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SEIT 1386

Small(est) quantum fluid(s)

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Heidelberg University

ECT*, Trento, 22nd April 2024

European Research Council

PASQuanS

ISOQUANT SFB1225

STRUCTURES CLUSTER OF EXCELLENCE

Baden-Württemberg Stiftung
WIR STIFTEN ZUKUNFT

Outline

I. Emergence of fluid behaviour

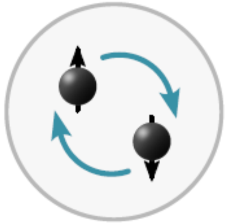
Emergence of interaction-driven elliptic flow

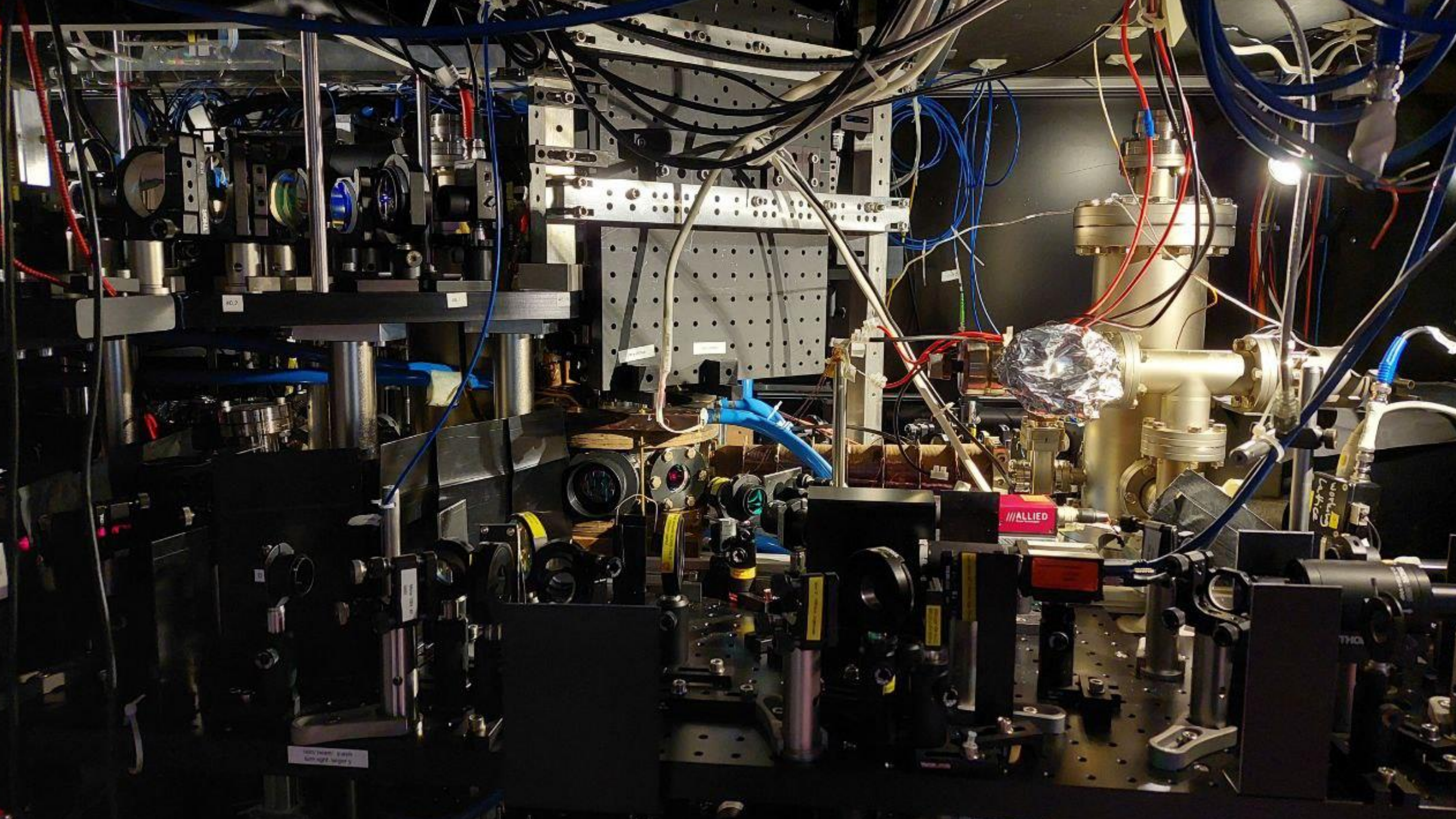
Brandstetter, Lunt et al., arXiv:2308.09699

II. Fractional quantum hall liquid:

Realisation of a Laughlin state of two rapidly rotating Fermions

Lunt et al., arXiv:2402.14814

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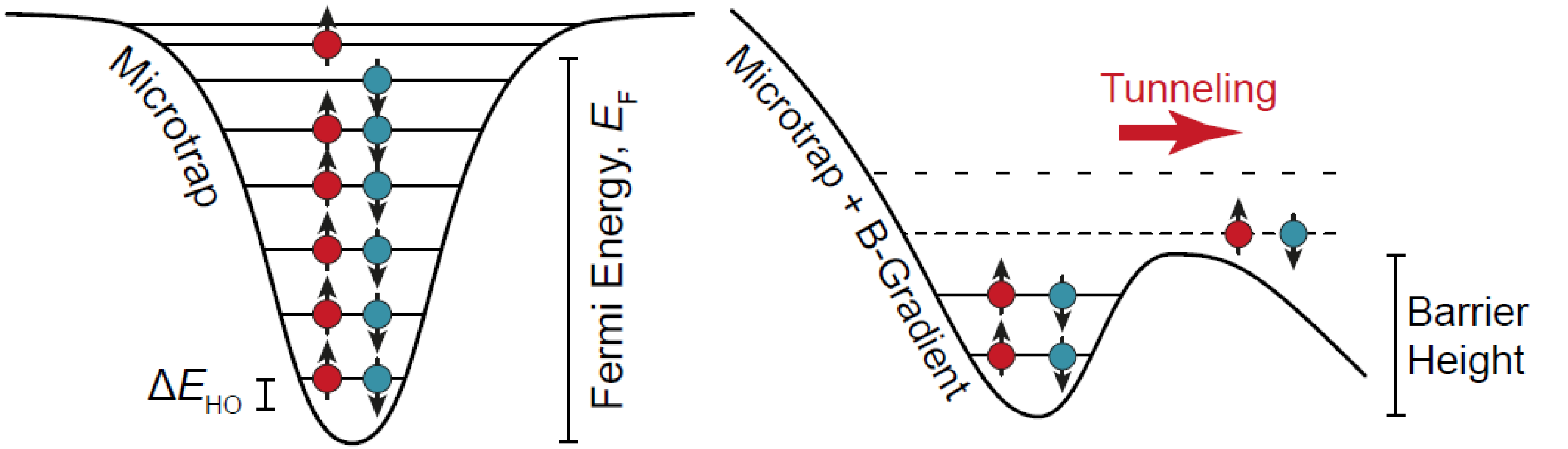
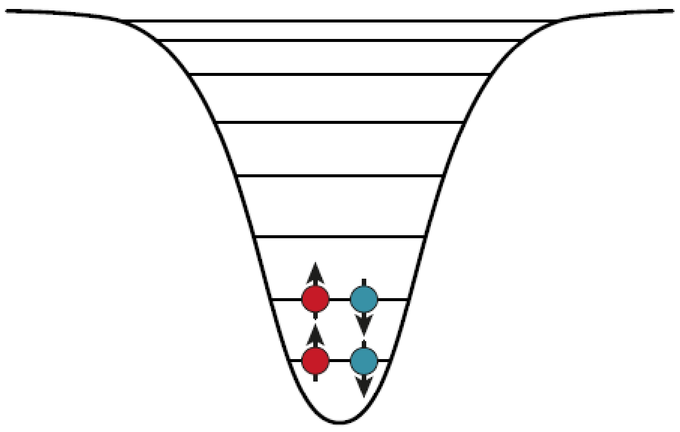
Our tools

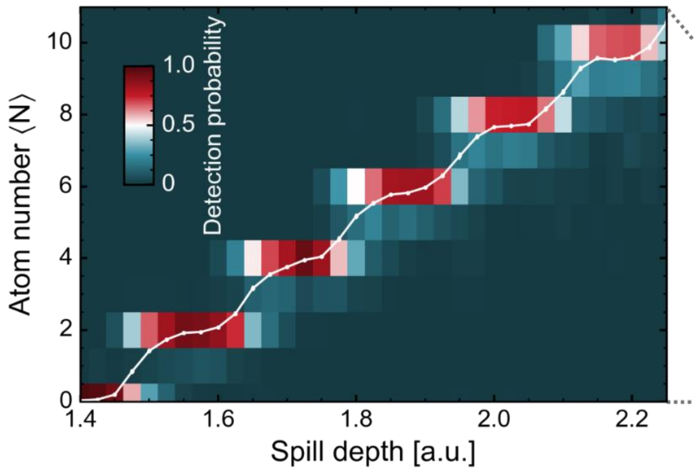
1. Deterministic ground state preparation





F. Serwane et al., **Science** Vol. 332., 6027, (2011)

1+1 Ground state fidelity: 98%



Our tools

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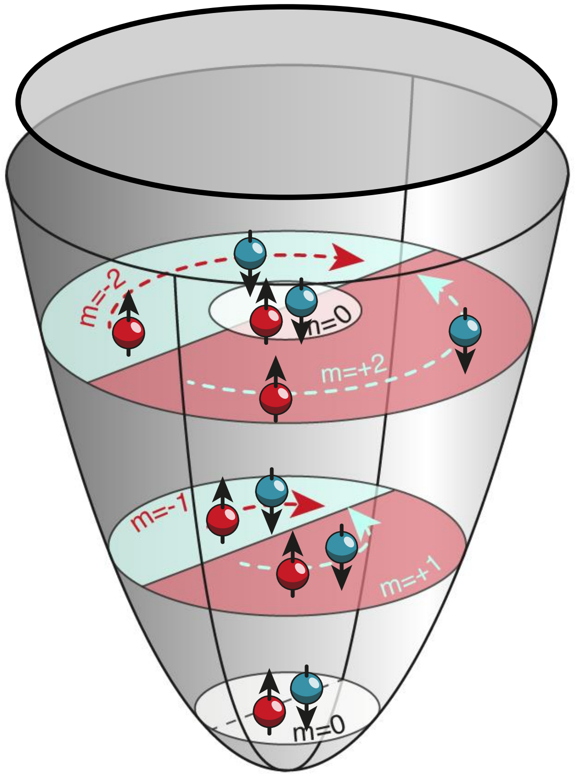
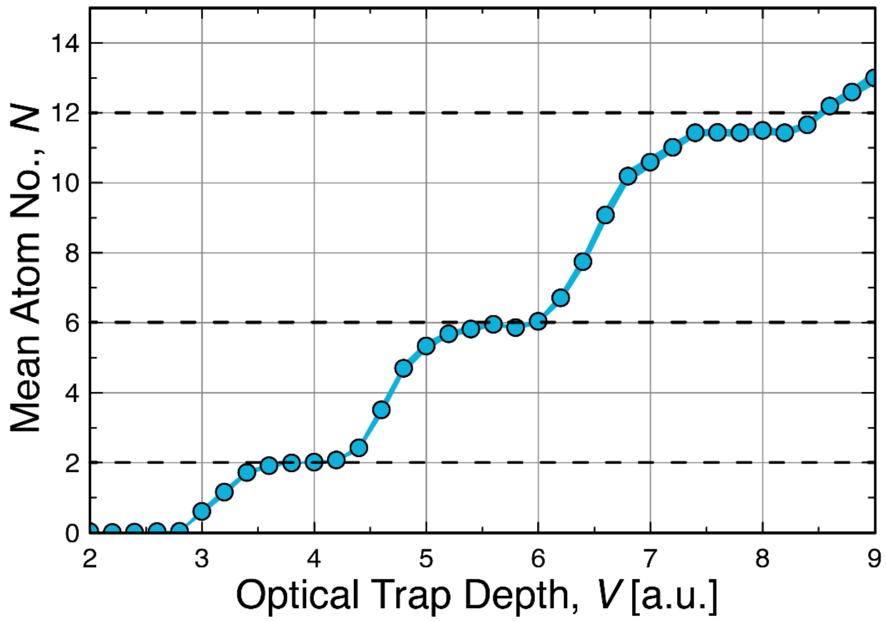
1. Deterministic ground state preparation (also in 2D)

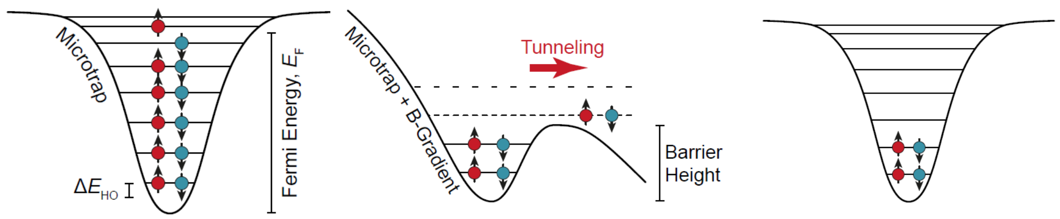




L Bayha et al., **Nature** 587, 583 (2020)

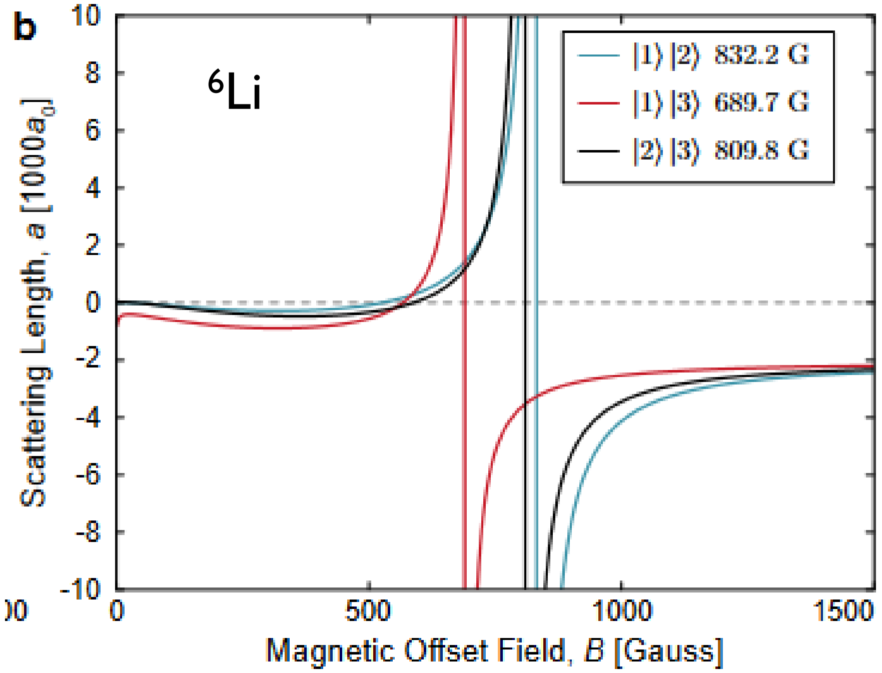
Our tools

1. Deterministic ground state preparation



2. Interaction tuneability

- Feshbach resonance
- Fast interaction turn on/off by changing the internal states



G. Zürn, et al., **Phys. Rev. Lett**. 110, 135301 (2013)

Our tools

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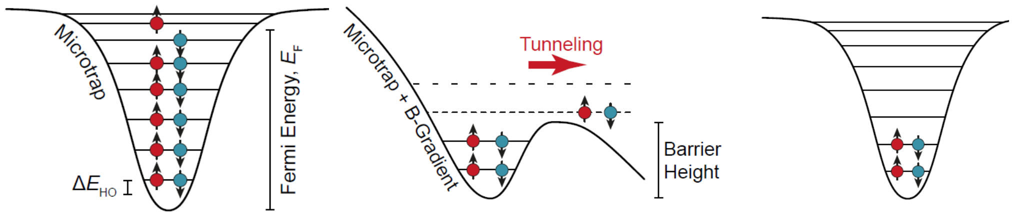
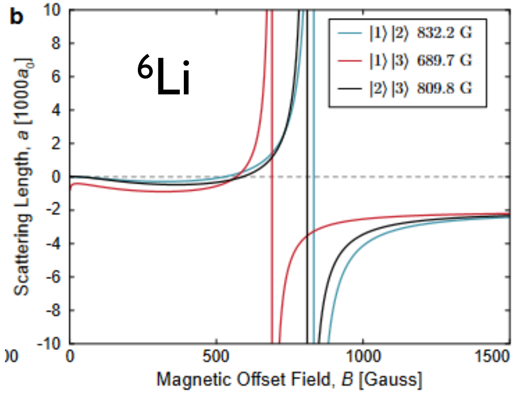
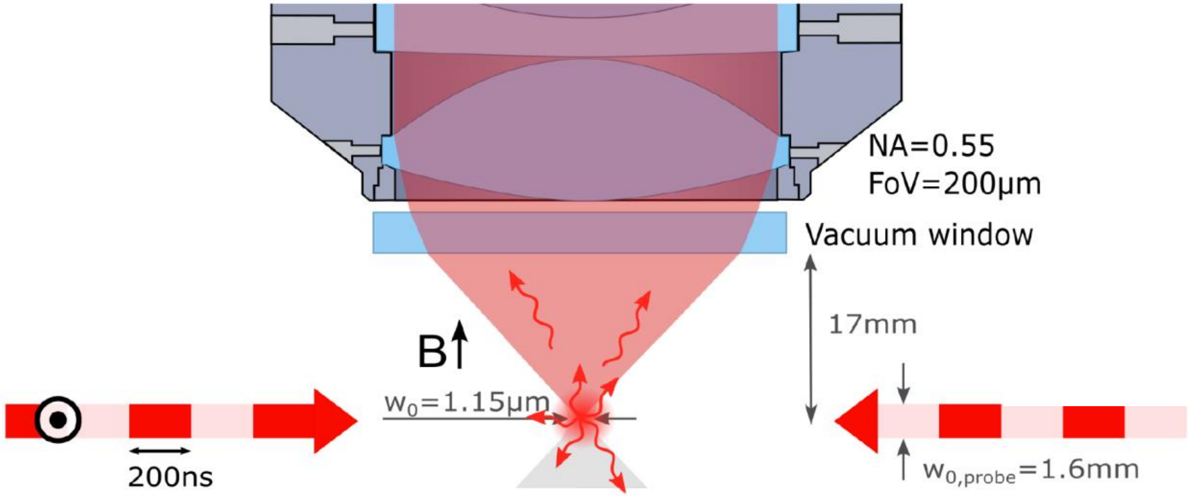


2. Interaction tuneability



3. Single-atom imaging/microscopy



A Bergschneider et al., **Phys. Rev. A** 97, 063613 (2018)

System size $\sim x\ \mu\text{m} \sim \lambda_{\text{Imaging Light}}$

- Microscopy

Single-atom microscopy

- In harmonic trap after T/4

$$x(T_1/4) = \frac{p(0)}{m\,\omega_1}$$

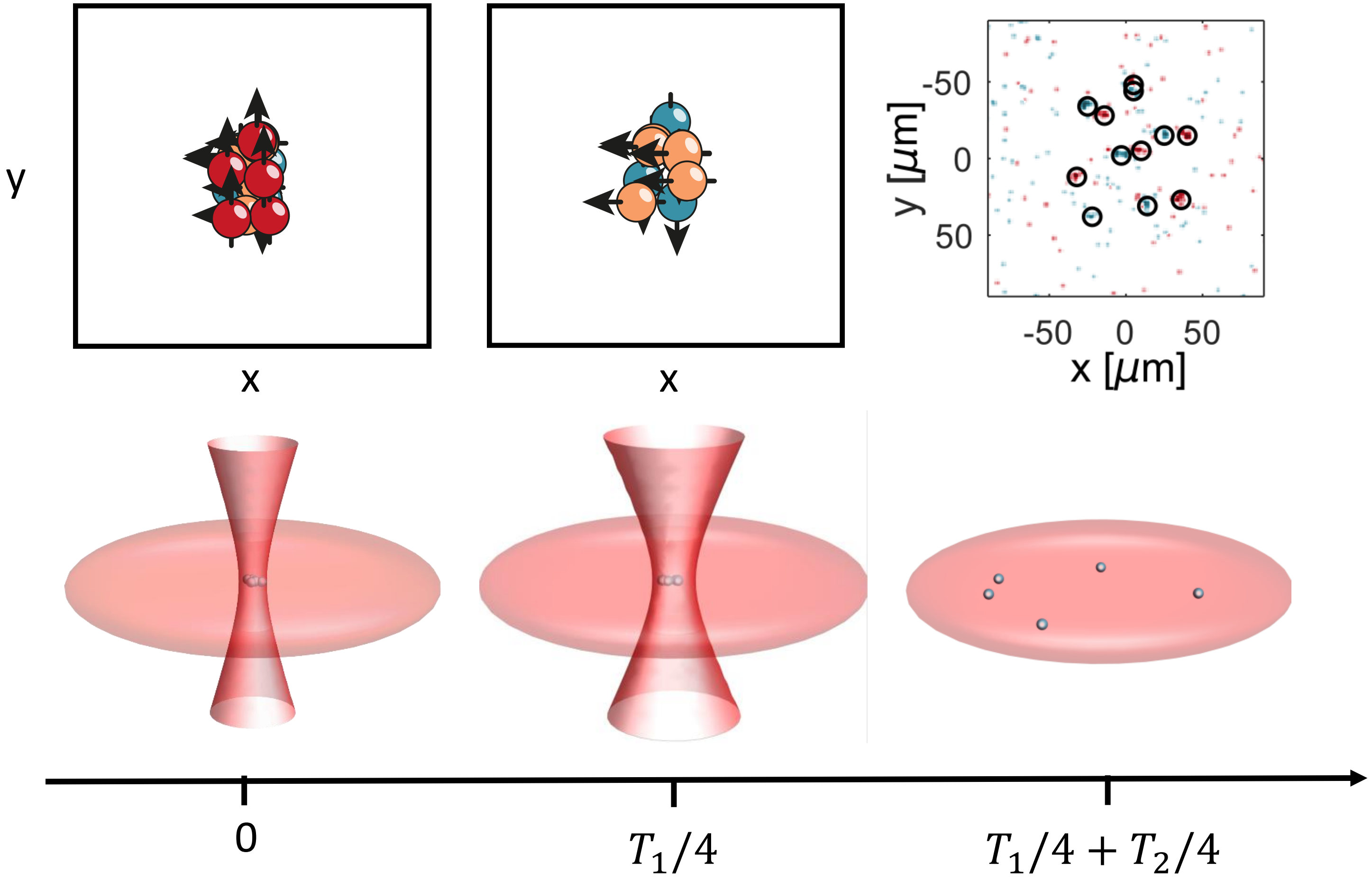
$$p(T_1/4) = -m\omega_1 x(0)$$

- Using two T/4 expansions in different traps

$$x(T_1/4 + T_2/4) = -\frac{\omega_1}{\omega_2}x(0)$$

Magnification

- Resolution ~300 nm



Asteria *et al*, *Nature* ***599***, *571–575 (2021)*
Murthy *et al*., *Phys. Rev. A* **90** (2014), 043611

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Our tools

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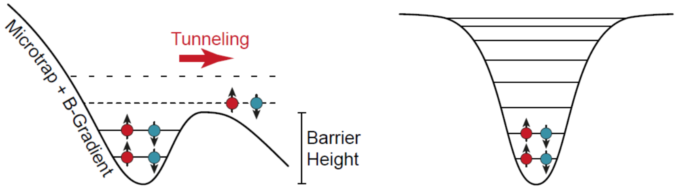
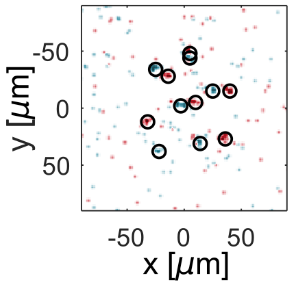
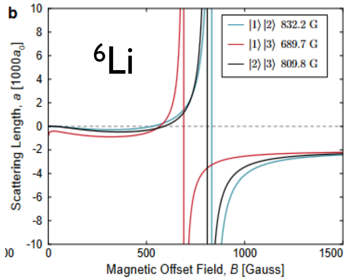
1. Deterministic preparation



3. Single-atom microscopy

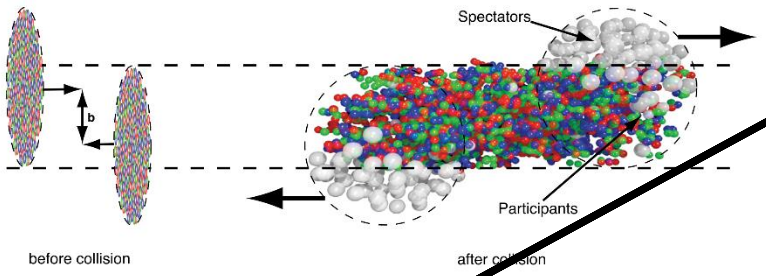


2. Interaction tuneability

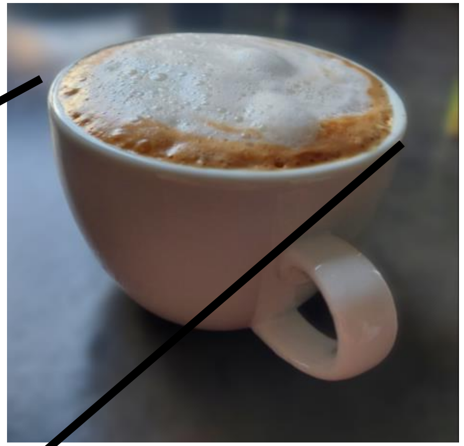


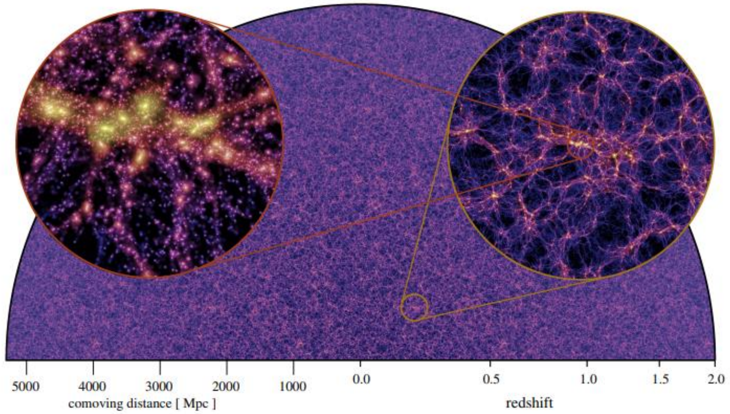
Part 1: What is a fluid?

Size

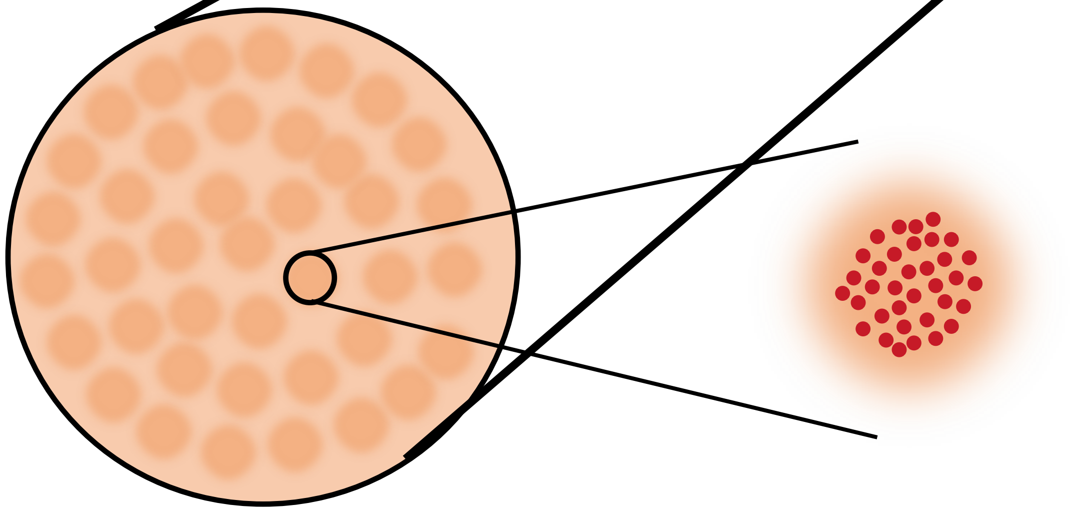


R. Snellings, New Journal of Physics 13 (2011) 055008





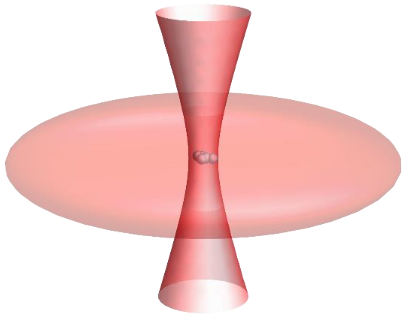
https://arxiv.org/abs/2210.10059

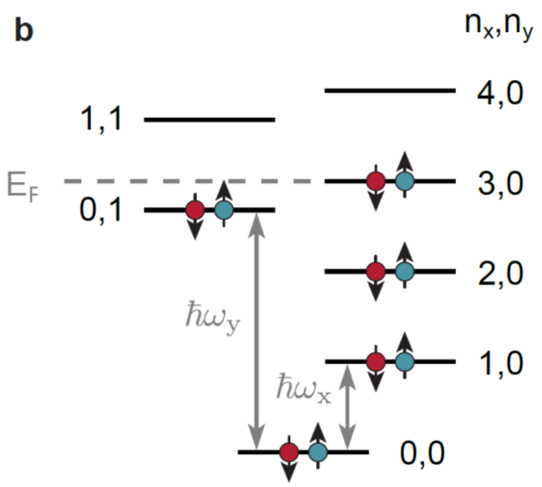


- Continuous medium
- Separation of scales

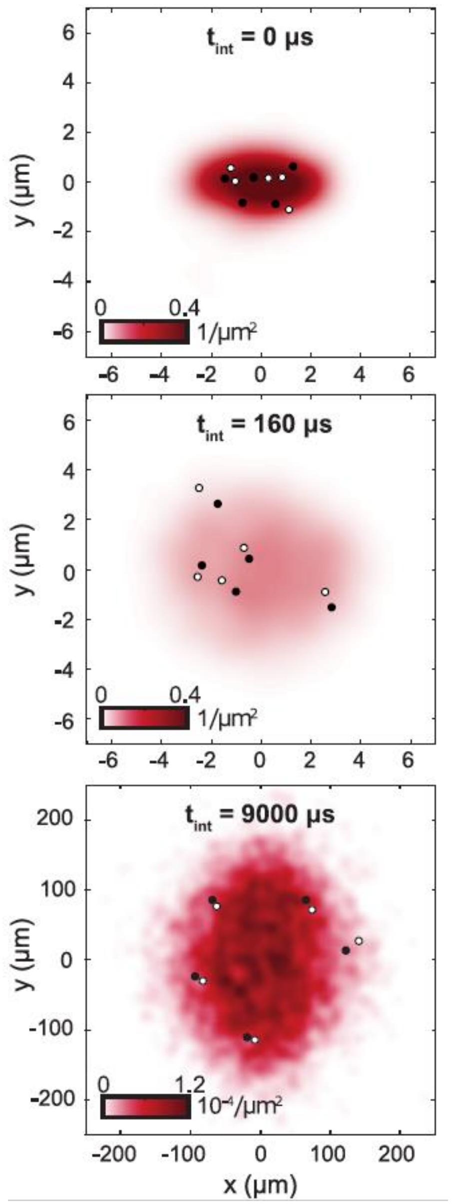
Elliptic flow of 10 particles

- Prepare in 2D elliptic trap
  $\frac{\omega_y}{\omega_x} \approx 3,\ \omega_x/(2\pi)\ \approx 1\ \text{kHz}$
  $\omega_z/(2\pi) \approx 7\ \text{kHz}$
- 5+5 atoms
- Strongly interacting

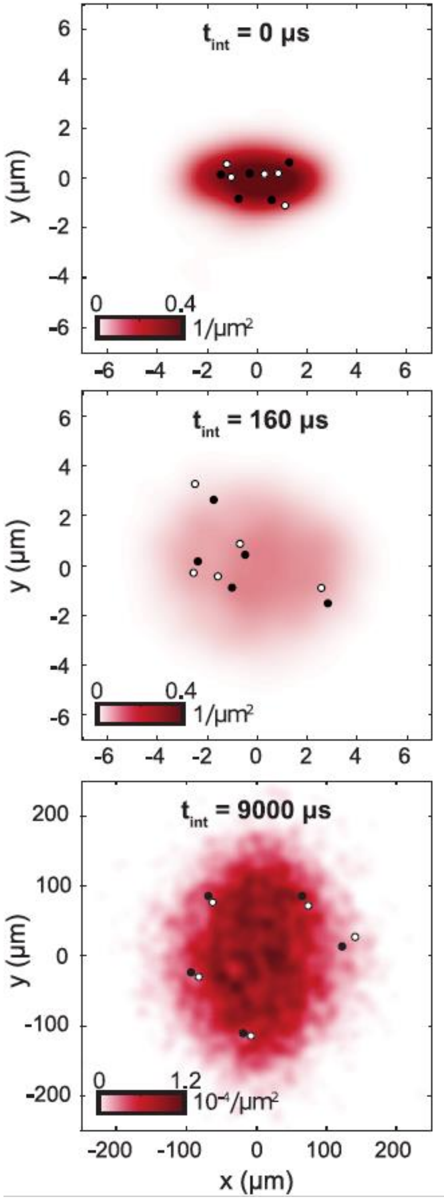


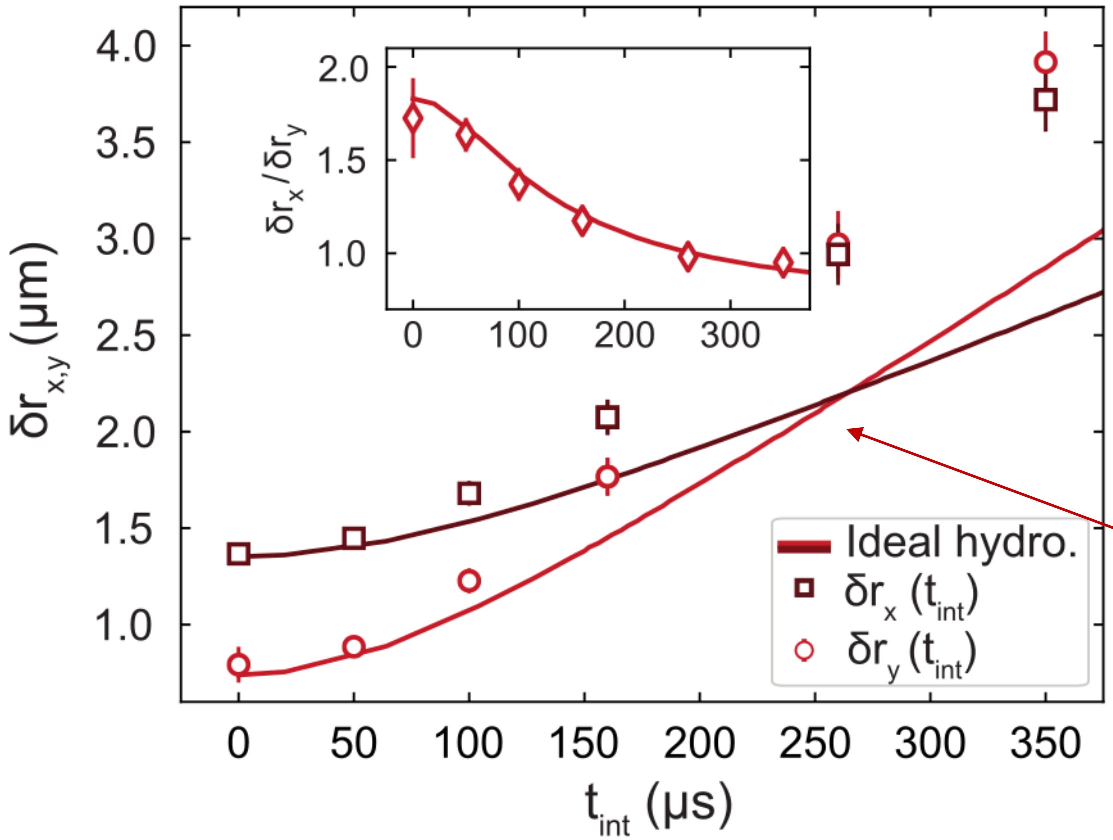


- ≈1000 samples reveal density



Real space evolution

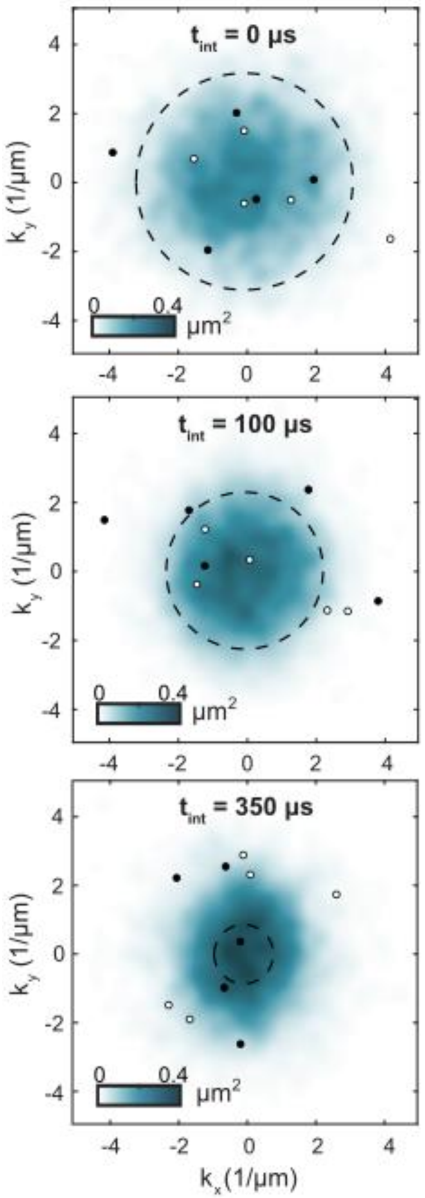


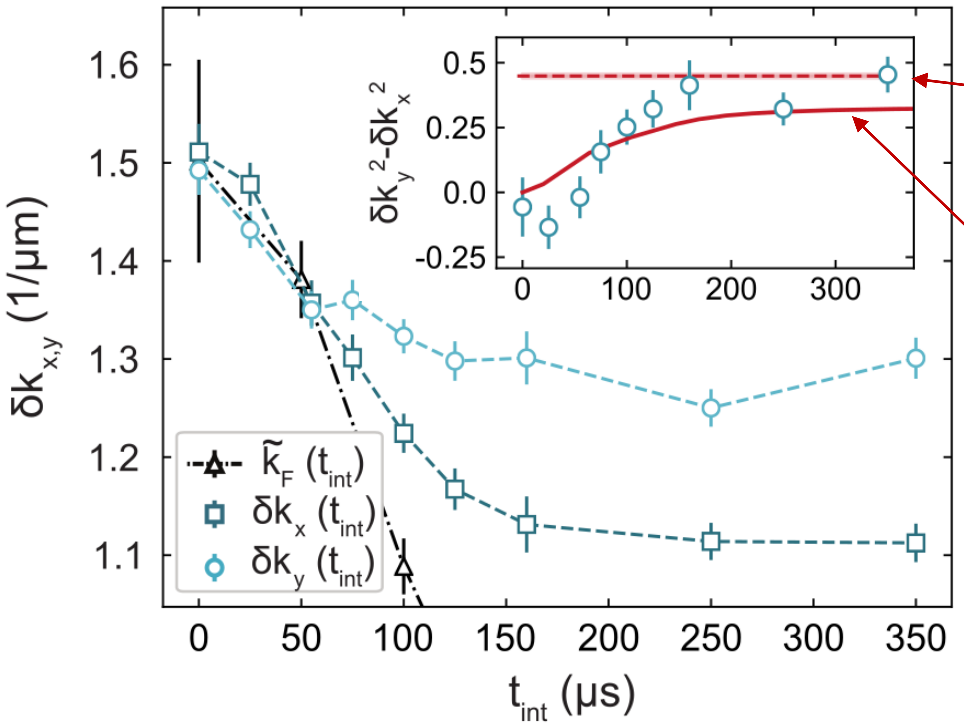


Ideal hydrodynamics +
Many-body equation of state

Vasiliy Makhalov et al.,
Phys. Rev. Lett. 112, 045301 (2014)

Momentum evolution





Long-time limit

Comparison to ideal hydrodynamics

$$\frac{m}{2N}\int_{\mathbf{x}} \rho(t,\mathbf{x})\left[v_y^2(t,\mathbf{x}) - v_x^2(t,\mathbf{x})\right] =$$

$$= (\delta p_y)^2 - (\delta p_x)^2$$

Tuning the interaction strength

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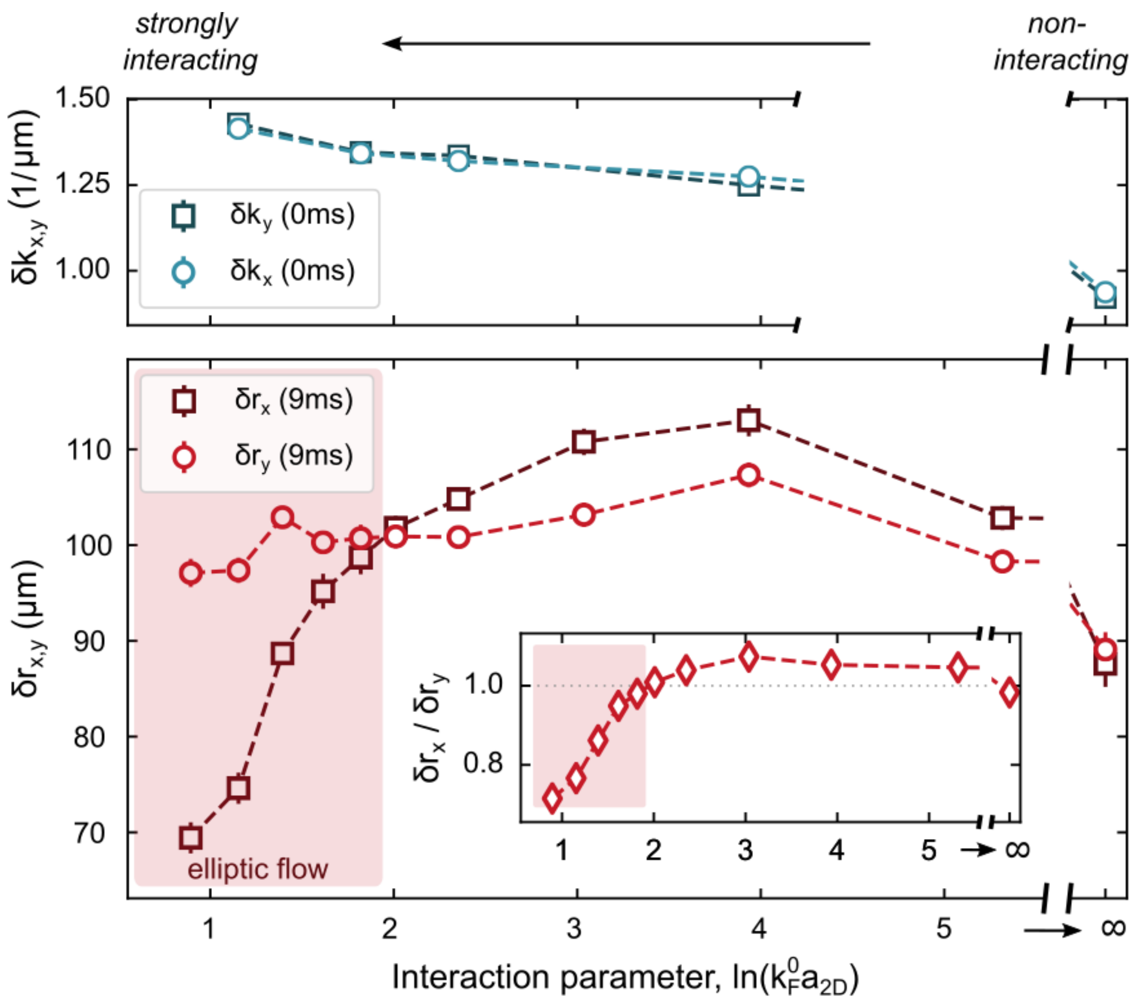


$$k_F^0 = \sqrt{2mE_F}/\hbar \qquad a_{2D} = \ell_z \sqrt{\frac{\pi}{0.905}} \exp\left(-\ell_z/a_{3D}\sqrt{\pi/2}\right)$$

- Symmetric insitu momenta
- Higher interaction strength → stronger ellipticity
- Below critical interaction strength
  - No inversion
  - Initial ellipticity maintained

  Collisionless mean-field

C. Menotti et al., Phys. Rev. Lett. 89, 250402 (2002)

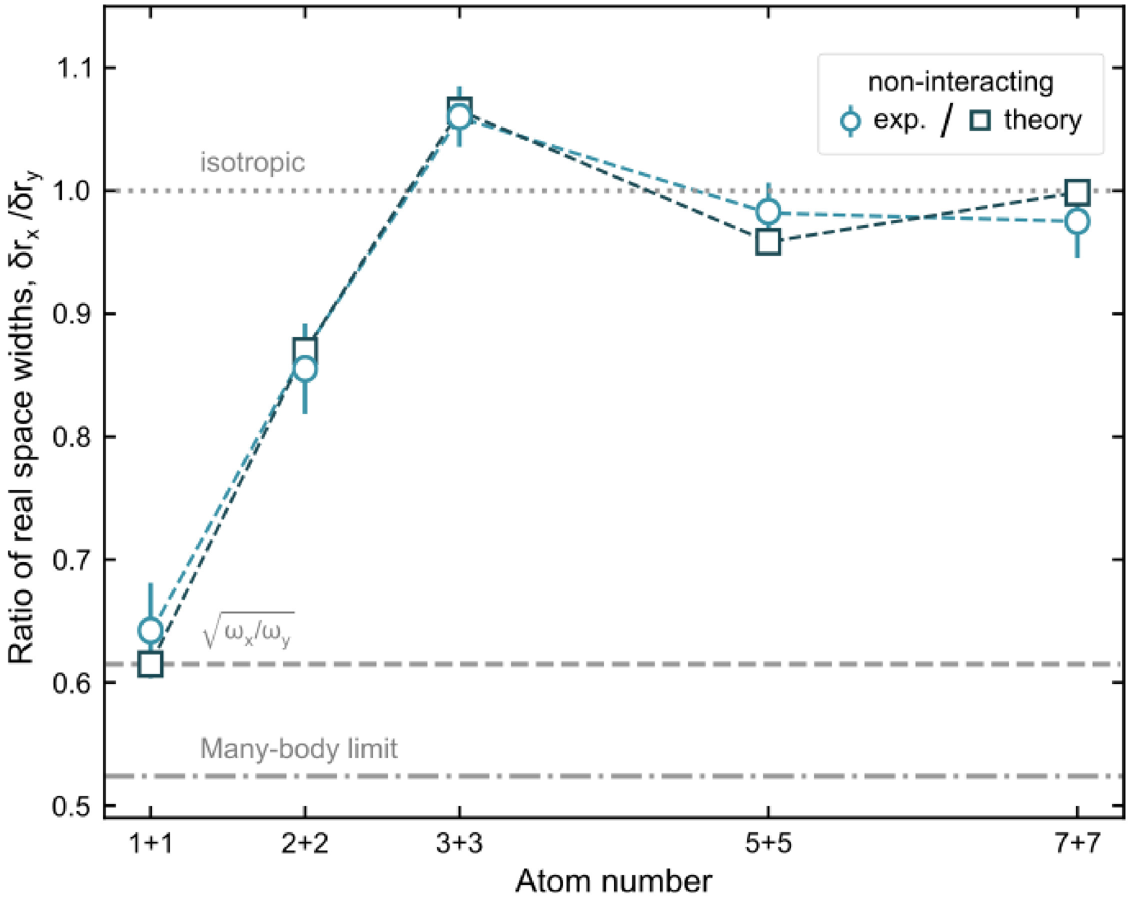
Building the fluid atom-by-atom

Noninteracting:

- Higher N → formation of round fermi surface
- Comparison to analytic solution of $|\psi(p_x,p_y)|^2$

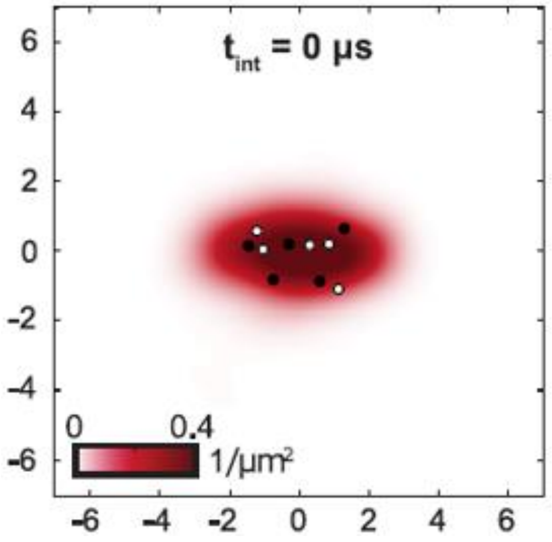
Interacting:

- Higher N → stronger ellipticity
- Inversion requires interactions

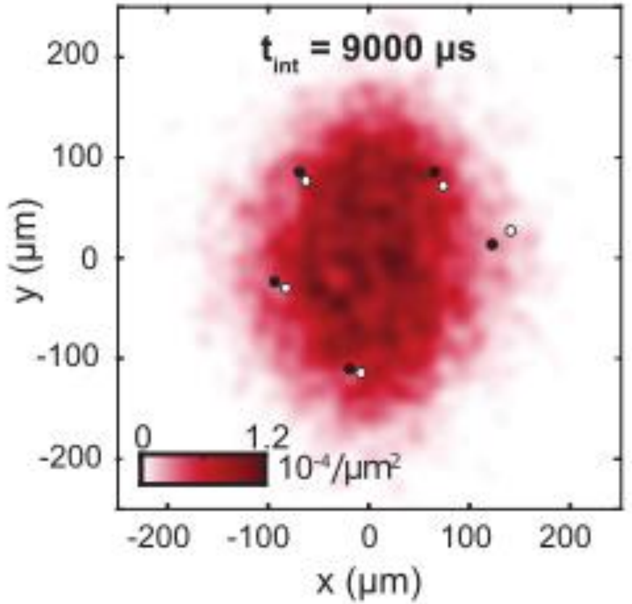


Conclusion/outlook

- Observation of the emergence of in interaction driven **elliptic flow**
- Redistribution of momentum distribution

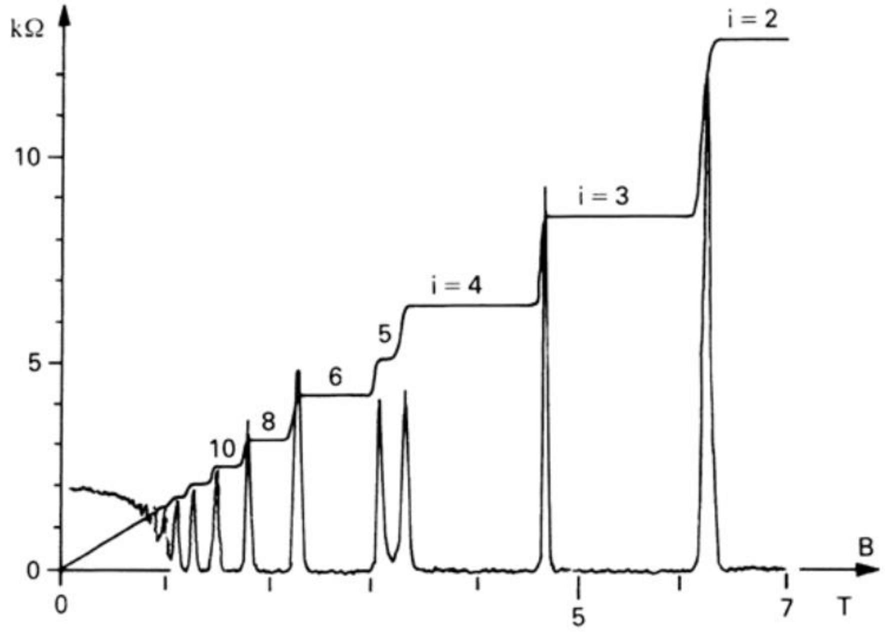


- Pair formation during interacting expansion
- Freeze out radius – HBT
- What happens if we turn on interactions during the expansion?

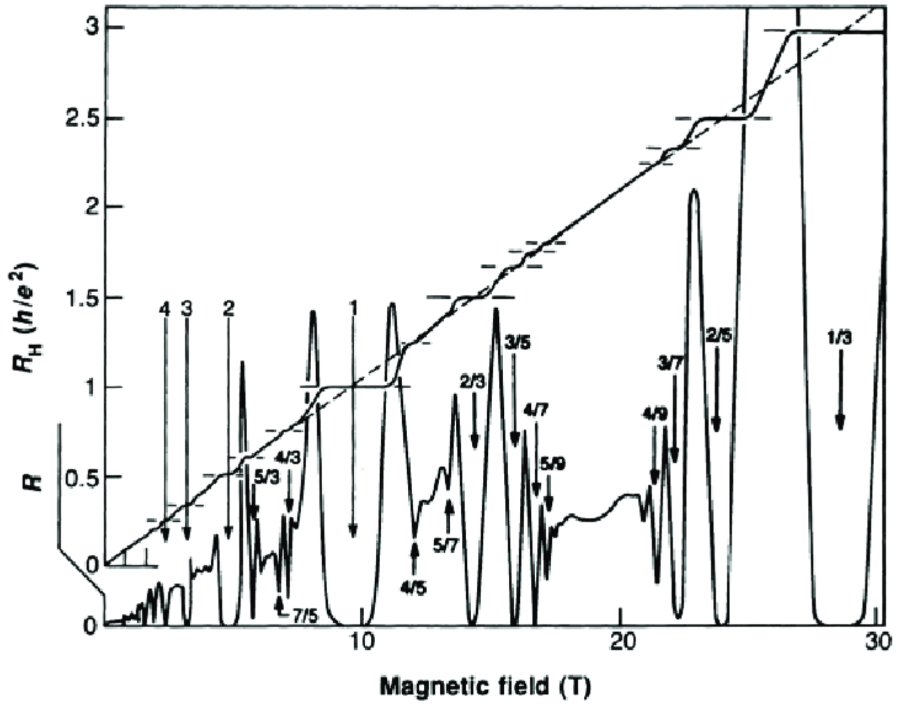


Part 2: Quantum Hall Physics

- 2D electron gas subjected to strong magnetic field
- Integer and Fractional quantum Hall effect



D. Tong, The Quantum Hall effect, lecture notes



H. Stormer, Physica B: Condensed Matter 177.1 (1992)

Part 2: Quantum Hall Physics

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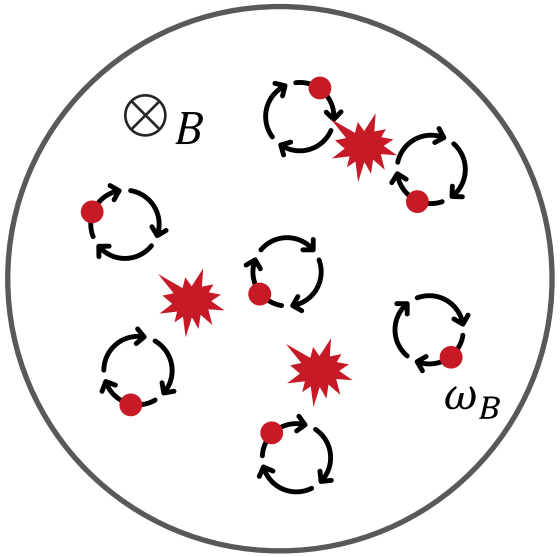
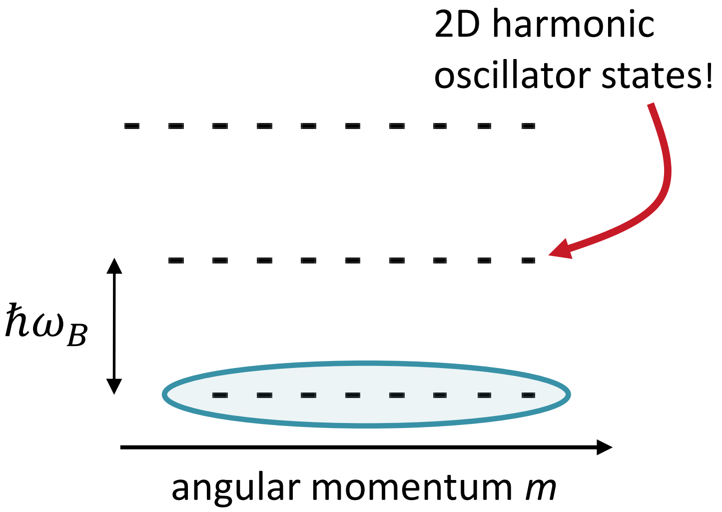
- 2D electron gas subjected to strong magnetic field

cyclotron motion



Landau levels



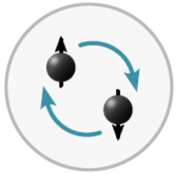
- states in LLL $|\mathrm{m}\rangle$: $\psi_m \sim r^{|m|} e^{im\varphi} = z^m$
- Coulomb interaction lifts degeneracy

Laughlin wavefunction

$$\Psi_{1/m} \sim \prod_{i<j} (z_i - z_j)^m$$

- suppresses repulsive interaction
- approximate ground state
- exact for contact interaction:
  - **non-interacting**
  - **eigenstates of free system**

$$\Psi_{1/m} \sim (z_1 - z_2)^m$$



Artificial magnetic fields

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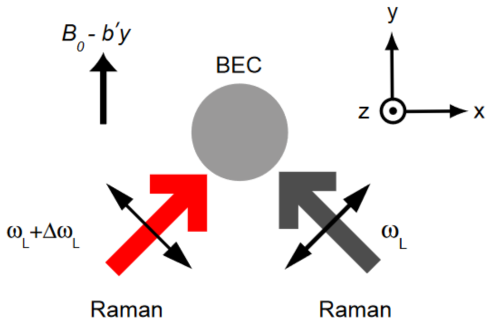
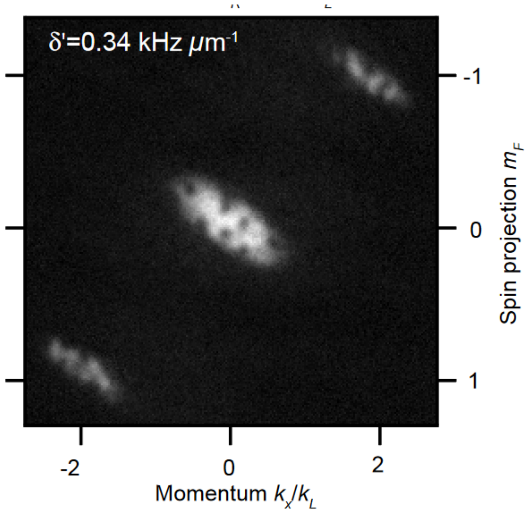
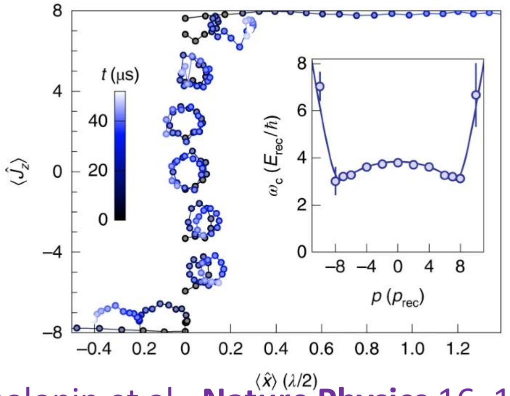
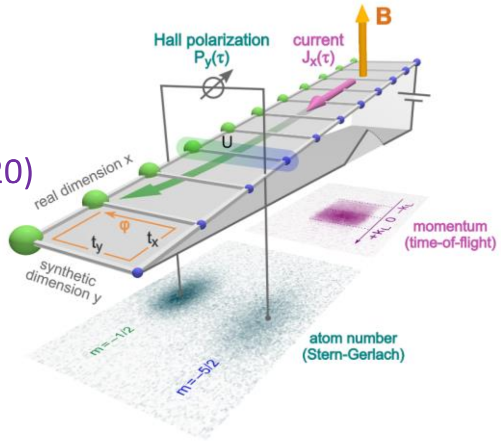




Y.J. Lin et al., **Nature** 462, 628 (2009)



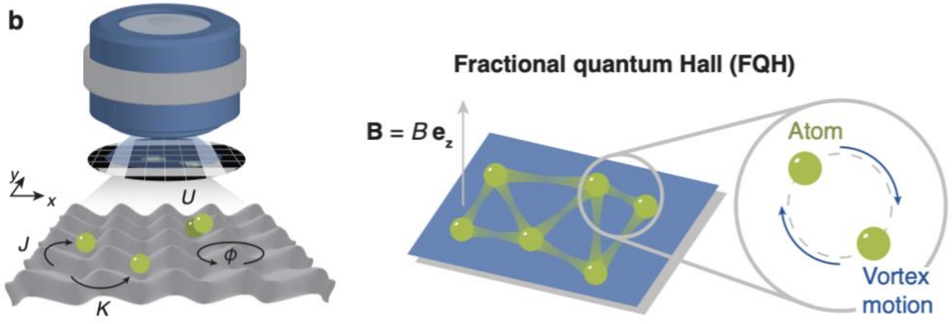
T Chalopin et al., **Nature Physics** 16, 1017 (2020)



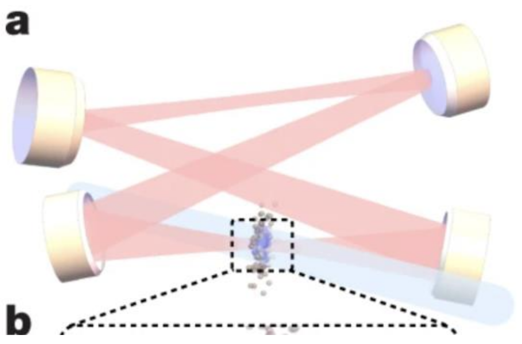
- Spin-orbit coupling
- Synthetic dimensions
- Floquet engineering

TW Zhou et al., **Science** 381 (2022)

Artificial gauge fields + interactions



J. Leonard et al., **Nature** *619*, 495 (2023)



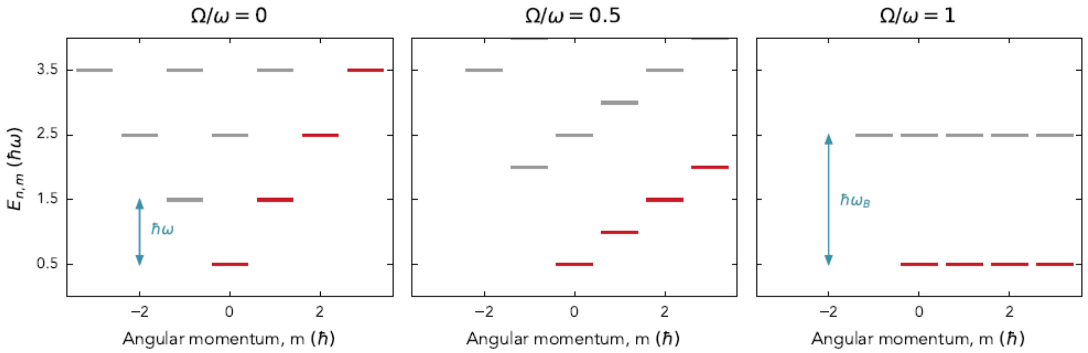
L. Clark et al., **Nature** *582*, 41 (2020)

Rotating ultracold gases

Lorentz force	$\boldsymbol{F}_L = -qB\,\boldsymbol{e}_z \times \boldsymbol{v}$
Coriolis force	$\boldsymbol{F}_C = -2m\Omega\,\boldsymbol{e}_z \times \boldsymbol{v}$

$$\mathcal{H}_\Omega = \mathcal{H} - \Omega L_z$$

$$\mathcal{H}_\Omega = \frac{(\boldsymbol{p} - m_a \boldsymbol{\Omega} \times \boldsymbol{r})^2}{2m_a} + \frac{m_a}{2}\left(\omega^2 - \Omega^2\right) r^2.$$



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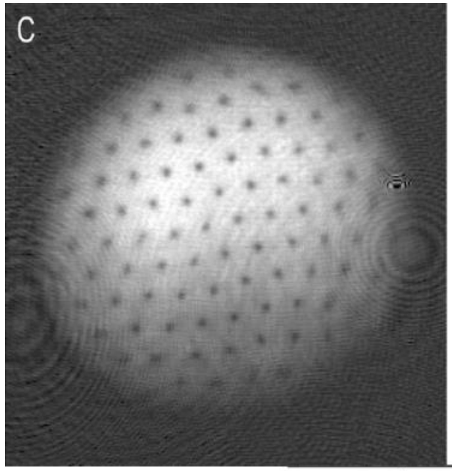
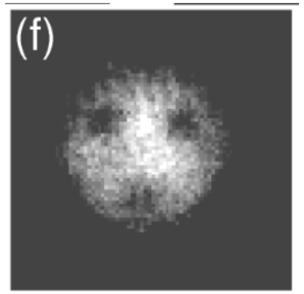
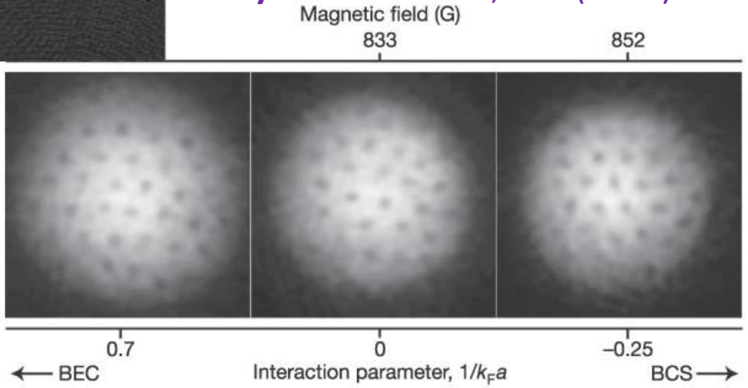




KW Madison et al.,
Phys. Rev. Lett. 84, 806 (2000)

JR Abo-Shaeer et al.,
Science 292, 476 (2001)

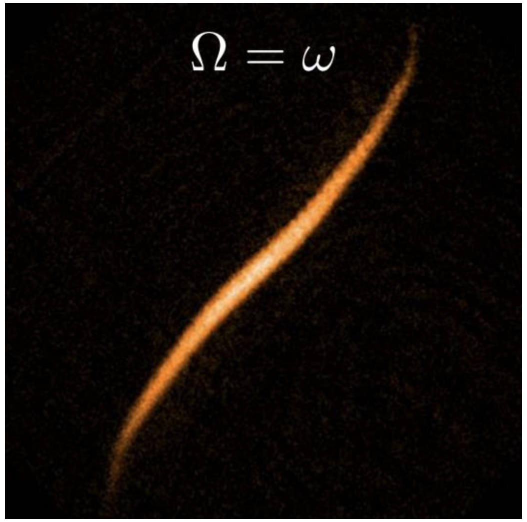


MW Zwierlein et al., **Nature** 435, 1047 (2005)

Experiment: W. Ketterle, E. Cornell , S. Chu, J. Dalibard, M. Zwierlein ...

More recently

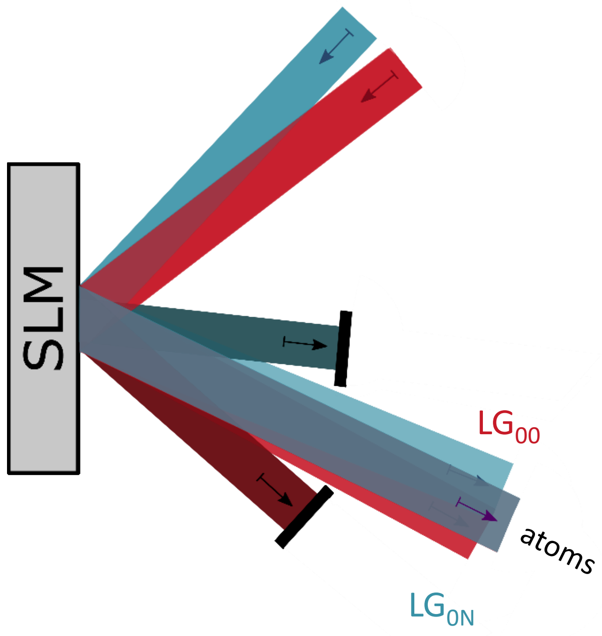
Preparing BEC in a lowest Landau level

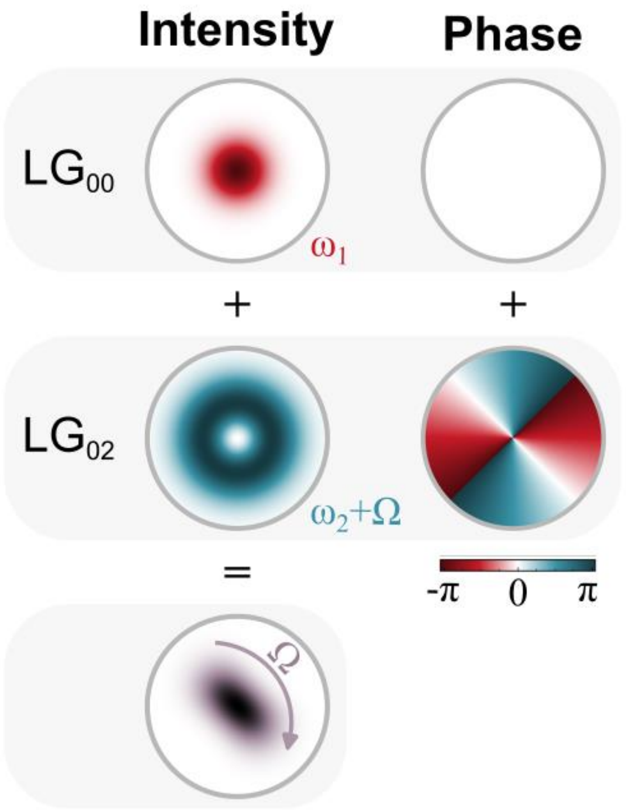


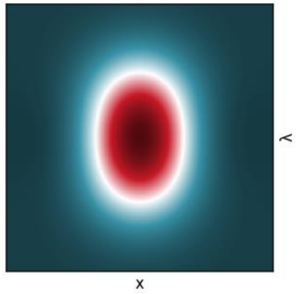
RJ Fletcher et al., **Science** 372, 1318 (2021)

Rotating traps

Interference of Laguerre-Gaussian beams



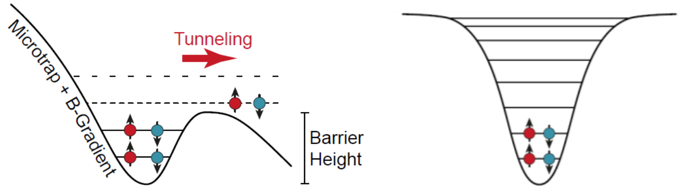




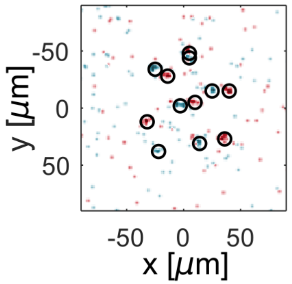
- Coupling states with different angular momentum
- With LG_{02}:
  - engineers operator $H_p \sim z^l e^{i\Omega t} + h.c.$
  - $|m\rangle \leftrightarrow |m+l\rangle$

Our tools

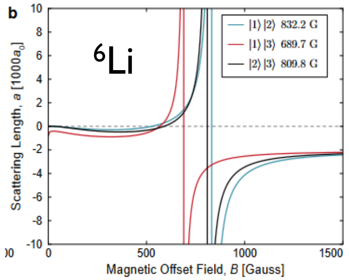
1. Deterministic preparation



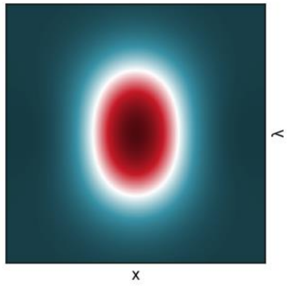
3. Single-atom microscopy



2. Interaction tuneability



4. Rotation



Rotating a single particle

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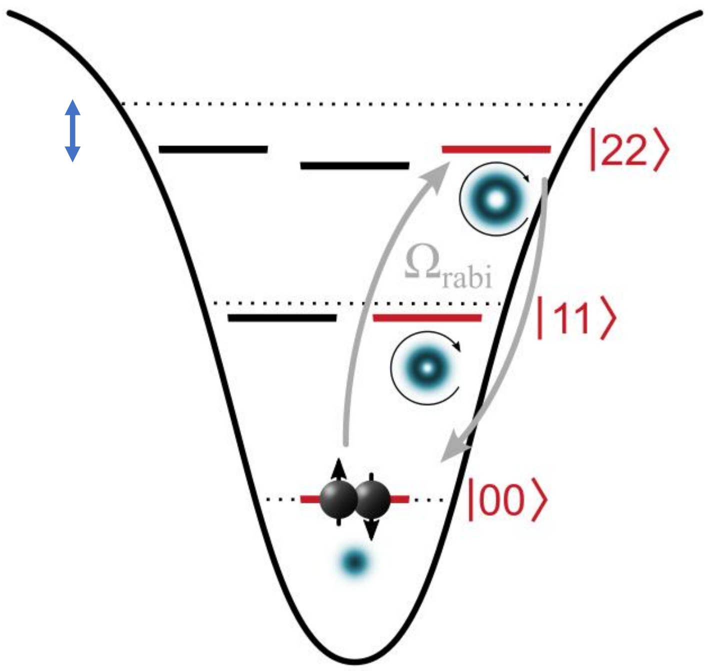
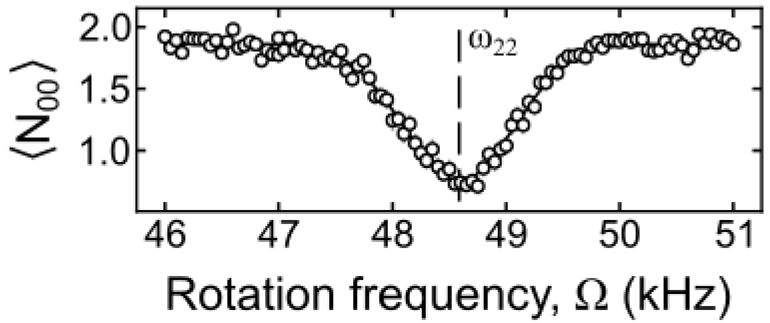
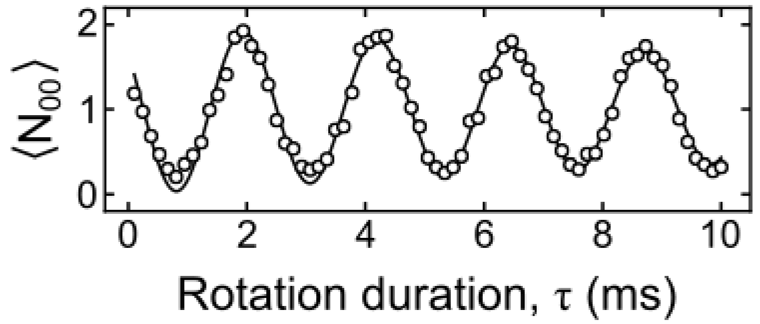
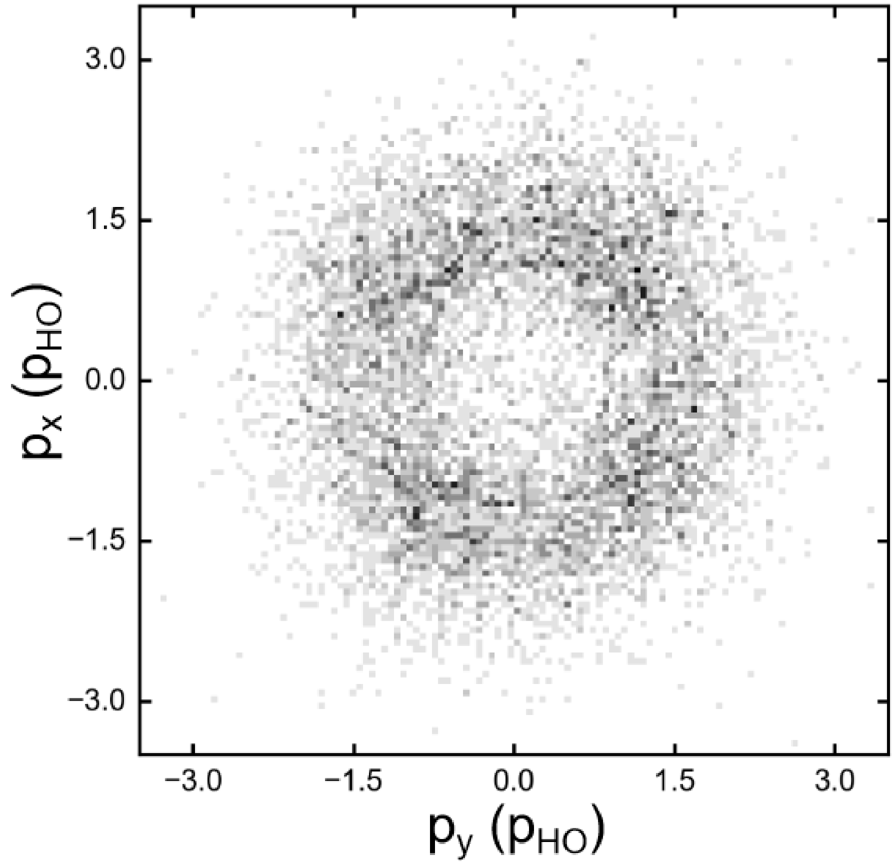
- start with 2 non-interacting particles in the ground state
  $\frac{\omega_x}{2\pi} = \frac{\omega_y}{2\pi} \approx 56\ \mathrm{kHz},\ \frac{\omega_z}{2\pi} \approx 8\ \mathrm{kHz}$
- perturbation couples to $|22\rangle$
- transition $|22\rangle \rightarrow |44\rangle$ off-resonant due to anharmonicity

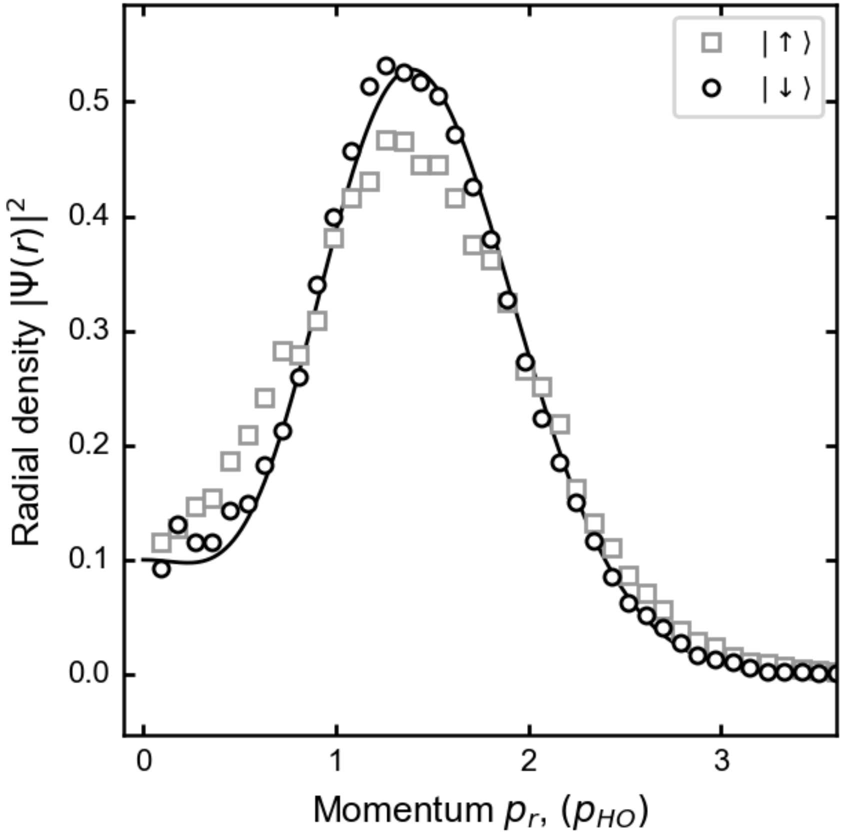






Single particle rotation – probability density





Do we really rotate?

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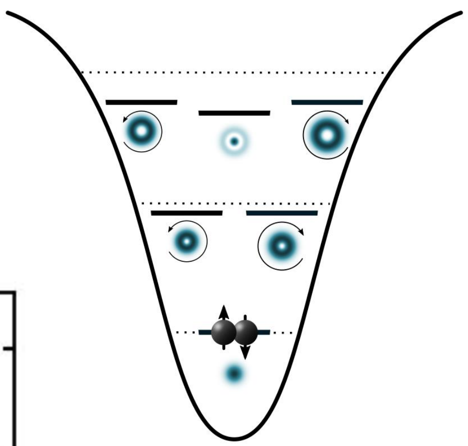
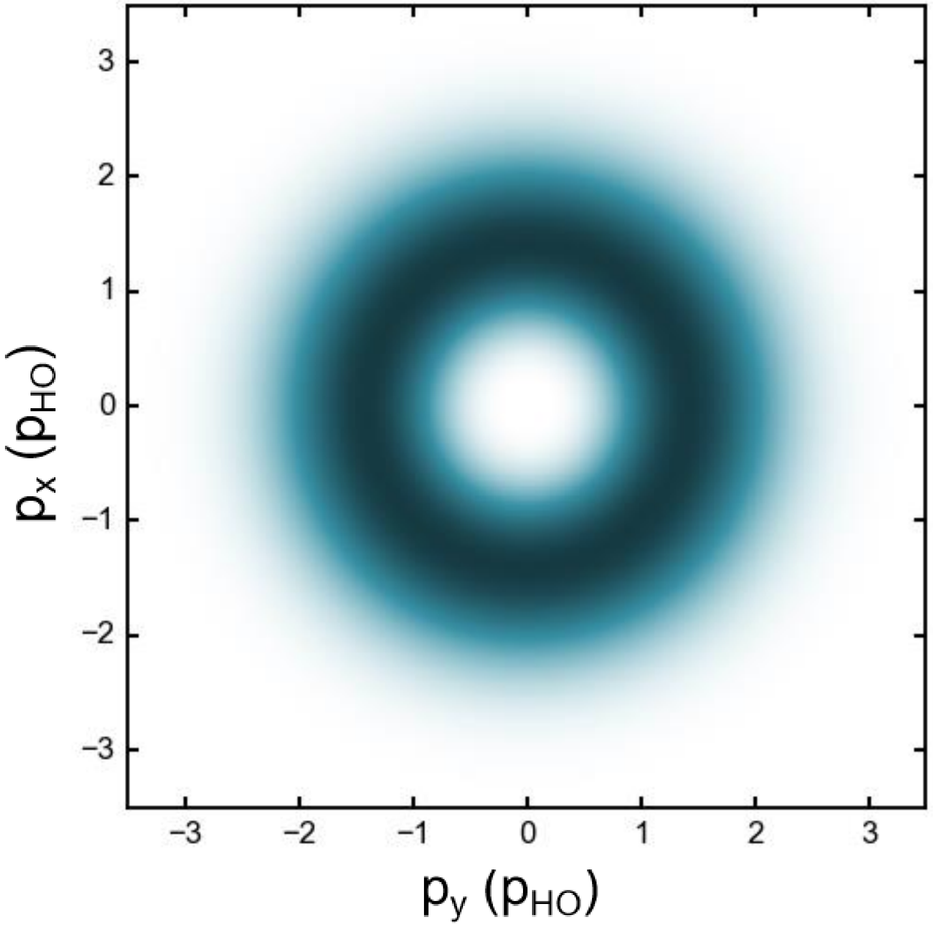
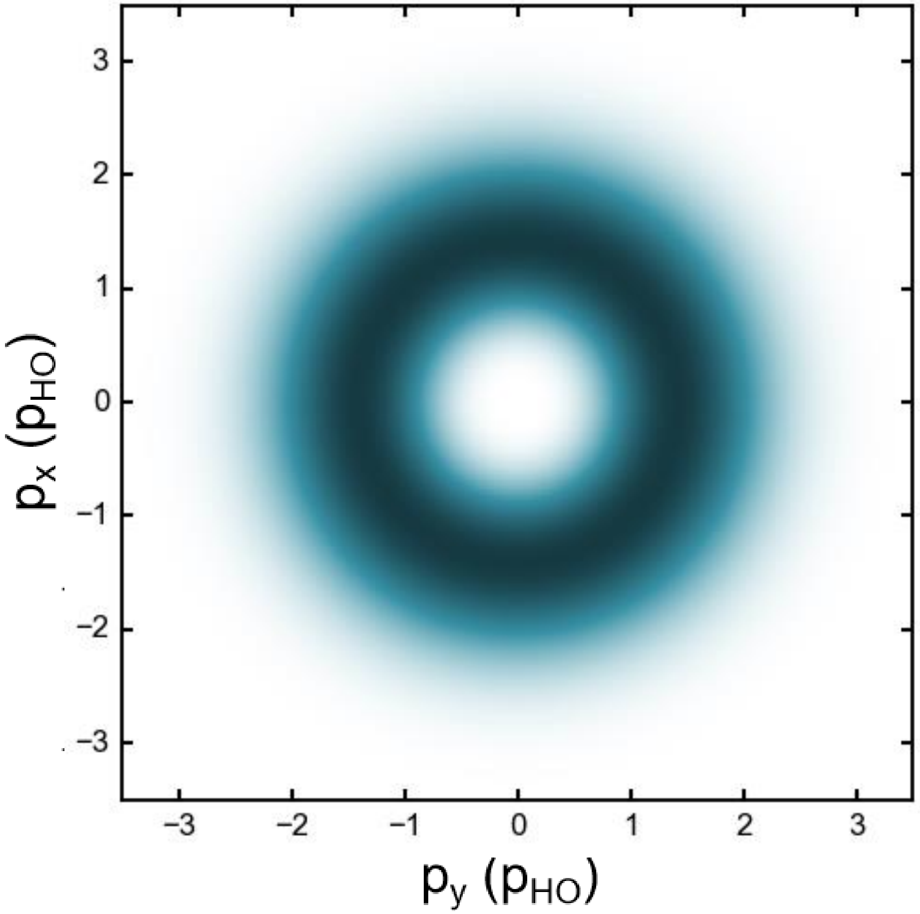


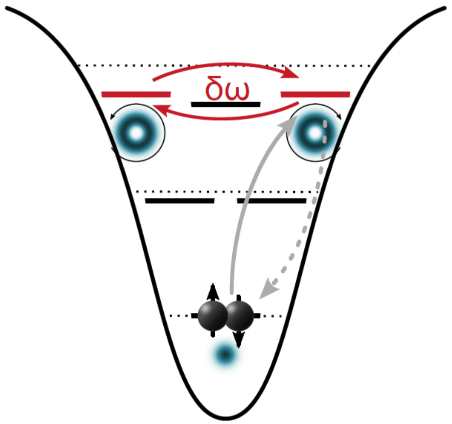
Superposition of $|00\rangle - |20\rangle$

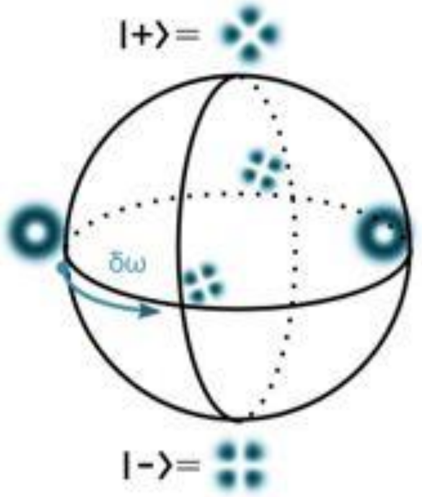


Incoherent mixture of $|\pm\rangle = |2,-2\rangle \pm |2,2\rangle$



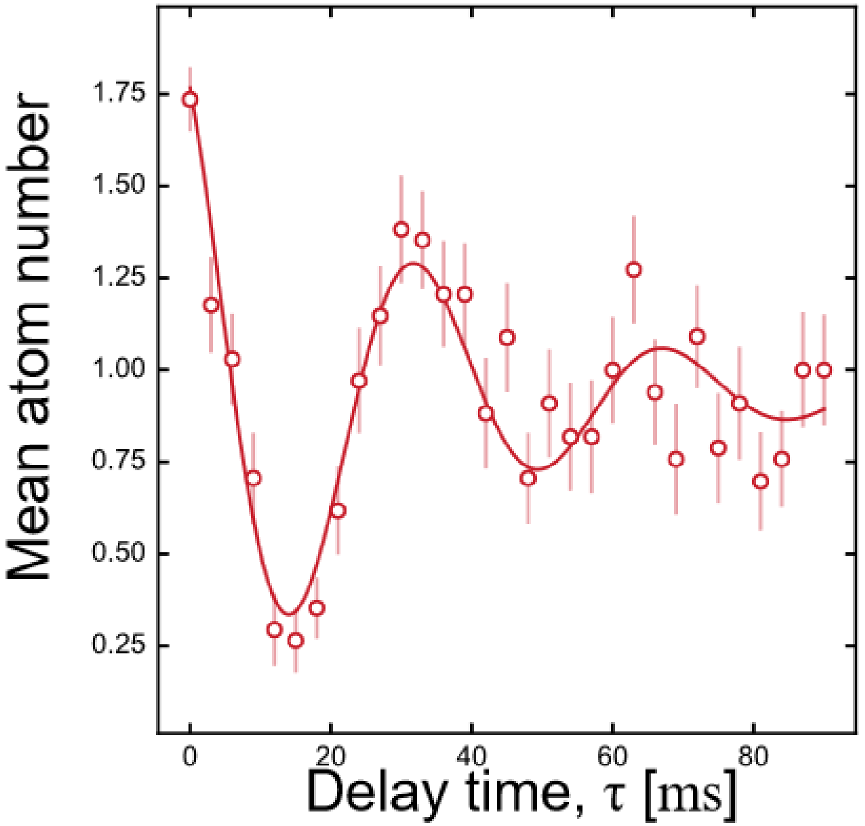
Sense of rotation





$\nu_r \sim 25kHz, \nu_x - \nu_y \sim 25Hz$

A slight anisotropy lifts degeneracies:
Angular momentum states are no longer eigenstates!



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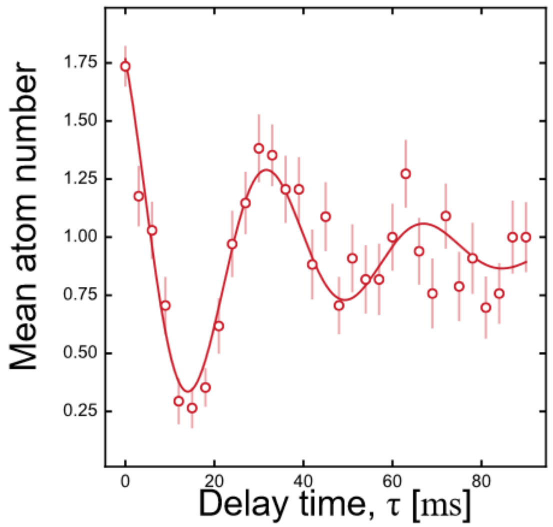

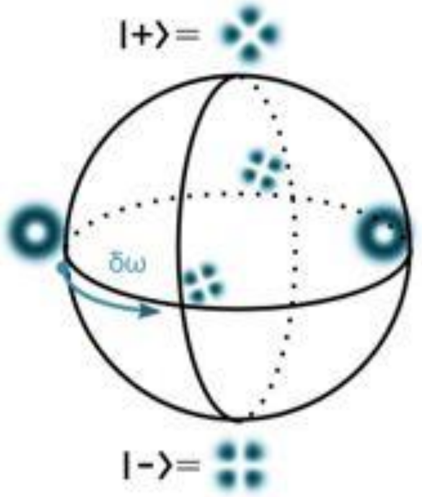

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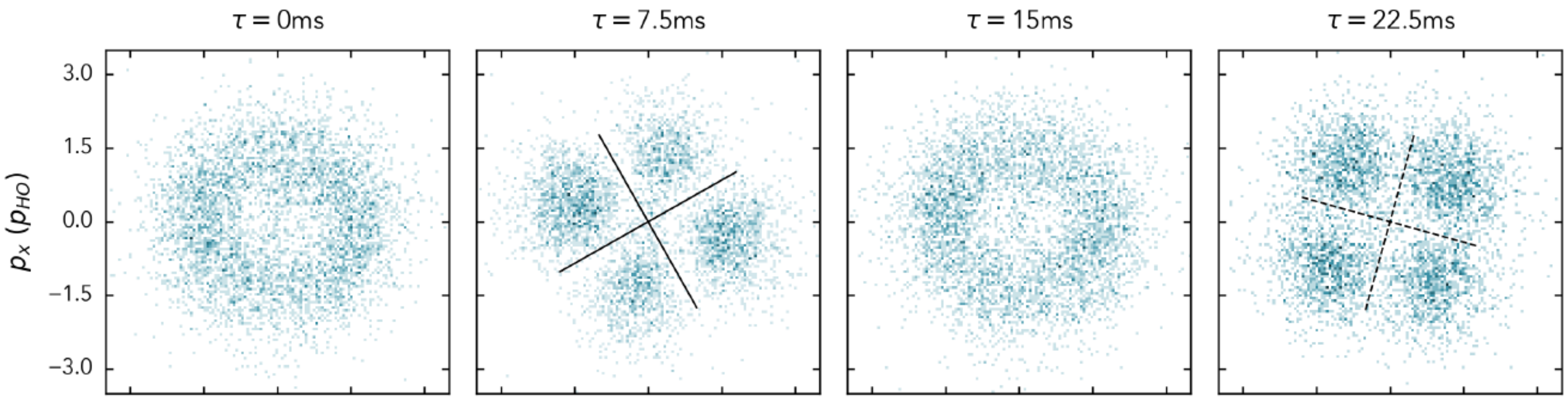
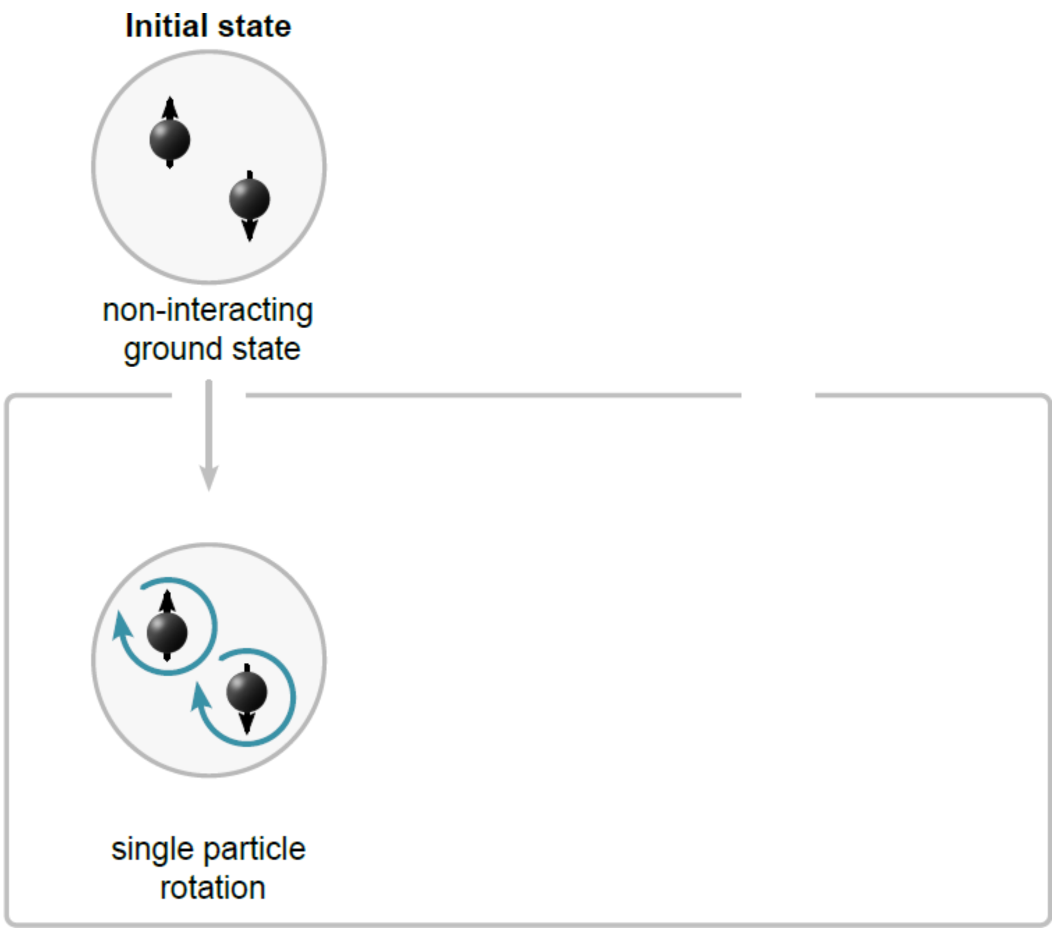


Rotating in the opposite direction

Conceptual path to a Laughlin state



Preparing a Laughlin State: Conceptual Idea

2 particles in a harmonic trap

$$H = H_{\uparrow}^{\text{ho}} + H_{\downarrow}^{\text{ho}} + g\delta^{(3)}(\boldsymbol{r}_{\uparrow} - \boldsymbol{r}_{\downarrow})$$

$$= H_{\text{com}}^{\text{ho}} + H_{\text{rel}}^{\text{ho}} + g\delta^{(3)}(\boldsymbol{r})$$

Preparing a Laughlin State: Conceptual Idea

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Preparing a Laughlin State: Conceptual Idea

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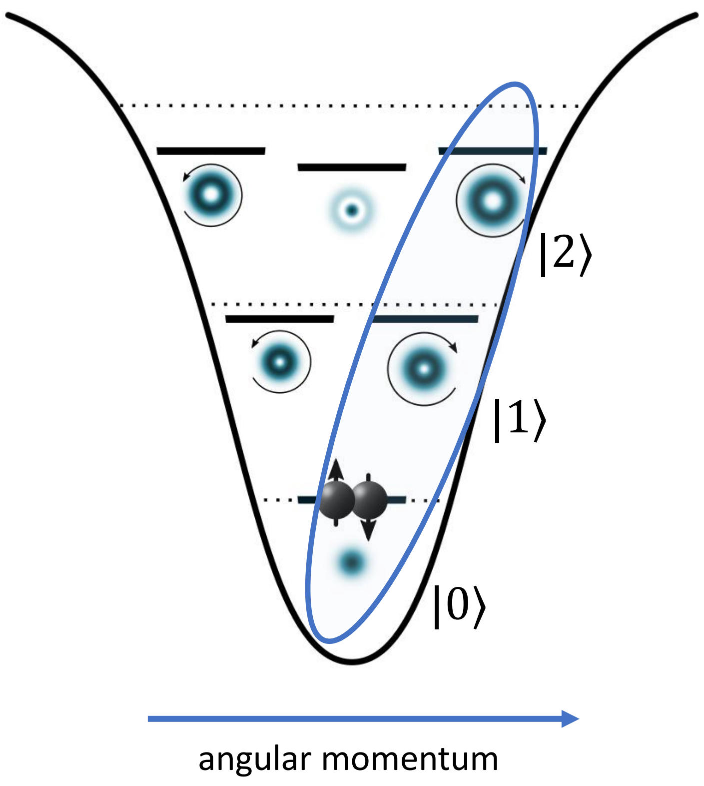
2 particles in a harmonic trap

non-interacting, 2D



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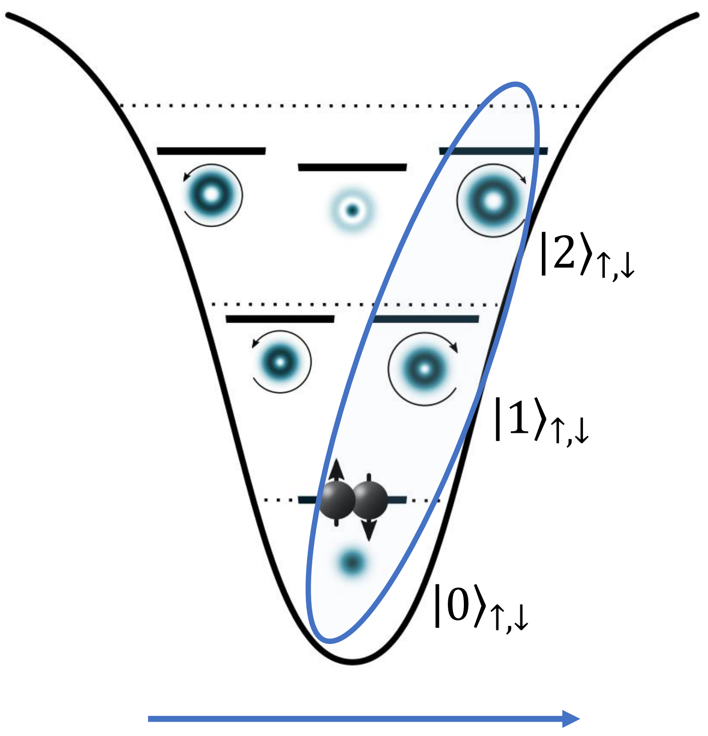

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Preparing a Laughlin State: Conceptual Idea

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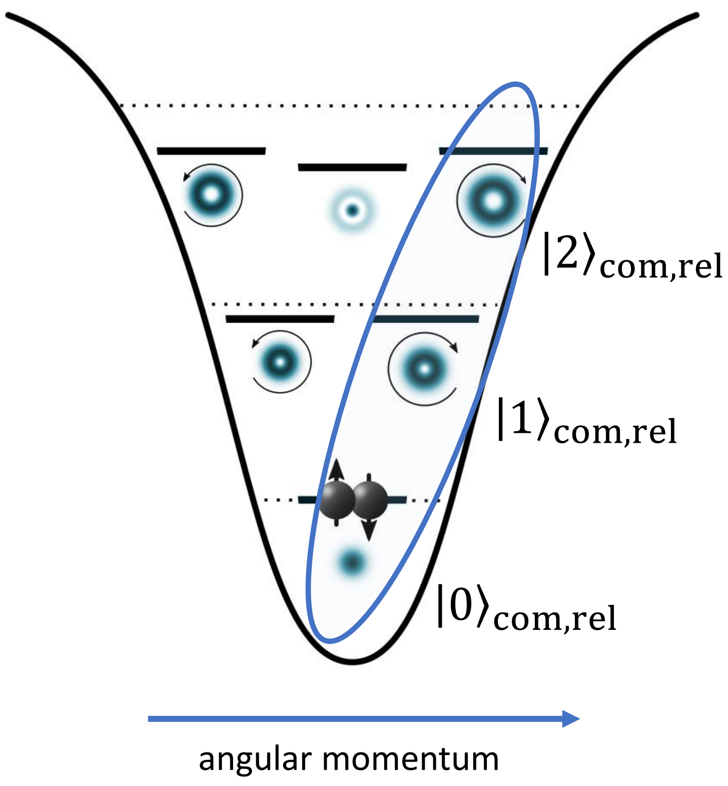

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Preparing a Laughlin State: Conceptual Idea

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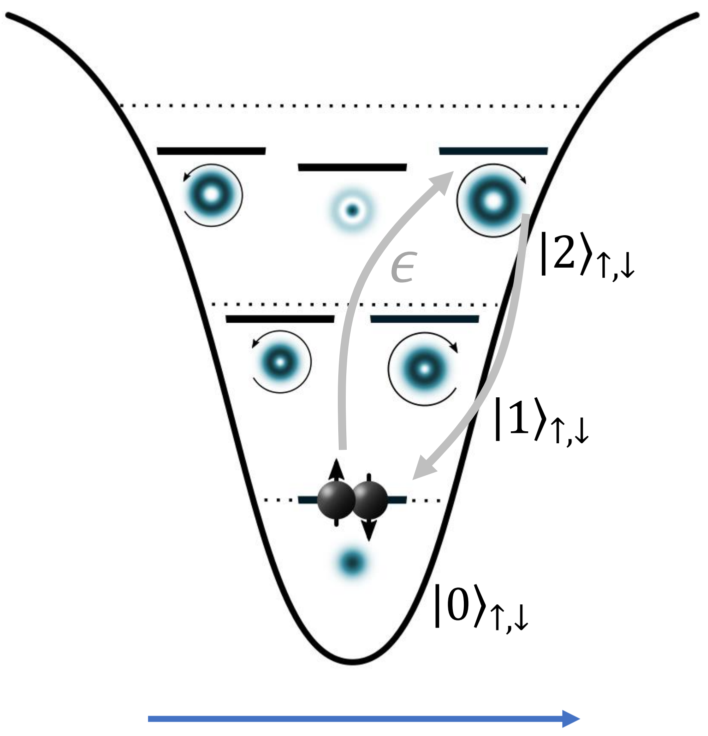

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$$= H_{\text{com}}^{\text{ho}} + H_{\text{rel}}^{\text{ho}} + g\delta^{(3)}(\boldsymbol{r})$$

analogously for perturbation

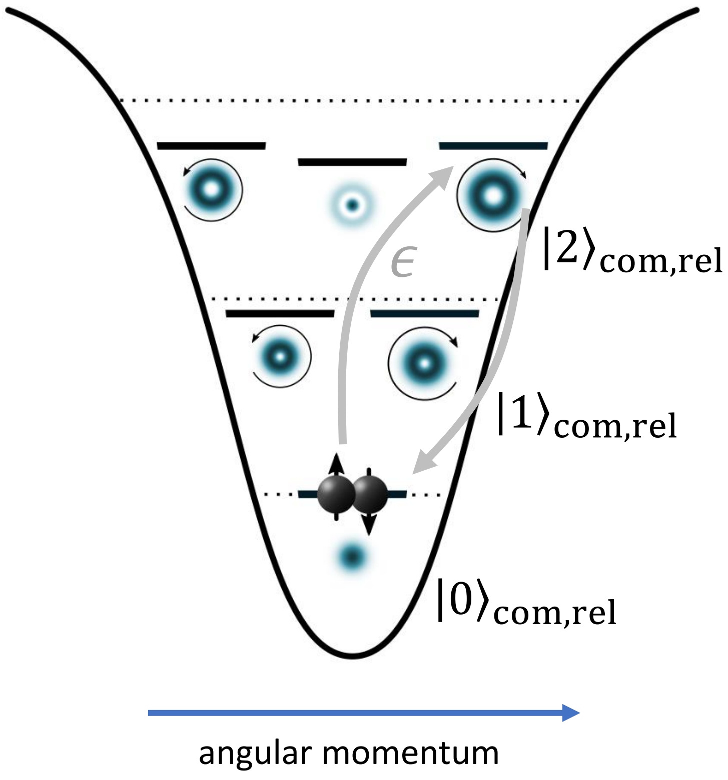
$$\sim \epsilon\left(z_\uparrow^2 e^{i\Omega t} + z_\downarrow^2 e^{i\Omega t} + h.c.\right)$$

$$= \epsilon\left(z_{com}^2 e^{i\Omega t} + z_{rel}^2 e^{i\Omega t} + h.c.\right)$$

Preparing a Laughlin State: Conceptual Idea

2 particles in a harmonic trap

non-interacting, 2D



$$H = H_{\uparrow}^{\text{ho}} + H_{\downarrow}^{\text{ho}} + g\delta^{(3)}(\boldsymbol{r}_{\uparrow} - \boldsymbol{r}_{\downarrow})$$

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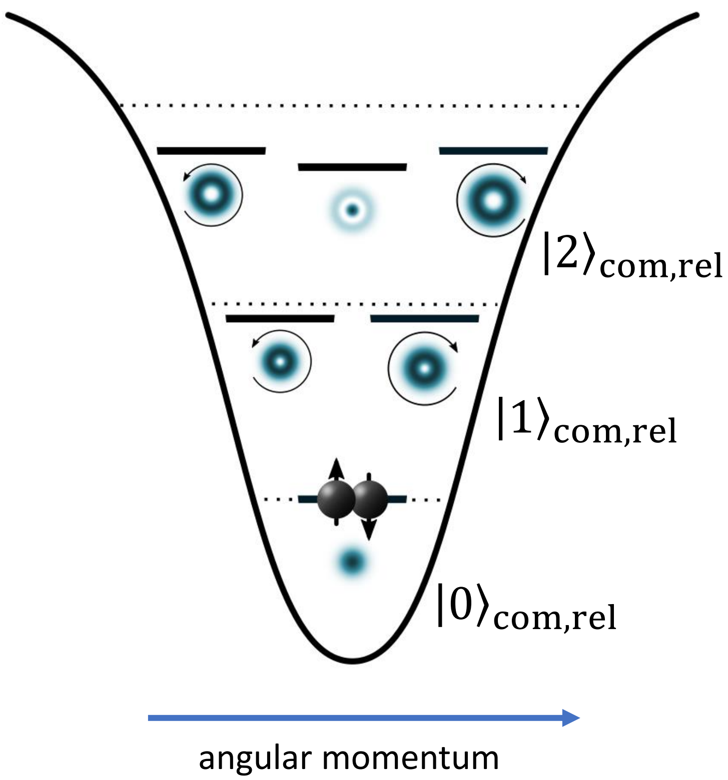
Preparing a Laughlin State: Switching on Interactions



Focus on states

Preparing a Laughlin State: Switching on Interactions

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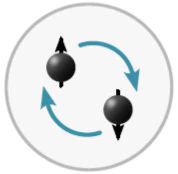
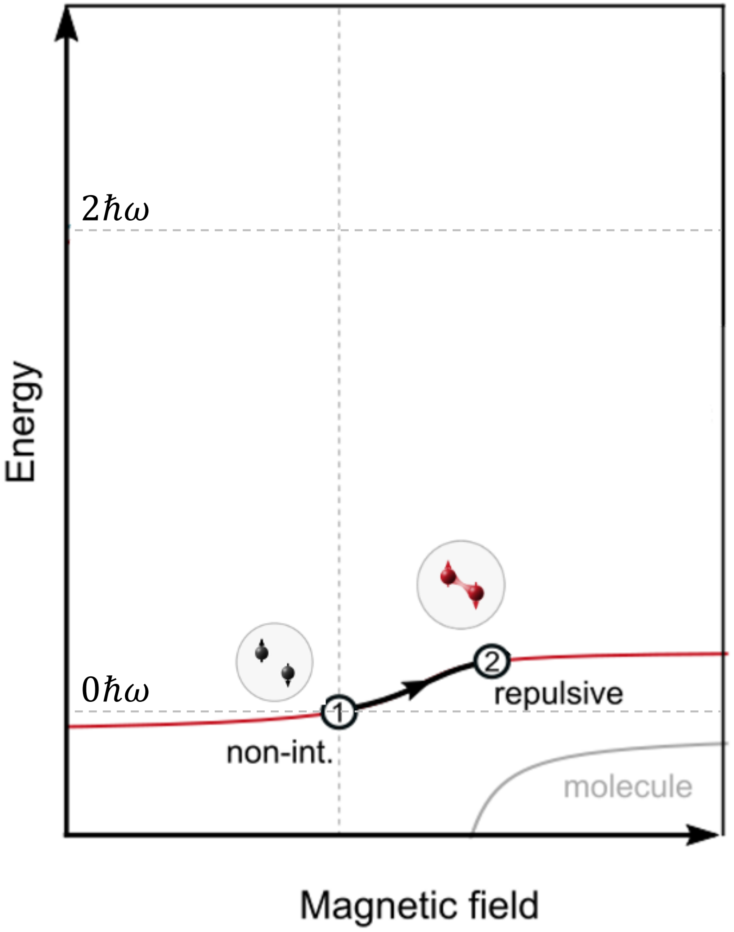
Focus on states

$|2\rangle_{\mathrm{com}}|0\rangle_{\mathrm{rel}}$

$|0\rangle_{\mathrm{com}}|2\rangle_{\mathrm{rel}}$



$|0\rangle_{\mathrm{com}}|0\rangle_{\mathrm{rel}}$



Preparing a Laughlin State: Switching on Interactions

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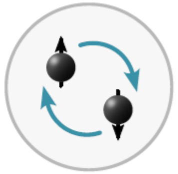

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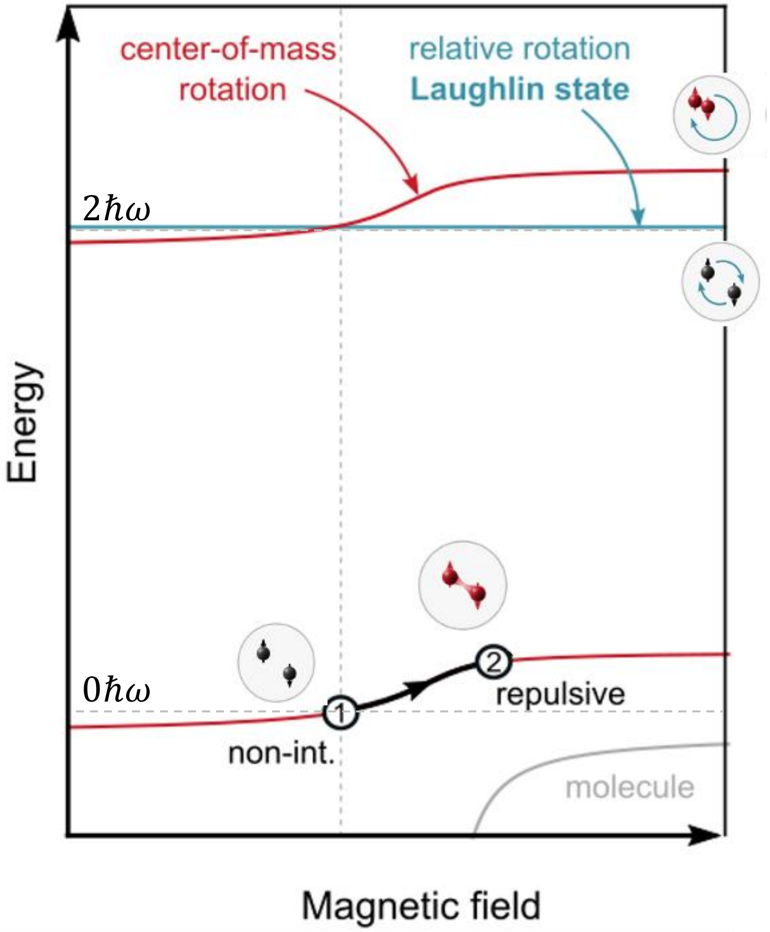

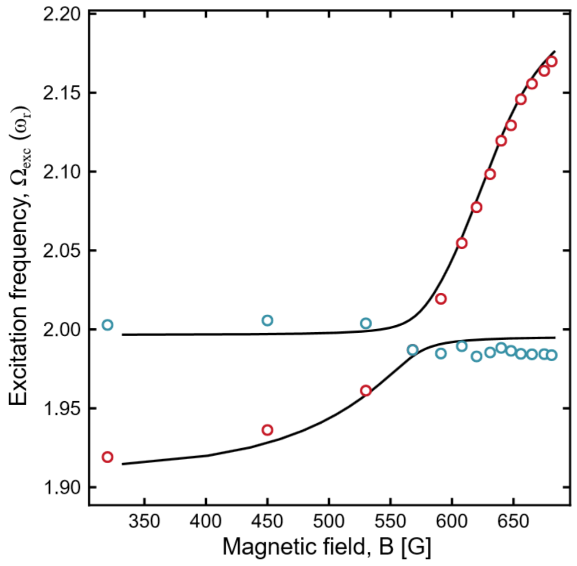

Preparing a Laughlin State: Switching on Interactions

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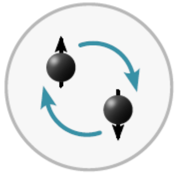

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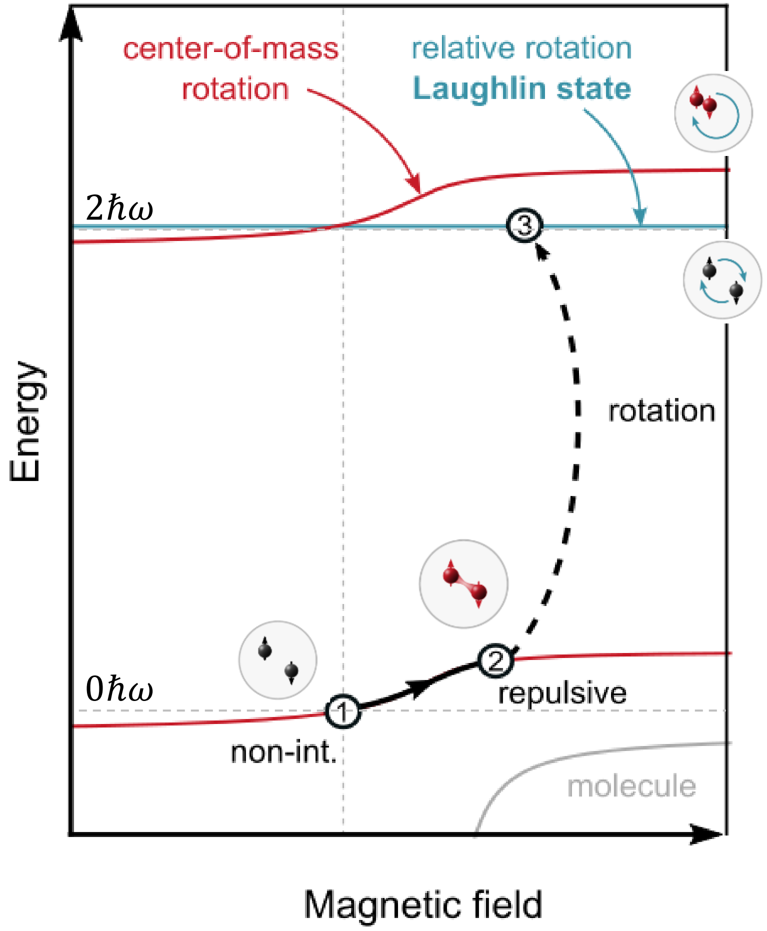
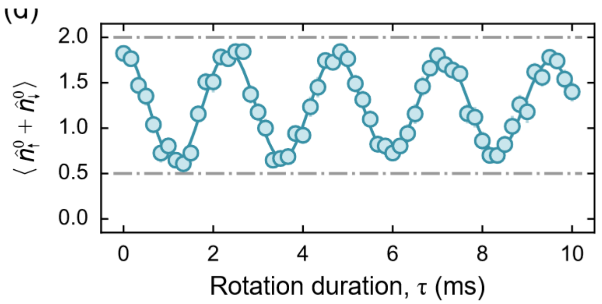
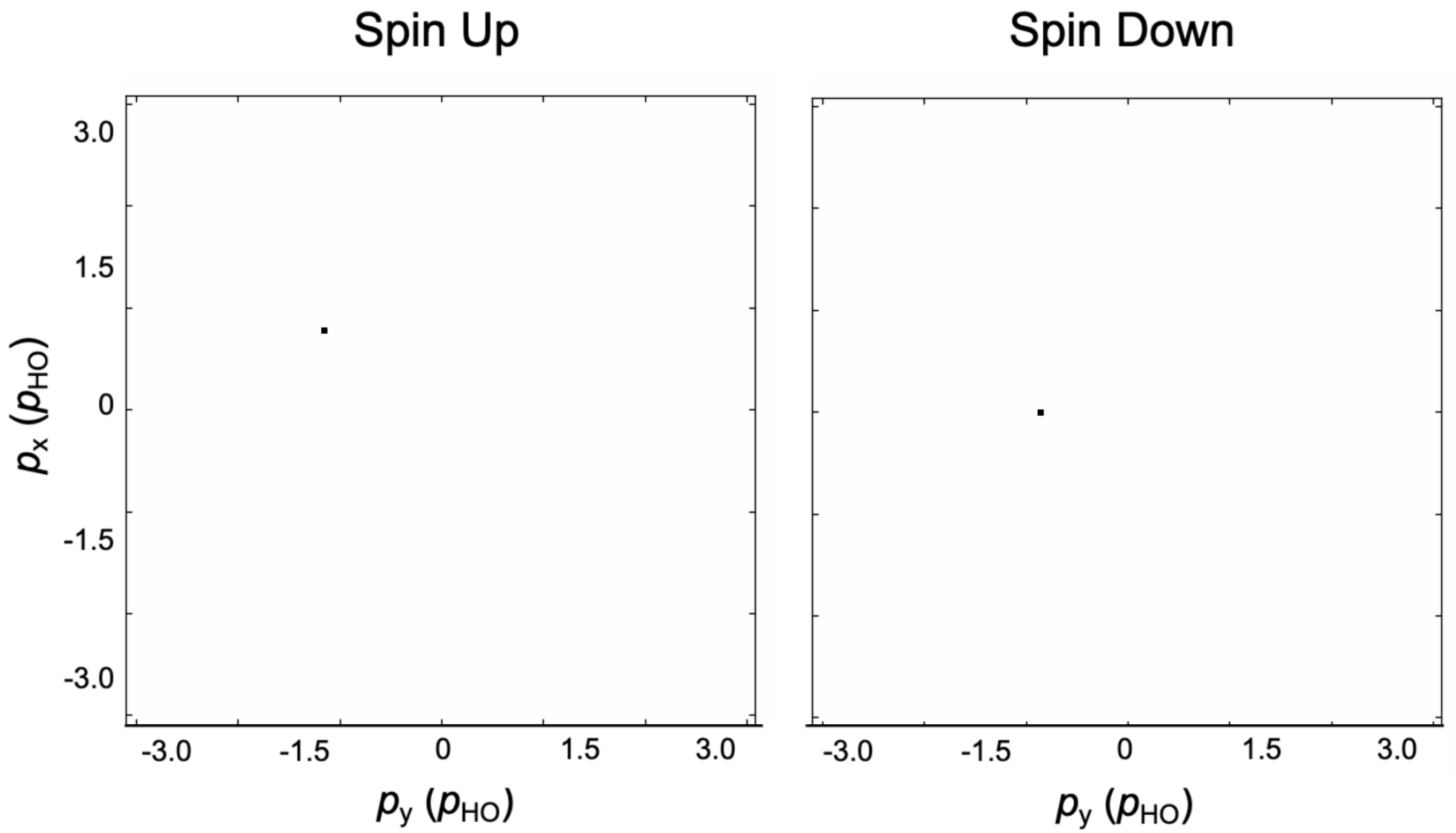




Two particle Laughlin state – single particle basis



Two particle Laughlin state – single particle basis

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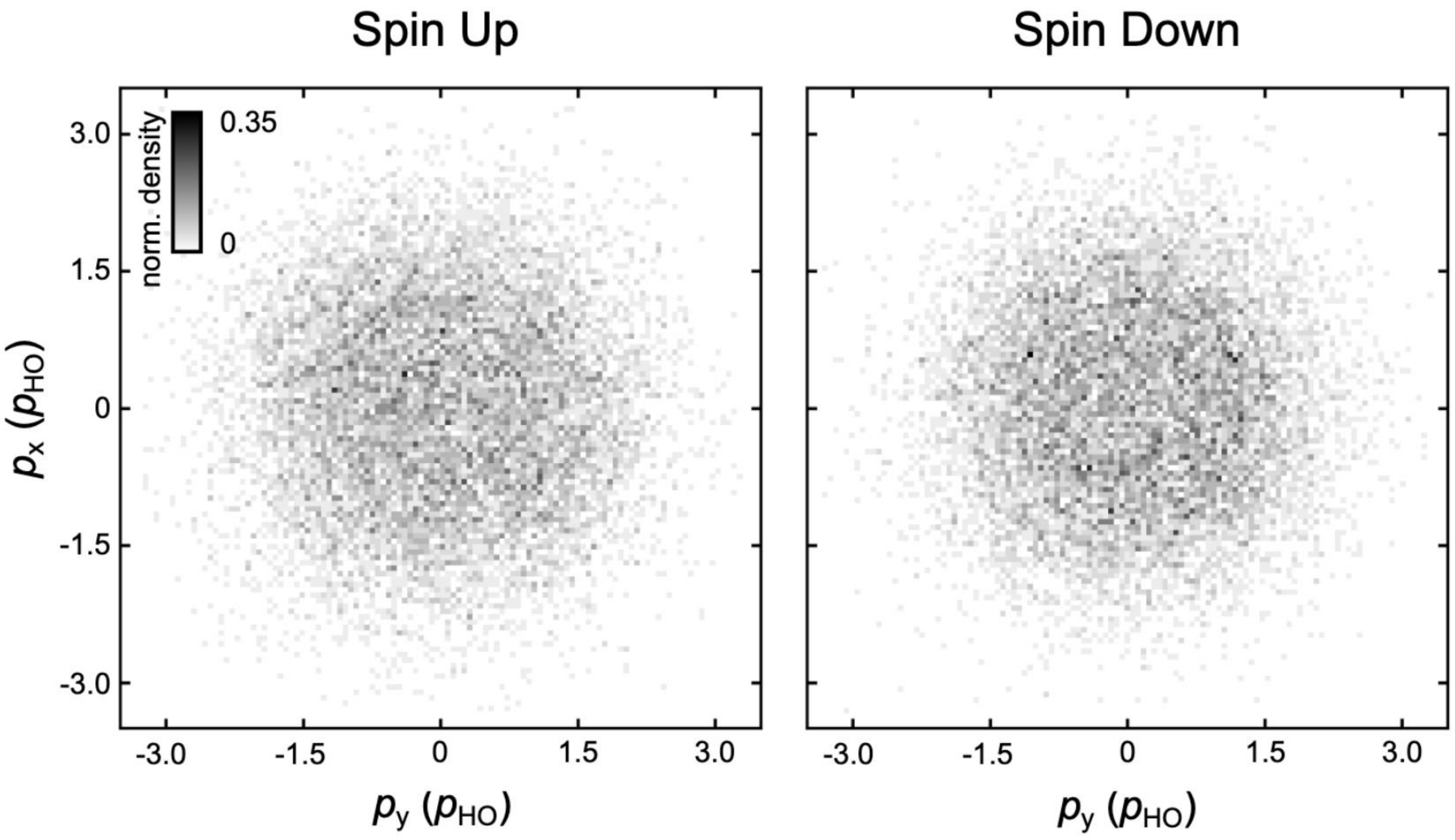
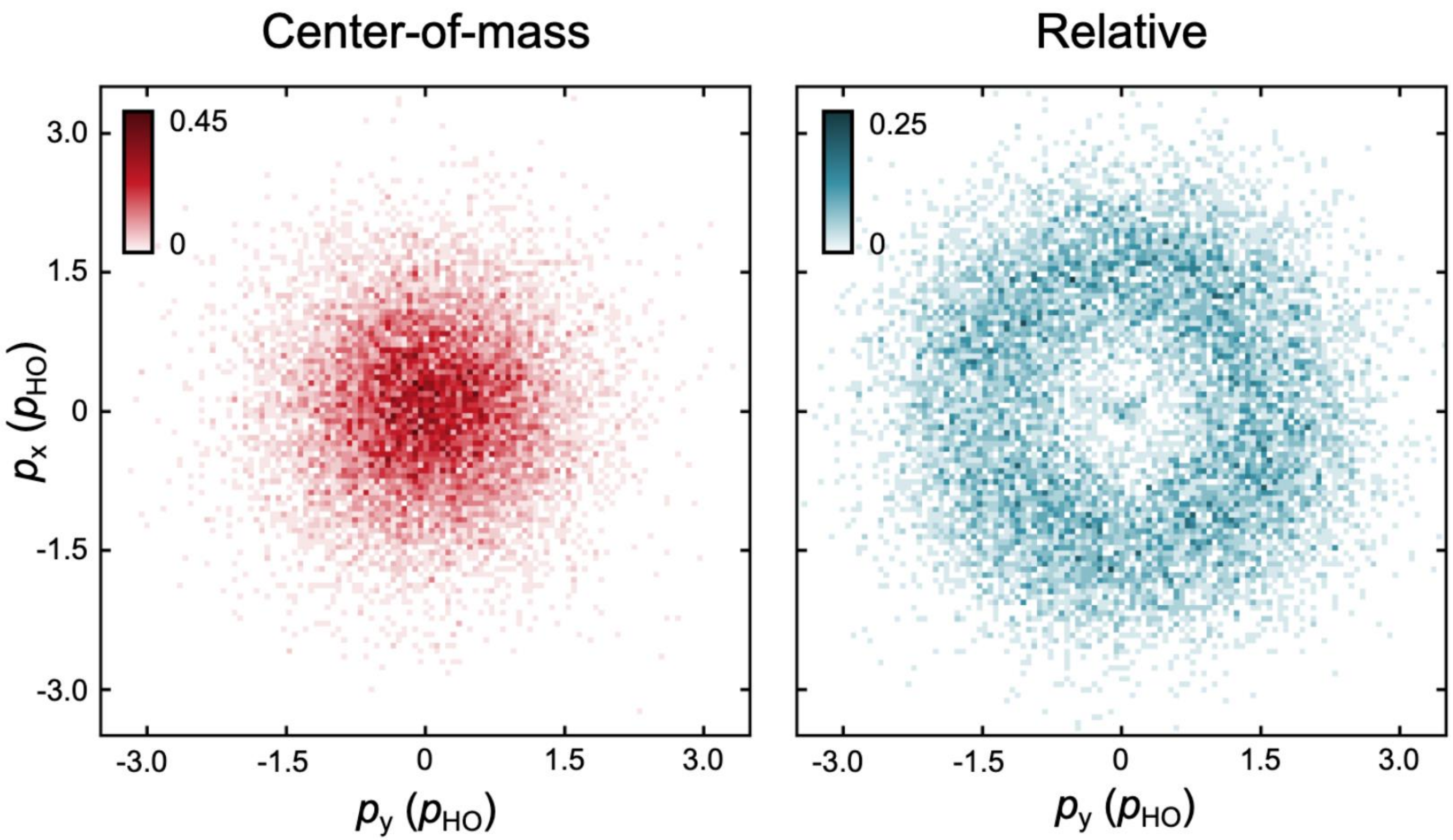


Two particle Laughlin state – *com* and *relative* basis



$\boldsymbol{p}_{com} = 1/\sqrt{2}(\boldsymbol{p}_\uparrow + \boldsymbol{p}_\downarrow)$

$\boldsymbol{p}_{rel} = 1/\sqrt{2}(\boldsymbol{p}_\uparrow - \boldsymbol{p}_\downarrow)$

Radial densities

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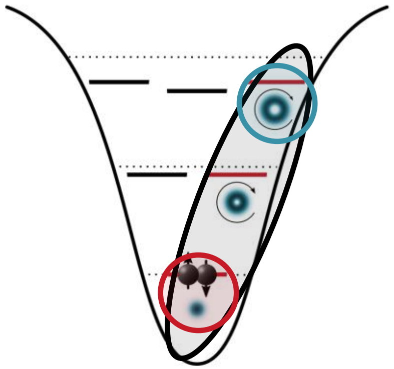
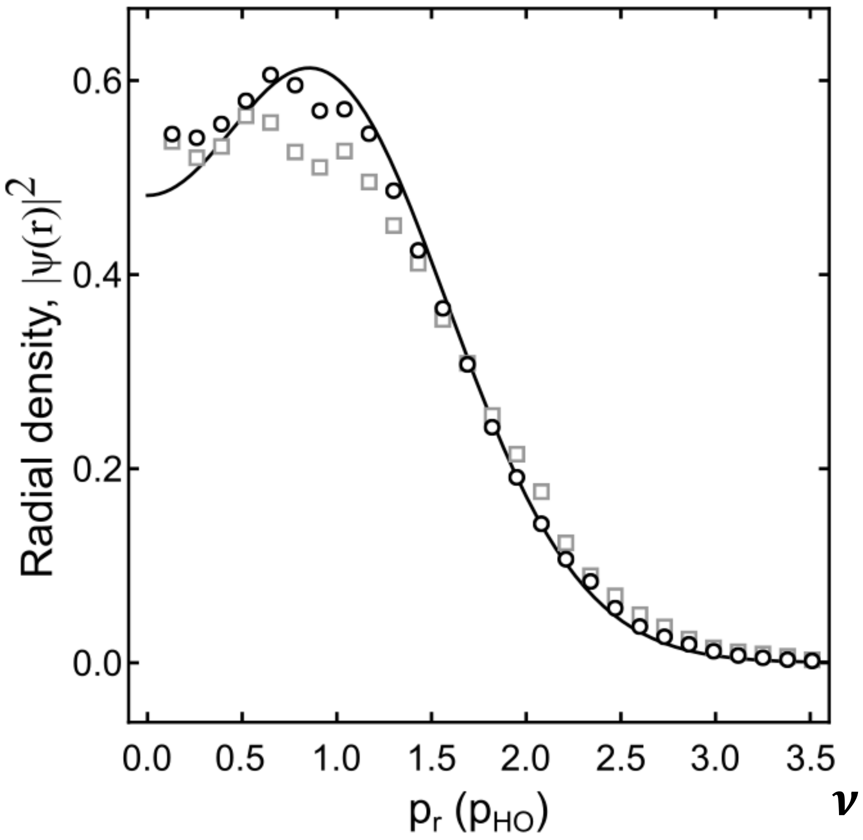
$$\Psi_{1/2} \sim (z_\uparrow - z_\downarrow)^2 = |0\rangle_{\text{com}} |2\rangle_{\text{rel}}$$

$$= \frac{1}{2}|0\rangle_\uparrow\, |2\rangle_\downarrow + \frac{1}{2}|2\rangle_\uparrow |0\rangle_\downarrow - \frac{1}{\sqrt{2}}|1\rangle_\uparrow |1\rangle_\downarrow$$

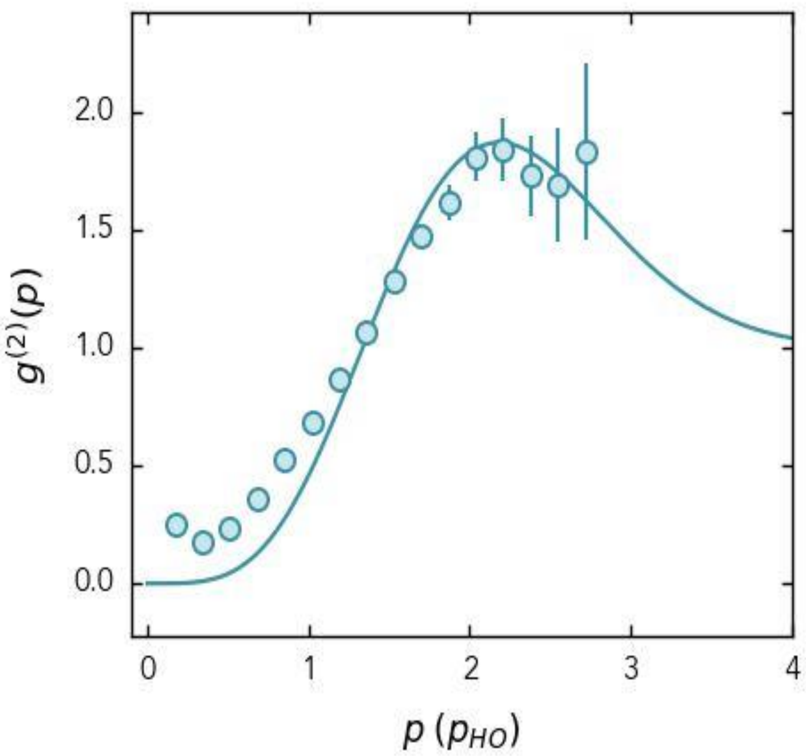


Single particle basis



Center-of-mass and relative basis

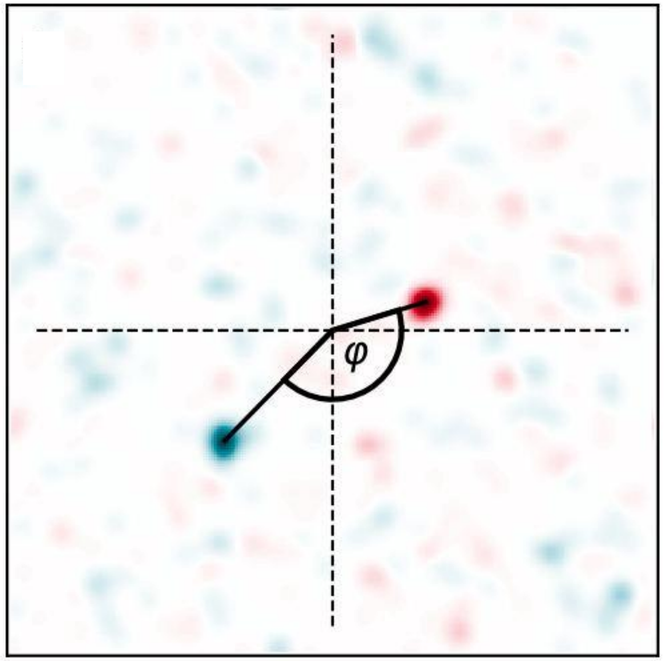
$$g^{(2)}(p) = \frac{\langle n(0) n(p) \rangle}{\langle n \rangle \langle n \rangle}$$



$\nu = 1/2$ **Laughlin s**

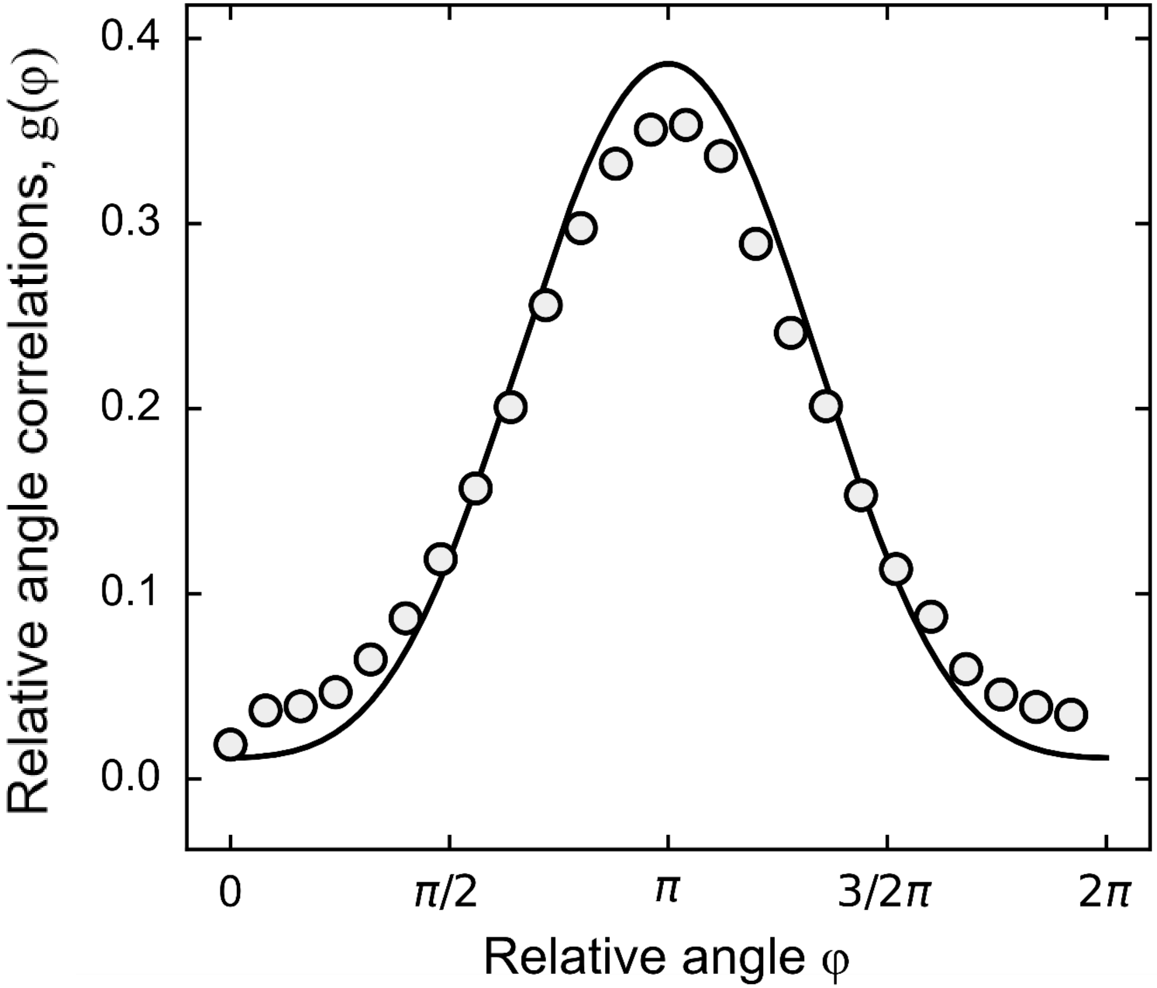
Angle correlations

Single snapshot



Determine relative angle of spin up and spin down particle

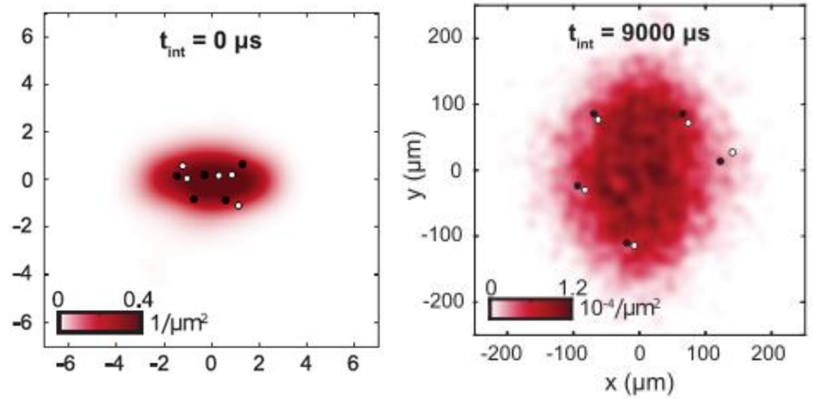
$$g_{1/2}(\varphi) = \frac{6 - 3\pi\cos(\varphi) + 4\cos^2(\varphi)}{16\pi}$$



Conclusion

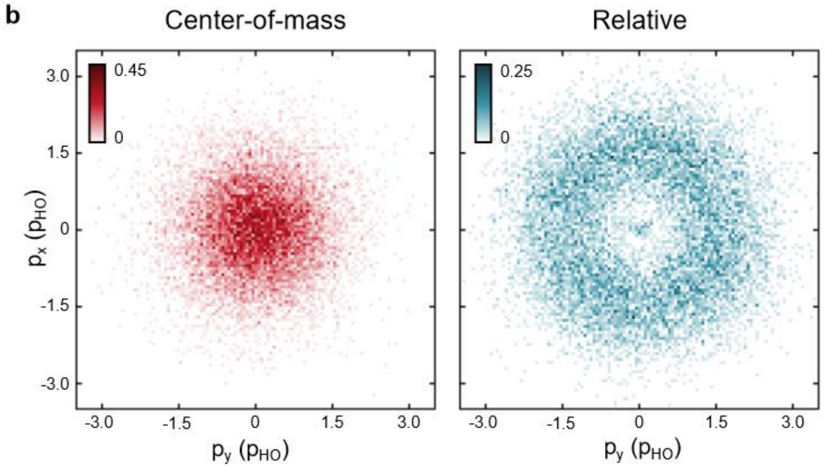
- Observation of the emergence of in interaction driven **elliptic flow**

- Redistribution of momentum distribution

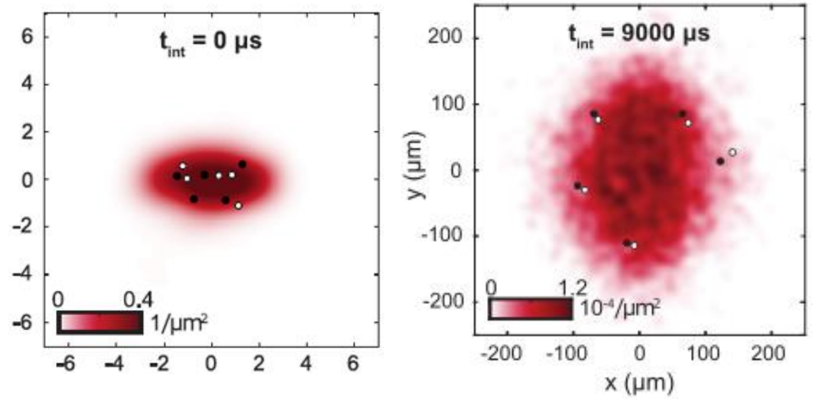


- **Motional control** of a **single particle** with specific angular momentum

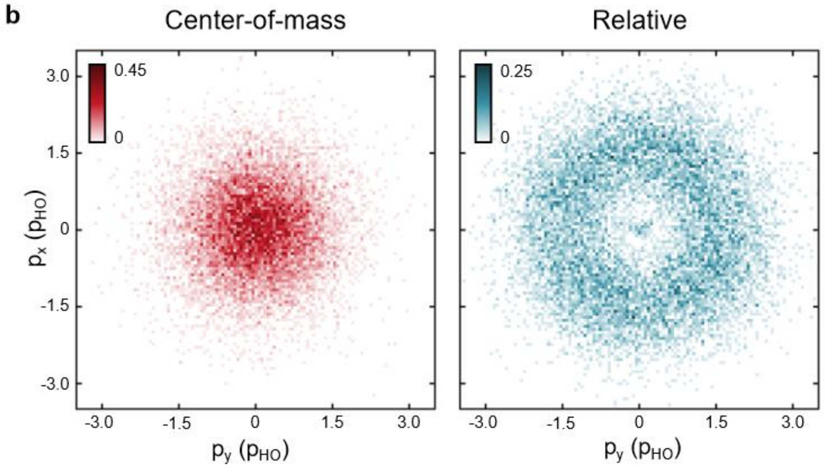
- Realization of the $\boldsymbol{\nu = 1/2}$ **Laughlin state** of two rapidly rotating fermions



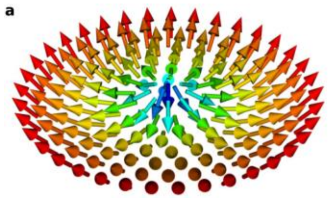
Outlook



- Pair formation during interacting expansion
- Freeze out radius – HBT
- What happens if we turn on interactions during the expansion?



- Scaling to larger particle numbers
- Quasi-hole excitations
- Bosonic fractional quantum Hall effect
- Skyrmion spin textures ?



L. Palm et al. *New J. Phys.* **22** 083037 (2020)

Thank you for your attention!