Ultracold Fermi gases as benchmark platform for neutron star studies: density functional theory approach

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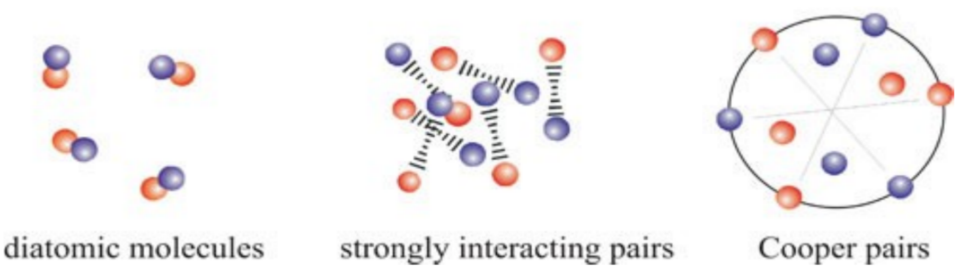
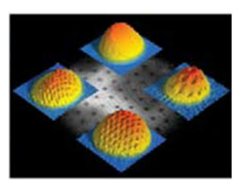




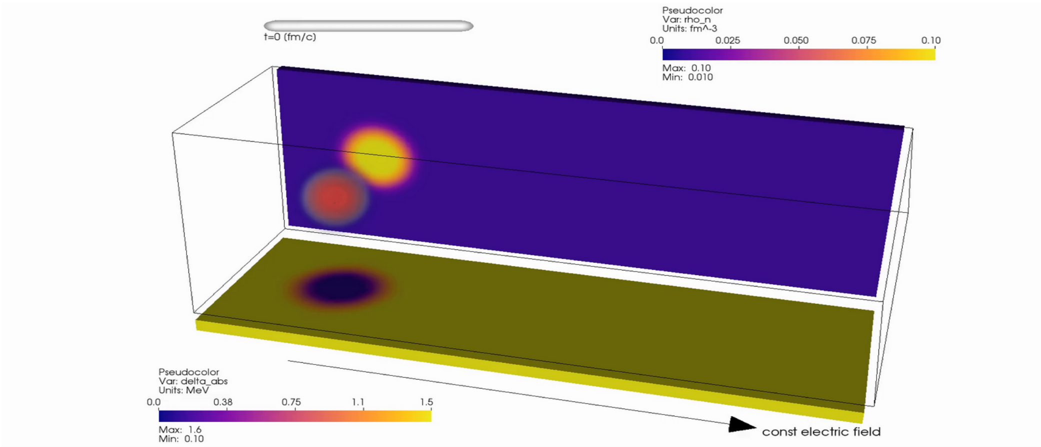
PL GRID LUMI

NATIONAL SCIENCE CENTRE POLAND

The physics of strongly interacting matter: neutron stars, cold atomic gases and related systems

Apr. 22 – 26, 2024, ECT* (Trento, Italy)

Context: superfluid dynamics in neutron stars

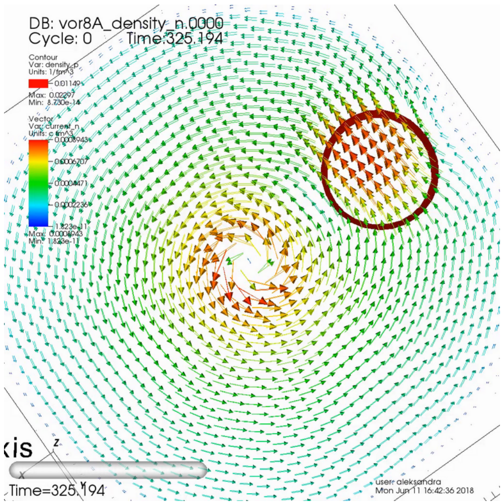


Nuclear impurity in superfluid neutron matter
(see talk by Daniel Pęcak)

and in vicinity of quantum vortex:

Source: D.Pęcak, et. al, arXiv:2403.17499

To what extent can we trust these types of calculations?

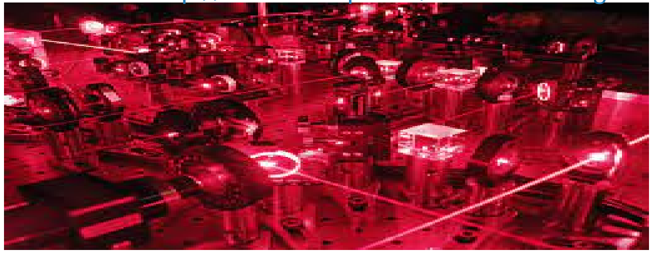


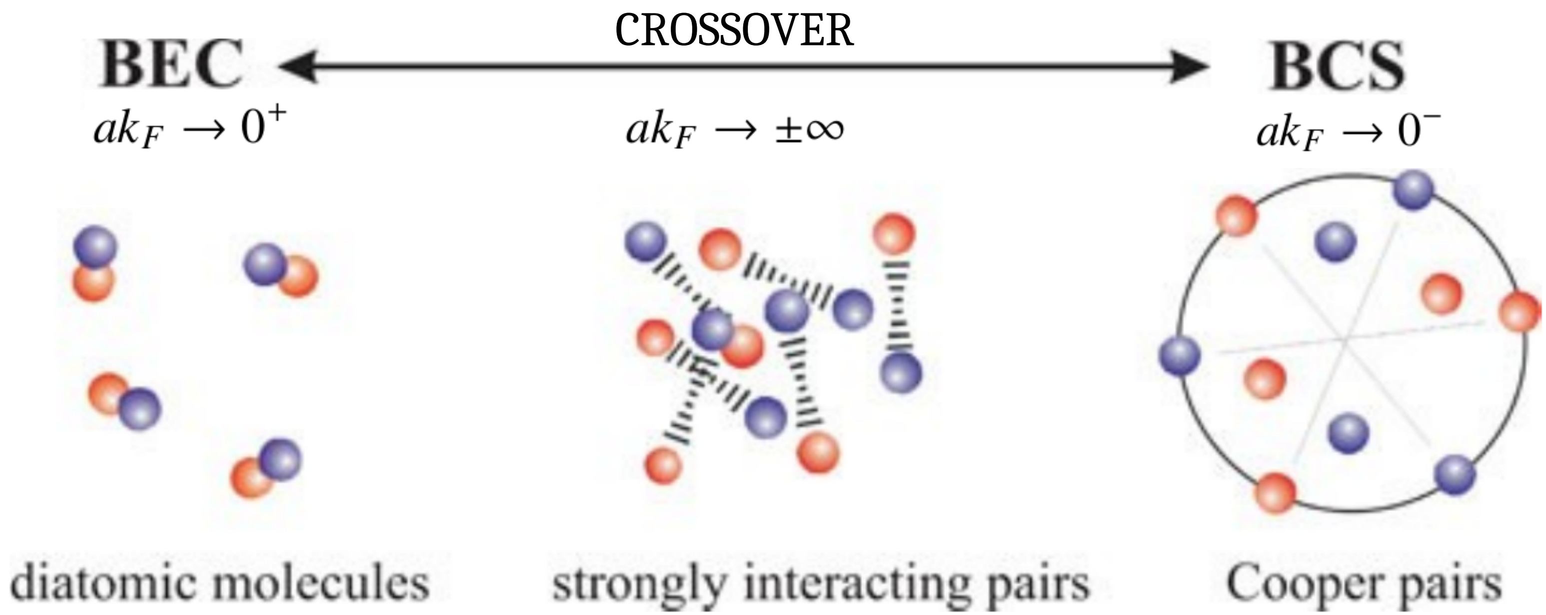
Overview:
*1. Method → DFT**
2. DFT for ultracold atoms and nuclear systems
3. Testing predictive power of DFT with ultracold gases
4. Long term challenges

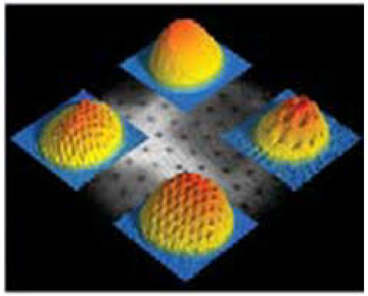
(*) Note: Many formal aspects of the theory will be presented superficially. Only general formulas...

- **Ultracold atomic** systems offer possibility to test predictive power of many-body methods.
- The (bare) interaction is simple $V(r-r')=g\delta(r-r')$…
- … but the interaction strength g can be tuned at will!

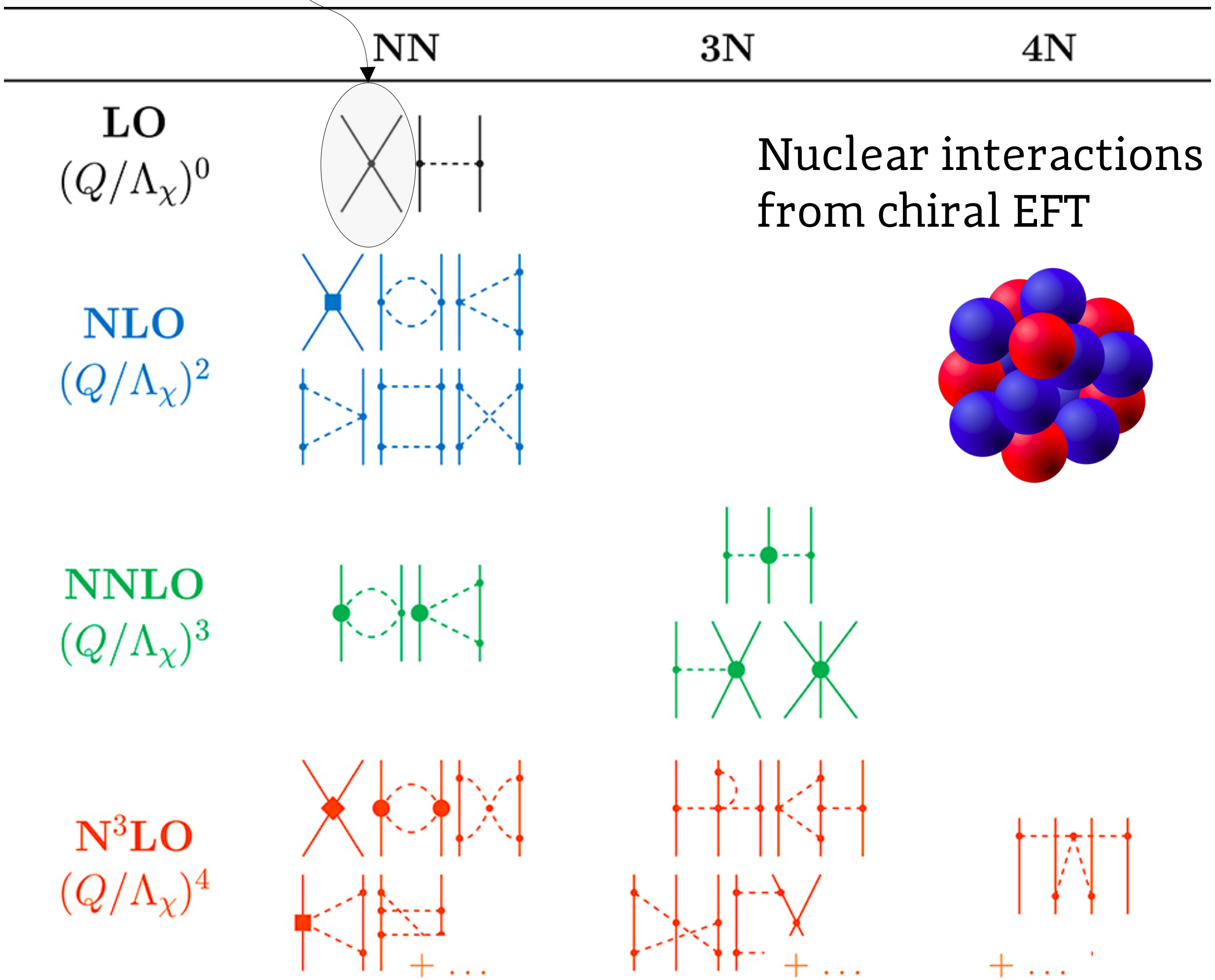
Photo from: http://www.lkb.upmc.fr/ultracoldfermigases/







$V(r-r') = g\delta(r-r')$
contact interaction



Nuclear interactions from chiral EFT

Fig from: Hergert H, Front. Phys. 8:379 (2020); doi:10.3389/fphy.2020.00379

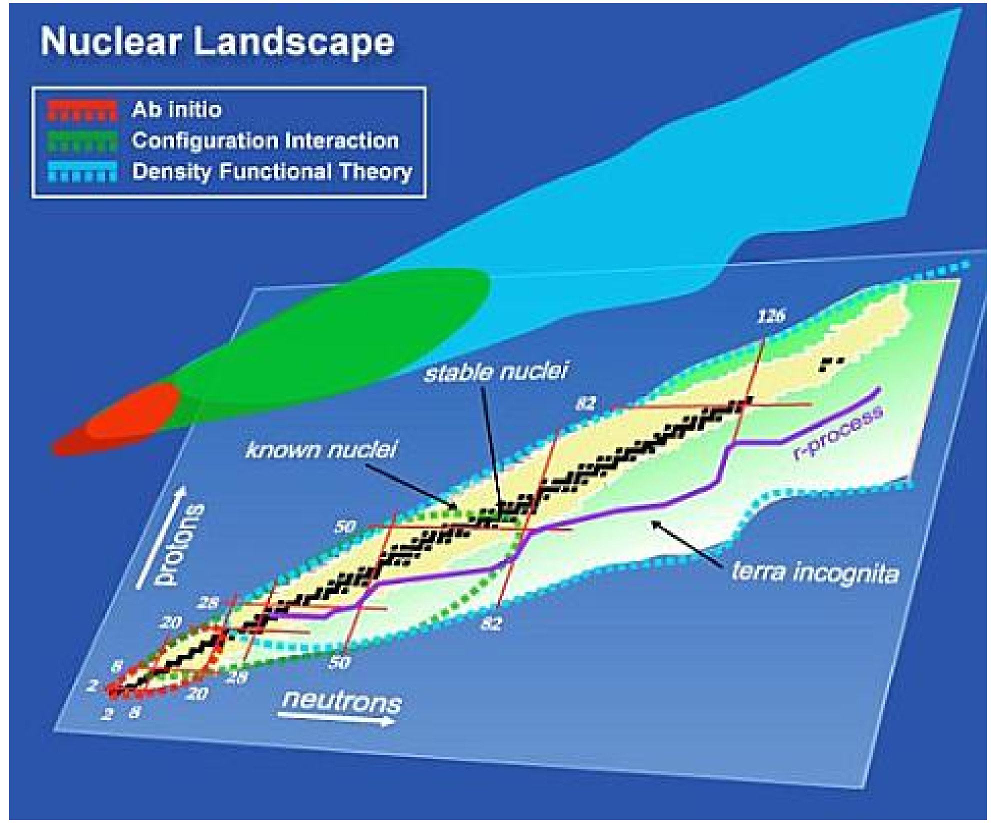
Expectation: the method when applied to “ultra-cold” atomic system should provide higher accuracy results as compared to applications in context of “nuclear” systems…

… upper limit for predictive power of the method.

Density Functional Theory (DFT):

Workhorse for ...

- Solid-state physics
- Quantum chemistry
- Condensed-matter physics



… also important tool for

- Nuclear physics
- (Nuclear) astrophysics
- Plasma physics ...

DFT is, in principle, exact theory
(due to Hohenberg-Kohn theorem)…
… in practice not – we need to construct the energy functional (no mathematical recipe how to derive it)

Many extensions: time-dependent formalism, finite temperature, normal/superconducting systems, non-relativistic/relativistic, …

General purpose framework

Nature 514, 550 (2014)

… *Twelve papers on the top-100 list relate to it* [DFT], *including 2 of the top 10.*

PHYSICS.WUT

SLDA-type functional

$$E_0 = \int \mathcal{E}[n_\sigma(\boldsymbol{r}), \tau_\sigma(\boldsymbol{r}), \boldsymbol{j}_\sigma, \nu(\boldsymbol{r})]\, d\boldsymbol{r}$$

The Fermi-Dirac distribution function

normal density

$$n_\sigma(\boldsymbol{r}) = \sum_{|E_n|<E_c} |v_{n,\sigma}(\boldsymbol{r})|^2 f_\beta(-E_n),$$

kinetic density

$$\tau_\sigma(\boldsymbol{r}) = \sum_{|E_n|<E_c} |\nabla v_{n,\sigma}(\boldsymbol{r})|^2 f_\beta(-E_n),$$

current density

$$\boldsymbol{j}_\sigma(\boldsymbol{r}) = \sum_{|E_n|<E_c} \mathrm{Im}[v_{n,\sigma}(\boldsymbol{r})\nabla v^*_{n,\sigma}(\boldsymbol{r})] f_\beta(-E_n),$$

anomalous density

$$\nu(\boldsymbol{r}) = \frac{1}{2} \sum_{|E_n|<E_c} \left[u_{n,a}(\boldsymbol{r})v^*_{n,b}(\boldsymbol{r}) - u_{n,b}(\boldsymbol{r})v^*_{n,a}(\boldsymbol{r})\right] f_\beta(-E_n).$$

Energy cut-off scale (need for regularization)

Denisties are **parametrized** via Bogoliubov quasiparticle wave functions

quasiparticle = mixture of hole particle

$$\varphi_\eta(\boldsymbol{r},t) = [u_\eta(\boldsymbol{r},t), v_\eta(\boldsymbol{r},t)]^T$$

$$\int \varphi^\dagger_\eta(\boldsymbol{r},t)\varphi_{\eta'}(\boldsymbol{r},t)\, d^3\boldsymbol{r} = \delta_{\eta,\eta'}$$

+ orthonormality condition (Pauli principle)

Additional density required by DFT theorem for systems with broken U(1) symmetry

Density-Functional Theory for Superconductors

L. N. Oliveira, E. K. U. Gross, and W. Kohn
Phys. Rev. Lett. **60**, 2430 – Published 6 June 1988

SLDA-type functional

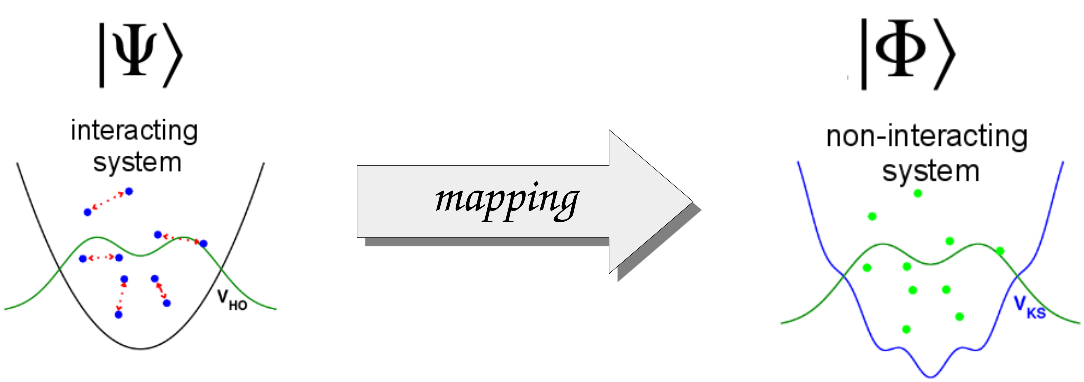
$$E_0 = \int \mathcal{E}[n_\sigma(\boldsymbol{r}), \tau_\sigma(\boldsymbol{r}), \boldsymbol{j}_\sigma, \nu(\boldsymbol{r})]\, d\boldsymbol{r}$$

↓ minimization

By construction minimization of the SLDA-type functional leads to equations that are mathematically equivalent to BdG or HFB equations

$$\begin{pmatrix} h_\uparrow(\boldsymbol{r}) - \mu_\uparrow & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_\downarrow^*(\boldsymbol{r}) + \mu_\downarrow \end{pmatrix} \begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r}) \\ v_{n,\downarrow}(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r}) \\ v_{n,\downarrow}(\boldsymbol{r}) \end{pmatrix}$$

$$h_\sigma = -\boldsymbol{\nabla} \frac{\delta E_0}{\delta \tau_\sigma} \boldsymbol{\nabla} + \frac{\delta E_0}{\delta n_\sigma} - \frac{i}{2} \left\{ \frac{\delta E_0}{\delta \boldsymbol{j}_\sigma}, \boldsymbol{\nabla} \right\}, \quad \Delta = -\frac{\delta E_0}{\delta \nu^*}.$$



DFT method from practical point of view:

DFT method allows for the description of many-body quantum systems with higher accuracy than the mean-field method while keeping the computational complexity at the same level as for the mean-field method.

EoS (typically from QMC)

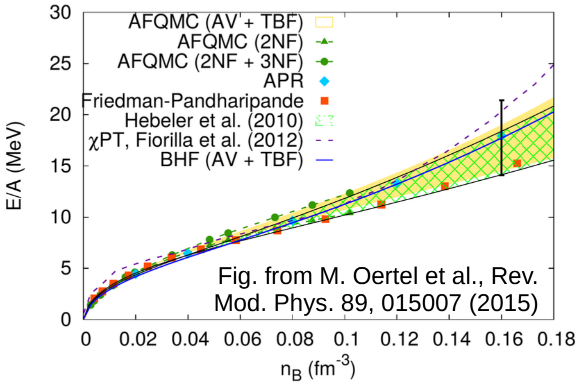
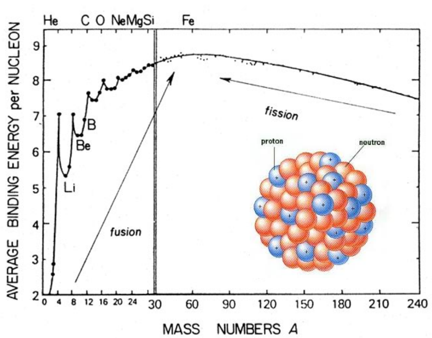
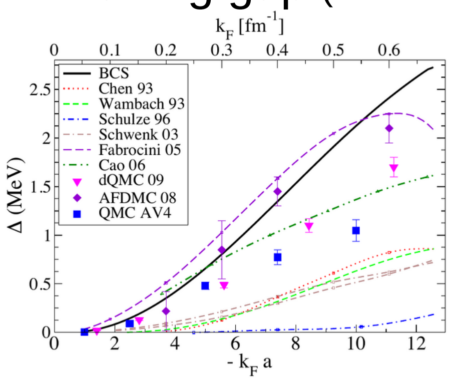


Fig. from M. Oertel et al., Rev. Mod. Phys. 89, 015007 (2015)

Exp. data for nuclei
(masses, radii, ...)



Pairing gap (s-wave)



Dimensional arguments, renormalizability, Galilean invariance, and symmetries (translational, rotational, gauge, parity, ...)

Postulate

Input 1

Input 2

Input 3

Energy Density Functional E[n,...]

Validation against other quantities

Predictions...

Note: EDF→EoS but EoS↛EDF

The quality of DFT results strongly depends on the quality of the functional, … which in turn depends on the quality of input data.

Brussels Skyrme functionals BSk(G)

We have fitted a series of nuclear energy-density functionals with full HFB calculations using extended Skyrme functionals

Experimental data/constraints:

- $\sim$ 2300 atomic masses (rms $\sim 0.5 - 0.6$ MeV$/c^2$)
- $\sim$ 900 nuclear charge radii (rms $\sim$ 0.03 fm)
- symmetry energy $29 \leq J \leq 32$ MeV
- incompressibility $K_v = 240 \pm 10$ MeV (giant resonances in nuclei)

Many-body ab initio calculations:

- equation of state of pure neutron matter
- $^1S_0$ pairing gaps in nuclear matter
- effective masses in nuclear matter (+giant resonances in nuclei)
- stability against spin and spin-isospin fluctuations

Grams et al., Eur. Phys. J. A 59, 270 (2023)

Slide from Nicolas Chamel's talk, ECT* Workshops, Trento, Apr. 2024

SLDA-type functional
for ultra-cold atoms

$$E_0 = \int \mathcal{E}[n(\boldsymbol{r}), \tau(\boldsymbol{r}), \boldsymbol{j}(\boldsymbol{r}), \nu(\boldsymbol{r})] d\boldsymbol{r}$$

Dimensionless functional parameters

$$\{A_\lambda, B_\lambda, C_\lambda\}$$

Densities $n(\boldsymbol{r}), \tau(\boldsymbol{r}), \nu(\boldsymbol{r})$ are defined via $[u_\eta(\boldsymbol{r},t), v_\eta(\boldsymbol{r},t)]^T$

$$\lambda = ak_F$$

$$\mathcal{E} = \frac{A_\lambda}{2}\left(\tau - \frac{\boldsymbol{j}^2}{n}\right) + \frac{3}{5}B_\lambda n \varepsilon_F + \frac{C_\lambda}{n^{1/3}}|\nu|^2 + \frac{\boldsymbol{j}^2}{2n}$$

dimensional analysis + symmetries

Kinetic term

Potential term

Pairing term

Center of mass motion

Units:
$\hbar = m = 1$

SLDA-type functional

for ultra-cold atoms

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dimensional analysis + symmetries

Kinetic term | Potential term | Pairing term | Center of mass motion

Units: $\hbar = m = 1$

Example: The simplest choice

BdG (mean-field)

$$A_\lambda \to 1$$

$$B_\lambda \to 0$$

$$C_\lambda \to \frac{4\pi\hbar^2}{(3\pi^2)^{1/3} m} \underbrace{ak_F}_{\lambda}$$

$$\mathcal{E}_{\mathrm{BdG}} = \frac{\tau}{2} + 4\pi a |\nu(\boldsymbol{r})|^2$$

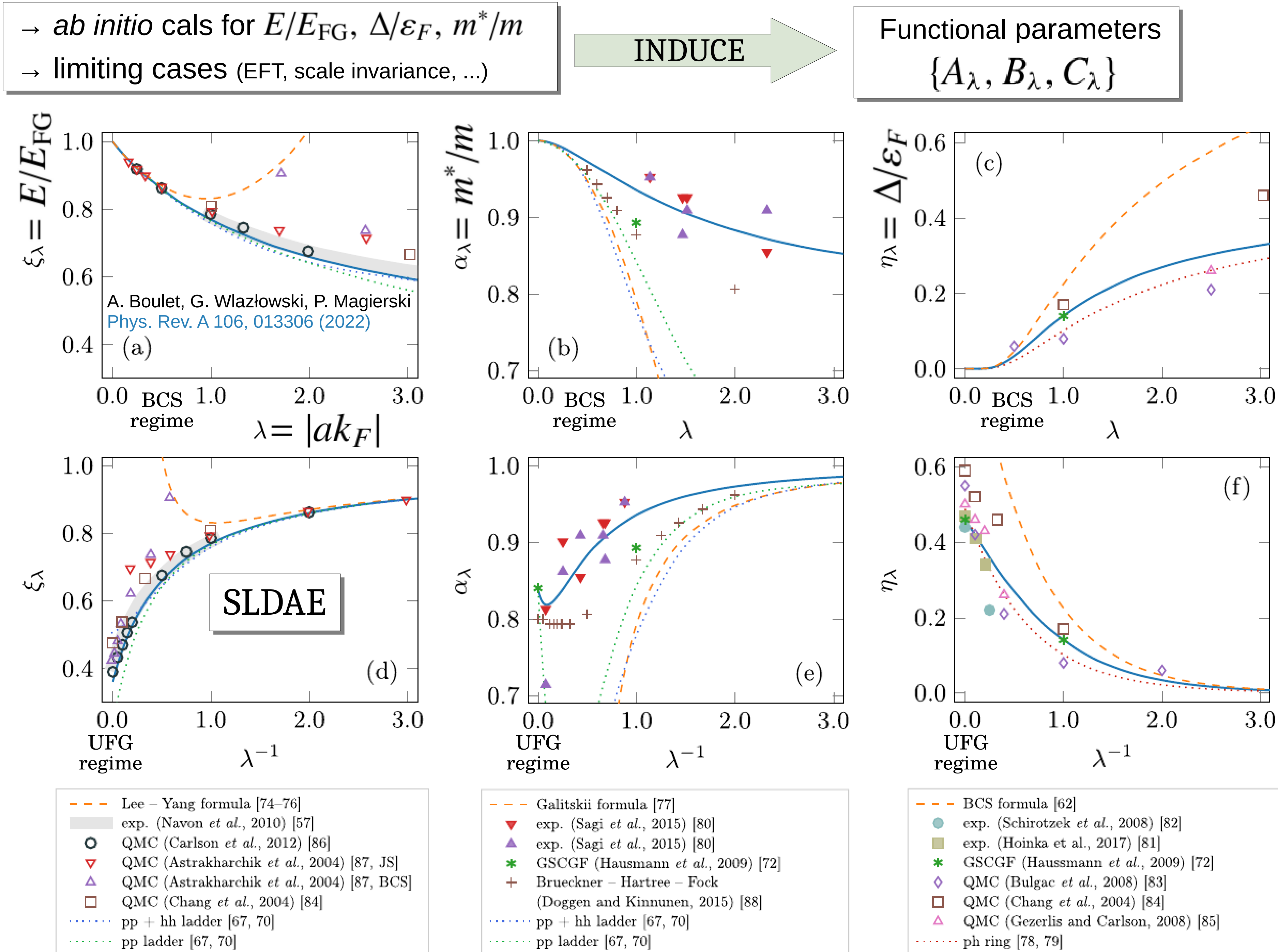
minimization

$$\begin{pmatrix} -\frac{\hbar^2}{2}\nabla^2 - \mu & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & \frac{\hbar^2}{2}\nabla^2 + \mu \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix}$$

$$\Delta = -4\pi a \sum_{|E_n|<E_c} u_n(\mathbf{r}) v_n^*(\mathbf{r}) \frac{f_\beta(-E_n) - f_\beta(E_n)}{2}$$

There always exists a functional that after minimization provides equations identical to the mean-field equations (zeroth order).

PHYSICS.WUT


→ ab initio cals for E/E_{FG}, Δ/ε_F, m^*/m
→ limiting cases (EFT, scale invariance, ...)
INDUCE
Functional parameters
$\{A_\lambda, B_\lambda, C_\lambda\}$
$\xi_\lambda = E/E_{FG}$
$\alpha_\lambda = m^*/m$
$\eta_\lambda = \Delta/\varepsilon_F$
A. Boulet, G. Wlazłowski, P. Magierski
Phys. Rev. A 106, 013306 (2022)
(a)
(b)
(c)
(d)
(e)
(f)
BCS regime
$\lambda = |ak_F|$
λ
UFG regime
λ^{-1}
ξ_λ
α_λ
η_λ
SLDAE
Lee – Yang formula [74–76]
exp. (Navon et al., 2010) [57]
QMC (Carlson et al., 2012) [86]
QMC (Astrakharchik et al., 2004) [87, JS]
QMC (Astrakharchik et al., 2004) [87, BCS]
QMC (Chang et al., 2004) [84]
pp + hh ladder [67, 70]
pp ladder [67, 70]
Galitskii formula [77]
exp. (Sagi et al., 2015) [80]
exp. (Sagi et al., 2015) [80]
GSCGF (Hausmann et al., 2009) [72]
Brueckner – Hartree – Fock
(Doggen and Kinnunen, 2015) [88]
pp + hh ladder [67, 70]
pp ladder [67, 70]
BCS formula [62]
exp. (Schirotzek et al., 2008) [82]
exp. (Hoinka et al., 2017) [81]
GSCGF (Haussmann et al., 2009) [72]
QMC (Bulgac et al., 2008) [83]
QMC (Chang et al., 2004) [84]
QMC (Gezerlis and Carlson, 2008) [85]
ph ring [78, 79]

Towards time-dependent problems

$$\begin{pmatrix} h_\uparrow(\boldsymbol{r}) - \mu_\uparrow & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_\downarrow^*(\boldsymbol{r}) + \mu_\downarrow \end{pmatrix} \begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r}) \\ v_{n,\downarrow}(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r}) \\ v_{n,\downarrow}(\boldsymbol{r}) \end{pmatrix}$$

From point of view of DFT this step represents uncontrolled approximation,
called *adiabatic approximation*

$$\begin{pmatrix} h_\uparrow(\boldsymbol{r},t) - \mu_\uparrow & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -h_\downarrow^*(\boldsymbol{r},t) + \mu_\downarrow \end{pmatrix} \begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r},t) \\ v_{n,\downarrow}(\boldsymbol{r},t) \end{pmatrix} = i\hbar \frac{\partial}{\partial t} \begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r},t) \\ v_{n,\downarrow}(\boldsymbol{r},t) \end{pmatrix}$$

Towards time-dependent problems

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Density-Functional Theory for Time-Dependent Systems

Erich Runge and E. K. U. Gross
Phys. Rev. Lett. **52**, 997 – Published 19 March 1984

Time-Dependent Density-Functional Theory for Superconductors

O. -J. Wacker, R. Kümmel, and E. K. U. Gross
Phys. Rev. Lett. **73**, 2915 – Published 21 November 1994

There exits analog of Hohenberg-Kohn theorem for time-dependent problems...

... but for time-dependent case the “exact” functional is in general different from the one that is used in static calculations...

Towards time-dependent problems

$$\begin{pmatrix} h_\uparrow(\boldsymbol{r}) - \mu_\uparrow & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h^*_\downarrow(\boldsymbol{r}) + \mu_\downarrow \end{pmatrix} \begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r}) \\ v_{n,\downarrow}(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_{n,\uparrow}(\boldsymbol{r}) \\ v_{n,\downarrow}(\boldsymbol{r}) \end{pmatrix}$$

From point of view of DFT this step represents uncontrolled approximation,
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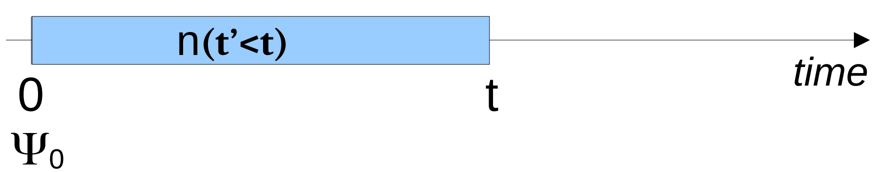
$$E(t) = E[\Psi(t=0), n(\boldsymbol{r},t' \leqslant t), \ldots]$$



$$E(t) = \int_V d\boldsymbol{r}\, \mathcal{E}[n(\boldsymbol{r},t), \ldots]$$

Adiabatic approximation

In general integro-differential equations ←

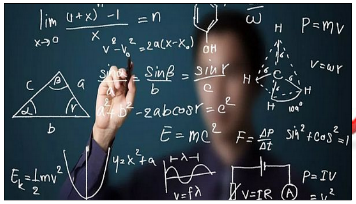
$$E(t) = \int_0^t dt' \int_V d\boldsymbol{r}\, \mathcal{E}[\Psi_0, n(\boldsymbol{r},t'), \ldots]$$

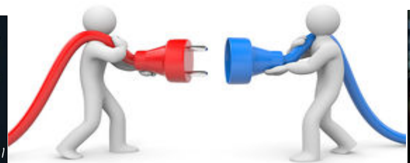
There exits analog of Hohenberg-Kohn theorem for time-dependent problems...

... but for time-dependent case the “exact” functional is in general different from the one that is used in static calculations...

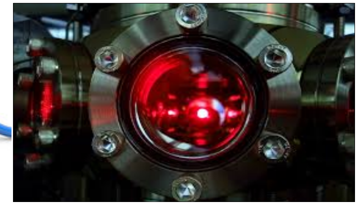
...if the evolution is slow (adiabatic), then the system follows instantaneous ground state
→ use the functional taken from static considerations.

Theoretical method





Experiment



Computer code

LUMI @ CSC's data center in Kajaani (Finland) - One of the pan-European pre-exascale supercomputers.

DFT method
is typically delivered
to community
in form of a code

Abinit

Frequently High Performance Computing (HPC) is involved

Piz Daint @ Swiss National Supercomputing Centre (Switzerland) - Access is granted thanks to PRACE.

VASP

JuKKR

QUANTUM ESPRESSO

siesta

WIEN2k

Tsubame3.0 @ Global Scientific Information and Computing Center, Tokyo Institute of Technology (Japan)

...and many others, see here:
https://en.wikipedia.org/wiki/List_of_quantum_chemistry_and_solid-state_physics_software

Functionalities | Download | Gallery | Publications and Materials | Related projects | Info | EN

Warsaw University of Technology | **W-SLDA Toolkit** / **W-BSk Toolkit**

W-SLDA Toolkit

Self-consistent solver of mathematical problems which have structure formally equivalent to Bogoliubov-de Gennes equations.

static problems: st-wslda

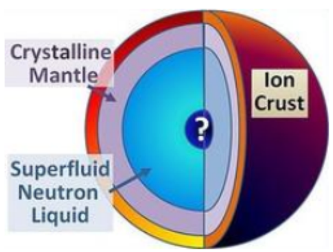
$$\begin{pmatrix} h_a(\boldsymbol{r})-\mu_a & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -h_b^*(\boldsymbol{r})+\mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix} = E_n \begin{pmatrix} u_n(\boldsymbol{r}) \\ v_n(\boldsymbol{r}) \end{pmatrix}$$

time-dependent problems: td-wslda

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_a(\boldsymbol{r},t)-\mu_a & \Delta(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & -h_b^*(\boldsymbol{r},t)+\mu_b \end{pmatrix} \begin{pmatrix} u_n(\boldsymbol{r},t) \\ v_n(\boldsymbol{r},t) \end{pmatrix}$$

- Extension to nuclear matter in neutron stars
- Unified solvers for static and time-dependent problems
- Dimensionalities of problems: 3D, 2D and 1D

Extension to nuclear matter in neutron stars



The W-SLDA Toolkit has been expanded to encompass nuclear systems, now available as the W-BSk Toolkit.

ALL FUNCTIONALITIES →

- Integration with VisIt: visualization, animation and analysis tool
- Speed-up calculations by exploiting High Performance Computing
- Functionals for studies of BCS and unitary regimes

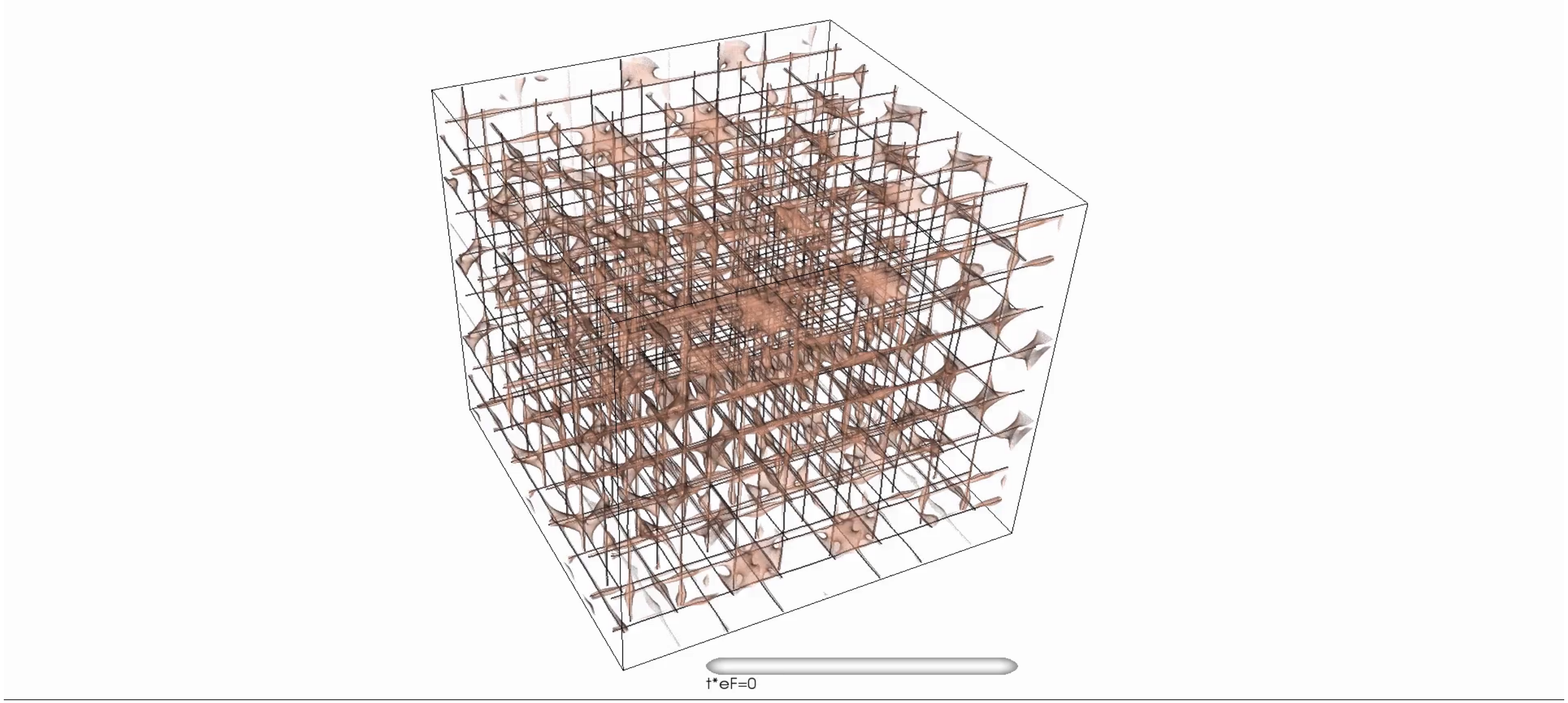
High Performance Computing

can run on “small” computing clusters as well as leadership supercomputers
(depending on the problem size)

NVIDIA CUDA | AMD ROCm | open source

… we follow this good practice also in case of developments for cold atoms and neutron stars…

http://wslda.fizyka.pw.edu.pl/

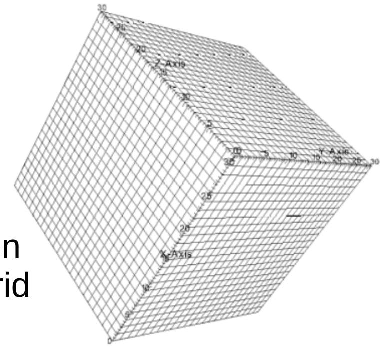


System: *unitary Fermi gas*
3D simulation on lattice 100^3

number of atoms = 26,790
number of quasi-particle states = 582,898
number of PDEs = 1,165,796

Quantum turbulence
in the unitary Fermi gas
PNAS Nexus, pgae160 (2024)

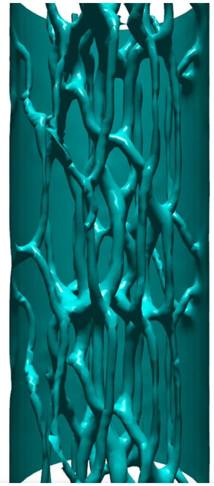
Computation
on spatial grid

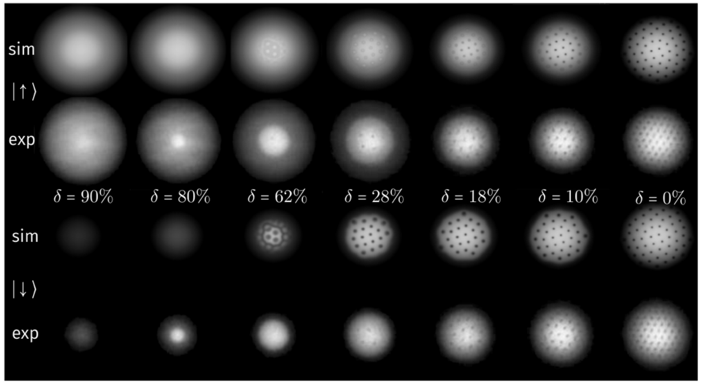


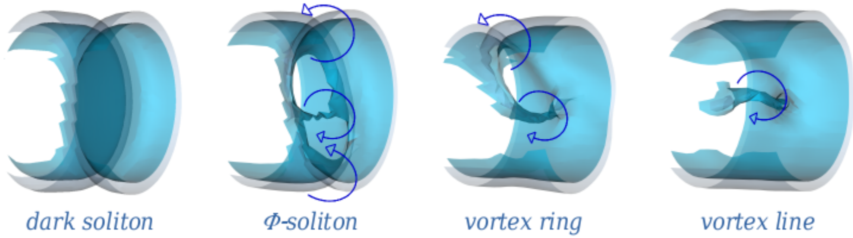
(the largest system in 3D we considered had 108,532 atoms)

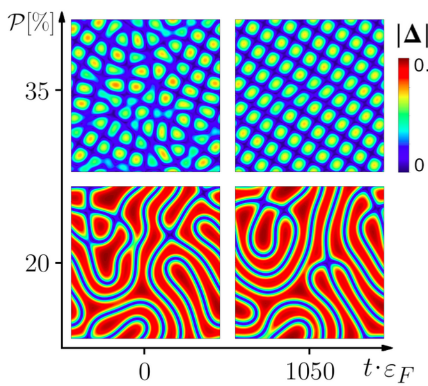
Applications in a broad context: getting knowledge about predictive power.

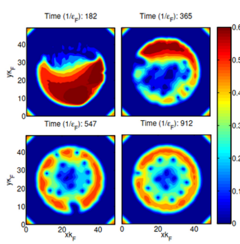
- Quantum vortices
  PRL. 130, 043001 (2023); PRA 106, 033322 (2022); PRA 104, 053322 (2021) PRA 103, L051302 (2021); PRL 112, 025301 (2014); Science 332, 1288 (2011)
- Quantum turbulence
  PRA 91, 031602 (2015); PRA 105, 013304 (2022); PNAS Nexus, pgae160 (2024)
- Spin-polarized impurities
  PRA 100, 033613 (2019); PRA 104, 033304 (2021); ...
- Solitonic cascades
  PRL. 120, 253002 (2018)
- Higgs/amplitude mode
  Sci. Rep. 13, 11285 (2023); PRL 102, 085302 (2009)
- Josephson junction
  PRL, 023003 (2023)
- Phase diagram of spin-imbalanced systems
  New J. Phys. 25, 033013 (2023); PRL, 101, 215301 (2008)
- Shock waves
  Phys. Rev. Lett. 108, 150401 (2012)

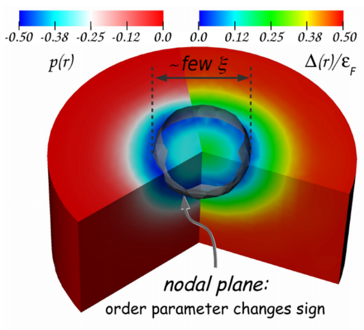


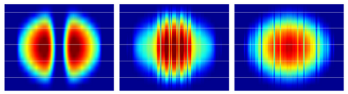


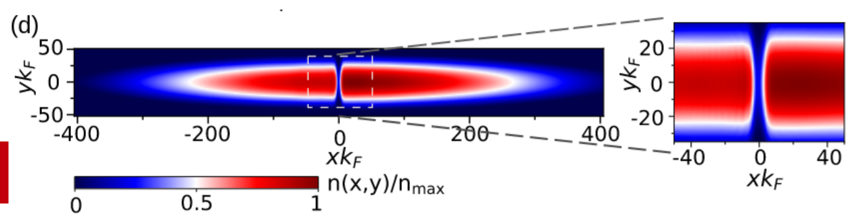






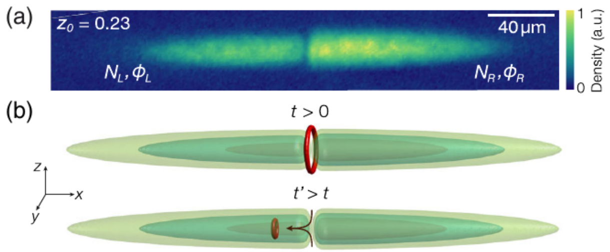






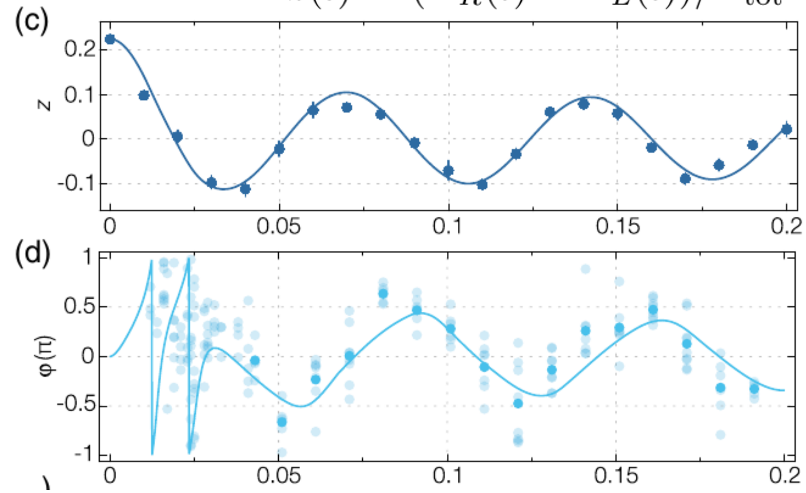
Example #1: Fermionic Josephson Junction

Inspired by LENS ^{6}Li setup (G. Roati's group):
[1] G. Valtolina, et.al., Science **350**, 1505, (2015);
[2] A. Burchianti, et.al., Phys. Rev. Lett. **120**, 025302 (2018)
[3] K. Xhani, et.al., Phys. Rev. Lett. **124**, 045301 (2020)



Figs from [2]

$$z(t) = (N_R(t) - N_L(t))/N_{\text{tot}}$$



$$\Delta\phi(t) \equiv \phi_L(t) - \phi_R(t)$$ t [s]

Experiment

Simulation

Example #1: Fermionic Josephson Junction

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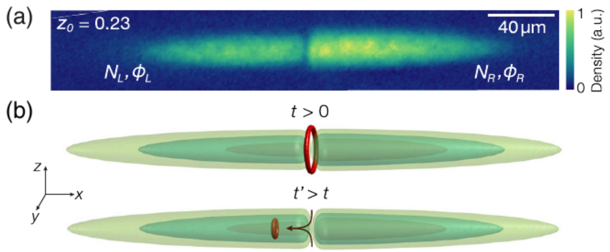

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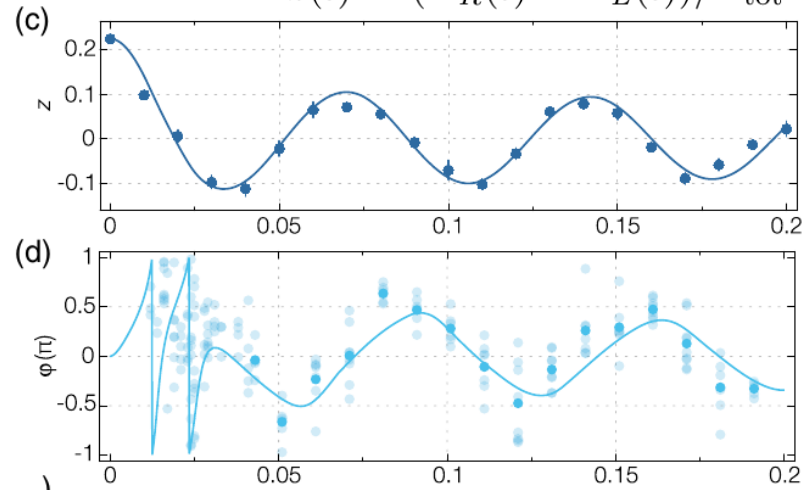

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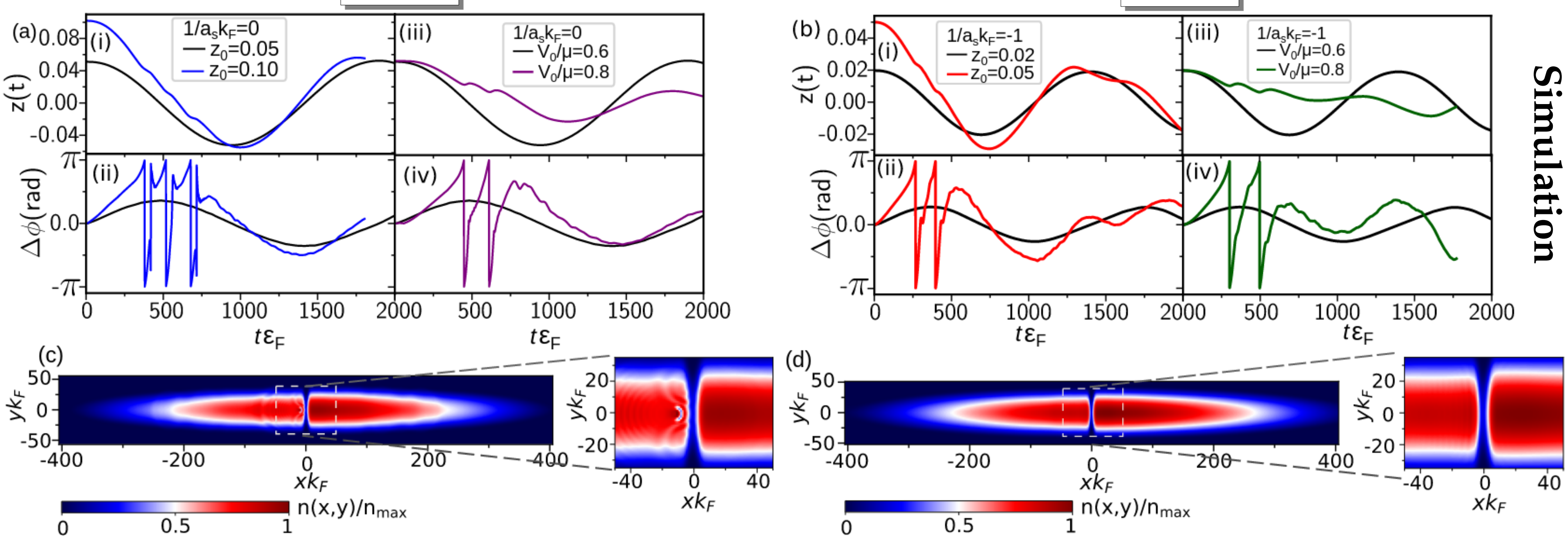
Figs from [2]

Experiment

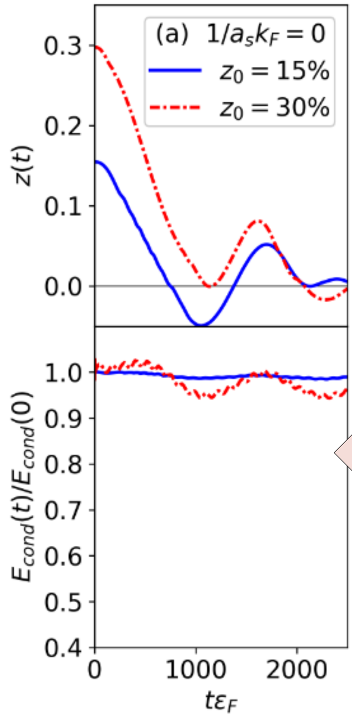
G. Wlazłowski, et.al.,
Phys. Rev. Lett. 130, 023003 (2023)

UFG

BCS

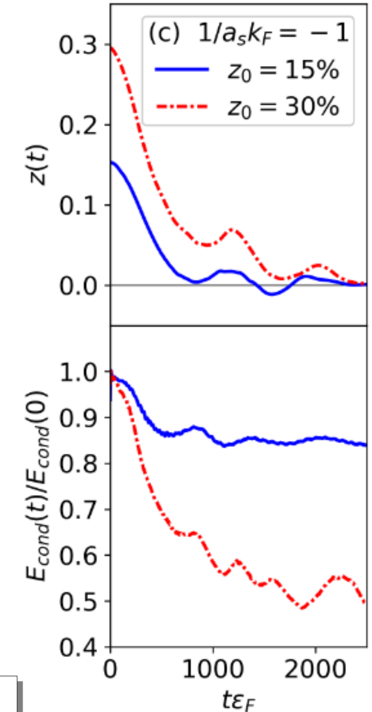


Simulation



$$E_{\rm BCS} = E_{\rm FG} - \frac{3|\Delta|^2}{8\varepsilon_F}N$$

$$E_{\rm cond} = \int \frac{3}{8}\frac{|\Delta(\mathbf{r})|^2}{\varepsilon_F(\mathbf{r})} n(\mathbf{r}) d\mathbf{r}$$

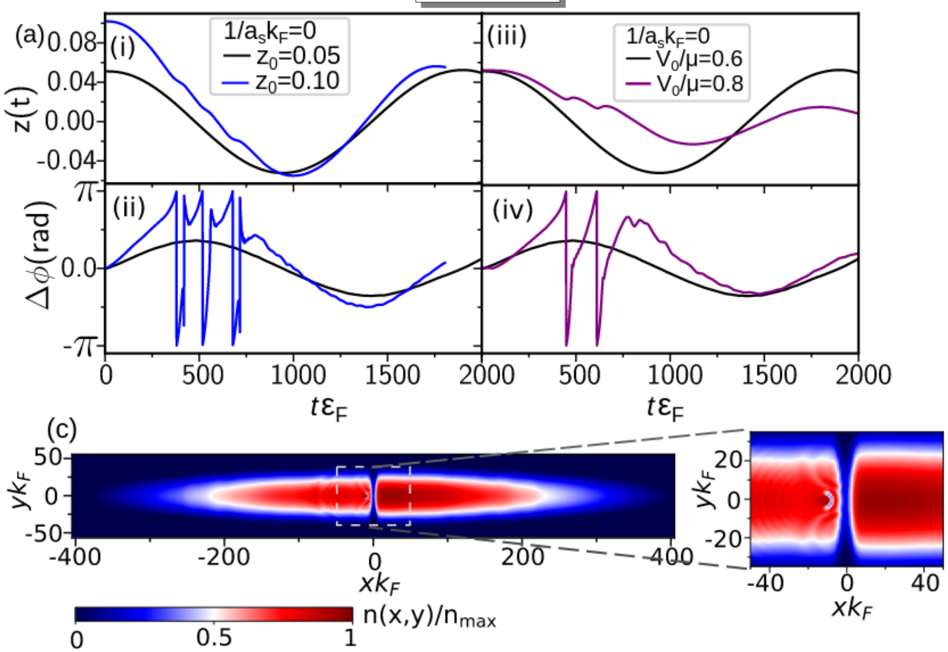


pair breaking is negligible

pair breaking!

G. Wlazłowski, K. Xhani, M. Tylutki, N. Proukakis, M. Magierski, Phys. Rev. Lett. 130, 023003 (2023)

UFG



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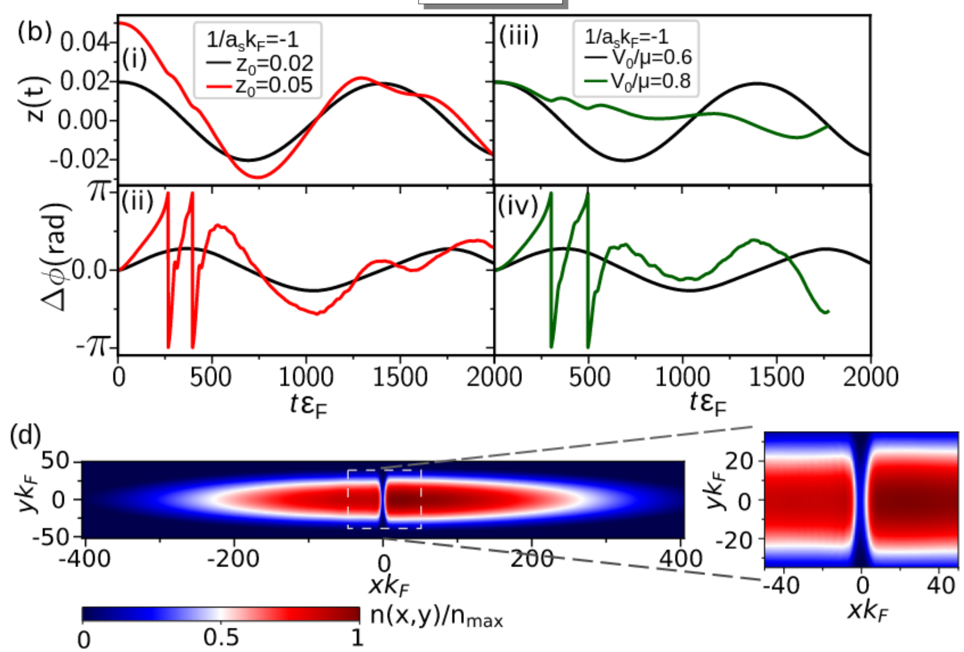

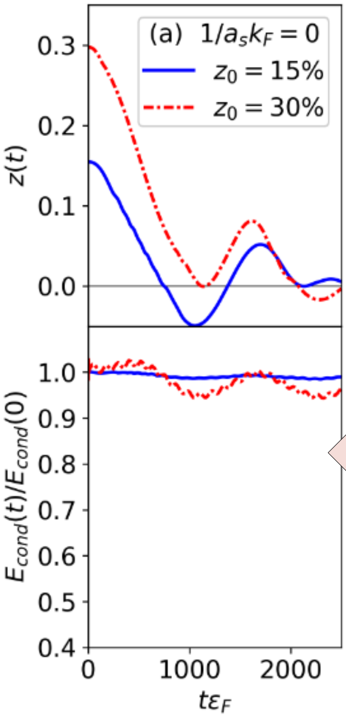

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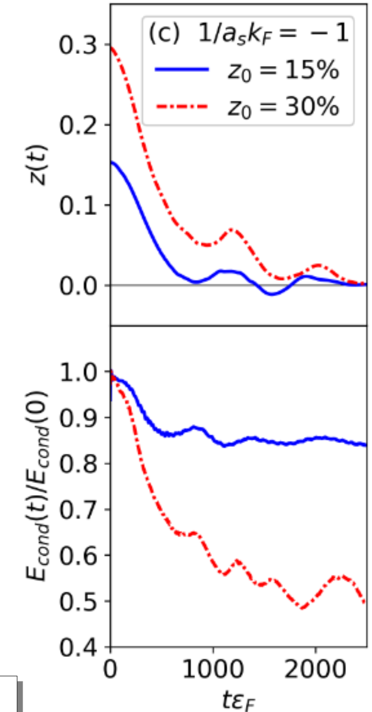
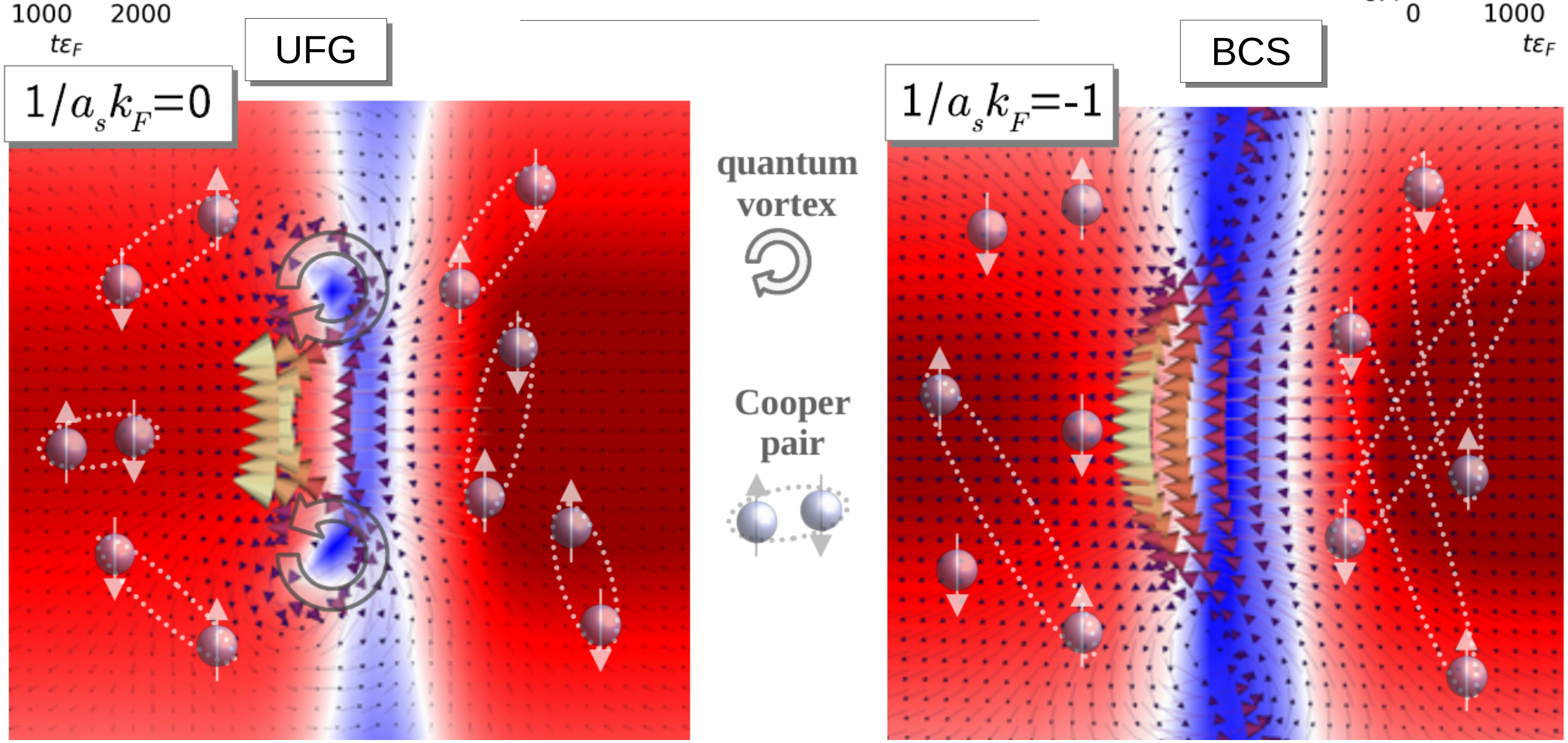


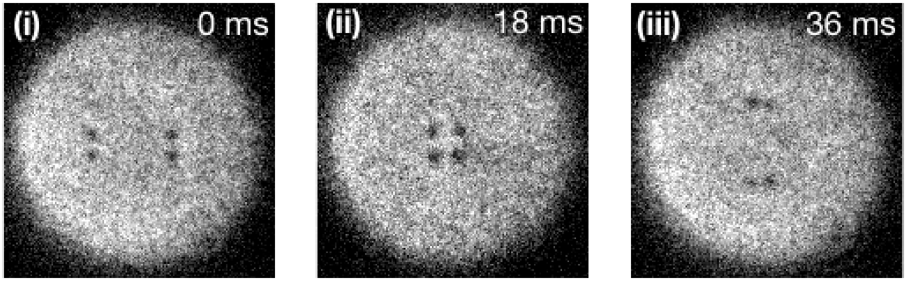
G. Wlazłowski, K. Xhani, M. Tylutki, N. Proukakis, M. Magierski, Phys. Rev. Lett. 130, 023003 (2023)

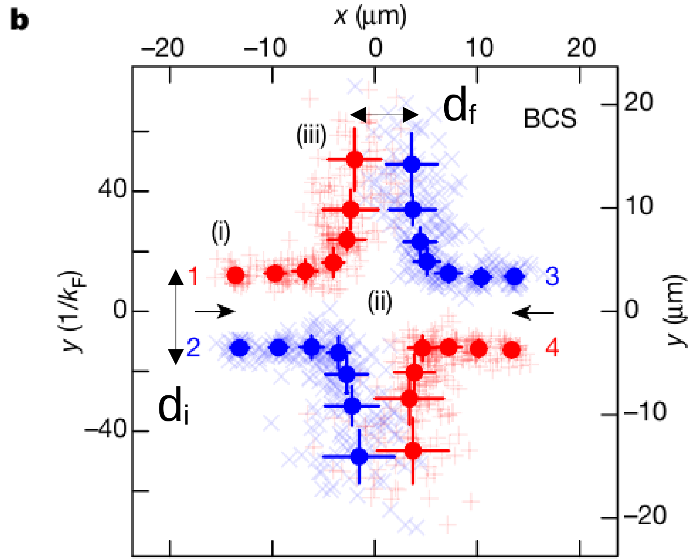


Example #2: Vortex collisions

Inspired by LENS ^{6}Li setup (G. Roati's group):
[1] W. J. Kwon, et.al., Nature 600, 64-69 (2021)

Figs from [1]





Experiment

Dissipation during the collision is reflected in $d_f/d_i < 1$.

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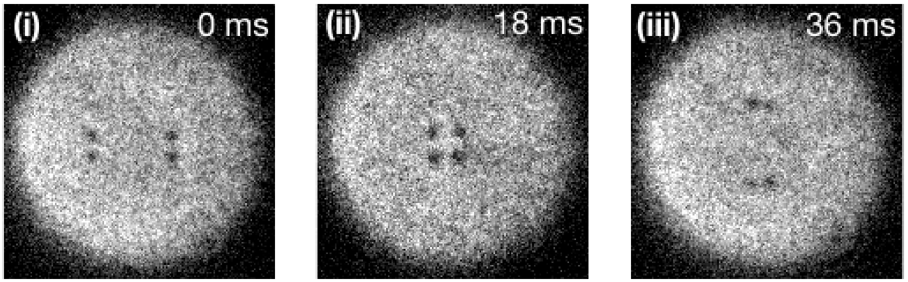
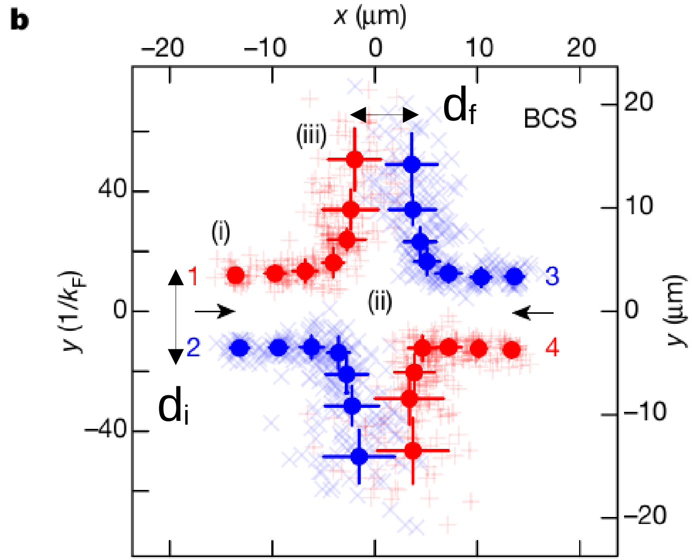
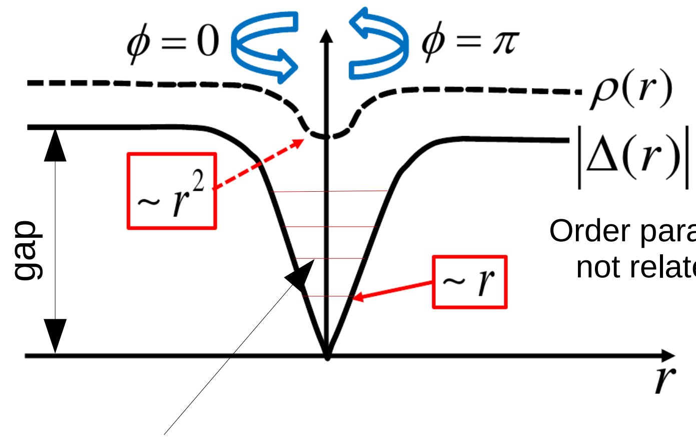




Experiment

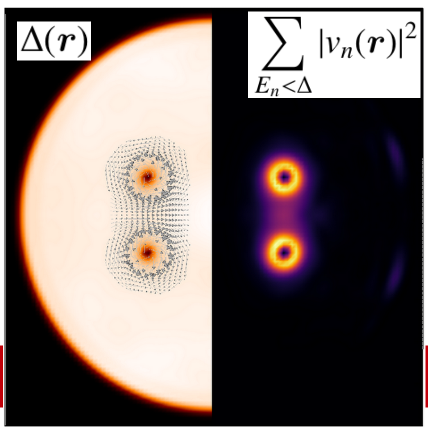
Dissipation during the collision is reflected in $d_f/d_i < 1$.

Vortex solution: Fermi gas → BdG



Order parameter:
not related directly to density

Occupation of Andreev states give rise to significant particle density inside the core.



Do the internal structure of vortices contribute to the dissiption?

Prediction [M. Silaev, PRL. 108 (2012)]:

→ Andreev quasiparticles can be excited (effective increase of the vortex core temperature), ...

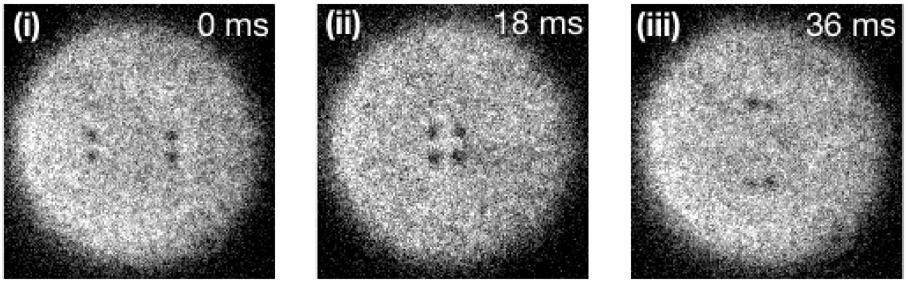
→ and eventually converted into delocalized states

→ the impact of this process gets stronger as we move towards BCS regime

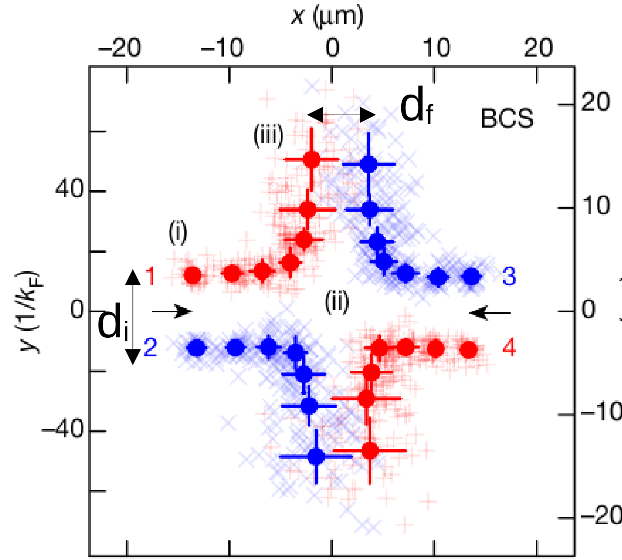
Example #2: Vortex collisions

Inspired by LENS ^{6}Li experiment (G. Roati's group):
[1] W. J. Kwon, et.al., Nature 600, 64-69 (2021)

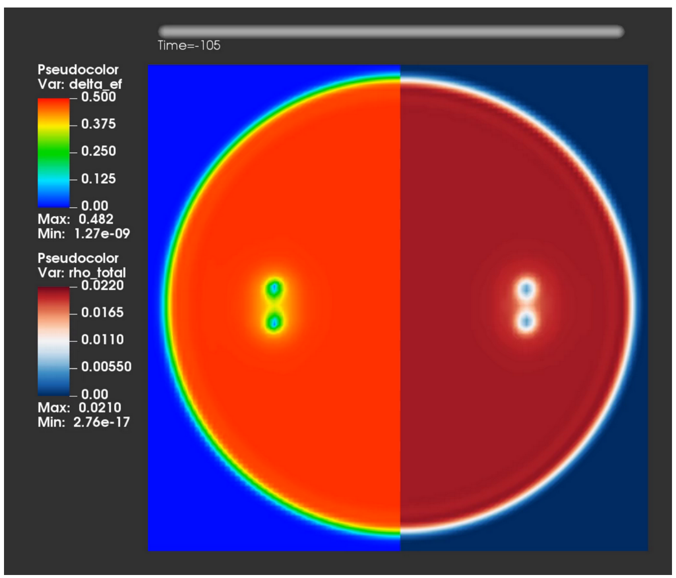
Figs from [1]

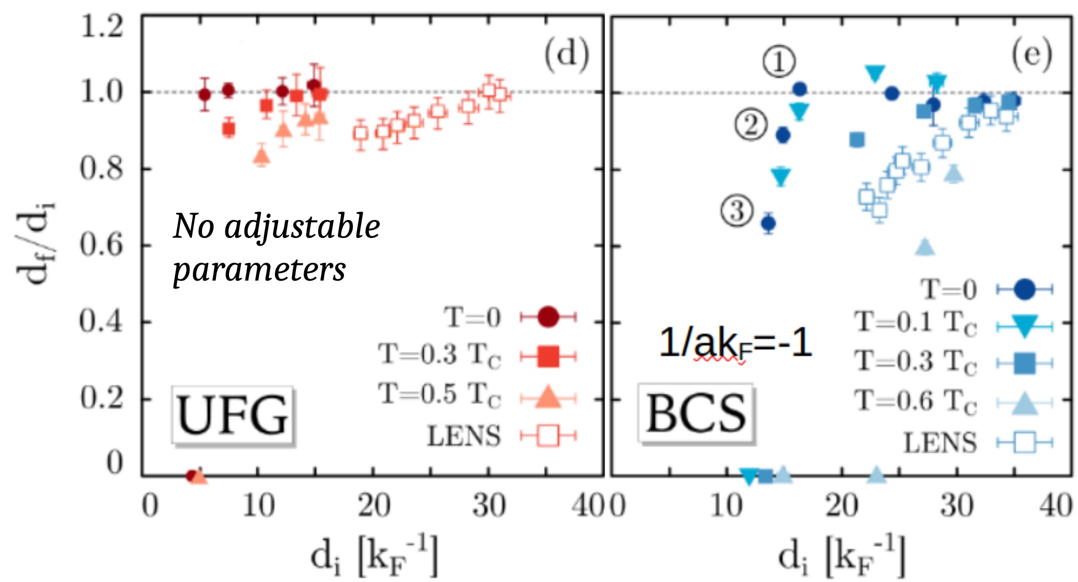


$T_{exp}=(0.3\text{-}0.4)T_c$



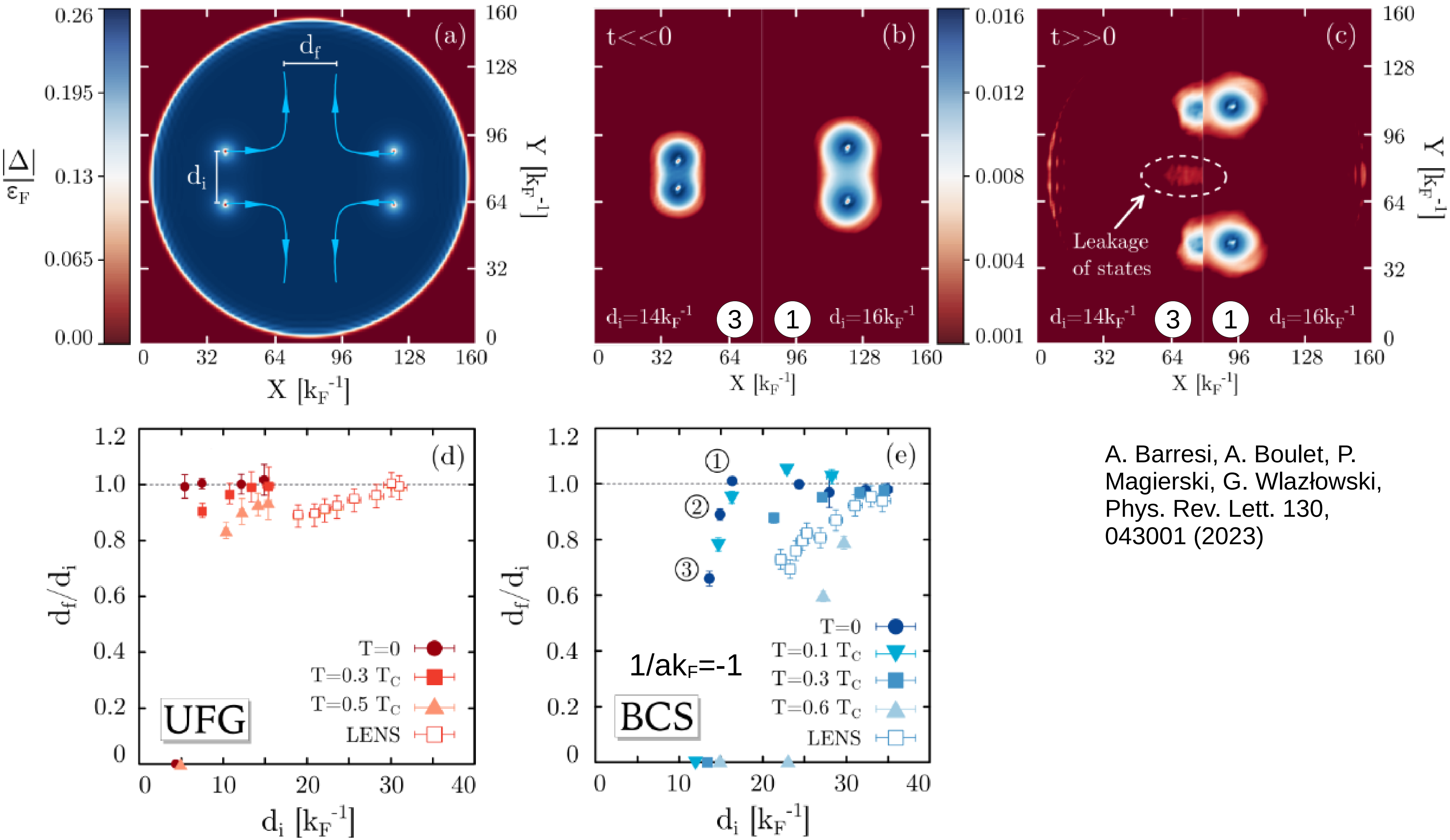
Experiment





Dynamical problem + strong interactions + superfludity + finite temperature

A. Barresi, A. Boulet, P. Magierski, G. Wlazłowski,
Phys. Rev. Lett. 130, 043001 (2023)



A. Barresi, A. Boulet, P. Magierski, G. Wlazłowski, Phys. Rev. Lett. 130, 043001 (2023)

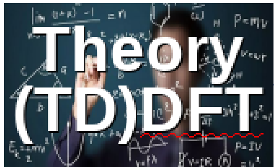
→ *the dissipation due to Andreev states is detected in BCS regime*

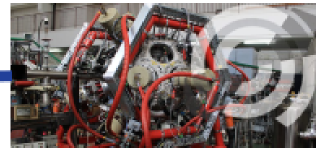
(it can be interpreted as effective increase of the vortex core temperature)

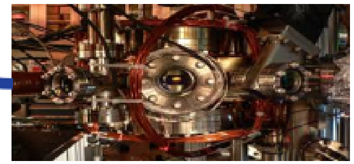
→ *the effect is to weak to explain the experimental measurements*

→ *significant sensitivity of the results to the temperature*

Towards effective model of neutron star...





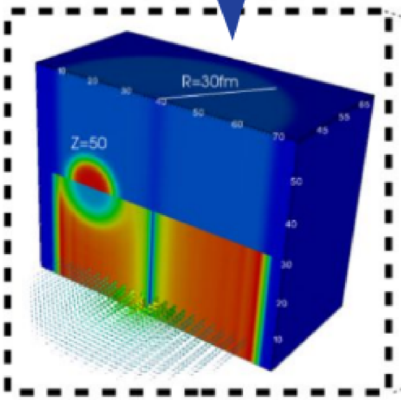


Low energy nuclear physics

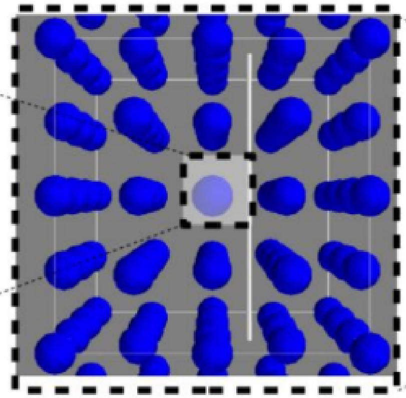
ab-initio

Ultra-cold atomic gases

Comparison with observations



feed



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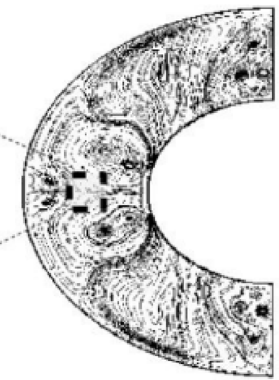
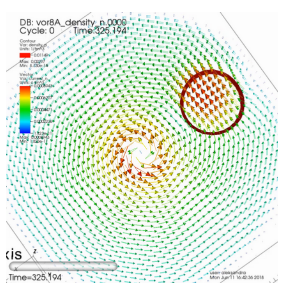
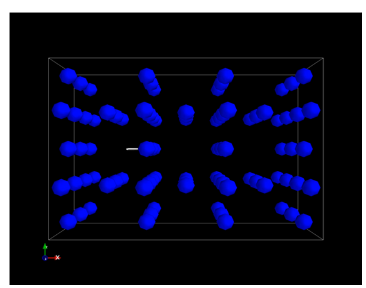


Method: TDDFT
DoF: neutrons and protons.
Scale: ~10^{-13}m

Method: Vortex Filament Model
DoF: impurities and vortices
Scale: ~10^{-9}m

Method: Hydrodynamics
DoF: fluid elements
Scale: ~size of star





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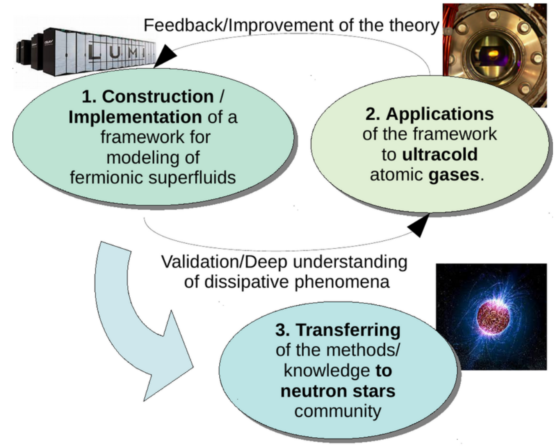
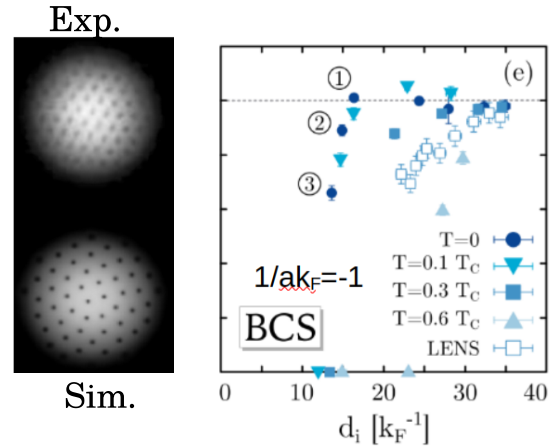
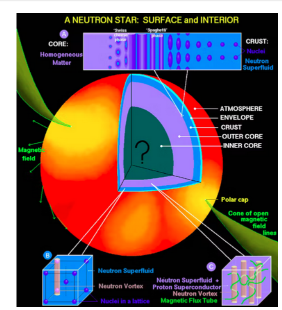
SUMMARY

- Ultracold Fermi gases and neutron matter share a lot of similarities. UFG regime can be used as a benchmark platform for testing the predictive power of many-body techniques, which are subsequently used for neutron star studies.
- (TD)DFT is general purpose framework: it overcomes limitations of mean-field approach, while keeping numerical cost at the same level as (TD)HFB calculations.
- For problems that have been (so far) contrasted with experimental measurements: *Predictions by functionals for ultracold Fermi gases (SLDA), created within similar methodology as for nuclear systems, are at least at the qualitative level in agreement with the measurements, ... in many cases, good quantitative agreement is obtained.*
- (TD)DFT and its implementations reached the level of maturity that allows for providing predictions for large and complex systems: neutron star's crust structure and its dynamics, transport coefficients, …







Collaborators: P. Magierski, D. Pęcak, M. Tylutki, A. Barresi, A. Boulet (WUT); N. Chamel (U. Bruxelles); M. Forbes, S. Sarkar,(WSU); A. Bulgac (UW); K. Xhani (LENS); N. Proukakis (Newcastle U.); A. Marek (MPCDF), M. Szpindler (Cyfronet).

physics wut

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