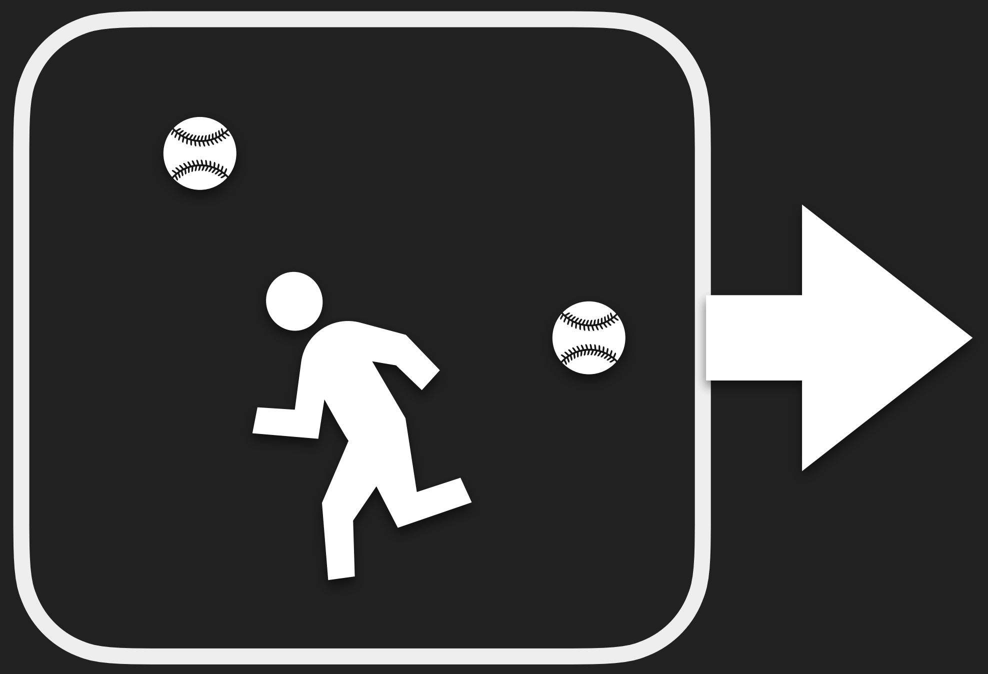
Gravity from distant objects:

Principle of equivalence

Uniform field: everything falls together - no measurement



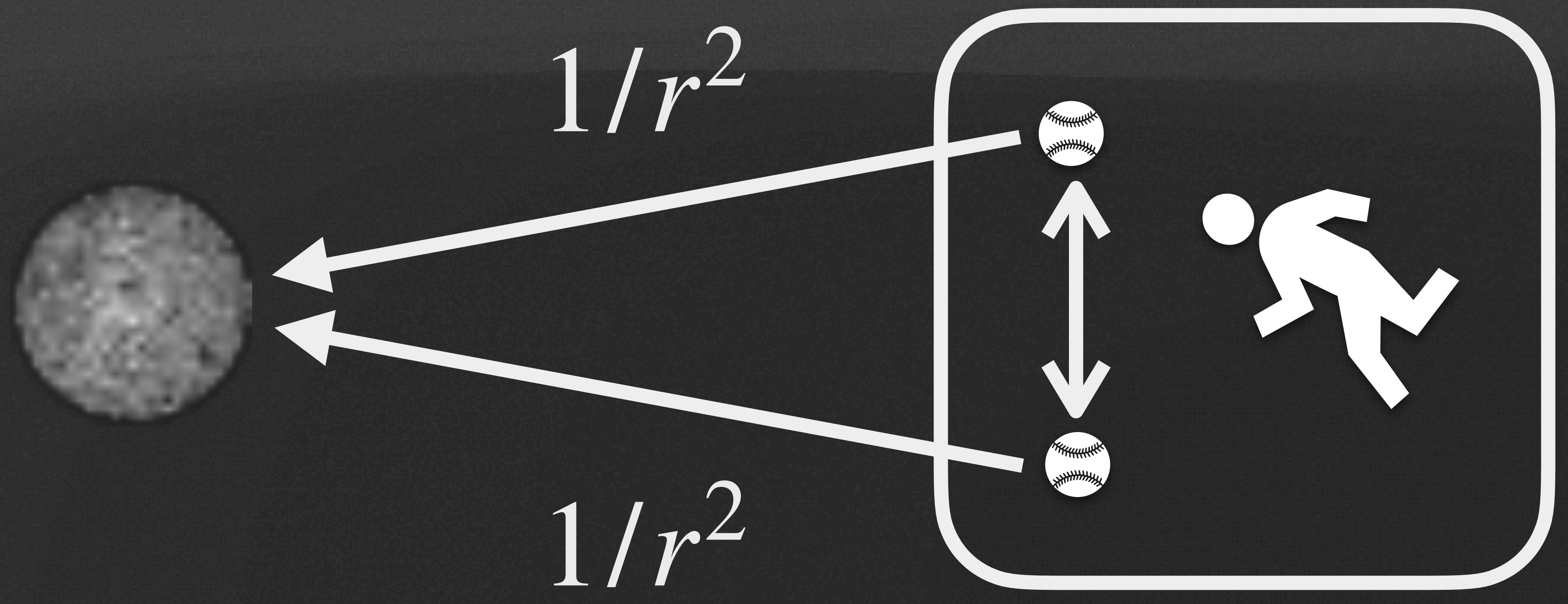
(Also: at small scales, “locally flat”, in gravitational fields)

Gravitational Waves Demystified, T. Creighton

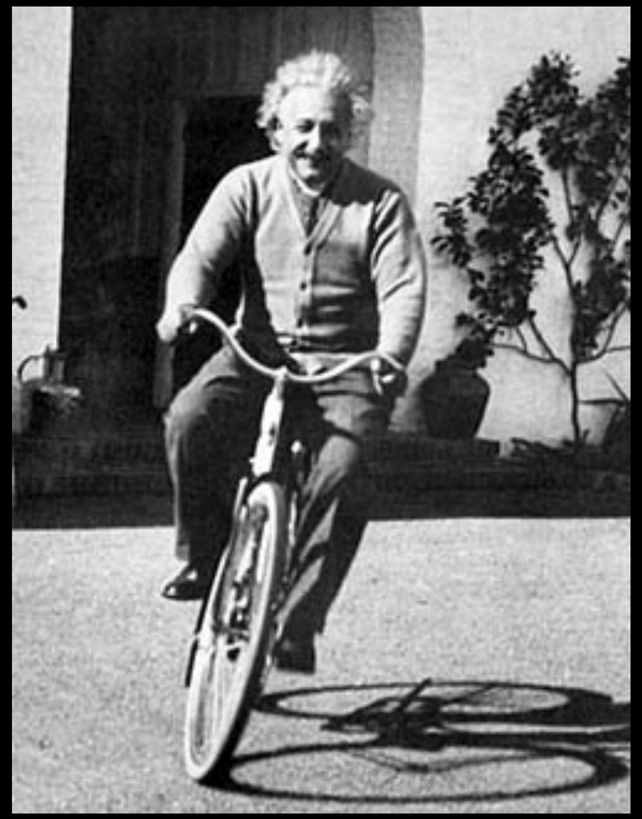
Gravity from distant objects:

Tidal field: differential acceleration

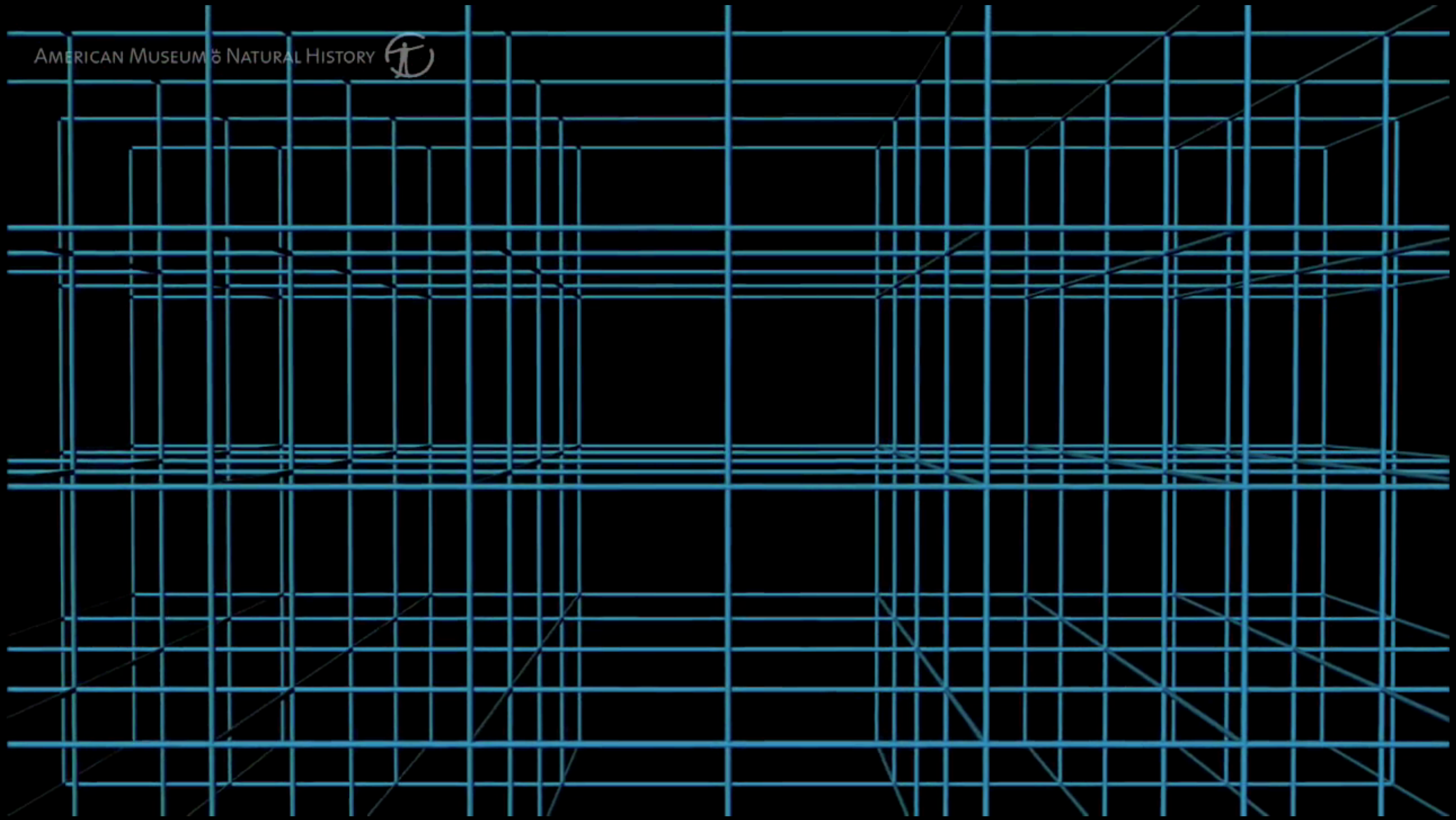
Gravity gradient: $$g' = \frac{\text{change in gravity}}{\text{separation}} \sim GM\frac{1}{r^3}$$



Gravitational Waves Demystified, T. Creighton



“Matter tells space-time how to curve and space-time tells matter how to move.”
- John A. Wheeler



American Museum of Natural History
“Gravity: Making Waves”

Gravity when the mass is in motion

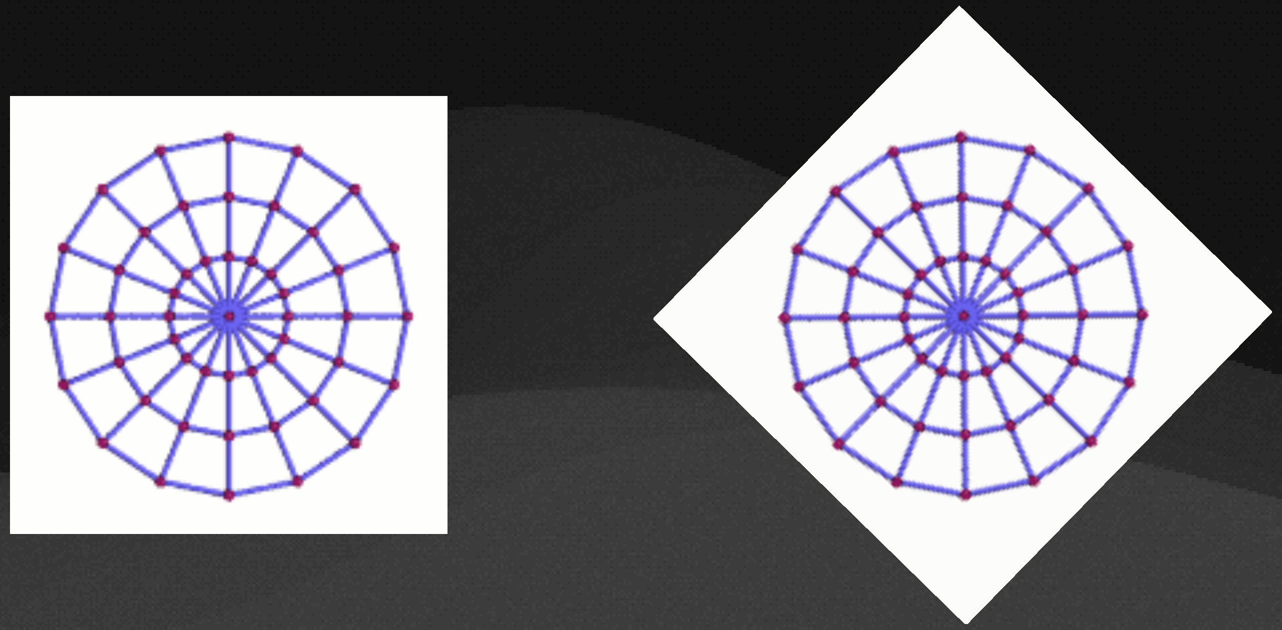
Moon passing Earth
as seen from NASA's DSCOVR spacecraft (NASA/NOAA)
at the L1 Point between the Earth and the Sun, 5 light seconds from Earth

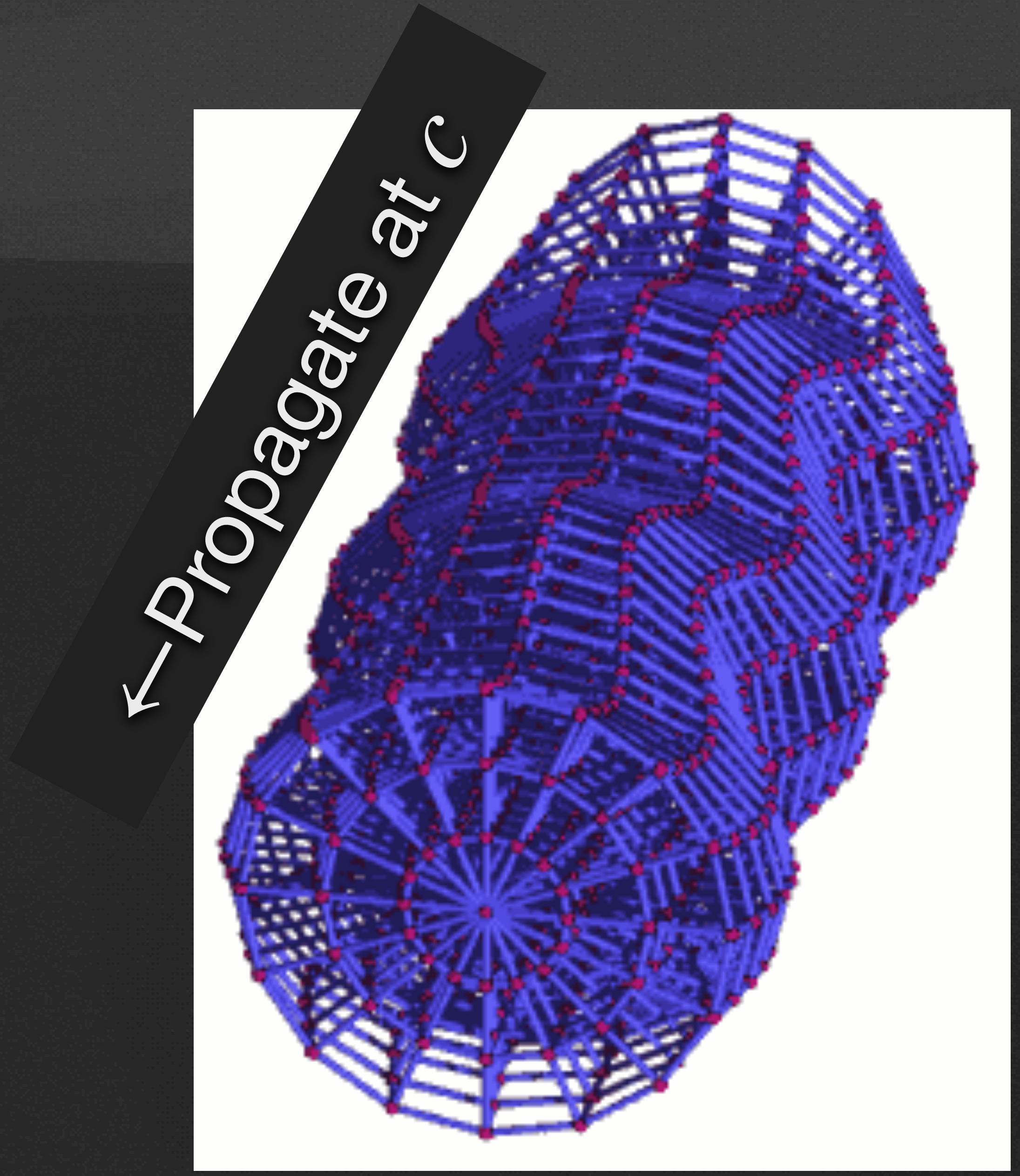
Gravitational waves

Basic principles

- Curvature of spacetime changes around moving objects
  → change propagates at speed of light
- Linearize Einstein Equations of General Relativity → wave equation
- Waves stretch and squeeze the distance between freely-falling objects (Pirani 1957)

$+$ polarization $\times$ polarization





Electromagnetic radiation

Charge monopole

$Q = \Sigma_k q_k$

$$E \sim \frac{Q}{r^2} \xrightarrow[\text{Faraway source, varying in time}]{Q \sim f(t - r/c)} E \sim \frac{\cancel{\dot{Q}}}{r}$$

Charge conservation;
No monopole radiation

Charge dipole

$P_i = \Sigma_k q_k s_{ki}$

$$E \sim \frac{P}{r^3} \xrightarrow[\text{Faraway source, varying in time}]{P \sim f(t - r/c)} E \sim \frac{\dot{P}}{r^2}, \quad E \sim \frac{\ddot{P}}{r}$$

Dipole
Radiation

Gravitational Waves Demystified, T. Creighton

Gravitational radiation

Mass monopole

$M = \Sigma_k m_k$

$$g' \sim \frac{M}{r^3} \xrightarrow[\text{Faraway source, varying in time}]{M \sim f(t - r/c)} g' \sim \frac{\cancel{\dot{M}}}{r^2}$$

Mass conservation at $1/r^2$;
No monopole radiation

Mass dipole

$P_i = \Sigma_k m_k s_{ki}$

$$g' \sim \frac{P}{r^4} \xrightarrow[\text{Faraway source, varying in time}]{P \sim f(t - r/c)} g' \sim \frac{\dot{P}}{r^3}\ ,\ g' \sim \frac{\ddot{P}}{r^2} = \frac{\Sigma_i \cancel{m_i a_i}}{r^2}$$

Momentum conservation at $1/r^2$;
No dipole radiation

Mass quadrupole

$Q_{ij} = \Sigma_k m_k s_{ki} s_{kj}$

$$g' \sim \frac{Q}{r^4} \xrightarrow[\text{Faraway source, varying in time}]{Q \sim f(t - r/c)} g' \sim \frac{\dot{Q}}{r^4}\ ,,,\ g' \sim \frac{\dddot{Q}}{r}$$

Gravitational radiation from quadrupole

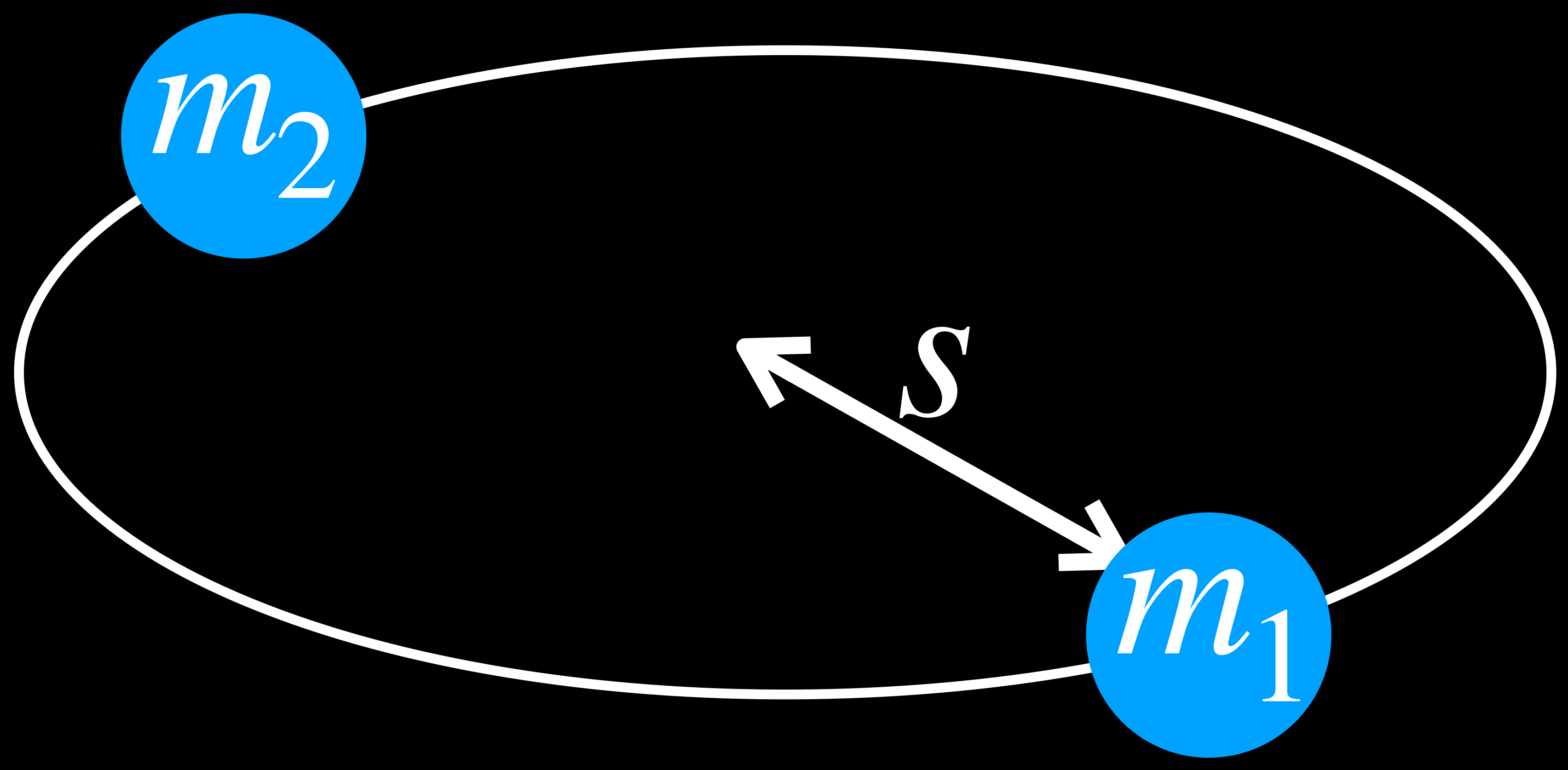
<u>Gravitational Waves Demystified, T. Creighton</u>

Sources of gravitational waves

What makes gravitational waves?

Oscillatory source d away,
mass M, size s, frequency f:
Quadrupole moment $Q \sim Ms^2$

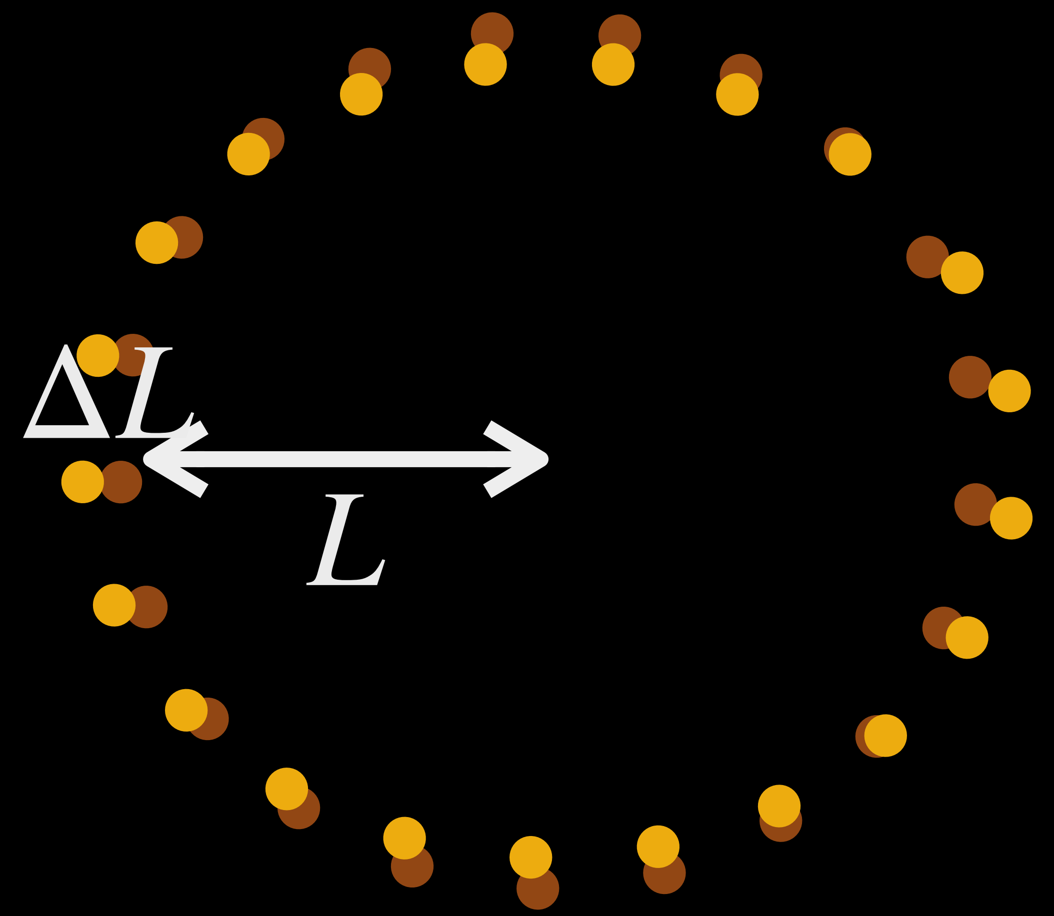
$$g' \sim \frac{\ddddot{Q}}{r} \sim GM\frac{f^4}{c^4}\frac{s^2}{r}$$

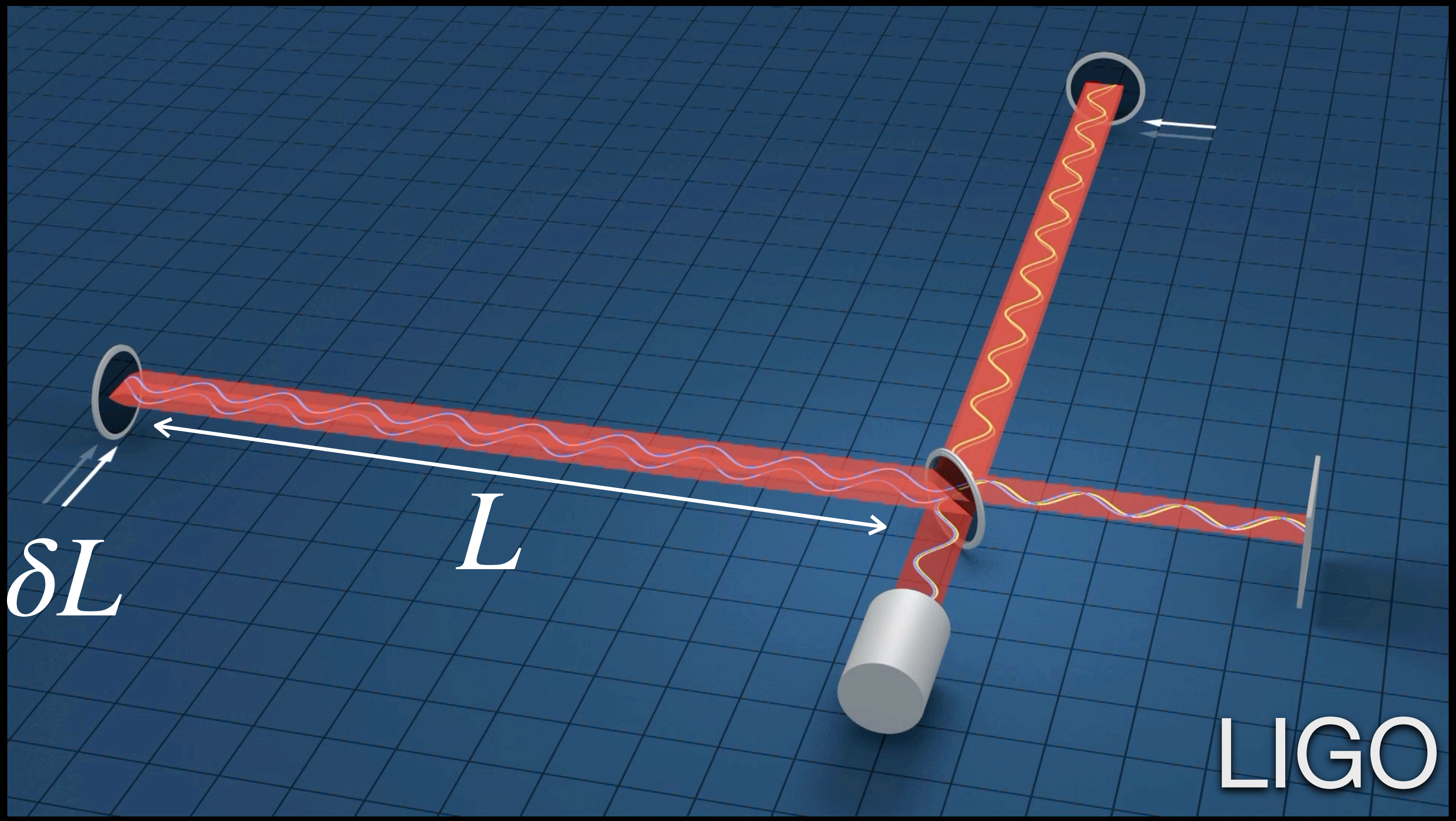


e.g. binary orbit with:
mass M, size s, orbital frequency f

Gravitational-wave strain

What the observatories measure





Test particles: differential acceleration

$$\Delta a = g' L \ = d^2(\Delta L)/dt^2$$

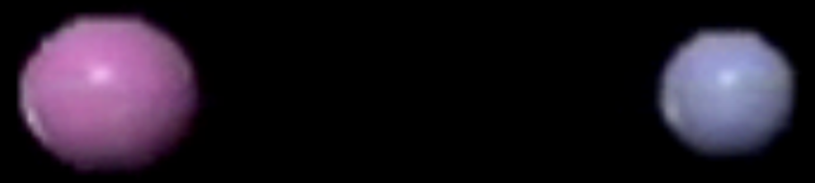
Relative displacement from integration

$$h \equiv 2\frac{\Delta L}{L} \sim GM\frac{f^2}{c^2}\frac{s^2}{r} \sim \frac{G}{c^2}\frac{M}{r}\frac{v_\perp^2}{c^2}$$

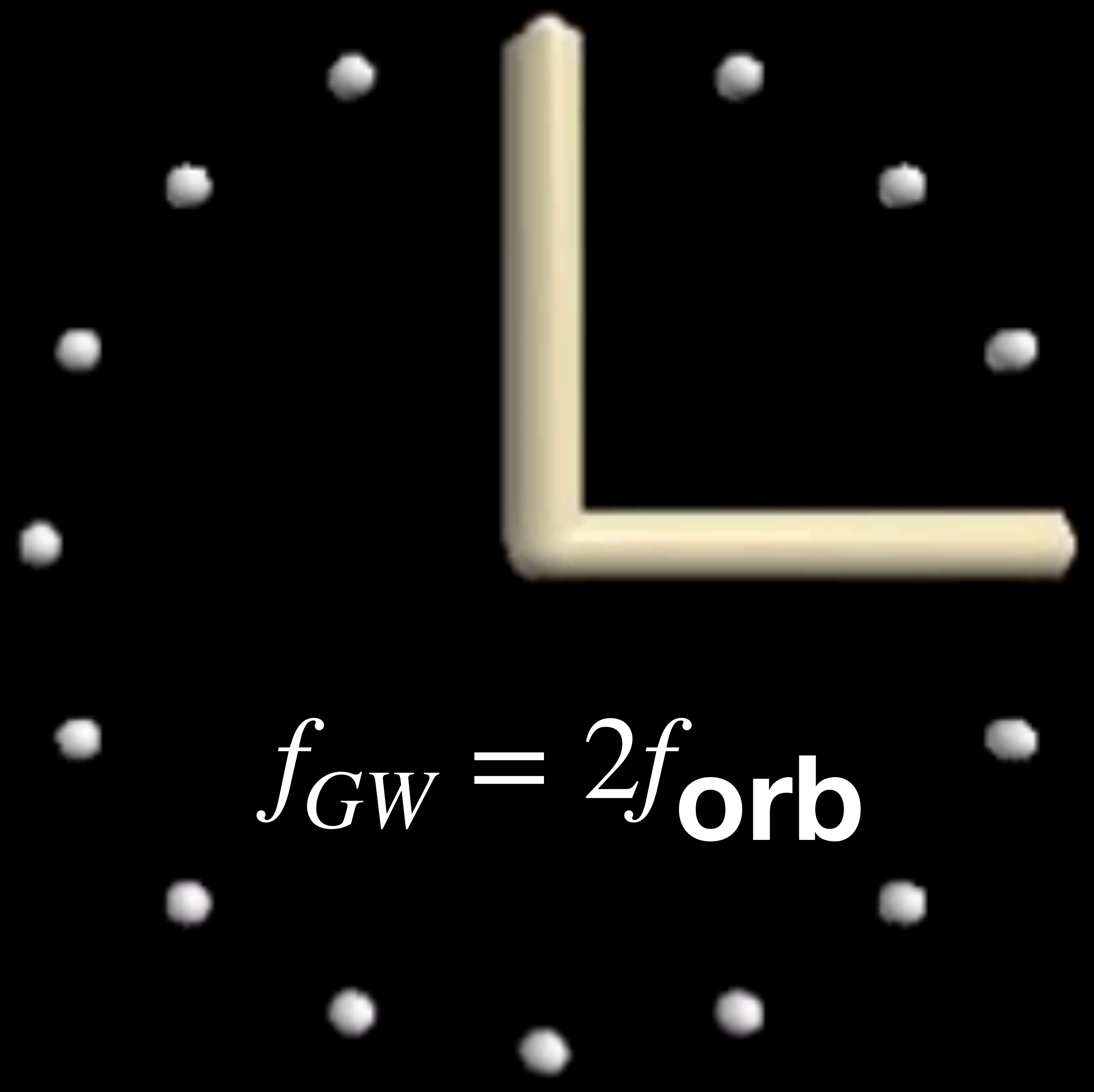
<u>Gravitational Waves Demystified, T. Creighton</u>

Gravity from the Ground Up, B. Schutz

Binary Orbit



Kepler's law $f^2 \propto G\frac{M}{s^3}$



$f_{GW} = 2f_{\textbf{orb}}$

$$h = 2\frac{\delta L}{L} \sim 10^{-21}\frac{100\ \text{Mpc}}{d_L}\frac{M}{1.4\,M_\odot}\frac{R_S}{s}$$

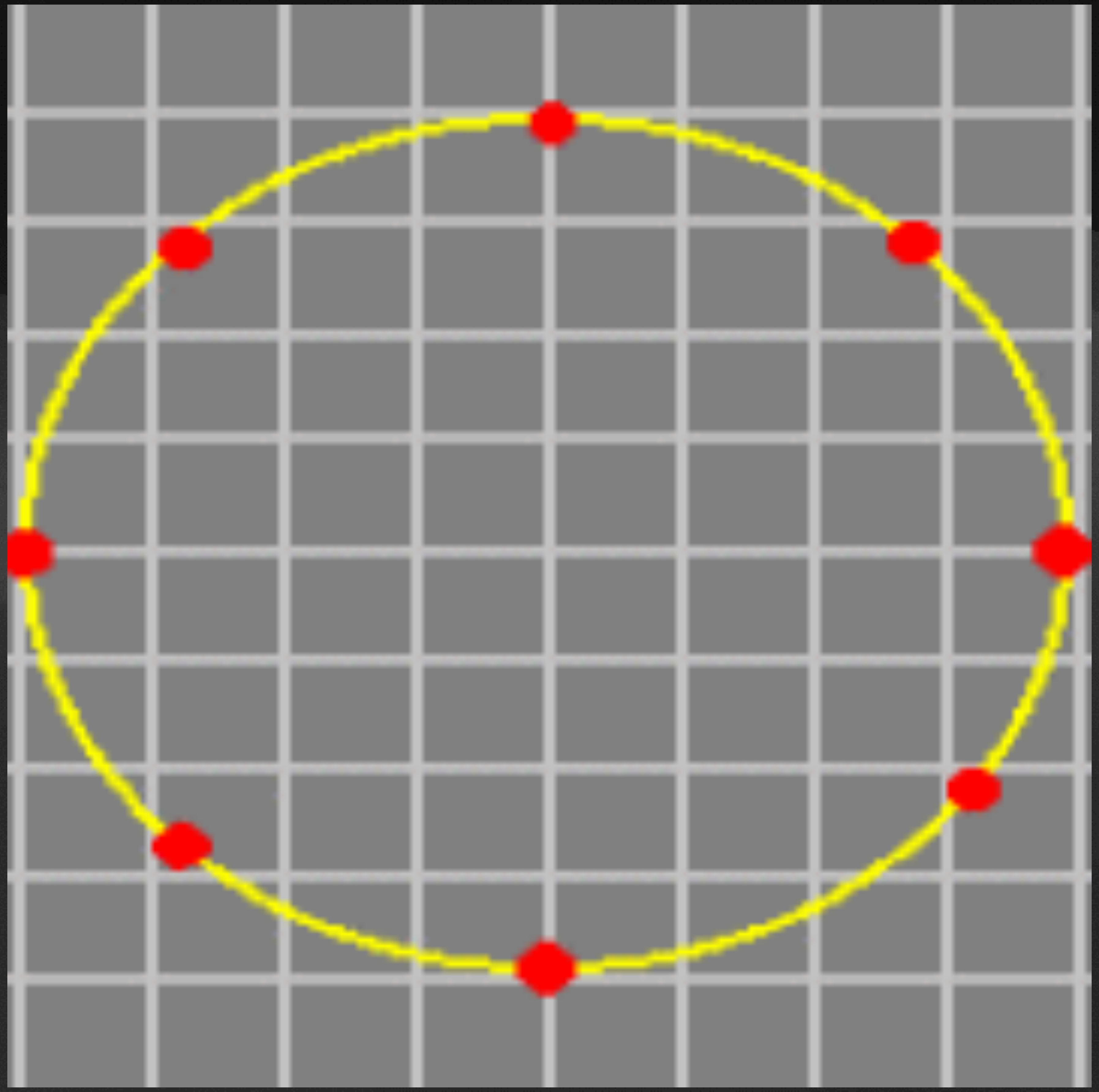
R_S the Schwarzschild radius of the orbiting mass ($R_S = 2GM/c^2$)

Movie by
Megan Loh, CSUF

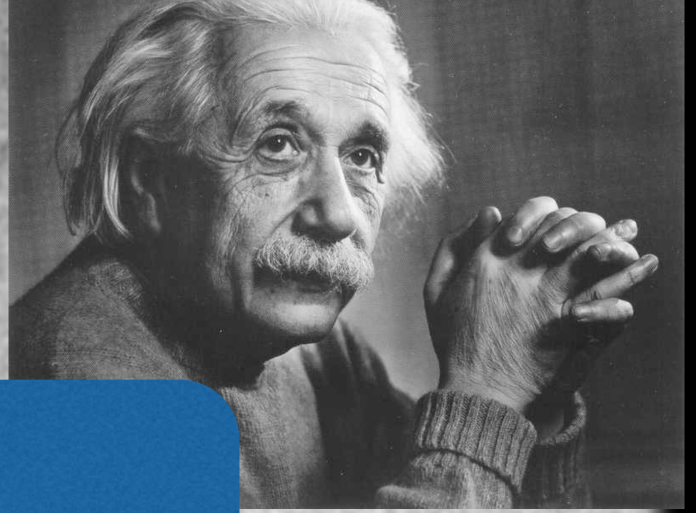
https://www.youtube.com/watch?v=loZIghbMoW8

Effects of gravitational waves

- Fractional change shown 10%) $h \sim 10^{-1}$
- Fractional change from gravitational waves arriving at Earth is $h \sim 10^{-21}$
- Suppose circle radius = 4 km, position change is 10^{-18} m ($\approx r_{\text{proton}}/1000$)



Freely falling massless test particles



Nä... n

One obtains the radiated energy of the system per unit time… sees that it must have in all conceivable situations a practically vanishing value.

Von A. Einstein.

ein. Man erhält aus ihm also die Ausstrahlung A des Systems pro Zeiteinheit durch Multiplikation mit $4\pi R^2$:

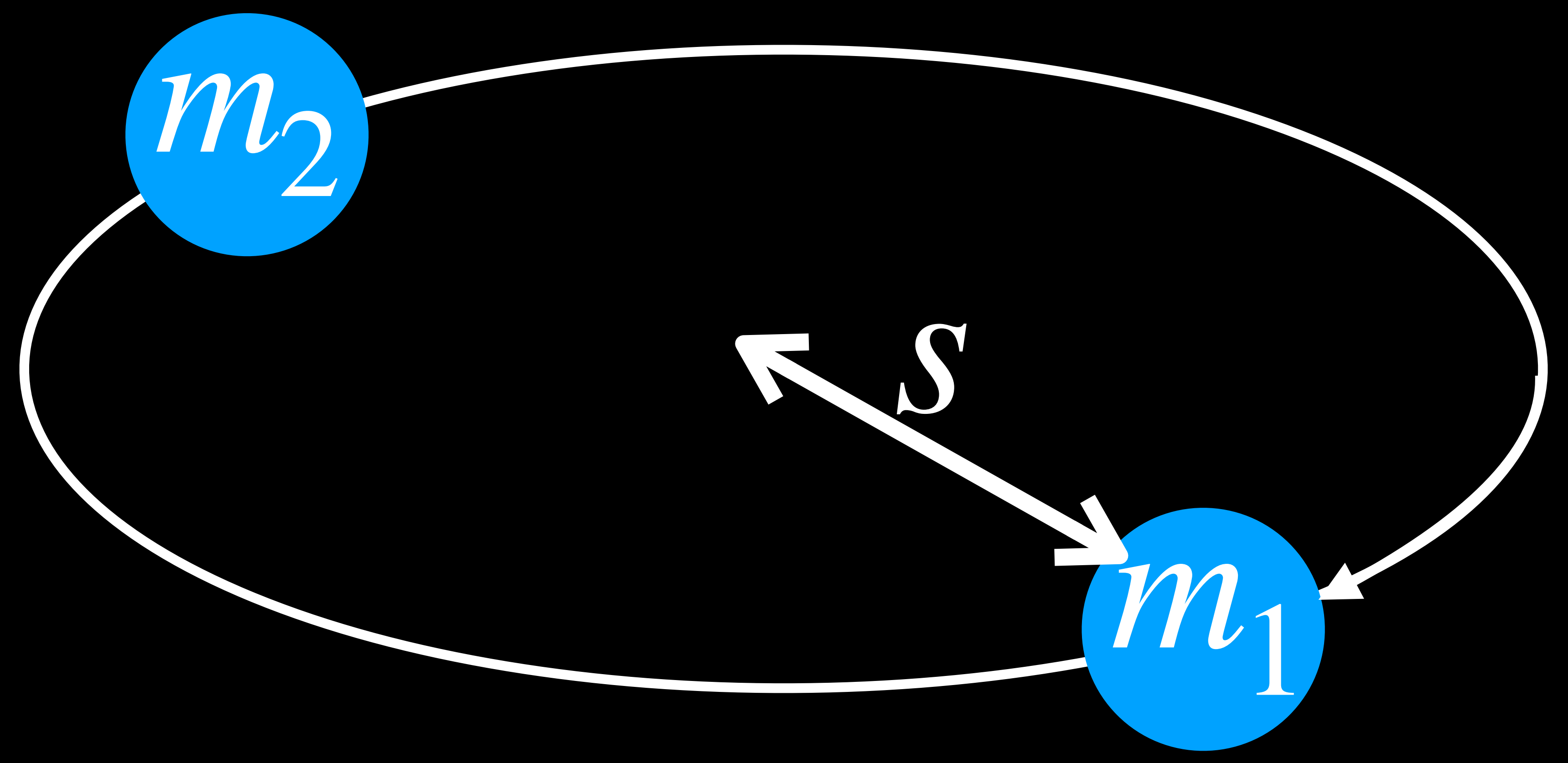
$$A = \frac{\varkappa}{24\pi} \sum_{\alpha\beta} \left(\frac{\partial^3 J_{\alpha\beta}}{\partial t^3}\right)^2. \qquad (21)$$

Würde man die Zeit in Sekunden, die Energie in Erg messen, so würde zu diesem Ausdruck der Zahlenfaktor $\frac{1}{c^4}$ hinzutreten. Berücksichtigt man außerdem, daß $\varkappa = 1.87 \cdot 10^{-27}$, so sieht man, daß A in allen nur denkbaren Fällen einen praktisch verschwindenden Wert haben muß.

Gravitational-wave luminosity

GW energy flux $\sim f^2 h^2$, integrate over sphere $\sim d^2$

$$\mathcal{L}_{GW} \sim \left(\frac{c^5}{G}\right) f^2 d^2 \left(GM \frac{f^4}{c^4} \frac{s^2}{d} \right)^2$$



Binary orbit example:

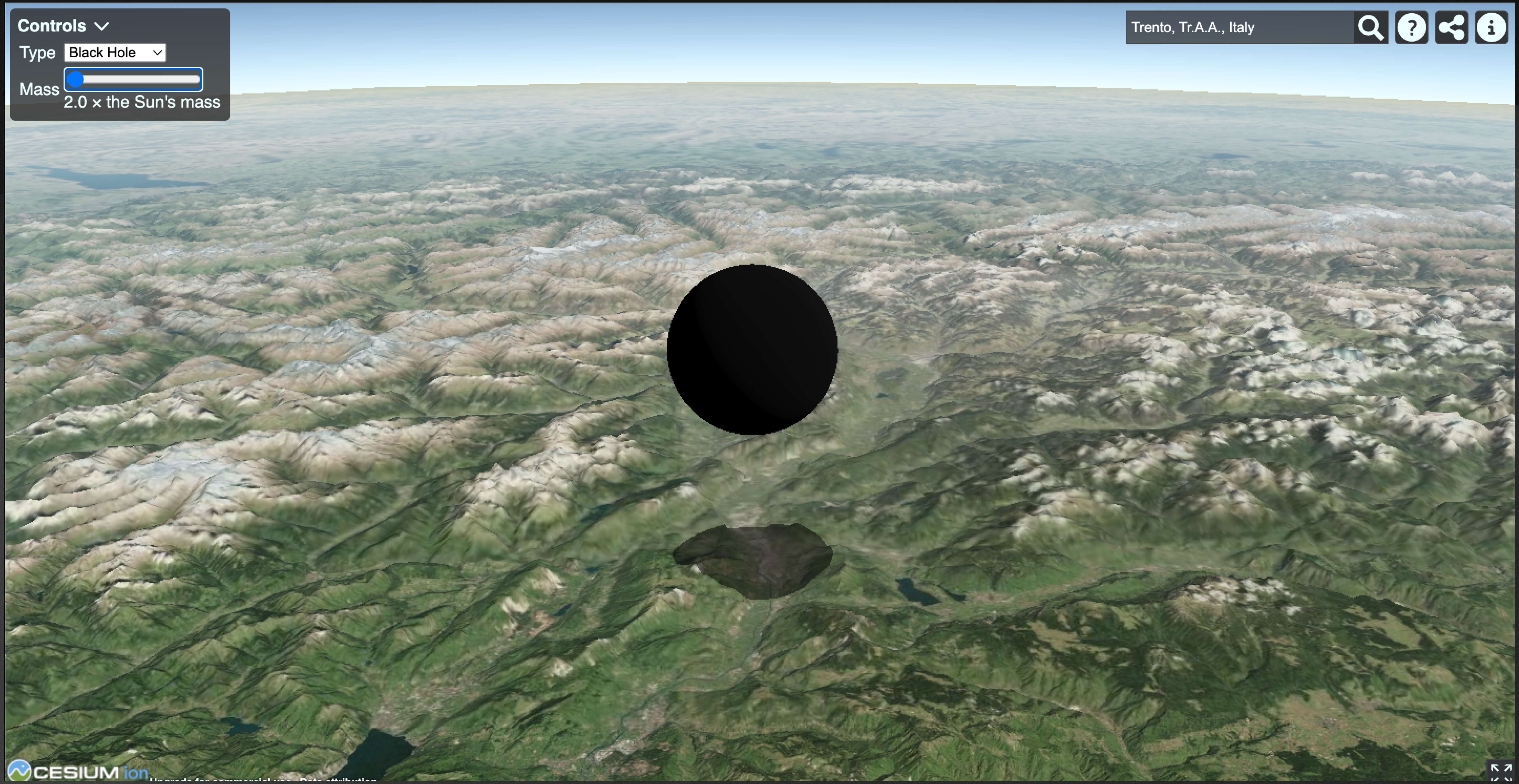
Keplerian orbits, $R_S = 2GM/c^2$

$$\mathcal{L}_{GW} \sim \frac{c^5}{G} \left(\frac{R_S}{s}\right)^2 \left(\frac{v}{c}\right)^6 \sim 10^{59}\ \text{erg s}^{-1} \frac{R_S}{s}$$

Compact object size scale

2.0 $M_\odot$ Black hole

$(R_S = 2GM/c^2)$

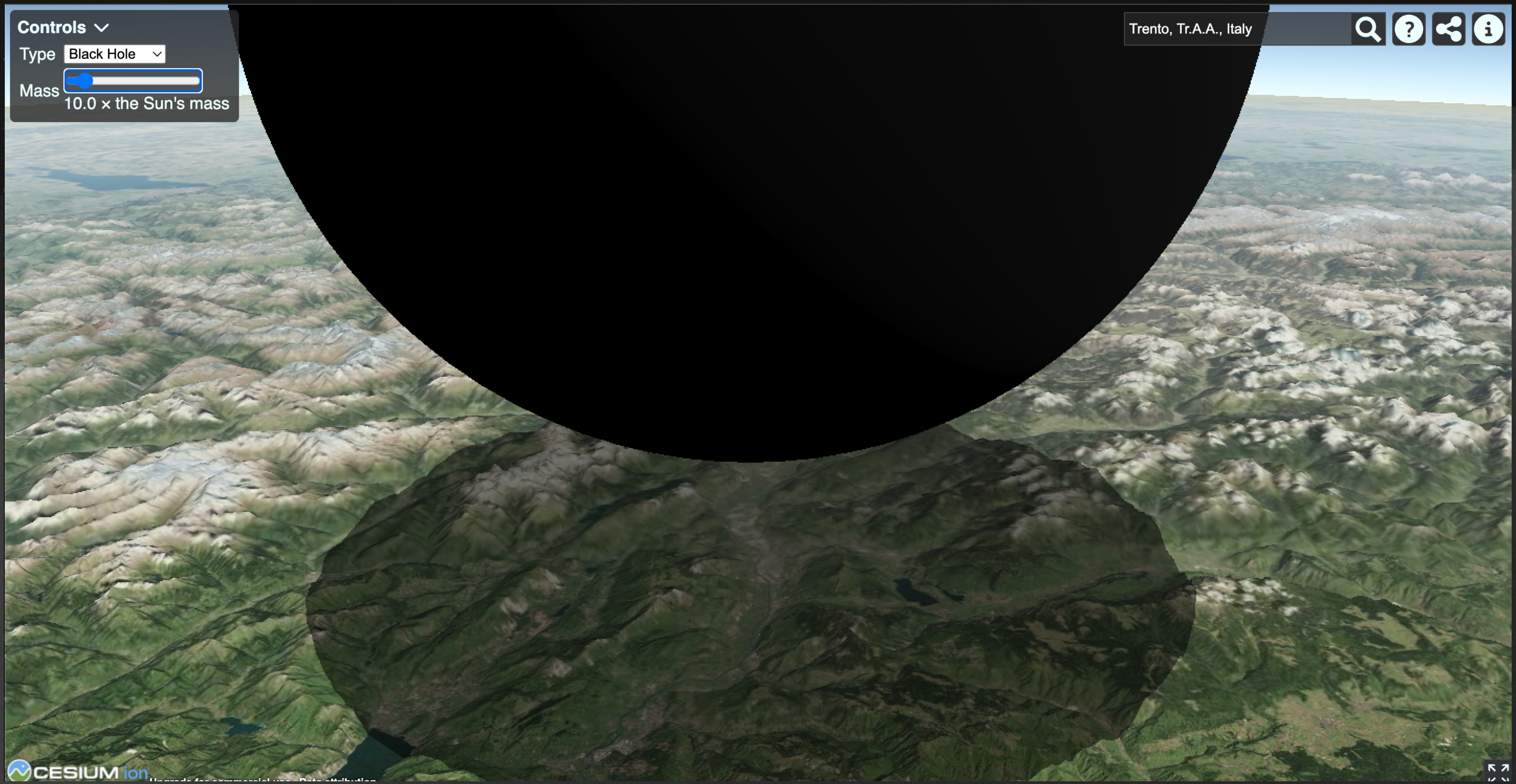


https://ns-in-my-city.daniel-wysocki.info/

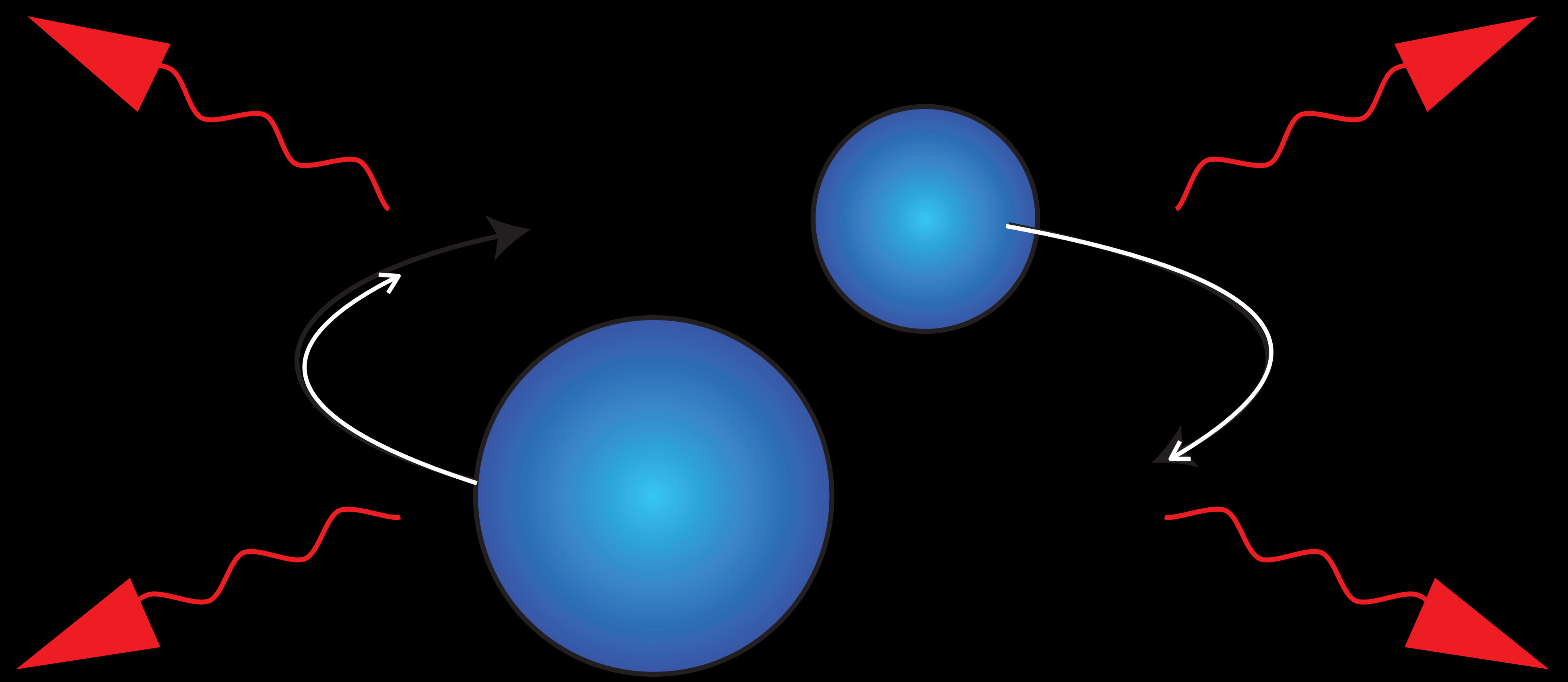
Compact object size scale

10 $M_\odot$ Black hole

$(R_S = 2GM/c^2)$



The “chirp”

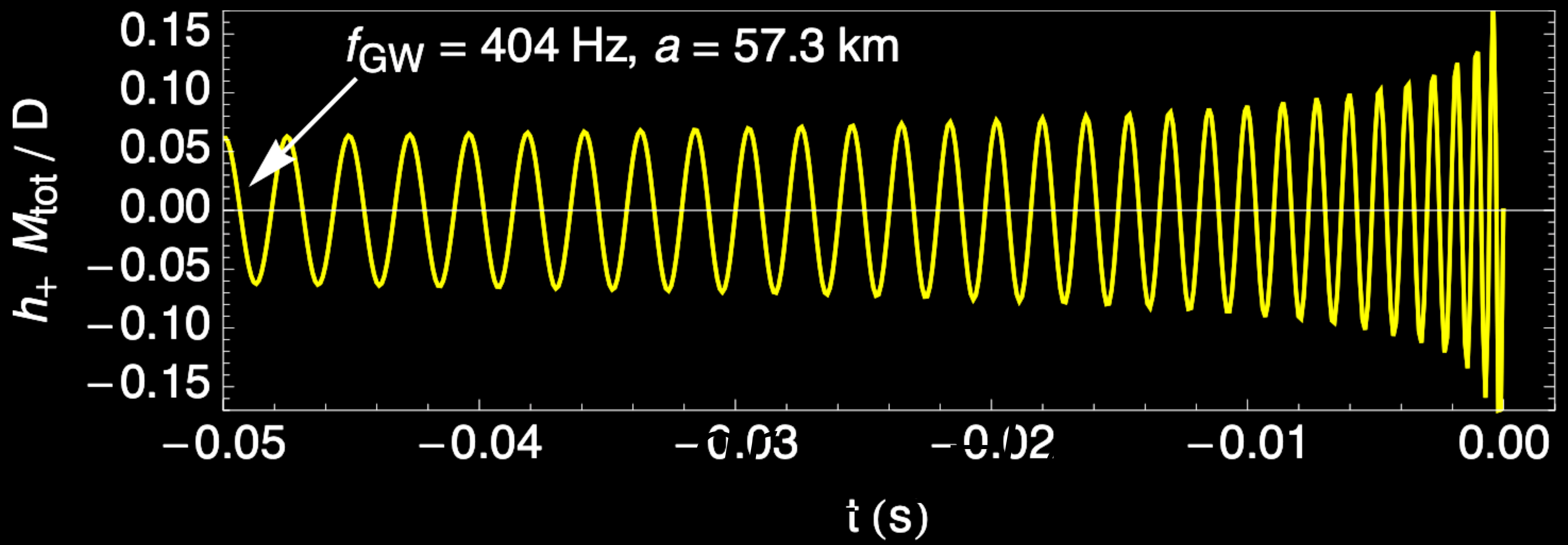


$$E_{orb} = \frac{1}{2}\left(m_1 v_1^2 + m_2 v_2^2\right) - G\frac{m_1 m_2}{a} = -G\frac{m_1 m_2}{2a} \qquad \frac{dE}{dt} = -\mathscr{L}_{GW}$$

$$\frac{da}{dt} = \frac{-\mathscr{L}_{GW}}{dE_{orb}(a)/da}$$

Solve for $a(t)$

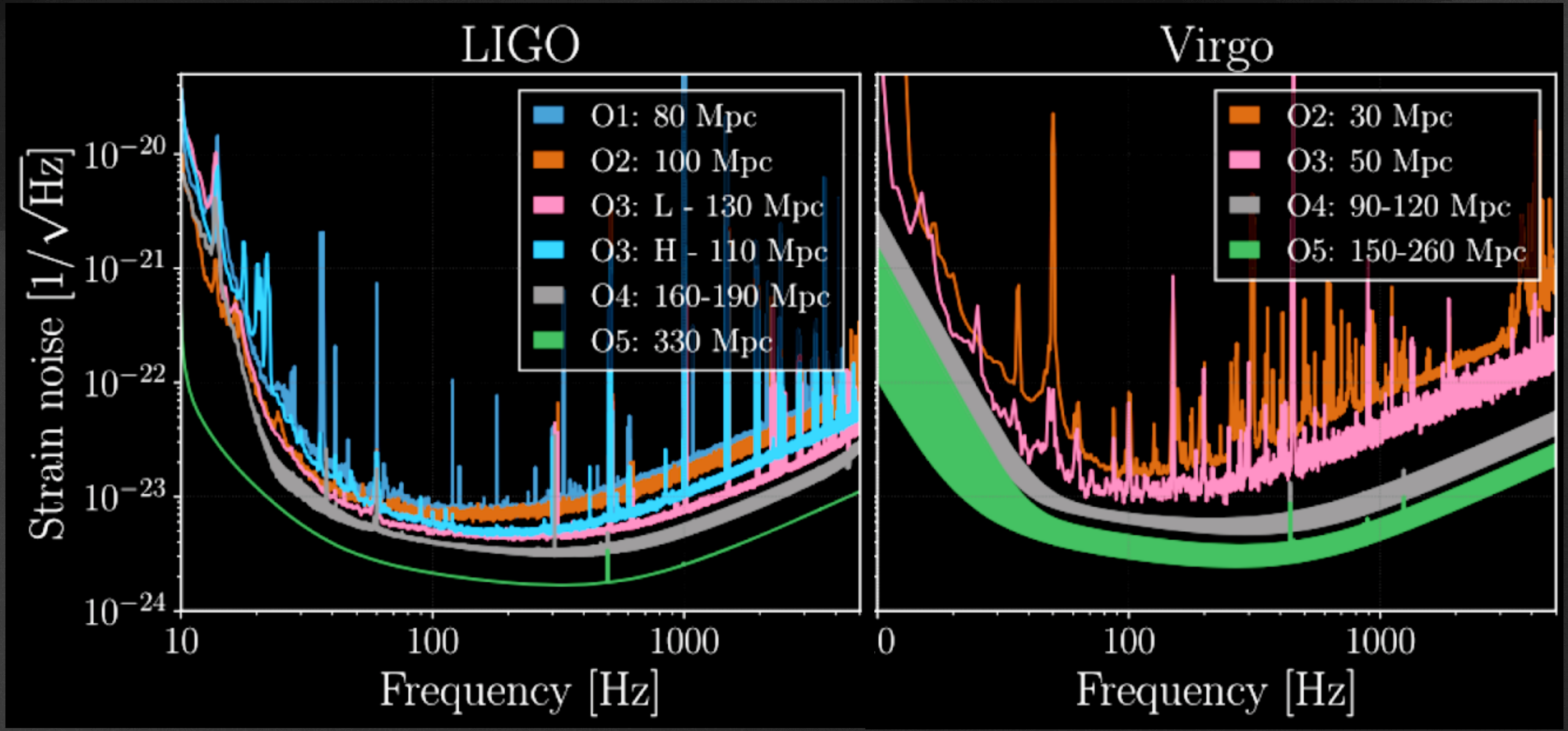
$a(t) \leftrightarrow \mathscr{L}_{GW}(t) \leftrightarrow h(t)$



International network of gravitational-wave observatories

Strain noise: detector sensitivity

- steady signal amplitude h_0 at f has signal-to-noise ratio $\propto h_0\sqrt{T}/\sqrt{S_n(f)}$
- compare strain noise $\sqrt{S_n(f)}$ to GW characteristic strain $|\tilde{h}(f)|f^{-1/2}$



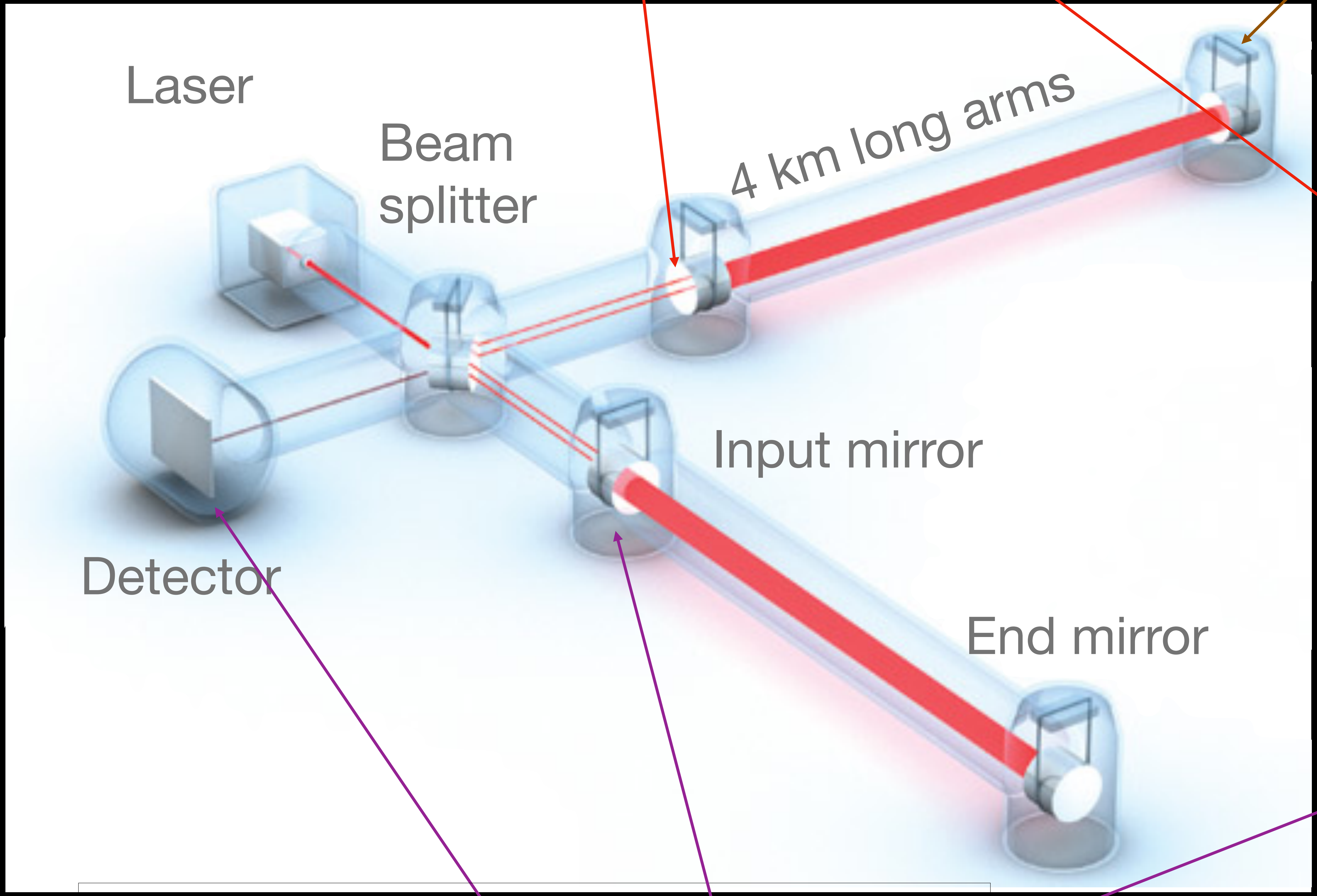
LIGO-Virgo-KAGRA Observing Scenarios Living Rev Relativ 23, 3 (2020)

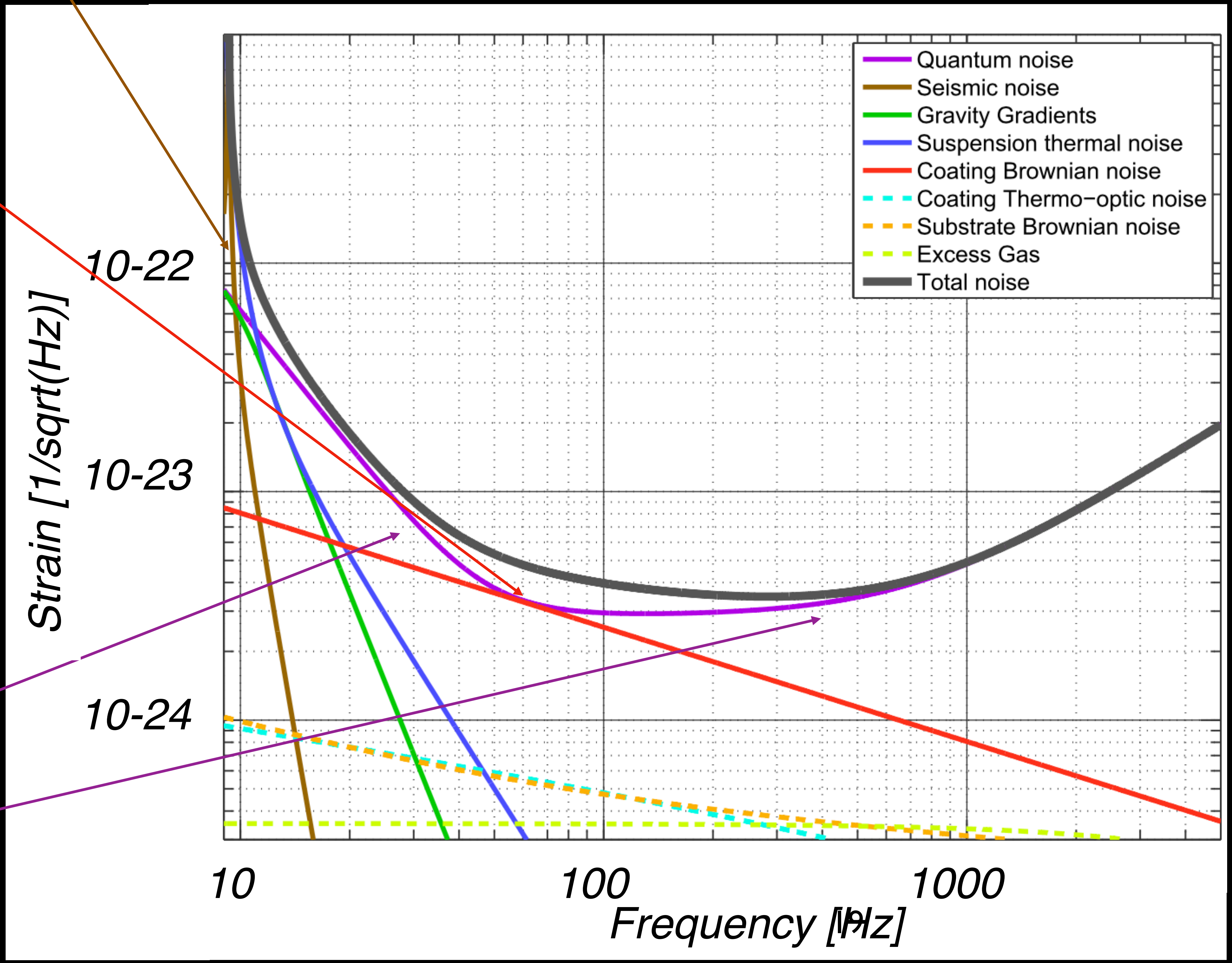
Noise background: limiting sources

Thermal motion of the mirror coatings

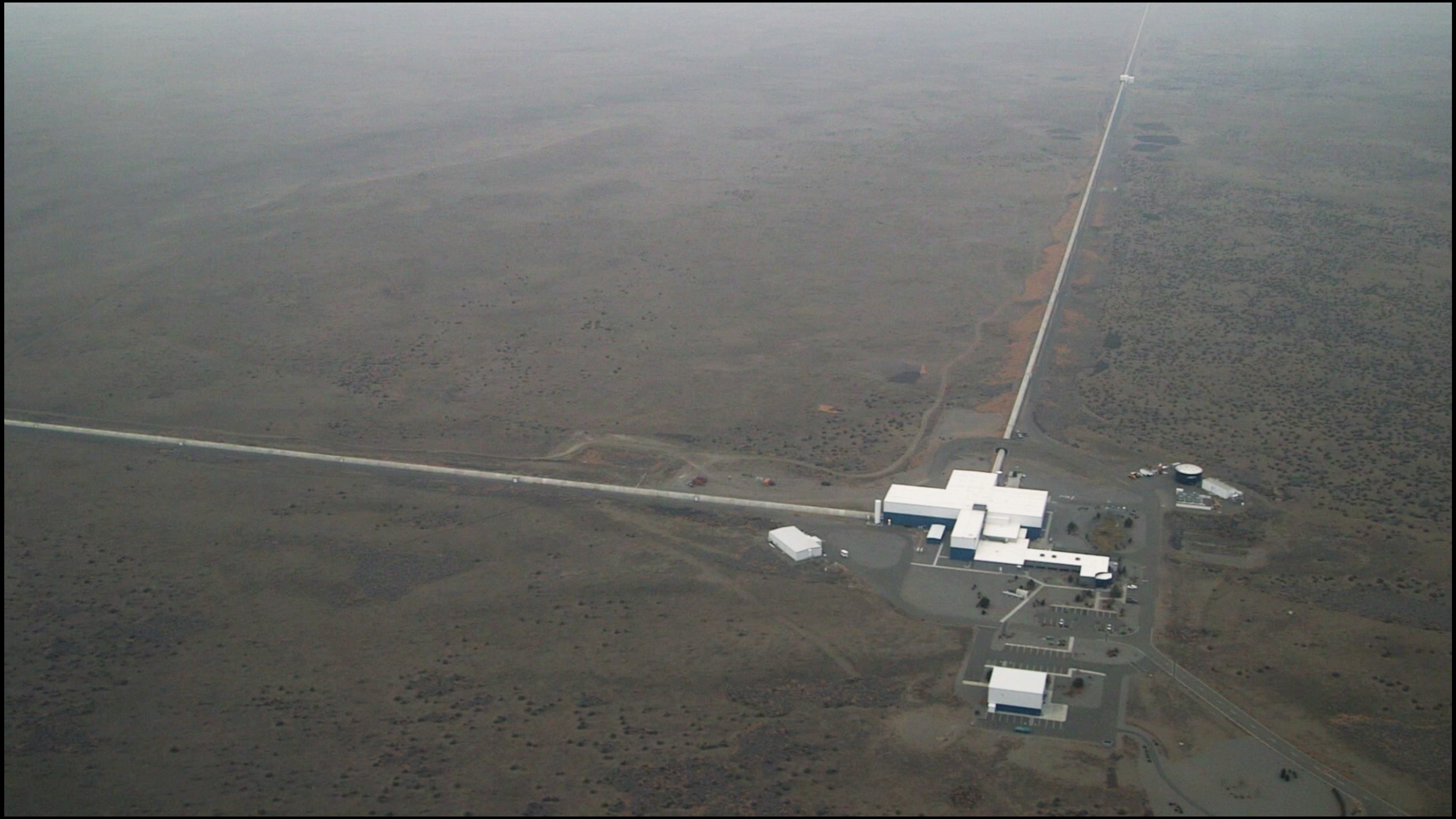
Seismic motion shakes mirrors

Class. Quantum Grav. 32 (2015) 074001





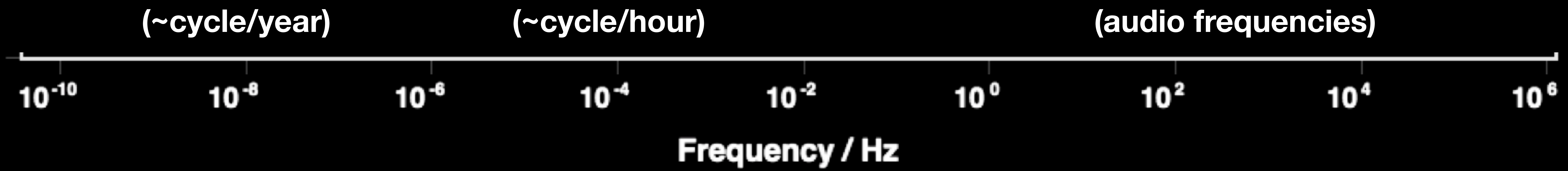
Quantum radiation pressure and shot noise

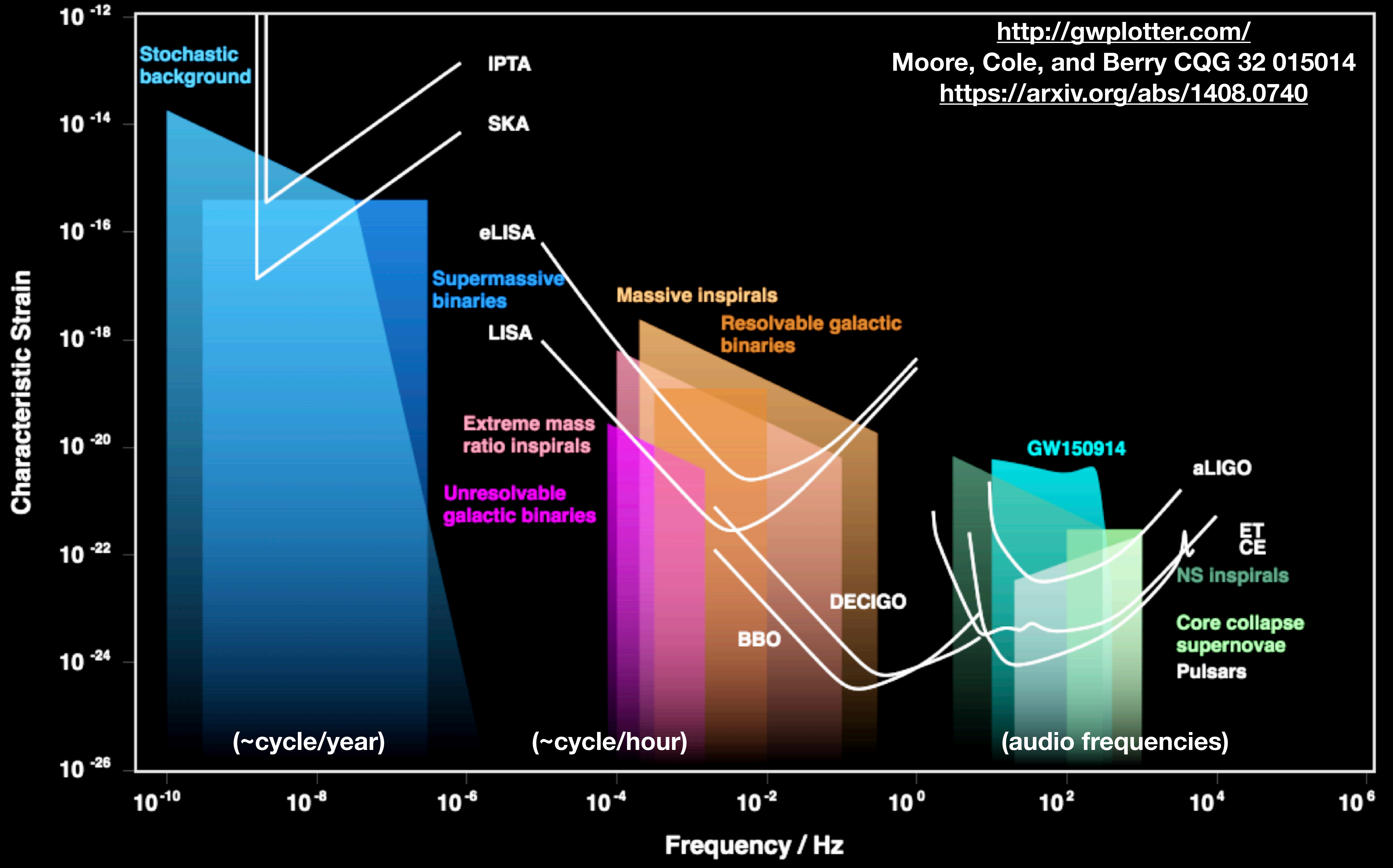


- Input laser light boosted to 35W+, circulating power goal 400 kW (Cahillane and Mansell https://arxiv.org/abs/2202.00847)

- Shot-noise squeezing of up to 6 dB (https://dcc.ligo.org/LIGO-T2300411/public)

The Gravitational-wave Spectrum

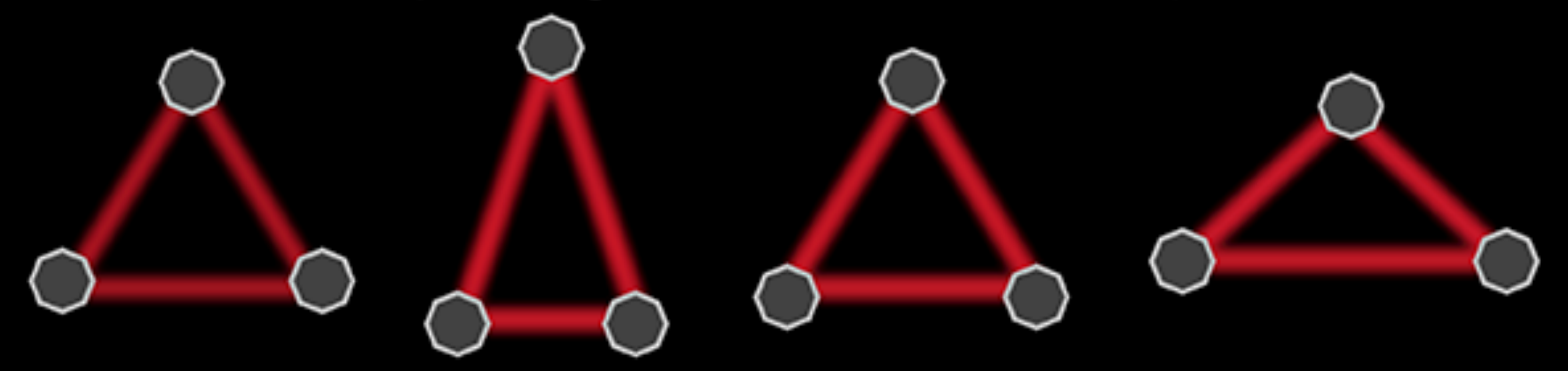



http://gwplotter.com/
Moore, Cole, and Berry CQG 32 015014
https://arxiv.org/abs/1408.0740
Characteristic Strain
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
10^{-26}
Stochastic background
IPTA
SKA
eLISA
Supermassive binaries
LISA
Massive inspirals
Resolvable galactic binaries
Extreme mass ratio inspirals
Unresolvable galactic binaries
GW150914
aLIGO
ET
CE
NS inspirals
Core collapse supernovae
Pulsars
DECIGO
BBO
(~cycle/year)
(~cycle/hour)
(audio frequencies)
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
Frequency / Hz

LISA - LASER INTERFEROMETER SPACE ANTENNA

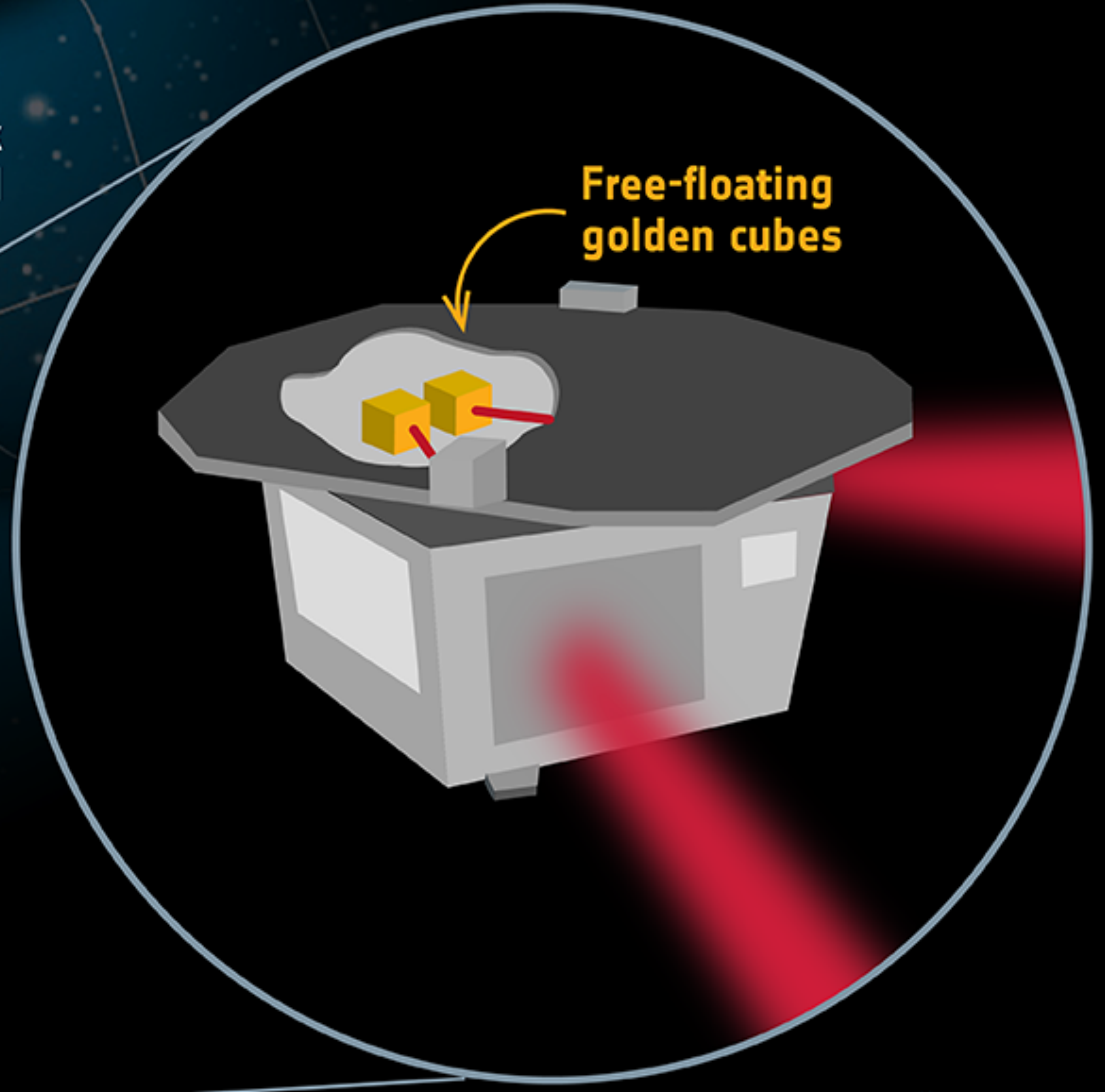
Gravitational waves are ripples in spacetime that alter the distances between objects. LISA will detect them by measuring subtle changes in the distances between **free-floating cubes** nestled within its three spacecraft.

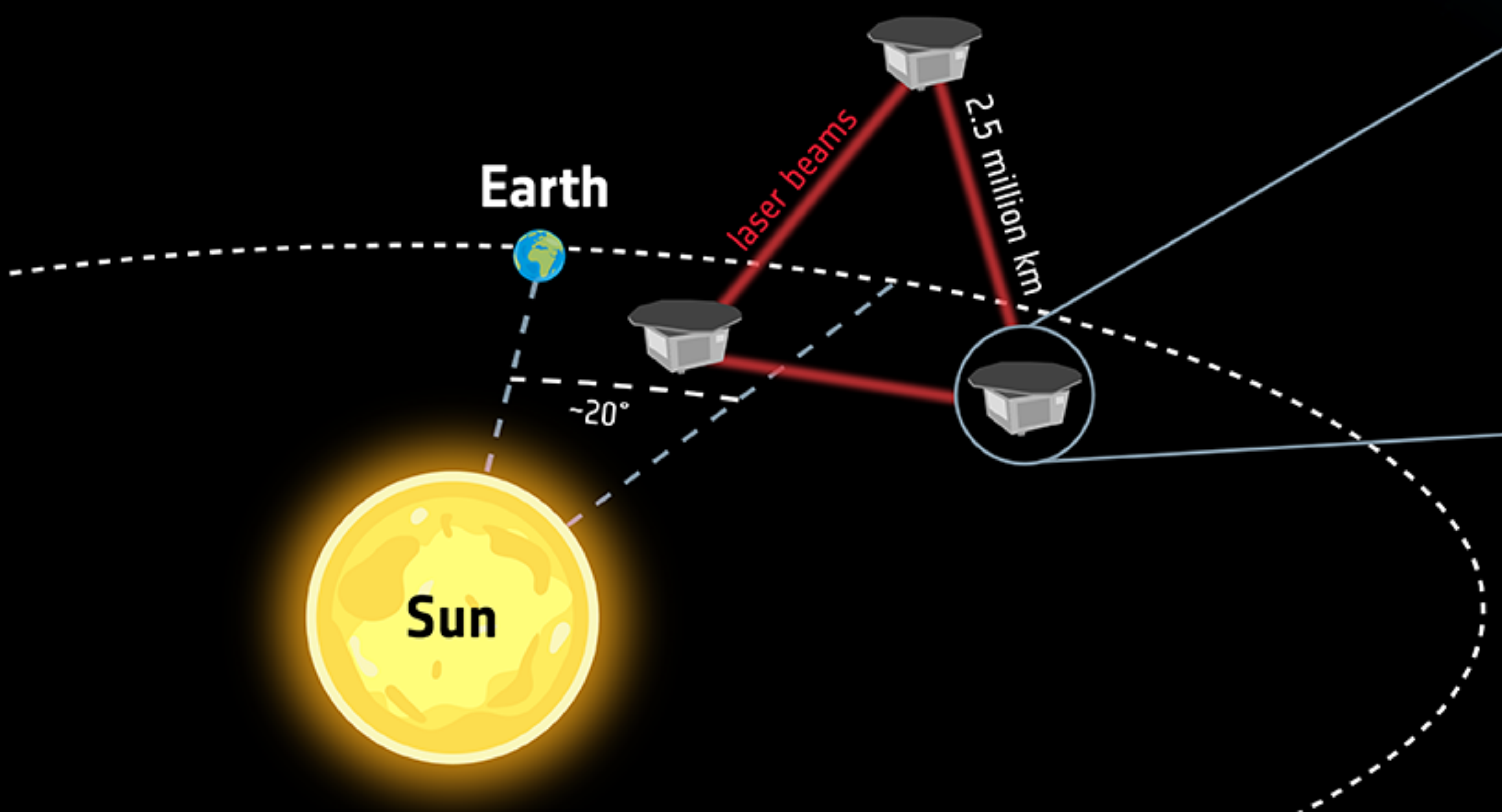
3 identical spacecraft exchange **laser beams**. Gravitational waves change the distance between the **free-floating cubes** in the different spacecraft. This tiny change will be measured by the laser beams.



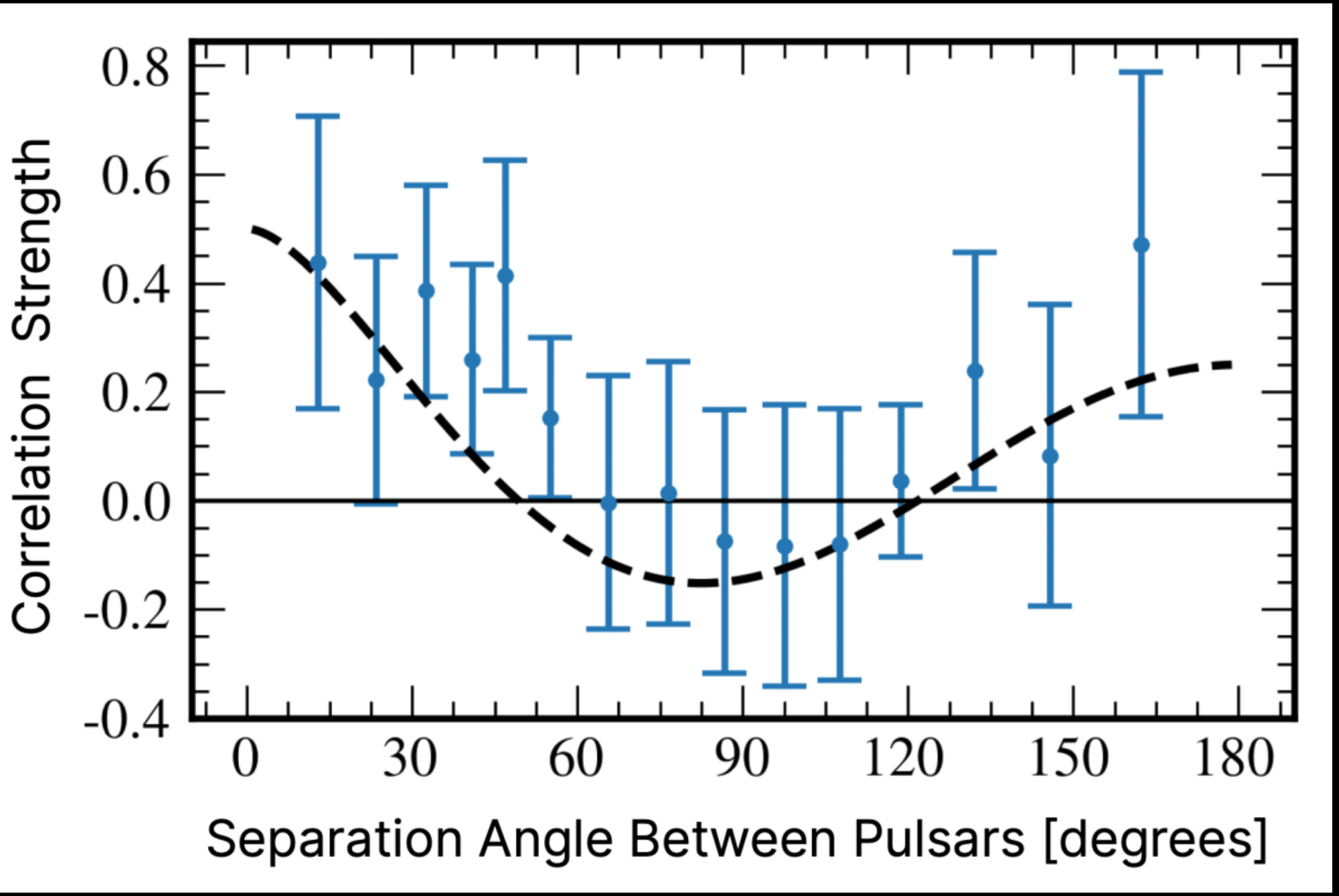
* *Changes in distances travelled by the laser beams are not to scale and extremely exaggerated*

Powerful events such as **colliding black holes** shake the fabric of spacetime and cause gravitational waves





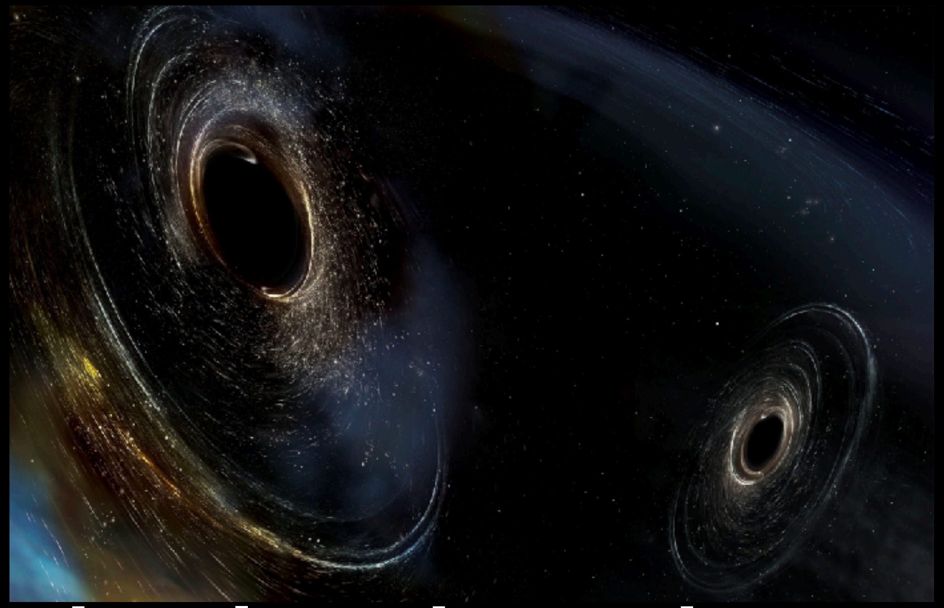
https://www.esa.int/Science_Exploration/Space_Science/LISA_factsheet



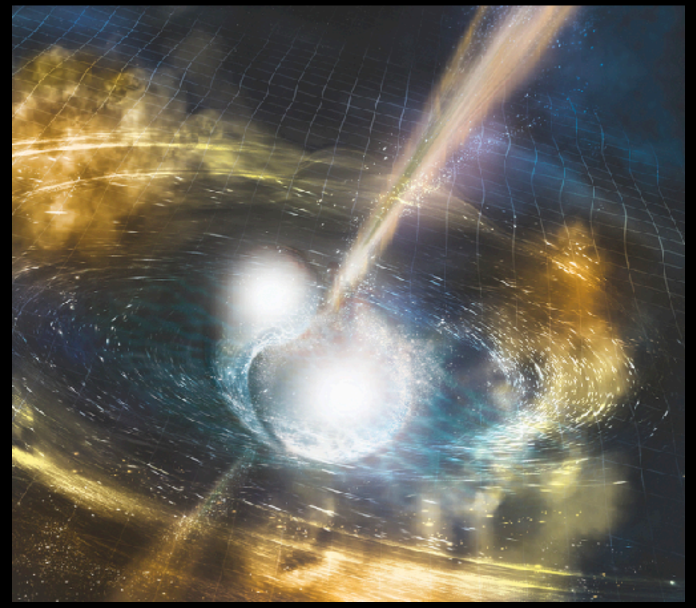
Agazie et al., 2023, The NANOGrav 15-year Data Set: The Stochastic Gravitational-Wave Background. https://doi.org/10.3847/2041-8213/acdac6

Potential sources of ~ 10 – 4000 Hz GW

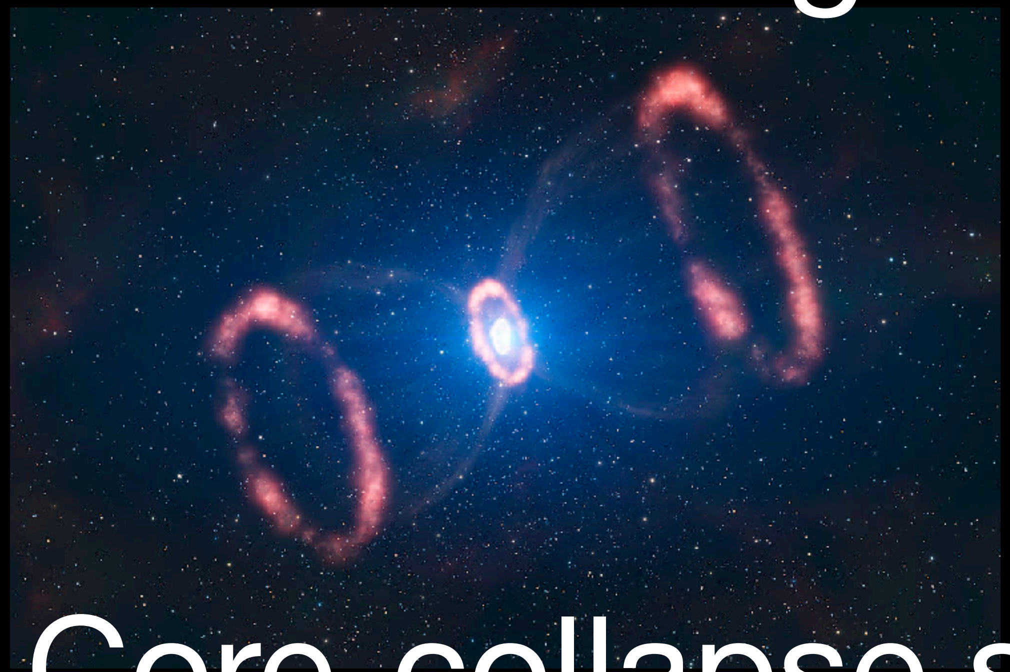
Black hole inspiral



Neutron star inspiral

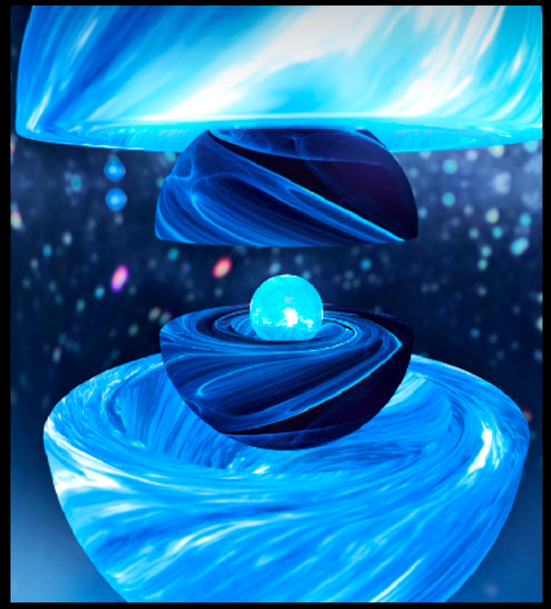


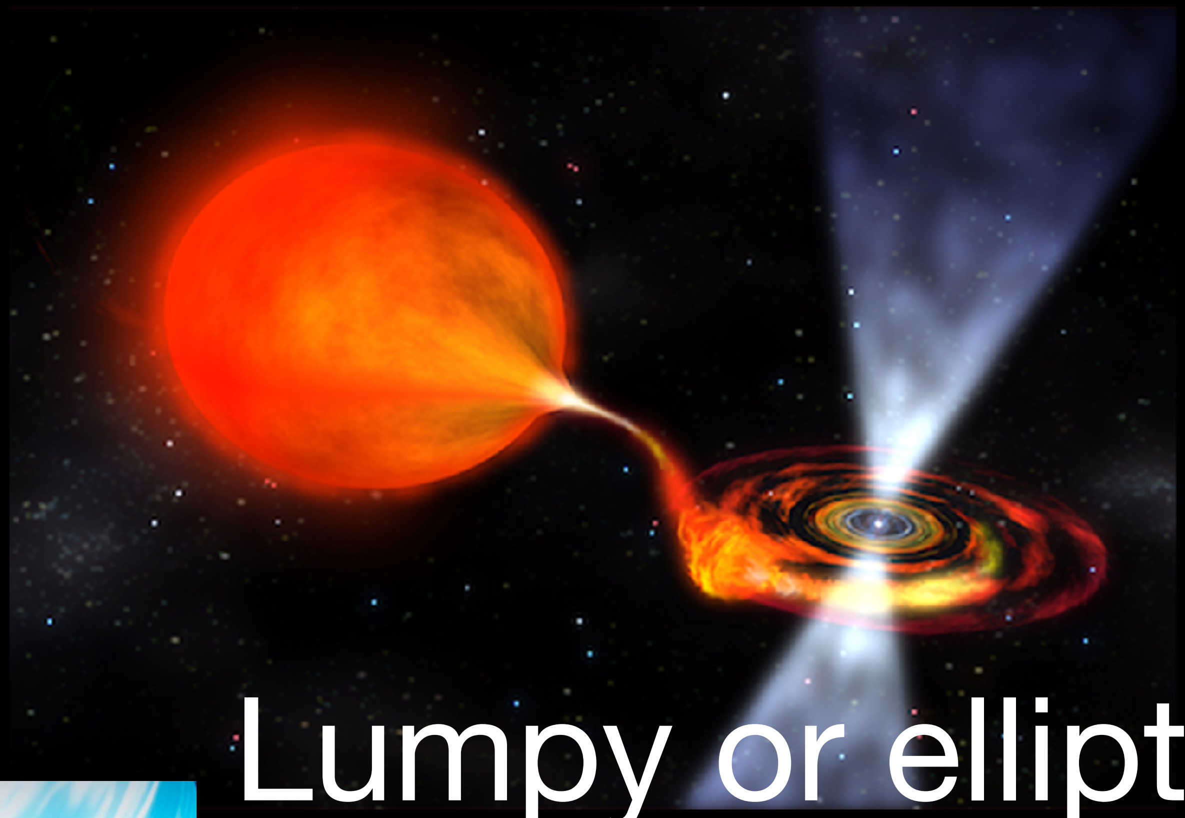
Black hole ringdown



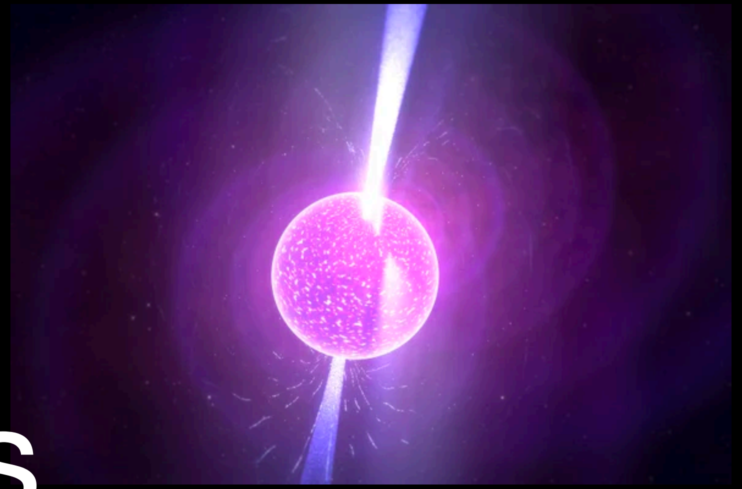
Neutron star post-merger

Core-collapse supernovae

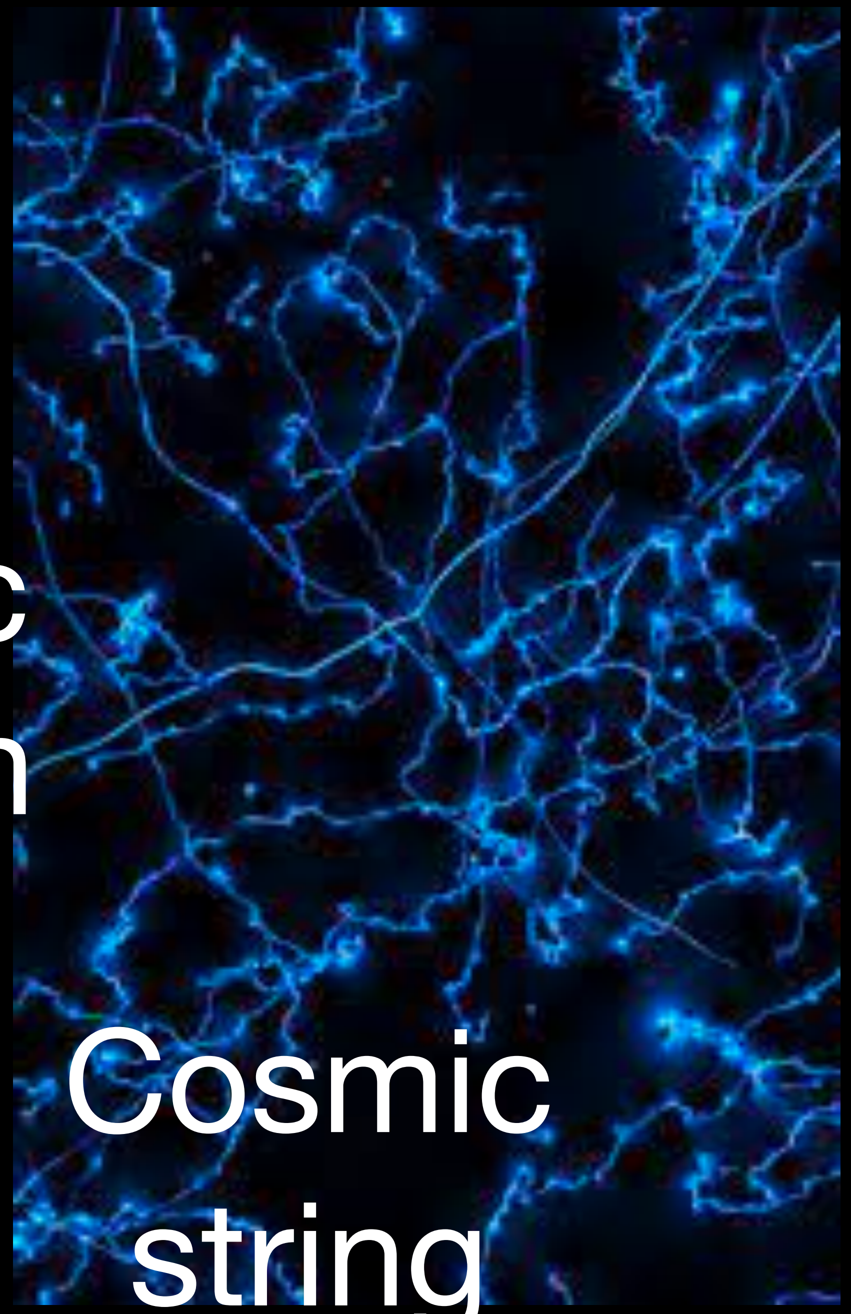




Lumpy or elliptic spinning neutron stars



Glitching neutron stars



Cosmic string fluctuations

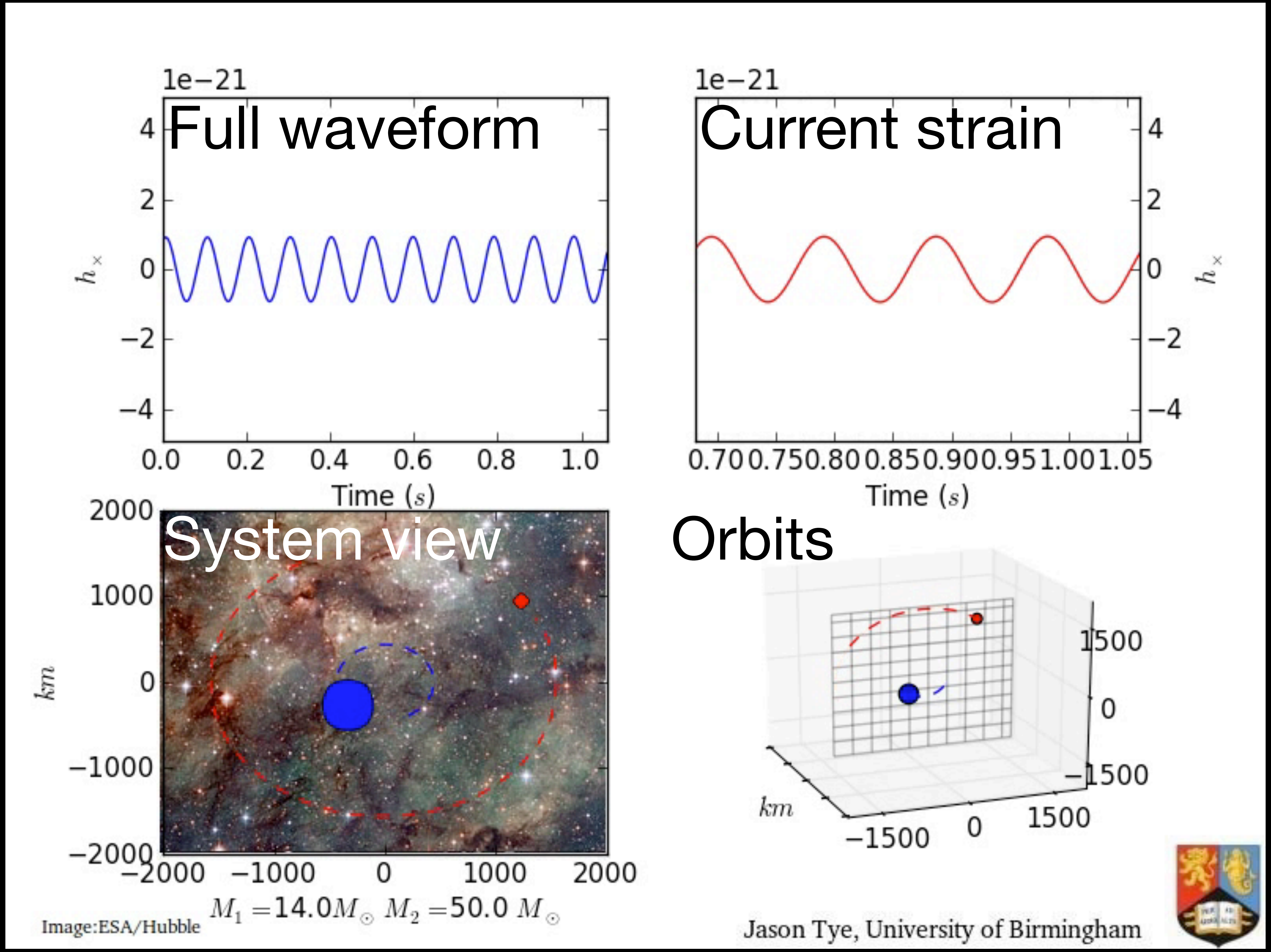
milliseconds — minutes — days — years

Duration of in-band gravitational-wave emission

Images: A. Simonnet/Sonoma State/LIGO, ESO/L. Calçada

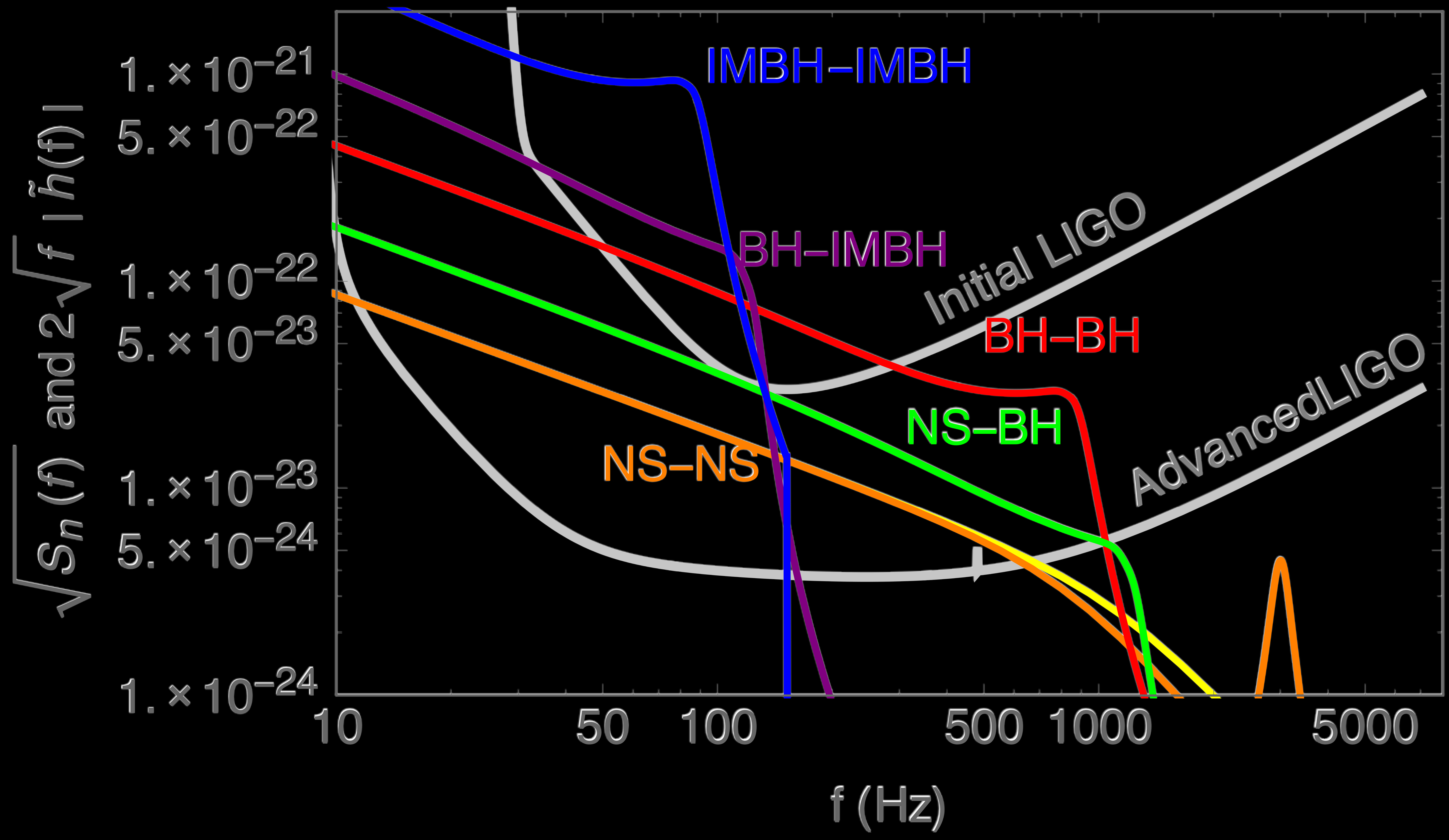
Images: Carl Knox/OzGrav, Kevin Gill Flickr (CC by 2.0)

Orbits and Strain



- Movie:14 M⊙ and 50 M⊙ black holes
- slowed down by a factor of 4 to see/hear detail
- BBH waveform model includes merger/ringdown

Binary mergers



Systems at 200 Mpc

Sky-averaged GW amplitude

(roughly Advanced LIGO NS-NS limit)

NS of 1.35 $M_\odot$

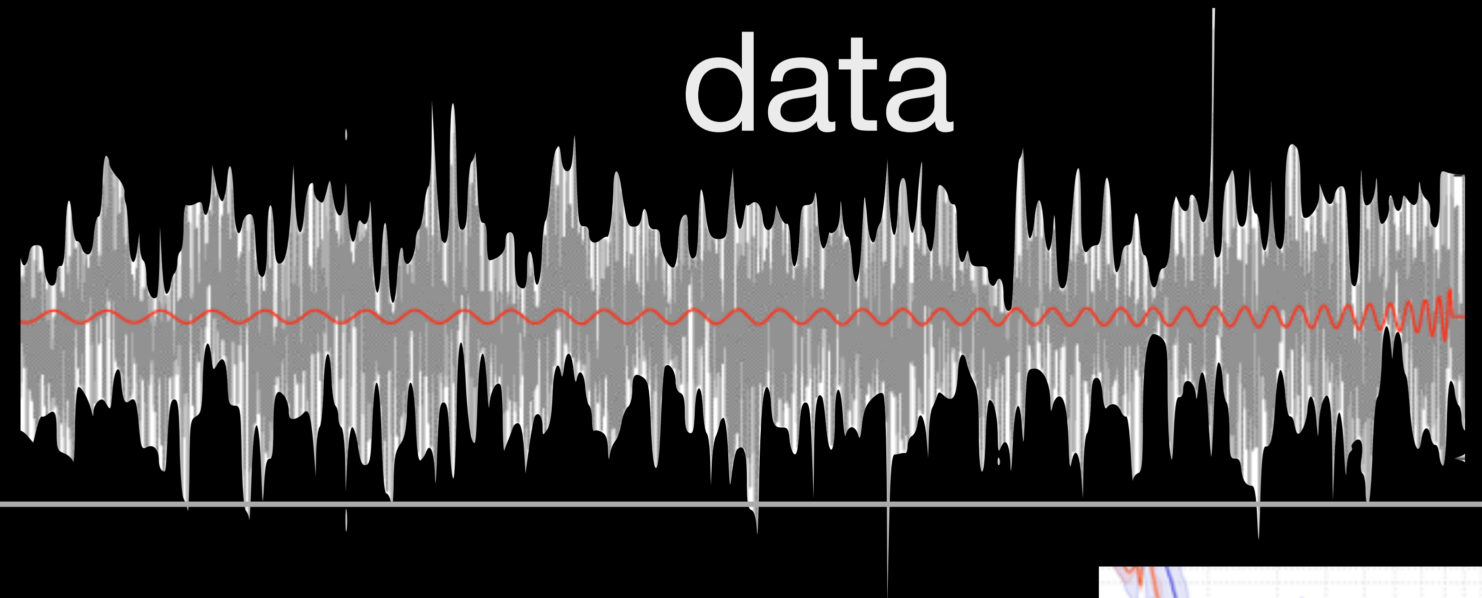
BH of 10 $M_\odot$

IMBH of 100 $M_\odot$

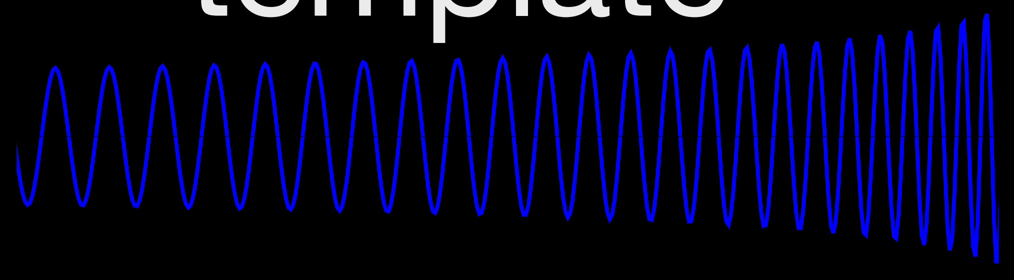
Matched-filter search

- Cross-corellate signal predictions against data over many cycles

data

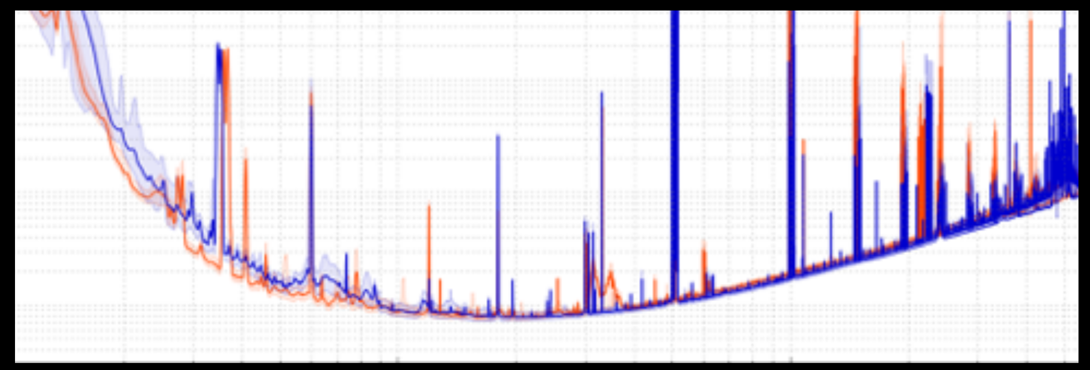


× template

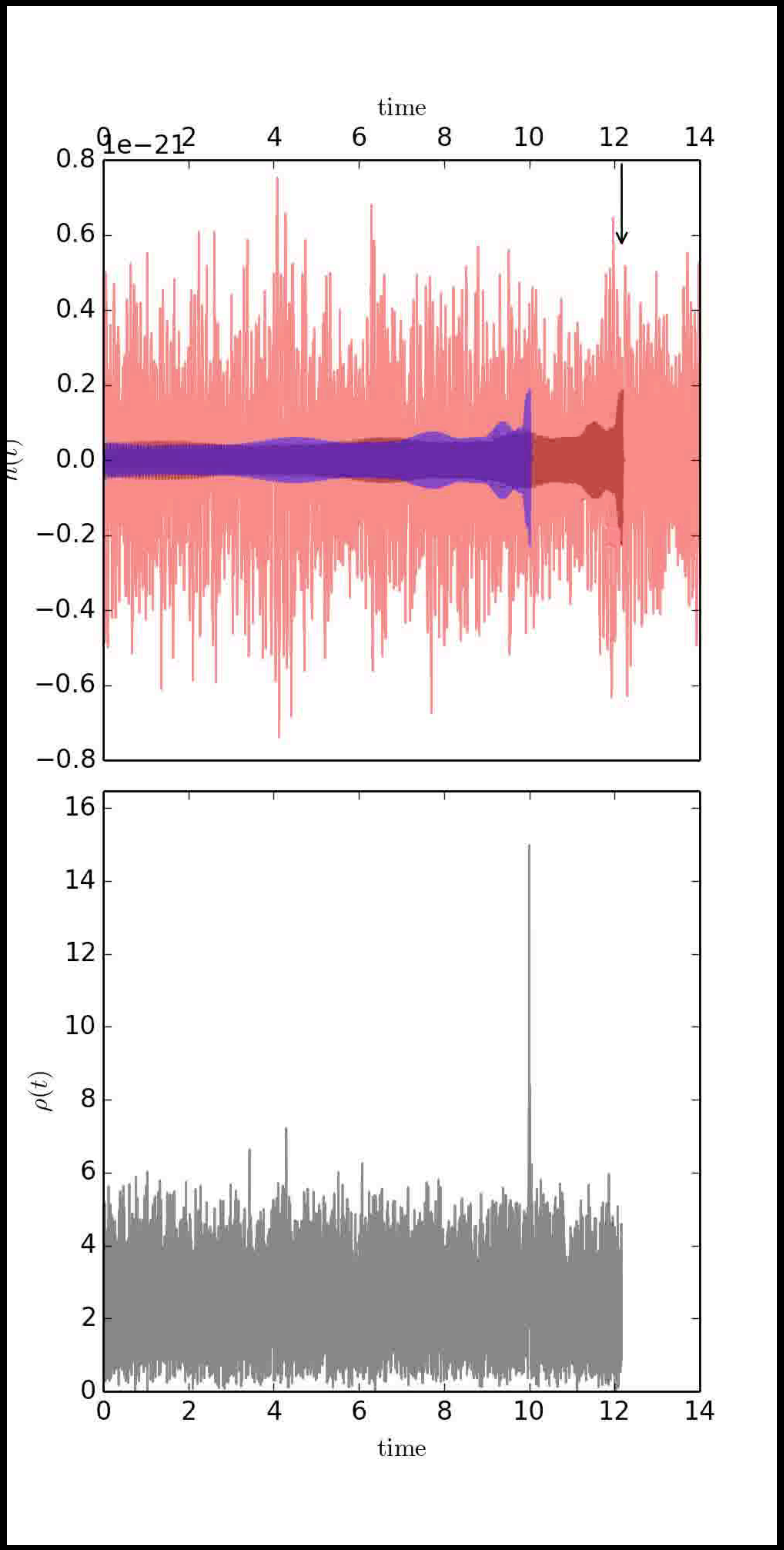


SNR =

noise



Animation by Salvatore Vitale



Bayesian parameter estimation

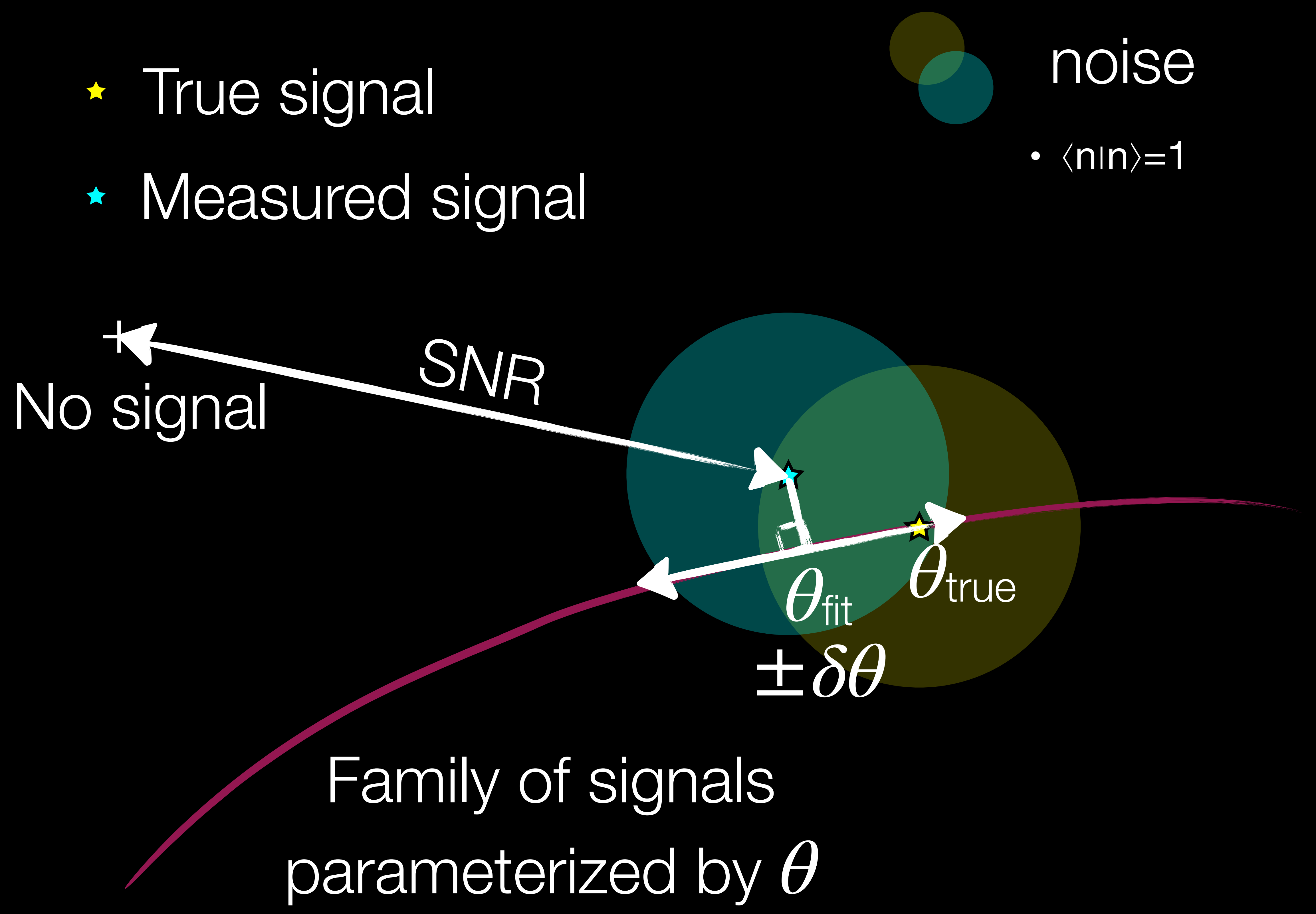
Recorded data

Model prediction

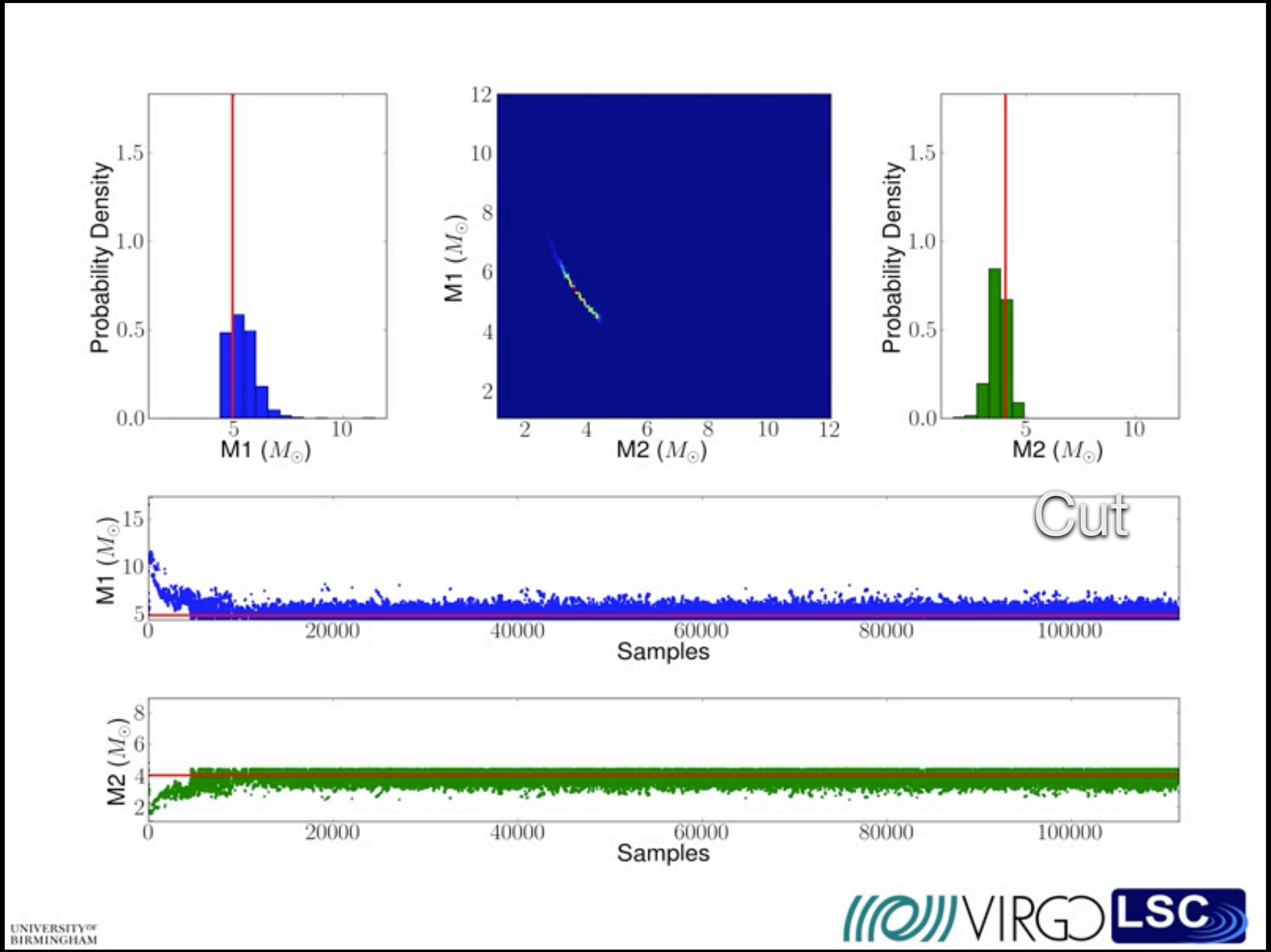
$$\mathscr{L}(d\,|\,\theta) \propto \exp\left(-\sum_k \frac{2\left|d_k - h_k(\theta)\right|}{S_k}\right)$$

Sum over frequency

Noise spectrum (PSD)



See e.g.: Talbot and Thrane arXiv:1809.02293



- LEFT: example estimate of masses http://arxiv.org/abs/1304.1775
- Precision in the "chirp mass":

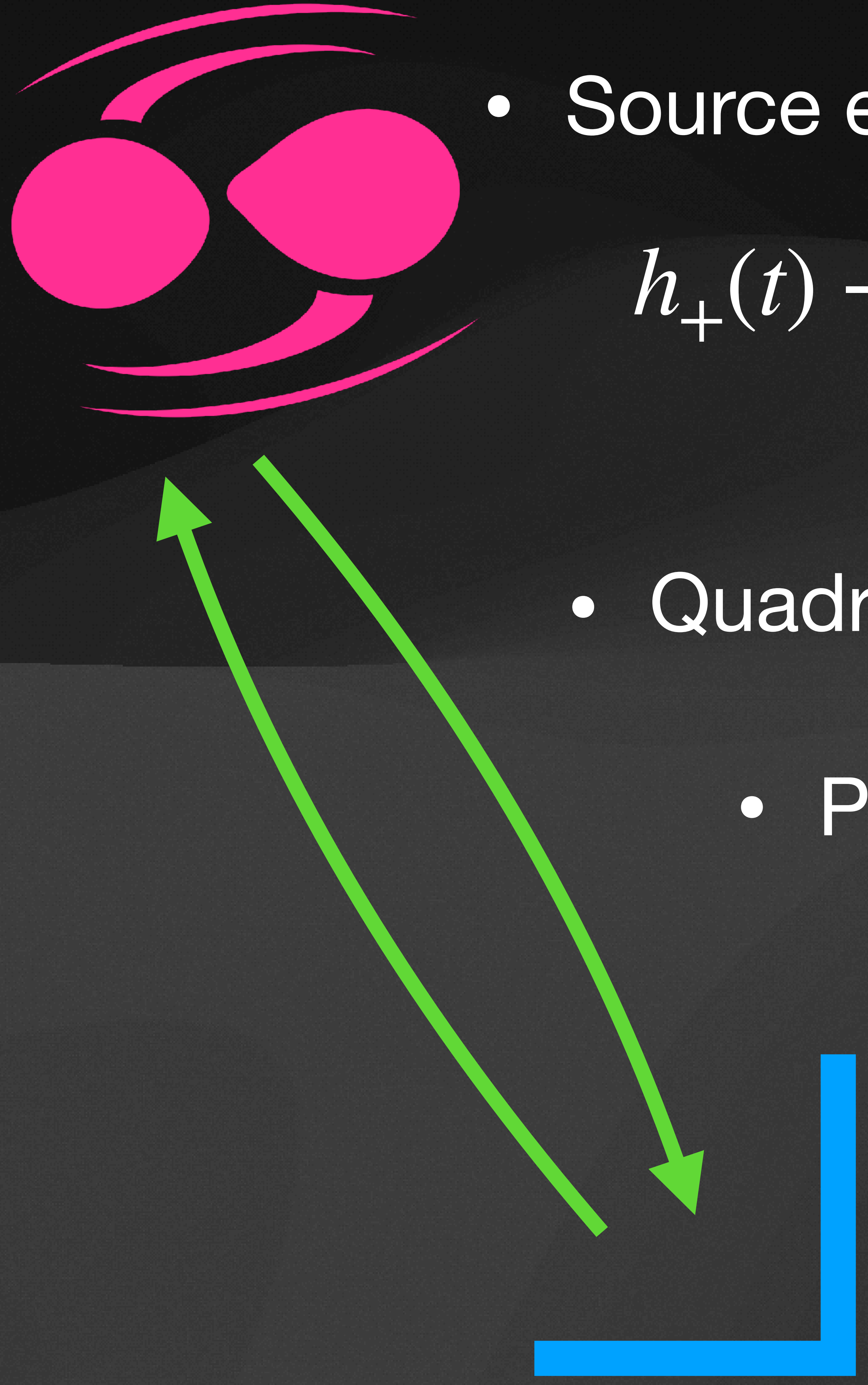
$$\mathscr{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

- Why?

$$\frac{df}{dt} \propto \left(\frac{G\mathscr{M}}{c^3}\right)^{5/3} f^{11/3}\left[1 + \ldots\right]$$

see e.g. Cutler and Flangan, Phys. Rev. D 49, 2658 (1994)

From Source to Strain



- Source emission model: Multipole expansion

$$h_+(t) - ih_\times(t) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} h_{\ell m}(t) Y_{-2}^{\ell m}(\iota, \varphi)$$

- Quadrupole-dominant: $h_{22}(t) = \mathscr{A}(t) e^{i\psi(t)}$

  - Projected onto detectors:

    $$h(t) = F_+(\alpha, \delta, \psi_p) h_+(t) + F_\times(\alpha, \delta, \psi_p) h_\times(t)$$

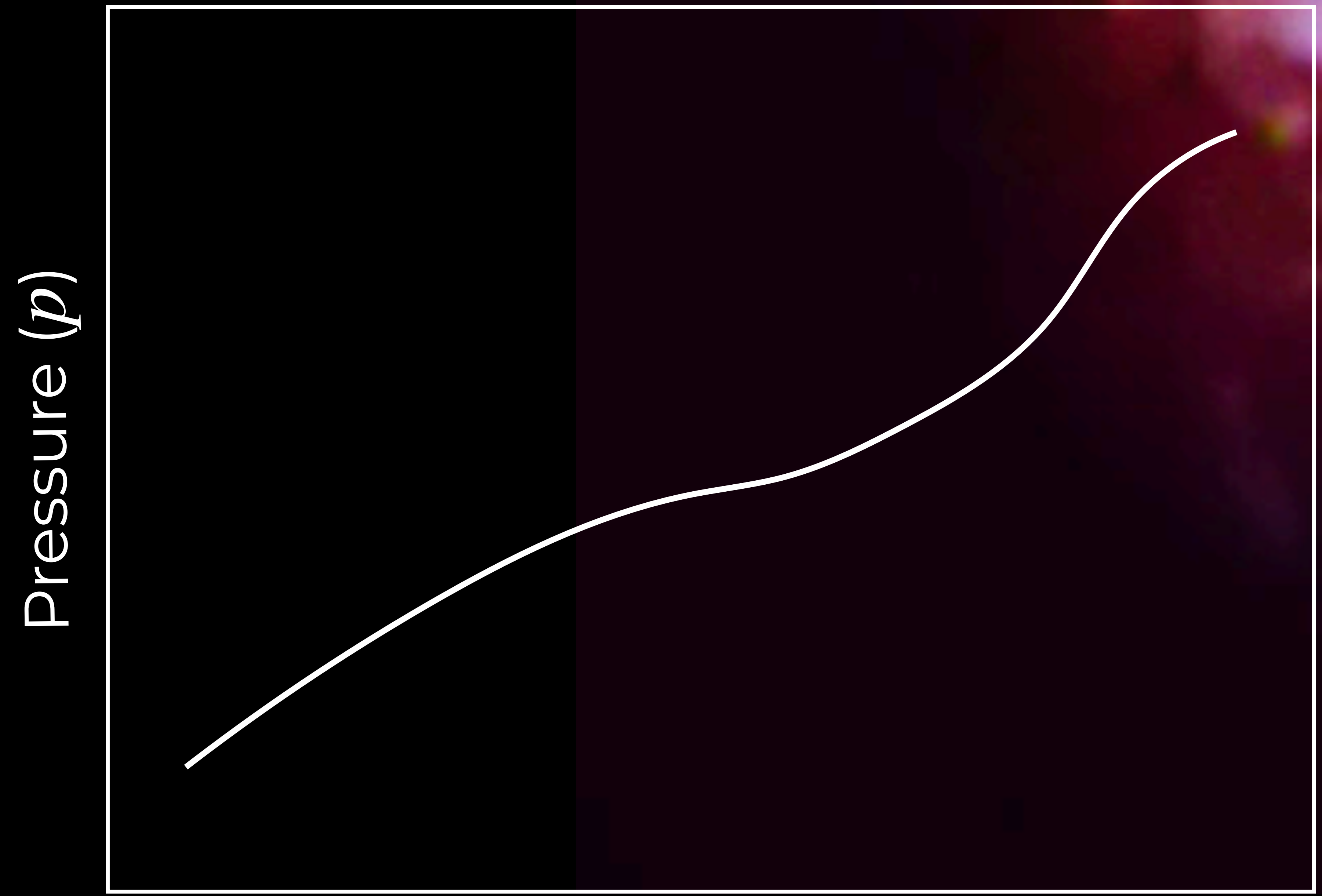
Sky location, orientation, inclination

- Measured strain: $h(t) = \dfrac{Q(\alpha, \delta, \iota, \psi)}{d_L} \mathscr{A}(t) e^{i\psi(t)}$

“Intrinsic properties”

When a star collapses, what stops its fall?

Equation of State



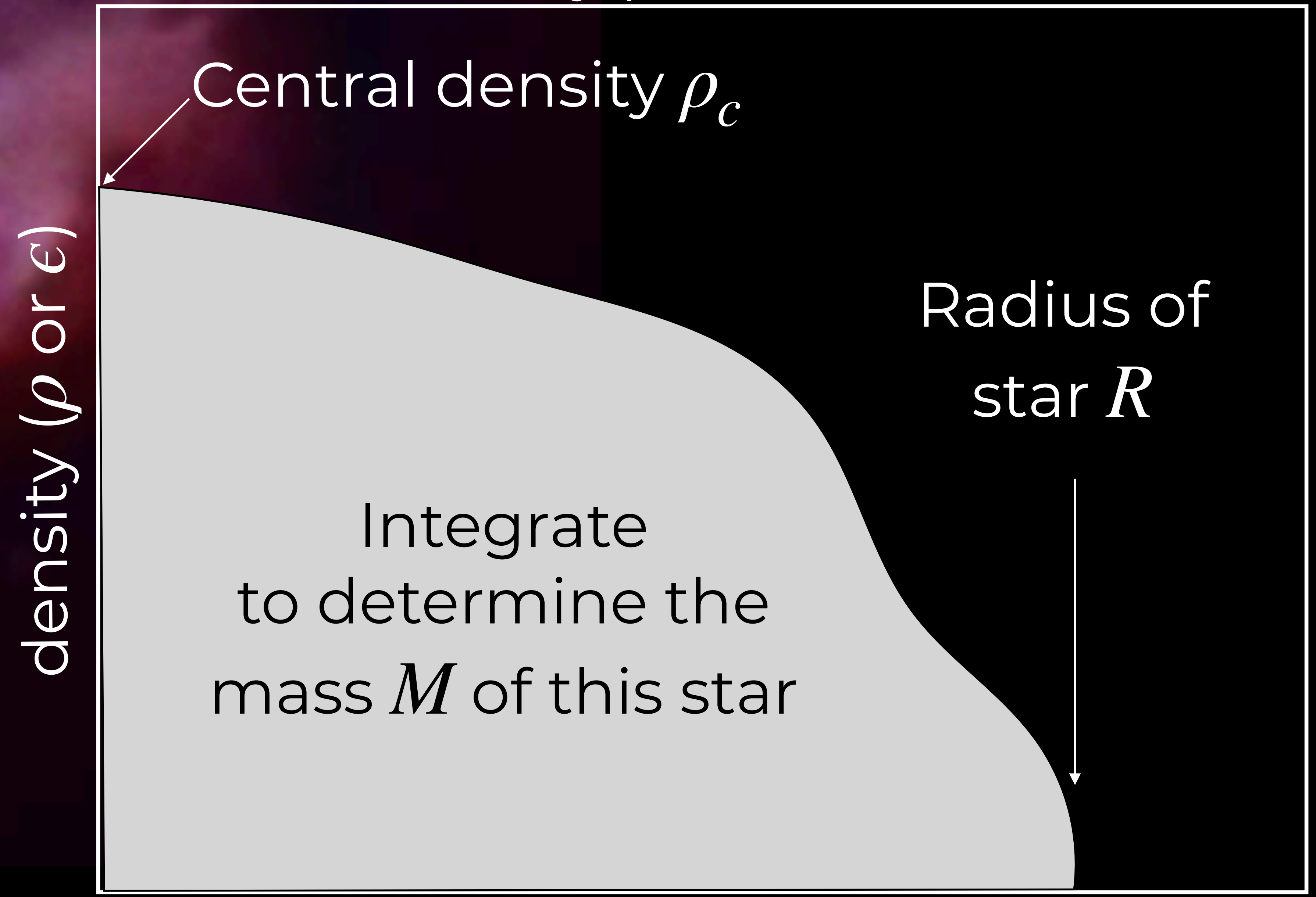
density (n, ρ or ϵ depending)

Hydrostatic Equilibrium

$$\frac{dP}{dr} = -\rho(r)\frac{Gm(r)}{r^2}$$

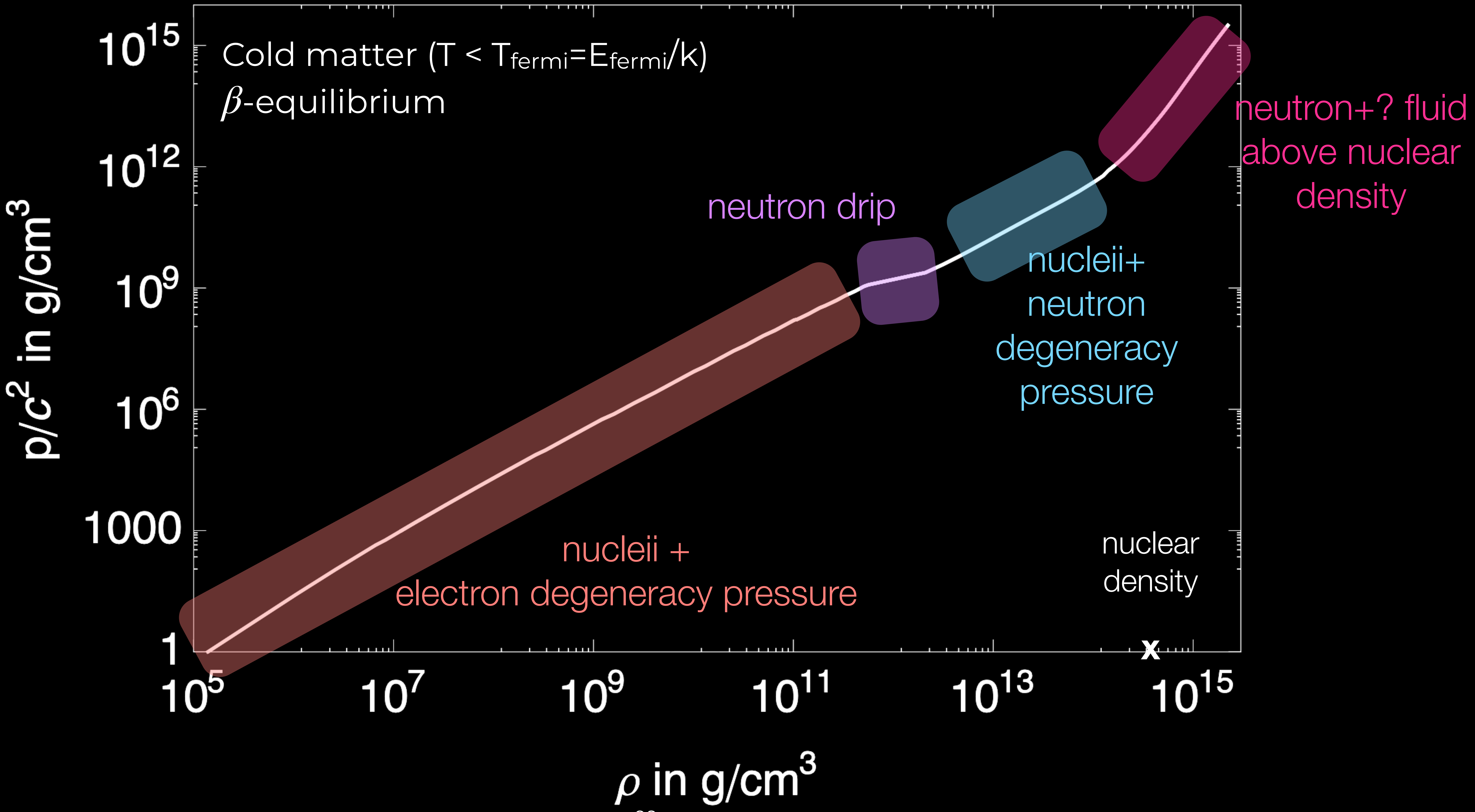
(Newtonian)

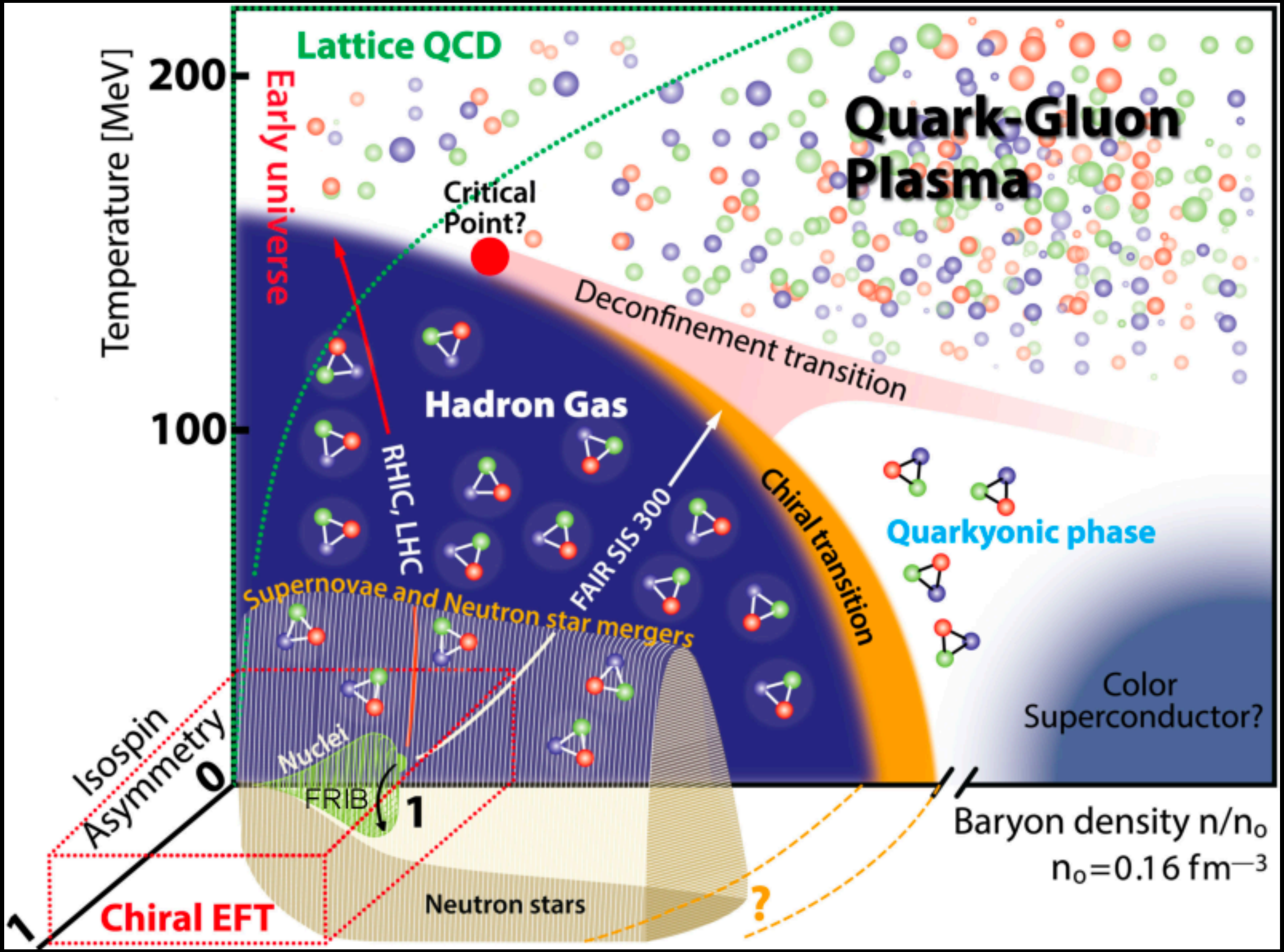
Density profile of a star



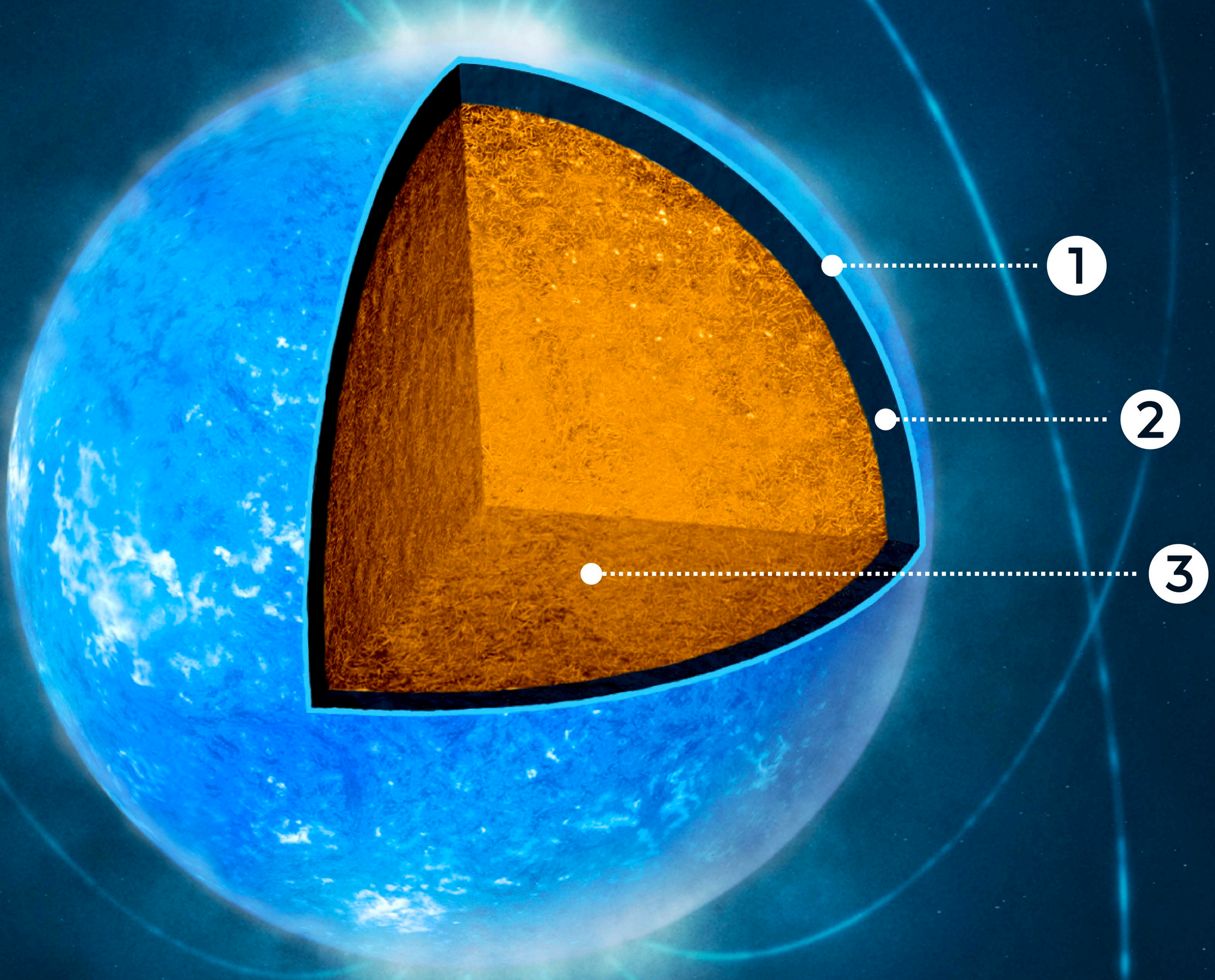
Distance from center r in km

Image: Kes 75, NASA/CXC/M. Gonzalez/F. Gavriil/P. Slane


Cold matter (T < Tfermi=Efermi/k)
β-equilibrium
neutron drip
nucleii+
neutron
degeneracy
pressure
neutron+? fluid
above nuclear
density
nucleii +
electron degeneracy pressure
nuclear
density
x
p/c² in g/cm³
1
1000
10⁶
10⁹
10¹²
10¹⁵
10⁵
10⁷
10⁹
10¹¹
10¹³
10¹⁵
ρ in g/cm³



Drischler, C., Holt, J.W., & Wellenhofer, C. Ann. Rev. Nucl. Part. Sci. 71:403-432 (2021)



1 | **OUTER CRUST** (0.1 km)

NUCLEI
ELECTRONS

2 | **INNER CRUST** (0.5 km)

NUCLEI
ELECTRONS
SUPERFLUID NEUTRONS

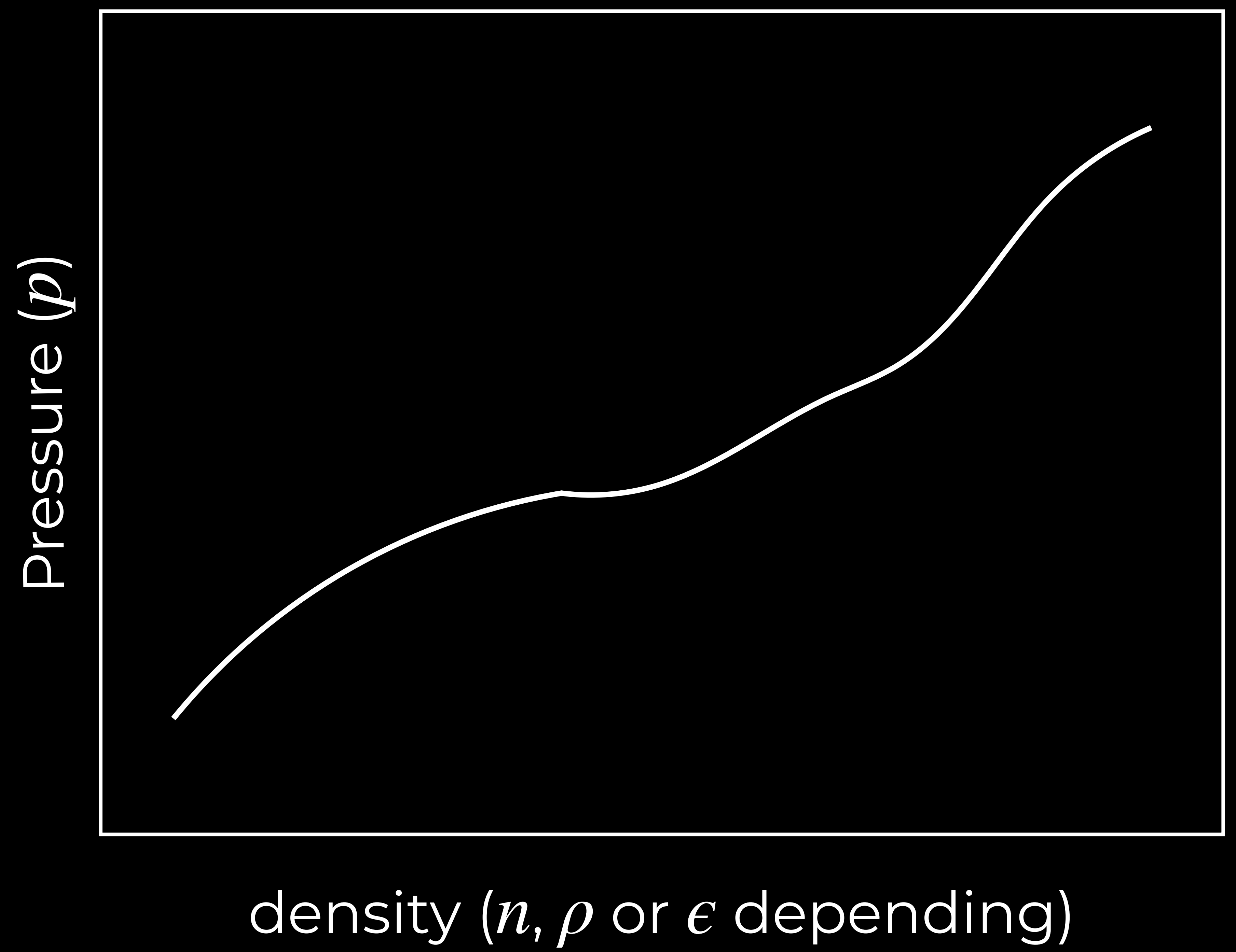
3 | **CORE** (10-13 km?)

SUPERFLUID NEUTRONS
SUPERCONDUCTING PROTONS
HYPERONS?
DECONFINED QUARKS?
COLOR SUPERCONDUCTOR?

Watts et al Rev. Mod. Phys. 88, 021001 (2016)

Family of stars

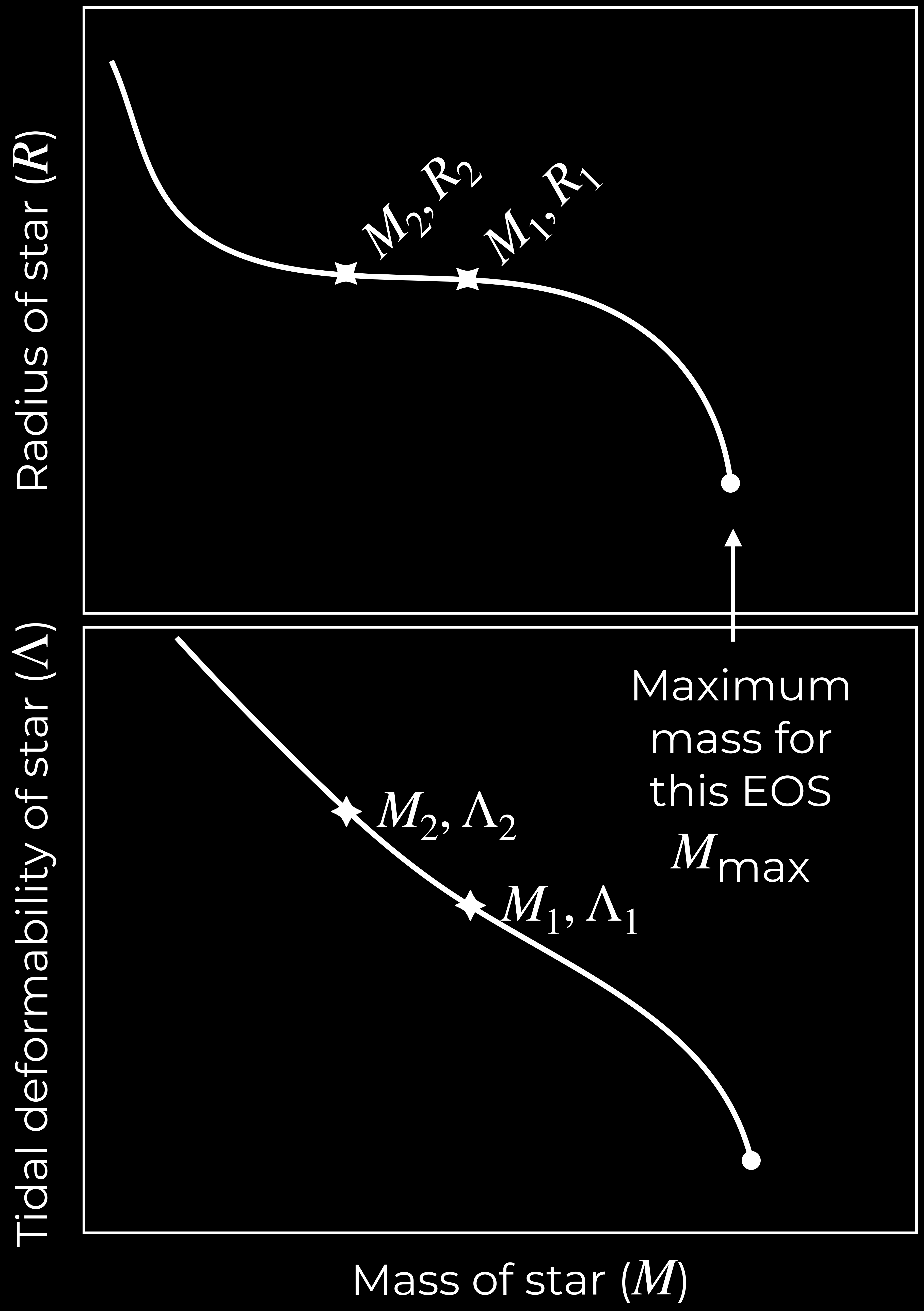
Equation of State

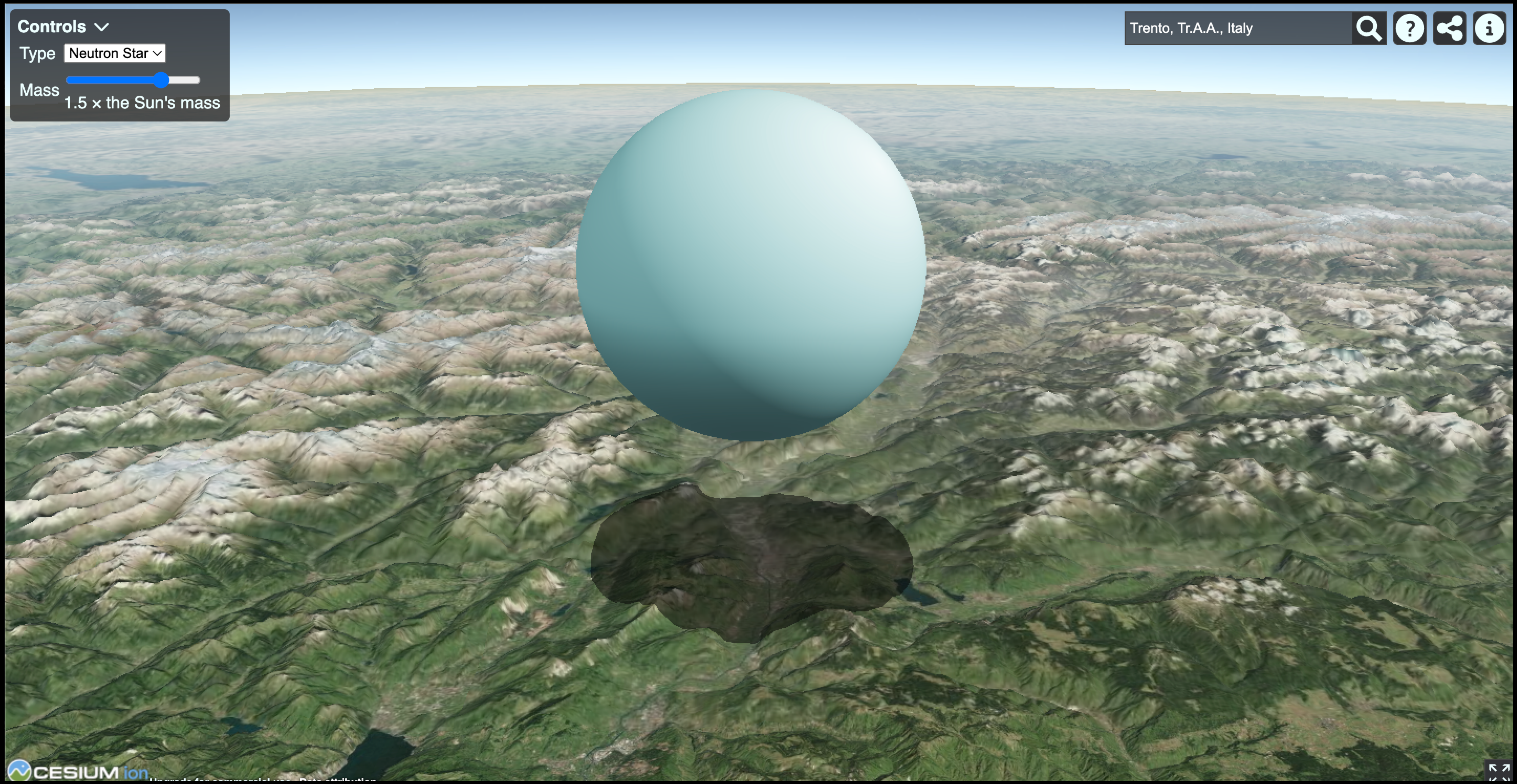


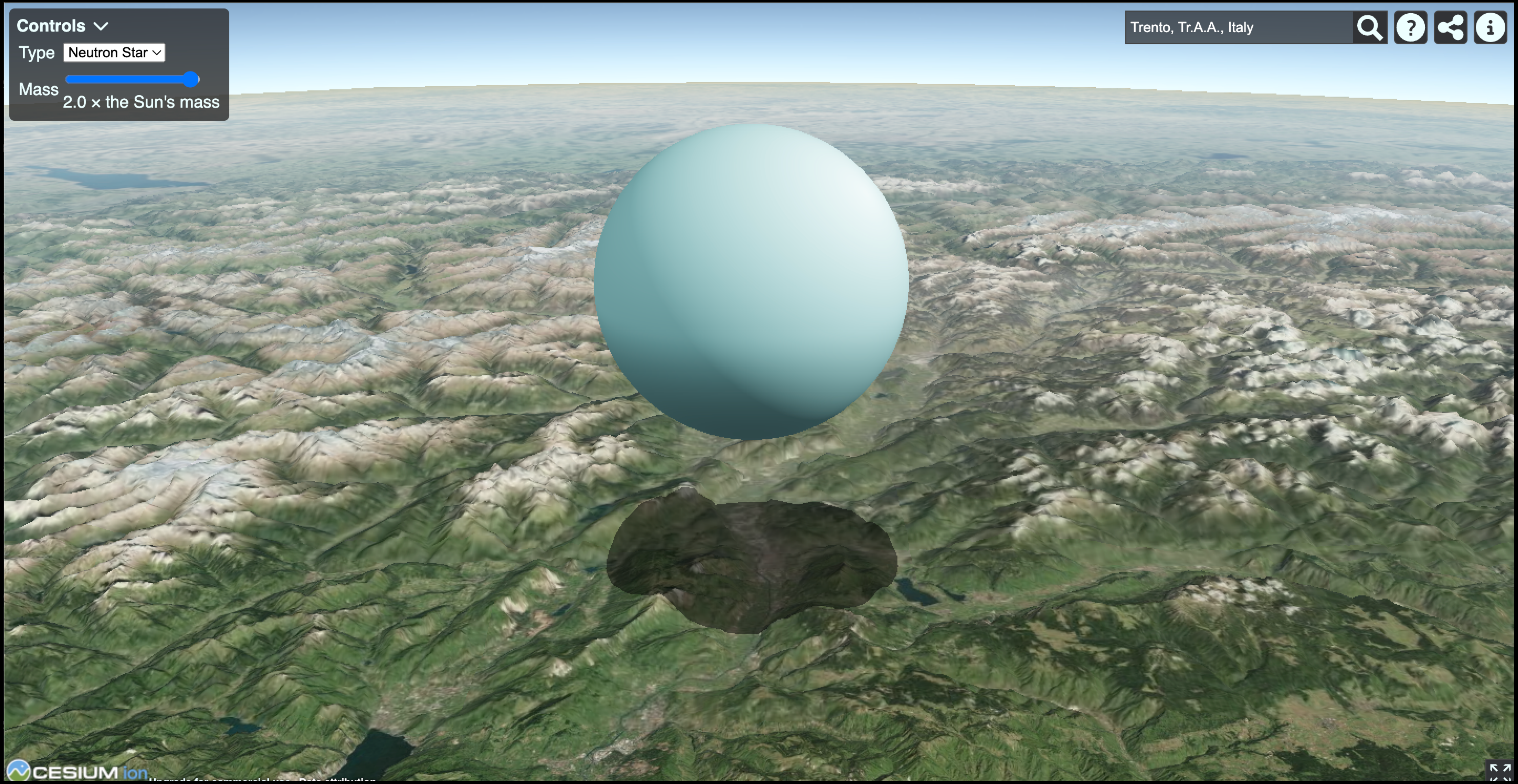
Equilibria for range of central densities, giving range of M

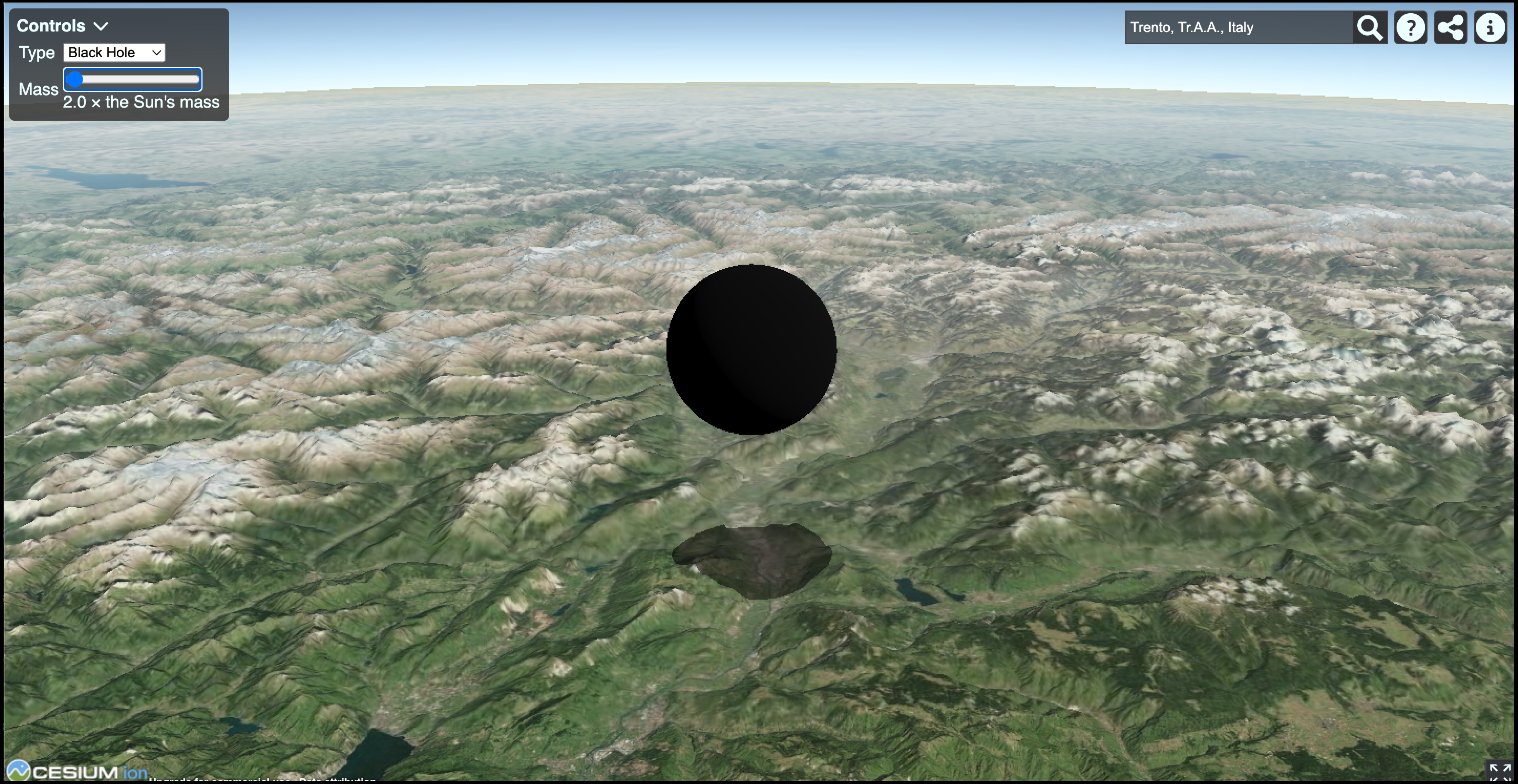
1-1 mapping

Stable stars for a given EOS



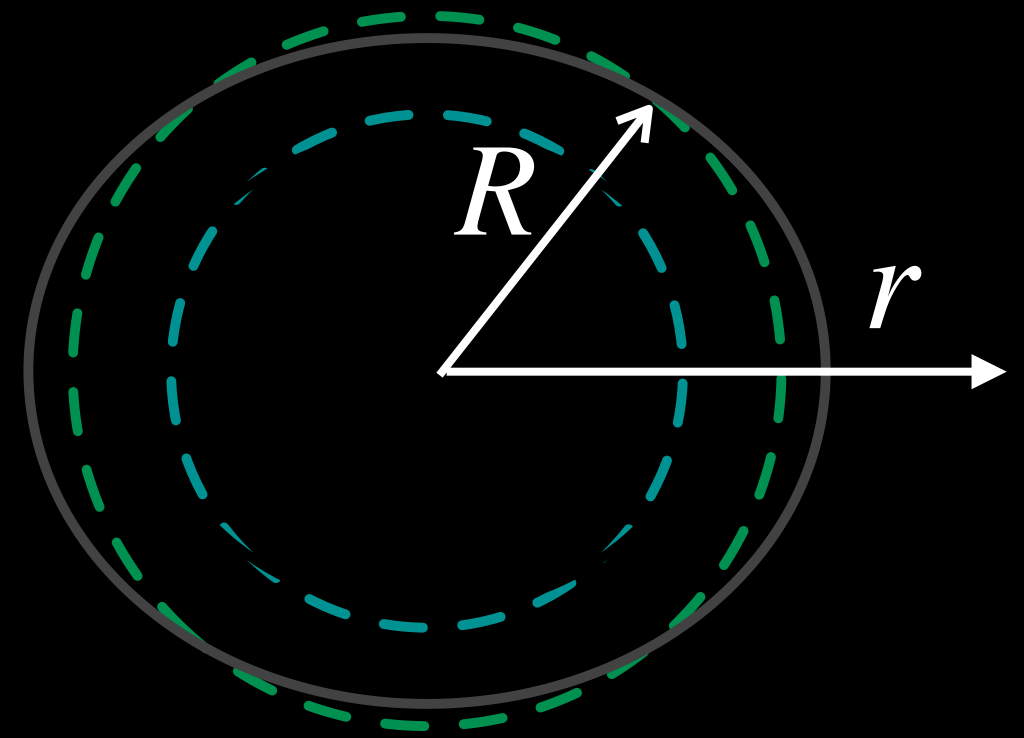

Controls
Type
Neutron Star
Mass
1.5 × the Sun's mass
Trento, Tr.A.A., Italy
CESIUM ion


Controls
Type
Neutron Star
Mass
2.0 × the Sun's mass
Trento, Tr.A.A., Italy
CESIUM ion


Controls
Type
Black Hole
Mass
2.0 × the Sun's mass
Trento, Tr.A.A., Italy
CESIUM ion

Tides: matter responds to a companion

$$\lambda_i = \frac{\text{size of quadrupole deformation}}{\text{strength of external tidal field}} = \frac{2}{3} k_2 R_i^5$$



- quadrupole deformation:

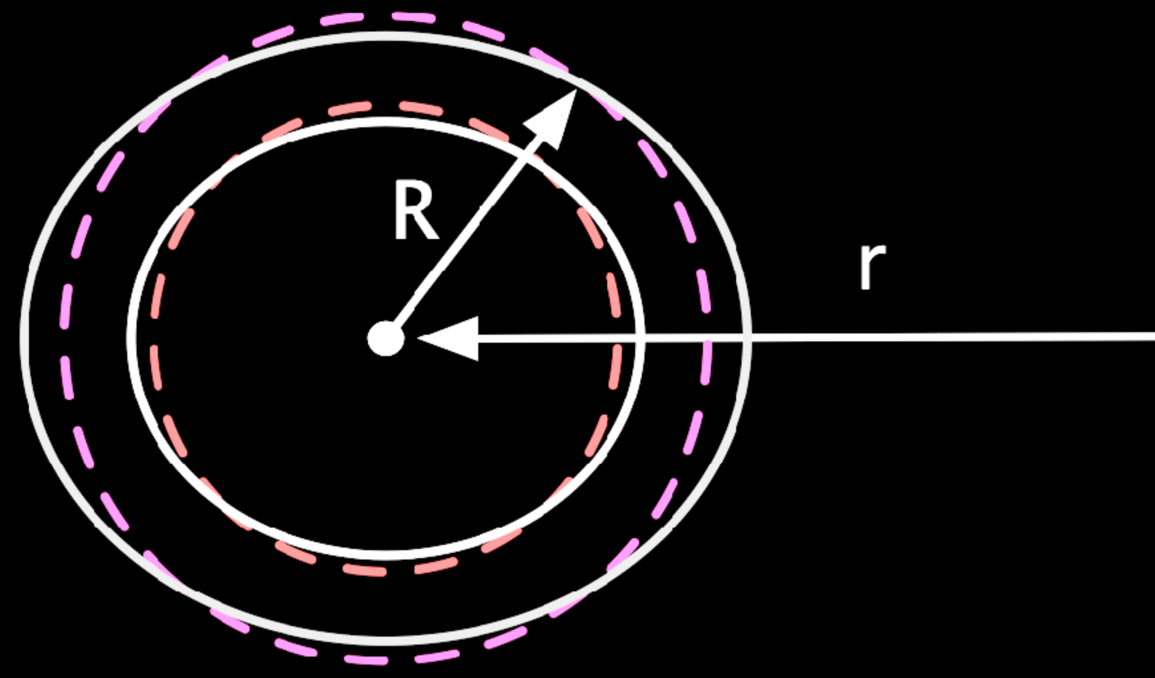
  $\ell = 2$ term in the star's gravitational potential $\sim \dfrac{1}{r^3}$

- k_2 relativistic Love number ≃ 0.05–0.15 ($k_2 = 0$ for BH)
  - Mass distribution inside the star (polarization), not just surface R

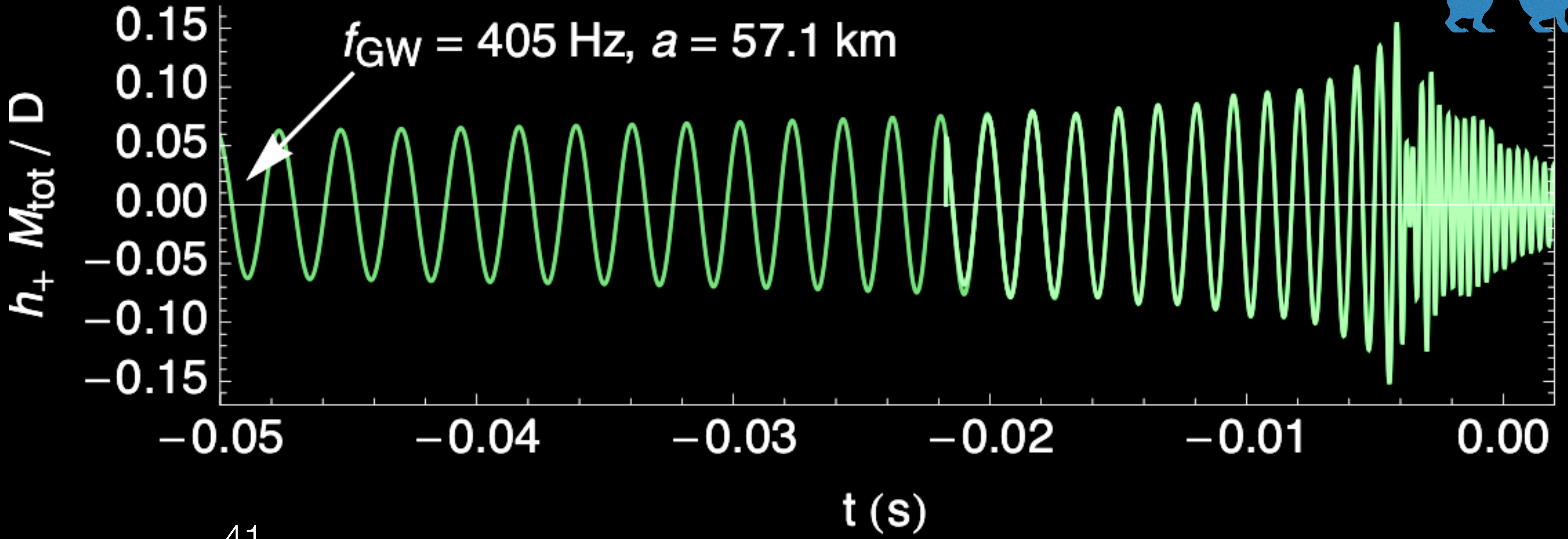
Dimensionless form: $$\Lambda_i = \frac{\lambda_i}{m_i^5} = \frac{2}{3} k_2 \left(\frac{R_i}{m_i}\right)^5$$

Imprint of matter

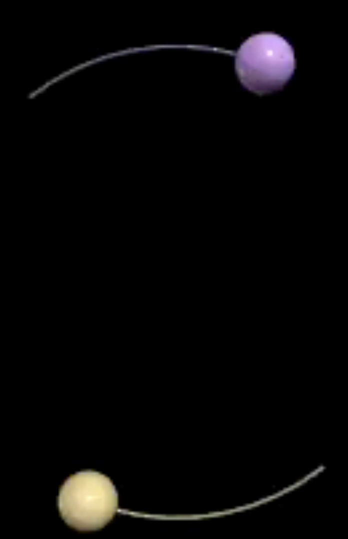
- Additional orbital energy lost to the deformation of the stars
- Tidal bulges add a little extra quadrupole, GW luminosity

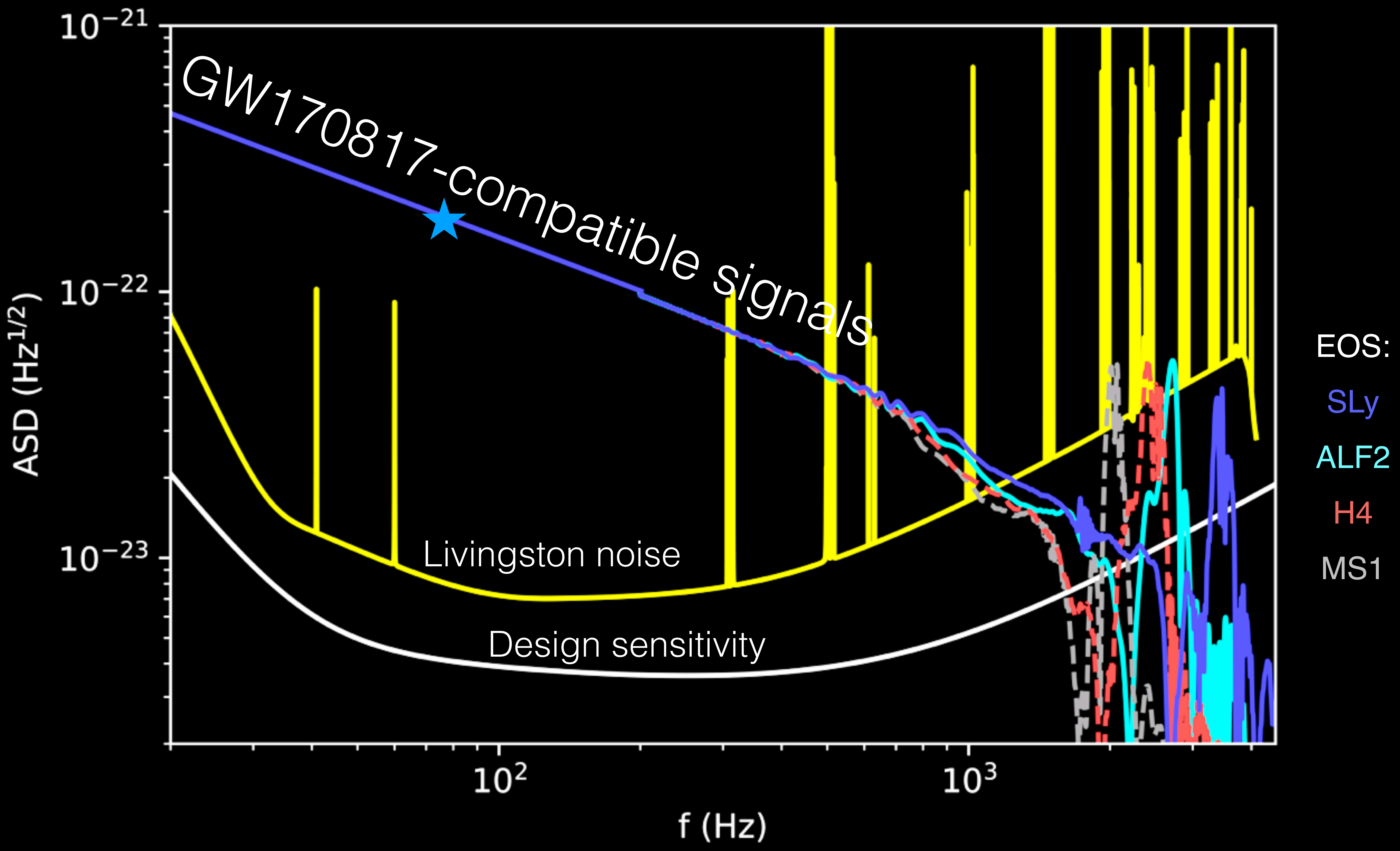


$$\frac{da}{dt} = \frac{-\mathscr{L}_{GW} - \mathscr{L}_{GW,def}}{dE_{orb}(a)/da + dE_{def}(a)/da}$$



Observing neutron star mergers



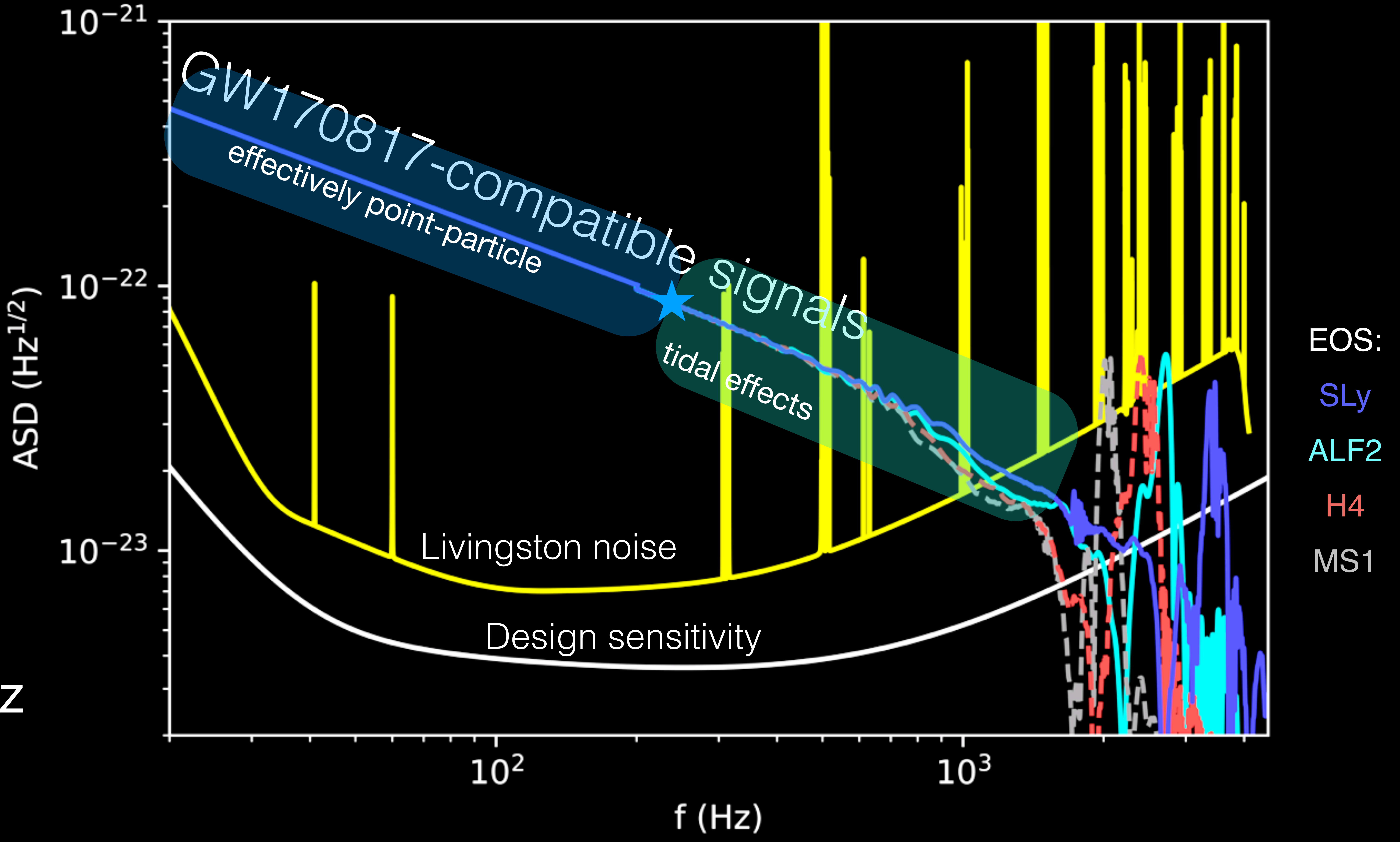


5 seconds before merger
orbital distance ~190 km
GW frequency ~70 Hz

Movie by
Megan Loh, CSUF

E. Leon/LIGO/Virgo. Noise curves from LIGO-P1800061-v11. Effective distance from GraceDB.
Numerical simulation data (above ~500 Hz) courtesy Tim Dietrich (AEI/FSU/BAM Collaboration)
Simulations published in Phys. Rev. D95(12):124006 and Phys. Rev. D95(2):024029

Observing neutron star mergers



Final 0.25 seconds
orbital distance 90 – 24 km
GW frequency 210 – 1600 Hz

Movie by
Megan Loh, CSUF

E. Leon/LIGO/Virgo. Noise curves from LIGO-P1800061-v11. Effective distance from GraceDB.
Numerical simulation data (above ~500 Hz) courtesy Tim Dietrich (AEI/FSU/BAM Collaboration)
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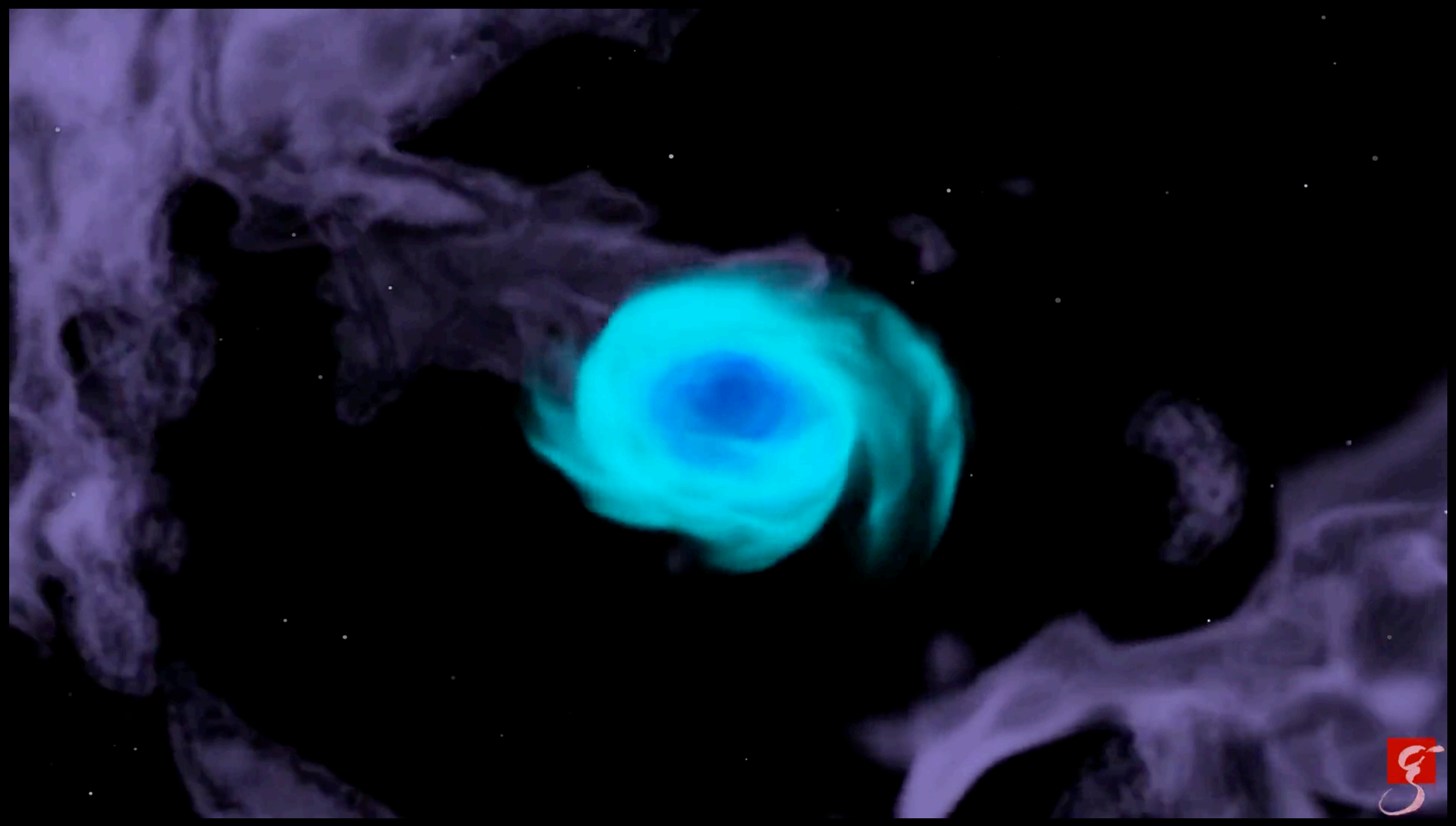
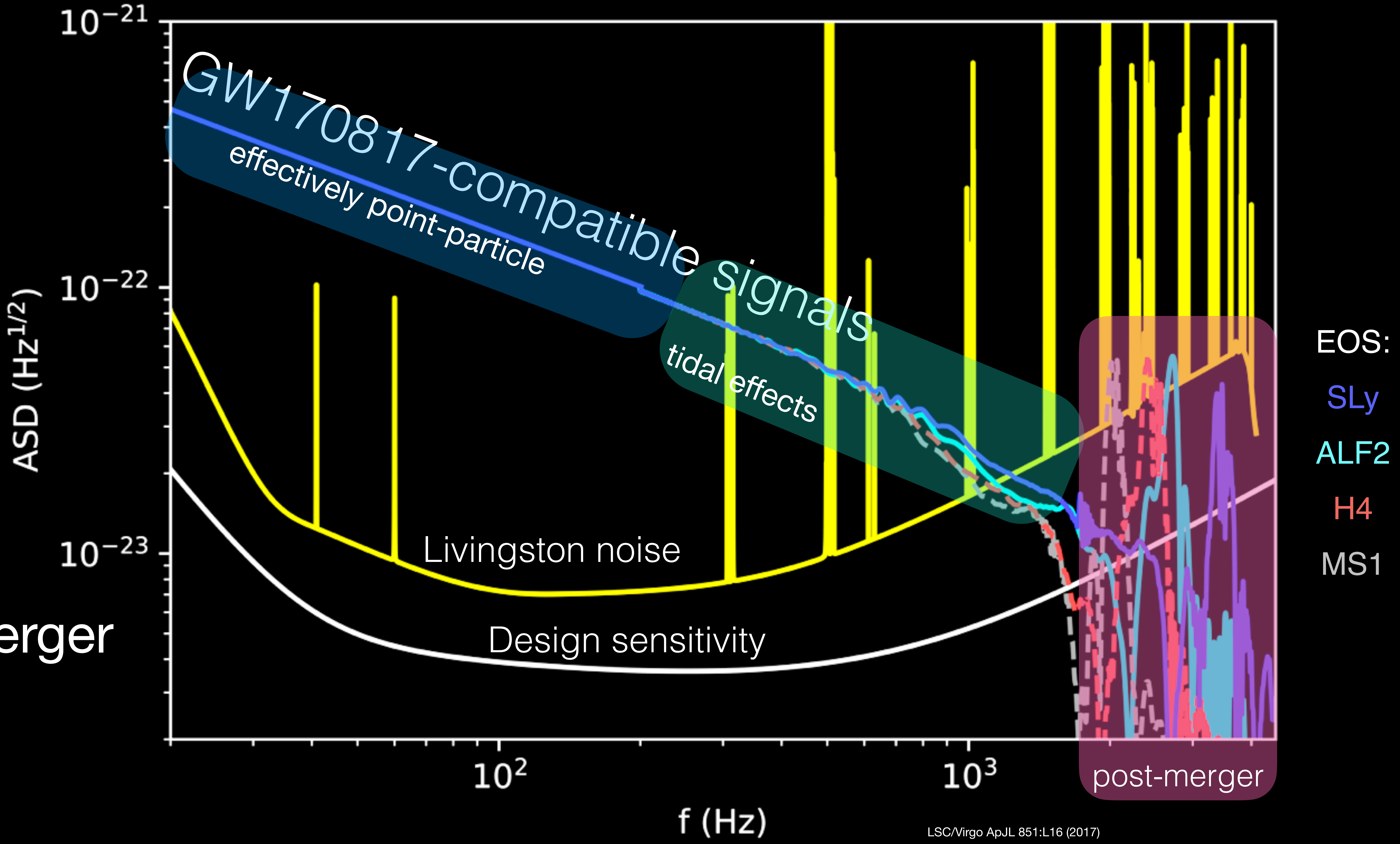
Observing neutron star mergers





Final 40 milliseconds
of inspiral, tens of ms post merger
orbital distance $\lesssim$ 30 km
GW frequency >1000 Hz

Neutron-star merger simulation:
T. Dietrich, S. Ossokine,
H. Pfeiffer, A. Buonanno (AEI)

E. Leon/LIGO/Virgo. Noise curves from LIGO-P1800061-v11. Effective distance from GraceDB.
Numerical simulation data (above ~500 Hz) courtesy Tim Dietrich (AEI/FSU/BAM Collaboration)
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