(\rho_q, \vec{\nabla}\rho_q, \nu_q, \tau_q, \boldsymbol{j}_q)$$


t=4090 (fm/c)
Pseudocolor
Var: rho_n
Units: fm^-3
0.0
0.028
0.055
0.083
0
Max: 0.11
Min: 0.0032
Pseudocolor
Var: delta_abs
Units: MeV
0.0
0.33
0.65
0.98
1.3
Max: 1.3
Min: 0.047
const electric field

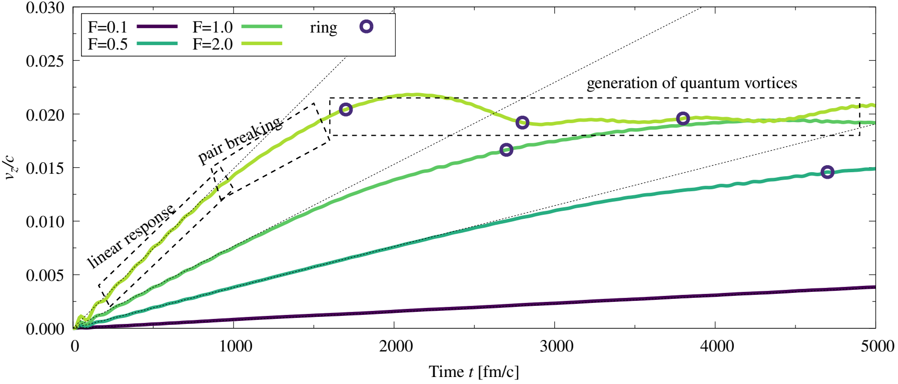


F=0.1
F=0.5
F=1.0
F=2.0
ring
0.030
0.025
0.020
0.015
0.010
0.005
0.000
v_z/c
0
1000
2000
3000
4000
5000
Time t [fm/c]



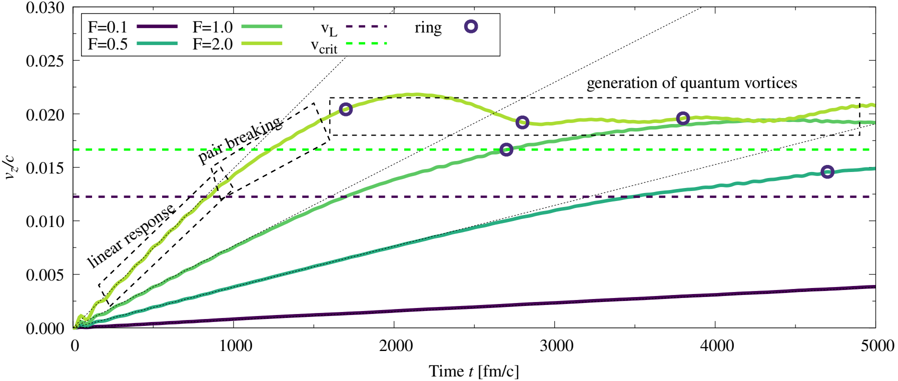
$$v_z = a_z t$$

$$M_{eff}^{(d)} = \frac{F}{a_z}$$



$$v_z = a_z t$$

$$M_{eff}^{(d)} = \frac{F}{a_z}$$

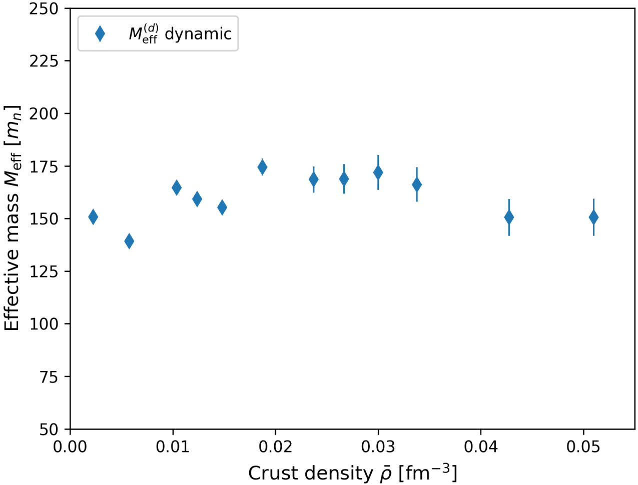


$$v_z = a_z t$$

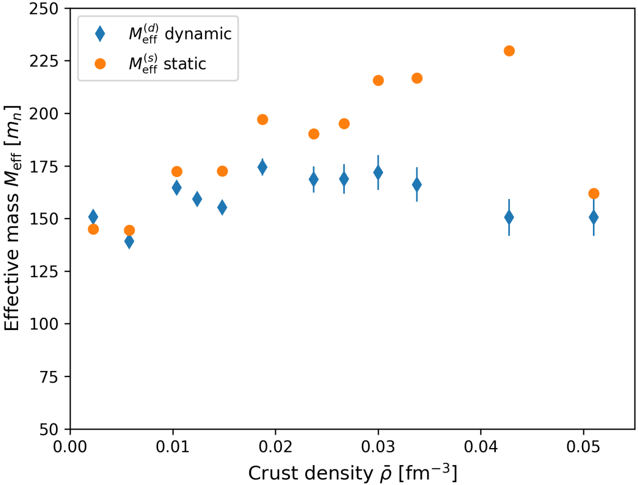
$$M_{eff}^{(d)} = \frac{F}{a_z}$$

$$v_L = \frac{\Delta}{\hbar k_F}$$

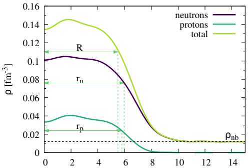
$$v_{\rm crit} = \frac{e}{2}\frac{\Delta}{\hbar k_F}$$



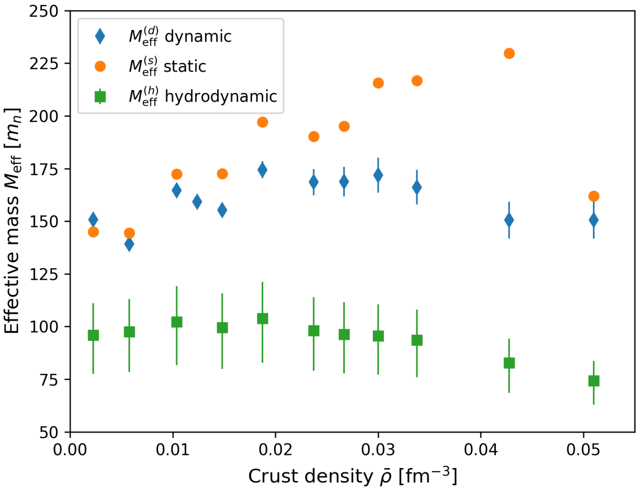
$M_{\mathrm{eff}}^{(d)}$ dynamic
Effective mass M_{eff} [m_n]
250
225
200
175
150
125
100
75
50
0.00
0.01
0.02
0.03
0.04
0.05
Crust density $\bar{\rho}$ [fm^{-3}]



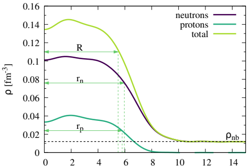
250
225
200
175
150
125
100
75
50
0.00
0.01
0.02
0.03
0.04
0.05
$M_{\rm eff}^{(d)}$ dynamic
$M_{\rm eff}^{(s)}$ static
Effective mass $M_{\rm eff}$ $[m_n]$
Crust density $\bar{\rho}$ [fm^{-3}]



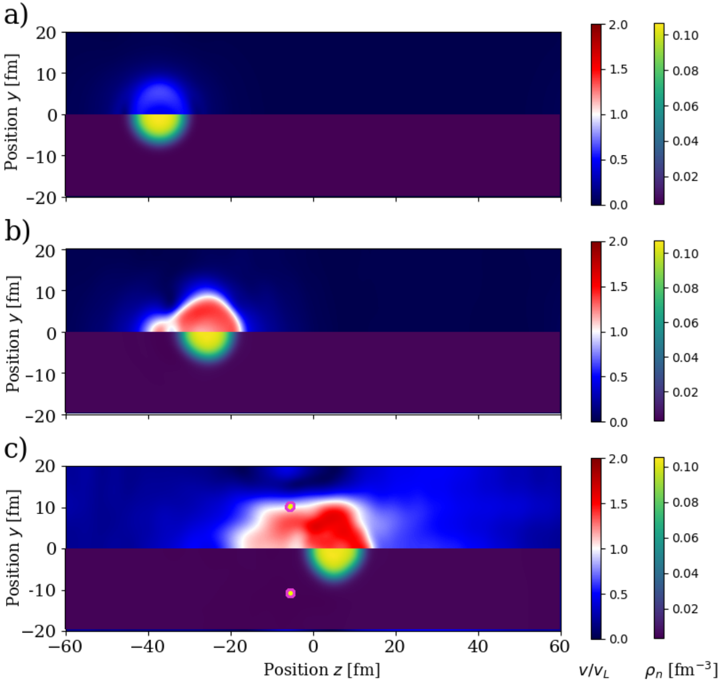
neutrons
protons
total
R
r_n
r_p
ρ_{nb}
ρ [fm^{-3}]



$$M_h = \frac{4}{3}\pi R^3 M \frac{(\rho_{\rm in} - \rho_{\rm out})^2}{\rho_{\rm in} + 2\rho_{\rm out}}.$$

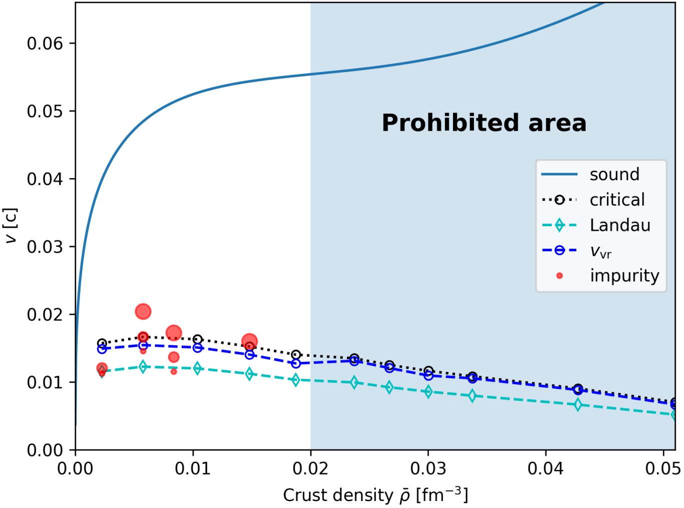


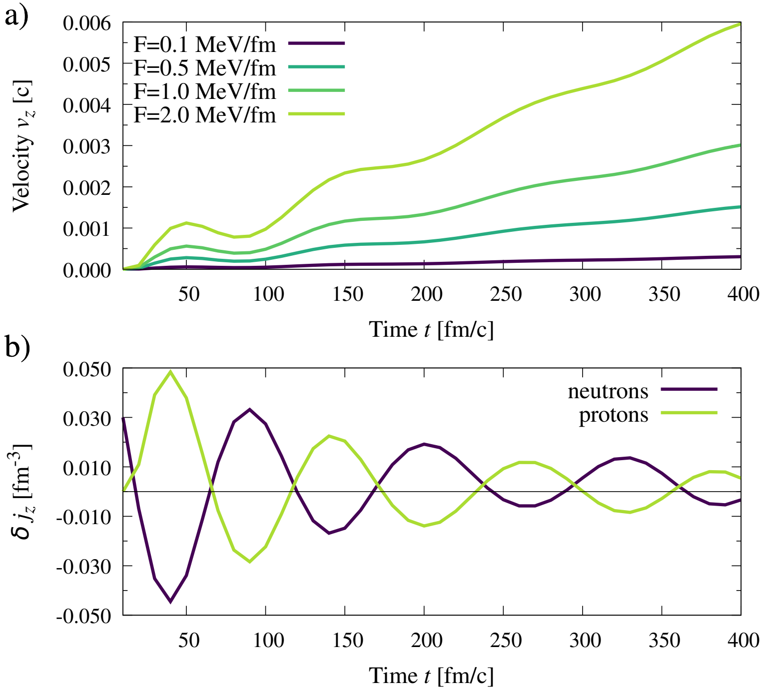
P. Magierski, Int. J. Mod. Phys. E 13 , 371-374 (2004)



a)
b)
c)
Position y [fm]
Position z [fm]
v/v_L
ρ_n [fm^{-3}]

$$v_{vr} = \frac{1}{2\pi R}\frac{\hbar c}{M_n c^2}\left(\ln\frac{8R}{a_{core}} - \alpha\right)$$




a)
0.006
0.005
0.004
0.003
0.002
0.001
0.000
Velocity v_z [c]
F=0.1 MeV/fm
F=0.5 MeV/fm
F=1.0 MeV/fm
F=2.0 MeV/fm
50
100
150
200
250
300
350
400
Time t [fm/c]
b)
0.050
0.030
0.010
-0.010
-0.030
-0.050
δj_z [fm^{-3}]
neutrons
protons
50
100
150
200
250
300
350
400
Time t [fm/c]

Summary

- fully self-consistent 3D (TD)HFB calculations
- BSk31 Energy Density Functional
- effective parameters can be extracted
- effective mass
- dissipation channels
- creating vortex rings
- giant dipole resonance

More details

arxiv:2403.17499

Thank you!