Natural swarms in 3.99 dimensions

A renormalization group approach to biological systems

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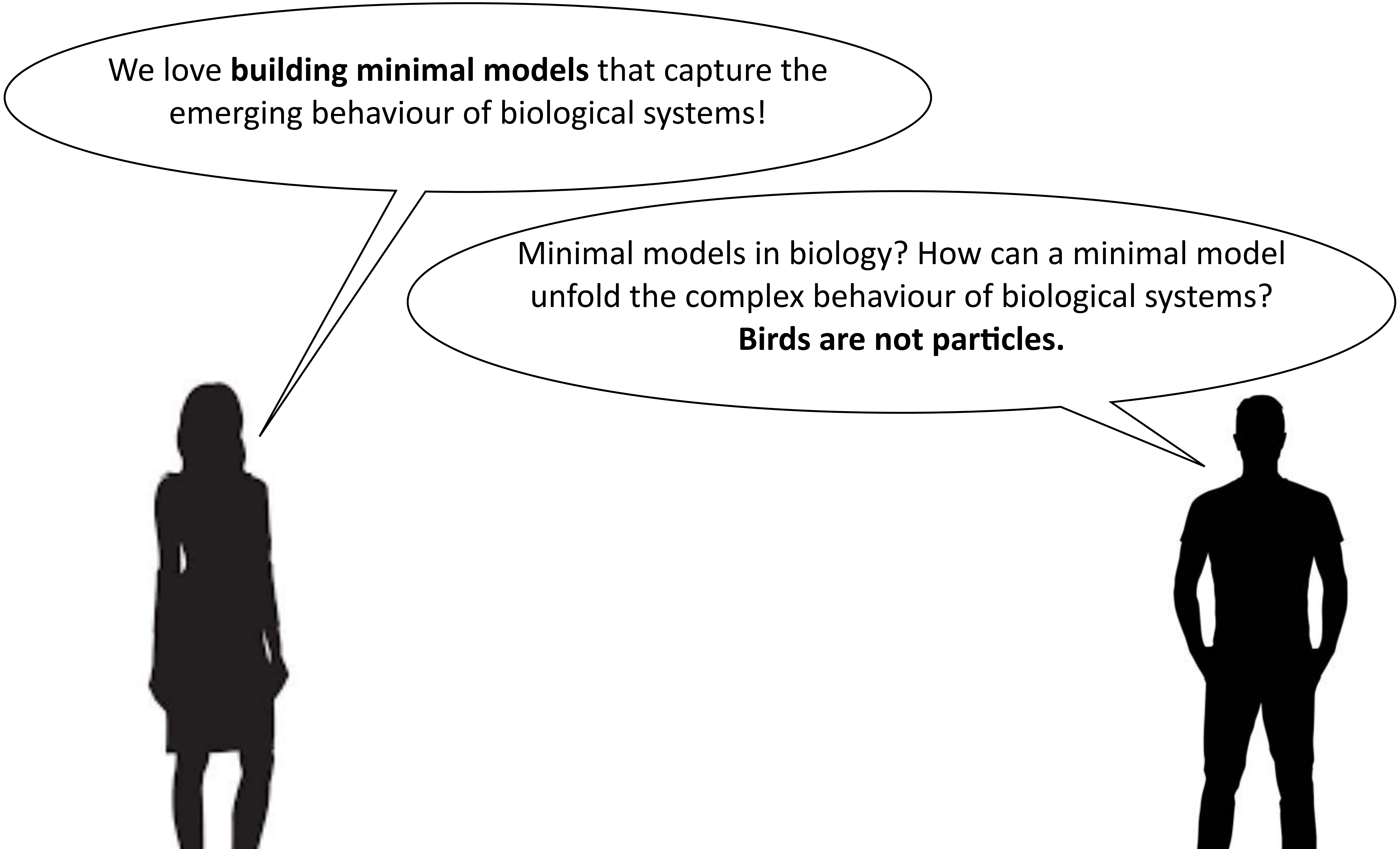
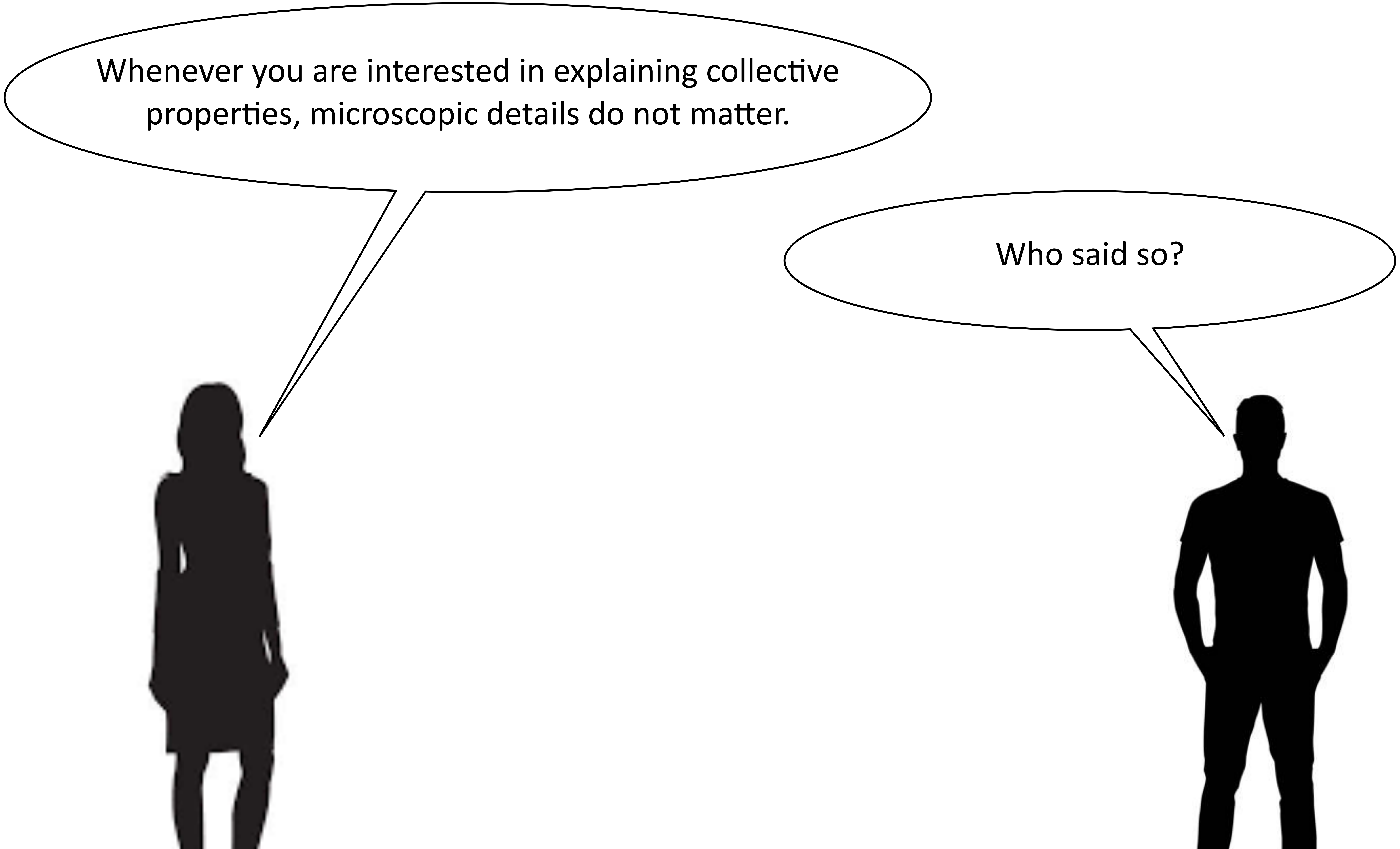
April 16, 2024 – Trento

Physicist’s approach to biology



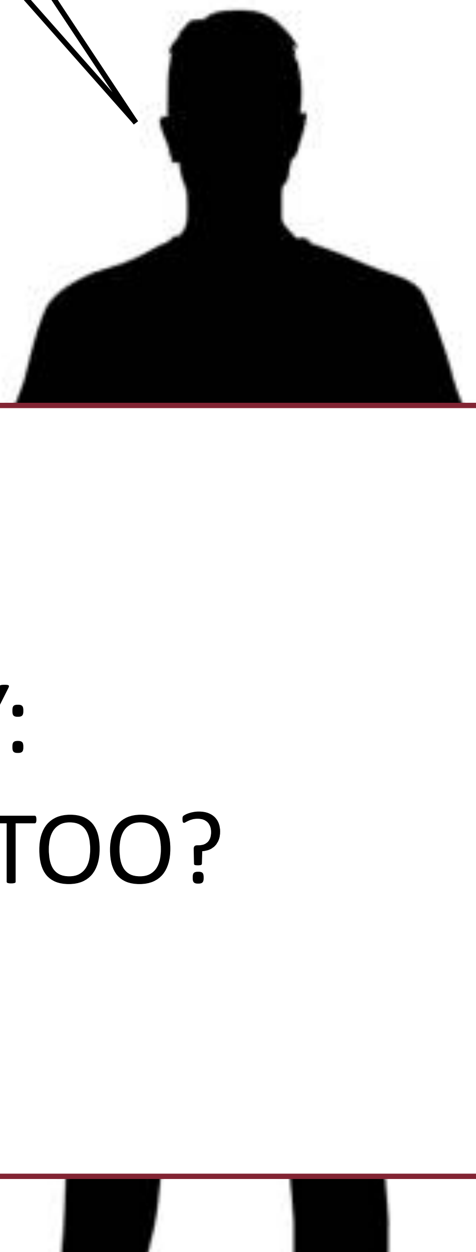
Physicist's approach to biology



Physicist's approach to biology

Whenever you are interested in explaining collective properties, microscopic details do not matter.

Who said so?



IN PHYSICS:
THE RENORMALIZATION GROUP

IN BIOLOGY:
MAYBE THE RG TOO?

Roadmap

Experiments

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Theoretical “ingredients”

↓

Renormalization Group

Experiments

↓

Theoretical “ingredients”

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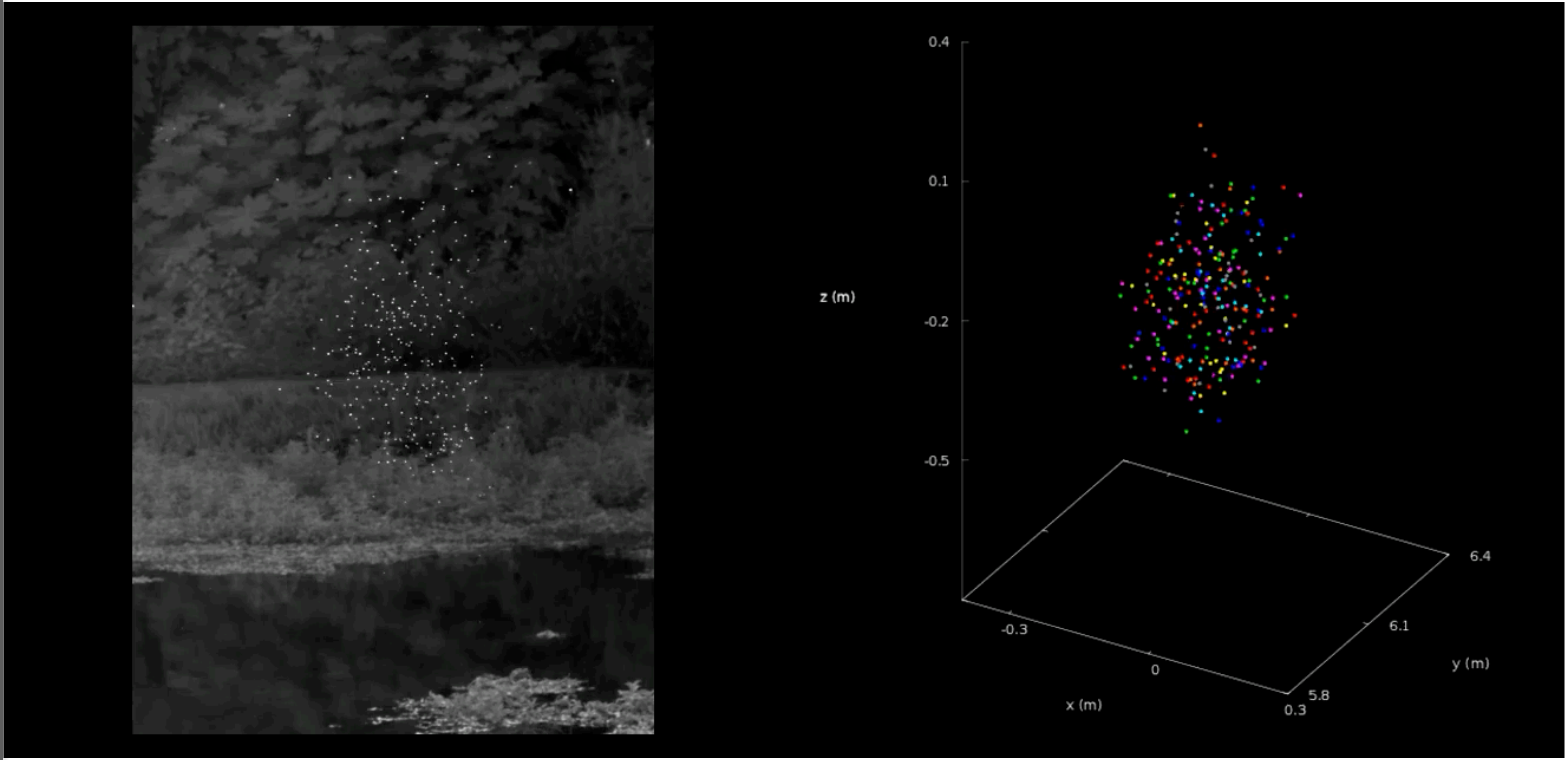
Renormalization Group

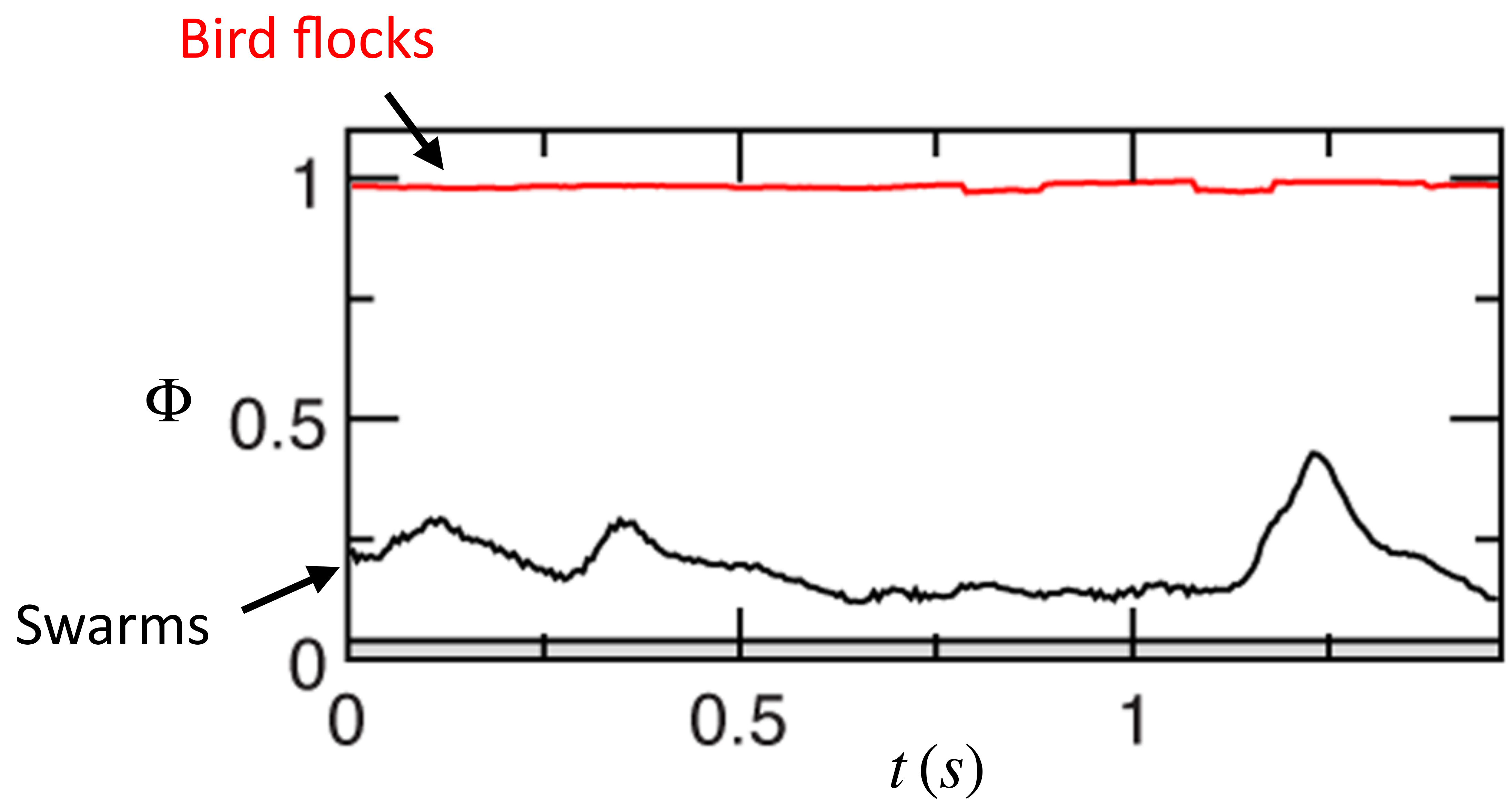
Experiments



Experiments

No global order

No net group motion: swarms are in a **disordered phase**



$$\Phi = \frac{1}{N} \sum_i \frac{v_i}{|v_i|}$$

A. Attanasi et al., PLoS Computational Biology, 2014

Experiments

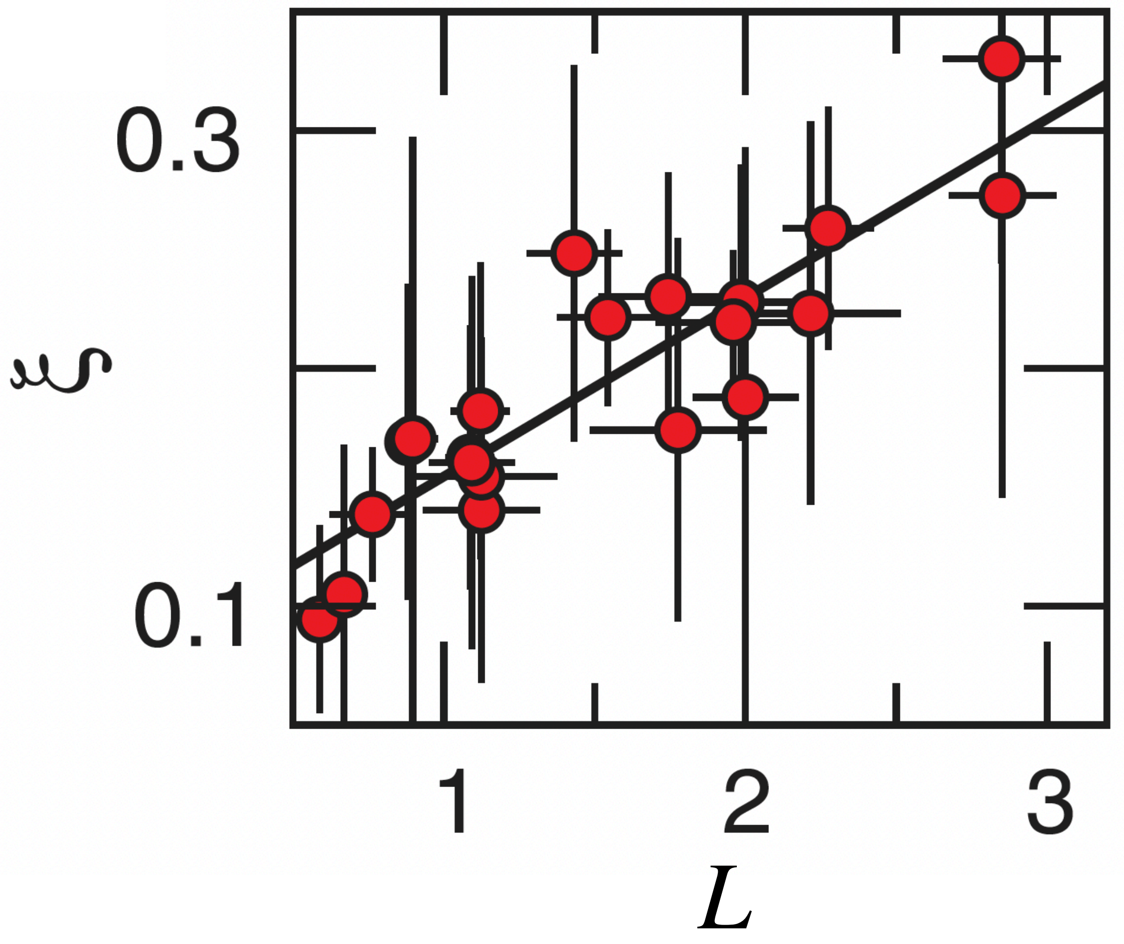
Scale-free spatial correlations

No net group motion: swarms are in a **disordered phase**

Velocity-velocity correlations

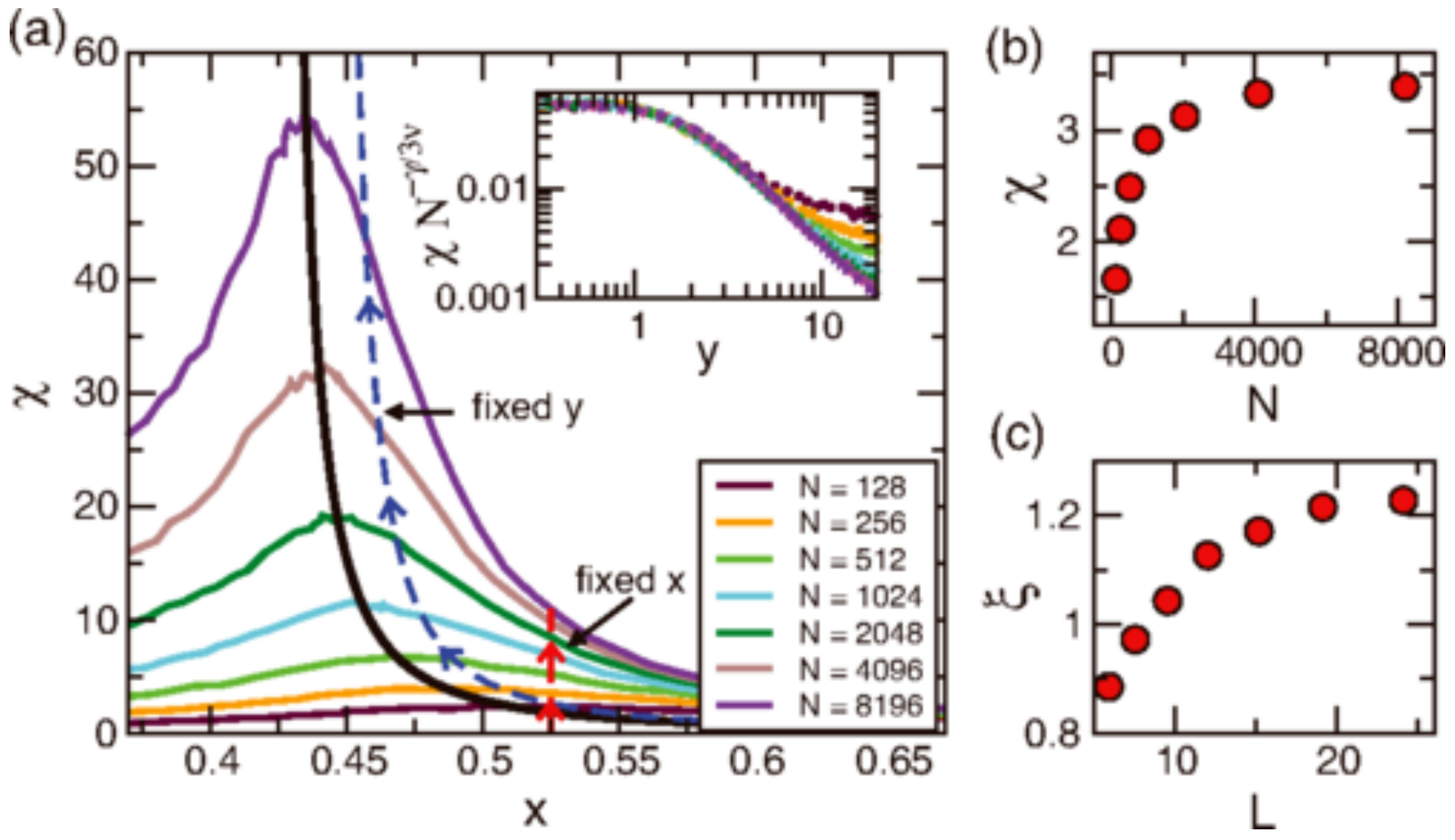
$$C(r) = \langle \delta \boldsymbol{v}(x_0, t_0) \cdot \delta \boldsymbol{v}(x_0 + r, t_0) \rangle_{x_0, t_0}$$

are strong and **scale-free**



$(\xi \sim L)$

exhibit **finite-size scaling**



A. Attanasi et al., PLoS Computational Biology, 2014 - A. Attanasi et al., PRL, 2014

Experiments

Dynamic scaling

No net group motion: swarms are in a **disordered phase**

Velocity-velocity correlations are **strong** and **scale-free** $(\xi \sim L)$ and exhibit **finite-size scaling**

Swarms behave as systems near a critical order-disorder transition

A. Cavagna et al., Nature Physics, 2017 - A. Cavagna et al., Nature Physics, 2023

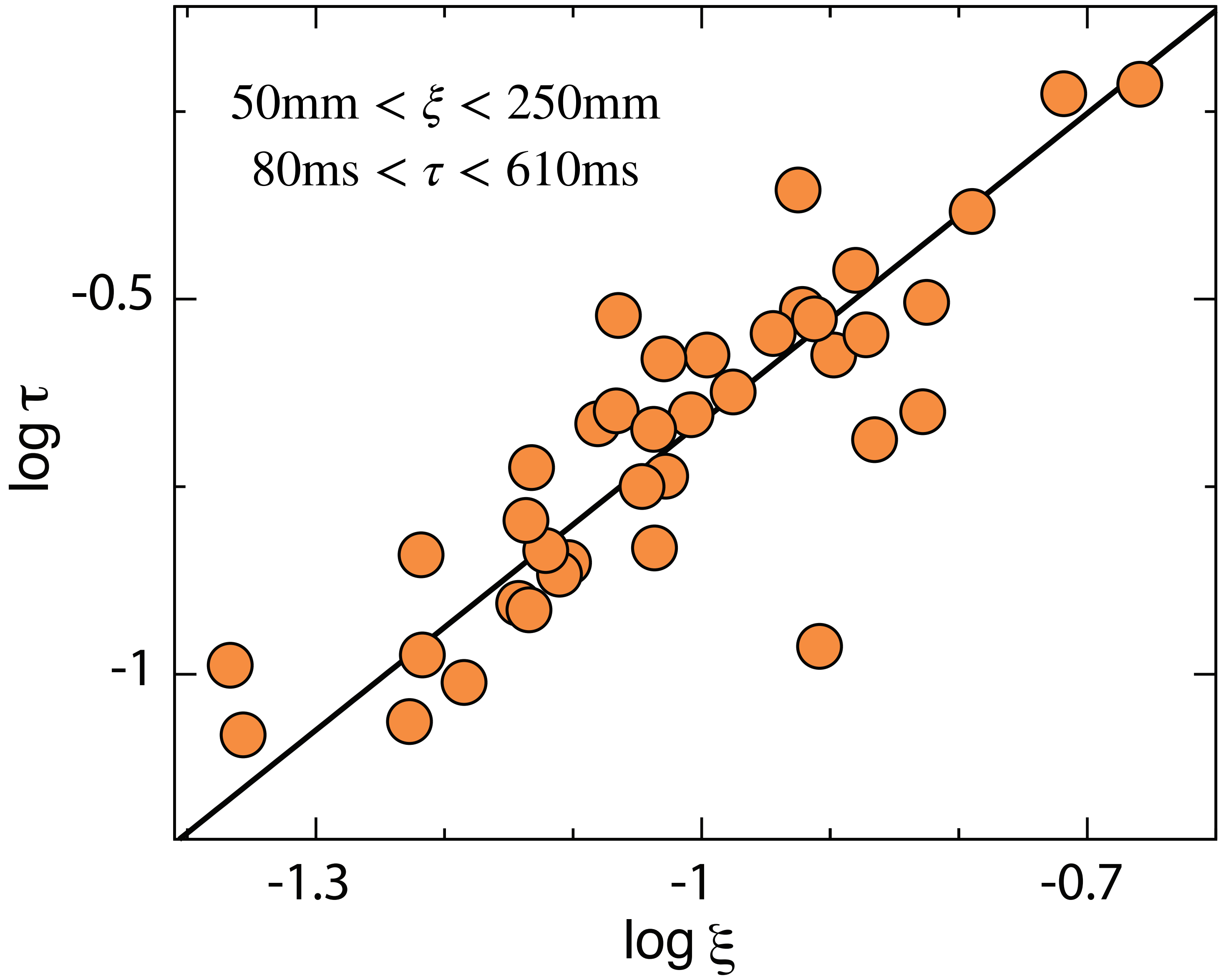
Experiments

Dynamic scaling

No net group motion: swarms are in a **disordered phase**

Velocity-velocity correlations are **strong** and **scale-free** $(\xi \sim L)$ and exhibit **finite-size scaling**

The **dynamic** correlation function exhibits **scaling** behaviour

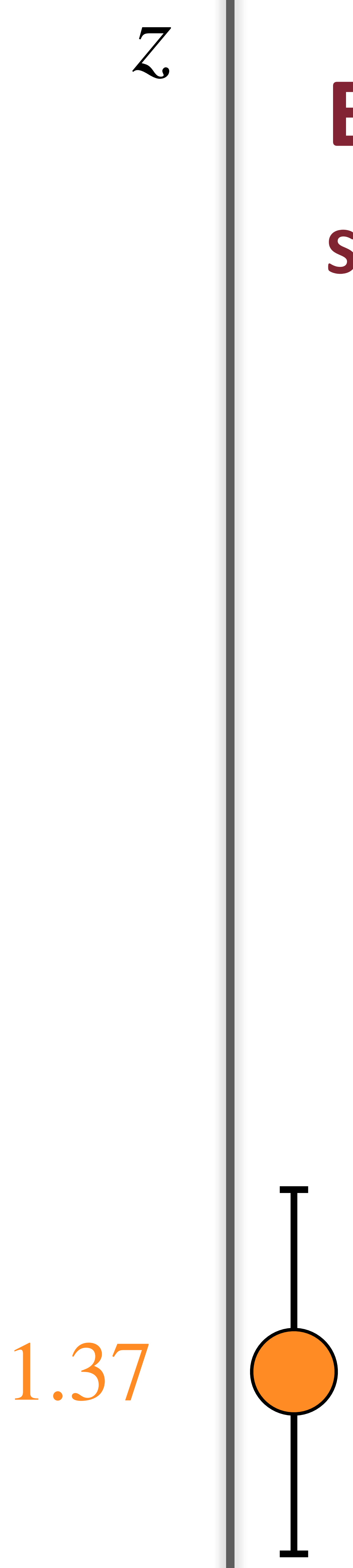


$$\tau \sim \xi^z$$

with a **dynamic critical exponent**

$$z = 1.37 \pm 0.11$$

A. Cavagna et al., Nature Physics, 2017 - A. Cavagna et al., Nature Physics, 2023



Experiments

Summary

No net group motion: swarms are in a **disordered phase**

Velocity-velocity correlations are **strong** and **scale-free** $(\xi \sim L)$ and exhibit **finite-size scaling**

The **dynamic** correlation functions exhibit **scaling** behaviour with $z = 1.37 \pm 0.11$

My goal: understanding the physics behind swarm's dynamic collective behaviour

A. Attanasi et al., PLoS Computational Biology, 2014 - A. Attanasi et al., PRL, 2014 - A. Cavagna et al., Nature Physics, 2023

Experiments

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Theoretical “ingredients”

↓

Renormalization Group

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Theoretical "ingredients"

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Renormalization Group

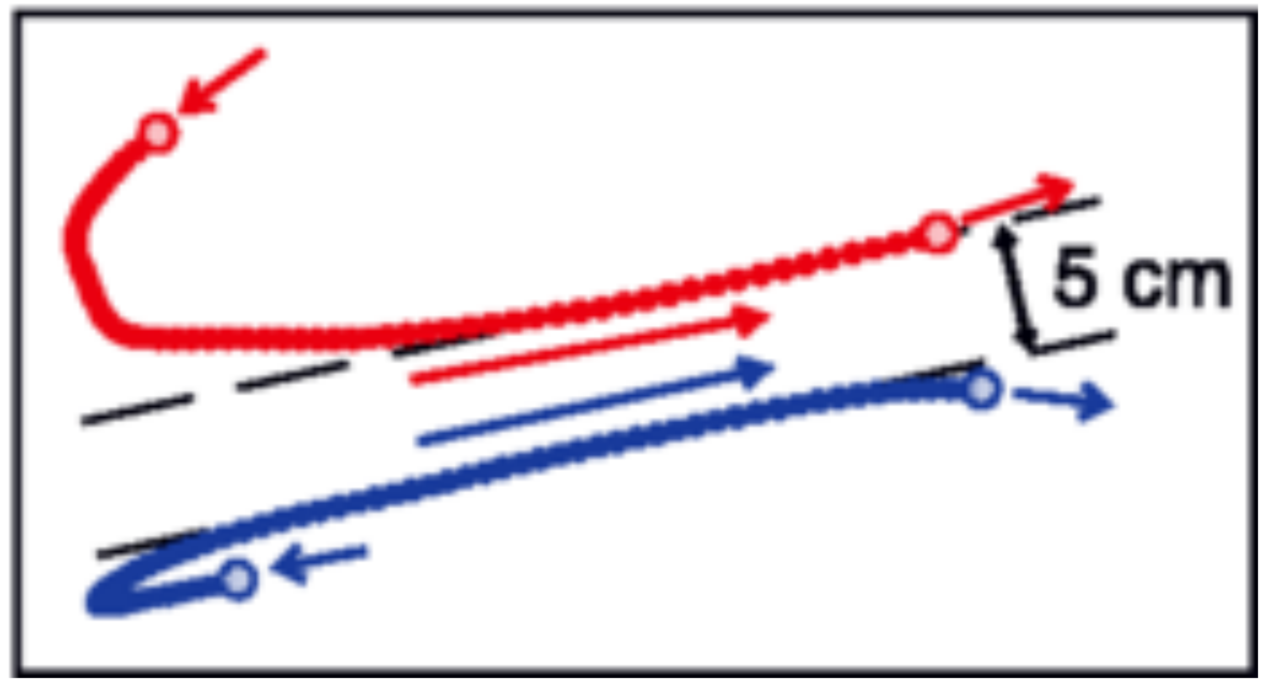
z

Imitative interactions

Simple ferromagnets

Experimental observation

Effective **alignment** between trajectories



Theoretical modelling

Imitative **ferromagnetic** behaviour

Model A (Dynamic Landau-Ginzburg)

$$\partial_t \boldsymbol{\psi} = -\Gamma \frac{\delta \mathscr{H}}{\delta \boldsymbol{\psi}} + \boldsymbol{\theta}$$

$$\mathscr{H} = \int d^d x \, \frac{1}{2} \left(\nabla \psi \right)^2 + \frac{r}{2} \psi^2 + \frac{u}{4} \psi^4$$

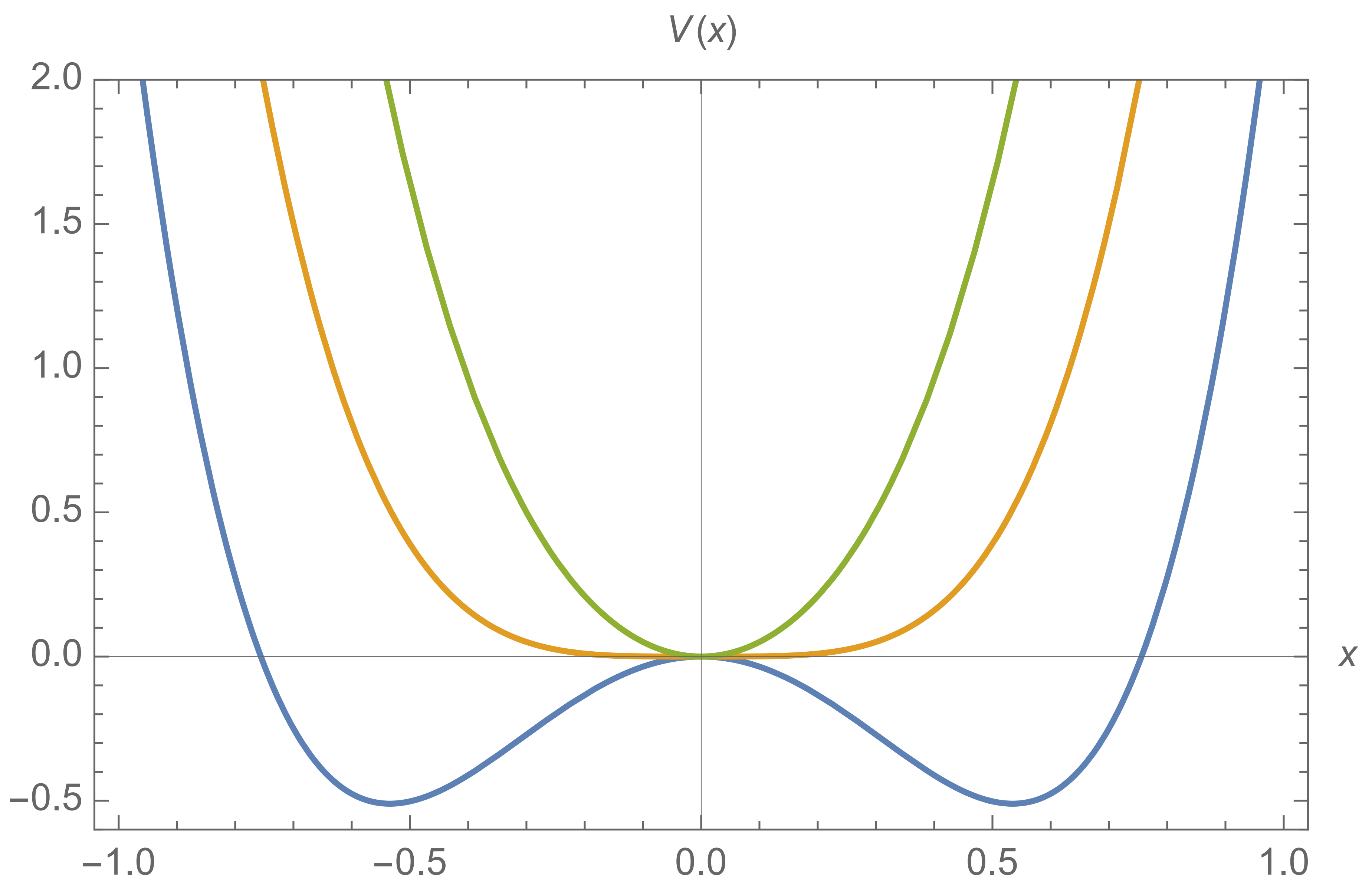
1.37

A. Attanasi et al., PRL, 2014 - P.C. Hohenberg and B.I. Halperin, RMP 1977

z

Imitative interactions

Simple ferromagnets



Mean-field phase diagram

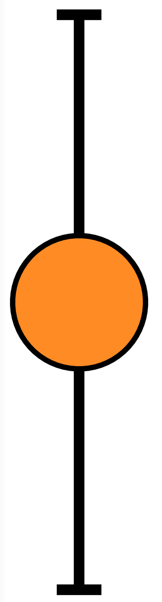
$r > 0 \quad \rightarrow$ disordered

$r < 0 \quad \rightarrow$ ordered

Model A (Dynamic Landau-Ginzburg)

$$\partial_t \boldsymbol{\psi} = -\Gamma \frac{\delta \mathcal{H}}{\delta \boldsymbol{\psi}} + \boldsymbol{\theta}$$

$$\mathcal{H} = \int d^d x \, \frac{1}{2} (\nabla \psi)^2 + \overbrace{\frac{r}{2}\psi^2 + \frac{u}{4}\psi^4}^{V(\psi)}$$



1.37

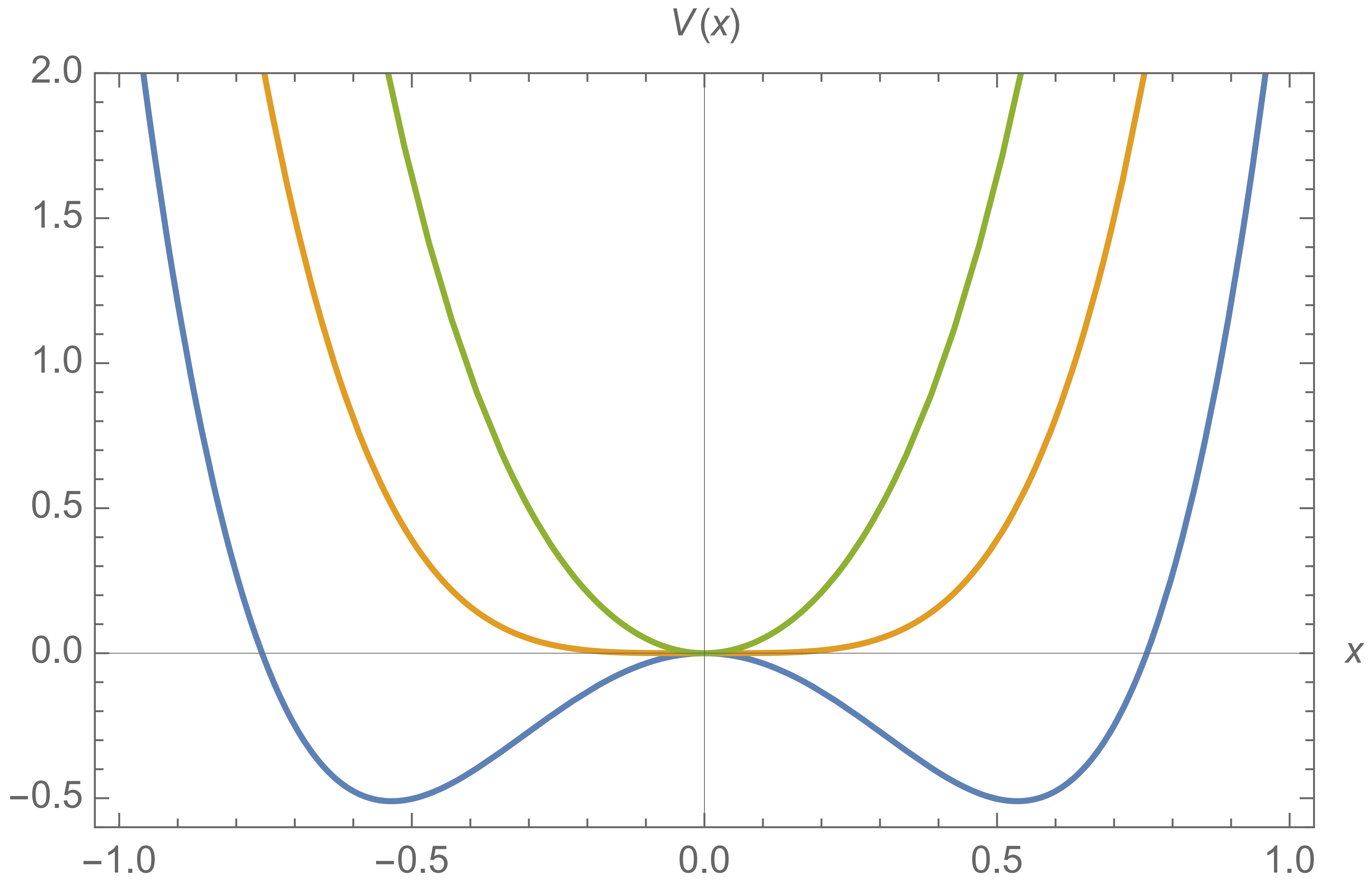
P.C. Hohenberg and B.I. Halperin, RMP 1977

z

2

Imitative interactions

Simple ferromagnets



Mean-field phase diagram

$r = 0 \quad \rightarrow$ Critical (Swarm's phase)

Model A (Dynamic Landau-Ginzburg)

$$\partial_t \boldsymbol{\psi} = -\Gamma \frac{\delta \mathcal{H}}{\delta \boldsymbol{\psi}} + \boldsymbol{\theta}$$

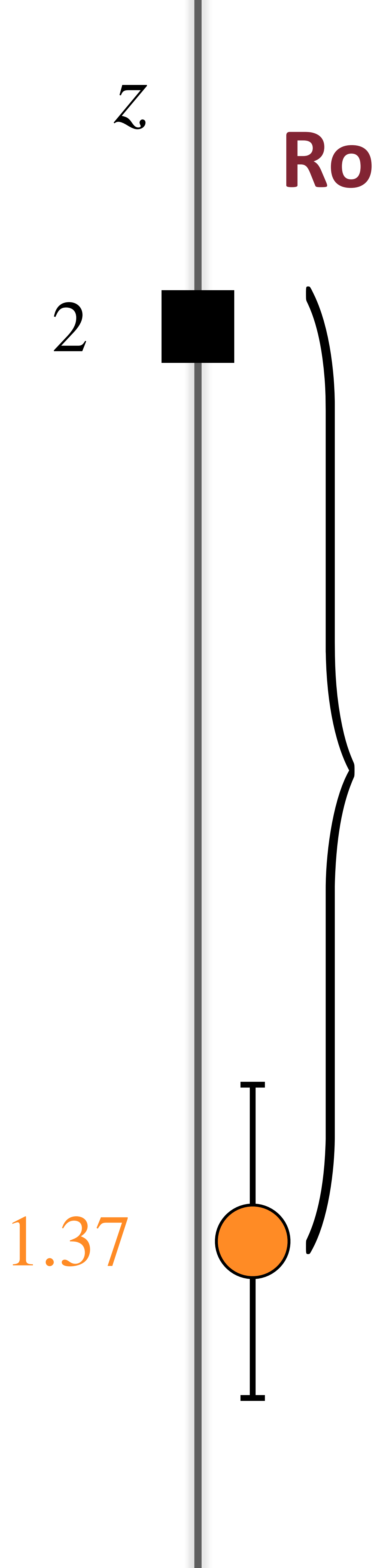
$$\mathcal{H} = \int d^d x \, \frac{1}{2} \left(\nabla \psi \right)^2 + \overbrace{\frac{r}{2}\psi^2 + \frac{u}{4}\psi^4}^{V(\psi)}$$

1.37

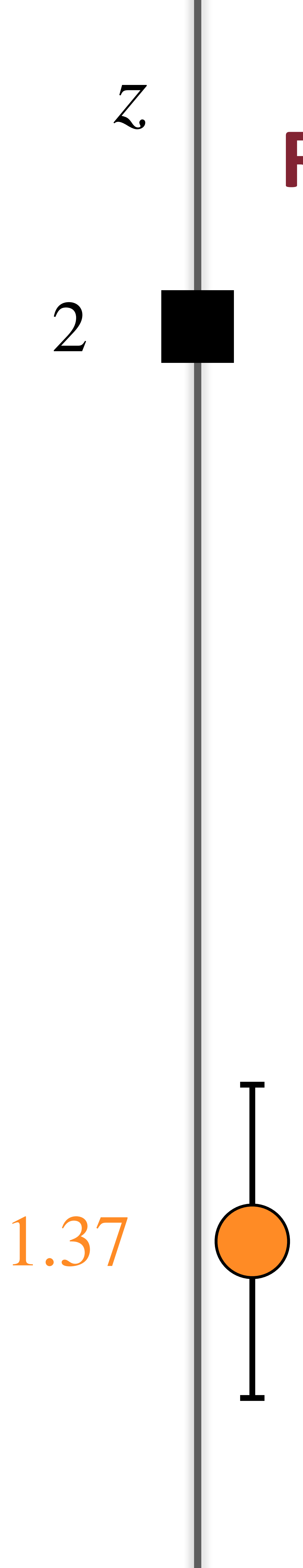
Critical Model A: $z \approx 2$

P.C. Hohenberg and B.I. Halperin, RMP 1977

Role of information propagation



What does my theory need to bridge this gap?



Role of information propagation

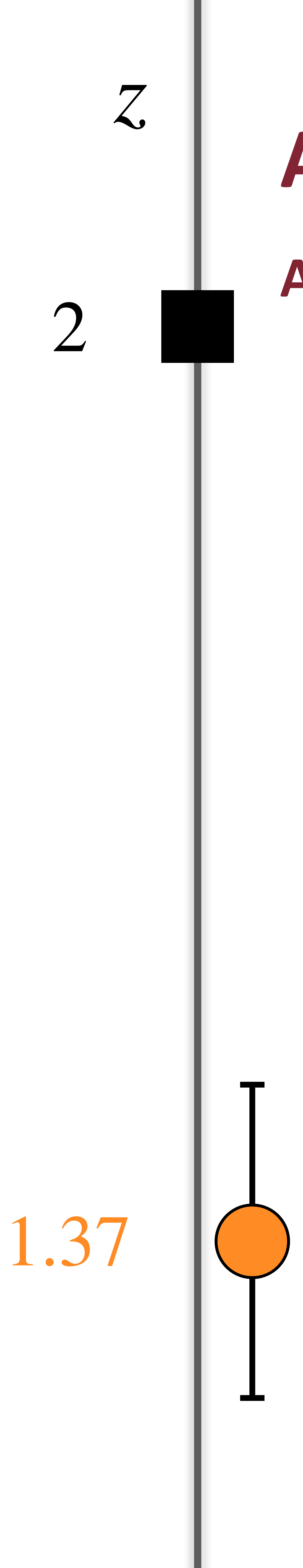
$$\tau \sim \xi^z \quad \longleftrightarrow \quad \omega(k) \sim k^z$$

$$\text{time} \sim \text{space}^z$$

The smaller z, the faster information is propagated

In ferromagnets $z = 2$ $\rightarrow$ information propagates diffusively

What does enhance information propagation?

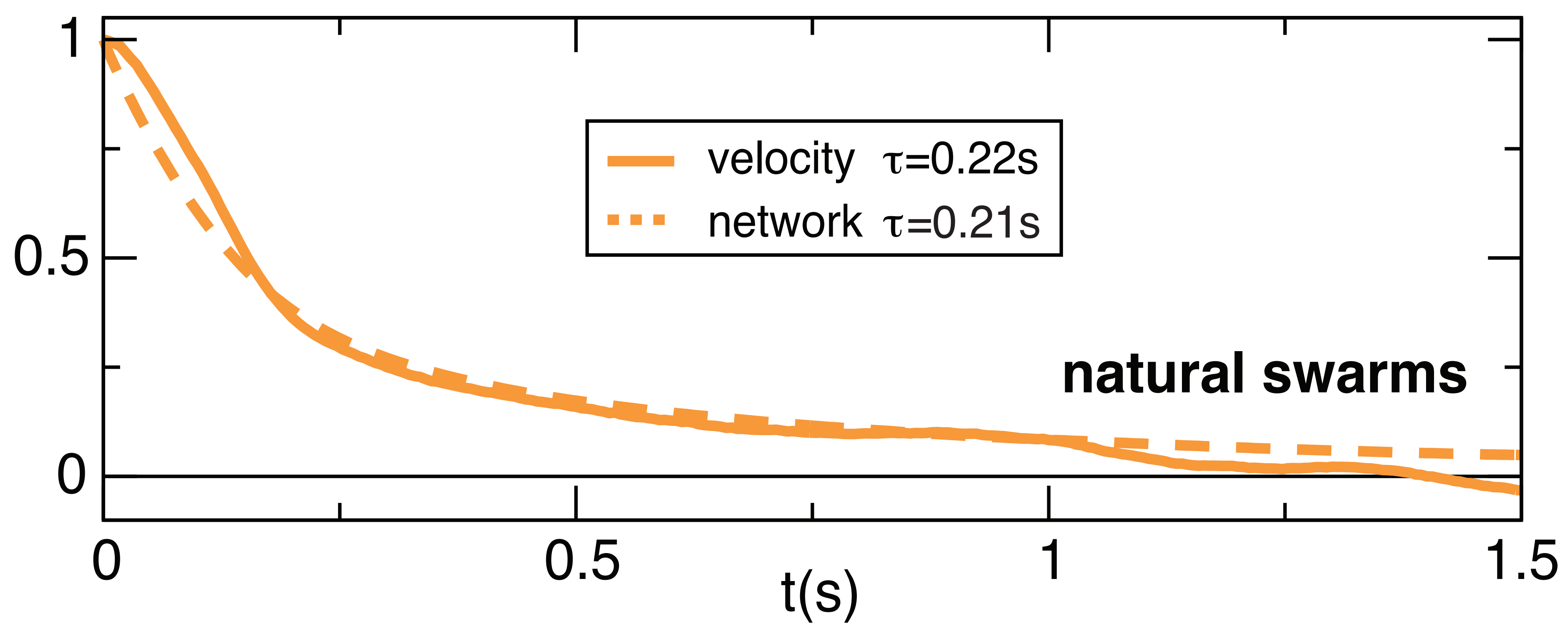


Active self-propulsion

Activity meets alignment: Toner and Tu theory

Experimental observation

Network reshuffling occurs on same times-scales as velocity alignment



Theoretical modelling

The order parameter is a velocity field

$$\boldsymbol{v} = v_0 \boldsymbol{\psi}$$

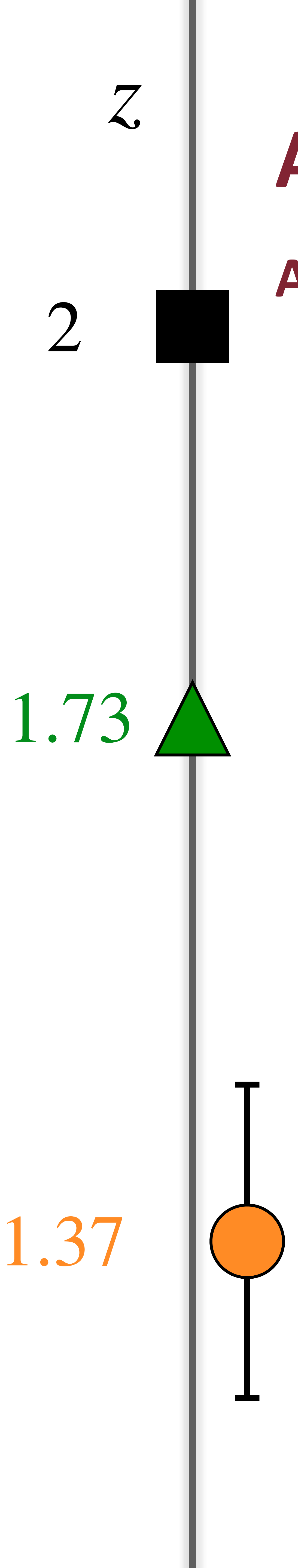
Toner and Tu theory

$$\begin{cases} D_t \boldsymbol{\psi} = -\Gamma \dfrac{\delta \mathcal{H}}{\delta \boldsymbol{\psi}} - \nabla P + \boldsymbol{\theta} \\ D_t = \partial_t + v_0 \boldsymbol{\psi} \cdot \nabla \end{cases}$$

Incompressible case: $\nabla \cdot \boldsymbol{\psi} = 0$

(Let's keep things simple)

T. Vicsek et al., PRL 1995 - J. Toner and Y. Tu, PRL 1995 - A. Cavagna et al., Nature Physics, 2023



Active self-propulsion

A new active fixed point: Chen Toner Lee 2015

Toner and Tu theory

$$\begin{cases} D_t \boldsymbol{\psi} = -\Gamma \dfrac{\delta \mathcal{H}}{\delta \boldsymbol{\psi}} - \nabla P + \boldsymbol{\theta} \\ D_t = \partial_t + v_0 \gamma_v \boldsymbol{\psi} \cdot \nabla \end{cases}$$

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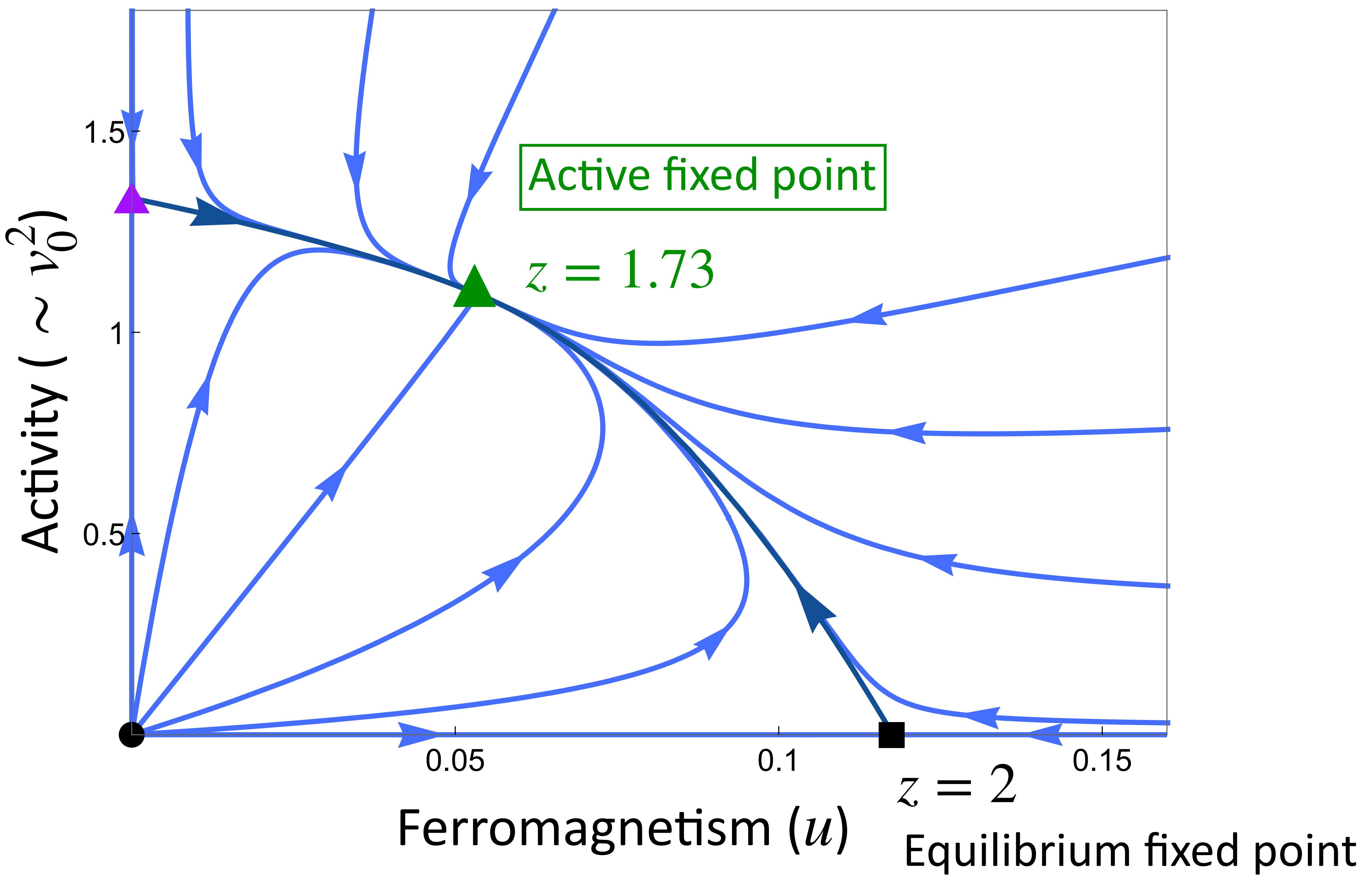
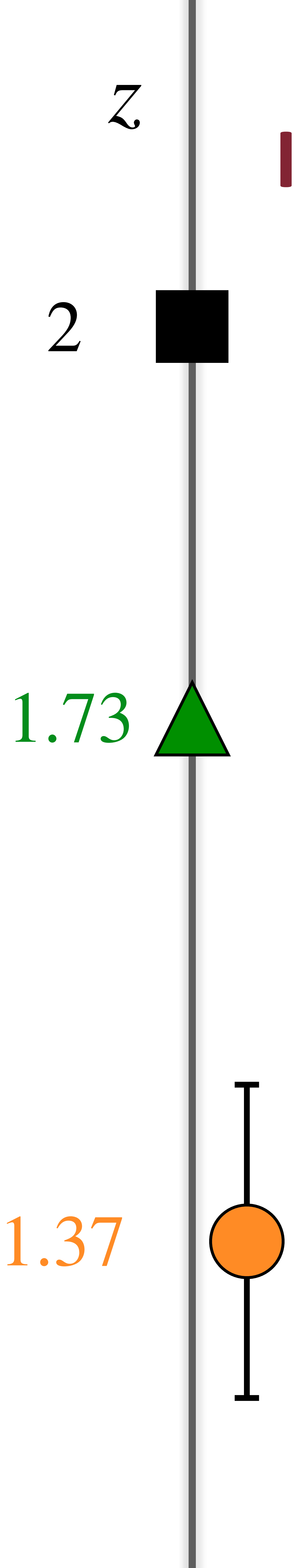


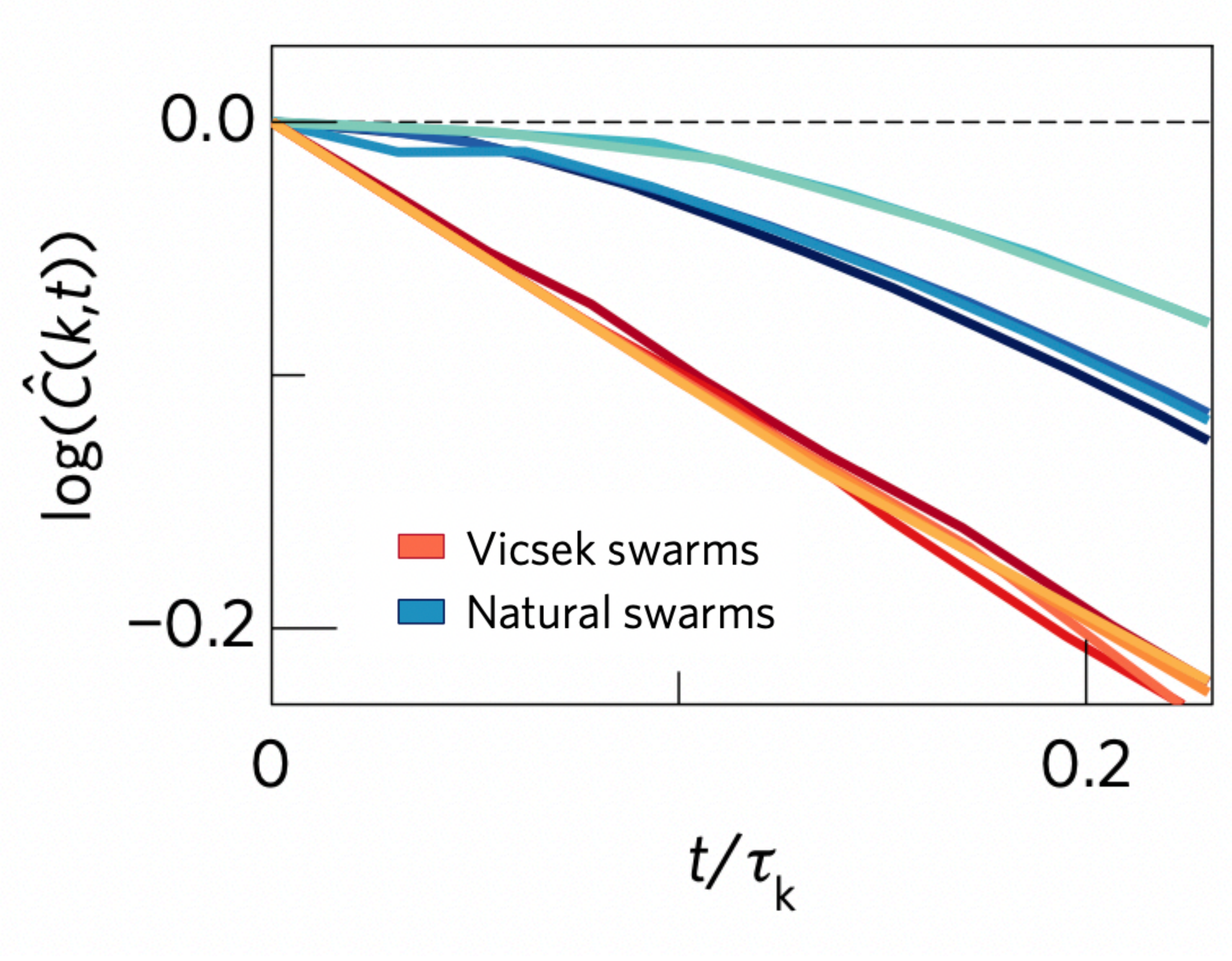
L. Chen et al., NJP 2015 - A. Cavagna et al., PRResearch 2021



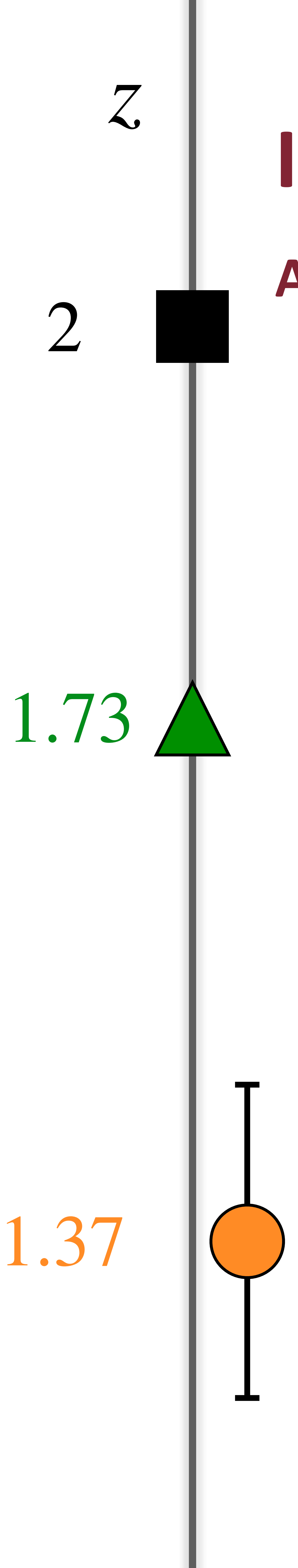
Inertial behaviour

Temporal correlations in swarm have an **inertial** shape

$$C(r,t) = \langle \delta \boldsymbol{v}(x_0, t_0) \cdot \delta \boldsymbol{v}(x_0 + r, t_0 + t) \rangle$$



A. Cavagna et al., Nature Physics, 2017



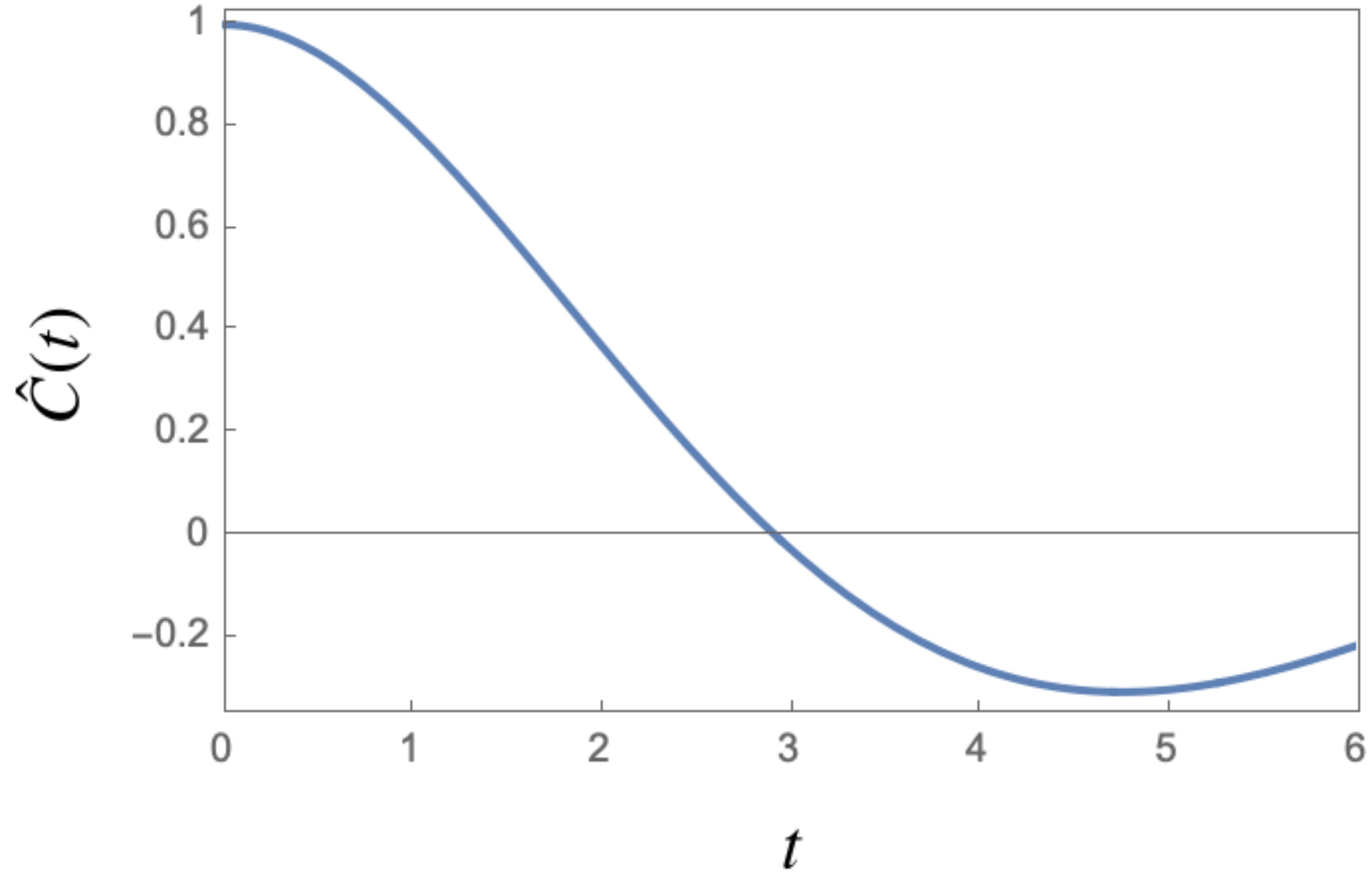
Inertial behaviour

A toy model: the stochastic harmonic oscillator

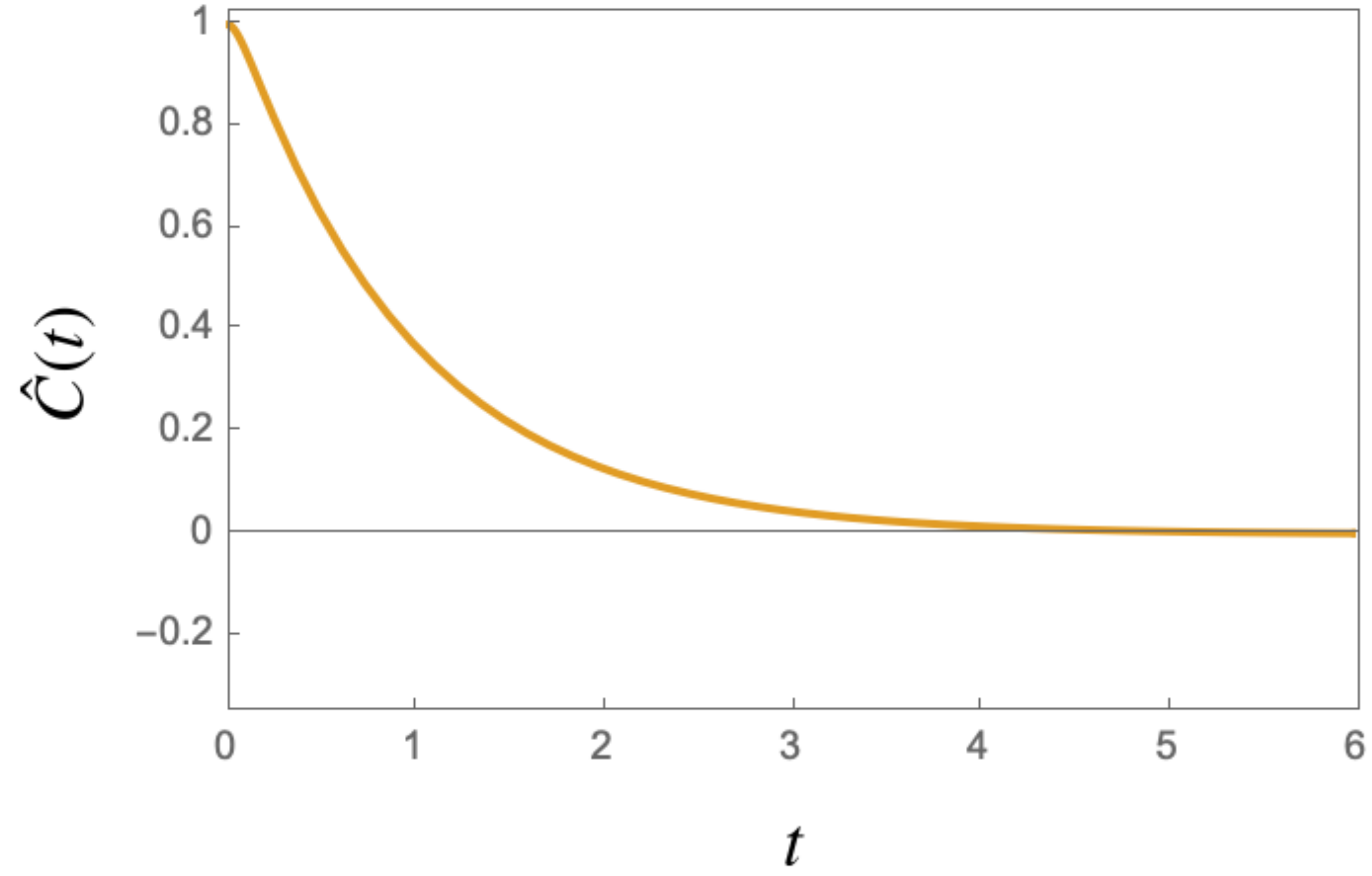
$$m\ddot{q} = -\kappa q - \eta\,\dot{q} + \sqrt{2T\eta}\,\zeta \quad \longrightarrow \quad \eta\dot{q} = -\kappa q + \sqrt{2T\eta}\,\zeta$$

$$\tau_{\mathrm{diss}} = \eta^{-1} m \ll \Delta t$$

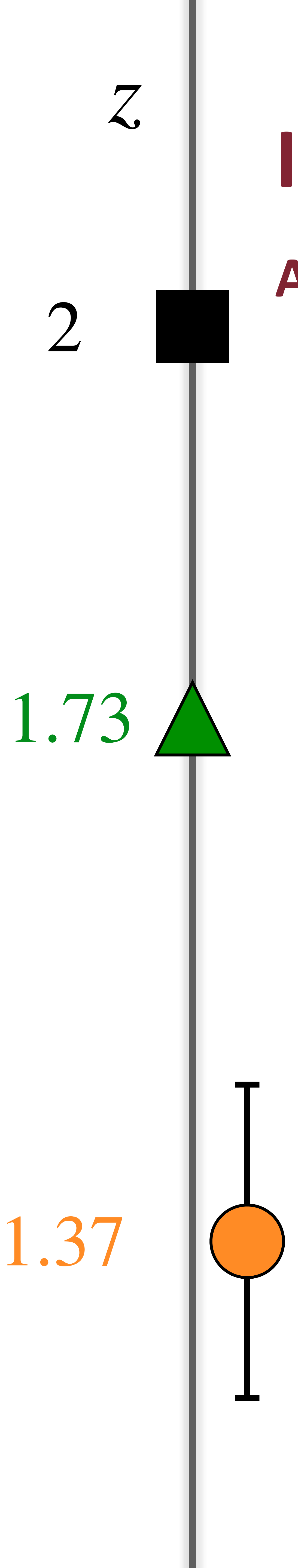
Underdamped



Overdamped



A. Cavagna et al., Nature Physics, 2017



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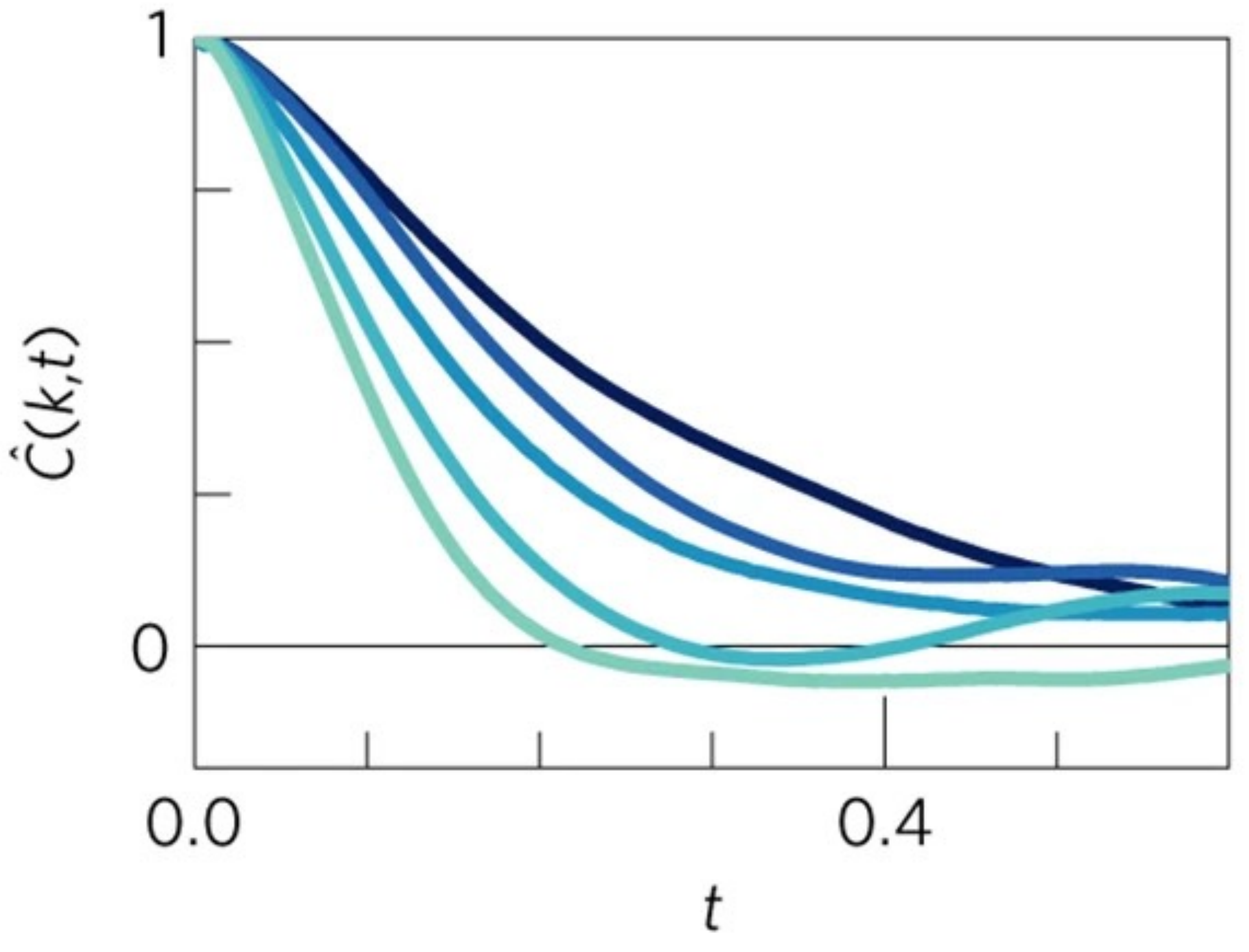
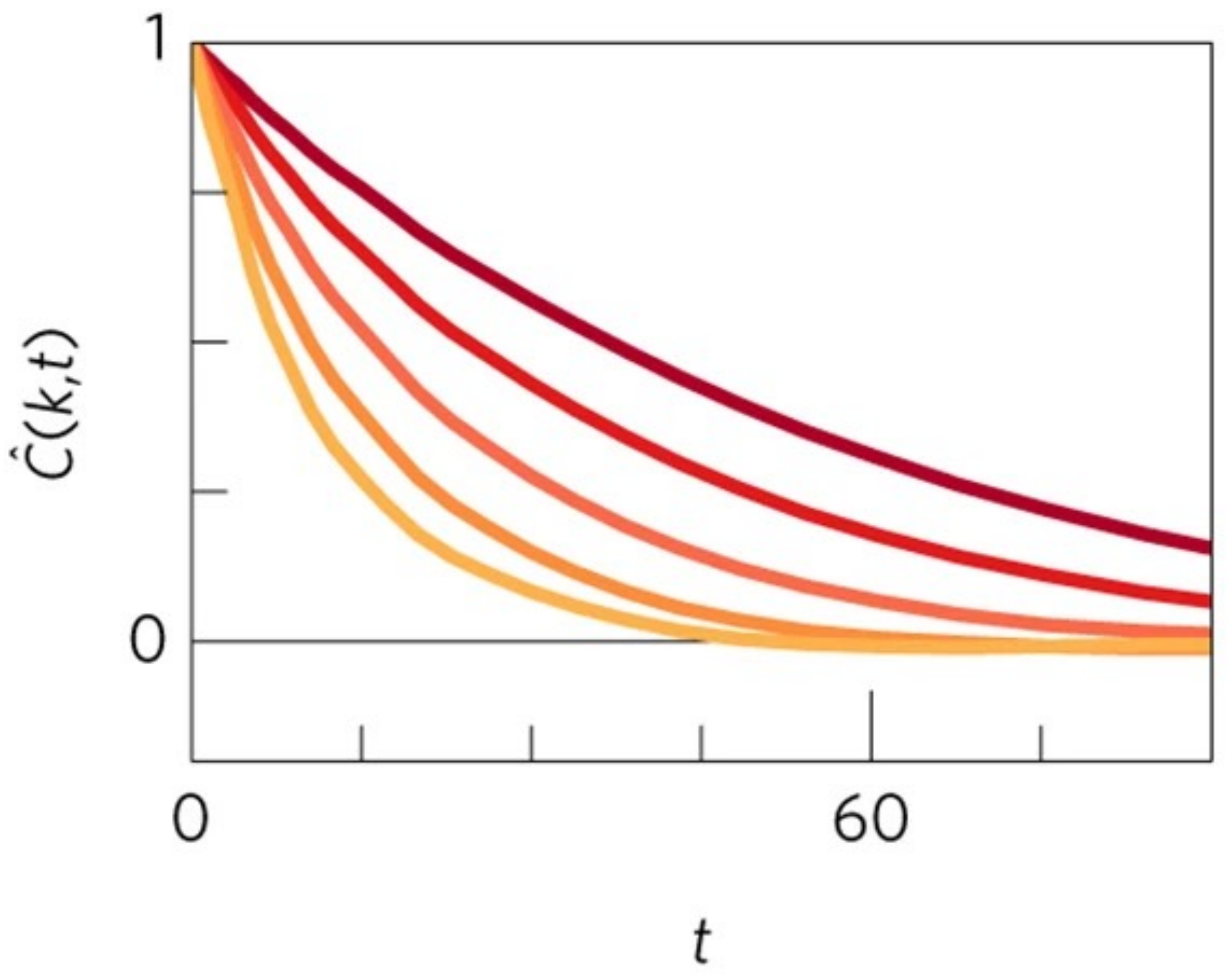


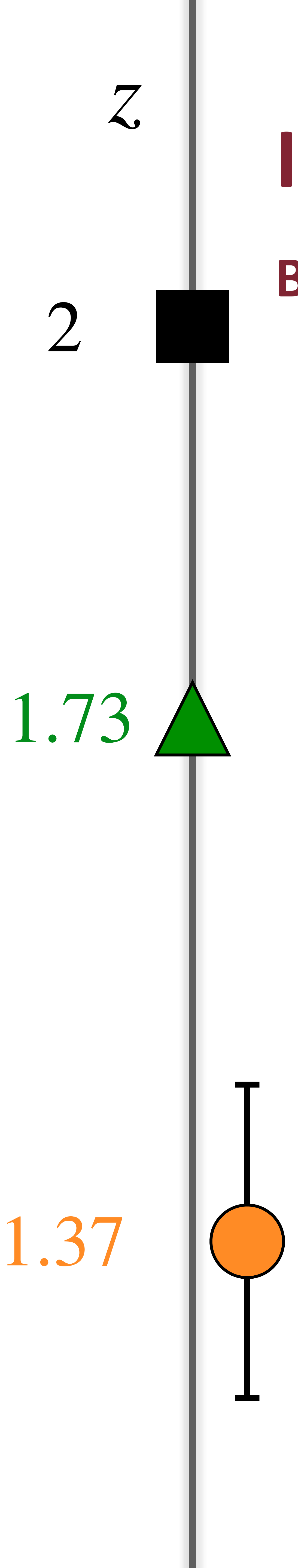
Natural swarms

Overdamped



Vicsek simulations

A. Cavagna et al., Nature Physics, 2017



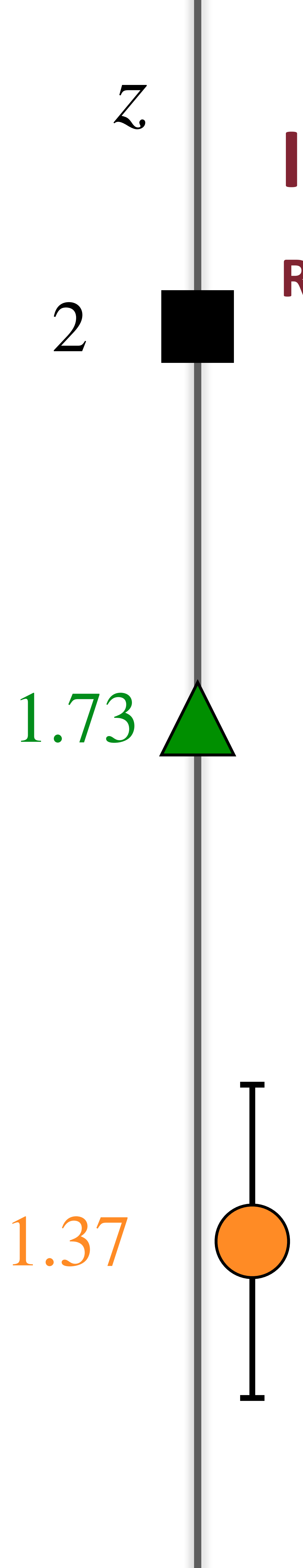
Inertial behaviour

But how does this help lowering z?

Imitative (alignment) forces are a **laplacian** in coarse-grained theories

Underdamped	Overdamped
$\partial_t^2 \boldsymbol{\psi} \sim \Gamma \nabla^2 \boldsymbol{\psi}$	$\partial_t \boldsymbol{\psi} \sim \Gamma \nabla^2 \boldsymbol{\psi}$
$\omega^2 \sim k^2$	$\omega \sim k^2$
$z \sim 1$	$z \sim 2$

A. Cavagna et al., J. Stat. Phys. 2015- A. Cavagna et al., Nature Physics 2017



Inertial behaviour

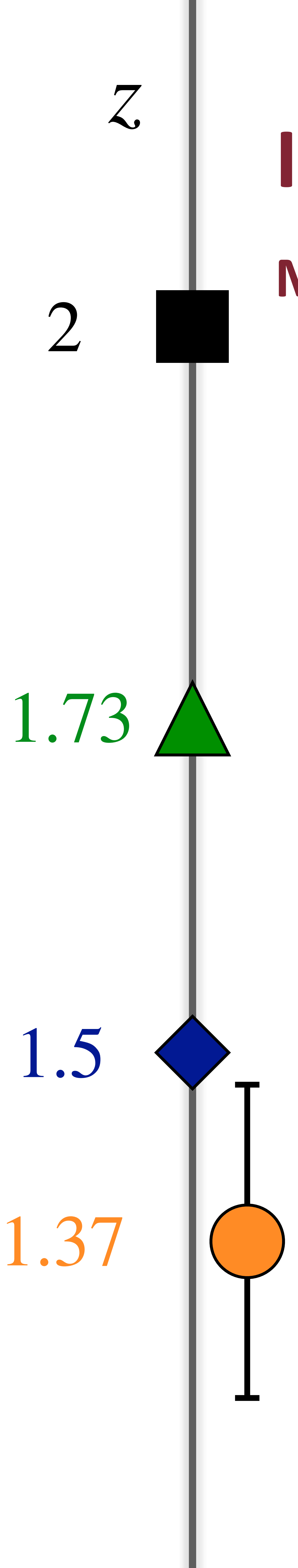
Recovering underdamped dynamics

To recover underdamped inertial behaviour we can use **symmetries**.

Symmetry → Noether Theorem → Conservation law

Rotational invariance of ψ
(All directions are equivalent)
→
Conservation of
generator of rotations of ψ

A. Cavagna et al., Nature Physics, 2017



Inertial behaviour

Model G

To recover underdamped inertial behaviour we can use **symmetries**.

$$\left\{ s_\alpha, \psi_\beta \right\} = g\, \epsilon_{\alpha\beta\gamma} \psi_\gamma$$

$\boldsymbol{\psi} \rightarrow$ order parameter

$\boldsymbol{s} \rightarrow$ spin field (generator of rotations of $\boldsymbol{\psi}$)

Overdamped (Model A)

$$\partial_t \boldsymbol{\psi} = -\Gamma \frac{\delta \mathscr{H}}{\delta \boldsymbol{\psi}} + \boldsymbol{\theta}$$

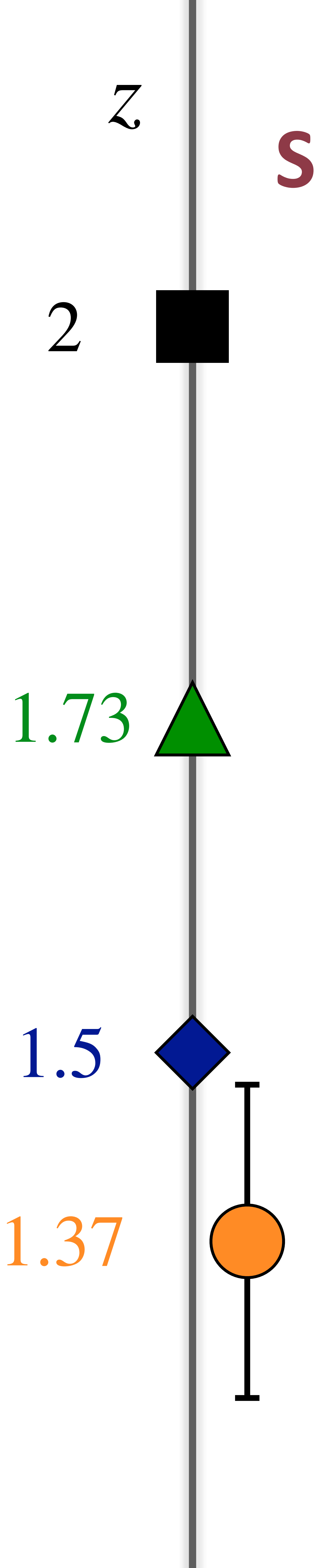
Underdamped (Model G)

$$\begin{cases} \partial_t \boldsymbol{\psi} = -\Gamma \dfrac{\delta \mathscr{H}}{\delta \psi} + \boxed{g \boldsymbol{\psi} \times \dfrac{\delta \mathscr{H}}{\delta s}} + \boldsymbol{\theta} \\ \partial_t \boldsymbol{s} = \lambda \nabla^2 \dfrac{\delta \mathscr{H}}{\delta s} + \boxed{g \boldsymbol{\psi} \times \dfrac{\delta \mathscr{H}}{\delta \psi}} + \boldsymbol{\zeta} \end{cases}$$

Inertia

Inertial exponent: $z = 1.5$

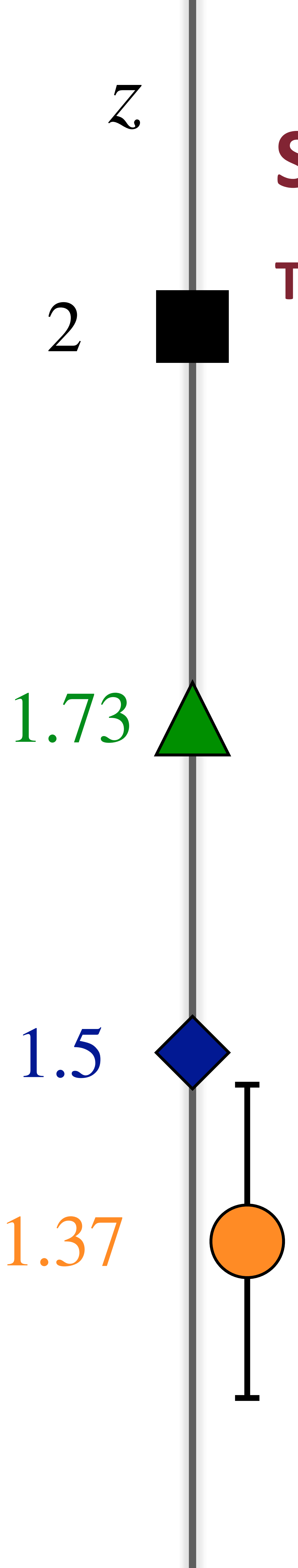
P.C. Hohenberg and B.I. Halperin, RMP 1977 - A. Cavagna et al., PRL 2019



Summary of the “ingredients”

Alignment	Ferromagnetism	$\mathscr{H} = \int d^d x \, \frac{1}{2} \left(\nabla \psi \right)^2 + \frac{r}{2} \psi^2 + \frac{u}{4} \psi^4$
Activity	Navier-Stokes	$\partial_t \rightarrow D_t = \partial_t + v_0 \boldsymbol{\psi} \cdot \nabla$
Inertia	Mode-coupling	$\left\{ s_\alpha, \psi_\beta \right\} = g \, \epsilon_{\alpha\beta\gamma} \psi_\gamma$

P.C. Hohenberg and B.I. Halperin, RMP 1977 - FNS, PRL 1977



Self-Propelled Model G

The dynamic field theory for swarms

The field theory for swarms is build by merging Model G with Navier Stokes equations

$$\partial_t \boldsymbol{\psi} + v_0 \gamma_v (\boldsymbol{\psi} \cdot \nabla) \boldsymbol{\psi} = -\Gamma \frac{\delta \mathcal{H}}{\delta \boldsymbol{\psi}} + g \boldsymbol{\psi} \times \frac{\delta \mathcal{H}}{\delta \mathbf{s}} - \nabla P + \boldsymbol{\theta}$$

$$\partial_t \mathbf{s} + v_0 \gamma_s (\boldsymbol{\psi} \cdot \nabla) \mathbf{s} = \lambda \nabla^2 \frac{\delta \mathcal{H}}{\delta \mathbf{s}} + g \boldsymbol{\psi} \times \frac{\delta \mathcal{H}}{\delta \boldsymbol{\psi}} + \boldsymbol{\zeta}$$

$$\nabla \cdot \boldsymbol{\psi} = 0$$

$$\mathcal{H} = \int d^d x \left[\frac{1}{2} (\nabla \boldsymbol{\psi})^2 + \frac{r}{2} \psi^2 + \frac{u}{4} \psi^4 + \frac{1}{2} s^2 \right]$$

A. Cavagna et al., Nature Physics 2023

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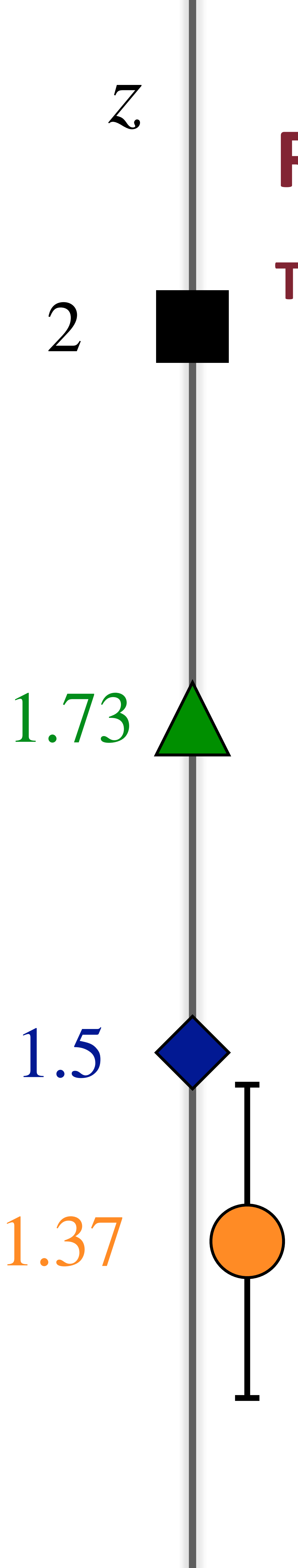
Renormalization Group

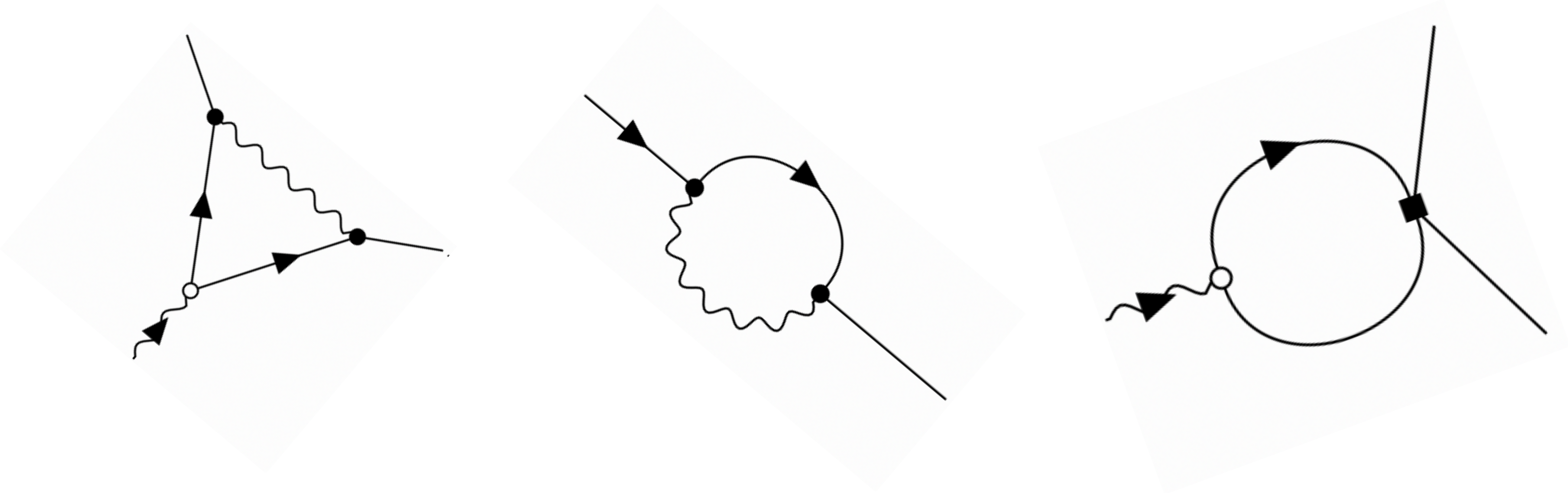


Renormalization Group

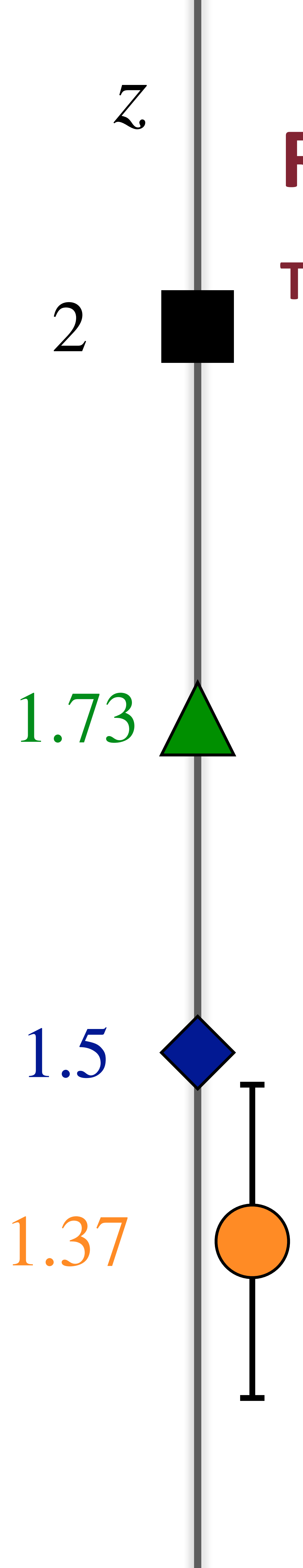
The Momentum-Shell RG

What is the RG?
A tool to obtain an **effective description** of the **large-scale physics**.

In a nutshell: it is a coarse-graining of short wavelength details.



K.G. Wilson, PRB 1971 - K.G. Wilson and M.E. Fisher, PRL 1972 - N. Goldenfeld, *Lectures on Phase Transitions and the Renormalization Group*, 1992



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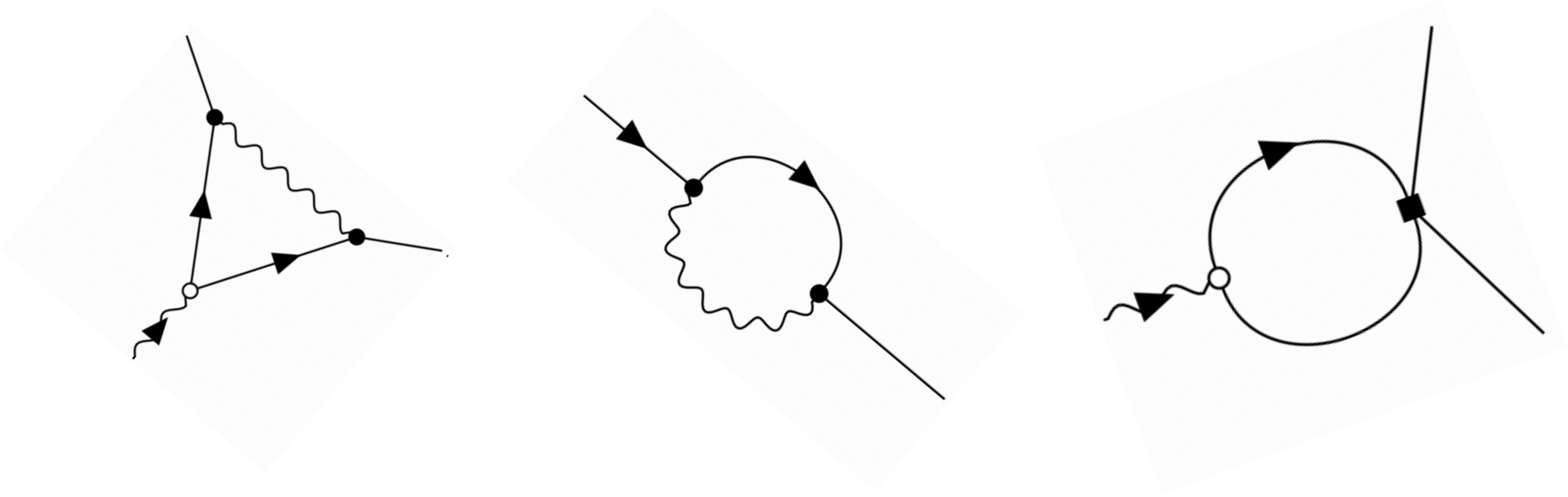
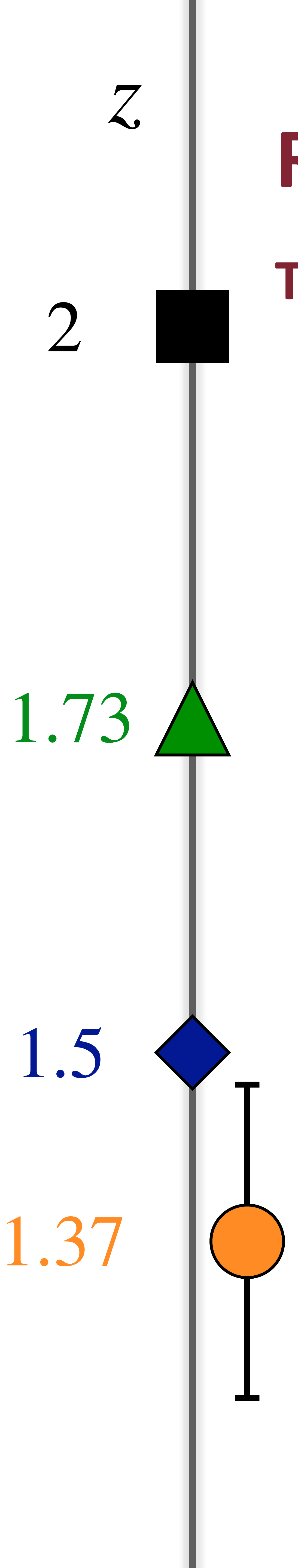


$$x \to bx \qquad t \to b^z t \qquad \varphi \to b^\chi \varphi$$

K.G. Wilson, PRB 1971 - K.G. Wilson and M.E. Fisher, PRL 1972 - N. Goldenfeld, *Lectures on Phase Transitions and the Renormalization Group*, 1992



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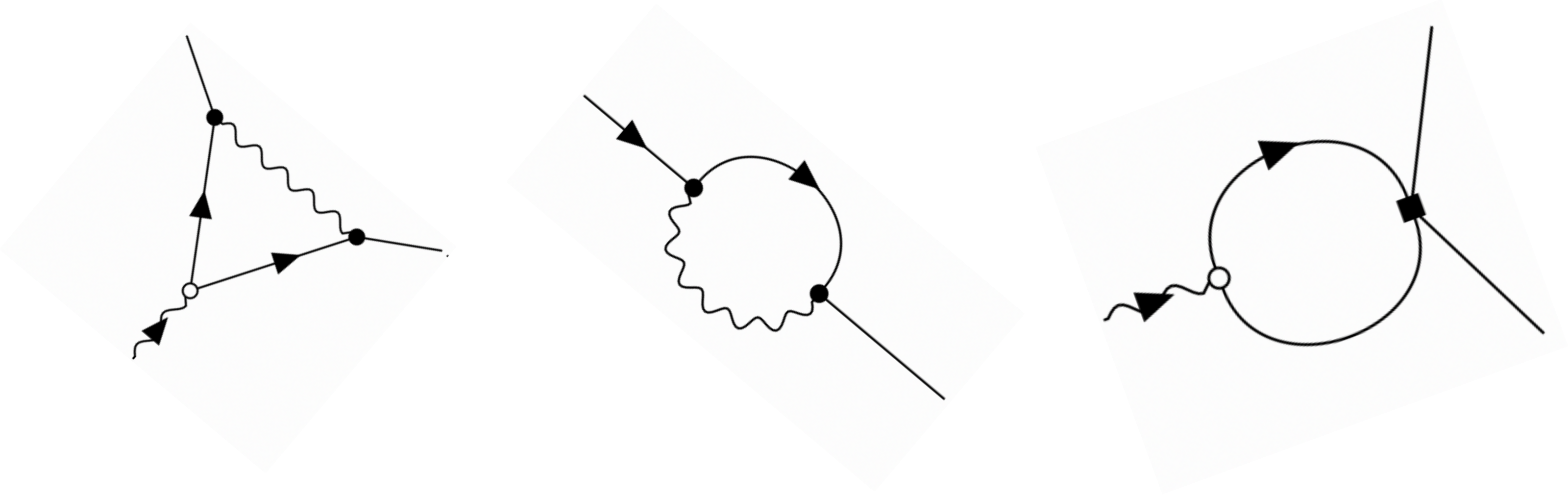

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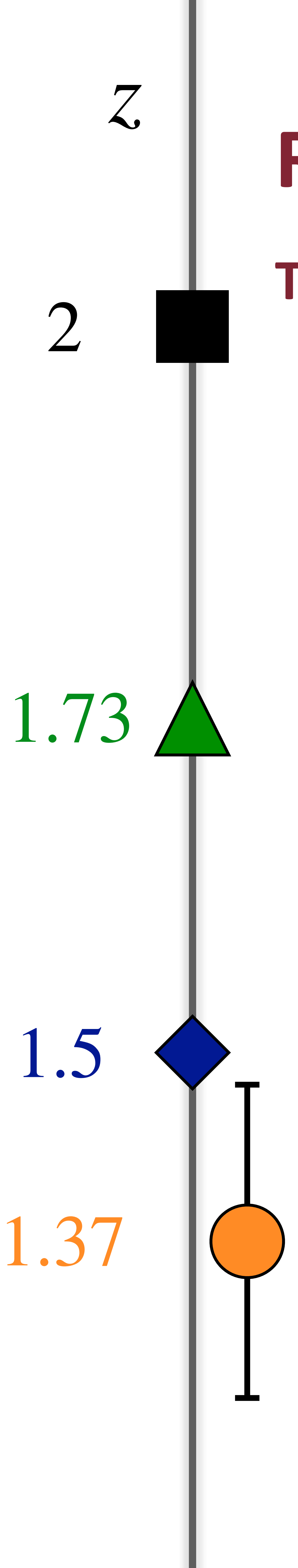

Renormalization Group

The Momentum-Shell RG

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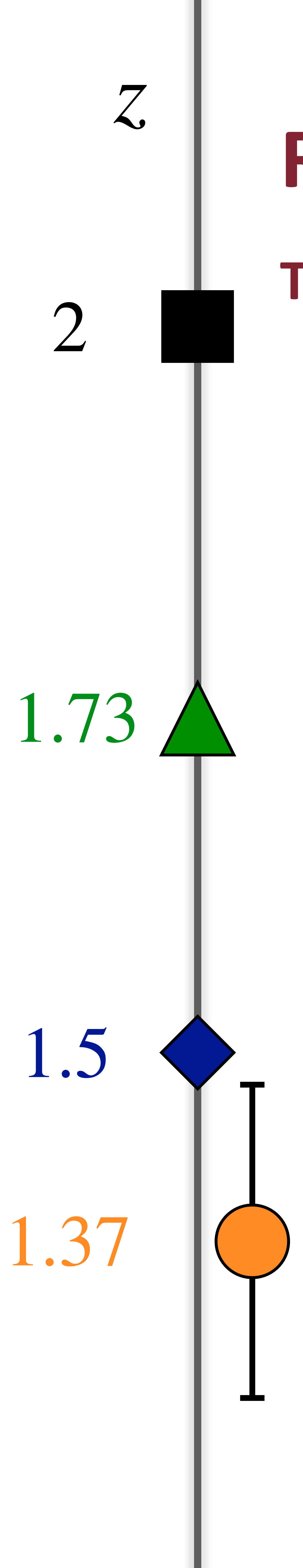
In a nutshell: it is a coarse-graining of short wavelength details.

$$S_0 \overset{RG}{\rightarrow} S_1 \overset{RG}{\rightarrow} S_2 \overset{RG}{\rightarrow} \ldots \overset{RG}{\rightarrow} S^*$$

K.G. Wilson, PRB 1971 - K.G. Wilson and M.E. Fisher, PRL 1972 - N. Goldenfeld, *Lectures on Phase Transitions and the Renormalization Group*, 1992



Renormalization Group

The Momentum-Shell RG

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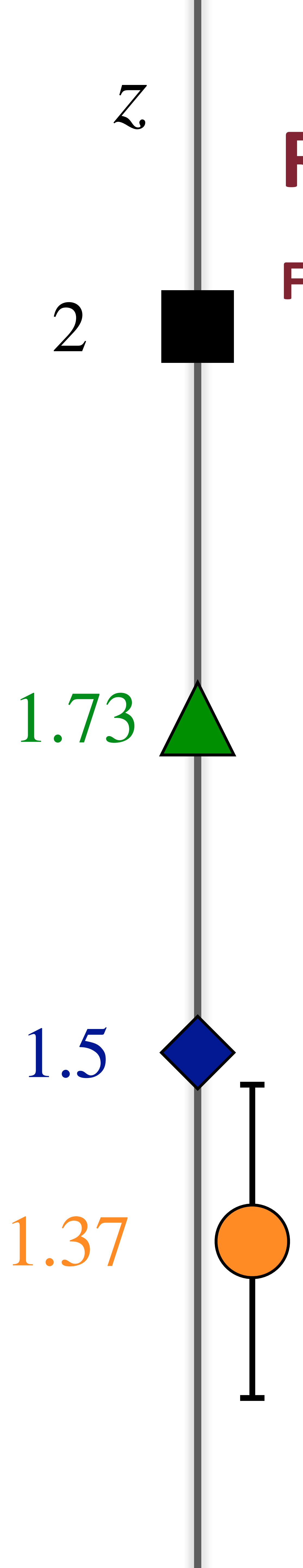
A tool to obtain an **effective description** of the **large-scale physics**.

In a nutshell: it is a coarse-graining of short wavelength details.

$$S_0 \overset{RG}{\rightarrow} S_1 \overset{RG}{\rightarrow} S_2 \overset{RG}{\rightarrow} \dots \overset{RG}{\rightarrow} S^*$$

Universality: different bare theories are described by the same fixed point S^*!

K.G. Wilson, PRB 1971 - K.G. Wilson and M.E. Fisher, PRL 1972 - N. Goldenfeld, *Lectures on Phase Transitions and the Renormalization Group*, 1992



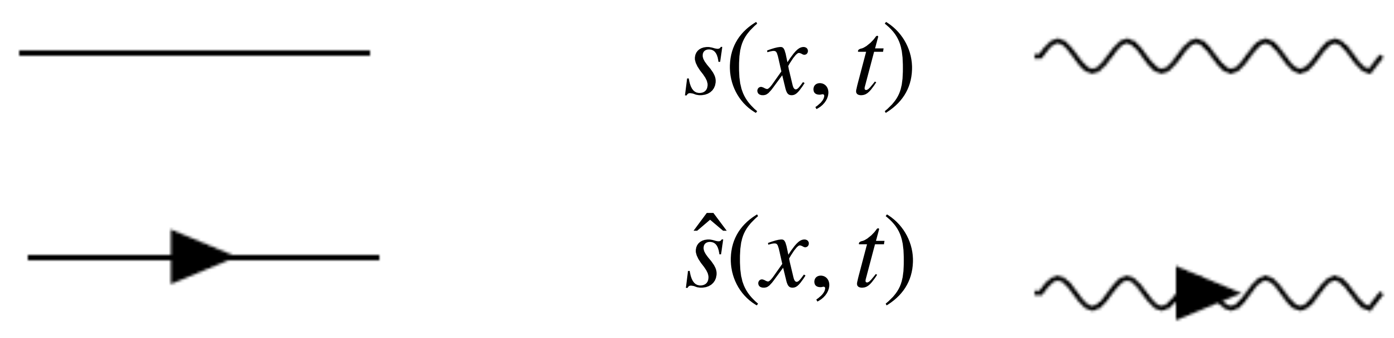
Renormalization Group

Fields and vertices

Use a Janssen-De Dominicis/Martin-Siggia-Rose formalisms

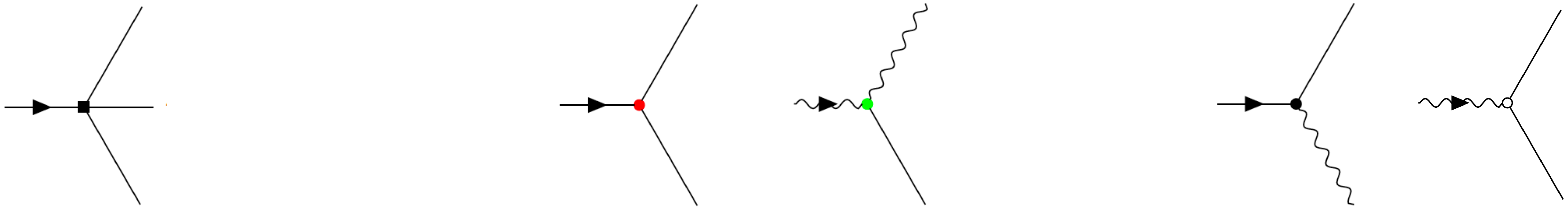
$\psi(x,t)$ $s(x,t)$

$\hat{\psi}(x,t)$ $\hat{s}(x,t)$

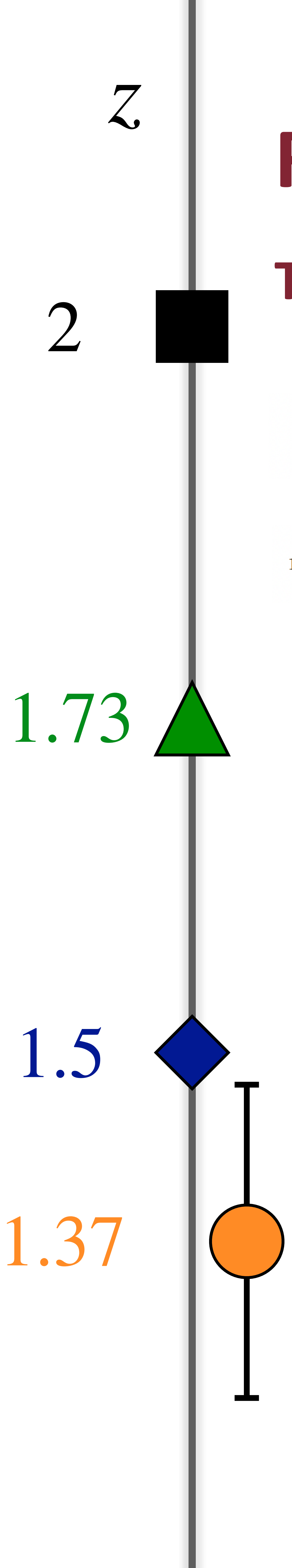


Perturbative expansion of the non-linear dynamic interactions:

Model A | Navier-Stokes | Model G

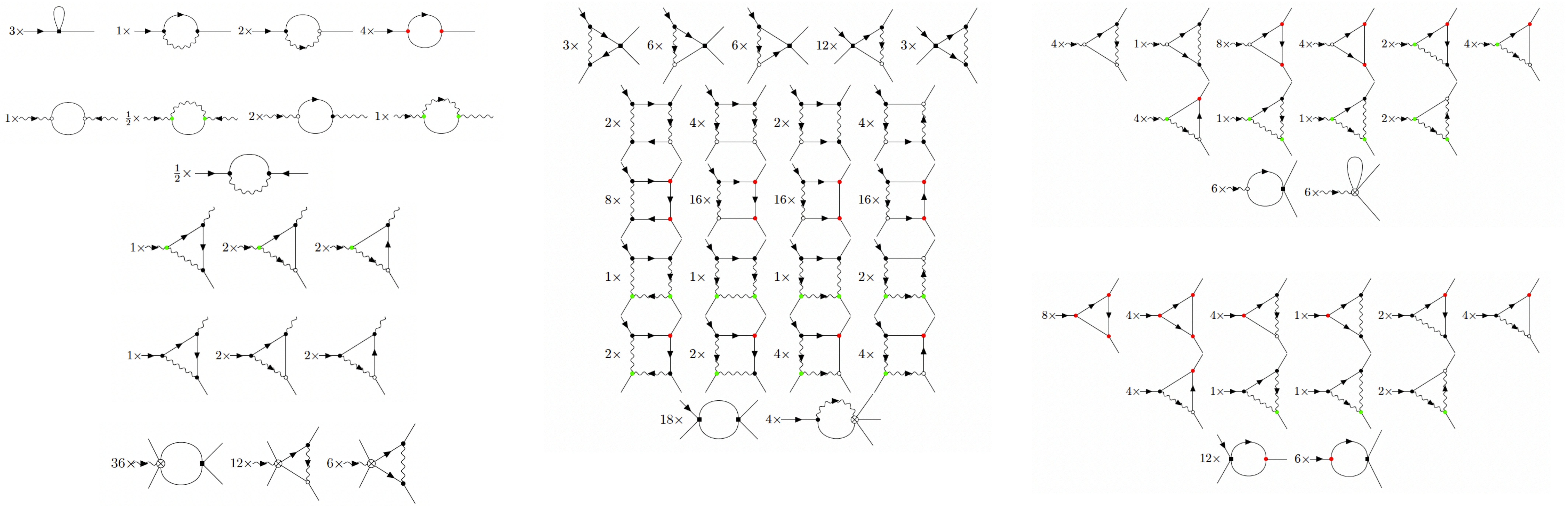


P.C. Martin et al., PRA 1973 - H.K. Janssen, Z Physik B 1976 - C. De Dominicis, J. Phys. Colloques 1976



Renormalization Group

The Feynman Diagrams

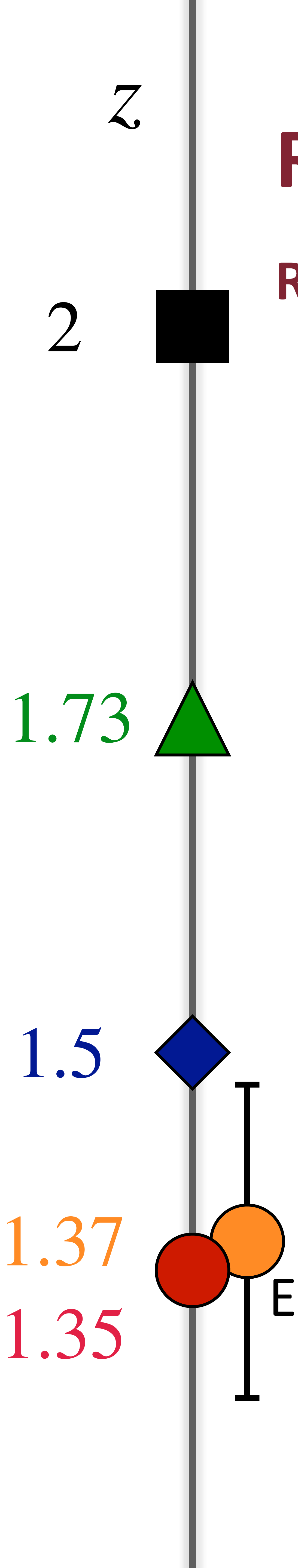


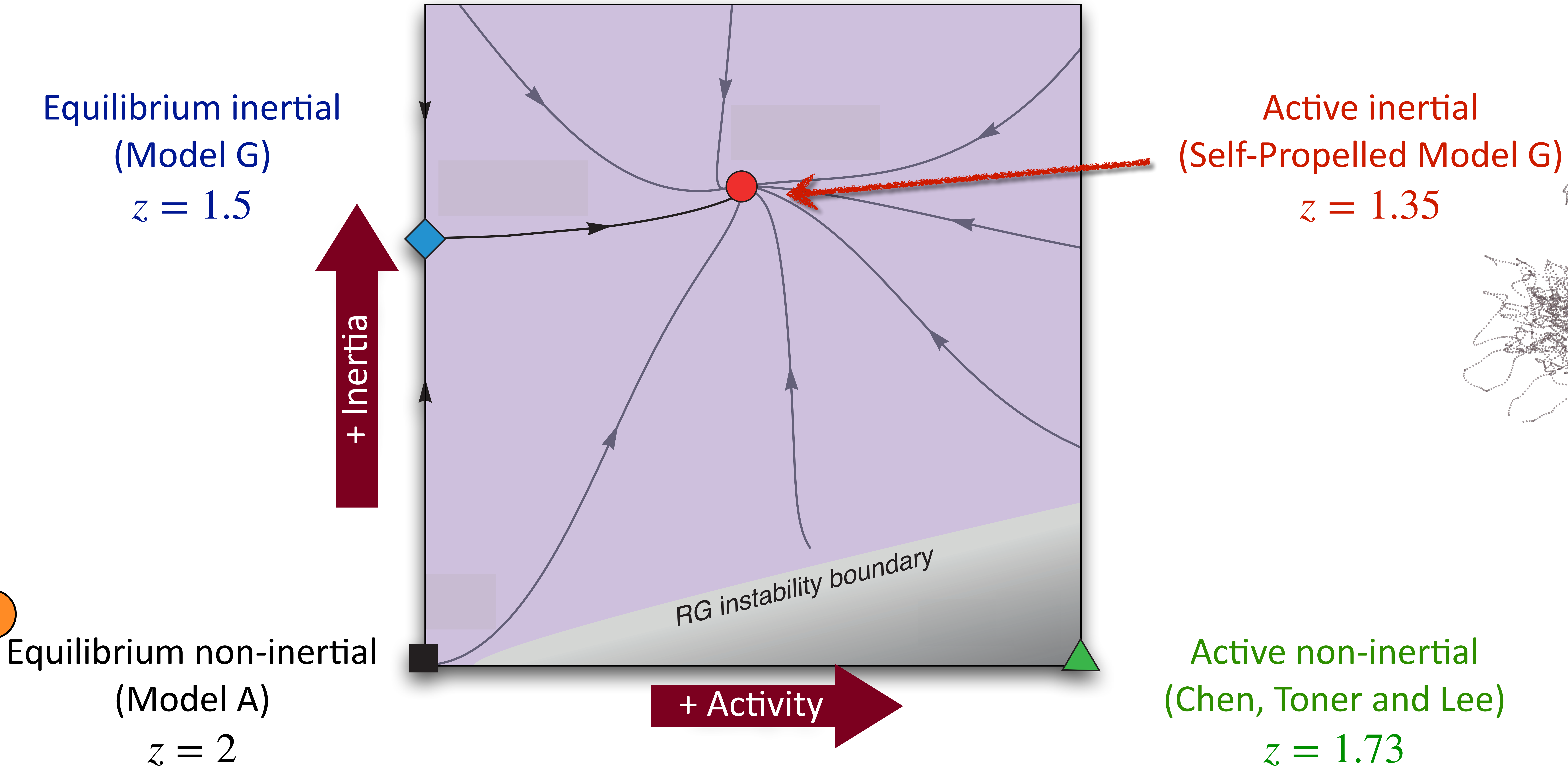
These are the diagrams at first order in $\epsilon = 4 - d$

A. Cavagna et al., Nature Physics 2023

Renormalization Group

RG flow and new fixed point





A. Cavagna et al., Nature Physics 2023

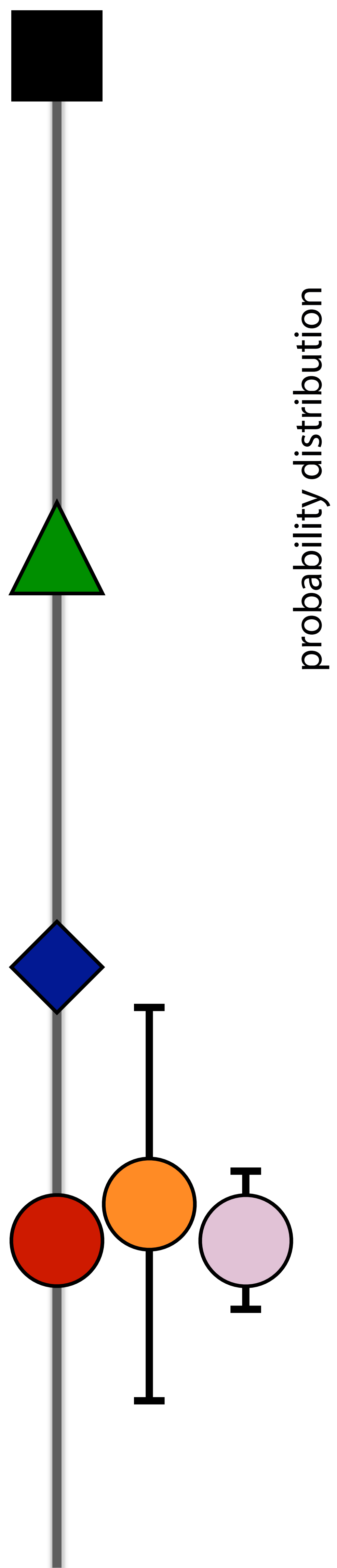
A new active inertial universality class

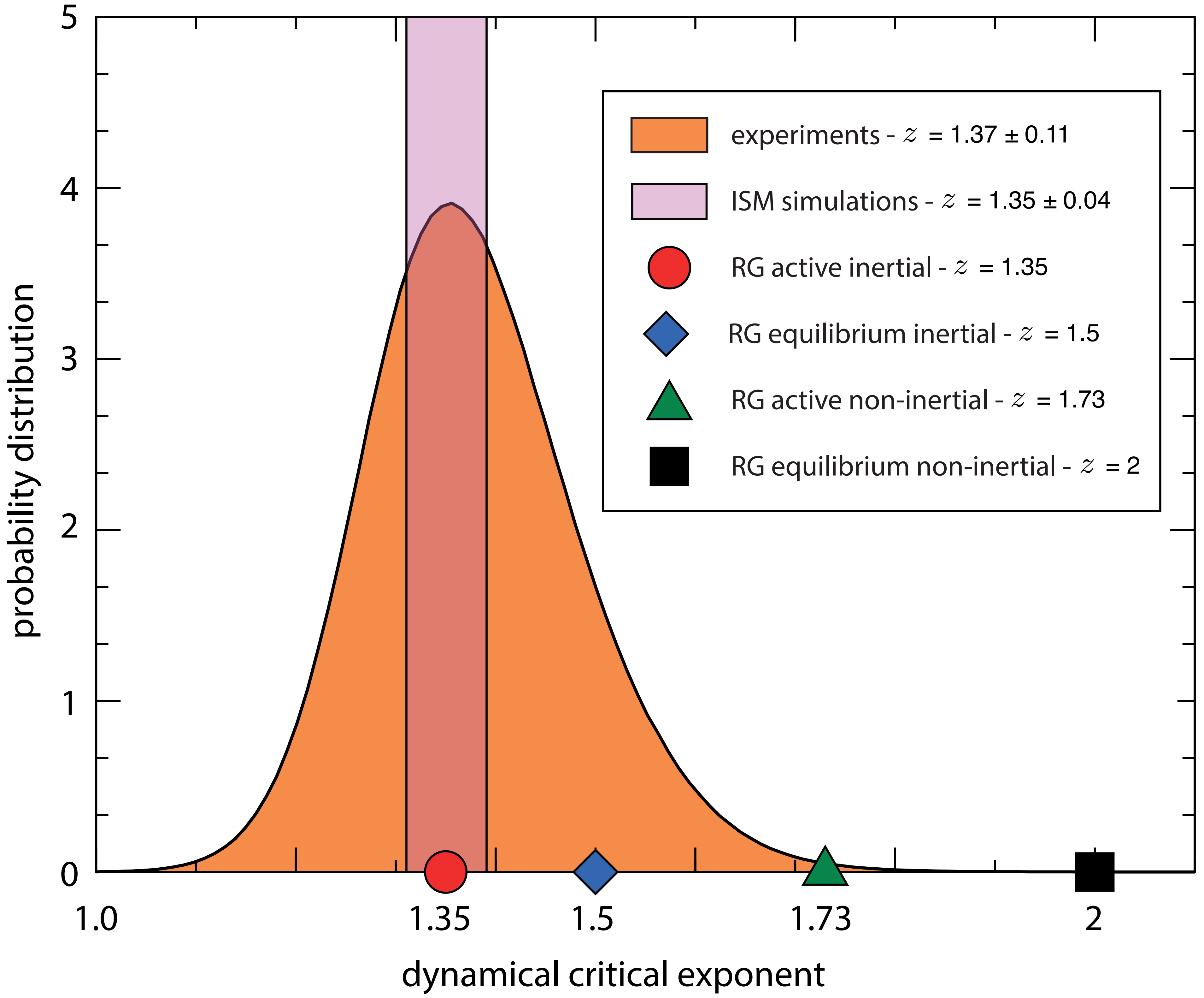
Equilibrium non-inertial (Model A) 2

Active non-inertial (Chen, Toner and Lee) 1.73

Equilibrium inertial (Model G) 1.5

Active inertial (Self-Propelled Model G) 1.35





1.37 ± 0.11 Natural swarms

1.35 ± 0.04 Numerical simulations

Take-home messages

- Renormalization Group is a powerful tool in describing true biological systems
- Universality might hold in the collective behaviour of biological systems
- New non-equilibrium universality classes are relevant in explaining biophysical data
- Inertial behaviour and activity are essential ingredients in the description of swarms

Bibliography

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Natural swarms in 3.99 dimensions, *Nature Physics* 19, 1043–1049 (2023)