The large-charge EFT and neutron matter

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U.S. DEPARTMENT OF ENERGY Office of Science

ECT* Trento, April 2024

Swiss National Science Foundation

I will discuss work done in collaboration with

<u>Domenico Orlando</u> (INFN, Torino and Bern) and <u>Susanne Reffert</u> (Bern)

arXiv:2403.18898

In progress

(Chowdhuri, Mishra,Son,arXiv:2309.15177)

Outline

- Motivation
- Fundamental theory: fermions near unitarity
- Superfluid EFT and large charge expansion
- Unnuclear physics at large charge
- Summary

Motivation

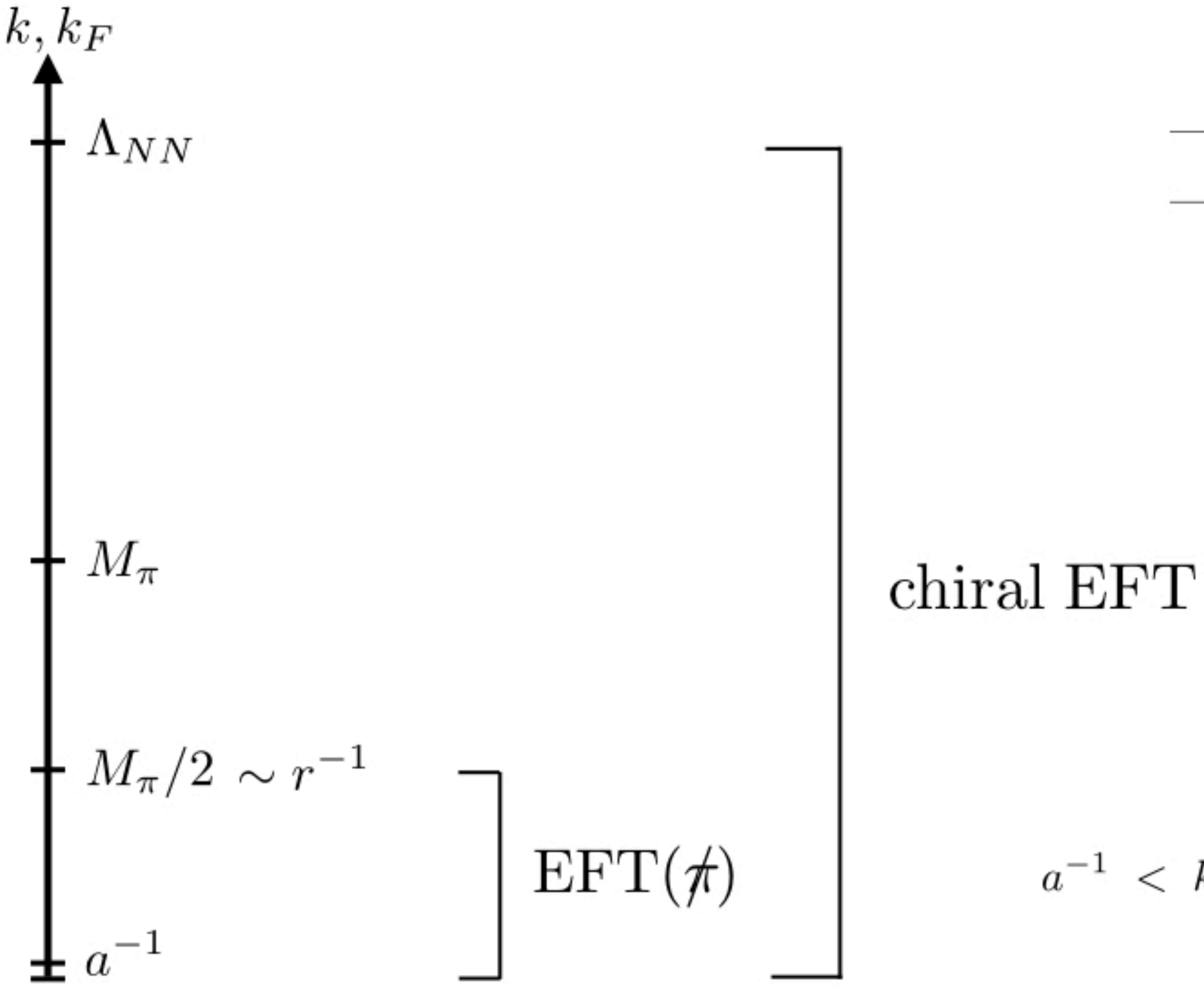
Can systems of many neutrons be described by a deformation of a non-relativistic conformal field theory?

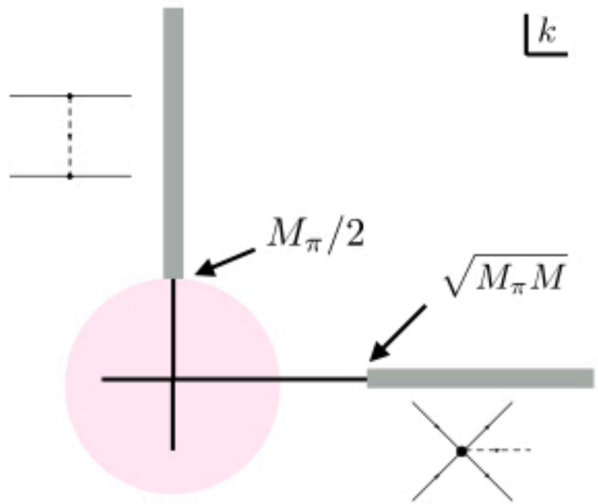
s-wave singlet neutron-proton scattering $\mathrm{EFT}(\not\pi)$

$$k\cot\delta(k) \;=\; -\frac{1}{a} \;+\; \frac{1}{2}rk^2 \;+\; v_2k^4 \;+\; v_3k^6 \;+\; \ldots$$

$a \sim -24\,\mathrm{fm}$ 🙂 $r \sim 3\ \mathrm{fm}$ 😭 $v_2 \sim 0.4\,\mathrm{fm}^3$ 🙂 $v_3 \sim 0.7\,\mathrm{fm}^5$ 🙂

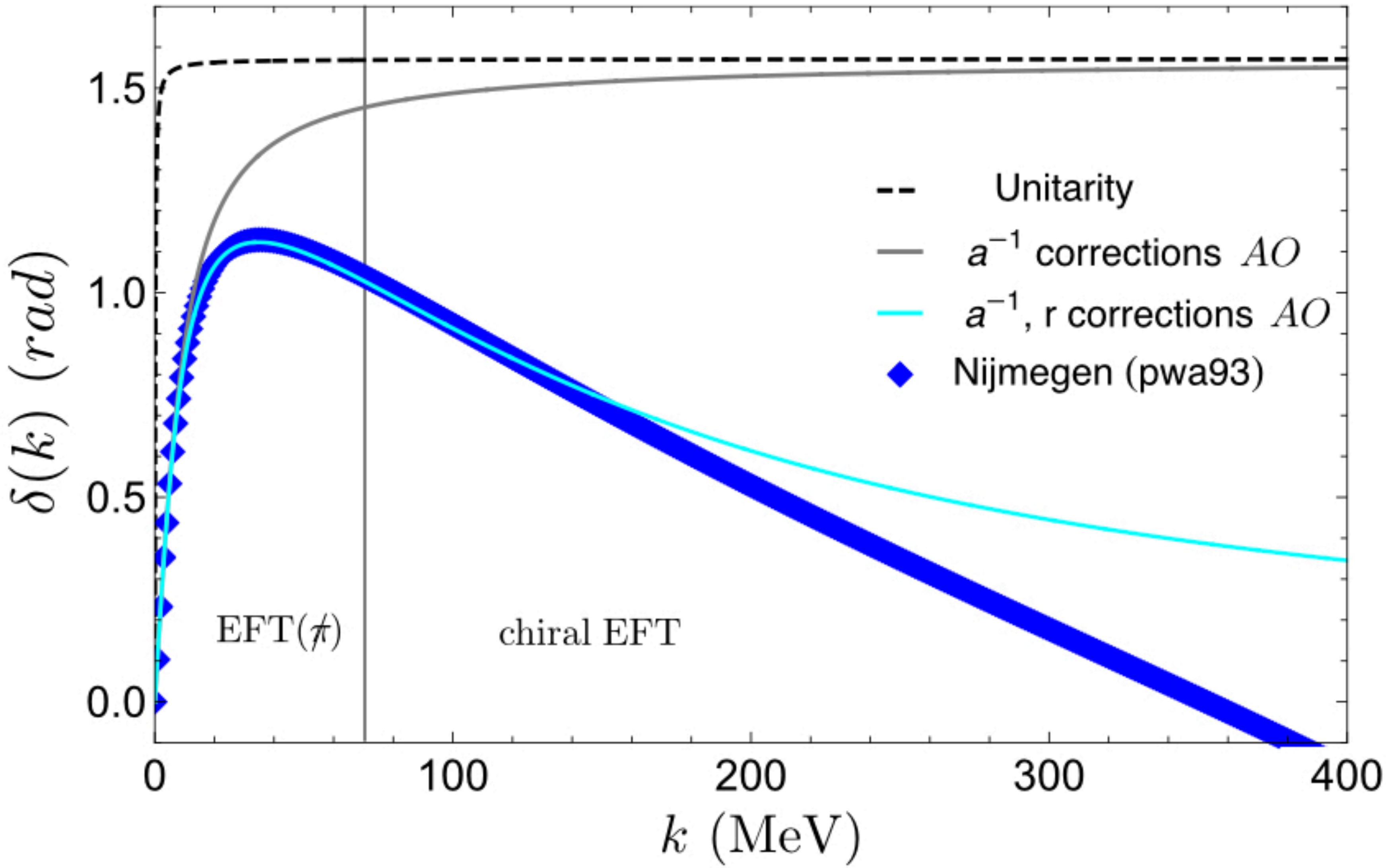
Fundamental theory: characteristic scales



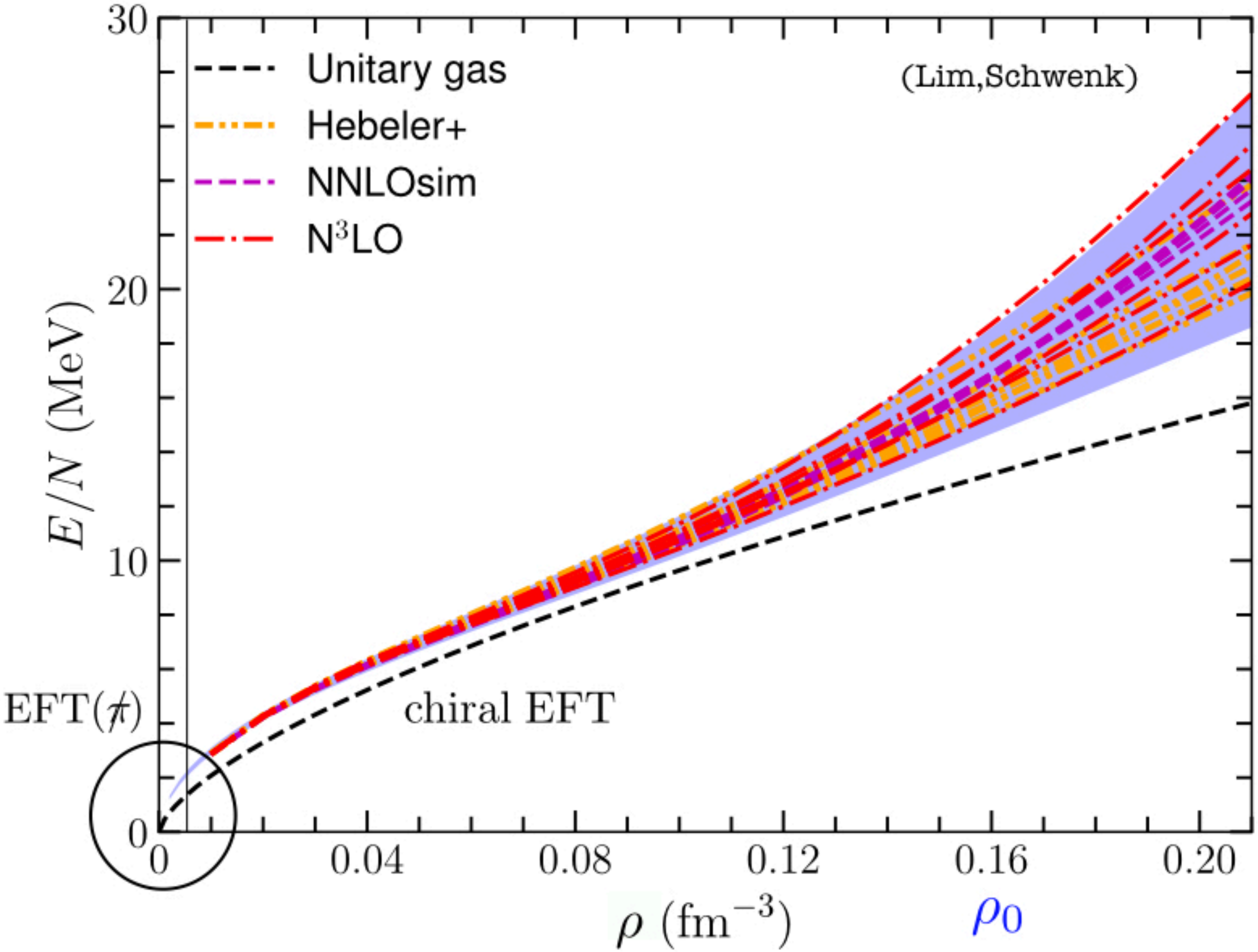


$a^{-1} < k, k_F < r^{-1}$

Neutron-proton singlet s-wave phase shift

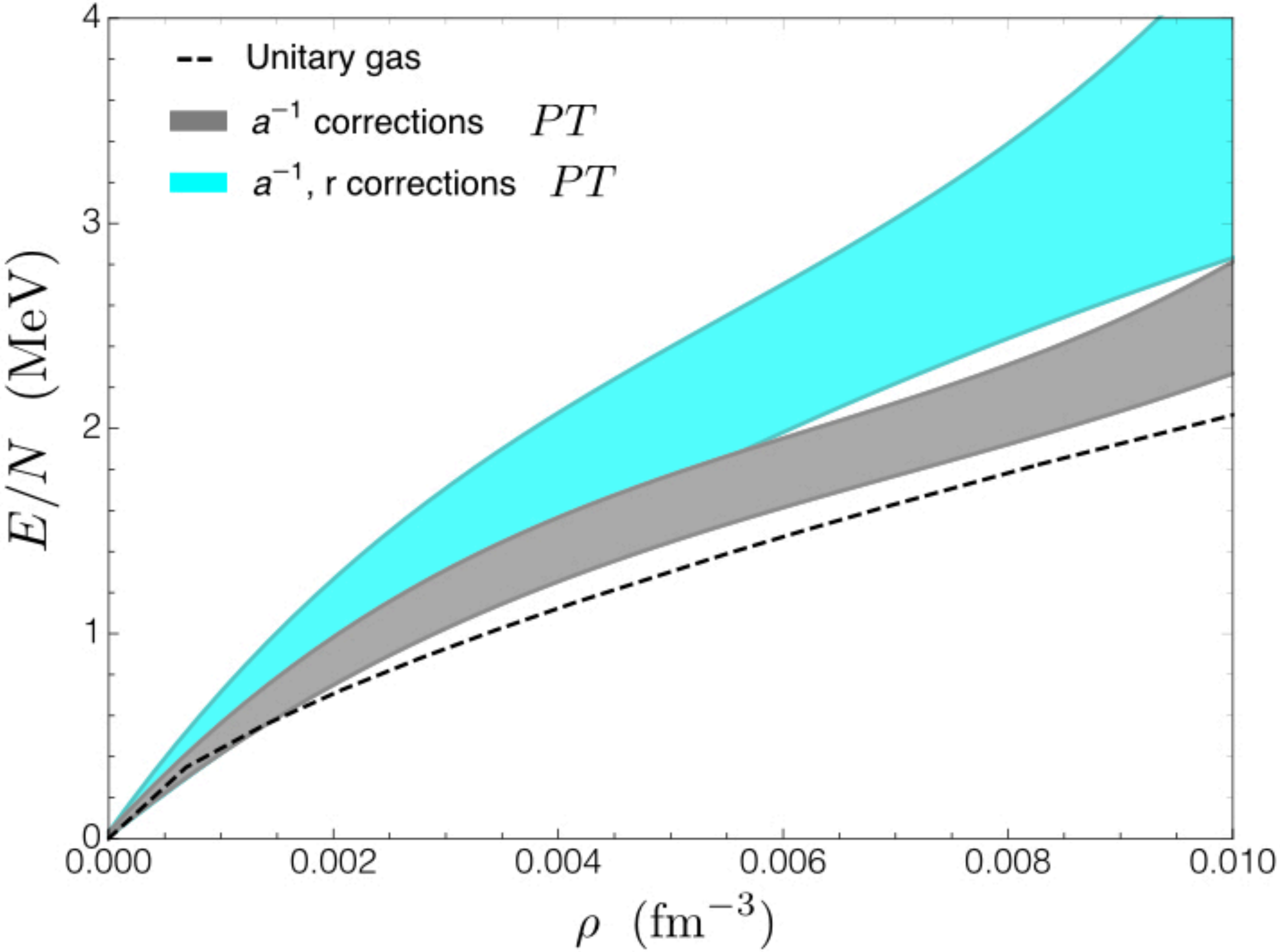


Energy of neutron matter



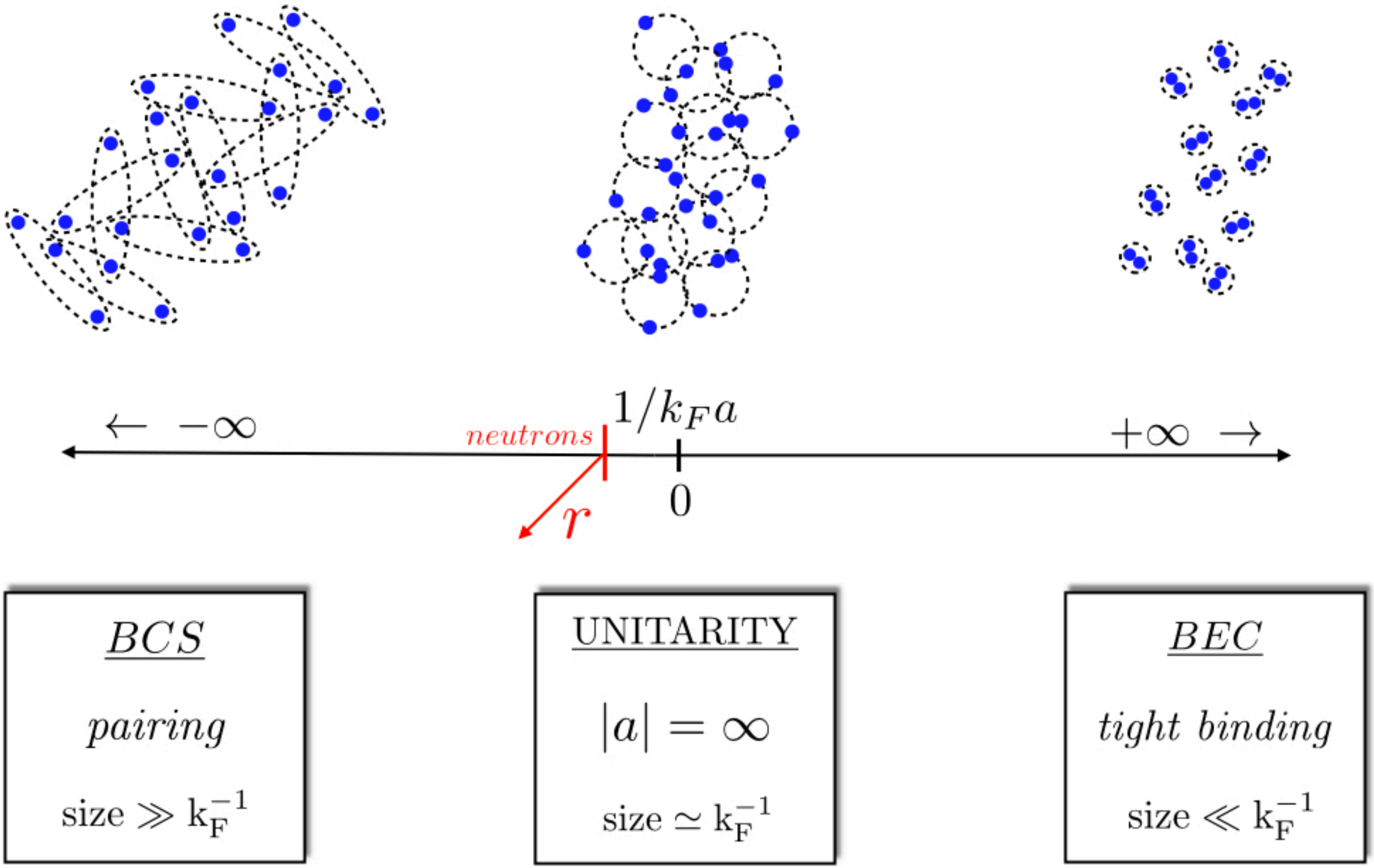
$$E/N = \frac{3}{5}\frac{k_{\rm F}^2}{2M}\left(\xi - \frac{\zeta}{k_{\rm F}a} - \frac{\zeta_2}{k_{\rm F}^2a^2} + \ldots + \eta\, k_{\rm F} r + \ldots\right)$$

(J.Carlson et al)



How does one obtain this in EFT?

Many fermions near unitarity



Fundamental theory: EFT of contact operators

(van Kolck)
(Kaplan,Savage,Wise)

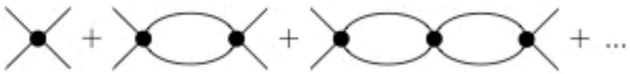
$$\mathcal{L} = \psi_\sigma^\dagger \left[i\partial_t + \frac{\overrightarrow{\nabla}^2}{2M} \right] \psi_\sigma - \frac{1}{2} C_0 (\psi_\sigma^\dagger \psi_\sigma)^2 \; + \; \ldots \qquad \sigma = \uparrow, \downarrow$$

EFT($\not\pi$)

Fermion-fermion scattering

$$T(k) \;=\; -\frac{4\pi}{M} \Big[k \cot \delta(k) - ik \Big]^{-1} \qquad k \cot \delta(k) \;=\; -\frac{1}{a} \;+\; \frac{1}{2} r k^2 \;+\; O(k^4)$$

Match with EFT



$$T(k) = C_0 + C_0^2 \, \mathbb{I}(k) + C_0^3 \, \mathbb{I}(k)^2 + \ldots \;=\; \left(\frac{1}{C_0} - \mathbb{I}(k) \right)^{-1}$$

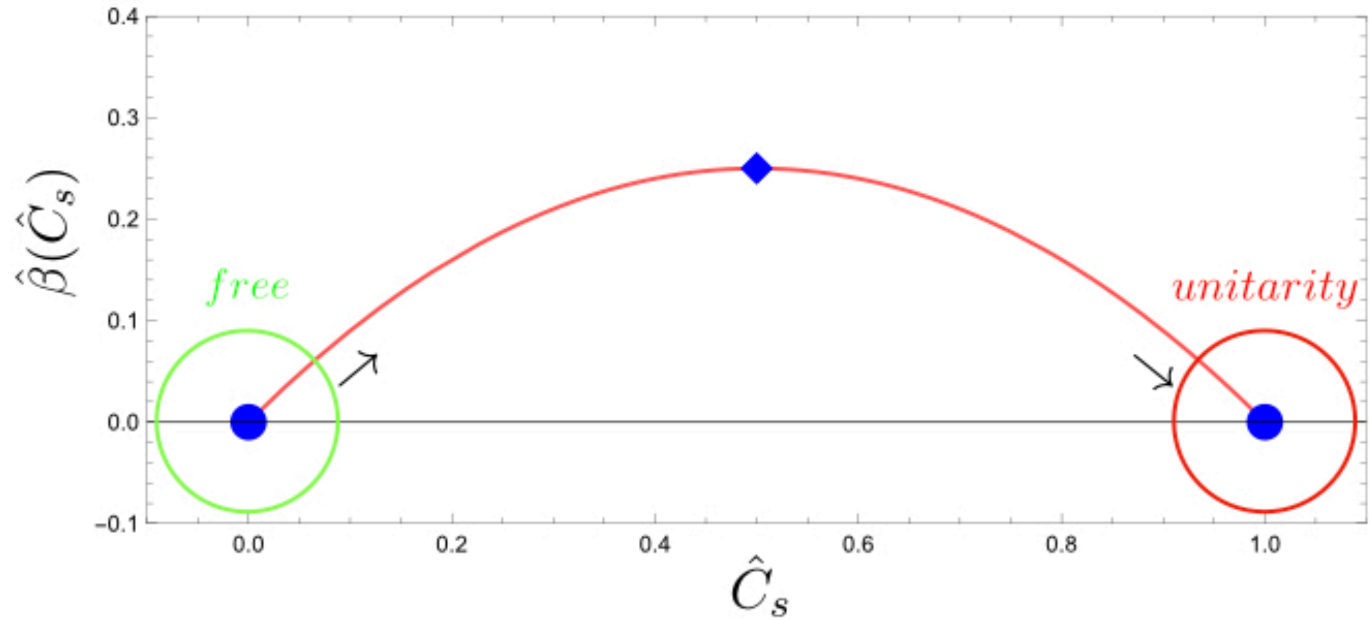
$$\mathbb{I}(k) \equiv \left(\frac{\nu}{2}\right)^{3-d} M \int \frac{d^d \mathbf{q}}{(2\pi)^d} \frac{1}{k^2 - q^2 + i\epsilon} \xrightarrow{\text{PDS}} -\frac{M}{4\pi} (\nu + ik)$$

RG flow of EFT contact operator coupling

$$C_0(\nu) = \frac{4\pi}{M}\frac{1}{1/a - \nu}$$

The beta function exhibits the two RG fixed points

$$\hat{\beta}(\hat{C}_0) = \mu\frac{d}{d\nu}\hat{C}_0(\nu) = -\hat{C}_0(\nu)\left(\hat{C}_0(\nu) - 1\right) \qquad \hat{C}_0 \equiv C_0/C_*$$



Fundamental theory: deformed CFT

Introducing auxiliary field s. At unitarity CFT defined by:

$$\mathcal{L}_{CFT} = \psi_\sigma^\dagger \left[i\partial_t + \frac{\overrightarrow{\nabla}^2}{2M} \right] \psi_\sigma + \frac{1}{C_\star} s^\dagger s + \psi_\downarrow^\dagger \psi_\uparrow^\dagger s + s^\dagger \psi_\uparrow \psi_\downarrow$$

Deformed CFT is defined by:

$$\mathcal{L} = \mathcal{L}_{CFT} + \frac{M}{4\pi a} s^\dagger s - \frac{M^2 r}{16\pi} s^\dagger \left(i\overleftrightarrow{\partial_t} + \frac{\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2}{4M} \right) s$$

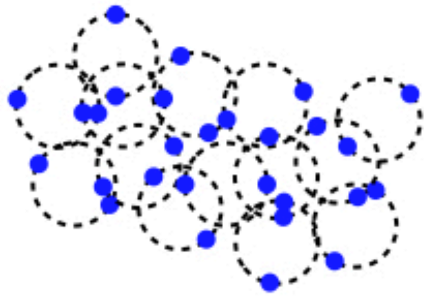
$[s] = 2$:

relevant deformation

irrelevant deformation

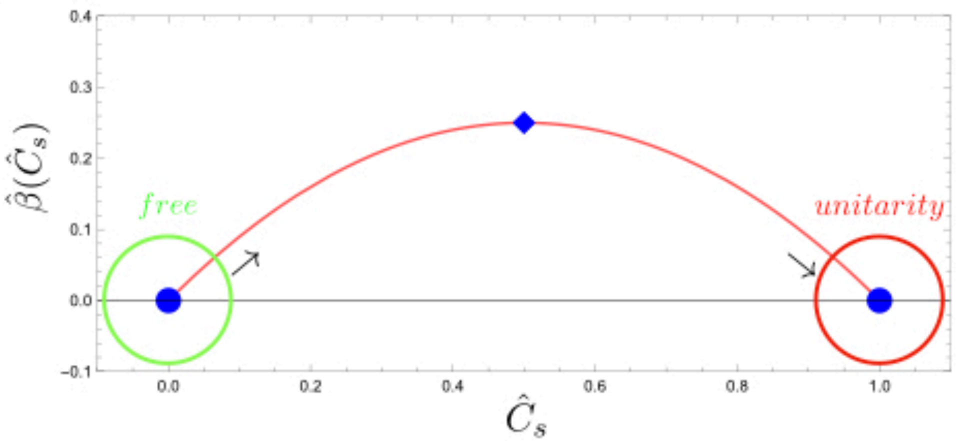
Fermi gas properties

$$\rho = \frac{k_F^3}{3\pi^2}$$



$$\mu = \frac{k_{\rm F}^2}{2M}$$

$$\mu = \xi \frac{k_{\rm F}^2}{2M}$$



$$E/N = \frac{3}{5}\frac{k_F^2}{2M}\left(1 + \frac{10}{9\pi}k_F a \ldots\right)$$

PT in fundamental theory (Lenz)

$$E/N = \frac{3}{5}\frac{k_{\rm F}^2}{2M}\left(\xi - \frac{\zeta}{k_{\rm F}a} + \ldots\right)$$

EFT?

Superfluid EFT: homogeneous ground state

(Son,Wingate)

(Greiter et al)

BCS instability + Fermi surface

$$U(1) \to \emptyset \quad : \quad \langle\psi\psi\rangle = |\langle\psi\psi\rangle|\, e^{-2i\theta}$$

Galilean invariant building block:

$$X = D_t\theta - \frac{(\partial_i\theta)^2}{2M} \quad , \quad D_t\theta = \dot{\theta} - A_0 \quad , \quad A_0 = M\omega^2 r^2/2$$

At unitarity have NR CFT:

$$\mathcal{L}_{LO} = c_0\, M^{3/2} X^{5/2} + \ldots \qquad c_0 = \frac{2^{5/2}}{15\pi^2\xi^{3/2}}$$

Grand canonical: homogeneous ground state

$$EOM: \quad \partial_t \rho - \frac{1}{M}\partial_i \left(\partial_i \theta \rho\right) = 0$$

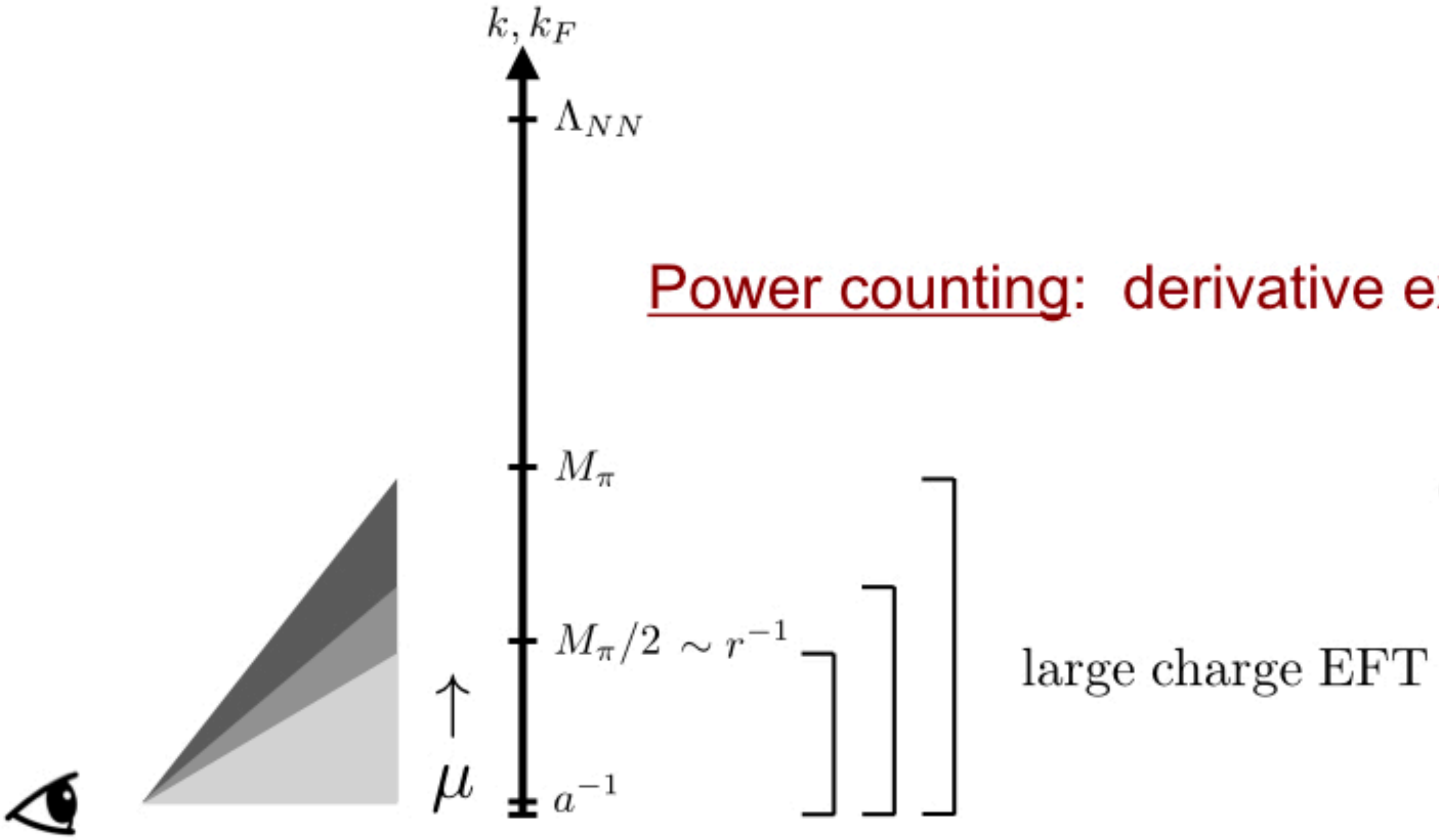
$$\Rightarrow \quad \theta(x) = \mu\, t - \phi(x)$$

$\mu\, t$ → symmetry breaking scale

$\phi(x)$ → phonon

$$\rho = \frac{5}{2} c_0\, M^{3/2} X^{3/2}$$

Power counting: derivative expansion or large μ



(Favrod,Orlando,Reffert)

(Kravec,Pal)

Recall s-wave scattering with only scattering length

$$S = e^{i2\delta(k)} = 1 - i\frac{kM}{2\pi}T(k) = \frac{1-iak}{1+iak}$$

$$a \leftrightarrow k \;\; invariance$$

Large k expansion is expansion about unitarity

$$T(k) = T_{CFT}(k)\Big[1 + i(ak)^{-1} + O((ak)^{-2})\Big]$$

But large k expansion fails for range corrections..

$$T(k) = T_{CFT}(k)\Big[1 + i(ak)^{-1} + O((ak)^{-2}) - \frac{i}{2}(rk)\Big]$$

Superfluid EFT: fixed charge Q

Now consider subsectors of fixed charge (canonical)

$$G(x_1, x_2, \ldots, x_N) \;=\; -i\langle 0|T\left(\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\ldots\mathcal{O}_N(x_N)\right)|0\rangle$$

n-point functions constrained by Schrödinger symmetry

$$G(x_1, x_2) = \delta_{Q_1,Q_2}\delta_{\Delta_1,\Delta_2}\,\theta(\tau_{12})\,\tau_{12}^{-\Delta_1}\,\exp\left(-\frac{Q_1 M \mathbf{x}_{12}^2}{2\tau_{12}}\right)\Psi_2 + \ldots$$

$$G(x_1, x_2, x_3) = \delta_{Q_1+Q_2,-Q_3}\,\theta(\tau_{13})\,\theta(\tau_{23})\,\tau_{13}^{-\Delta_{13,2}/2}\,\tau_{23}^{-\Delta_{23,1}/2}\,\tau_{12}^{-\Delta_{12,3}/2} \times \exp\left(-\frac{Q_1 M \mathbf{x}_{13}^2}{2\tau_{13}} - \frac{Q_2 M \mathbf{x}_{23}^2}{2\tau_{23}}\right)\Psi_3(v_{123}) + \ldots$$

$\tau_{ij} \equiv \tau_i - \tau_j,\ \mathbf{x}_{ij} \equiv \mathbf{x}_i - \mathbf{x}_j$ $\qquad \Delta_{ij,k} \equiv \Delta_i - \Delta_j - \Delta_k$ $\qquad v_{123} \equiv \dfrac{1}{2}\dfrac{(\mathbf{x}_{13}\tau_{23} - \mathbf{x}_{23}\tau_{13})^2}{\tau_{12}\tau_{13}\tau_{23}}$

Schrödinger symmetry = Galilean + conformal

$$\begin{aligned} H &: (x_i, t) \rightarrow (x_i, t+a) \\ P_i &: (x_i, t) \rightarrow (x_i + a_i, t) \\ K_i &: (x_i, t) \rightarrow (x_i + t v_i, t) \\ M_{ij} &: (x_i, t) \rightarrow (R_{ij} x_j, t) \end{aligned}$$

$$\begin{aligned} D &: (x_i, t) \rightarrow (\lambda x_i, \lambda^2 t) \\ C &: (x_i, t) \rightarrow \left(\frac{x_i}{1+at}, \frac{t}{1+at}\right) \end{aligned}$$

State-operator correspondence

The dimension of a primary operator = energy of state in harmonic potential

$$H_\omega \equiv H + \omega^2 C \quad : \quad (y_i, \tilde{t}) \rightarrow (y_i, \tilde{t} + a)$$

Two equivalent "frames"

$$\begin{aligned} \omega t &= \tan \omega \tilde{t} \\ x_i &= y_i \sec \omega \tilde{t} \end{aligned} \quad \longleftrightarrow \quad \begin{aligned} \omega \tilde{t} &= \arctan \omega t \\ y_i &= \frac{x_i}{\sqrt{1+\omega^2 t^2}} \end{aligned}$$

Exact large-charge solution can be found

Charge Q operator

$$\mathcal{O}_{\Delta,Q} = \mathcal{N}\; X^{\Delta/2} \exp\left(iQ\theta\right)$$

normalization

primary operator

Now need Euclidean path integral formulation

$$G_Q(x_1,x_2) \;=\; \int \mathcal{D}\theta\, \mathcal{O}_{\Delta,Q}(x_2)\mathcal{O}_{\Delta,-Q}(x_1)\, e^{-\int \mathrm{d}^4x \mathcal{L}_I} \;=\; \mathcal{N}^2\!\int \mathcal{D}\theta\, e^{-\int \mathrm{d}^4x \mathcal{L}}$$

$$\mathcal{L} = \mathcal{L}_I - \frac{\Delta}{2}\log X\left[\delta^4(x-x_2)+\delta^4(x-x_1)\right] - iQ\,\theta\left[\delta^4(x-x_2)-\delta^4(x-x_1)\right]$$

EOM: continuity equation with source

$$\partial_\tau \rho + \frac{1}{M}\partial_i \left(i\partial_i \theta \rho \right) = Q \left[\delta^4(x - x_2) - \delta^4(x - x_1) \right]$$

Saddle point solution: master field

(Orlando,Reffert,SB)
(Son,Stephanov,Yee)

$$\theta_s(\tau, \mathbf{x}) = \frac{i}{2}\gamma \log\left(\frac{\tau_1 - \tau}{\tau - \tau_2}\right) - \frac{i}{4}M\left[\frac{(\mathbf{x} - \mathbf{x}_2)^2}{(\tau - \tau_2)} - \frac{(\mathbf{x} - \mathbf{x}_1)^2}{(\tau_1 - \tau)}\right]$$

Emergent time-dependent harmonic trap

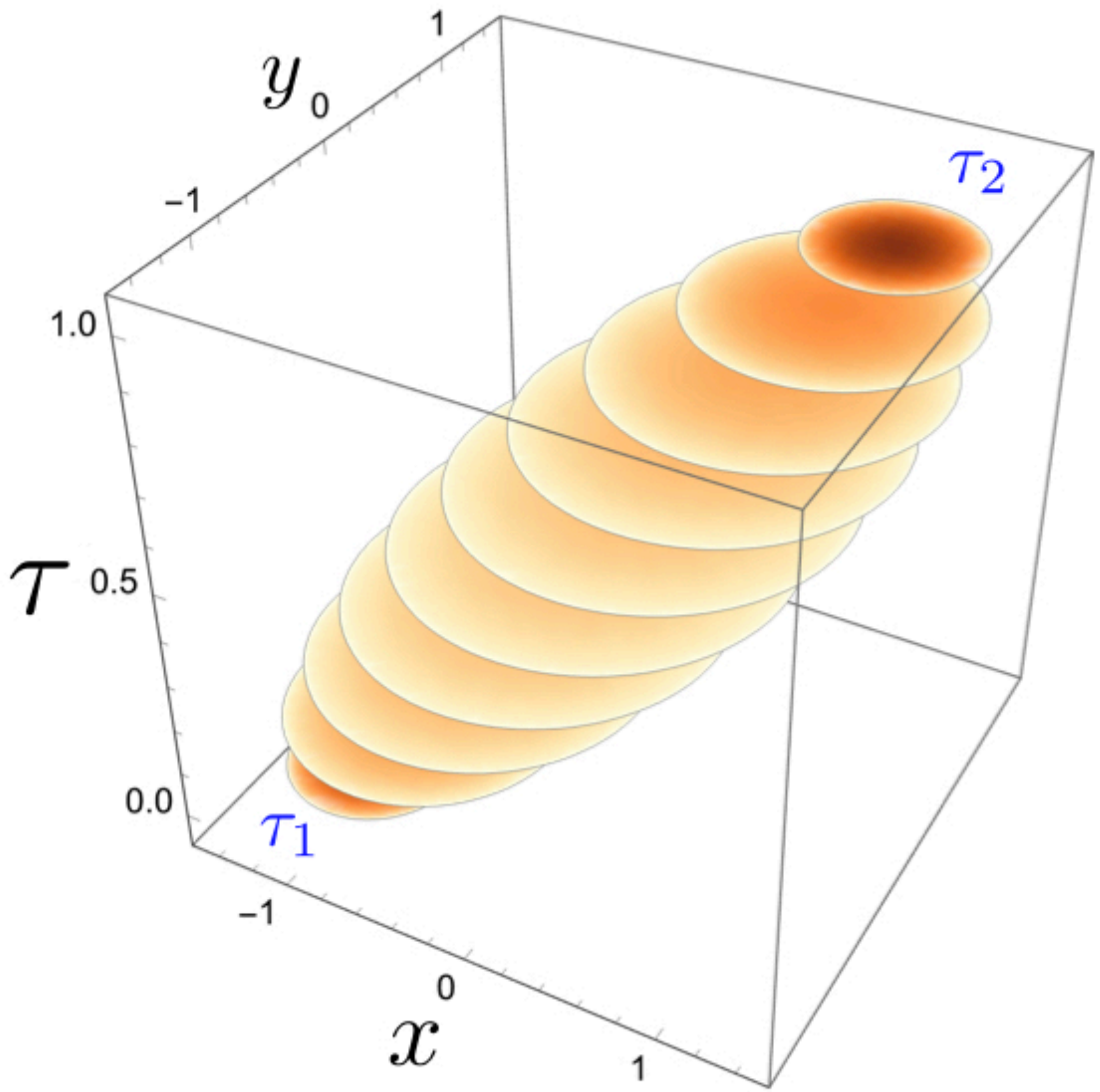
$$X = \bar{\mu}(\tau) - \frac{1}{2}M\bar{\omega}(\tau)^2 r^2 \qquad = 0 \quad @ \quad r = \bar{R}(\tau)$$

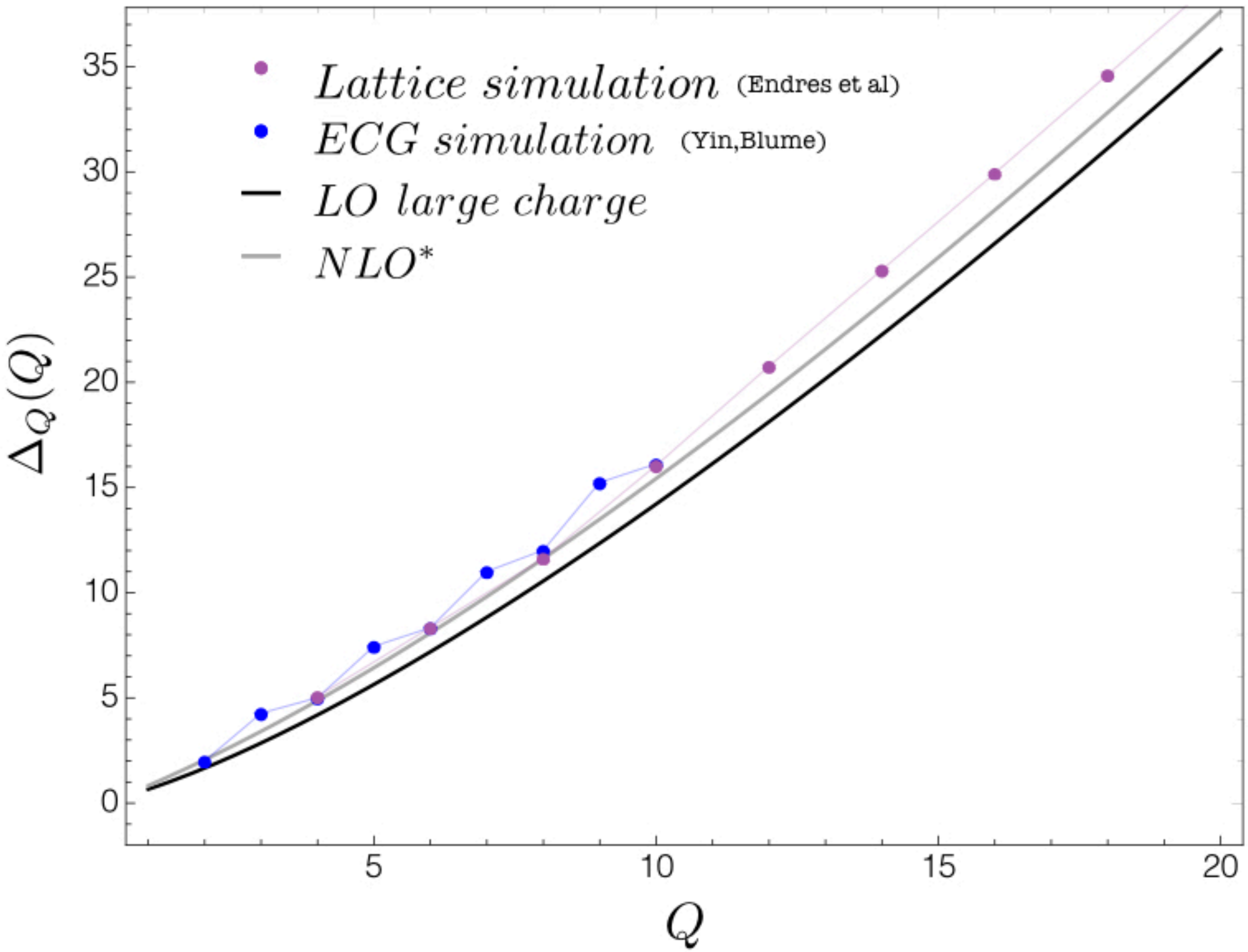
$$\bar{\mu}(\tau) = \frac{1}{2}\gamma \frac{(\tau_1 - \tau_2)}{(\tau - \tau_2)(\tau_1 - \tau)} \qquad \bar{\omega}(\tau) = \frac{1}{2}\frac{(\tau_1 - \tau_2)}{(\tau - \tau_2)(\tau_1 - \tau)}$$

Recovers state-operator correspondence

Master field (instanton) profile

$$\bar{R}(\tau) = \sqrt{\frac{2\bar{\mu}}{M}}\frac{1}{\bar{\omega}} = \frac{2}{\sqrt{M}}\left(\gamma\frac{(\tau-\tau_2)(\tau_1-\tau)}{(\tau_1-\tau_2)}\right)^{1/2}$$



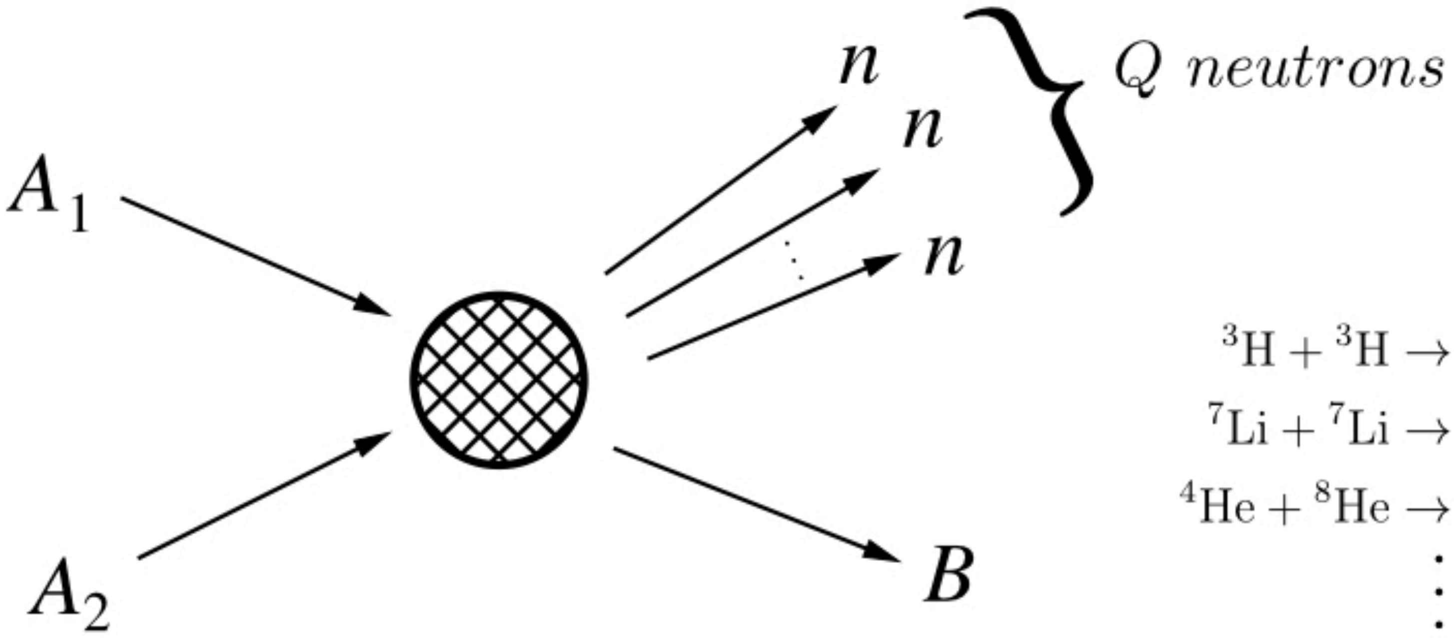


$$\Delta_Q(Q) \;=\; \frac{3^{4/3}}{4}\xi^{1/2}Q^{4/3} \;-\; 3^{2/3}\sqrt{2}\pi^2\xi\, c_{NLO}\, Q^{2/3} \;+\; O(Q^{5/9}) \;+\; \dots \;+\; \frac{1}{3\sqrt{3}}\log Q$$

(Son,Wingate) (Hellerman et al)

Unnuclear physics

(Hammer,Son)
(Chowdhuri, Mishra,Son)



$$^3\mathrm{H} + {}^3\mathrm{H} \to {}^4\mathrm{He} + 2n$$
$$^7\mathrm{Li} + {}^7\mathrm{Li} \to {}^{11}\mathrm{C} + 3n$$
$$^4\mathrm{He} + {}^8\mathrm{He} \to {}^8\mathrm{Be} + 4n$$
$$\vdots$$

$$\frac{d\sigma}{d\Omega} \sim (E_0 - E)^{\Delta_Q - 5/2}$$

$$\Delta_2 = 2$$
$$\Delta_3 \approx 4.27272$$
$$\Delta_4 \approx 5.0$$

Symmetry breaking: spurion analysis

$$\mathcal{L} = \mathcal{L}_{CFT} + \frac{M}{4\pi a} s^\dagger s - \frac{M^2 r}{16\pi} s^\dagger \left(i \overleftrightarrow{\partial_t} + \frac{\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2}{4M} \right) s$$

$$\mathcal{L}_{SB} = g_1 \, a^{-1} M X^2 + g_2 \, a^{-2} M^{1/2} X^{3/2} + h_1 \, M^2 X^3 r + h_2 \, M^{5/2} X^{7/2} r^2$$

Read off grand potential (M=1)

$$\Omega(\mu) = -c_0 \mu^{5/2} + g_1 a^{-1} \mu^2 + g_2 a^{-2} \mu^{3/2} + h_1 r \mu^3 + h_2 r^2 \mu^{7/2} + \ldots$$

Does large-charge EFT with range exist?

Schrödinger symmetry breaking at fixed charge

$$\Delta_2 = 2 - \sqrt{\frac{2}{\pi}}\frac{1}{a\sqrt{M\omega}} + \frac{1}{2\sqrt{2\pi}} r\sqrt{M\omega} + \ldots$$

$$\Delta_Q(Q) = -\frac{64\sqrt{2}\,3^{1/3}\pi\xi^2}{35}\left(\frac{g_0}{a\sqrt{\mu}} + \frac{2}{3}h_1 r\sqrt{\mu}\right) Q^{4/3} + \ldots$$

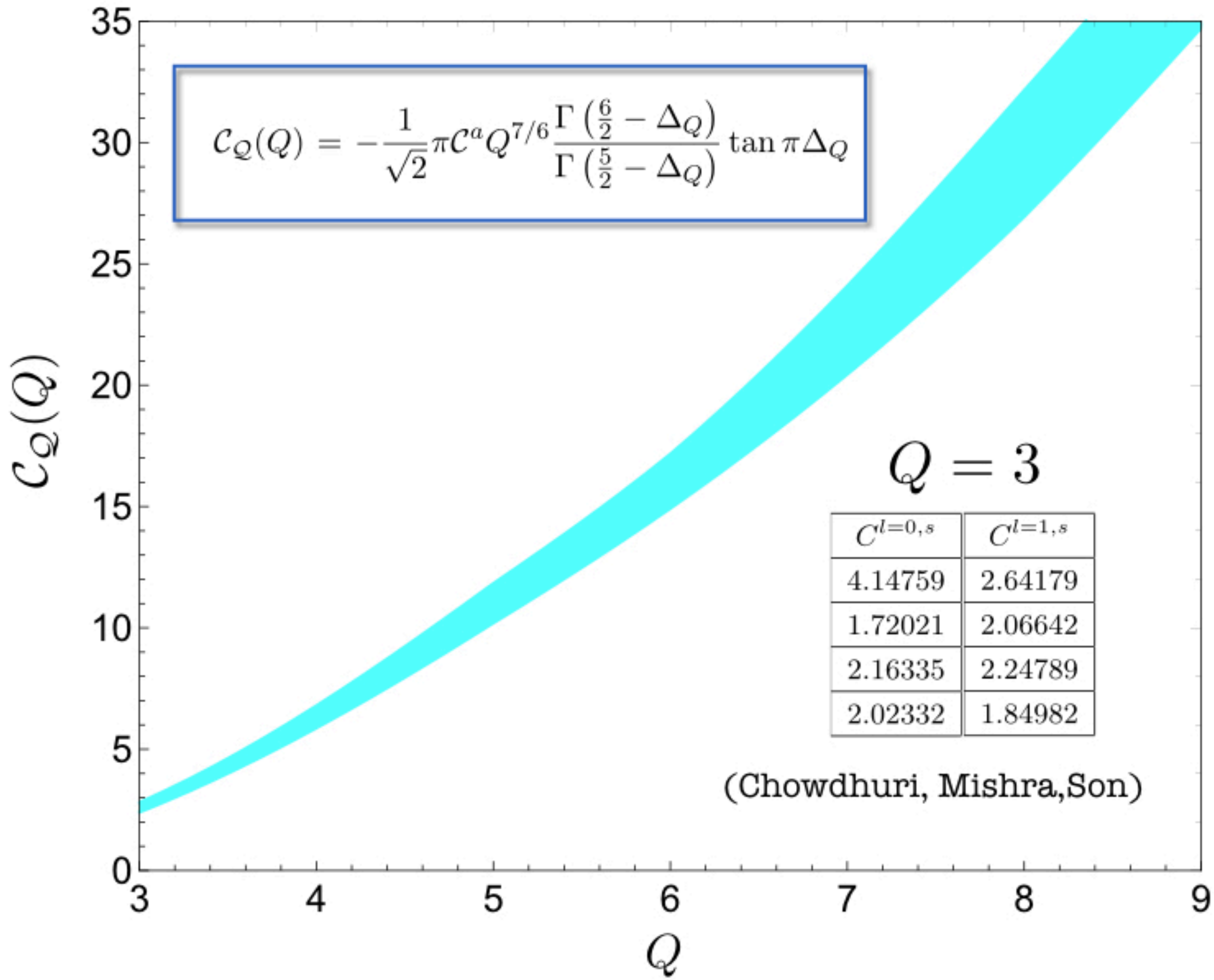
State-operator correspondence fails. Must compute matrix elements explicitly: conformal perturbation theory. Need few-body wavefunctions!

(Chowdhuri, Mishra,Son)

More efficient to use the large-charge EFT?

$$G(x_1, x_2) = G_{CFT}(x_1, x_2) e^{-S_{\rm SB}[\theta_S]}$$

$$S^a_{SB} = \frac{1}{\sqrt{2}}\pi\,\mathcal{C}^a M^{-1/2} Q^{7/6} a^{-1} \tau_{12}^{1/2} \qquad \mathcal{C}^a \equiv \frac{64\sqrt{2}\,3^{1/6}\pi\,\xi^{7/4} g_1}{35}$$



$$\mathcal{C}_Q(Q) = -\frac{1}{\sqrt{2}}\pi\mathcal{C}^a Q^{7/6}\frac{\Gamma\left(\frac{6}{2}-\Delta_Q\right)}{\Gamma\left(\frac{5}{2}-\Delta_Q\right)}\tan\pi\Delta_Q$$

$Q = 3$

$C^{l=0,s}$	$C^{l=1,s}$
4.14759	2.64179
1.72021	2.06642
2.16335	2.24789
2.02332	1.84982

(Chowdhuri, Mishra,Son)

$$\text{Im}\, G_Q(E,\mathbf{0}) = C_0\, E^{\Delta_Q - 5/2}\left[1 + \frac{\mathcal{C}_Q}{a\sqrt{ME}}\right]$$

Effective range corrections:

$$\mathrm{Im}\, G_Q(E,\mathbf{0}) \sim E^{\Delta_Q - 5/2}\left[1 \,+\, \mathcal{C}^r_Q\, r\sqrt{ME}\right]$$

$$S^r_{SB} \;=\; \int d^4x\, \mathcal{H}^r_{SB} = \mathcal{C}^r M^{1/2} Q^{3/2} r \frac{1}{2\sqrt{2}} \int_{\tau_2+\varepsilon}^{\tau_1-\varepsilon} d\tau \left(\frac{(\tau_1-\tau_2)}{(\tau-\tau_2)(\tau_1-\tau)}\right)^{3/2}$$

$$= \mathcal{C}^r M^{1/2} Q^{3/2} r \frac{2}{\sqrt{2}} \frac{1}{\varepsilon}$$

$$\mathcal{C}^r \equiv \frac{128}{35}\sqrt{\frac{2}{3}}\pi\,\xi^{9/4} h_1$$

$$\mathcal{C}^r_{\mathcal{Q}} = 0$$

(also occurs with Q=3!)

(Chowdhuri, Mishra,Son)

- Leading range corrections vanish: divergent droplet edge effect, no real part. Divergence removed by operator normalization.
- Next order in r also diverges but with real part. But there is also problem that r must be unnaturally small. Perhaps possible to resum r corrections to all orders.

Summary

- The large-charge EFT provides a systematic (far infrared) description of many fermions near unitarity. Applicability to neutron matter is complicated by large effective range effects in the neutron-neutron interaction.
- The state-operator correspondence is a powerful tool for computing the dimensions of operators in an interacting NR CFT. Including symmetry breaking is non-trivial.
- Large-charge master field is known; useful for computing Schrödinger-symmetry breaking corrections to correlation functions without detailed knowledge of few-body wavefunctions.
- Relevant deformations of non-relativistic CFT are under control. However, irrelevant deformations pose something of a puzzle. Resummation of range (and scattering length) effects may be possible.

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