

Tracking and Analysis of Active Droplet Dynamics

from image processing to non-equilibrium statistical physics

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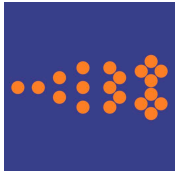
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Cellular, Computational and Integrative Biology



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Introduction

- **Active matter systems** can **harness energy** from their surroundings and **propel themselves** away from equilibrium.
- **Even if composed by "simple" individual entities**, they show **complex collective behaviour** of dynamical **self-assembly**.

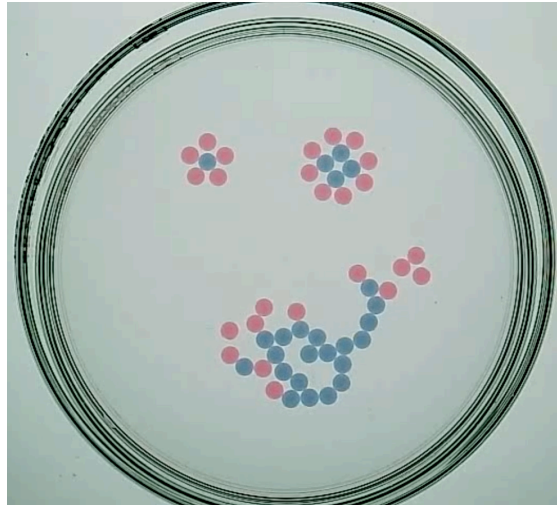
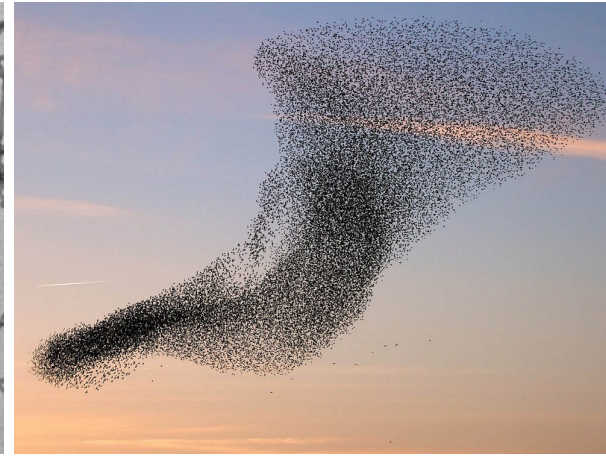
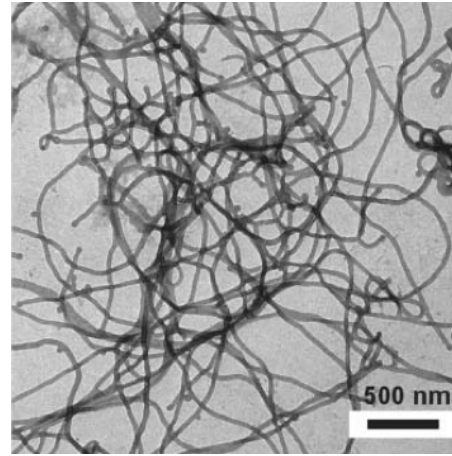


Introduction

Realizations:

- **Biological:** Span all levels of living organisms.
- **Synthetic:** Systems capable of dynamical self-propelled behaviour akin to that found in living matter

Countless **applications**, e.g. bacterial micromotors [1], soft robotics, design of novel materials

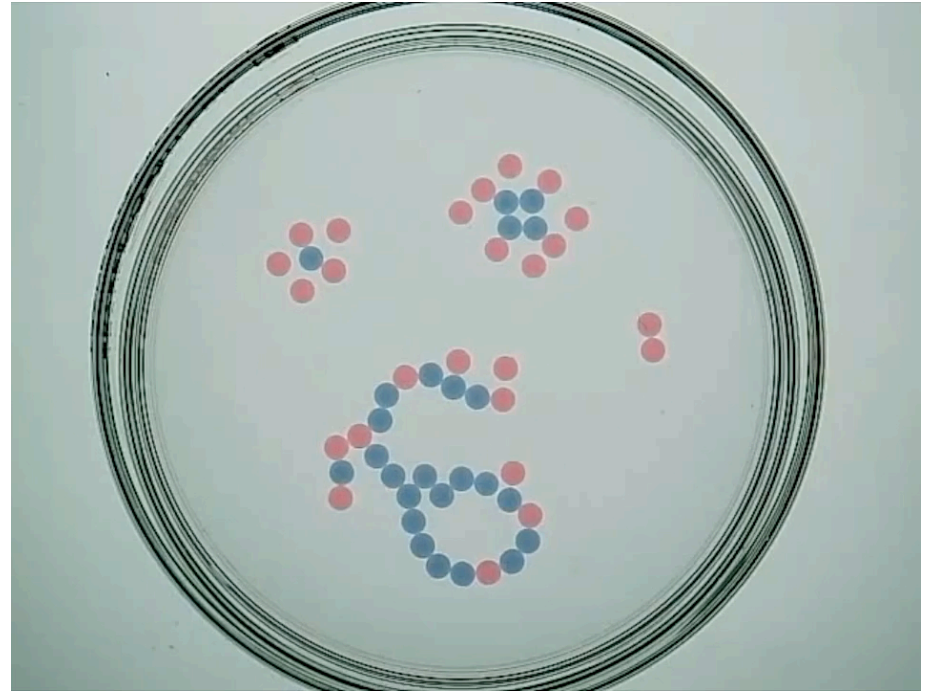


Introduction

The **system under study** in this project is made of **synthetic active matter** droplets immersed in a solution:

- **Droplets:** Ethil Silicite (ES) & Paraffin + Oil O Red dye for red droplets and Sudan B black dye for blue droplets [2], [3]
- **Solution:** Sodium dodecyl sulfate

Self propulsion arises due to the **evaporation of ES.**

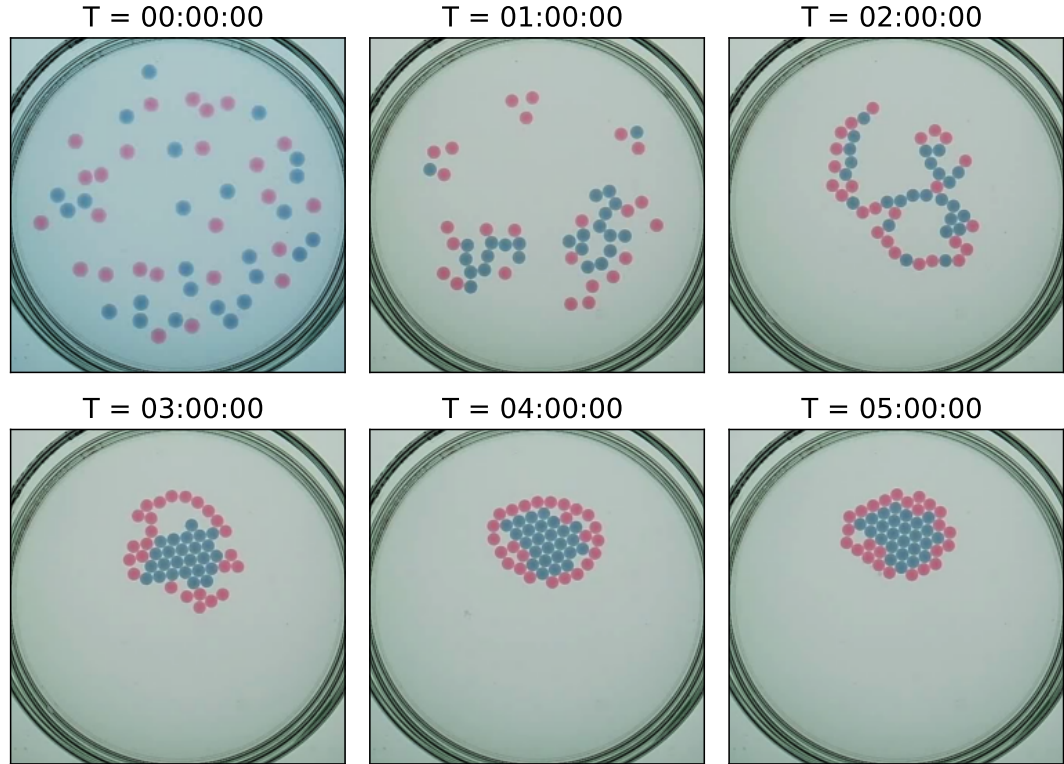


Video: dynamics of active droplets (25b25r) — 20 x

2. Muneyuki Matsuo, Hiromi Hashishita, Shinpei Tanaka, and Satoshi Nakata. Sequentially selective coalescence of binary self-propelled droplets upon collective motion. *Langmuir*, 39(5):2073–2079, 2023. PMID: 36692295.
3. R.J.G. Löffler. *New Materials for Studies on Nanostructures and Spatio-temporal Patterns Self-organized by Surface Phenomena*. 2021

Introduction

- **Stage 1:** Active "Brownian" motion with no structures
- **Stage 2:** Medium-sized semi-persistent structures
- **Stage 3:** Persistent arrangement in a quasi-regular structure



Objectives

The **long term goal** is to **predict large scale structures** and **dynamical assembly** of the droplets by **varying the system composition**

The **short term goal** is to **characterize the behaviour of the system** starting from the experimental video:



1. Tracking procedure
2. Dynamical analysis
3. Structural analysis

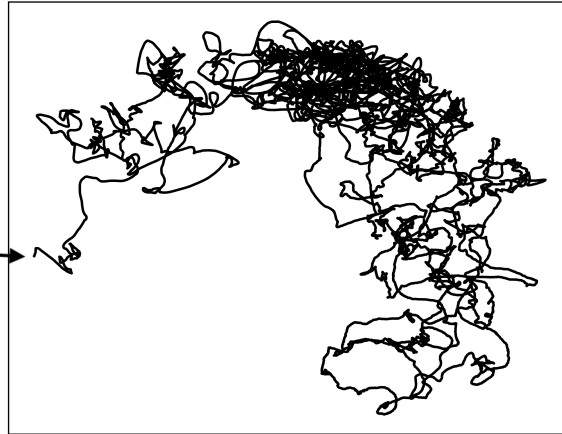
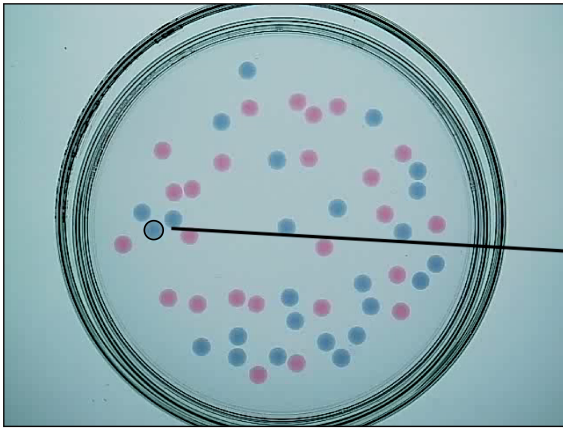
Tracking procedure

Tracking procedure

Accurate droplet positions
and radii over time



Quantitative characterization
of the dynamics



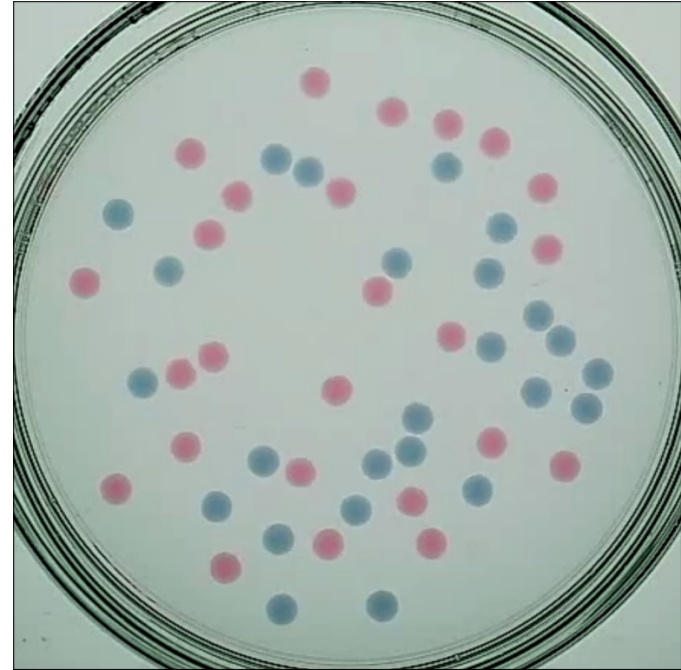
Steps of the pipeline:

1. Video preprocessing
2. Features detection
3. Linking

Tracking procedure - Video preprocessing

Preprocessing steps

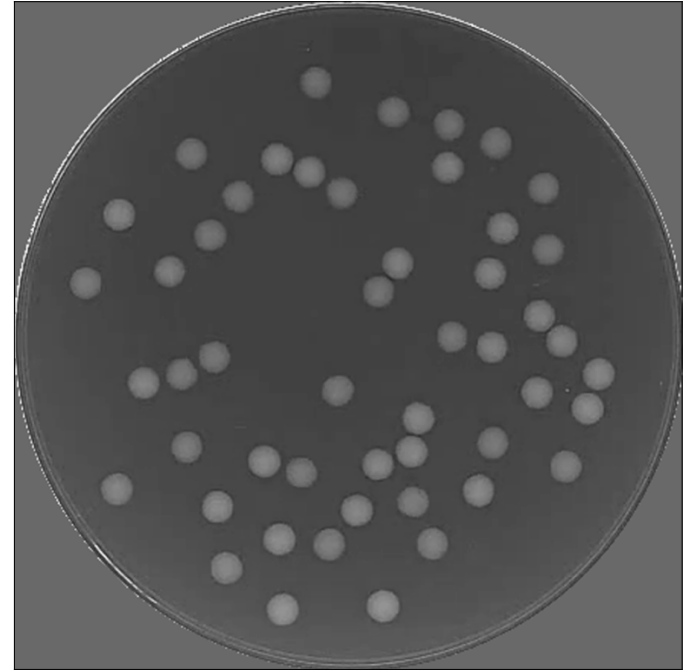
- **Grey scaling** to simplify data format
- **Circular crop** to remove the petri dish from the frame
- **Sharpen kernel** to enhance the droplets' borders



Tracking procedure - Video preprocessing

Preprocessing steps

- **Grey scaling** to simplify data format
- **Circular crop** to remove the petri dish from the frame
- **Sharpen kernel** to enhance the droplets' borders



Tracking procedure - Features detection

Deep learning solution:

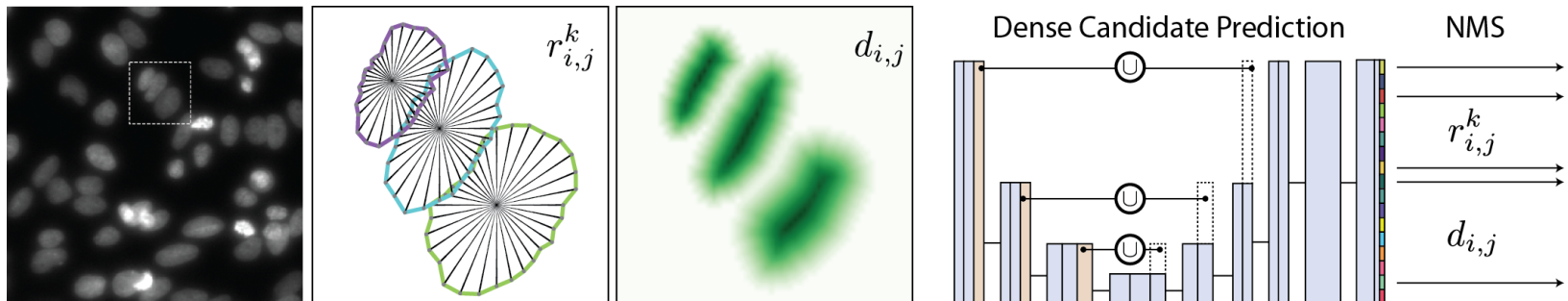
Stardist [4]



U-net architecture

+

Star convex prediction head



Tracking procedure - Features detection

Deep learning solution:

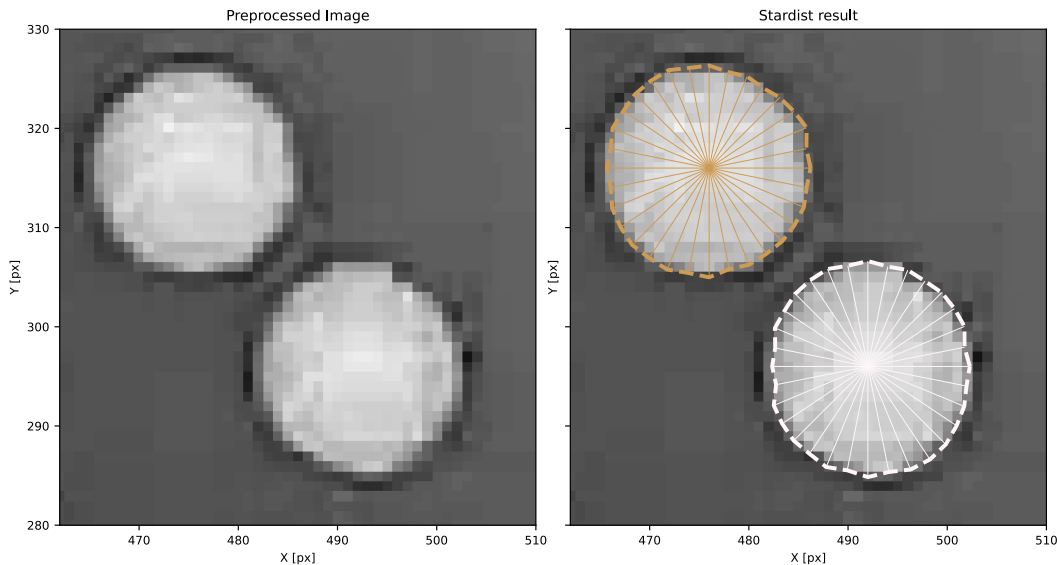


U-net architecture

+

Star convex prediction head

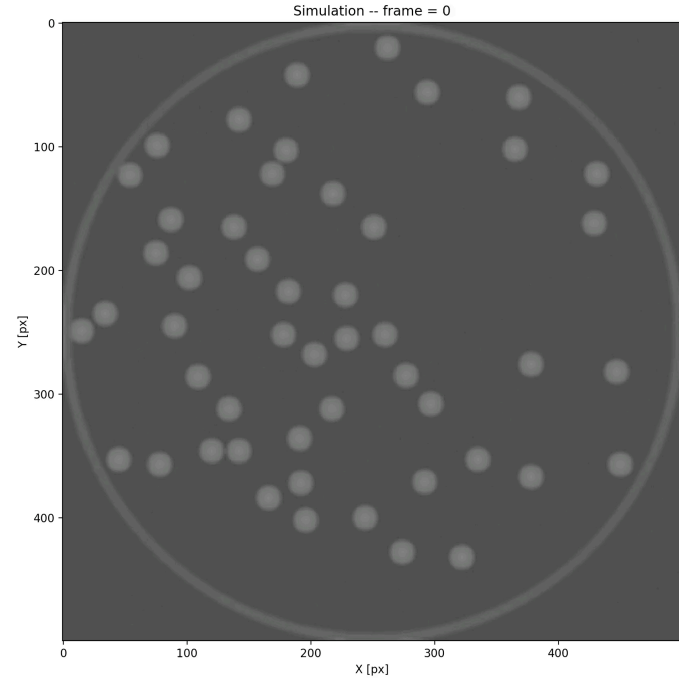
Stardist [4]



4. Martin Weigert, Uwe Schmidt, Robert Haase, Ko Sugawara, and Gene Myers. Star-convex polyhedra for 3d object detection and segmentation in microscopy. In The IEEE Winter Conference on Applications of Computer Vision (WACV), March 2020.

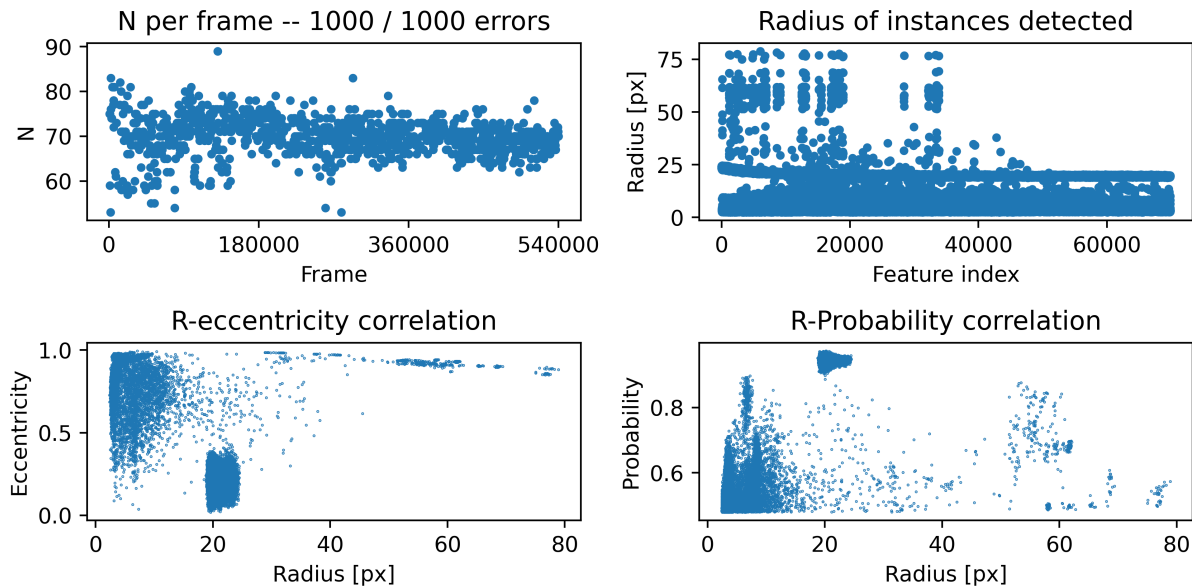
Tracking procedure - Features detection

To **train and/or optimize** the **Stardist network** we **simulated** an **interacting ABP system** and **generated synthetic images** resembling the post-processed data.



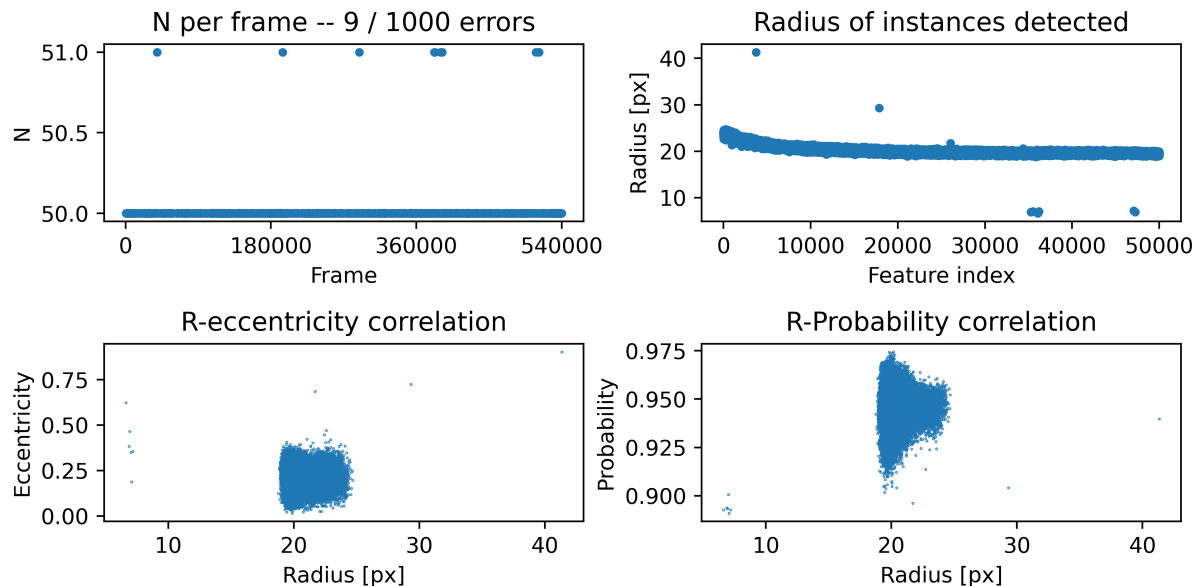
Tracking procedure - Features detection

Pretrained



Tracking procedure - Features detection

Pretrained + optimization



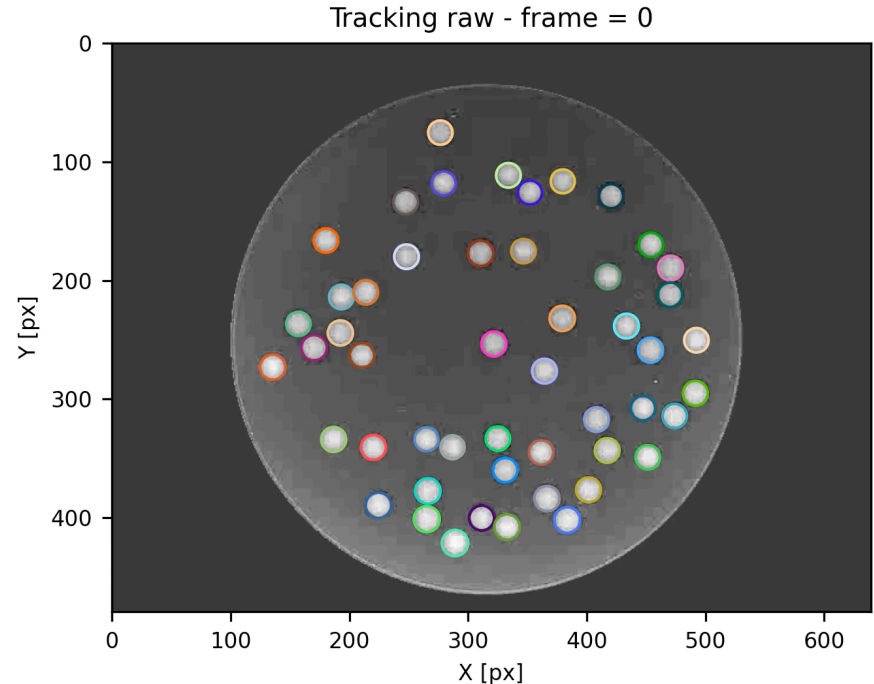
Tracking procedure - Linking

Instances' linking between frames, preserving droplets' identity

- **Probability** for the displacement of N non-interacting **Brownian particles**

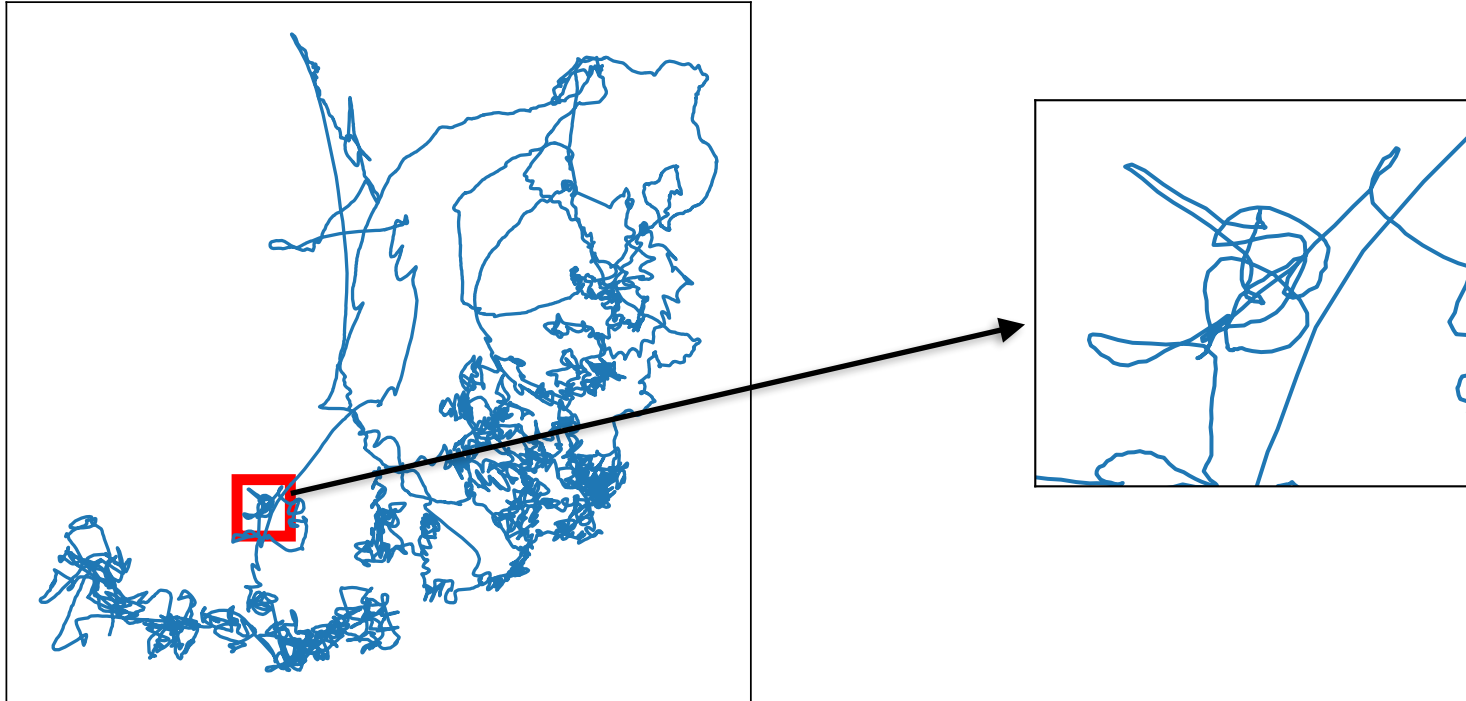
$$P(\{\delta_i\}|\tau) = \left(\frac{1}{4\pi D\tau}\right)^N \exp\left(-\sum_{i=1}^N \frac{\delta_i^2}{4D\tau}\right)$$

**most probable identity assignment
across frames maximizes the
probability**



Tracking procedure - Outcomes

Result of the tracking procedure: highly accurate trajectory of each droplet.



Analysis

Analysis - Introduction

System characterization:

- **Activity:** Droplets' depth in the solution
- **Dynamical properties:** MSD & Turning Angles distribution
- **Structural analysis:** Velocity Autocovariance & RDF

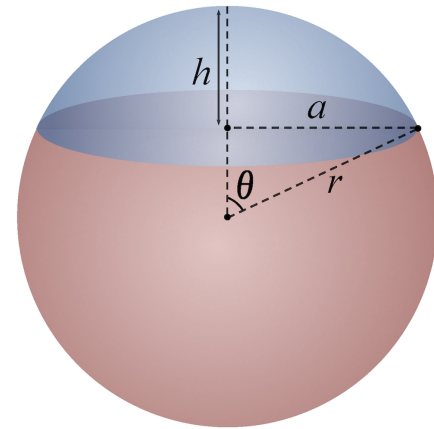
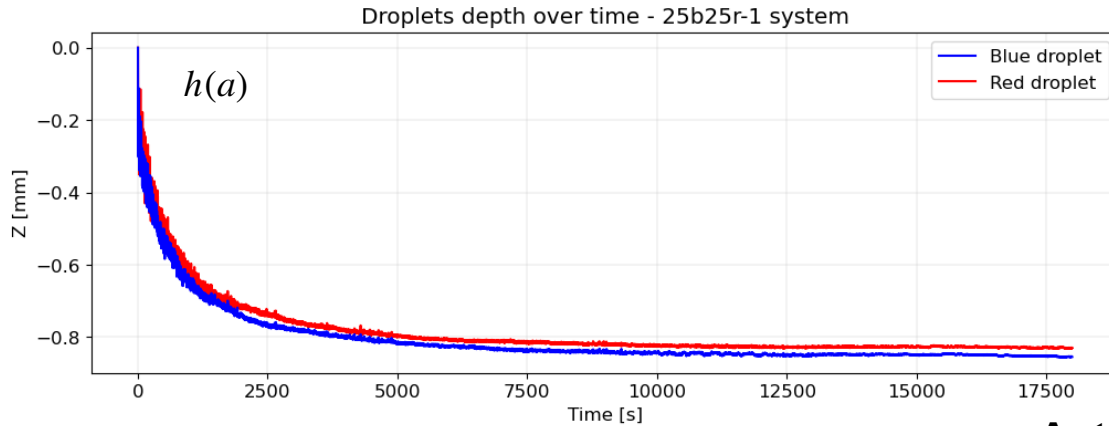
Analysis - Droplets' depth analysis

Assumptions:

- Perfectly **spherical droplets**
- **Droplets at frame 0 are half submerged**

Radius as seen from above $\longrightarrow a$

$$\begin{cases} A_{cap} = 2\pi r h \\ A_{cap} = \pi(a^2 + h^2) \end{cases} \implies h = h(a)$$



Activity decays as the droplet sinks

Analysis - Window-based analyses

**Global time average
analyses**

(MSD, Turning Angles & VACF)

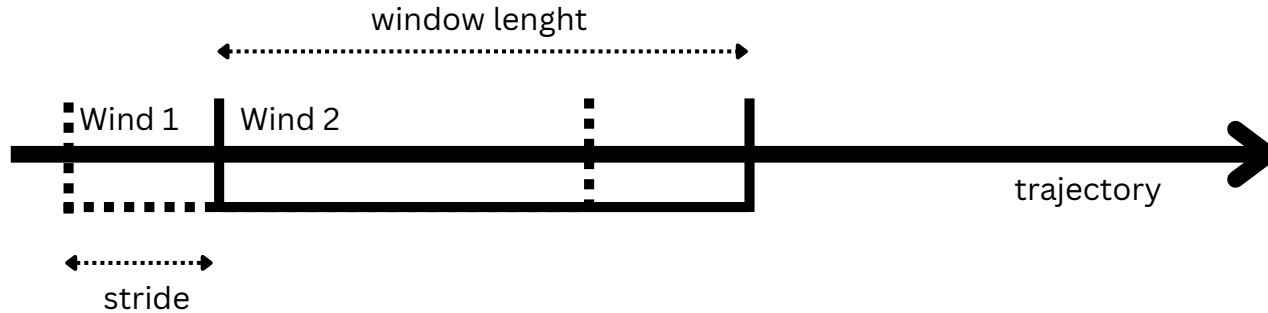


- **Not well-founded** since **time-translational invariance is not met** (activity decays over time)
- **Mixing different activity regimes**

For these reasons we perform a **window-based analysis**:

- **Trajectories are divided into windows** of 600 s
- **Window slides over the full trajectory** by a stride of 10 s

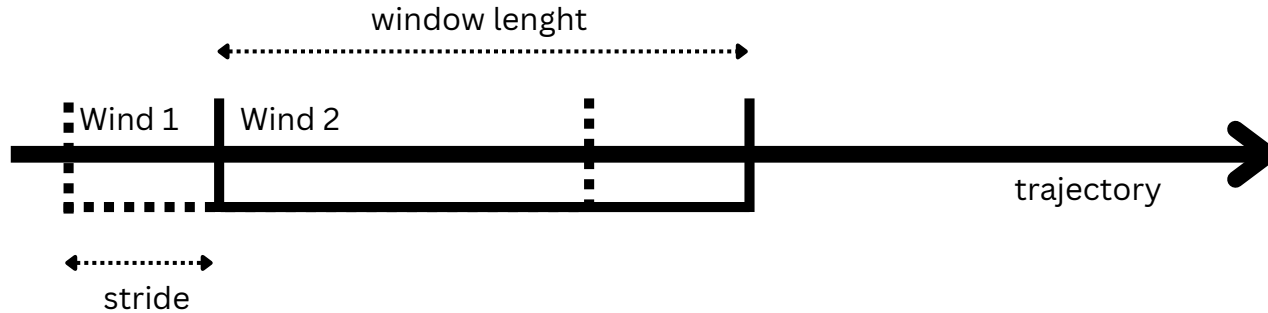
Analysis - Window-based analyses



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- **Trajectories are divided into windows** of 600 s
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Analysis - Window-based analyses



Window-based analyses assume that the macroscopic statistical properties (activity) do not change significantly over the window time extent.

Analysis - Mean Squared Displacement (MSD)

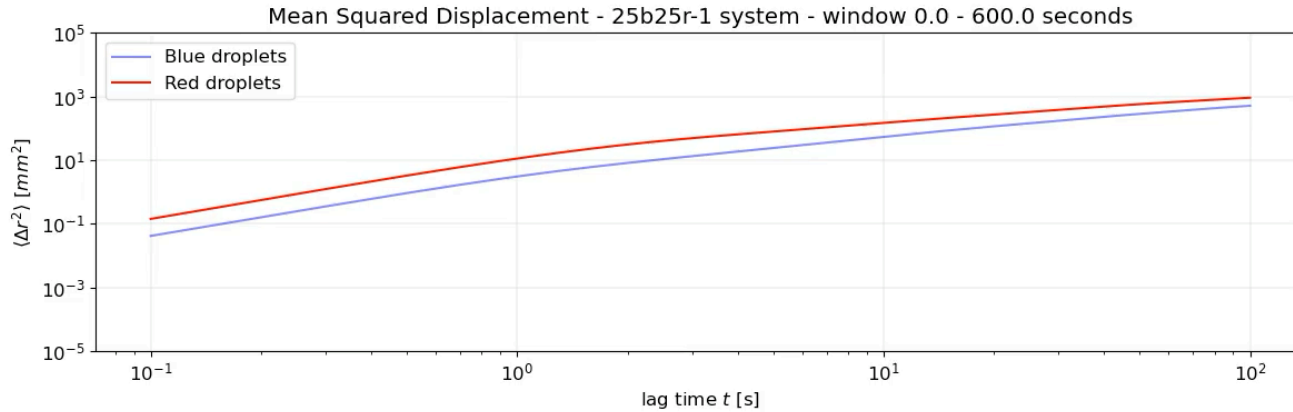
We compute the **time Average MSD** of droplets of same species over the window portion of the trajectory:

$$\langle \Delta \mathbf{r}^2(\tau) \rangle_\gamma = \frac{1}{N_\gamma} \sum_{k \in \gamma} \overline{\delta^2(\tau)}^k \quad \text{where} \quad \overline{\delta^2(\tau)}^k = \langle (\mathbf{r}_{t+\tau}^k - \mathbf{r}_t^k)^2 \rangle_{t \in T} = \frac{1}{T - \tau} \sum_{t=0}^{T-\tau} (\mathbf{r}_{t+\tau}^k - \mathbf{r}_t^k)^2$$

and perform **power law fit** in the [10 - 100] s region:

$$\langle \Delta \mathbf{r}^2(\tau) \rangle_\gamma = K_\alpha \tau^\alpha \quad \begin{cases} \text{diffusive for } \tau \gg 10 \text{ s} \\ \text{ballistic for } \tau \ll 10 \text{ s} \end{cases}$$

Analysis - Mean Squared Displacement (MSD)

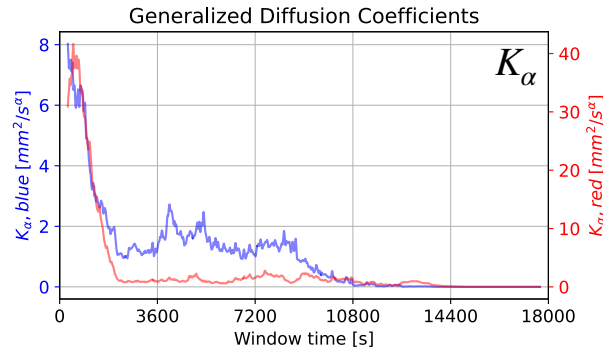
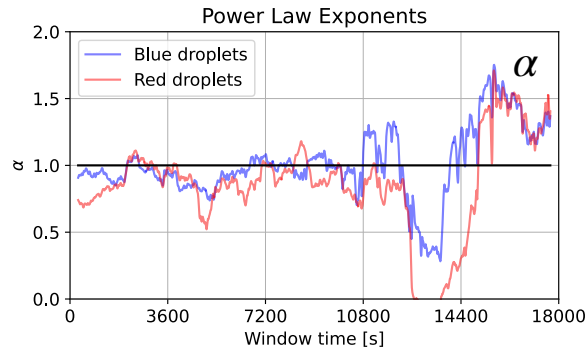


power law fit:

$$\langle \Delta \mathbf{r}^2(\tau) \rangle_{\gamma} = K_{\alpha} \tau^{\alpha}$$



Diffusive properties depend on the activity of the system.

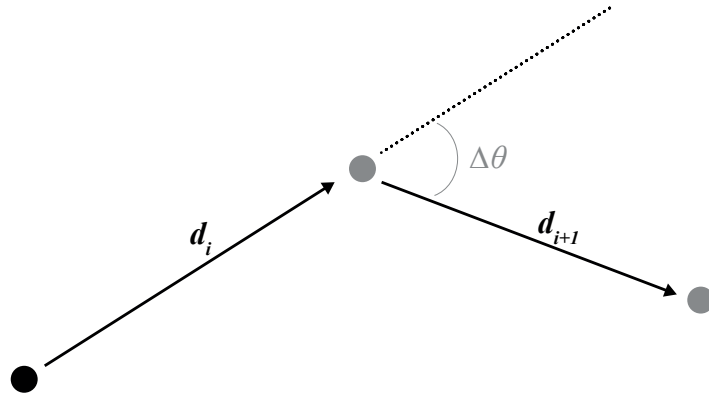


Analysis - Turning Angles Distribution

We characterize the **droplet's rotational behaviour** as a function of the activity.

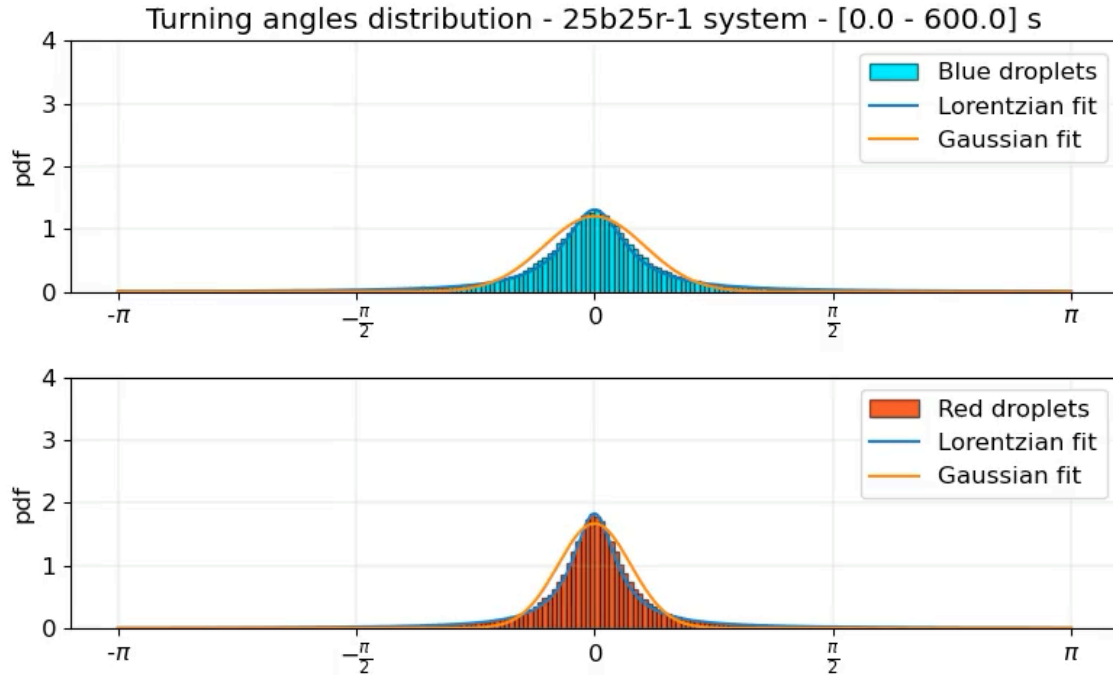
In the standard **ABP model** turning angles are **Gaussian distributed**:

$$P(\Delta\theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\Delta\theta - \mu)^2}{2\sigma^2}}$$



We perform the **window-based analysis** to **resolve** explicit dependencies of the **rotational diffusion** of the droplets.

Analysis - Turning Angles Distribution



**Gaussian distribution
not adequate**

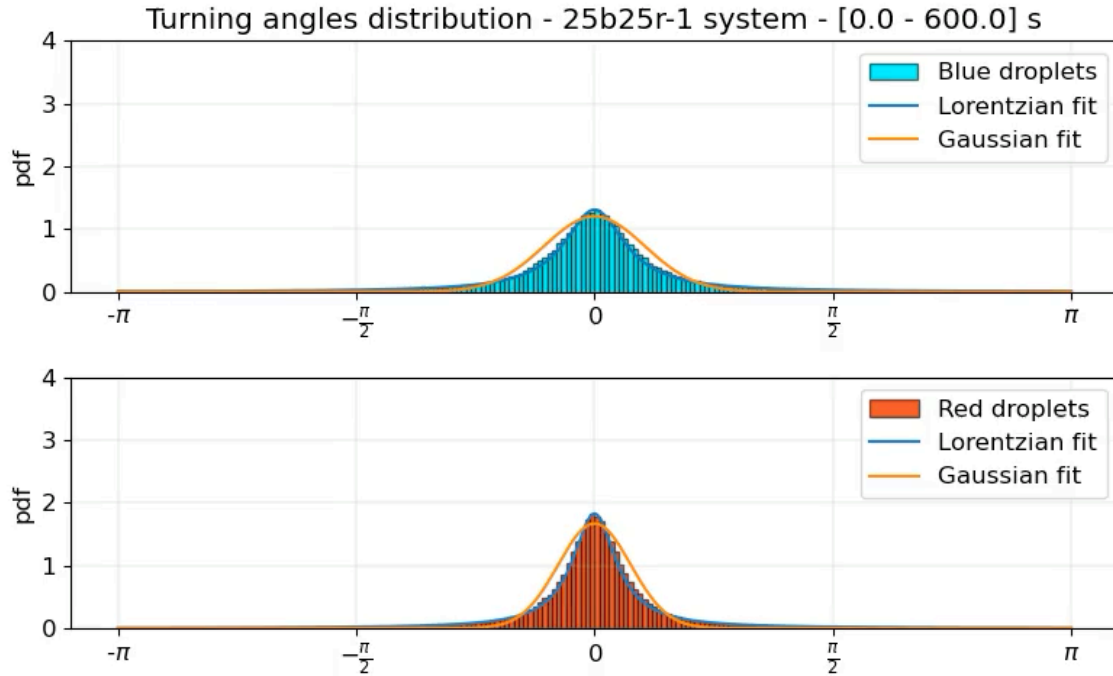
$$P(\Delta\theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\Delta\theta - \mu)^2}{2\sigma^2}}$$

**Lorentzian distribution
adequate**

$$P(\Delta\theta) = \frac{1}{\pi} \frac{\gamma}{(\Delta\theta - \mu)^2 + \gamma^2}$$

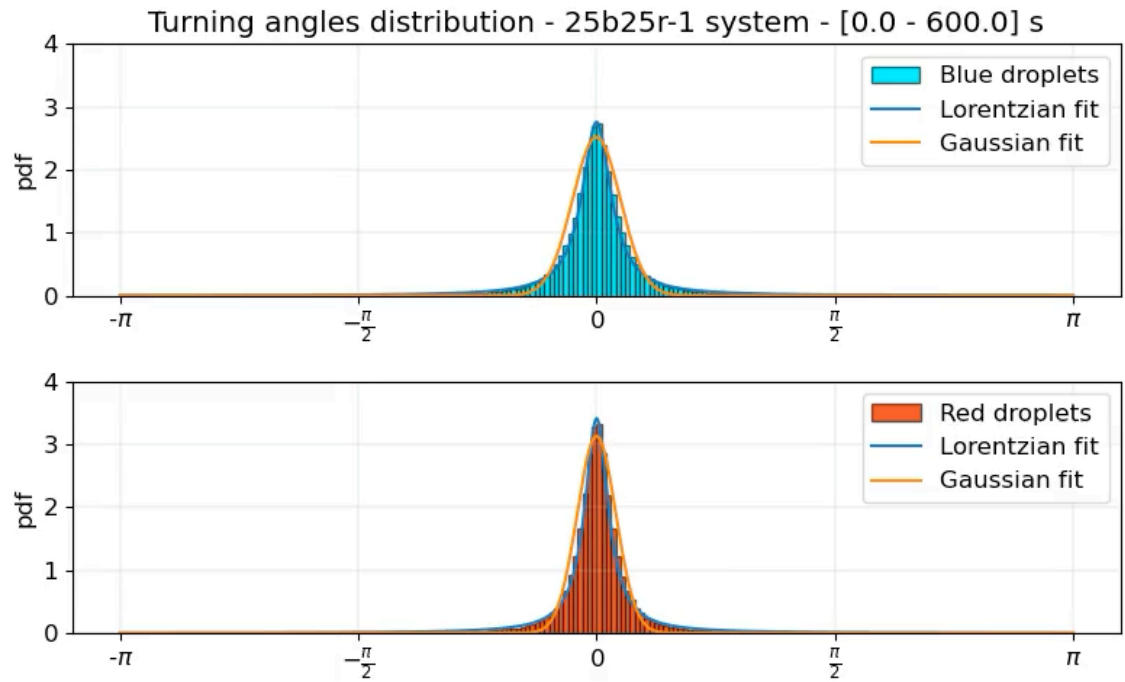
Lévy flight stochastic process
for the rotational diffusion ? [5]

Analysis - Turning Angles Distribution



Discrepancies arise
when droplets **move slow**
and the **uncertainty of**
detection becomes
relevant

Analysis - Turning Angles Distribution - smooth



Lorentzian distribution is preserved

Discrepancies arise
when droplets **move slow**
and the **uncertainty of**
detection becomes
relevant

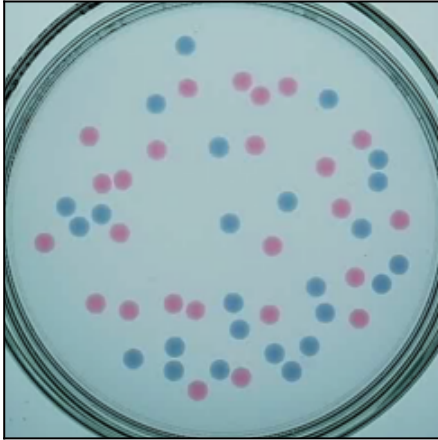


Savitzky-Golay filter
smoothing of trajectories
are applied

Analysis - Dynamical properties

Translational and **rotational diffusive properties** depend on the activity of the system

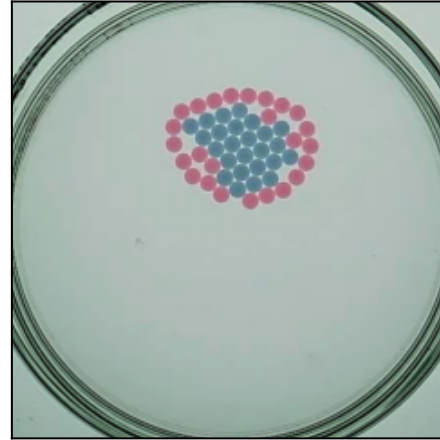
Stage 1 -- T = 00:00:00



Stage 2 -- T = 02:00:00



Stage 3 -- T = 04:00:00



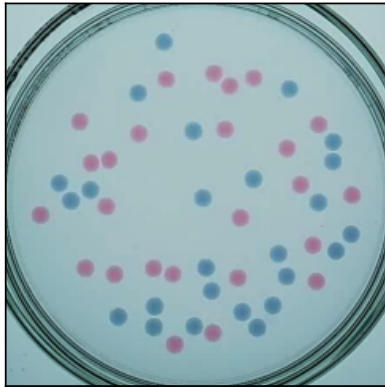
The next step is the characterization of the **relaxation dynamics** and **structure formation** through the means of VACF & RDF

Analysis - Velocity autocovariance function

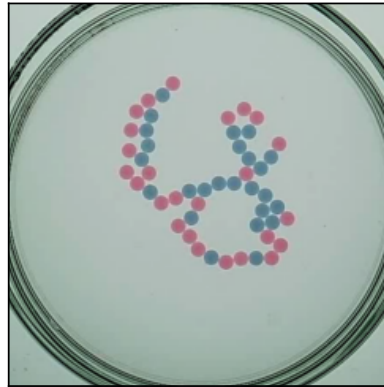
VACF are employed to **investigate the structural arrest of a droplets' motion**

$$K^a(\tau) = \frac{1}{\sigma_\alpha^2} \frac{1}{N_\alpha} \sum_{i \in \alpha} \langle (\mathbf{v}^i(t) - \langle \mathbf{v}^i(t) \rangle) \cdot (\mathbf{v}^i(t + \tau) - \langle \mathbf{v}^i(t + \tau) \rangle) \rangle$$

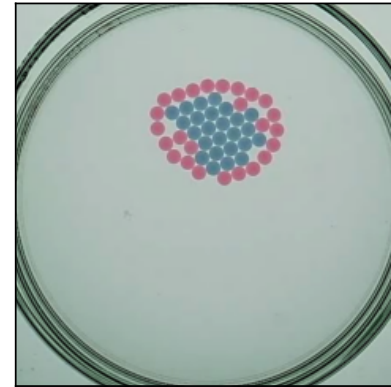
Stage 1 -- T = 00:00:00



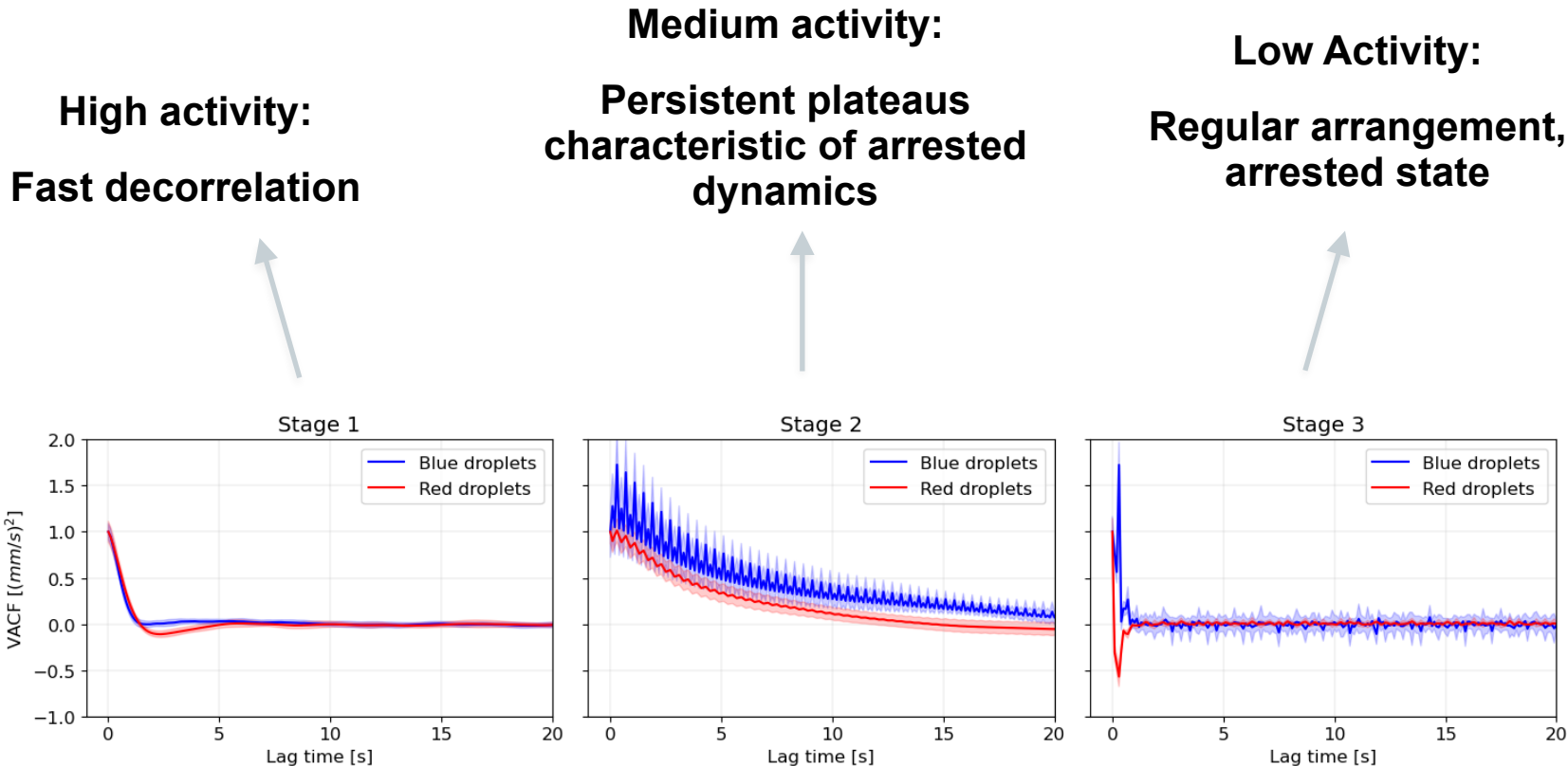
Stage 2 -- T = 02:00:00



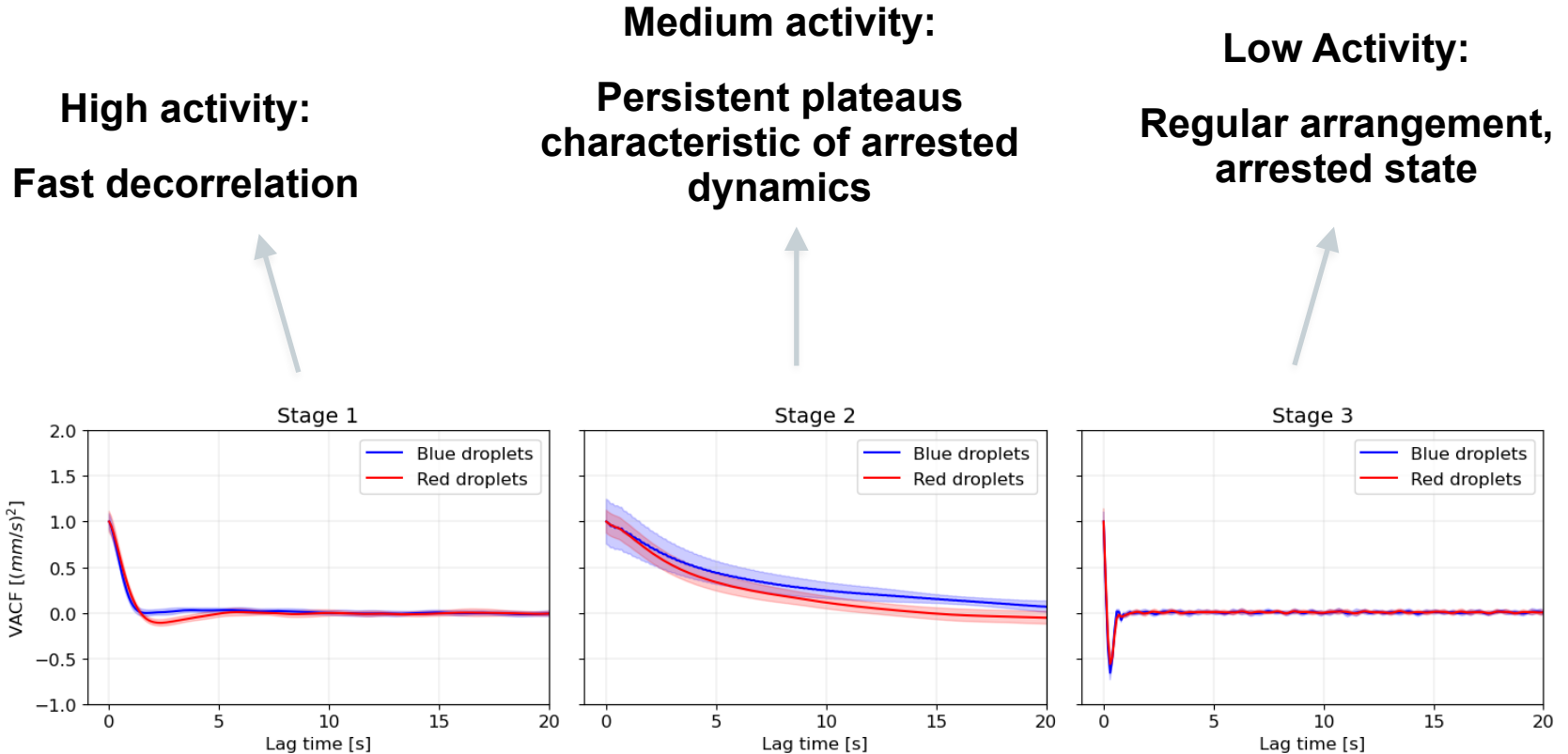
Stage 3 -- T = 04:00:00



Analysis - Velocity autocovariance function



Analysis - Velocity autocovariance function - smooth

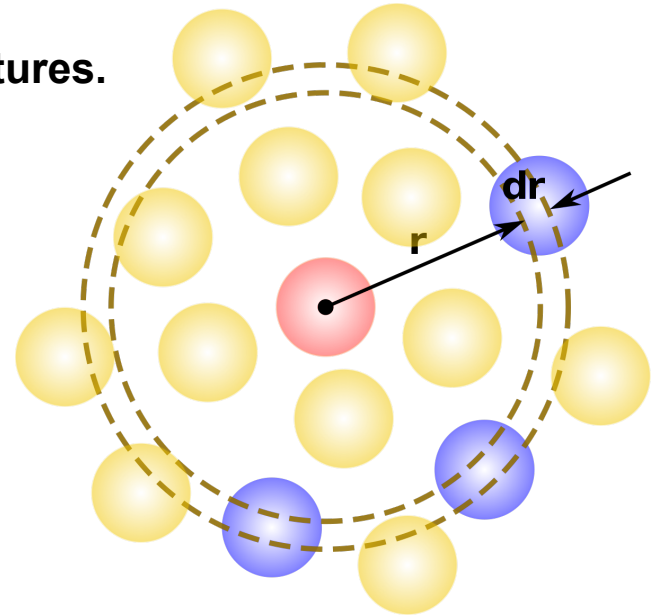


Analysis - Radial Distribution Function

Provides the **characterization of the spatial local structures**.

Approximation: Computed by dividing the average number of droplets at distance r by the the expected number of droplets assuming a homogeneous distribution.

$$g_{\alpha,\beta}(r) = \frac{\langle \rho_{\alpha,\beta}(r) \rangle}{N_{\beta}V} \quad \text{with} \quad V = \pi(\delta r^2 + 2r\delta r)$$



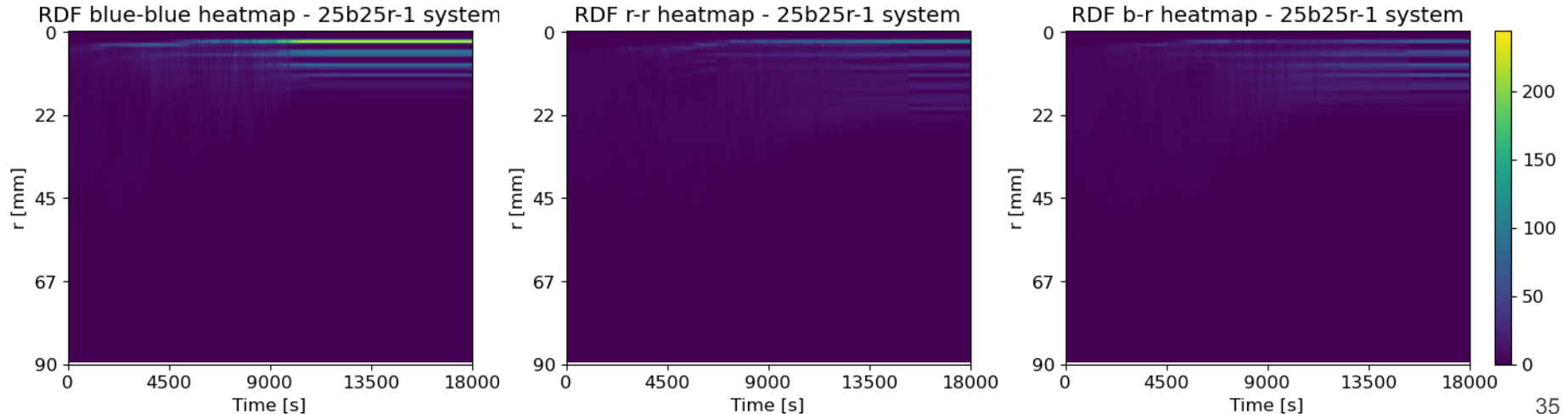
Analysis - Radial Distribution Function

Consistency with steric

interaction: $g_{\alpha,\beta}(r < d_d) = 0$

The first “**solvation**” shell appear after 2000 s for blue droplets, after 5000 s for red droplets.

Structure observed also in the mixed species RDF

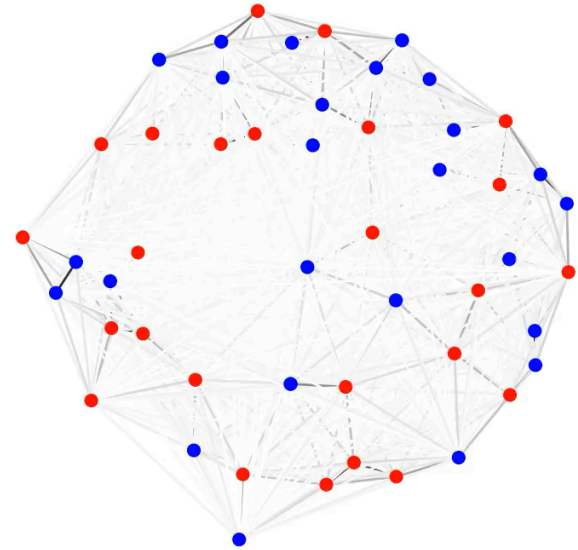


Future developments

The future developments from this point are multiple:

- **Improve smoothing via Kalman Filter**
- **Network-based analysis**
- **Orientation alignment and Velocity vector field analysis**

25b25r-1 at : 00:00:00

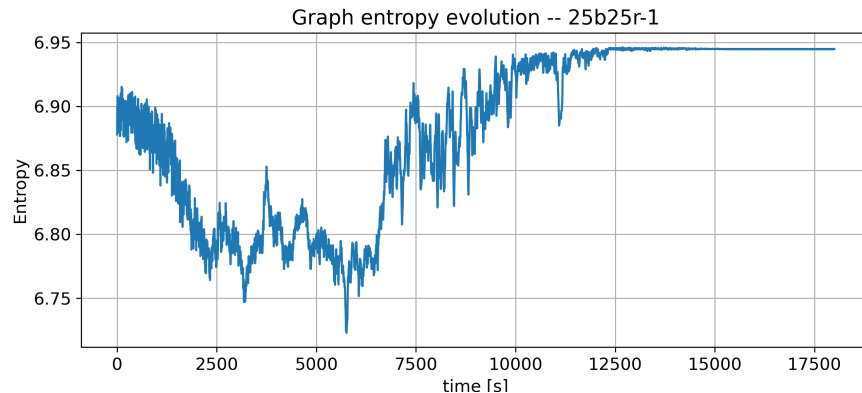


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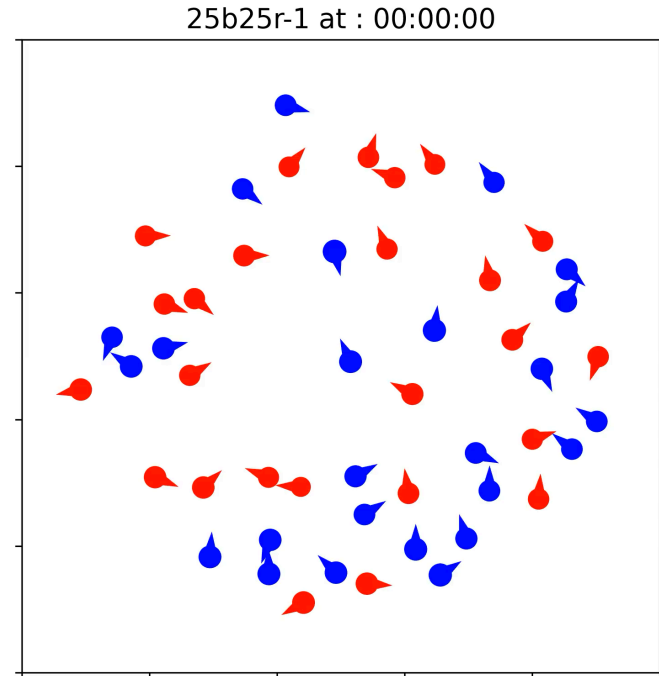
$$E(G, w) = - \sum_{uv \in E} p_{uv} \log(p_{uv}) \quad p_{uv} = \frac{w_{uv}}{\sum_{uv \in E} w_{uv}}$$



Future developments - orientation alignment

The future developments from this point are multiple:

- **Improve smoothing via Kalman Filter**
- **Network-based analysis**
- **Orientation alignment and Velocity vector field analysis**

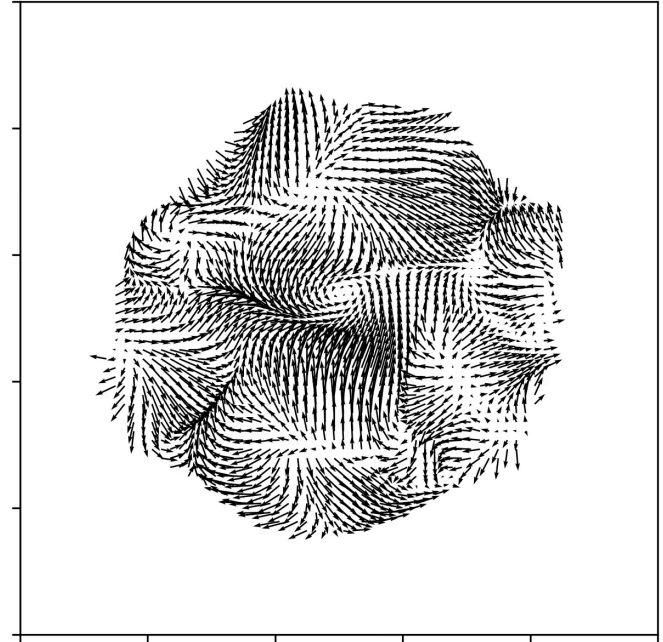


Future developments - velocity field

The future developments from this point are multiple:

- **Improve smoothing via Kalman Filter**
- **Network-based analysis**
- **Orientation alignment and Velocity vector field analysis**

25b25r-1 at : 00:30:08

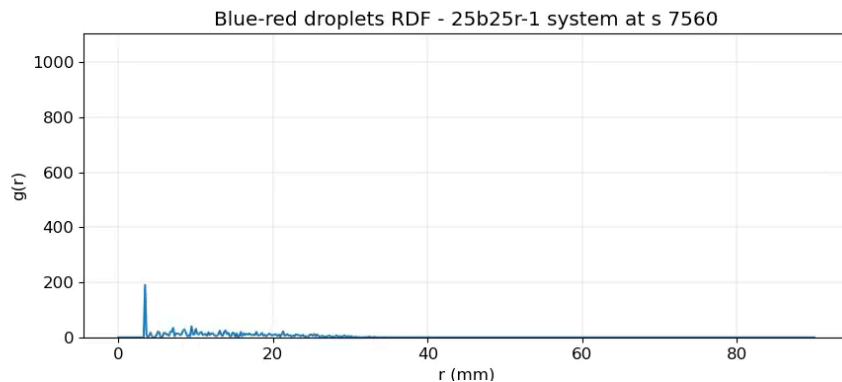
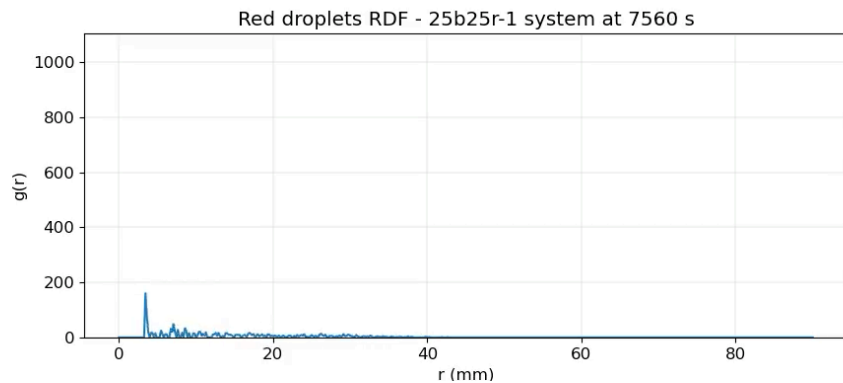
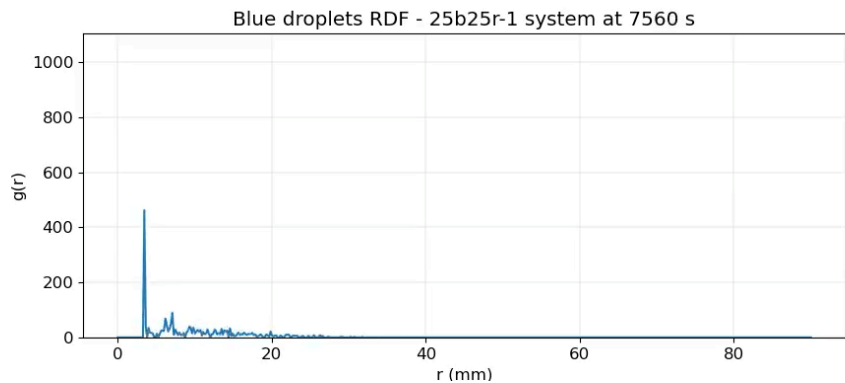


Thanks for your attention!

References

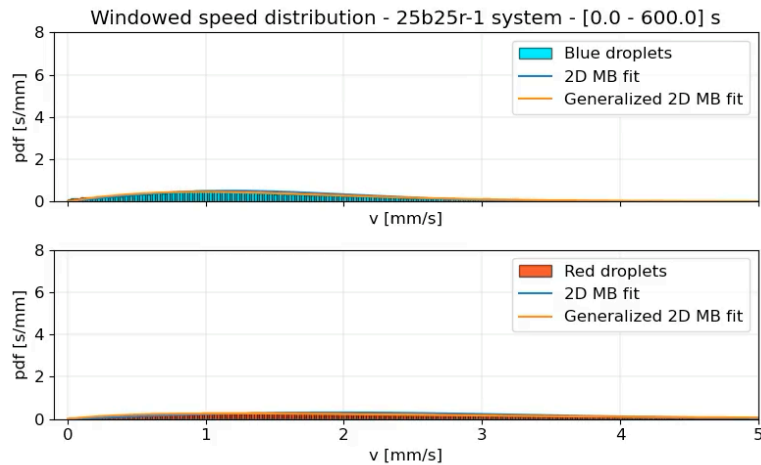
1. Vizsnyiczai, G., Frangipane, G., Maggi, C. *et al.* Light controlled 3D micromotors powered by bacteria. *Nat Commun* **8**, 15974 (2017).
2. Muneyuki Matsuo, Hiromi Hashishita, Shinpei Tanaka, and Satoshi Nakata. Sequentially selective coalescence of binary self-propelled droplets upon collective motion. *Langmuir*, 39(5):2073–2079, 2023. PMID: 36692295.
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4. Martin Weigert, Uwe Schmidt, Robert Haase, Ko Sugawara, and Gene Myers. Star-convex polyhedra for 3d object detection and segmentation in microscopy. In *The IEEE Winter Conference on Applications of Computer Vision (WACV)*, March 2020.
5. M.F. Shlesinger, G.M. Zaslavsky, and U. Frisch. *Lévy Flights and Related Topics in Physics: Proceedings of the International Workshop Held at Nice, France, 27–30 June 1994*. *Lecture Notes in Physics*. Springer Berlin Heidelberg, 1995

Extra Material - RDF

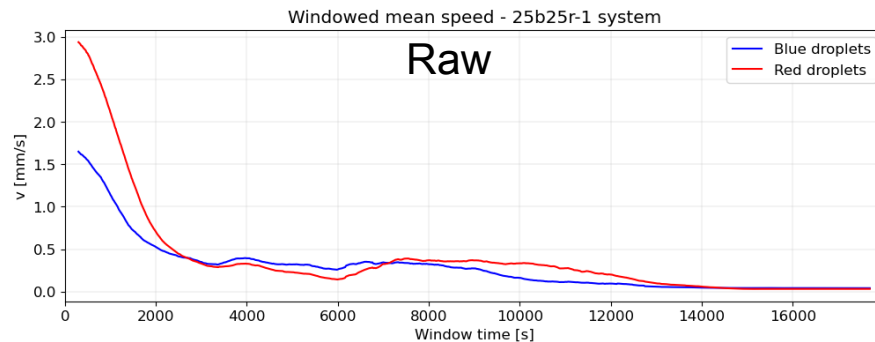
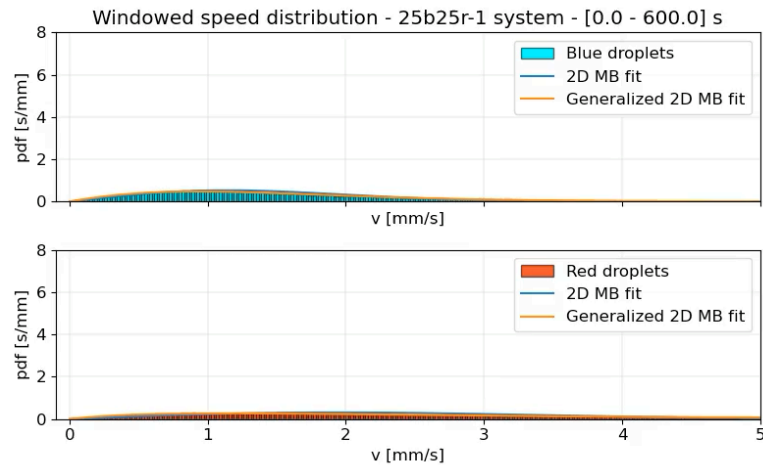


Extra Material - Speed distribution

Raw



Smooth



Extra Material - TDA on Graph

Persistent homology

Vietoris-Rips filtration