ECT* Workshop on
The physics of strongly interacting matter: neutron stars, cold atomic gases and related systems

Trento, 22-26 April 2024

Propagation of Sound in Density Modulated Superfluids

UNIVERSITÀ DI TRENTO

Sandro Stringari

CNR-INO ISTITUTO NAZIONALE DI OTTICA CONSIGLIO NAZIONALE DELLE RICERCHE

Pitaevskii Center on Bose-Einstein Condensation

In a uniform single component superfluid at **zero temperature** (superfluid He4, He3, neutron matter, Bose and Fermi degenerate gases) the superfluid density is expected to coincide with the total density and the velocity of sound at T=0 is fixed by the compressibility according to the hydrodynamic relation

$$\boxed{mc^2 = \kappa^{-1}}$$

where κ^{-1} is the inverse compressibility $\kappa^{-1} = n\partial\mu / \partial n$

Main motivations for studying density modulated superfluids:

- Many experiments available in cold atoms in the presence of **optical lattices** (e.g. Superfluid/Mott Insulator transition)

- Recent availability of **supersolid configurations** in ultracold atomic gases

- Fermi superfluidity in the **inner crust** of **neutron stars**

- Recent interest in **Leggett's bound** to superfluid fraction

Breaking of translational invariance can be the consequence of

- External potentials (ex. optical lattices, disorder)
- Spontaneous breaking of translational symmetry (supersolids)

Part 1 $[H, P_x] \neq 0$

- Translational invariance is broken by external periodic perturbation (**dilute BEC** and **unitary Fermi gases** in a **box**)
- Only a single class of Goldstone modes is present, due to spontaneous breaking of phase symmetry (**not** a supersolid)

$\Downarrow$

(superfluid phonons)

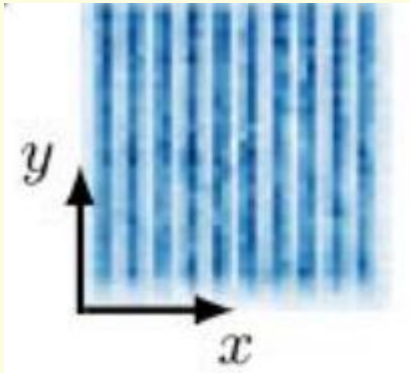
Part 2 $[H, P_x] = 0$

- Both phase symmetry and translational invariance are broken spontaneously (**supersolid BEC gas** in a **ring**)
- Two classes of Goldstone modes

$\Downarrow$

(phonons of mixed superfluid and crystal nature)

The case of a dilute Bose-Einstein condensate confined in a box



- Application of the 1D periodic perturbation $V(x) = V_0 \cos\left(x 2\pi / d\right)$ gives rise to **stripes**

- In a dilute BEC gas, described by Gross-Pitaevskii theory one can prove that the **superfluid fraction** (along x) **coincides with Leggett's upper bound** (1970,1998)

$$f_{S,x} \equiv \frac{\rho_{S,x}}{\bar{\rho}} = \left(\frac{\bar{n}}{L} \int \frac{dx}{n(x)} \right)^{-1}$$

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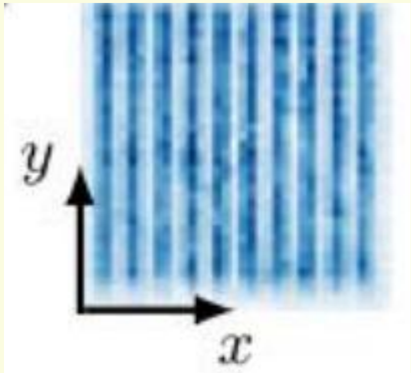

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- On the other hand hydrodynamic theory of superfluids predicts the anisotropic result for the **sound velocities**, yielding result

$$mc_x^2 = f_{S,x} / \kappa$$
$$mc_y^2 = 1 / \kappa$$

$$f_{S,x} = c_x^2 / c_y^2$$

for the superfluid fraction (avoiding determination of κ)

A recent exp/theory collaboration with Jean Dalibard's team at the Collège de France, has confirmed the **consistency** of the determination of the superfluid density based on the **independent** measurement of **Leggett's integral** and of the **sound velocities**

PHYSICAL REVIEW LETTERS **130,** 226003 (2023)

Editors' Suggestion

Superfluid Fraction in an Interacting Spatially Modulated Bose-Einstein Condensate

G. Chauveau, C. Maury, F. Rabec, C. Heintze, G. Brochier, S. Nascimbene, J. Dalibard, and J. Beugnon[*]
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and Trento Institute for Fundamental Physics and Applications, INFN, 38123 Trento, Italy

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PHYSICAL REVIEW LETTERS **131**, 163401 (2023)

Featured in Physics

Closely related paper

Observation of Anisotropic Superfluid Density in an Artificial Crystal

J. Tao,* M. Zhao,* and I. B. Spielman
Joint Quantum Institute, University of Maryland and National Institute of Standards and Technology, College Park, Maryland 20742, USA

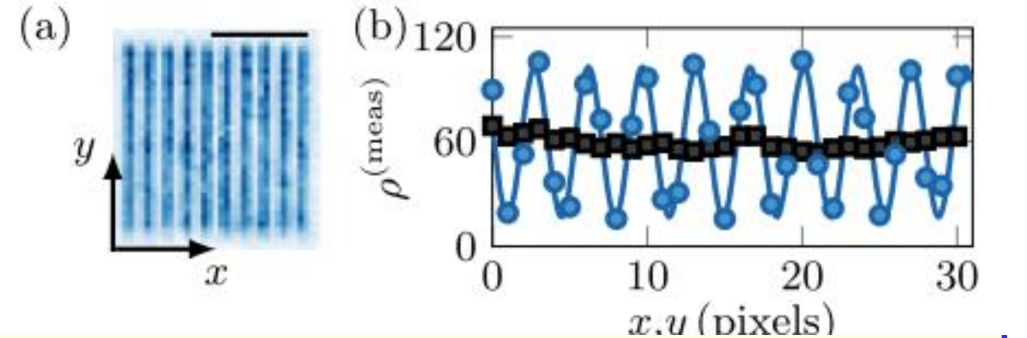
- Measurement of Leggett's integral

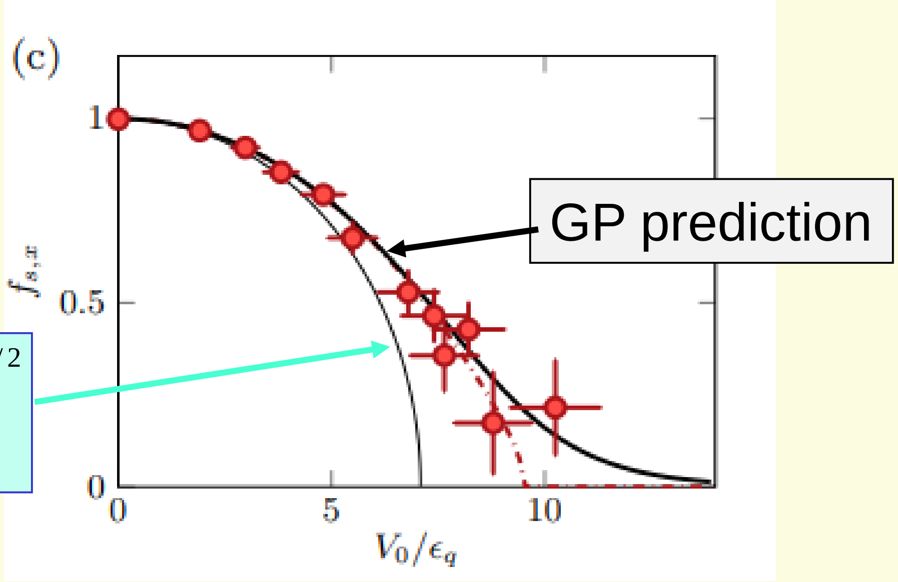
(Chauveau et al. PRL 133, 226003)

$$f_{S,x} = \frac{\rho_S}{\bar{\rho}} = \left(\frac{\bar{\rho}}{L} \int \frac{dx}{\rho(x)} \right)^{-1}$$

N=10^5 atoms a box of L= 40 microns

Due to large period of density modulations (3.94 microns) in-situ density distribution is measurable, accounting for finite optical resolution

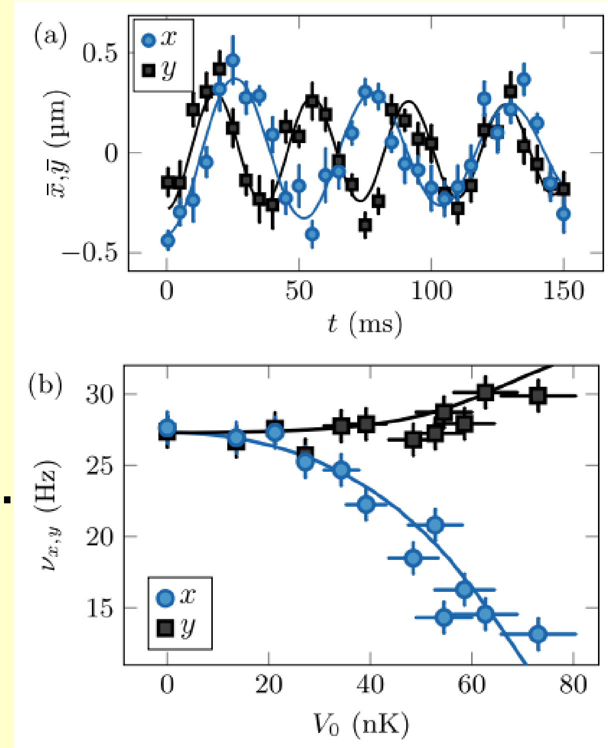




$$f_{S,x}^{LDA} = \left(1 - \frac{V_0}{\mu}\right)^{1/2}$$

- Measurement of sound velocities

(Chauveau et al. PRL 133, 226003)



Sound is excited by suddenly removing a weak linear perturbation generated along x or y and measuring the time evolution the center of mass of the cloud.

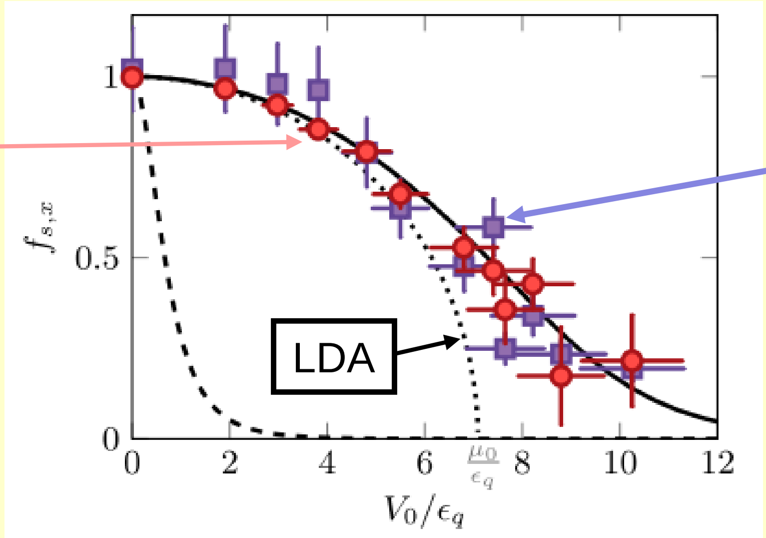
The speed of sound is determined by the HD relation $\boxed{c_{x,y} = 2L\upsilon_{x,y}}$, valid if both healing length and period of the potential are much smaller than phonon wave length 2L.

- Excellent agreement with theory predictions based on TDGP equation (full lines)

Comparison between experimental results for superfluid fraction obtained using

Leggett's approach and ratio of sound velocities

$$f_{Sx} = \left(\frac{\bar{n}}{L} \int \frac{dx}{n(x)} \right)^{-1}$$



$$f_{s,x} = \frac{c_x^2}{c_y^2}$$

provides a **consistent** understanding of the suppression of the superfluid fraction in the presence of a periodic potential, in agreement with the predictions of GP theory

Chauveau et al. PRL 133, 226003 (2023)

Validity of Leggett's bound $f_{S,x}^{L} = \left(\frac{\bar{n}}{L} \int \frac{dx}{n(x)} \right)^{-1}$, as a measure of superfluid fraction,
is however limited to dilute Bose gas and to factorized density profiles $n(x,y) = f(x)f(y)$

Important deviations between Leggett's bound and actual value of superfluid fraction take place

- in **Fermi superfluids**,

- if **density** profile is **not factorized**
  (e.g. triangular optical lattice, isotropic disorder)

- in systems **violating Galilean invariance**
  (e.g. spin-orbit coupled superfluids)

Actual value of superfluid density in density modulated Fermi superfluids can be relevant in the problem of the **inner crust of neutron stars**

PRL **119,** 062701 (2017) PHYSICAL REVIEW LETTERS week ending 11 AUGUST 2017

Superfluid Density of Neutrons in the Inner Crust of Neutron Stars: New Life for Pulsar Glitch Models

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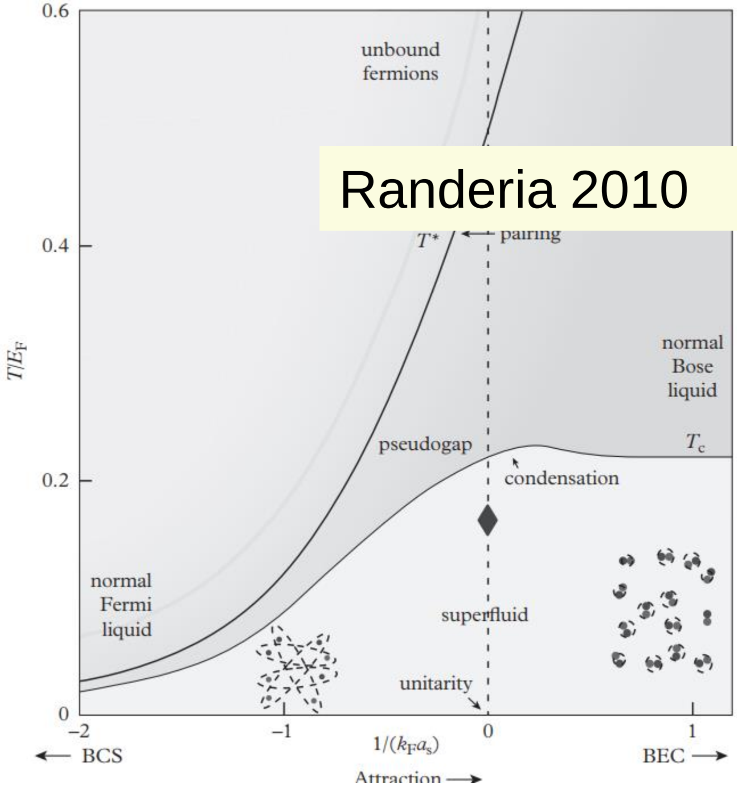
(Received 28 April 2017; revised manuscript received 23 June 2017; published 10 August 2017)

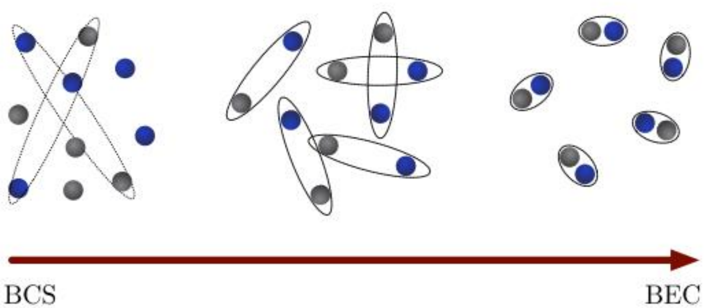
Can we get useful insight studying the behavior of superfluidity in strongly interacting atomic Fermi gases in the presence of an external periodic potential ?

Recent collaboration with
Giuliano Orso (Lab MPQ, France)

Fermi superfluidity along the BCS-BEC crossover



Randeria 2010



Cooper pairs along the crossover

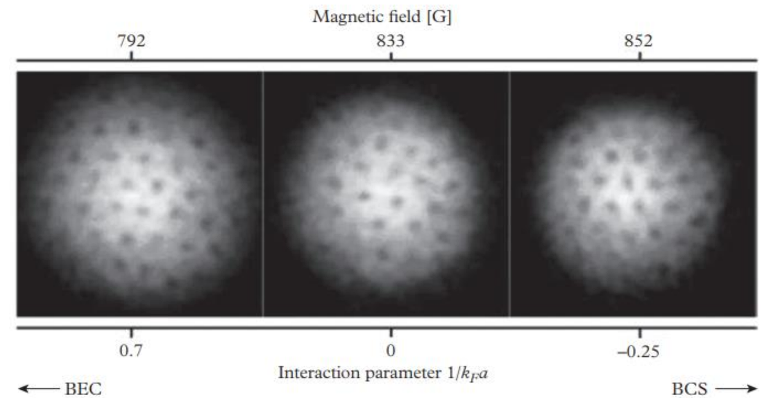


Figure 19.12 Experimental observation of quantized vortices in a superfluid Fermi gas along the BCS–BEC crossover. From Zwierlein et al. (2005b).

Measured vortices along the crossover

Rigorous results for superfluid fraction, based on phase twist method, and for Leggett's bound, in the presence of external periodic potentials available using sum rule techniques:

$$\boxed{\begin{aligned} f_{S,x} &= 1 - 2V_0^2 m_1(q) m_{-3}(q) \\ f_{S,x}^L &= 1 - 2V_0^2 (m_{-1}(q))^2 \end{aligned}}$$

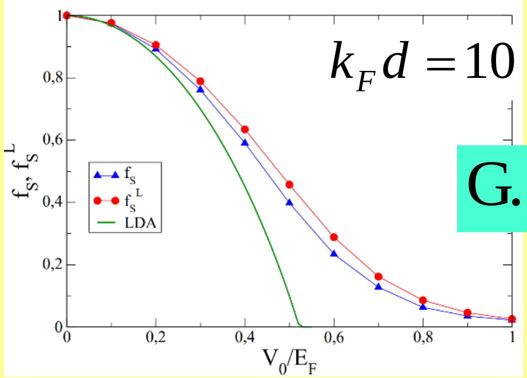
$$m_p(q) = \int_0^\infty d\omega \omega^p S(q,\omega)$$

Comparison shows that only if identity $\boxed{\frac{m_1(q)}{m_{-1}(q)} = \frac{m_{-1}(q)}{m_{-3}(q)}}$ holds will Leggett's bound coincide with actual value of superfluid fraction. This is the case of

- **Weakly interacting BEC gas** (Bogliubov excitation spectrum)
- **Phonon regime of small q** where all relevant sum rules are exhausted by phonon mode. True also for strongly interacting superfluids, including unitary Fermi gas and liquid Helium

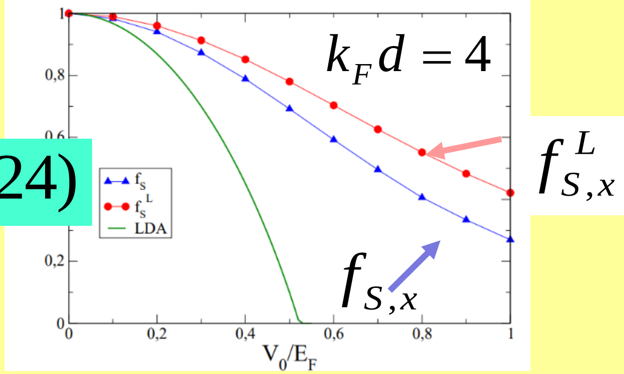
Results for **unitary Fermi gas** ($k_F |a_s| \gg 1$)

phonon regime



Leggett's bound is OK

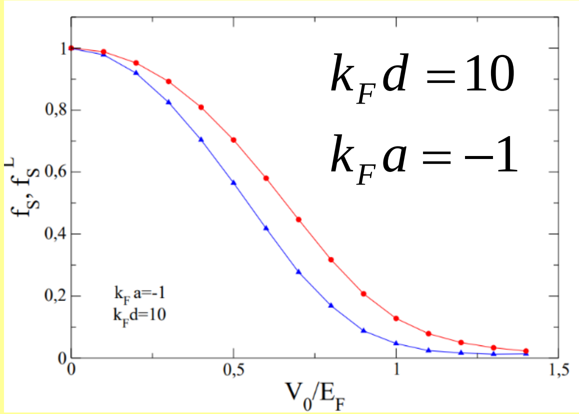
large value of $q = 2\pi/d$



Leggett's bound overestimates $f_{S,x}$

G. Orso and S.S. (2024)

Inadequacy of Leggett's bound is sizable even for $k_F d = 10$ in **BCS regime** of small and negative scattering length.
Phonon regime requires periods larger than size $k_F / m\Delta$ of Cooper pairs



Experimental measurement of **superfluid fraction** in Fermi superfluid is expected to be best obtained through measurement of phonon velocity thanks to hydrodynamic relations (holding in the presence of 1D periodic potentials)

$$\boxed{\begin{aligned} mc_x^2 &= f_{S,x}/\kappa \\ mc_y^2 &= 1/\kappa \end{aligned}}$$

Similar to the case of BEC gases, **ratio** of longitudinal and transverse sound velocities yields result

$$\boxed{f_{S,x} = c_x^2 / c_y^2}$$

independent of value of compressibility parameter

Waiting for new experiments !

Part 2 $\boxed{[H, P_x] = 0}$

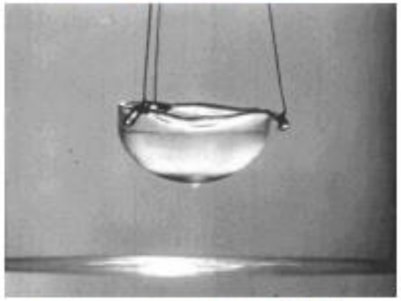
- Both phase symmetry and translational invariance are broken spontaneously (**supersolid BEC gas** in a **ring**)
- Two classes of Goldstone modes

$\Downarrow$

(two phonon modes of mixed superfluid and crystal nature)

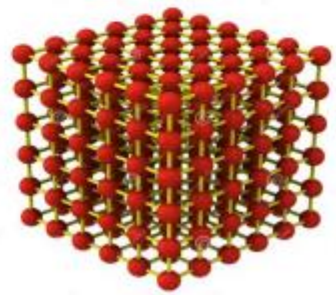
What is a supersolid?

SUPERFLUID



Spontaneous U(1) symmetry breaking

SOLID



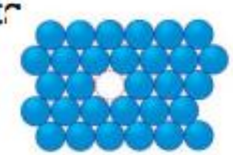
Spontaneous translational symmetry breaking

SUPERSOLID

Spontaneous and simultaneous breaking of both symmetries

First approach: SUPERFLUID SOLID

- Theory: condensation of delocalized zero-point defects



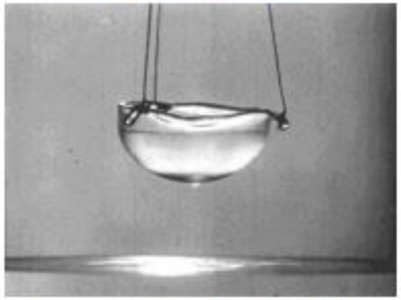
Andreev & Lifshitz (1969), Chester (1970), Leggett (1970)

Can a solid be superfluid ?

(Leggett 1970)

What is a supersolid?

SUPERFLUID



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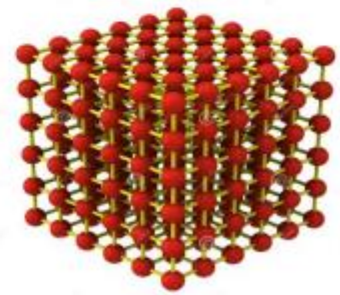

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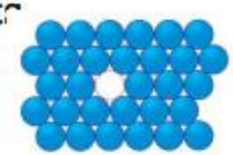
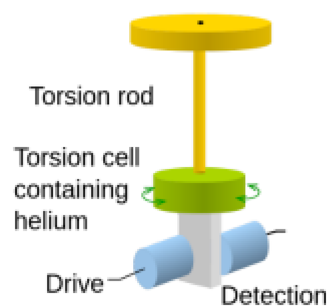
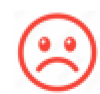


Andreev & Lifshitz (1969), Chester (1970), Leggett (1970)

- Experiment on ^{4}He

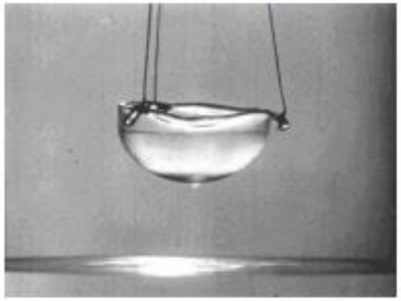


- 2004 
- 2012 

Kim & Chan, Nature (2004), (2012)

What is a supersolid?

SUPERFLUID



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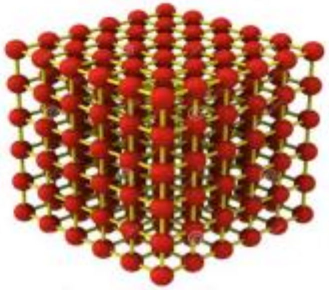


Spontaneous translational symmetry breaking

SUPERSOLID

Spontaneous and simultaneous breaking of both symmetries

Different approach: SOLID SUPERFLUID

→ Ultracold quantum gases

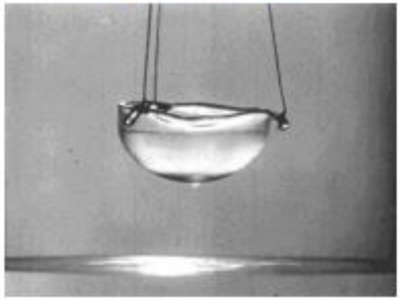
Gross, Annals Physics (1960) Pomeau & Rica, PRL (1994)

- Weakly interacting dilute gas
- Tunable interparticle interactions
- Many particles per site

Can a gas behave like a crystal?

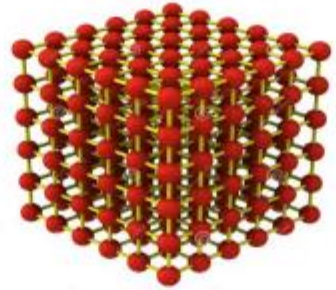
What is a supersolid?

SUPERFLUID



Spontaneous U(1)
symmetry breaking

SOLID



Spontaneous translational
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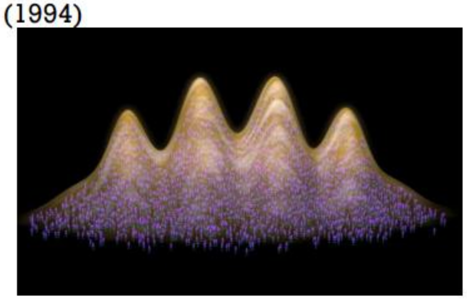


Image from IQOQI

In Part 1 we have understood that if only phase symmetry is spontaneously broken, the superfluid fraction of a density modulated superfluid determines the velocity of sound according to the T=0 hydrodynamic relation

$$\boxed{mc^2 = f_S / \kappa}$$

This **relation cannot hold** in the presence of additional **spontaneous breaking of translational symmetry** since Goldstone theorem predicts two sound velocities both propagating along the same direction.

Can we measure the superfluid fraction through the measurement of the sound velocities ?

What is the link between the superfluid fraction and the sound velocities in the case of a **supersolid** at zero temperature?

Determination of superfluid fraction from measurement of sound velocities has pioneering example:

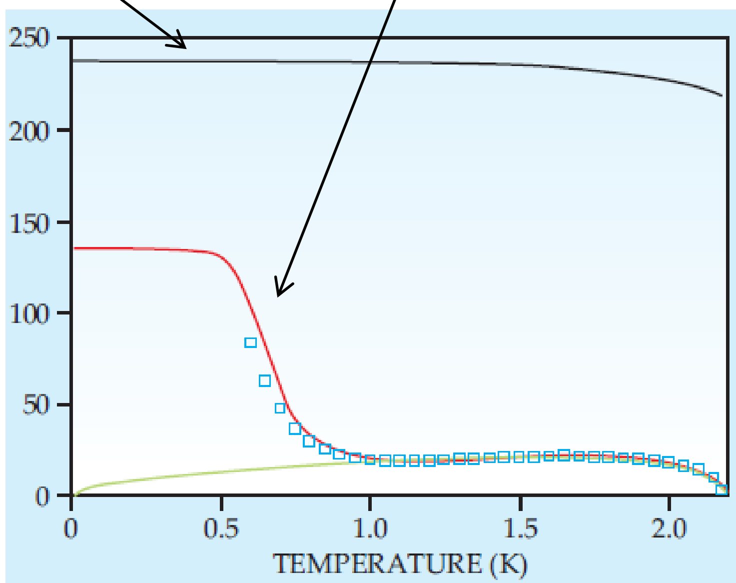
Superfluid Helium at finite temperature

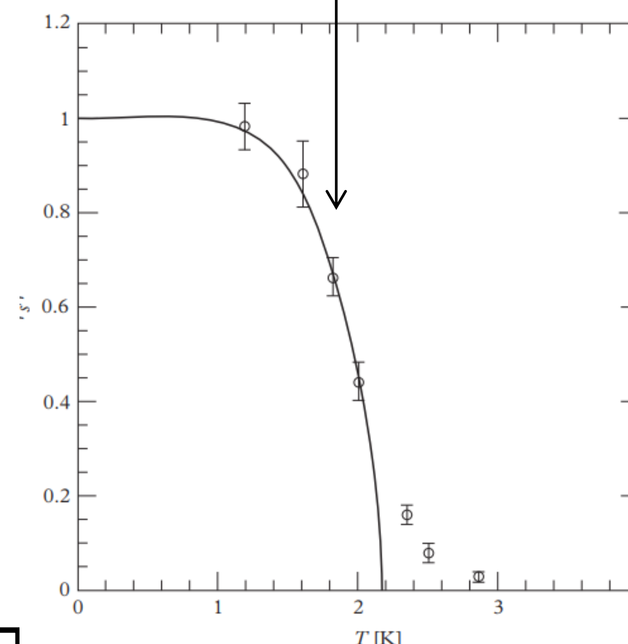
First and second sound velocities and superfluid fraction in **superfluid He** (almost incompressible fluid)

$$c_1^2 = \tfrac{1}{m}\left(\frac{\partial P}{\partial n}\right)_S$$

$$c_2^2 = \tfrac{1}{m}\frac{\rho_s T s^2}{\rho_n C_P}$$

$$f_S = \frac{\rho_S}{\rho}$$

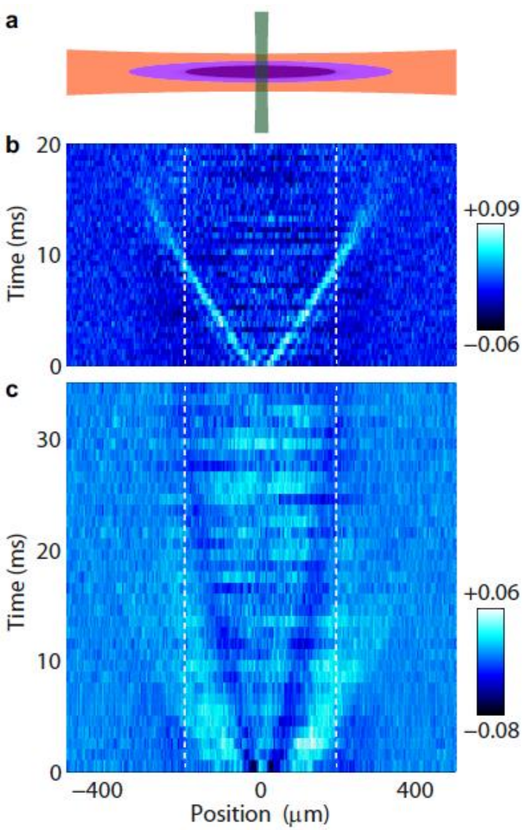


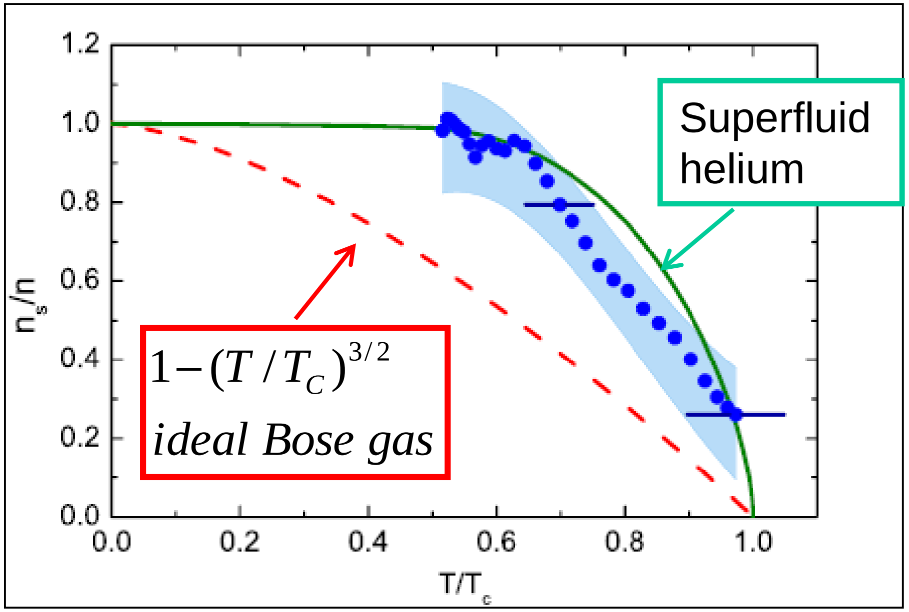


Liquid He
(experiment, Peshkov 1946)

The case of the **unitary Fermi gas** at finite temperature

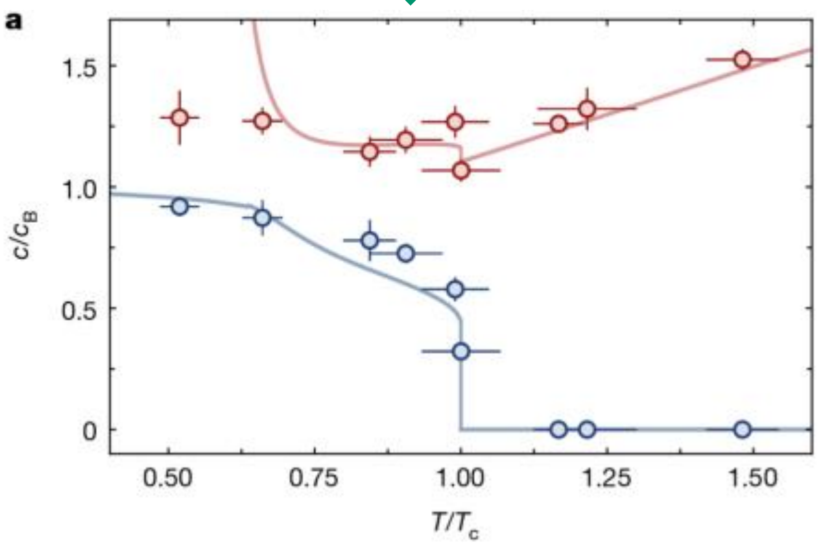
From measurement of first and second sound velocities in the unitary Fermi gas one can reconstruct the temperature dependence of the **3D superfluid fraction**
(Innsbruck-Trento collaboration, Sidorenkov et al., Nature 2013)

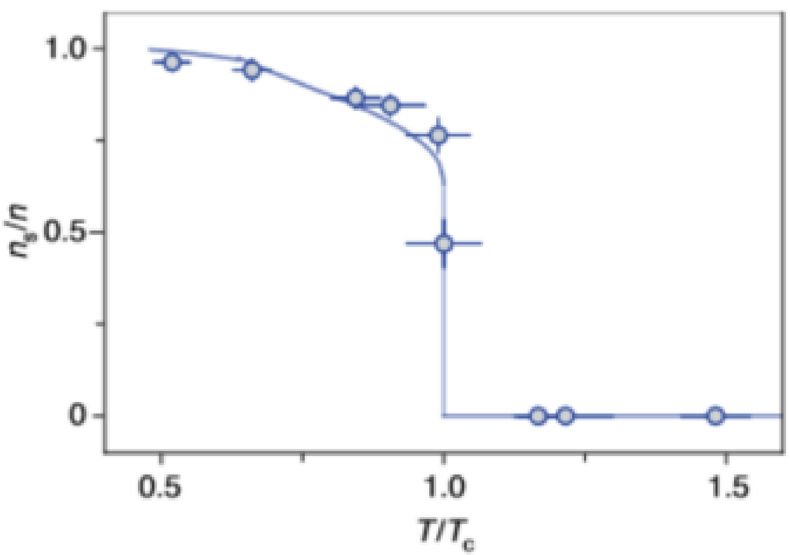




First measurement of superfluid density **in a Fermi superfluid**

Temperatue dependence of superfluid fraction in a 2D Kosterlitz-Thouless Bose gas, based on measurement of **first** and **second** sound velocities
(Christodoulou ….Hadzibabic, Nature 2021)





First experimental confirmation of the predicted (Ozawa and S.S. PRL 2014) **jump of second sound** velocity **at the BKT transition**

The case of a supersolid at T=0

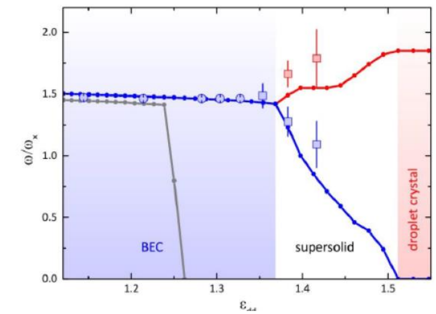
Measurement of the Goldstone modes has been already the object of experimental papers in a supersolid dipolar gas confined in harmonic trap (axial breathing modes)

LETTER

https://doi.org/10.1038/s41586-019-1568-6

Supersolid symmetry breaking from compressional oscillations in a dipolar quantum gas

L. Tanzi[1,2,3,6], S. M. Roccuzzo[4,5,6], E. Lucioni[1,2,3], F. Famà[1], A. Fioretti[1], C. Gabbanini[1], G. Modugno[1,2,3]*, A. Recati[4,5]* & S. Stringari[4,5]



LETTER

https://doi.org/10.1038/s41586-019-1569-5

The low-energy Goldstone mode in a trapped dipolar supersolid

Mingyang Guo[1,2,4], Fabian Böttcher[1,2,4], Jens Hertkorn[1,4], Jan-Niklas Schmidt[1,2], Matthias Wenzel[1,2], Hans Peter Büchler[2,3], Tim Langen[1,2] & Tilman Pfau[1,2]*

PHYSICAL REVIEW LETTERS **123**, 050402 (2019)

Editors' Suggestion | Featured in Physics

Excitation Spectrum of a Trapped Dipolar Supersolid and Its Experimental Evidence

G. Natale,[1] R. M. W. van Bijnen,[2] A. Patscheider,[1] D. Petter,[1] M. J. Mark,[1,2] L. Chomaz,[1] and F. Ferlaino[1,2,*]
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[2]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Technikerstraße 21a, 6020 Innsbruck, Austria*

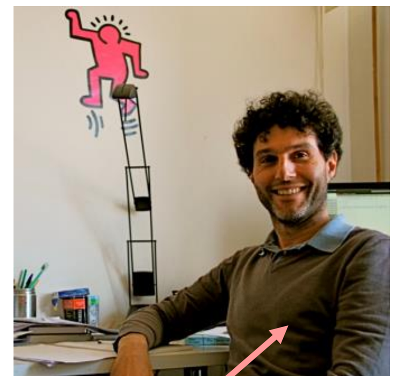
NEWS AND VIEWS (NATURE)
16 October 2019

Sounds of a supersolid detected in dipolar atomic gases for the first time

Ultracold gases of dipolar atoms can exhibit fluid and crystalline oscillations at the same time, illuminating the ways in which different kinds of sound propagate in the quantum state of matter known as a supersolid.

However **the explicit connection** between the propagation of **sound** and the **superfluid fraction** is **largely unexplored**

In a recent paper we have addressed the question of the link between the propagation of sound and the behavior of the superfluid fraction in the case of a **dipolar supersolid** gas confined in a **ring geometry** (hopefully of feasable experimental realization)



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Sound, Superfluidity, and Layer Compressibility in a Ring Dipolar Supersolid

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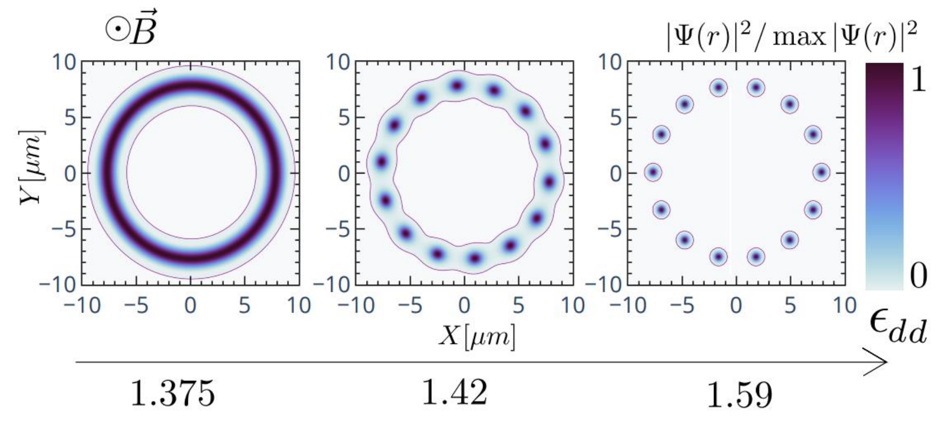
Atomic densities in the ring configuration

(Length of the ring: 49 microns), for different values of the relevant interaction parameter

$$\boxed{\varepsilon_{dd} = a_{dd} / a}$$

where $a_{dd} = \mu_0 \mu^2 / 12\pi\hbar^2$ is the dipolar length, and a is the s-wave scattering length.



For small values of ε_{dd} the system is in the uniform superfluid phase. By increasing ε_{dd} one enters the supersolid phase where droplets are formed over a sea of a superfluid gas. For even larger values of ε_{dd} one enters the crystal phase of well separated droplets

Protocol for exciting the Goldstone modes

By suddenly releasing a small periodic perturbation of the form

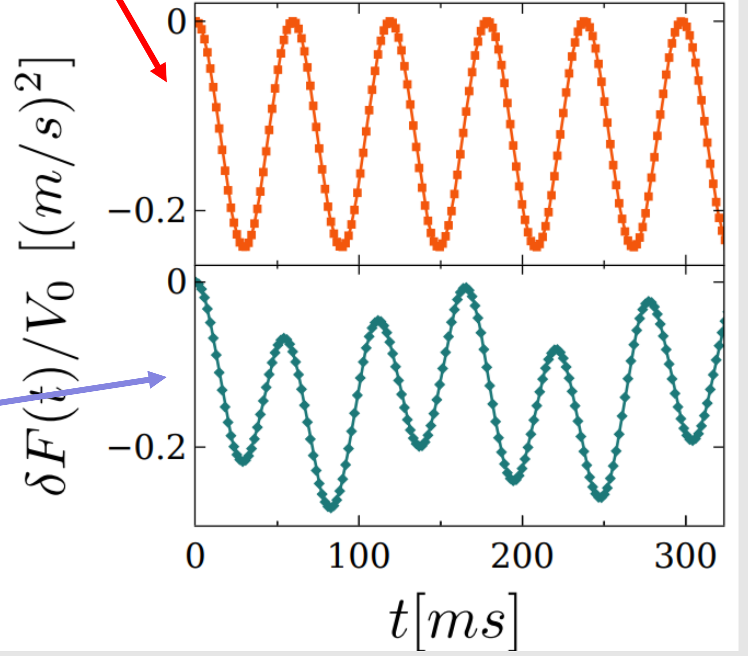
$$\boxed{V(\varphi) = V_0 \cos(\varphi)}$$

one explores the resulting time dependent oscillations of the quantity $F(t) = \langle \cos(\varphi) \rangle (t)$, obtained solving the extended GP eq.

In the superfluid phase one observes a single frequency of the excitation $\omega(k)$ spectrum of the elongated superfluid configuration, corresponding to

$$\boxed{k = 2\pi / L}$$

In the supersolid phase one instead observes a beating of two frequencies, corresponding to the excitation of the two Goldstone modes.



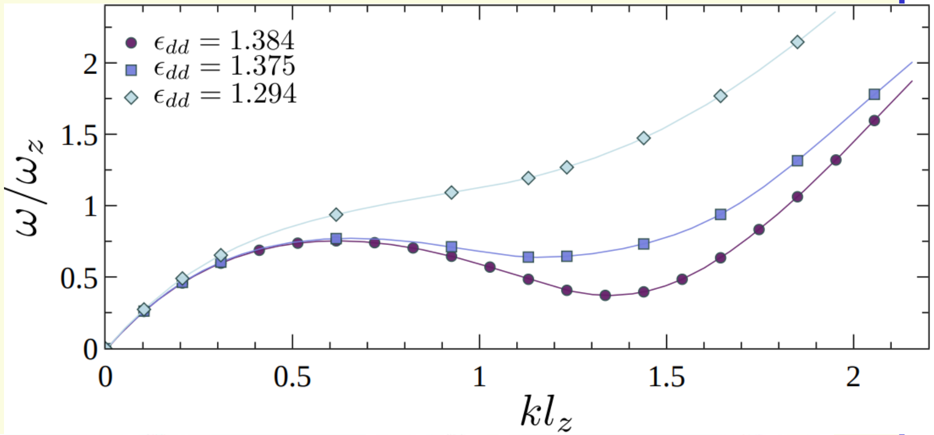
To appreciate the potential of the proposed protocol we have studied the resulting collective frequencies in the **superfluid** phase applying a perturbation of the form

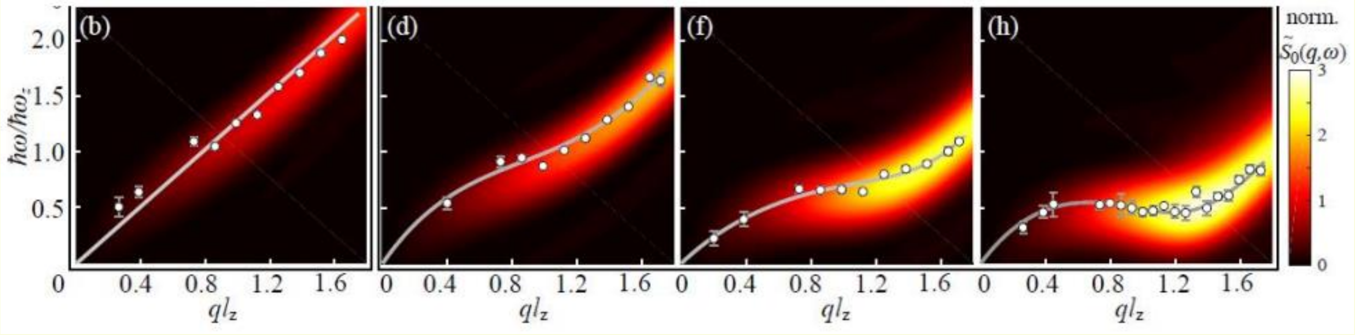
$$\boxed{V(\varphi) = V_0 \cos(n\varphi) \quad \text{with } n = 1,2,..}$$

allowing for the study of the excitation spectrum at higher wave vectors

$$k = 2\pi n / L$$

We observe the softening of the roton spectrum as one approaches the superfluid supersolid phase transition confirming IBK experiment (Petter et al. PRL 2019)





In the **supersolid** phase the observed signal has typical **beating** form

$$\boxed{\delta F(t) = V_0[\chi_1(q)\cos(\omega_1(q)t) + \chi_2(q)\cos(\omega_2(q)t)]}$$

with $\omega_1(q)$ and $\omega_2(q)$ approaching, for small q (and hence large L), the linear phonon dispersion

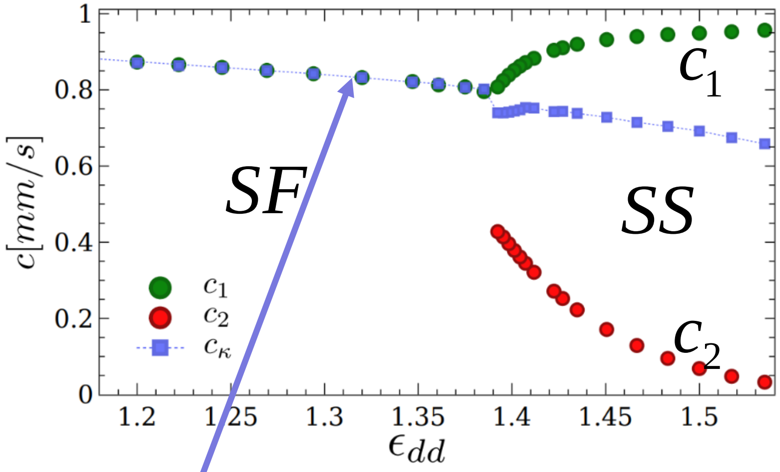
$$\omega_1(\mathrm{q}) = \mathrm{c}_1\mathrm{q} \text{ and } \omega_2(q) = c_2 q$$

with c_1 and c_2 the **first** and **second** sound velocities.

The sum of the weights $\chi_1(q)$ and $\chi_2(q)$ fixes the compressibility $\chi(q) = \chi_1(q) + \chi_2(q) \to N\kappa/2$ in the $\mathrm{q} \to 0$ limit, while the relative contribution of second sound to χ is related to the sound velocities by the V_0 independent relation

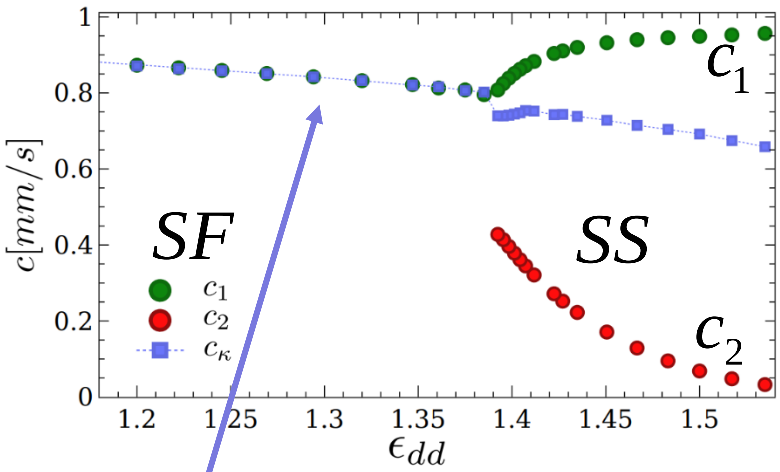
$$\boxed{R = \frac{\chi_2}{\chi} = \frac{c_1^2 - c_\kappa^2}{c_1^2 - c_2^2}}$$

Results for the sound velocities as a function of ε_{dd} in a ring trap



The usual hydrodynamic result $c_\kappa = \sqrt{1/m\kappa}$ is consistent with the observed velocity **only** in the superfluid phase.

Results for the sound velocities as a function of ε_{dd} obtained by solving the time dependent Gross-Pitaevskii eq. in a ring trap



The usual hydrodynamic result $c_\kappa = \sqrt{1/m\kappa}$ is consistent with the observed velocity **only** in the superfluid phase.

Similar results for the sound velocities are obtained in the case of an infinite tube potential
(Blakie et al., Phys. Rev. Res. 5, 033161 (2023);
Platt et al., arXiv: 2403.1915)

Hydrodynamic theory of a 1D supersolid

(Andreev and Lifschtz 1969) Josserand, Pomeau and Rica 2008, Yoo and Dorsey (2010))

In a recent paper Hofmann and Zwerger (2021) have investigated the applicability of the hydrodynamic theory of supersolids to a dipolar supersolid in **1D like configuration**, where the layered structure is reasonably well realized by the presence of droplets and the relavant parameters are the compressibility the superfluid fraction and the layer compressibility modulus

At zero temperature the theory predicts the following result for the two longitudinal sound velocities (Hofmann and Zwerger, J.Stat.Mech. 033104, 2021)

$$\boxed{\mathrm{c}_{1,2}^2 = \mathrm{c}_\kappa^2 \frac{1}{2}[1+\beta\kappa \pm \sqrt{(1+\beta\kappa)^2 - 4 f_S \beta\kappa}\,]}$$

where $c_\kappa^2 = \kappa^{-1}/m$, $\beta = B/\rho_N$ with B the layer compressibility modulus and f_S the superfluid fraction (strain density coupling set equal to zero)

Supersolid 1D hydrodynamics provides the **long sought relationship**

$$c_\kappa^2 f_S = \frac{c_1^2 c_2^2}{c_1^2 + c_2^2 - c_\kappa^2}$$

for the superfluid fraction in terms of the two sound velocities c_1^2 and c_2^2 and of $c_\kappa^2 = 1/m\kappa$

Supersolid relationship generalizes the usual relationship

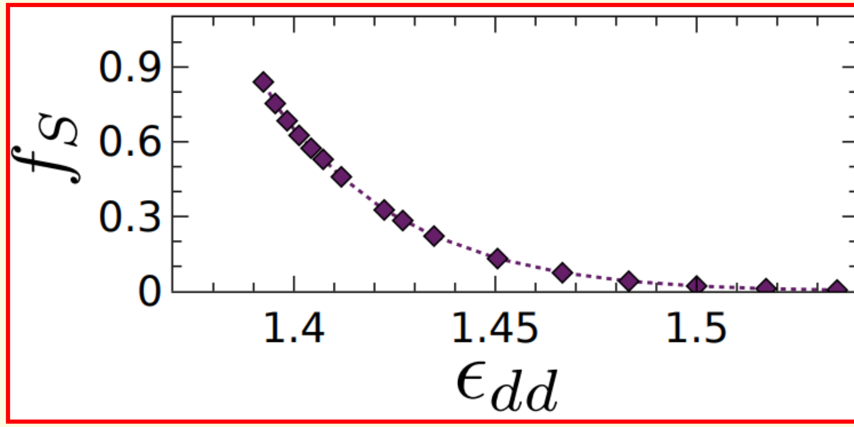
$$c_\kappa^2 f_S = c^2$$

holding in the presence of a single Goldstone mode

Using our results of GP simulation for the sound velocities c_1, c_2 and $c_\kappa = \sqrt{1/m\kappa}$

we can extract the value of **the superfluid fraction** in the supersolid phase.

f_S decreases as one increases the value of ε_{dd} approaching the transition to the crystal phase of independent droples, while it increases to unity at the transition to the superfluid phase.



How does the value of the superfluid fraction, extracted from the values of the sound velocities, compare with the value calculated using Leggett's prescription for the non classical fraction of the moment of inertia ?

$$f_S^L = 1 - \frac{\Theta}{\Theta_{rig}}$$

We calculate Θ applying a rotational contraint $-\Omega L_z$ and evaluating the corresponding value of angular momentum $< L_z >= \Omega\Theta$, with Θ_{rig} the rigid value of the moment of inertia.

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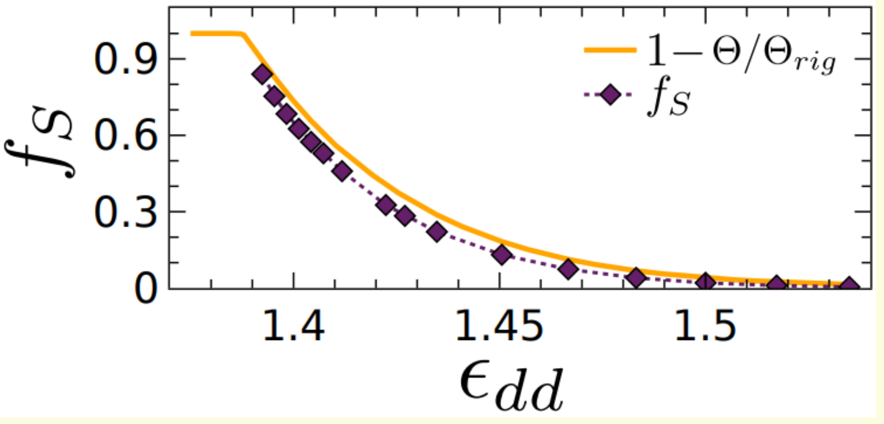
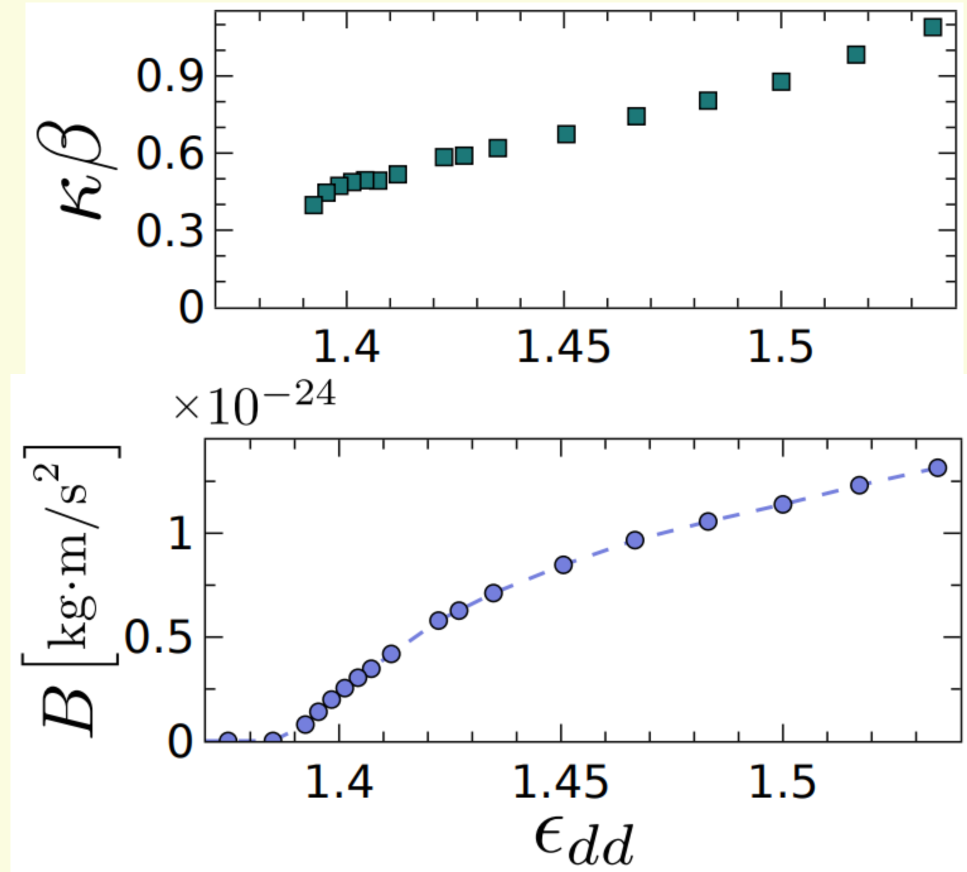


The excellent **agreement** confirms the **consistency** of the extended **Gross-Pitaevskii** theory used to calculate the sound velocities with the **hydrodynamic model for supersolids**.

Our approach also provides the **first quantitative evaluation of the layer compressibility modulus** of a supersolid dipolar gas

From the results for

$$\kappa\beta = \frac{c_1^2 + c_2^2}{c_\kappa^2} - 1$$



one extracts the layer compressibility modulus

$$B = \beta f_N$$

which vanishes at the transition to the superfluid phase

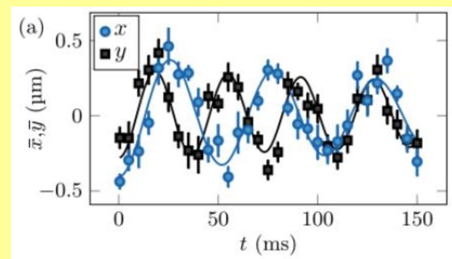
(see also Blakie et al. arXiv: 2403.1915)

Main conclusion

- Propagation of sound yields useful information on the superfluid denisty on density modulated quantum systems at T=0.

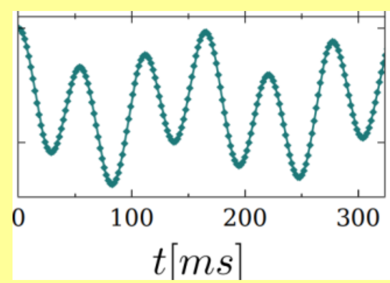
- Superfluid BEC gas in a box

$$f_{S,x} = c_x^2 / c_y^2$$



- Supersolid Dipolar BEC gas in a ring

$$c_\kappa^2 f_S = \frac{c_1^2 c_2^2}{c_1^2 + c_2^2 - c_\kappa^2}$$



- Theory predictions for interacting Fermi gases and dipolar supersolids in a ring are waiting for new experiments

Closely related projects running in Trento

- Effects of permanent currents and vortex lines on the propagation of sound in a supersolid

- Superfluid vs elastic oscillations in higher dimensional configurations

- Effects of disorder on the propagation of sound in a superfluid gas

The Trento BEC team (2019)

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