Theoretical Studies of Magnetic Electron Scattering from Nuclei

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Outline

- Basic formalism of elastic magnetic electron scattering
- Non-relativistic theory and relativistic theory
- Effects of velocity-dependent force on magnetic electron scattering
- Magnetic electron scattering from exotic nuclei
- Meson exchange current in RMF
- Quenching effect at low q-region
- Summary and Outlook

Basic formalism of Magnetic Electron Scattering

Magnetic Electron Scattering Cross section formula:

$$\frac{d\sigma}{d\Omega} = \sigma_M \left(\frac{1}{2} + \mathrm{tg}^2 \frac{\theta}{2}\right) \sum_{\lambda = \mathrm{odd}} |F_\lambda^M(q)|^2$$

Magnetic interaction is much weaker than Coulomb interaction, but Coulomb scattering cross section decrease sharply with scattering angle.

Magnetic scattering becomes more and more important at large angle: $\theta \rightarrow 180^\circ$

Mott cross section (from a point like particle):

$$\sigma_M = \left(\frac{\alpha \cos \frac{\theta}{2}}{2\, E \sin^2 \frac{\theta}{2}}\right)^2$$

Multipole Form factor (include the structure information):

$$F_\lambda^M(q) = \sqrt{\frac{4\pi}{2J_i + 1}} |\langle f \| T_\lambda^{mag} \| i \rangle|$$

Magnetic electron scattering can measure the current density in nuclei. The current includes two parts:

Current Operator: $$\boldsymbol{J}_{\text{eff}} = \boldsymbol{J}_c + \nabla \times \boldsymbol{\mu}$$

convection (proton) — **spin (proton /neutron)**

After mathematical derivation, the magnetic form factor can be written as the Fourier transform of the transition current density

$$F_L^M(q) = \int_0^\infty J_{L\,L}(r) j_L(qr) r^2 dr$$

Transition current density: $$J_{LL} = J_{LL}^c + J_{LL}^s$$

In the single particle approximation, the current density is mainly determined by the wave function of the unpaired nucleon. The wave function can be obtained by non-relativistic or relativistic nuclear structure theory.

Non-relativistic theory and relativistic theory

Non-relativistic theory:

$$
\begin{aligned}
J^c_{LL}(r) &= \frac{ie}{M}(-1)^{j-1/2} g_l (2L+1)(2l+1)(2j+1) \\
&\times \left(\frac{(2L-1)l(l+1)(2l+1)}{4\pi(L+1)}\right)^{1/2} \\
&\times \left\{ \begin{array}{ccc} l & j & 1/2 \\ j & l & L \end{array} \right\} \left\{ \begin{array}{ccc} L-1 & 1 & L \\ l & l & l \end{array} \right\} \\
&\times \left(\begin{array}{ccc} l & L-1 & l \\ 0 & 0 & 0 \end{array} \right) \frac{R^2_{nl}(r)}{r},
\end{aligned}
$$

The transition current can be calculated with the radial wave function of unpaired nucleon.

$$
\begin{aligned}
J^s_{LL}(r) &= -i\Big[\frac{L^{1/2}}{\hat{L}}\left(\frac{d}{dr} + \frac{L+2}{r}\right)\mu_{sLL+1}(r) \\
&+ \frac{(L+1)^{1/2}}{\hat{L}}\left(\frac{d}{dr} - \frac{L-1}{r}\right)\mu_{sLL-1}(r)\Big],
\end{aligned}
$$

$$
\begin{aligned}
\mu_{sLL'}(r) &= \frac{e}{2M}(-1)^l \mu_i (2l+1)(2j+1) \\
&\times \left(\frac{6(2L+1)(2L'+1)}{4\pi}\right)^{1/2} \\
&\times \left\{\begin{matrix} l & l & L' \\ 1/2 & 1/2 & 1 \\ j & j & L \end{matrix}\right\} \begin{pmatrix} l & L' & l \\ 0 & 0 & 0 \end{pmatrix} \boxed{R_{nl}^2(r),}
\end{aligned}
$$

Magnetic dipole moment (μ):
in the long-wavelength limit the form factor M1 can be reduced to magnetic dipole moment

$$
-iF_1^M(q \to 0) = \frac{q}{2M}\sqrt{\frac{(j+1)(2j+1)}{6\pi j}}\mu
$$

Schmidt formula:

$$
\mu = [g_l(j+\frac{1}{2}) \pm \lambda'] \frac{2j+1 \pm 1}{2(j+1)}, \qquad \text{for} \quad j = l \pm \frac{1}{2},
$$

Relativistic theory:

In the relativistic mean field (RMF) theory, the wave function of nucleon can be described as a two component form

spinor spherical harmonics

$$\psi_{n\kappa m} = \begin{bmatrix} i\,[G(r)\,/\,r]\,\Phi_{\kappa m}(\hat{r}) \\ -[F(r)\,/\,r]\,\Phi_{-\kappa m}(\hat{r}) \end{bmatrix} = \begin{bmatrix} i|n\kappa m\rangle \\ -\overline{|n\kappa m\rangle} \end{bmatrix} = \begin{bmatrix} i|nl\tfrac{1}{2}jm\rangle \\ -|nl'\tfrac{1}{2}jm\rangle \end{bmatrix}$$

G(r): upper (large) component *F(r)*: lower (small) component

$$j = |\kappa| - \frac{1}{2}, \qquad \begin{cases} l = \kappa, & l' = l - 1, \quad (\kappa > 0) \\ l = -(\kappa + 1), & l' = l + 1, \quad (\kappa < 0) \end{cases}$$

The magnetic multipole operator can be written in a block matrix form:

$$\hat{T}^{mag}_{LM}(\boldsymbol{r}) = \begin{bmatrix} iq(\lambda'/2M)\Sigma'^{M}_{L}(\boldsymbol{r}) & Q\Sigma^{M}_{L}(\boldsymbol{r}) \\ Q\Sigma^{M}_{L}(\boldsymbol{r}) & -iq(\lambda'/2M)\Sigma'^{M}_{L}(\boldsymbol{r}) \end{bmatrix}$$

orbital

spin

The transition current can be calculated with the upper and lower wave function *g(r)=G(r)/r, f(r)=F(r)/r*.

$$J^c_{LL}(r) = 2Q(-1)^{l'}(\frac{6}{4\pi})^{1/2}(2L+1)(2j+1)((2l+1)(2l'+1))^{1/2} \times \left\{ \begin{array}{ccc} l' & l & L \\ 1/2 & 1/2 & 1 \\ j & j & L \end{array} \right\} \left(\begin{array}{ccc} l' & L & l \\ 0 & 0 & 0 \end{array} \right) g(r)f(r)$$

$$\begin{aligned} J^s_{LL} = \; & \frac{\lambda'}{2M}\frac{(-1)^l}{q}(\frac{6}{4\pi})^{1/2}(2l+1)(2j+1) \\ & \times \Big[\left\{ \begin{array}{ccc} l & l & L+1 \\ 1/2 & 1/2 & 1 \\ j & j & L \end{array} \right\} \left(\begin{array}{ccc} l & L+1 & l \\ 0 & 0 & 0 \end{array} \right) \\ & \times (L(2L+3))^{1/2}(\frac{d}{dr} + \frac{L+2}{r})g^2(r) \\ & + \left\{ \begin{array}{ccc} l & l & L-1 \\ 1/2 & 1/2 & 1 \\ j & j & L \end{array} \right\} \left(\begin{array}{ccc} l & L-1 & l \\ 0 & 0 & 0 \end{array} \right) \\ & \times ((L+1)(2L-1))^{1/2}(\frac{d}{dr} - \frac{L-1}{r})g^2(r) \Big] \\ & - \frac{\lambda'}{2M}\frac{(-1)^{l'}}{q}(\frac{6}{4\pi})^{1/2}(2l'+1)(2j+1) \\ & \times \Big[\left\{ \begin{array}{ccc} l' & l' & L+1 \\ 1/2 & 1/2 & 1 \\ j & j & L \end{array} \right\} \left(\begin{array}{ccc} l' & L+1 & l' \\ 0 & 0 & 0 \end{array} \right) \\ & \times (L(2L+3))^{1/2}(\frac{d}{dr} + \frac{L+2}{r})f^2(r) \\ & + \left\{ \begin{array}{ccc} l' & l' & L-1 \\ 1/2 & 1/2 & 1 \\ j & j & L \end{array} \right\} \left(\begin{array}{ccc} l' & L-1 & l' \\ 0 & 0 & 0 \end{array} \right) \\ & \times ((L+1)(2L-1))^{1/2}(\frac{d}{dr} - \frac{L-1}{r})f^2(r) \Big]. \end{aligned}$$

The magnetic moment in RMF:

Long-wavelength limit: $-iF_1^M(q \to 0) = \frac{q}{2M}\sqrt{\frac{(j+1)(2j+1)}{6\pi j}}\mu$

$$\mu_{RMF} = [A_{\pm} g_l (j + \frac{1}{2}) \pm \lambda'] \frac{2j+1\pm 1}{2(j+1)}, \qquad \text{for} \quad j = l \pm \frac{1}{2},$$

$$A_{\pm} = \mp \frac{2Mc^2}{\hbar c} \frac{2}{2j+1\pm 1} \int dr r^3 g(r) f(r).$$

Same as the result obtained by other authors using different method

M. Bawin, C. A. Hughes, and G. L. Strobel, Phys. Rev. C 28, 456 (1983)

Two forms of magnetic moment formula

non-relativistic form:

$$\mu = [g_l(j+\frac{1}{2}) \pm \lambda']\frac{2j+1\pm 1}{2(j+1)}$$

relativistic form:

$$\mu_{RMF} = [A_{\pm} g_l(j+\frac{1}{2}) \pm \lambda']\frac{2j+1\pm 1}{2(j+1)}$$

the only difference

$$A_{\pm} = \mp\frac{2Mc^2}{\hbar c}\frac{2}{2j+1\pm 1}\int drr^3 g(r)f(r).$$

The magnetic moments for odd-N nucleus in nonrelativistic and relativistic theories are equal.
The relativistic magnetic moment of odd-Z nuclei depends on the details of model (e.g., different parameter sets of RMF will give different results).

Application of non-relativistic theory: Effects of Velocity-Dependent Force on Magnetic Scattering

In the non-relativistic theory, the spin-orbit interaction is introduced as a surface term:

$$V_{ls}(\boldsymbol{r}) = \sum_{i=1}^{A} V(r_i)\delta(\boldsymbol{r} - \boldsymbol{r}_i)\boldsymbol{\sigma}_i \cdot \boldsymbol{l}_i$$

$$V(r) \quad = \quad V_{so}\frac{1}{r}\frac{df}{dr}$$

Woods-saxon function: $f(r) = [1 + \exp(r - R)/a]^{-1}$

During electron scattering, the transverse Hamiltonian for the interaction between the electron and nucleus:

$$H_T = -\int \boldsymbol{J} \cdot \boldsymbol{A} d^3 r.$$

When a charged particle is moving in the external electromagnetic filed, the gauge invariance requires that the kinetic momentum *p* should be replaced by *p − eA*. Then there will be an additional term to the transverse Hamiltonian:

$$H'_T = -|e| \sum_i \varepsilon_i \int V(r_i)\delta(\boldsymbol{r} - \boldsymbol{r}_i)\boldsymbol{\sigma}_i \cdot \boldsymbol{r}_i \times \boldsymbol{A}(\boldsymbol{r}_i) d^3 r.$$

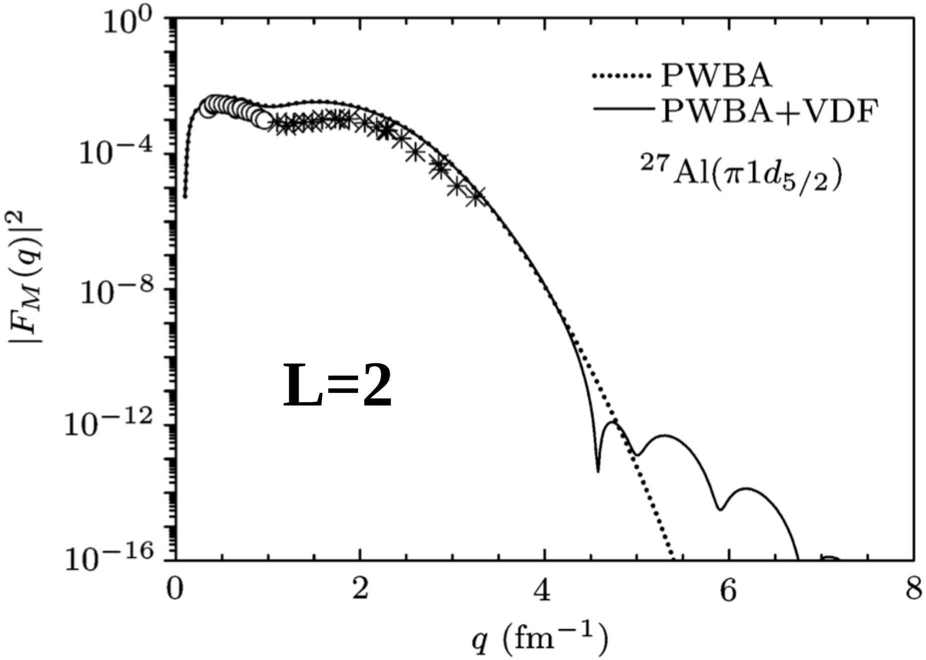
Total current $$\boldsymbol{J}_{\text{eff}} = \boldsymbol{J}_c + \nabla \times \boldsymbol{\mu} + \boldsymbol{J}_v$$

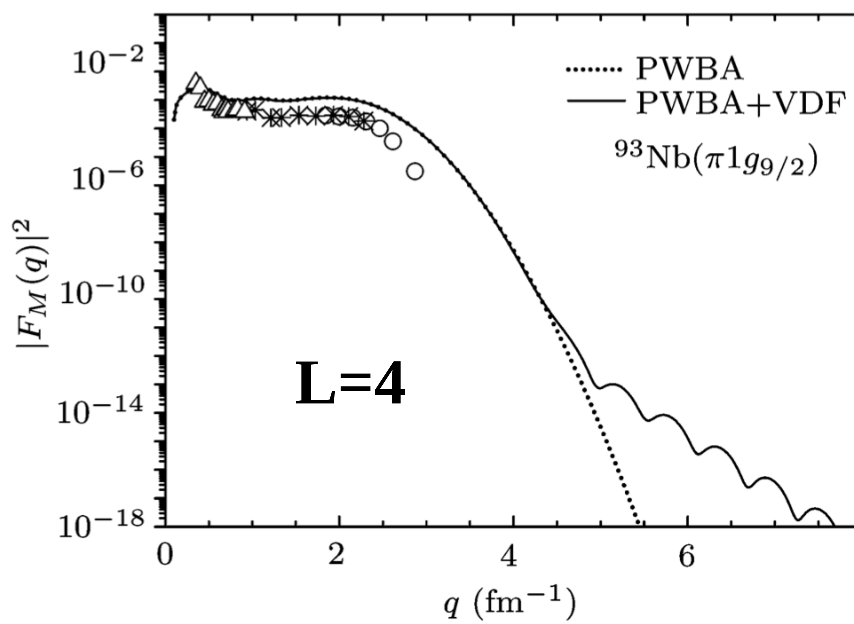
additional current $$\boldsymbol{J}_v = \sum_{i=1}^{A} \varepsilon_i V(r_i)\delta(\boldsymbol{r} - \boldsymbol{r}_i)\boldsymbol{\sigma}_i \times \boldsymbol{r}_i$$

The additional magnetic moment:

$$\mu' = \pm M \frac{(2j+1)}{2j+2} \langle r^2 V(r) \rangle, \quad \text{for} \quad j = l \pm \frac{1}{2}.$$

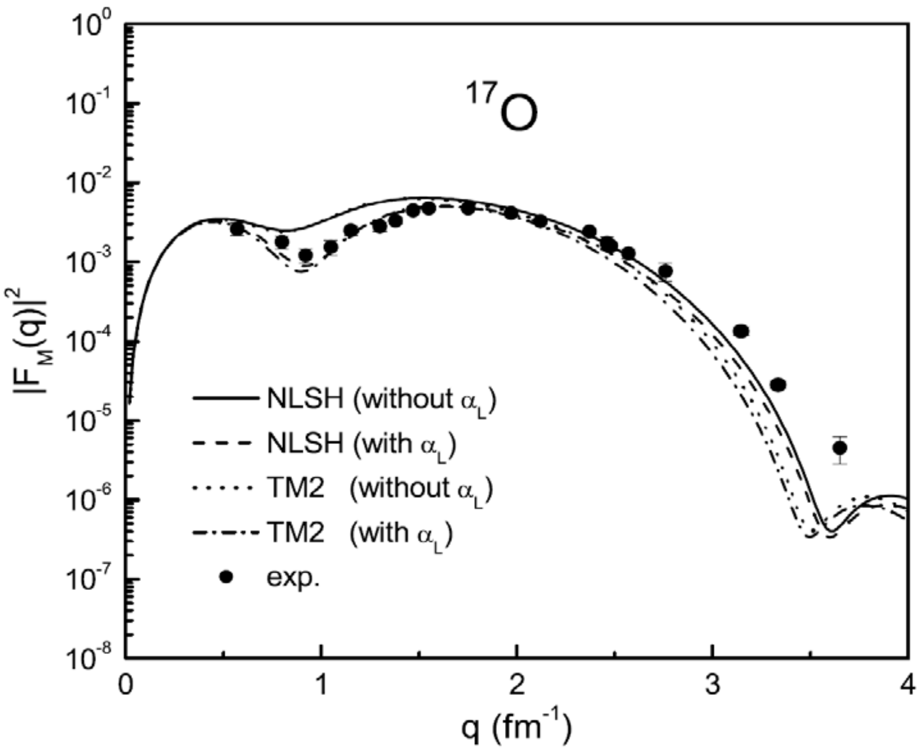
It is same as the result given by Jensen and Mayer with another method, Phys. Rev. 85, 1040 (1952)





- **In the smaller q region the contribution of VDF is much smaller than the normal form factor. But in the large q region the contribution of VDF appear.**
- **The velocity-dependent parts oscillate frequently.**
- **However the cross section at large q region is very small, it is a challenge.**

Application of relativistic theory: Magnetic electron scattering from exotic nuclei

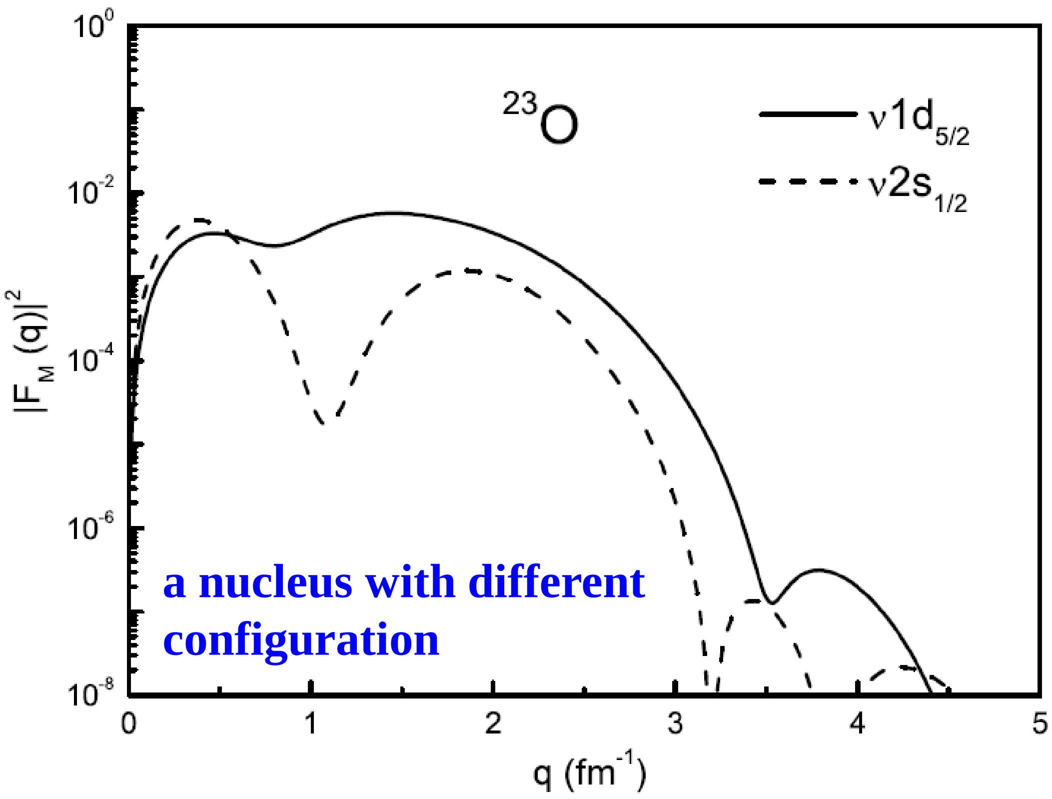


To test the computer code, some nuclei are calculated and compared with experimental data. It is found that the results agree with data though there are some discrepancies. These discrepancies can also be found when non-relativistic theory is used.

The discrepancies between the experimental and theoretical results were often treated by introducing the spectroscopic factors (quenching factor) α_L

$$F_M^2(q) = \sum_{L=1}^{\mathrm{odd}} \alpha_L^2 F_{ML}^2(q)$$

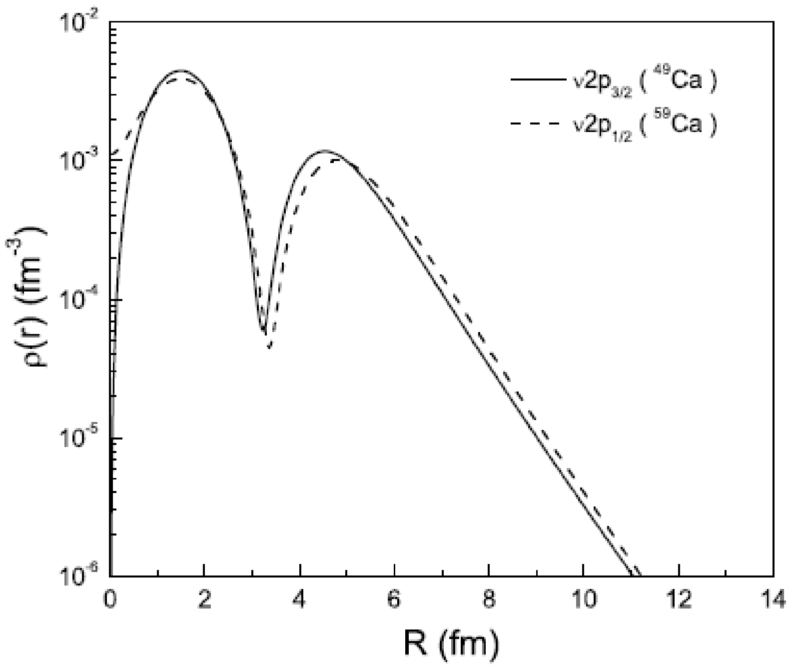
Magnetic electron scattering from exotic nuclei

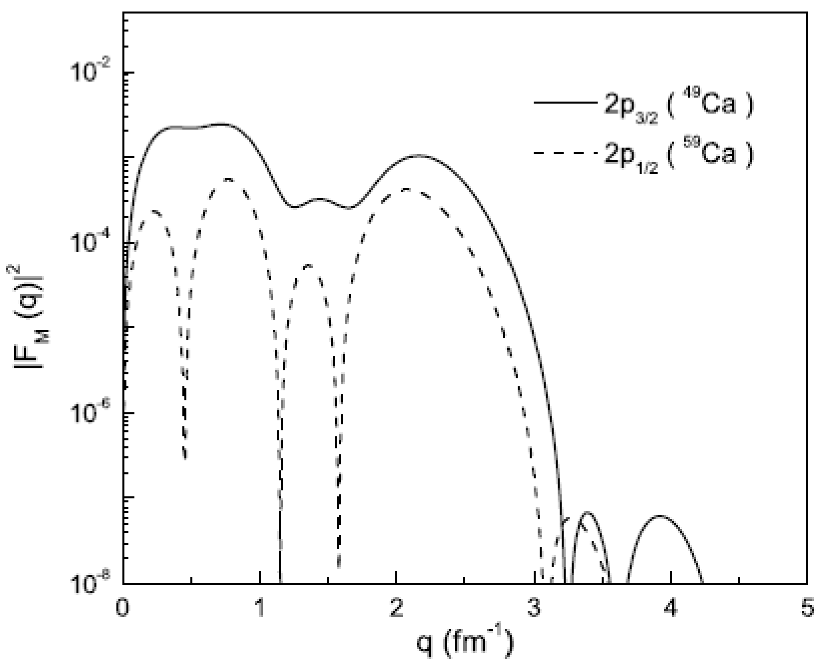


The node number, the orbital angular momentum, and the total angular momentum are all different.

The configuration of valence nucleon can be determined by magnetic electron scattering.

Magnetic form factor of ^{49}Ca and ^{59}Ca





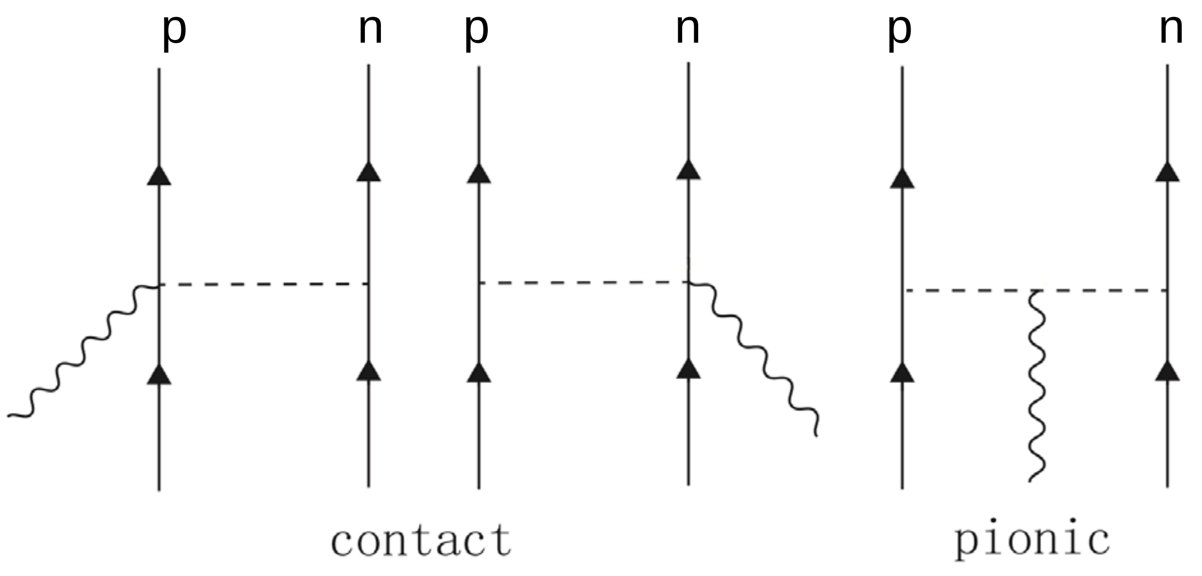
The unpaired neutrons occupy $2p_{3/2}$ and $2p_{1/2}$, respectively. The same node number, the same orbital angular momentum, only the total angular momentum are different.
The magnetic form factor is sensitive to the valence orbit with any different quantum number .

Meson exchange current in RMF

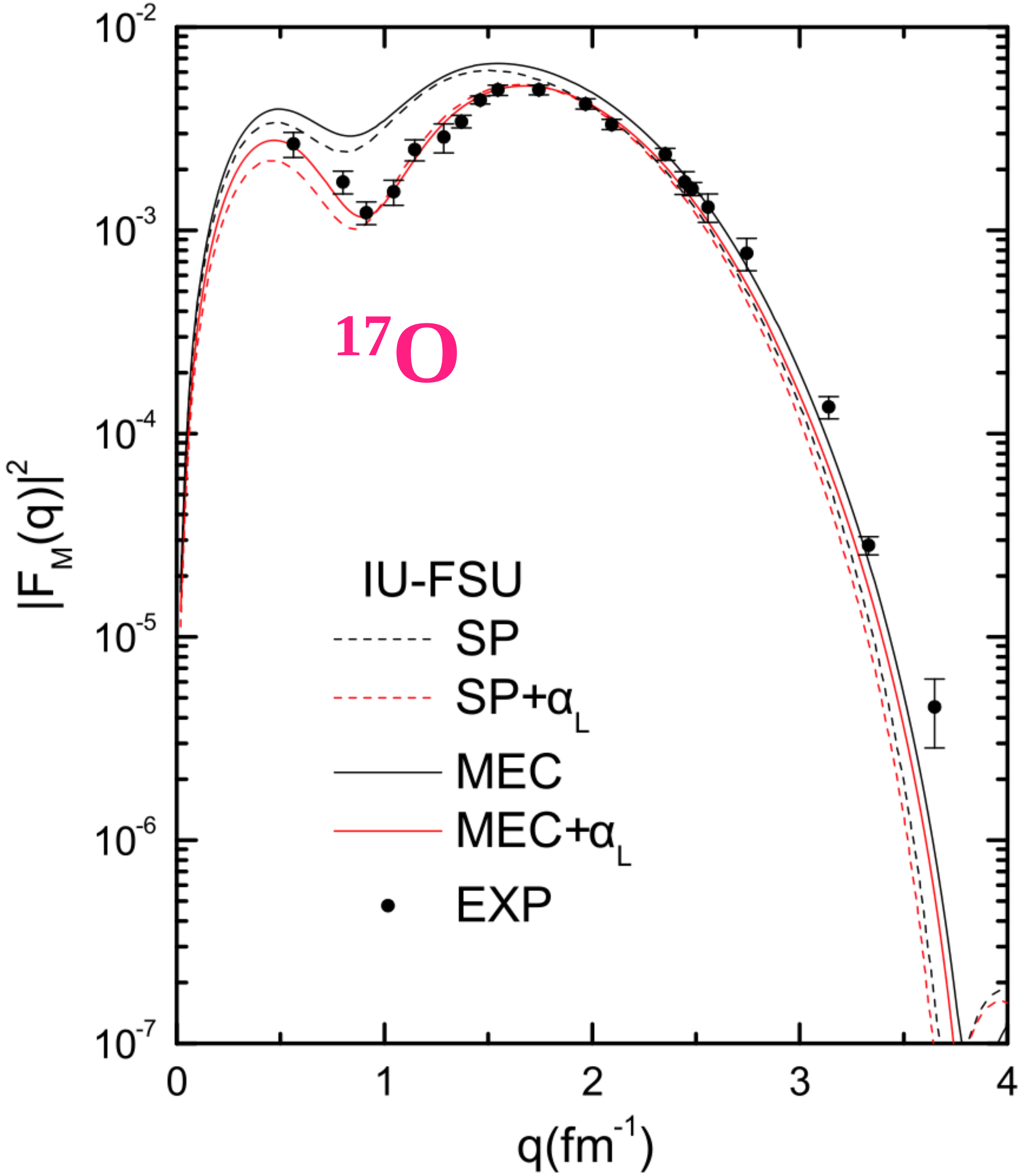
Only single pion exchange current is considered. We use the pseudo-vector coupling between pion and nucleon

$$\mathcal{L}_{N\pi} = \frac{g_\pi}{2M}\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi\cdot\partial^\mu\vec{\pi}$$

The one-pion exchange currents contain two terms: the 'contact' terms and the 'pionic' term. The current originates from the electromagnetic wave emitted during pion exchange.



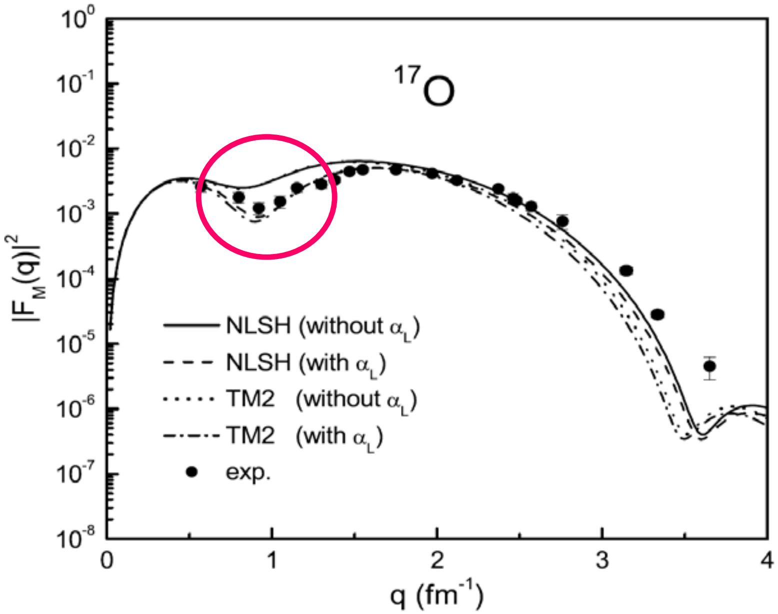
Cun Zhang *et al* 2016 *J. Phys. G: Nucl. Part. Phys.* 43 045103



The MEC can improve the form factor at large q region, but near the first minimum the difference is more larger. If we introduce the quenching factor the experimental data can be reproduced better in the whole q region.

Quenching effect at low q-region

Experimental form factor in the low-q region is lower than single particle results. This quenching effect is regarded as the result of many-body effects.



At present, the quenching factors are introduced, which can be obtained by the least-square fit.

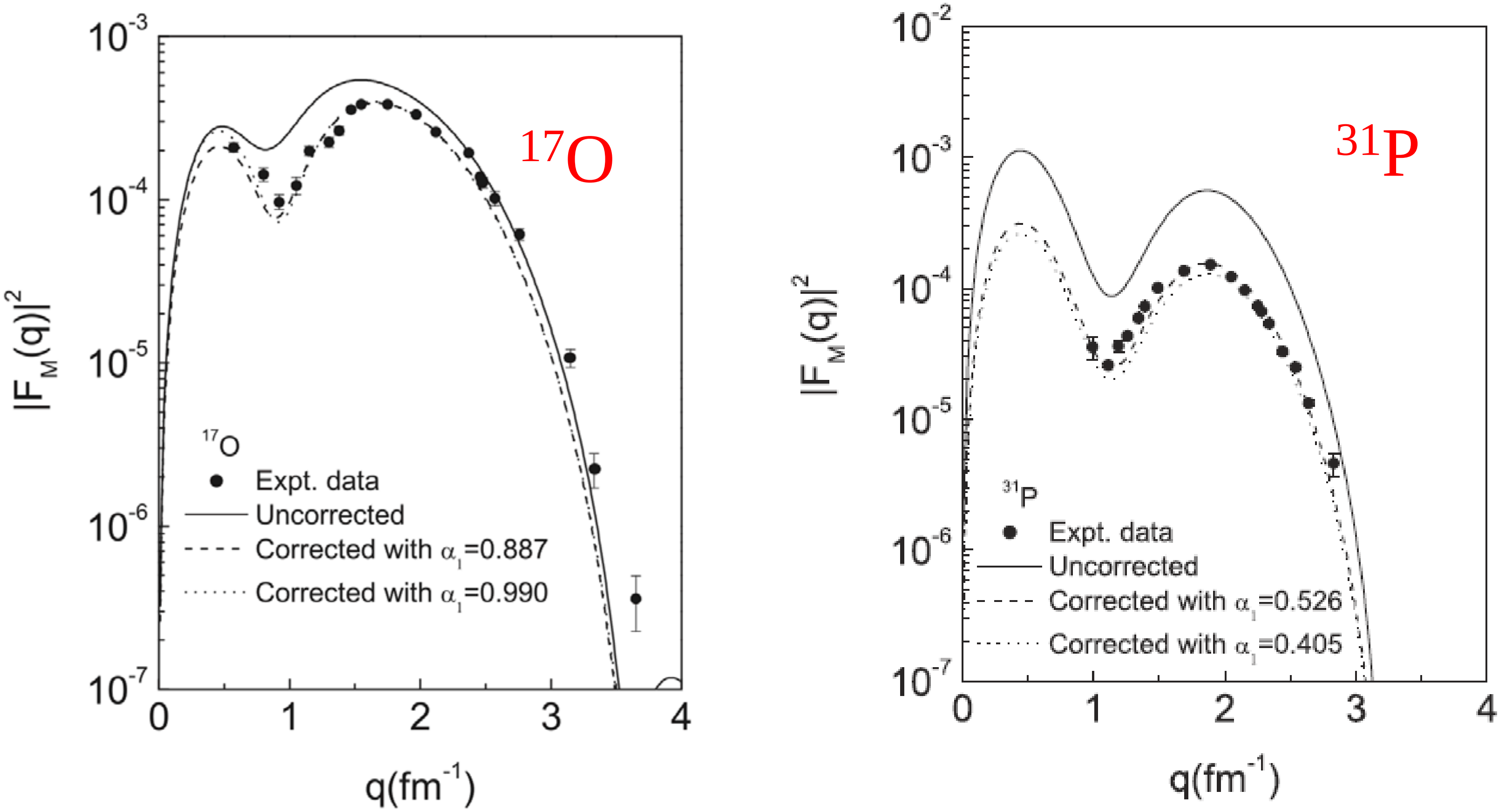
$$F_M^2(q) \;=\; \sum_{L=1}^{\text{odd}} \alpha_L^2 F_{ML}^2(q)$$

On the other hand, magnetic form factor in the low q region is contributed mainly by M1 ($|F_1(q)|^2$) . And the magnetic moment can be derived from the magnetic form factor at the long-wave limit:

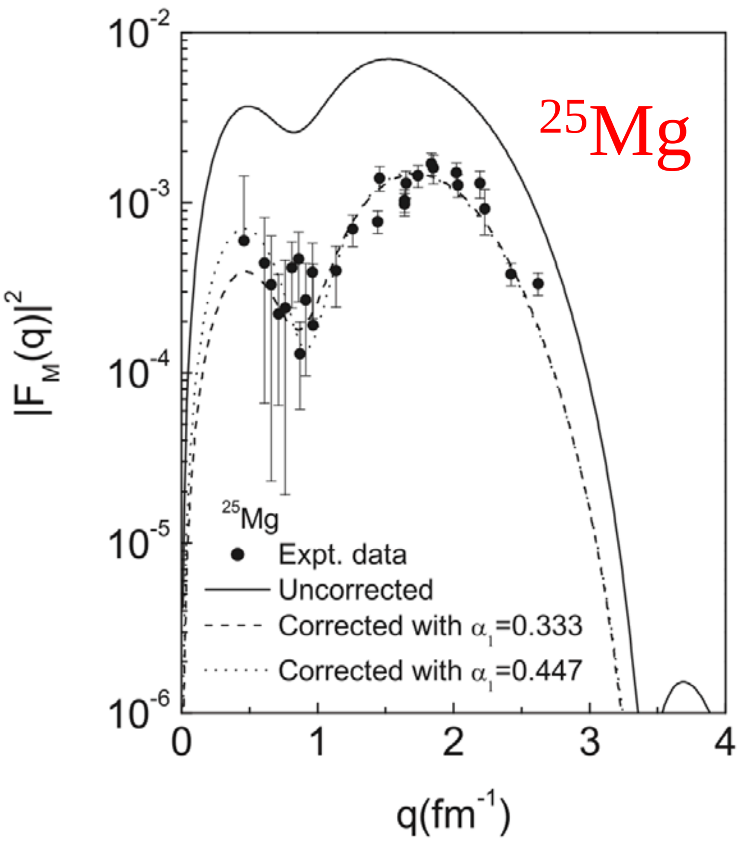
$$-iF_1^M(q \to 0) = \frac{q}{2M}\sqrt{\frac{(j+1)(2j+1)}{6\pi j}}\mu$$

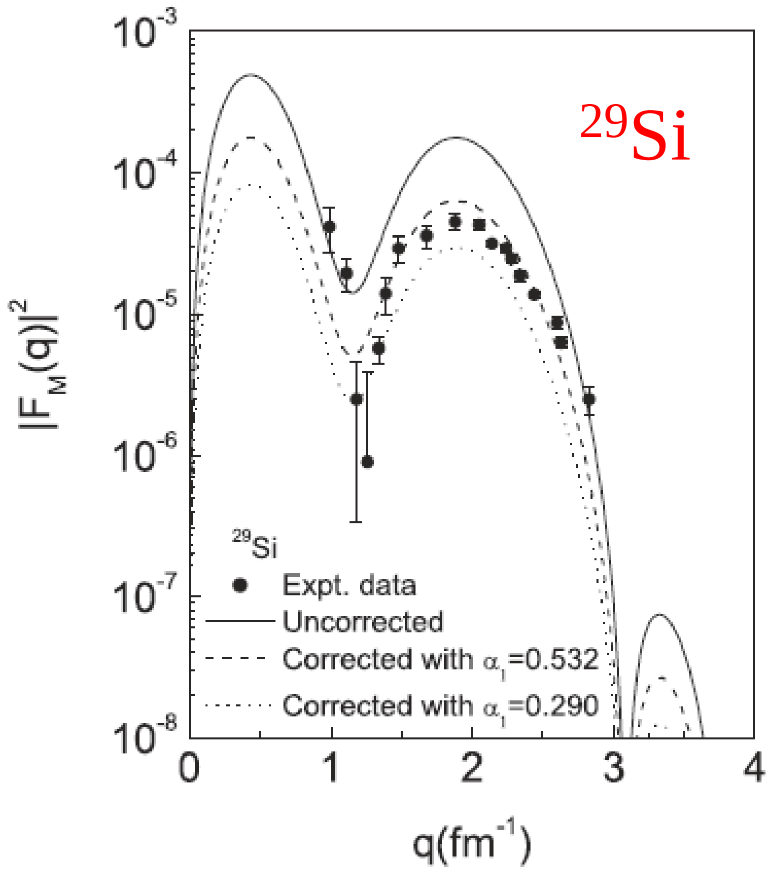
Therefore, the quenching factor of M1 can also be estimated from the ratio of experimental to single particle magnetic moment:

$$\alpha_1 = \frac{\mu_{\exp}}{\mu_{sp}}$$



For some nuclei with good data (cover large q region and small error bars), parameter α_1 obtained by fitting is close to the ratio of experimental to single particle magnetic moment.

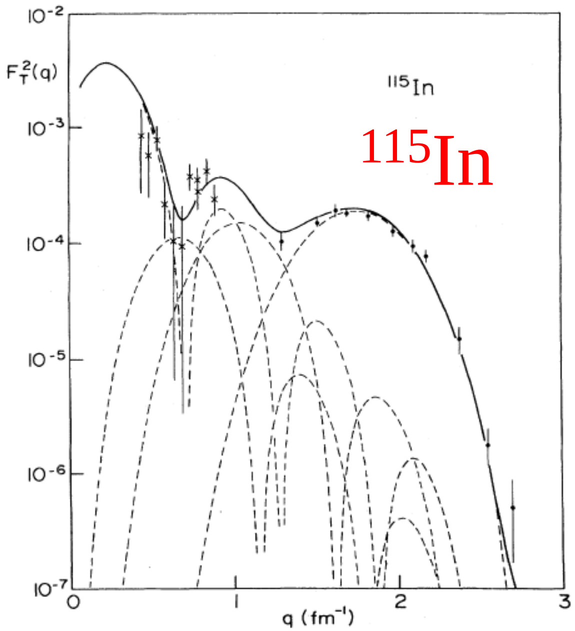


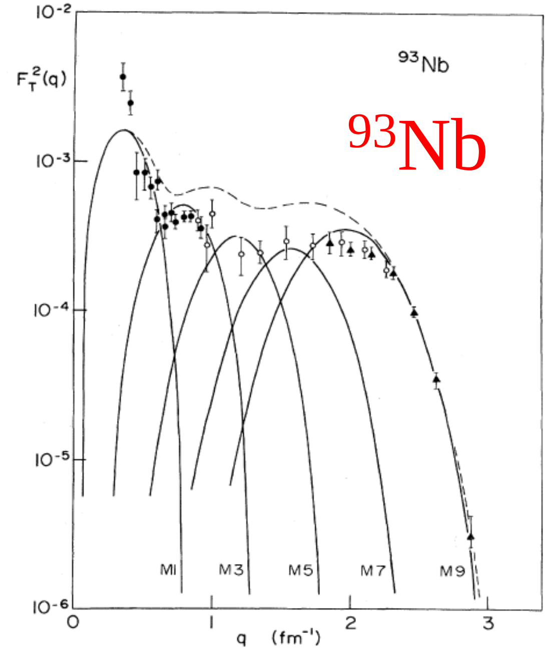


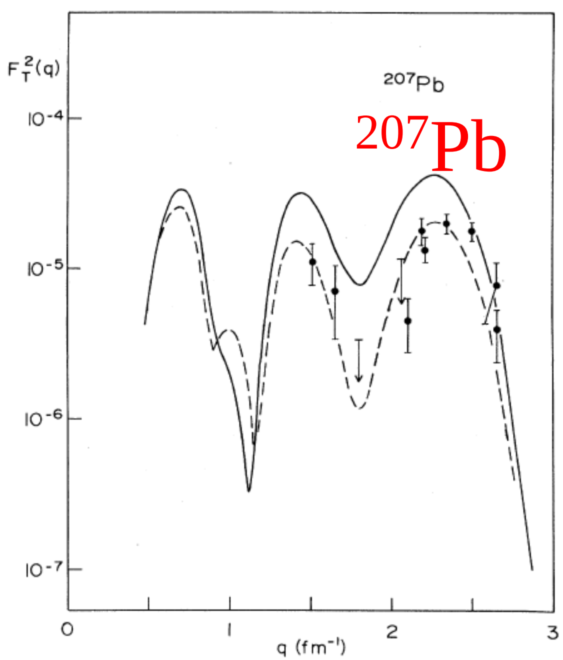
For some nuclei , the experimental data are not very good (large error bars), α_1 deviate large but it is difficult to judge which one is better.

The status of magnetic electron scattering

- **From 1960s-1990s, many nuclei including odd-A and odd-odd nuclei from ^{2}H to ^{209}Bi have been measured. As I know ^{41}Ca ($T_{1/2}$ =99 ky) is the last experiment.**
- **But for very few nuclei the data are good (cover large q region and with small error bars).**
- **Furthermore, for many nuclei it is difficult to explain the data even the corrections (e.g., meson exchange current and configuration mixing) are considered.**
- **In the 21st century, new electron scattering facilities may bring new opportunities. More wide and accurate data, modern nuclear structure models may solve many remained problems.**


115In
115In
F_T^2(q)
10-2
10-3
10-4
10-5
10-6
10-7
0
1
2
3
q (fm-1)


93Nb
93Nb
F_T^2(q)
10-2
10-3
10-4
10-5
10-6
M1
M3
M5
M7
M9
0
1
2
3
q (fm-1)


207Pb
207Pb
F_T^2(q)
10-4
10-5
10-6
10-7
0
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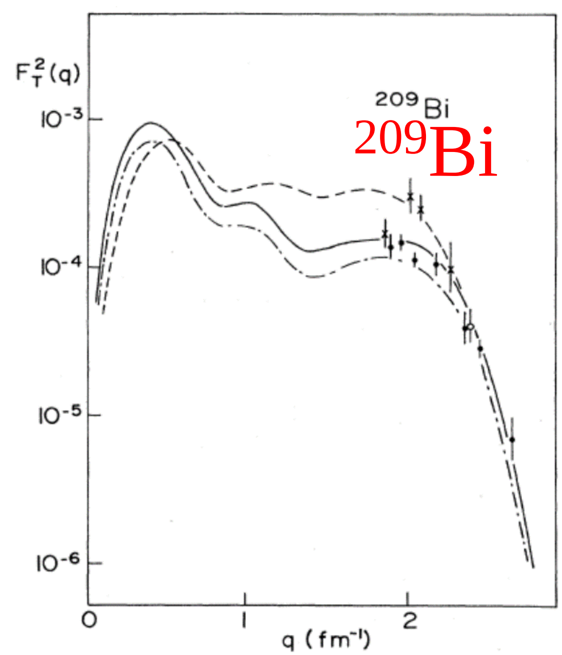
q (fm-1)


209Bi
209Bi
F_T^2(q)
10-3
10-4
10-5
10-6
0
1
2
q (fm-1)

Summary and Outlook

- We investigate the elastic magnetic electron scattering in both the non-relativistic and relativistic framework
- The contribution of one-pion exchange current has been considered in RMF
- There are still some problems, e.g., the quenching effects in RMF or other nuclear structure theory
- It is interesting to consider the magnetic electron scattering from some stable nuclei in new e-A colliders to get more accurate results.

Thank You！

$$\hat{\rho}(\boldsymbol{r}) = |e| \sum_{i=1}^{A} e_i \delta(\boldsymbol{r} - \boldsymbol{r}_i),$$

$$\hat{\boldsymbol{j}}_c(\boldsymbol{r}) = \frac{|e|}{2M} \sum_{i=1}^{A} e_i \{\delta(\boldsymbol{r} - \boldsymbol{r}_i)\boldsymbol{p}_i + \boldsymbol{p}_i \delta(\boldsymbol{r} - \boldsymbol{r}_i)\},$$

$$\hat{\mu}(\boldsymbol{r}) = \frac{|e|}{2M} \sum_{i=1}^{A} \mu_i \delta(\boldsymbol{r} - \boldsymbol{r}_i)\sigma_i,$$

$$e_i = \frac{1}{2}(1 + \tau_{iz})e_p + \frac{1}{2}(1 - \tau_{iz})e_n,$$

$$\mu_i = \frac{1}{2}(1 + \tau_{iz})\mu_p + \frac{1}{2}(1 - \tau_{iz})\mu_n,$$

$$e_p = 1, \quad e_n = 0, \quad \mu_p = 2.793, \quad \mu_n = -1.913.$$

Publications on Magnetic Electron Scattering

- **Dong Tie-Kuang, Ren Zhong-Zhou, Guo Yan-Qing, *Elastic magnetic form factors of exotic nuclei*, Physical Review C, 2007, 76:054602。**
- **Dong Tie-Kuang, Ren Zhong-Zhou, *Effects of Velocity-Dependent Force on the Magnetic Form Factors of Odd-Z Nuclei*, Chinese Physics Letters, 2008, 25(3):884-887。**
- **Wang Zai-Jun,Ren Zhong-Zhou, Dong Tie-Kuang, Xu *Chang,Spins and parities of the odd-A P isotopes within a relativistic mean-field model and elastic magnetic electron-scattering theory,* Physical Review C,2014,90(2):024307。**
- **Wang Zai-Jun,Ren Zhong-Zhou, Dong Tie-Kuang, *Probe the 2s(1/2) and 1d(3/2) state level inversion with electron-nucleus scattering,* Chinese Physics C,2014,38(2):024102。**
- **Wang Zai-Jun,Ren Zhong-Zhou,Dong Tie-Kuang,Guo Xiao-Yong, *Quenching of magnetic form factors of s-d shell nuclei O-17, Mg-25, Al-27, Si-29, and P-31 within the relativistic mean field model,* Physical Review C,2015,92:014309。**
- **Zhang Cun, Liu Jiang and Ren Zhongzhou, *One-pion exchange current effects on magnetic form factor in the relativistic formalism,* Journal of Physics G: Nuclear and Particle Physics 43 045103 (2016)。**