Dipole response in exotic nuclei

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Contents

- Time-dependent approaches to nuclear response
- Dipole response in neutron-halo systems
- Dipole response in medium-heavy exotic nuclei

- I do not talk about correlations with symmetry energy, slope parameter, etc.

Time-dependent approach

- Time-dependent equation

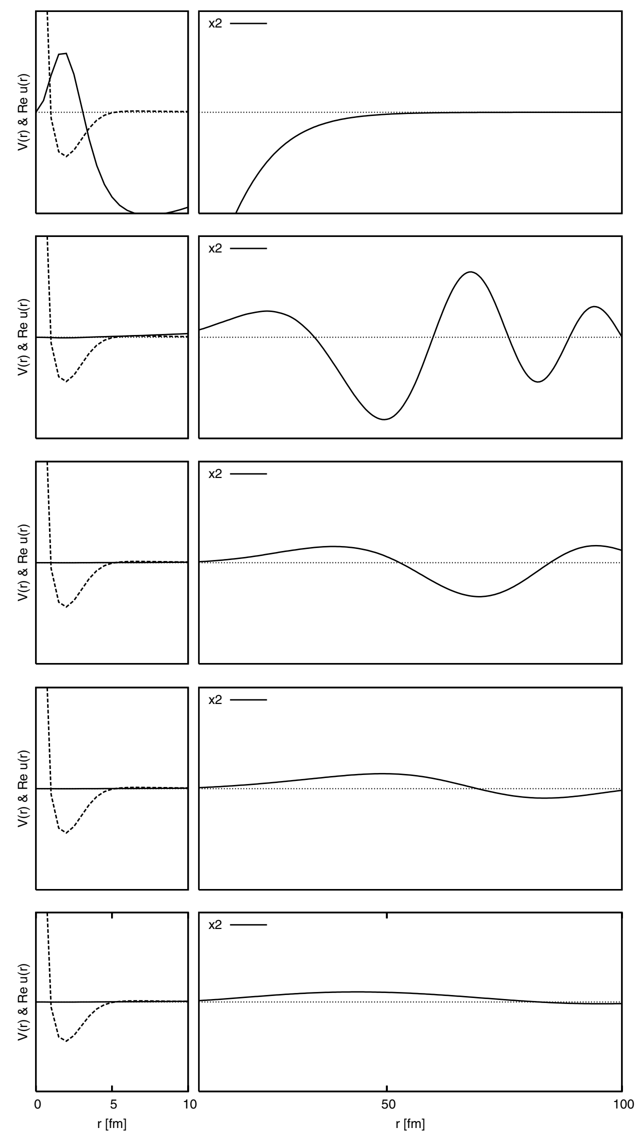
$$i\hbar \frac{\partial}{\partial t}\psi(\vec{r},t) = H\psi(\vec{r},t)$$

- Use of a wave-packet wave function
  - Superposition of many energy eigenstates
  - Energy resolution is limited by time duration
- Especially useful for investigation of bulk properties and continuum (scattering) states.

Dipole response of weakly-bound nuclei at low excitation

^{6}He, ^{11}Li

Nakatsukasa, Yabana, Ito,
Eur. Phys. J. Special Topics 156, 249–256 (2008)



We can rewrite E1 strength function as

$$\frac{dB(E1,E)}{dE} = \sum_{lm'} \int dE' \delta(E-E') \left| \left\langle \phi_{E',l=1,m'} \left| M_{1m} \right| \phi_0 \right\rangle \right|^2$$

$$= \left\langle \phi_0 \left| M_{1m}^{+} \delta\left(E-H\right) M_{1m} \right| \phi_0 \right\rangle$$

$$= -\frac{1}{\pi} \mathrm{Im} \left\langle \phi_0 \left| M_{1m}^{+} \frac{1}{E+i\varepsilon-H} M_{1m} \right| \phi_0 \right\rangle$$

$$\frac{dB(E1,E)}{dE} = -\frac{1}{\pi} \mathrm{Im} \sum_m \frac{1}{i\hbar} \int_0^\infty dt\, e^{iEt/\hbar} \underbrace{\left\langle \phi_0 \right| M_{1m}^{+}}_{\langle \psi(0) |} \underbrace{e^{-iHt/\hbar} M_{1m} \left| \phi_0 \right\rangle}_{| \psi(t) \rangle}$$

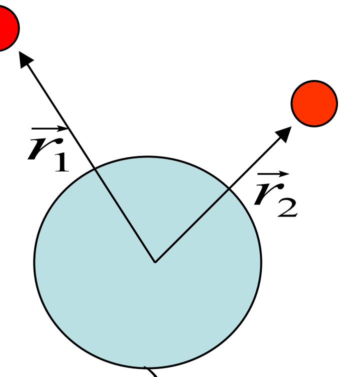
Equations to numerically solve

Continuum effect

$$|\psi(0)\rangle = M_{1m} |\phi_0\rangle$$

$$i \frac{\partial}{\partial t} |\psi(t)\rangle = \{H - iW_{abs}\} |\phi_0\rangle$$

Dipole strength of borromean nuclei: real-time calculation



3-body model

$$i\hbar \frac{\partial}{\partial t}\psi\left(\vec{r}_1,\vec{r}_2,t\right)=\left(-\frac{\hbar^2}{2m}\nabla_{r_1}^{\ 2}-\frac{\hbar^2}{2m}\nabla_{r_2}^{\ 2}+V_{nC}\left(r_1\right)+V_{nC}\left(r_2\right)+V_{nn}\left(\left|\vec{r}_1-\vec{r}_2\right|\right)\right)\psi\left(\vec{r}_1,\vec{r}_2,t\right)$$

Simplified treatment, ignoring recoil term

Time representation of response function

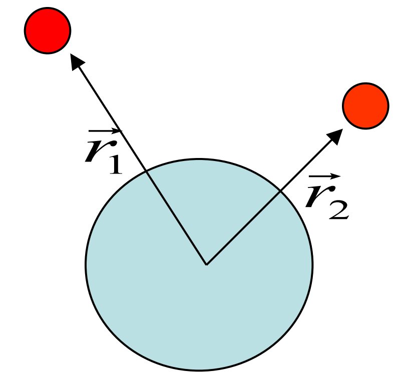
$$\frac{dB(E1)}{dE}=\frac{1}{\pi\hbar}\mathrm{Re}\int_0^\infty dt\, e^{iEt/\hbar}\sum_m \int d\vec{r}_1 d\vec{r}_2 \psi_{1m}^*\left(\vec{r}_1,\vec{r}_2,0\right)\psi_{1m}\left(\vec{r}_1,\vec{r}_2,t\right)$$

Initial wave function: (Dipole operator) x (3-body ground state)

$$\psi\left(\vec{r}_1,\vec{r}_2,t=0\right)=\left(z_1+z_2\right)\phi_0\left(\vec{r}_1,\vec{r}_2\right)$$

$$i\hbar\frac{\partial}{\partial t}\psi\left(\vec{r}_1,\vec{r}_2,t\right)=\left\{H-iW_{abs}(r_1)-iW_{abs}(r_2)\right\}\psi\left(\vec{r}_1,\vec{r}_2,t\right)$$

Detail of assumed Hamiltonian and numerical method



$$i\hbar\frac{\partial}{\partial t}\psi\left(\vec{r}_1,\vec{r}_2,t\right)=\left(-\frac{\hbar^2}{2m}\nabla_{r_1}{}^2-\frac{\hbar^2}{2m}\nabla_{r_2}{}^2+V_{nC}(r_1)+V_{nC}(r_2)+V_{nn}\left(\left|\vec{r}_1-\vec{r}_2\right|\right)\right)\psi\left(\vec{r}_1,\vec{r}_2,t\right)$$

Core-neutron: Central + Spin-Orbit (Woods-Saxon shape)
Neutron-neutron: Minnesota force
Orthogonality to occupied orbitals ($s_{1/2}$ for ^6He, $s_{1/2}$ and $p_{3/2}$ for ^{11}Li)
Recoil terms are ignored
Binding energy are set to 0.975MeV for ^6He, 0.295MeV for ^{11}Li
by adjusting depth of the central potential

$$\psi^{JM}\left(\vec{r}_1\sigma_1,\vec{r}_2\sigma_2,t\right)=\sum_{l_1l_2LS}\frac{u_{l_1l_2LS}{}^{JM}(r_1,r_2,t)}{r_1r_2}\left[\left[Y_{l_1}(\hat{r}_1)Y_{l_2}(\hat{r}_2)\right]_L\left[\chi(\sigma_1)\chi(\sigma_2)\right]_S\right]_{JM}$$

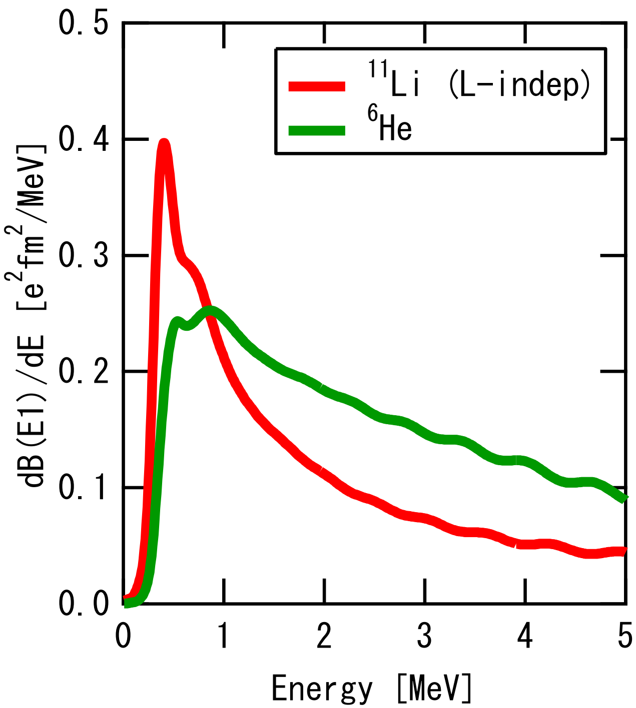
$l_1, l_2 \le 10$ $\quad 0 < r_1, r_2 < 90$ fm $\quad W_{abs}(r)$ for $r > 30$ fm $\quad \Delta r = 0.6$ fm

Lagrange mesh (Discrete Variables Representation) for differentiation
Taylor expansion for time-evolution

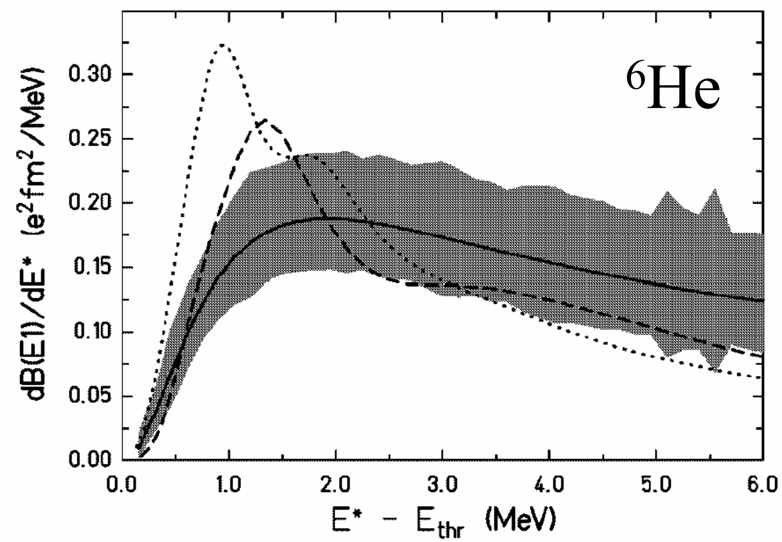
Dipole Strength of ^{6}He, ^{11}Li

Dipole strength from real-time calculation

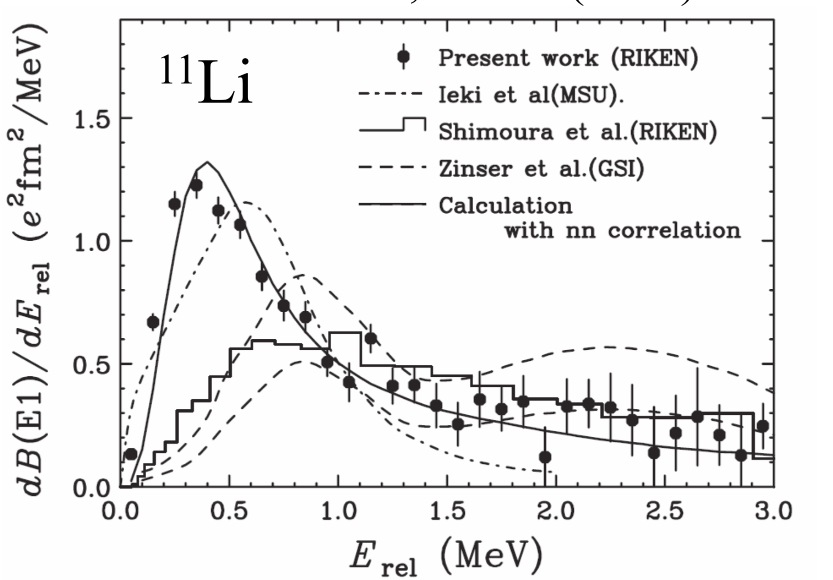
$$\frac{dB(E1)}{dE} = \left(\frac{Z}{A}\right)^2 \frac{3}{\pi} \int dt\, e^{iEt} \langle \Phi_0 | (z_1 + z_2) | \Psi(t) \rangle$$



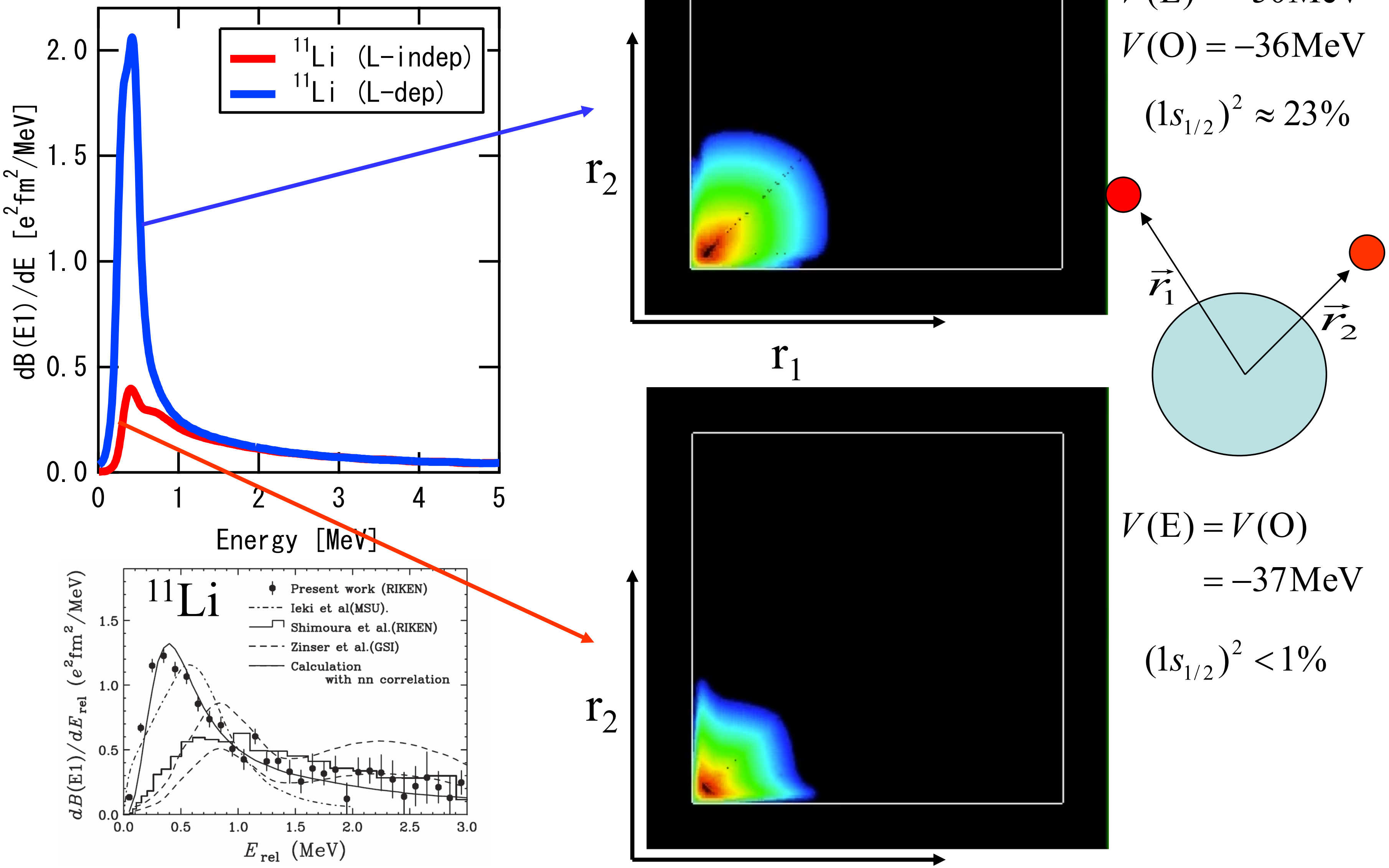
T. Aumann et.al, PRC59(1999)1252



T. Nakamura et.al, PRL96(2006)252502



Low strength is sensitive to the s-wave component $(1s_{1/2})^2$



$V(E) = -50\text{MeV}$

$V(O) = -36\text{MeV}$

$(1s_{1/2})^2 \approx 23\%$

$V(E) = V(O)$

$= -37\text{MeV}$

$(1s_{1/2})^2 < 1\%$

T. Nakamura et.al, PRL96(2006)252502

Energy density functionals

- Universal description of ground and excited states
- Reasonable computational time
- Time-dependent density-functional theory to calculate nuclear response

Time-dependent density functional theory (TDDFT) for nuclei

- Time-odd densities (current density, spin density, etc.)

$$E\left[\rho_q(t),\underbrace{\tau_q(t)}_{\text{kinetic}},\underbrace{\overleftrightarrow{J}_q(t)}_{\text{spin-current}},\underbrace{\vec{j}_q(t)}_{\text{current}},\underbrace{\vec{s}_q(t)}_{\text{spin}},\underbrace{\vec{T}_q(t)}_{\text{spin-kinetic}};\underbrace{\kappa_q(t)}_{\text{pair density}}\right]$$

- Time-dependent Kohn-Sham-Bogoliubov eq.

$$i\frac{\partial}{\partial t}\begin{pmatrix} U_\mu(t) \\ V_\mu(t) \end{pmatrix} = \begin{pmatrix} h(t)-\lambda & \Delta(t) \\ -\Delta^*(t) & -\left(h(t)-\lambda\right)^* \end{pmatrix}\begin{pmatrix} U_\mu(t) \\ V_\mu(t) \end{pmatrix}$$

Canonical-basis TDHFB

$$i\frac{\partial}{\partial t}|k(t)\rangle = \left(h(t)-\eta_k(t)\right)|k(t)\rangle,\quad i\frac{\partial}{\partial t}|\bar{k}(t)\rangle = \left(h(t)-\eta_{\bar{k}}(t)\right)|\bar{k}(t)\rangle$$

$$i\frac{\partial}{\partial t}\rho_k(t) = \kappa_k(t)\Delta_k^*(t) - \kappa_k^*(t)\Delta_k(t)$$

$$i\frac{\partial}{\partial t}\kappa_k(t) = \left(\eta_k(t)+\eta_{\bar{k}}(t)\right)\kappa_k(t) + \Delta_k(t)\left(2\rho_k(t)-1\right)$$

$$\rho_k(t) \equiv |v_k(t)|^2$$

$$K_k(t) \equiv u_k(t)v_k(t)$$

$\eta_k(t), \eta_{\bar{k}}(t)$: arbitrary real function of t

- Conserve the particle number and the total energy $\quad \frac{d}{dt}\langle N\rangle = \frac{d}{dt}E_{\mathrm{tot}} = 0$
- Conserve the orthonormality of canonical orbitals $\quad \frac{d}{dt}\langle k(t)|k'(t)\rangle = 0$
- Reduce to TDHF for Δ=0
- Its static limit coincides with the HF+BCS

In the small-amplitude limit,

- Nambu-Goldstone modes appear as the zero-energy modes.
- The pairing vibrations in the normal phase coincide with the pp- and hh-RPA

Real-time calculation of response functions

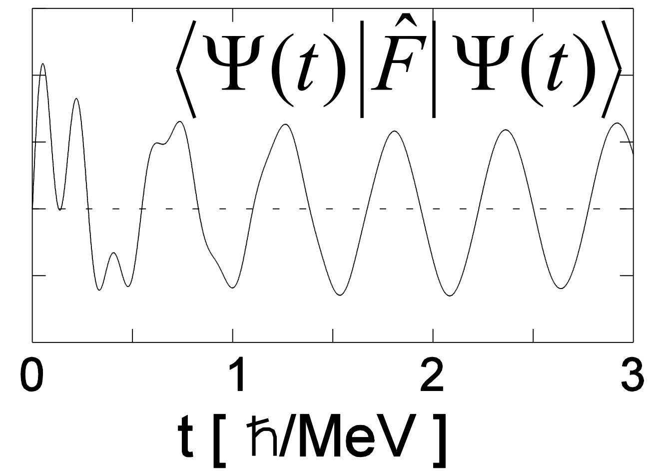
1. Weak instantaneous external perturbation

$$V_{\mathrm{ext}}(t) = \hat{F}\delta(t)$$

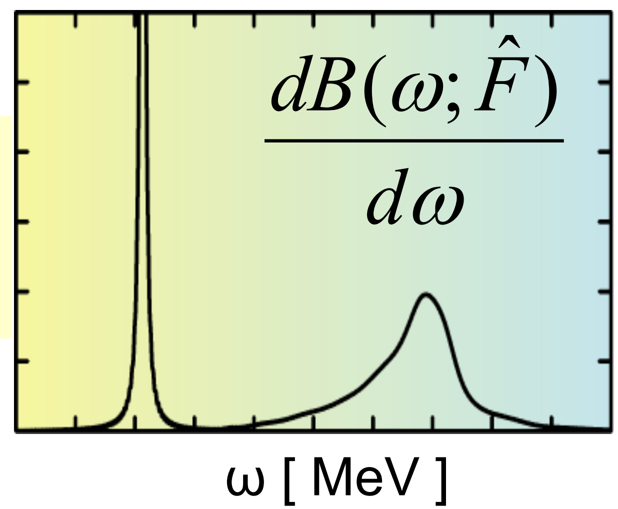
2. Calculate time evolution of

$$\left\langle \Psi(t) \middle| \hat{F} \middle| \Psi(t) \right\rangle$$

3. Fourier transform to energy domain



$$\frac{dB(\omega;\hat{F})}{d\omega} = -\frac{1}{\pi}\,\mathrm{Im}\int \left\langle \Psi(t) \middle| \hat{F} \middle| \Psi(t) \right\rangle e^{i\omega t}\,dt$$

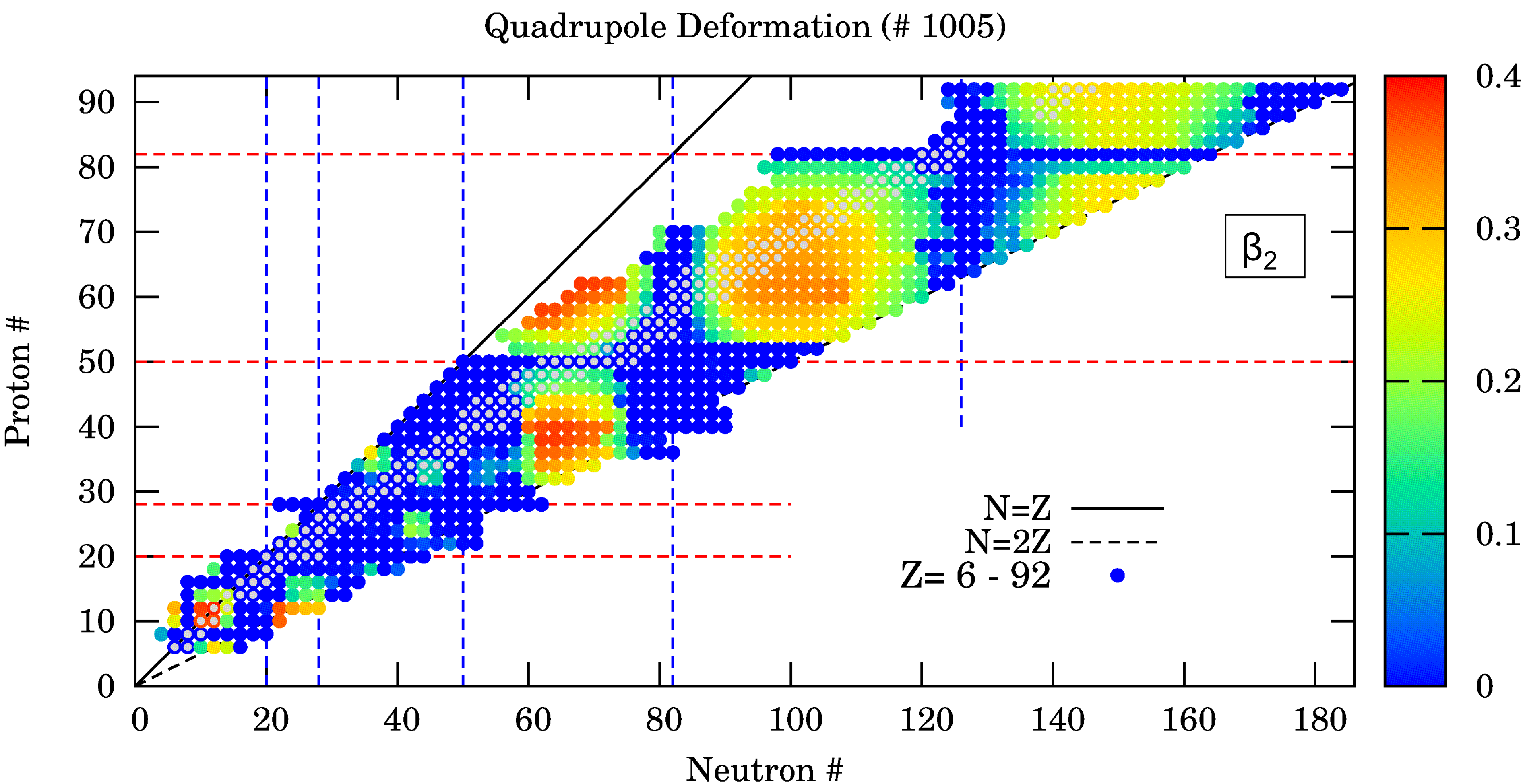


Numerical calculation with 3D coordinate mesh space with SkM* EDF

Ground-state deformation

Ebata, Nakatsukasa, Phys. Scr. 92, 064005 (2017)

- Systematic calculation with SkM*



Ground-state deformation

Ebata, Nakatsukasa, Phys. Scr. 92, 064005 (2017)

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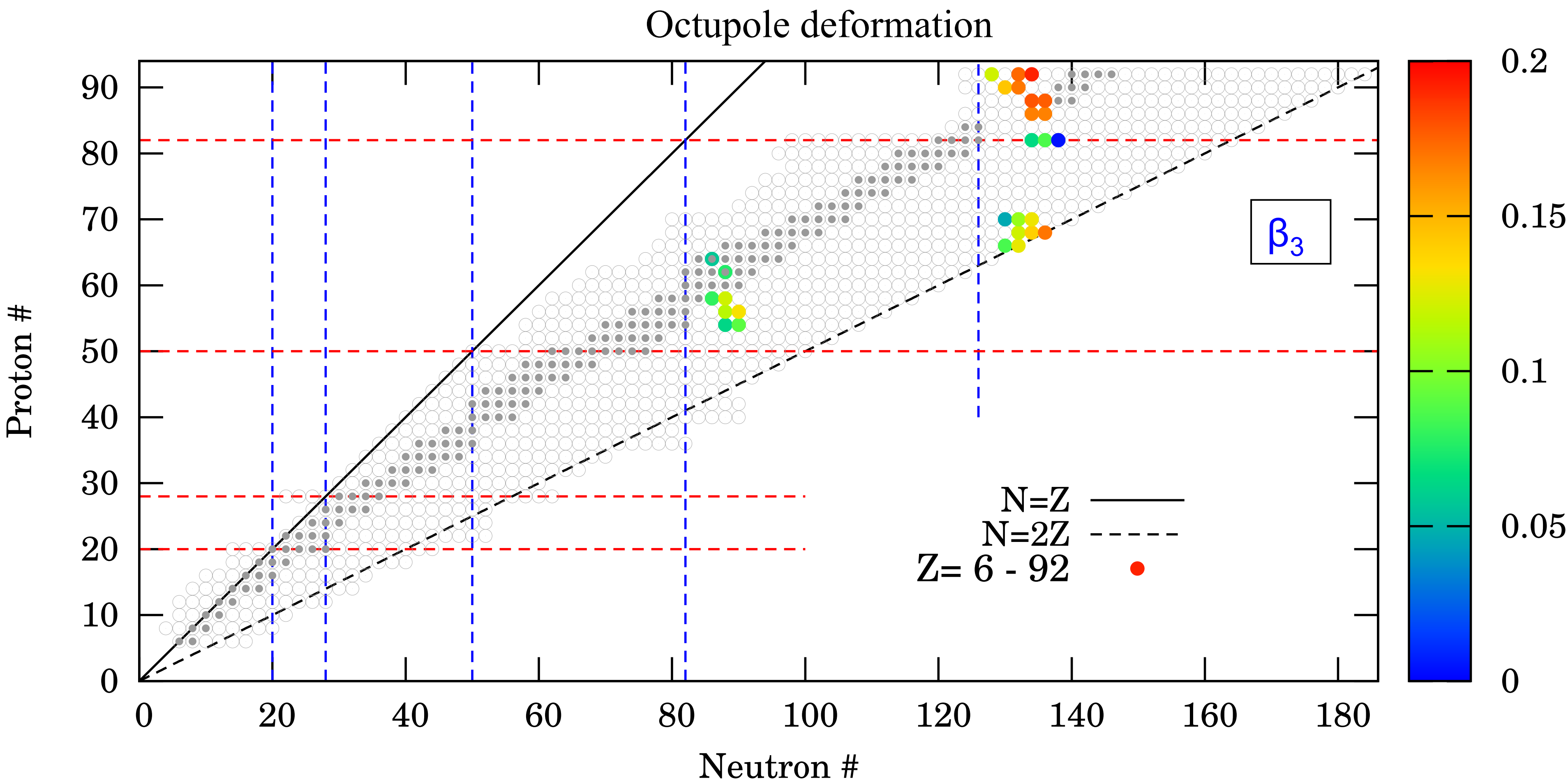
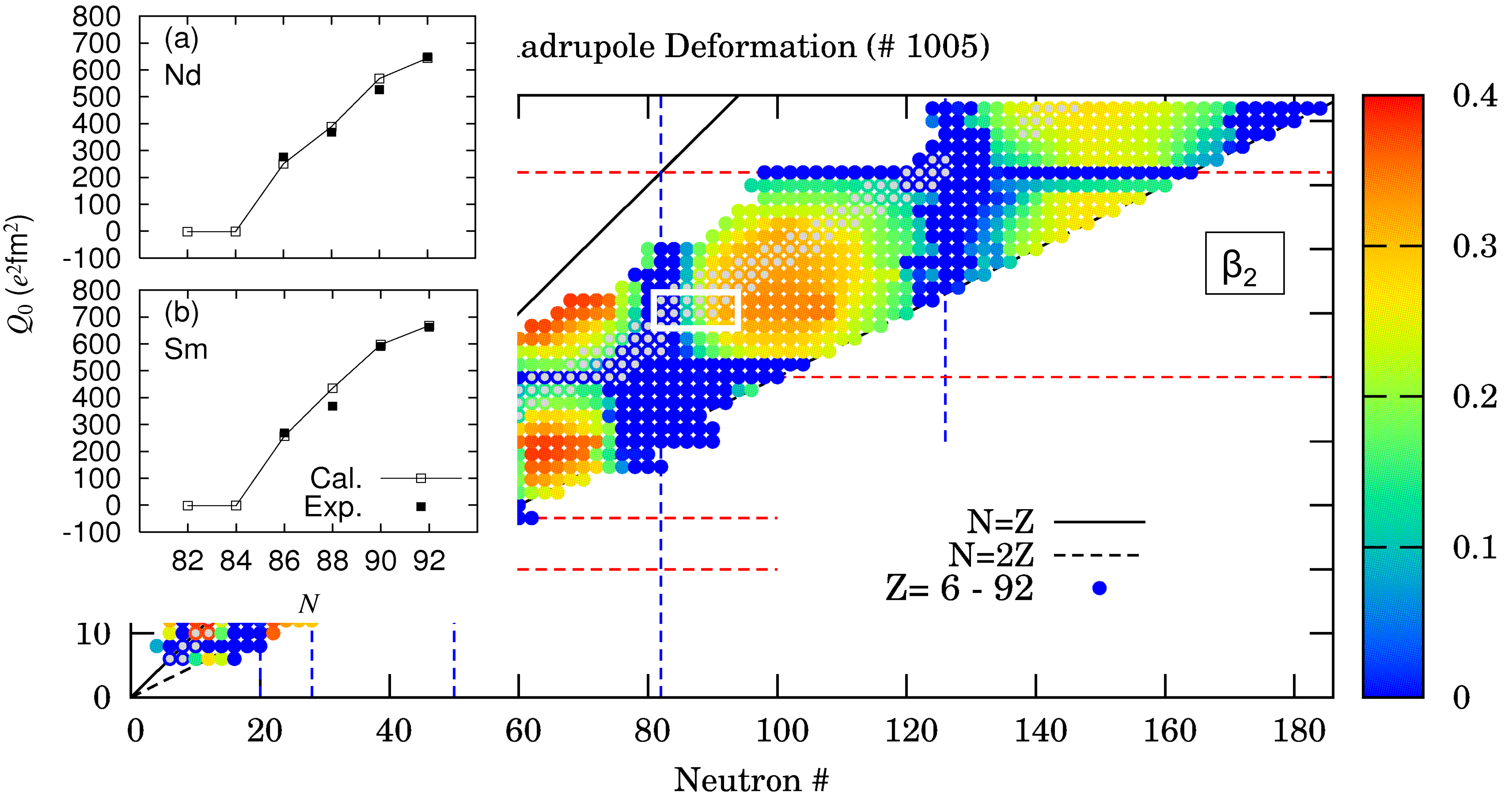


Ground-state deformation

Rare-earth region



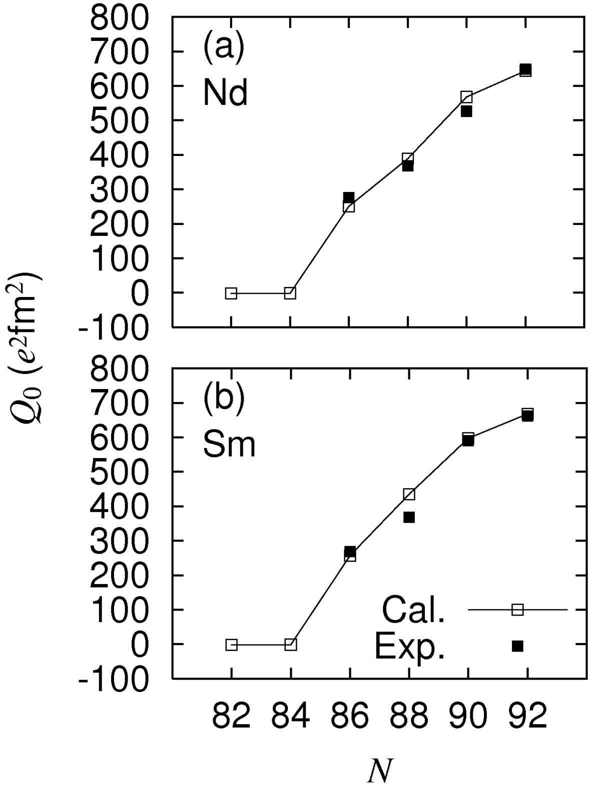
Deformation effects for photoabsorption cross section

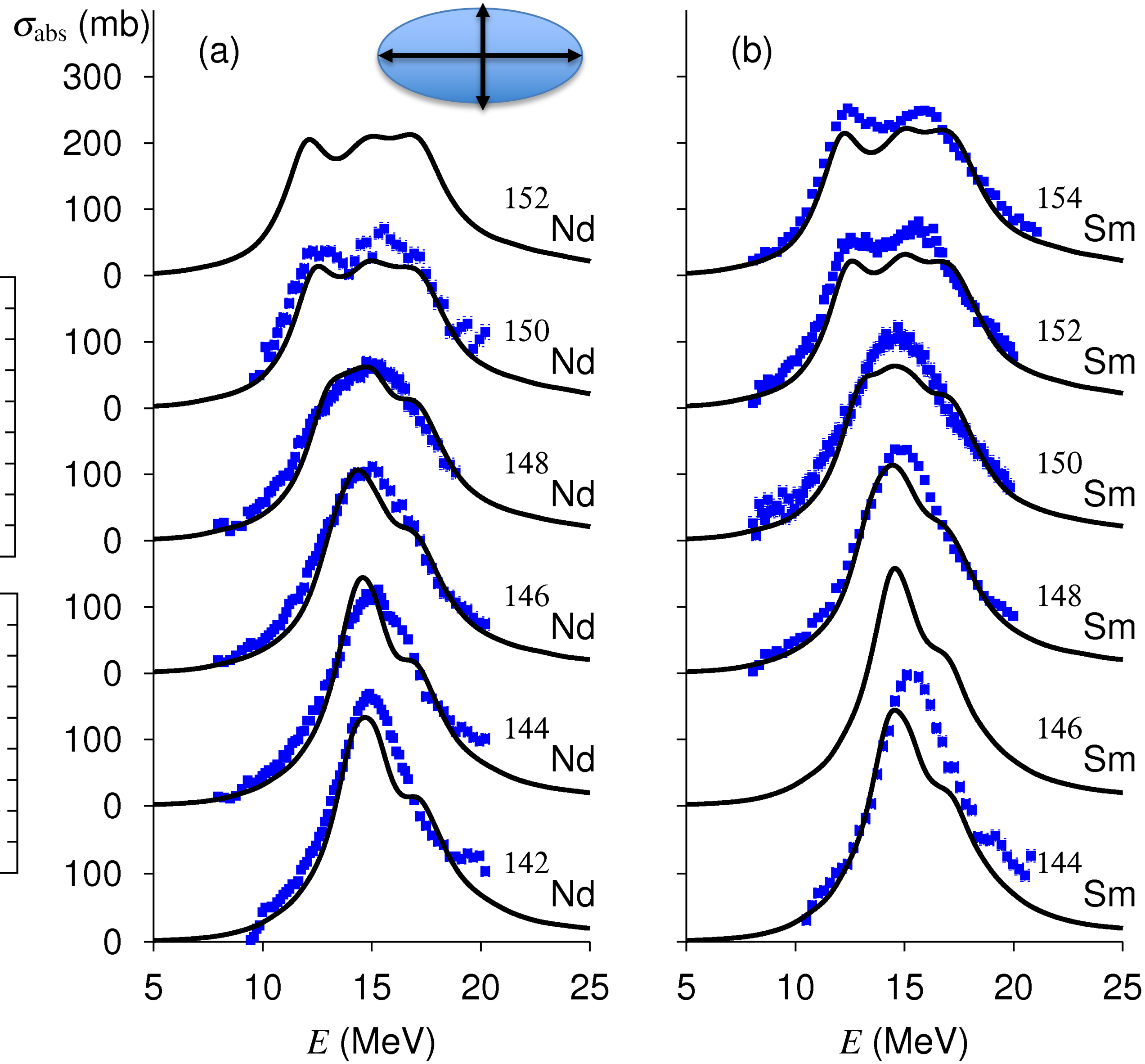
SkM* functional
(Γ = 2 MeV)

K.Yoshida and TN, PRC83, 021404 (2011).

Intrinsic Q moment

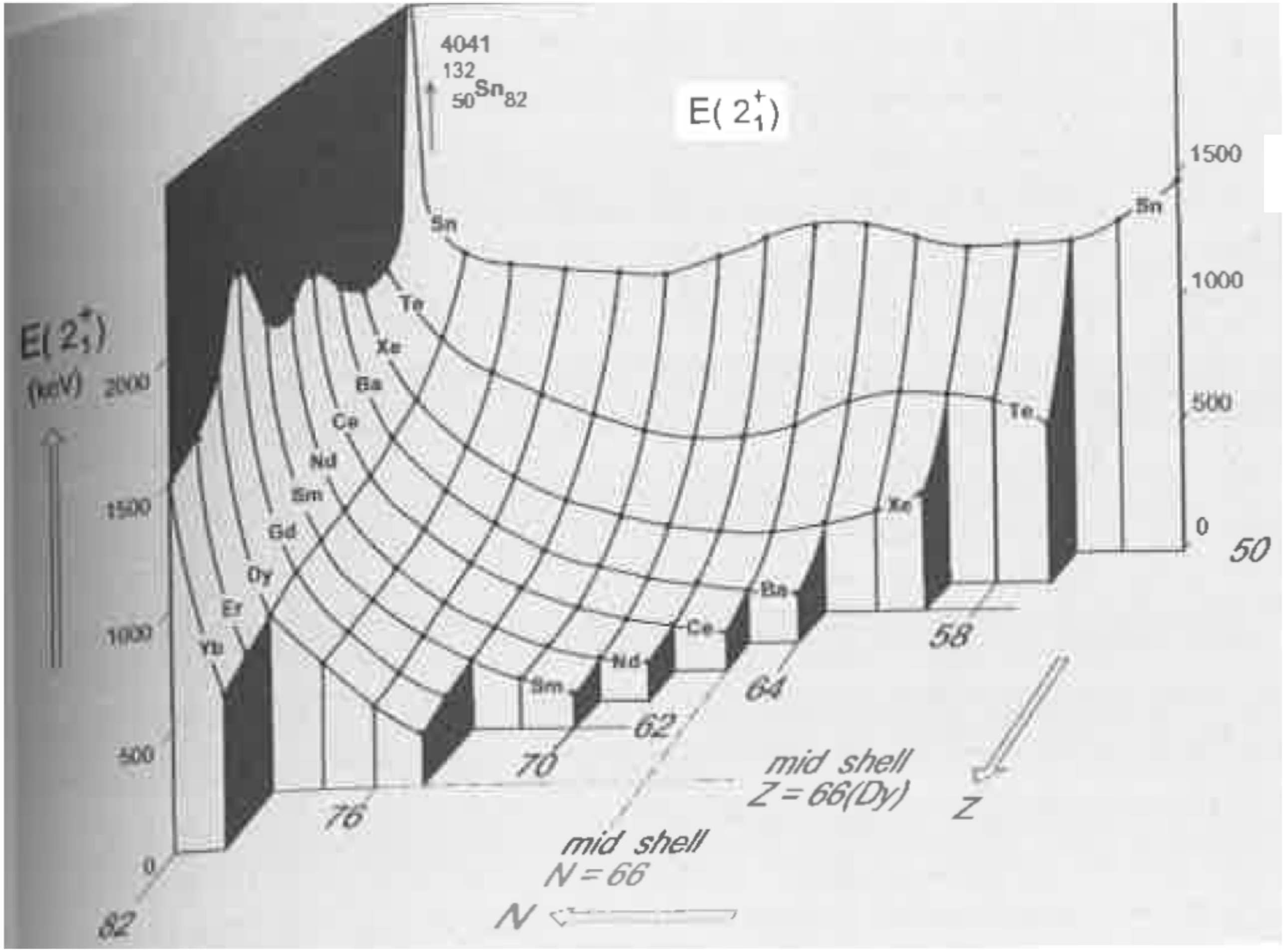
$$\langle \hat{Q}_{20} \rangle$$

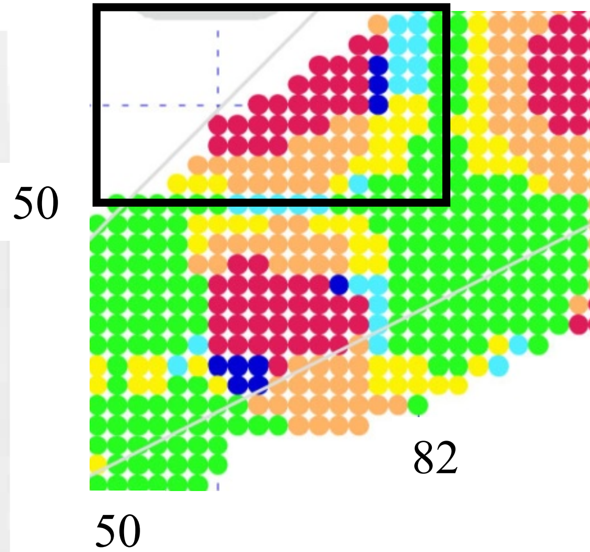




Deformation in open shell

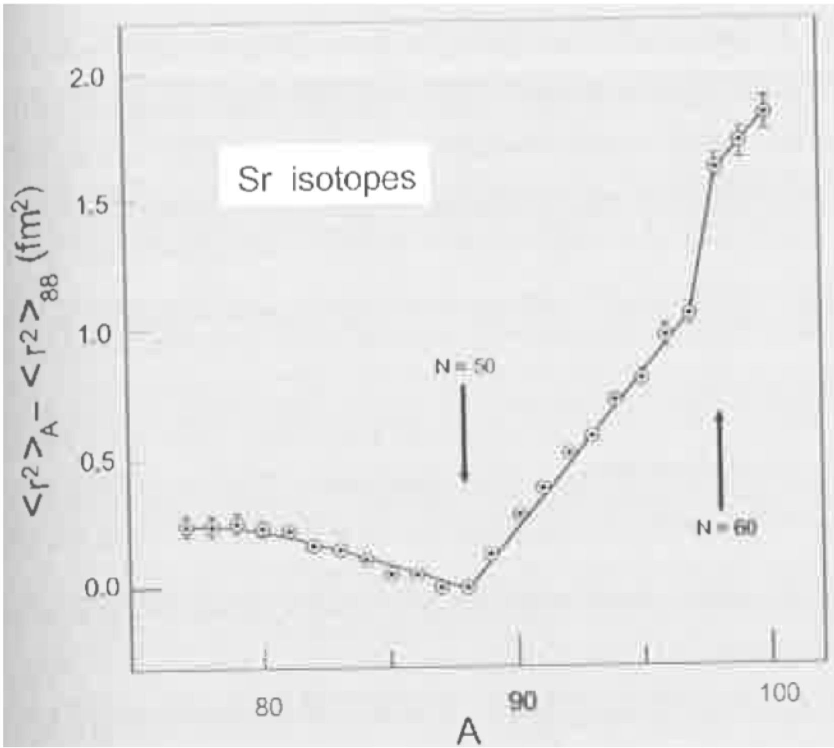
Rare-earth region

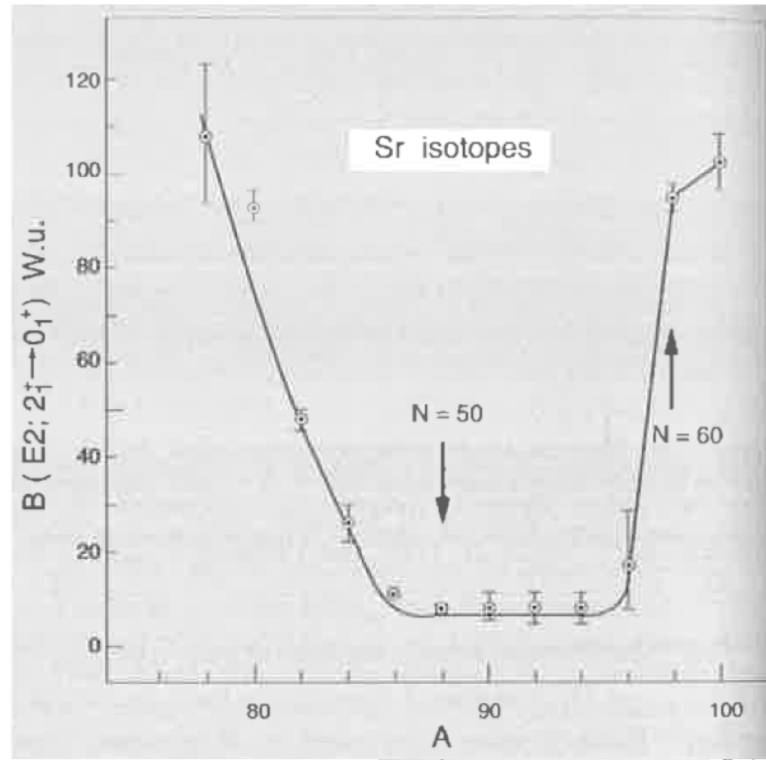


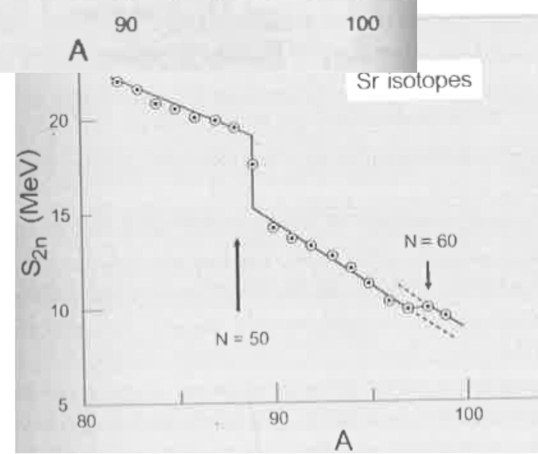


Rowe & Wood,
Fundamental Nuclear models

Sudden deformation onset in Sr isotopes

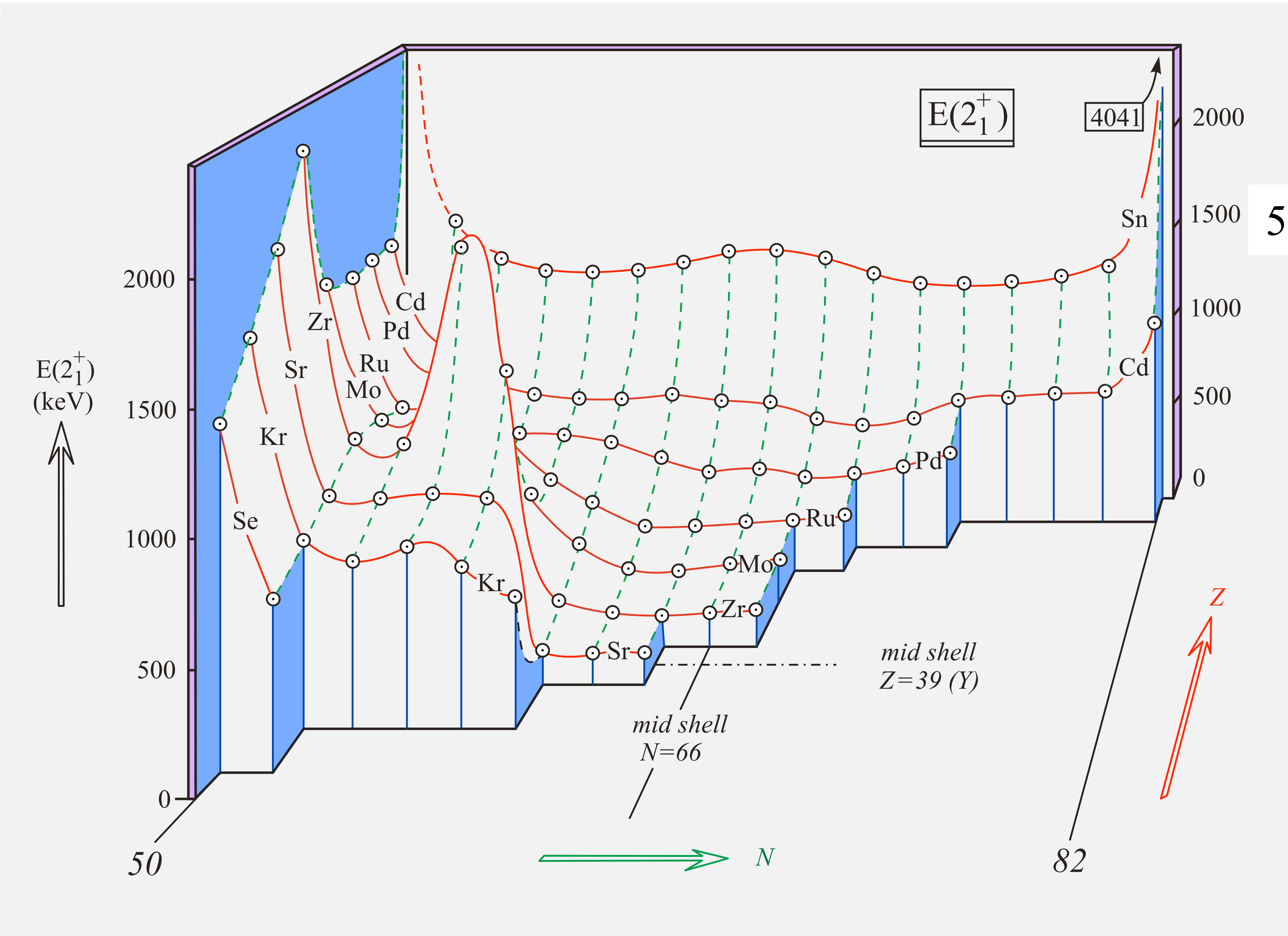


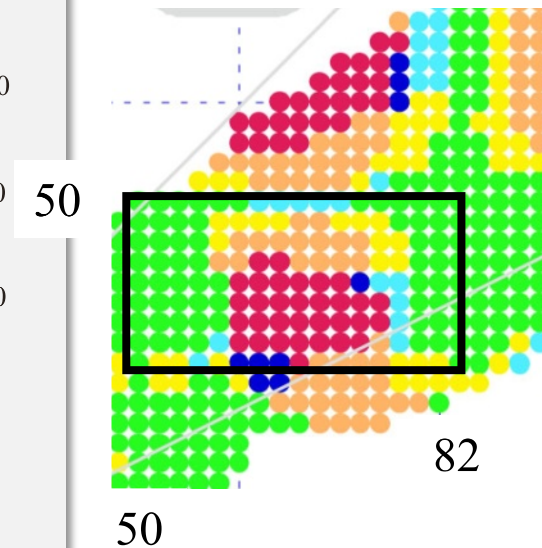




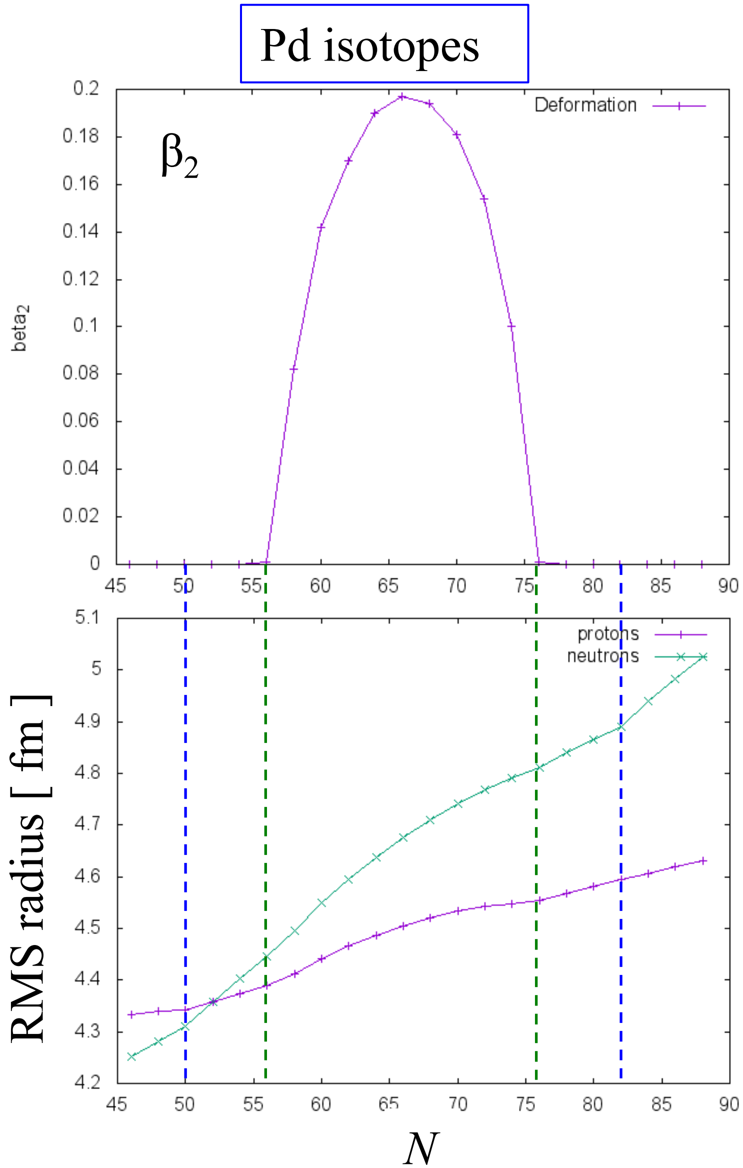
Rowe & Wood,
Fundamental Nuclear models

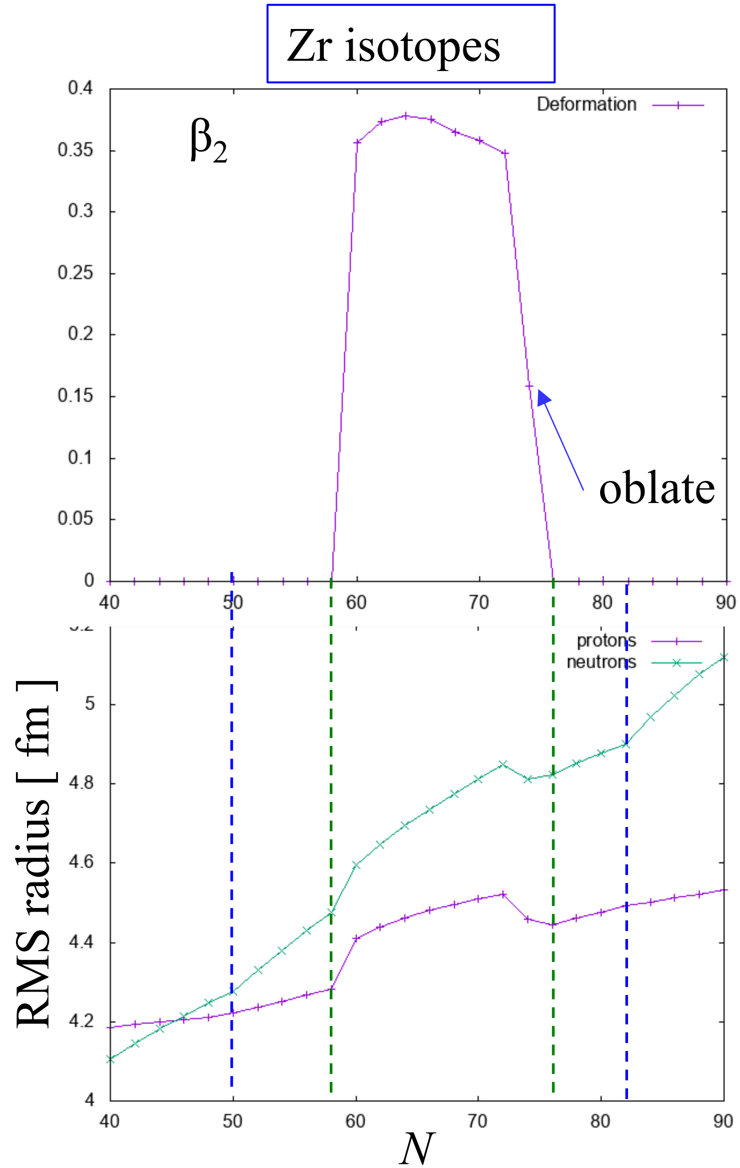
Deformation in open shell



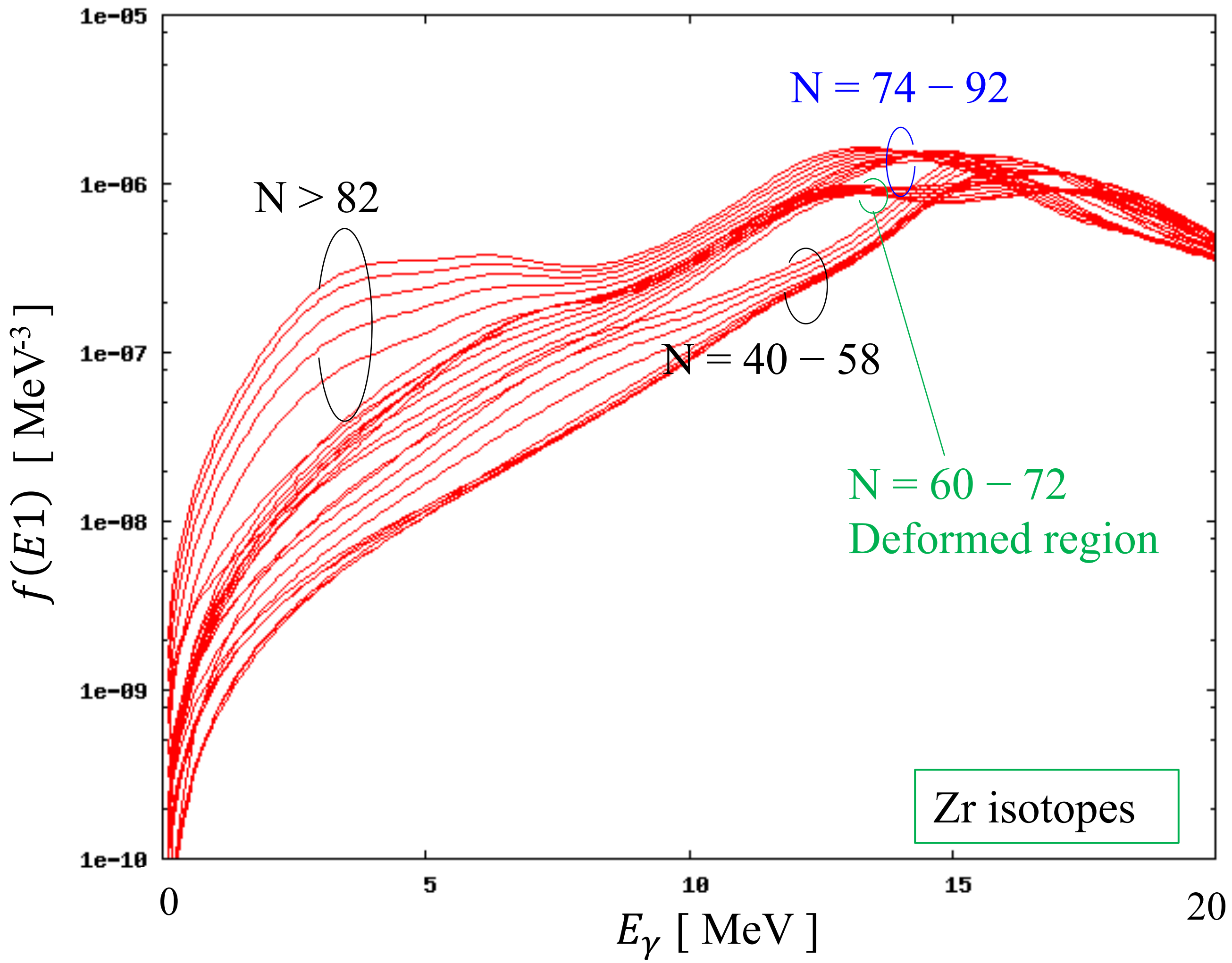


Deformation evolution

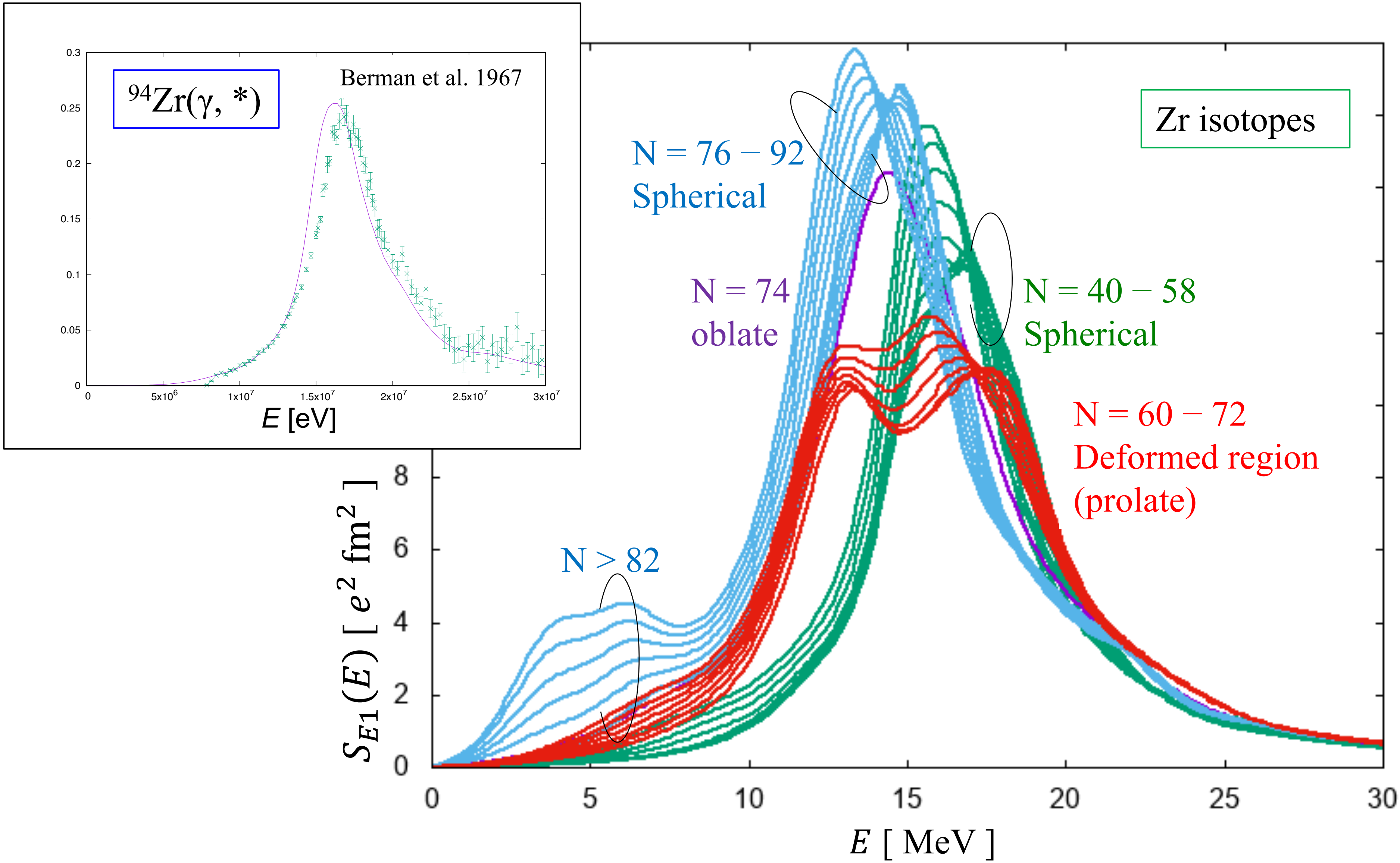




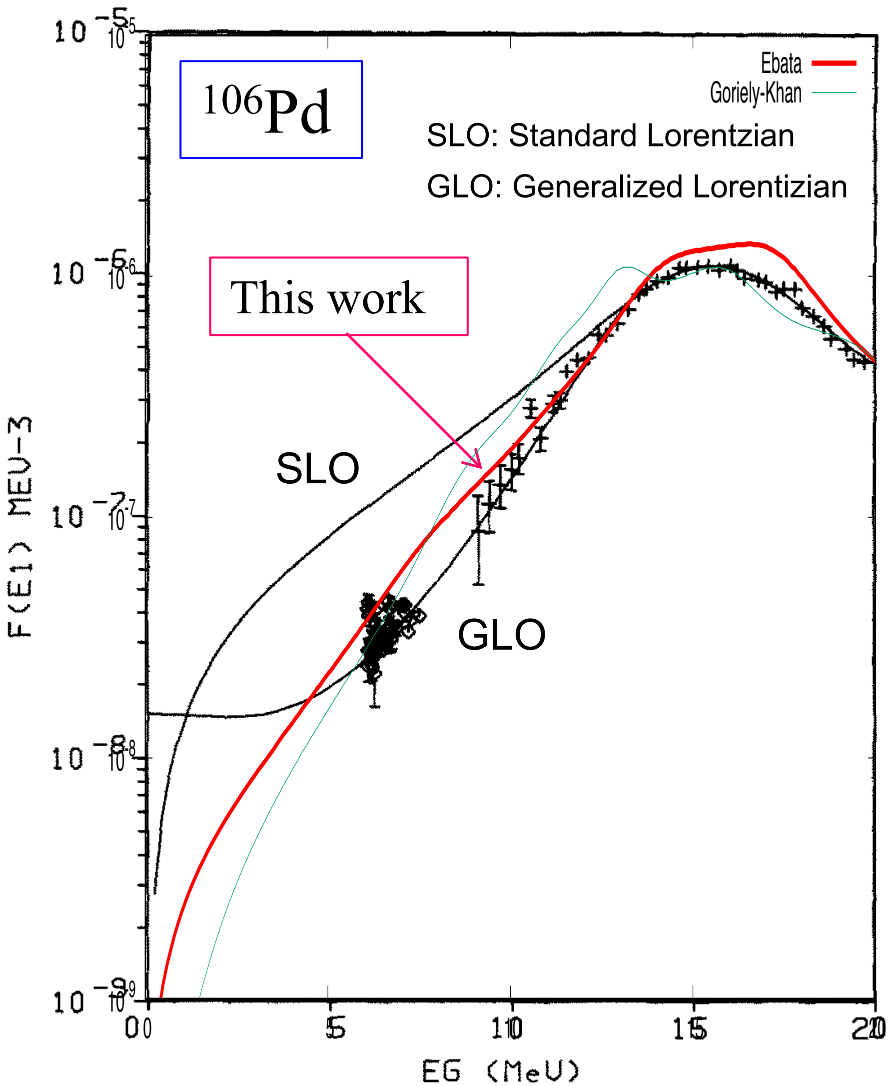
E1 oscillator strength and deformation

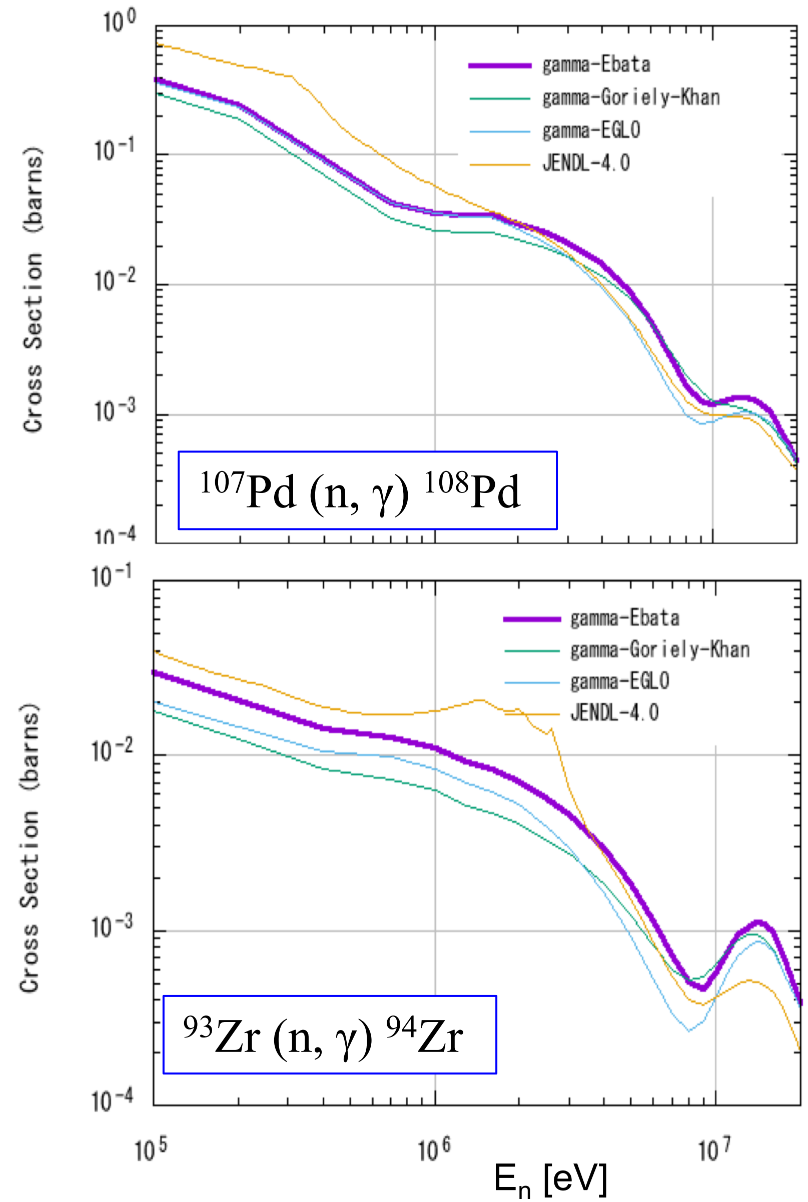


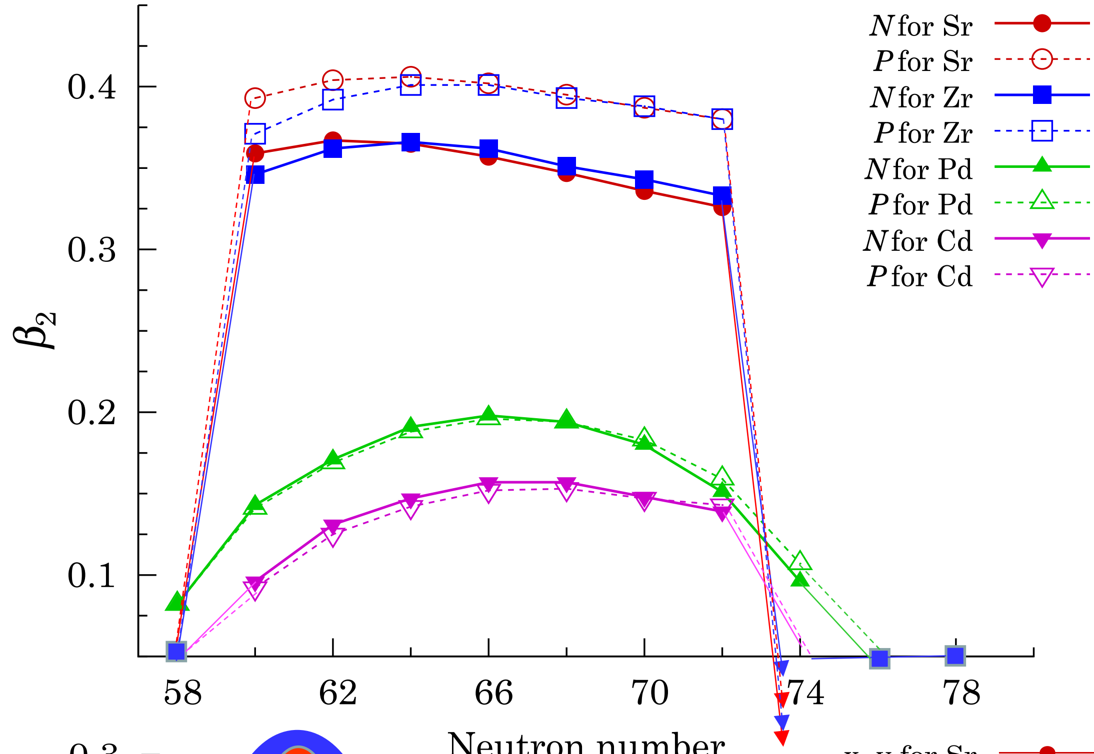
E1 strength in Zr isotopes



Neutron capture cross sections

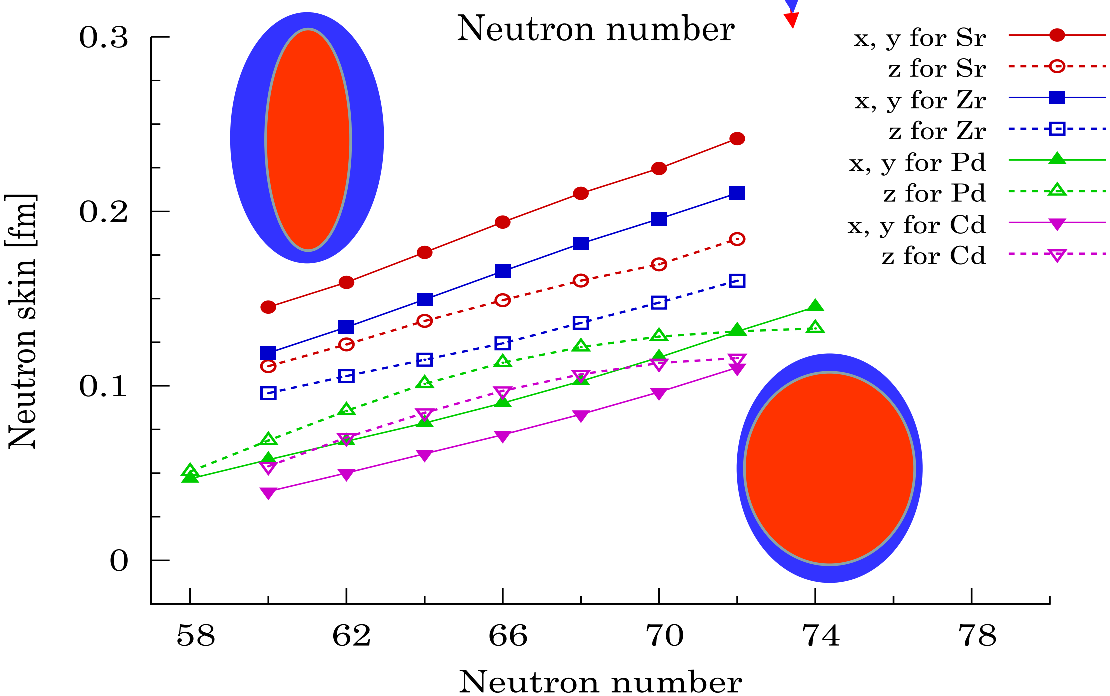






Ebata, Inakura, Nakatsukasa,
Phys. Rev. C 90, 024303 (2014)

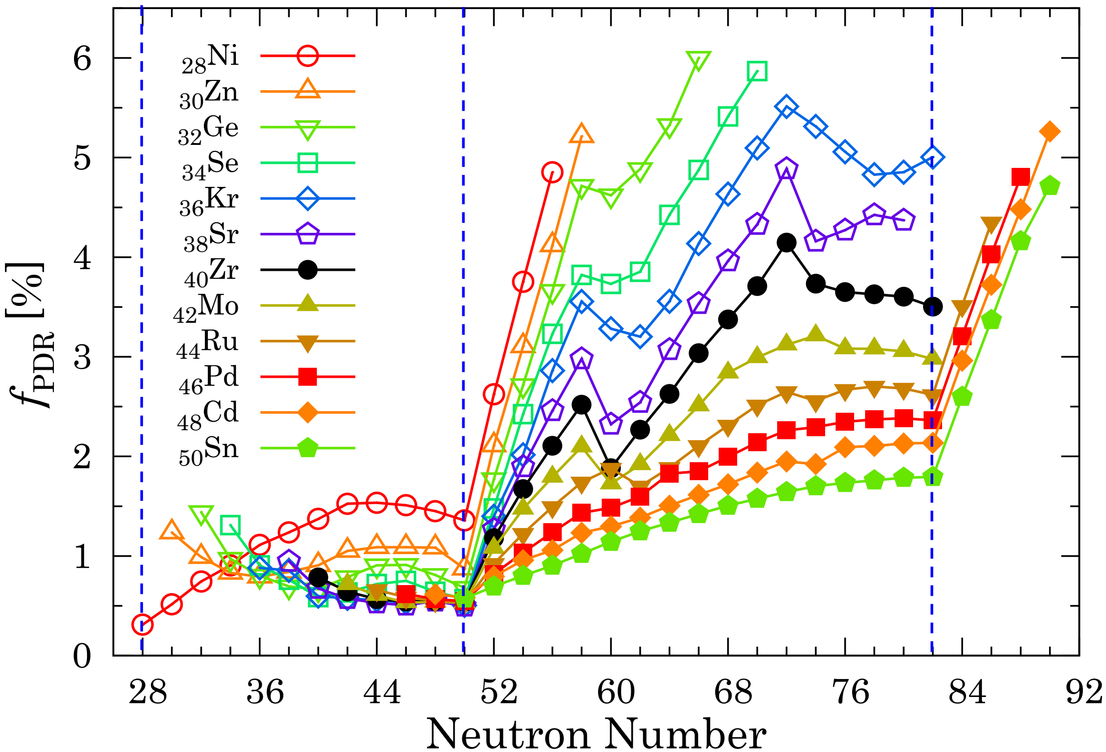
Deformation (p,n)



Neutron skin

$$\sqrt{\langle x^2\rangle_n} - \sqrt{\langle x^2\rangle_p}$$

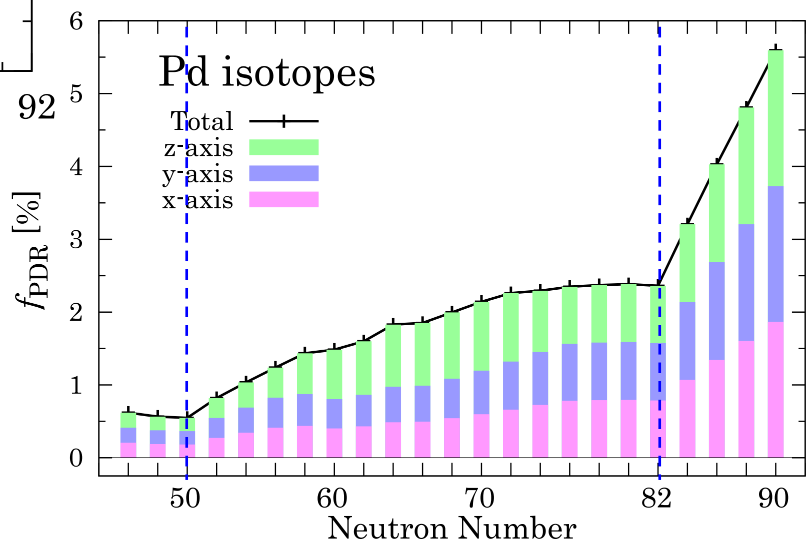
Low-energy E1 strengths

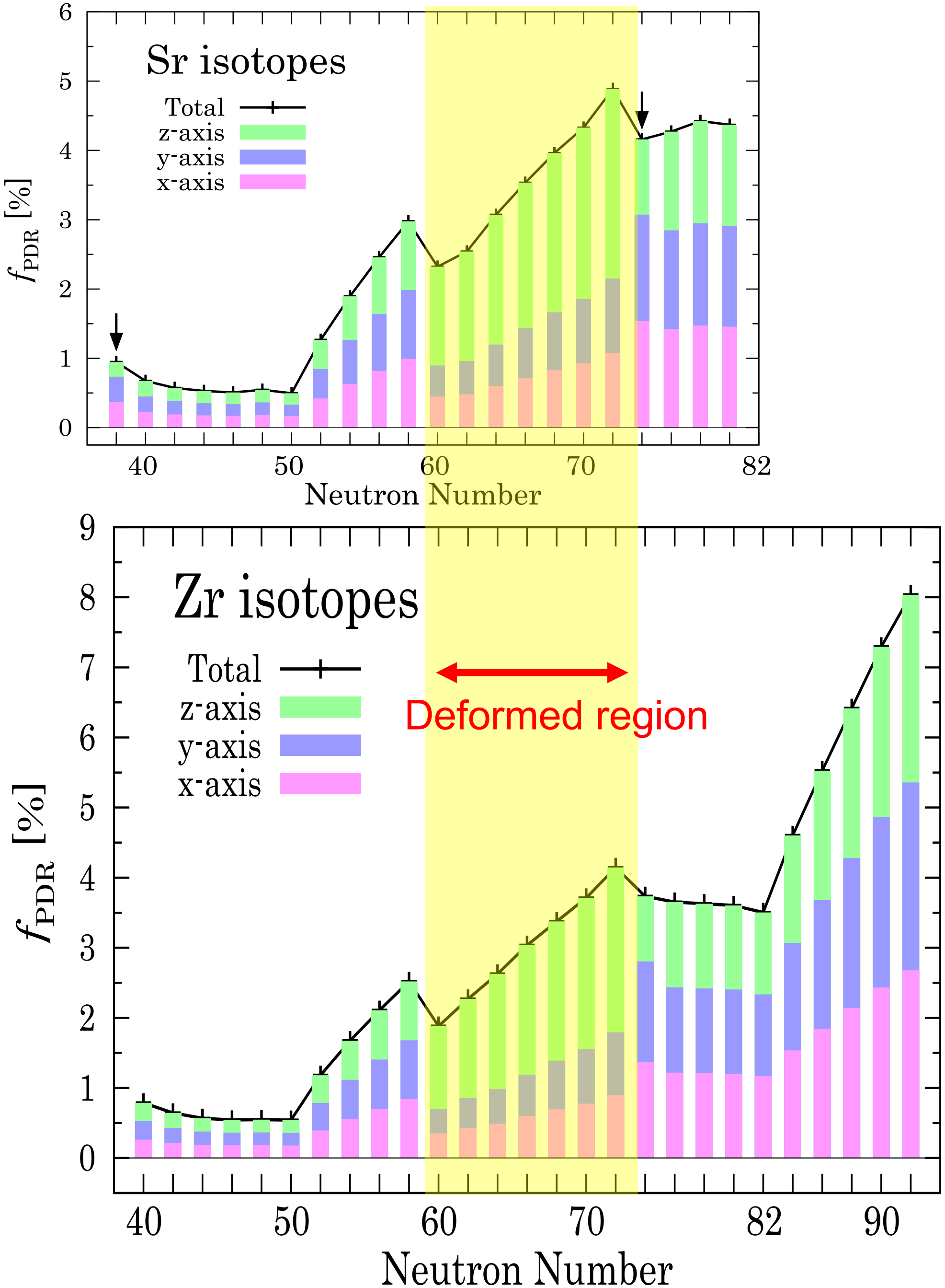


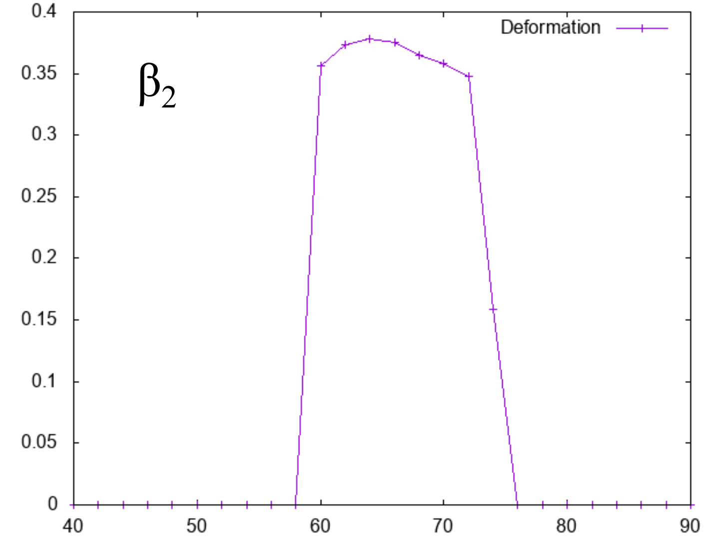
E1 PDR fraction

$$f_{\mathrm{PDR}} = \frac{m_1(E_c)}{m_1} \equiv \frac{\int^{E_c} E \times S(E1; E)dE}{\int E \times S(E1; E)dE},$$

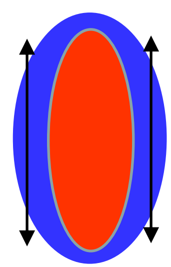
Ebata, Inakura, Nakatsukasa,
Phys. Rev. C 90, 024303 (2014)







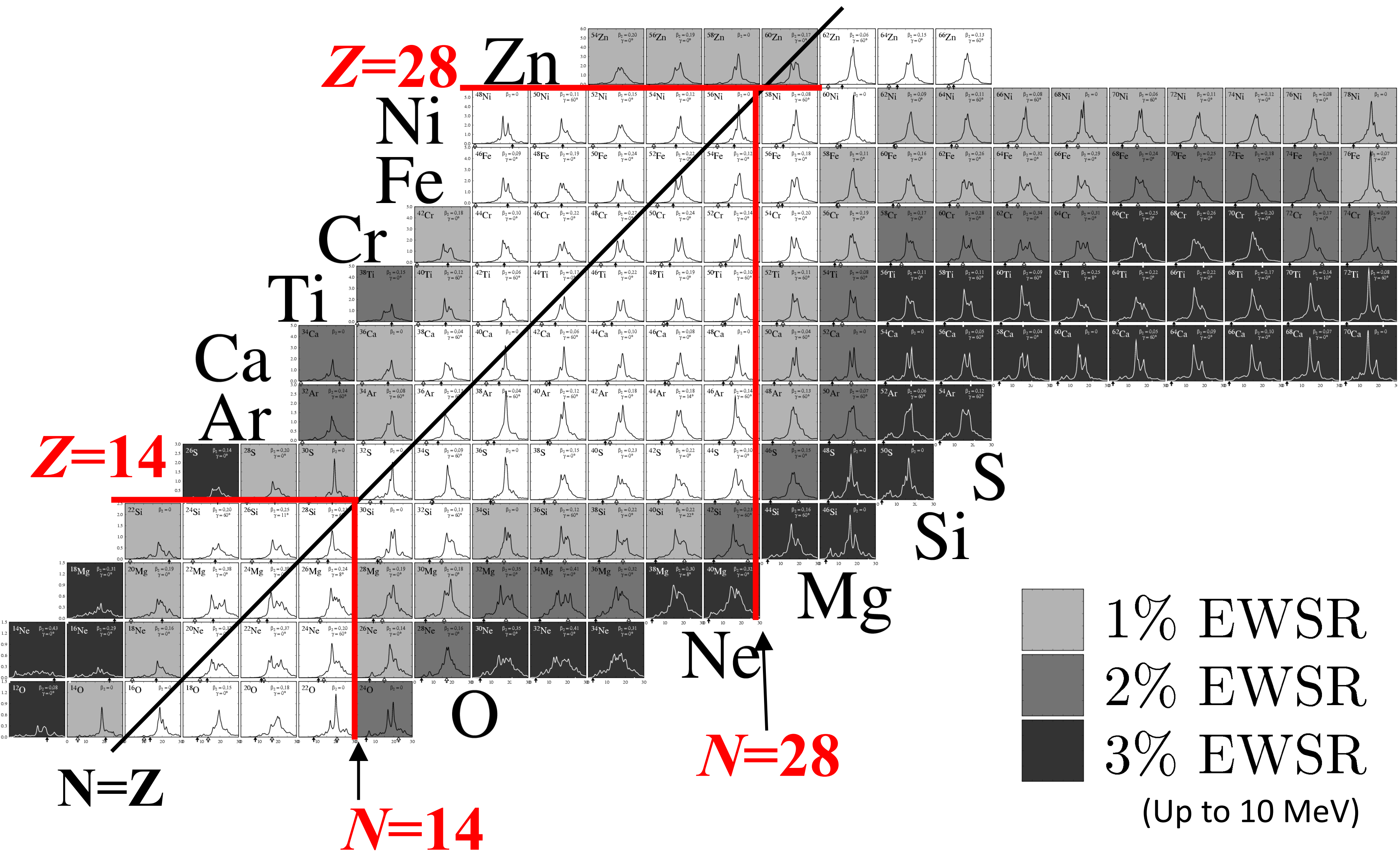
Onset of deformation at N=60
Prolate to oblate/spherical at N=74
→ Hindrance of PDR fraction



K=0 dominance
at the deformed region

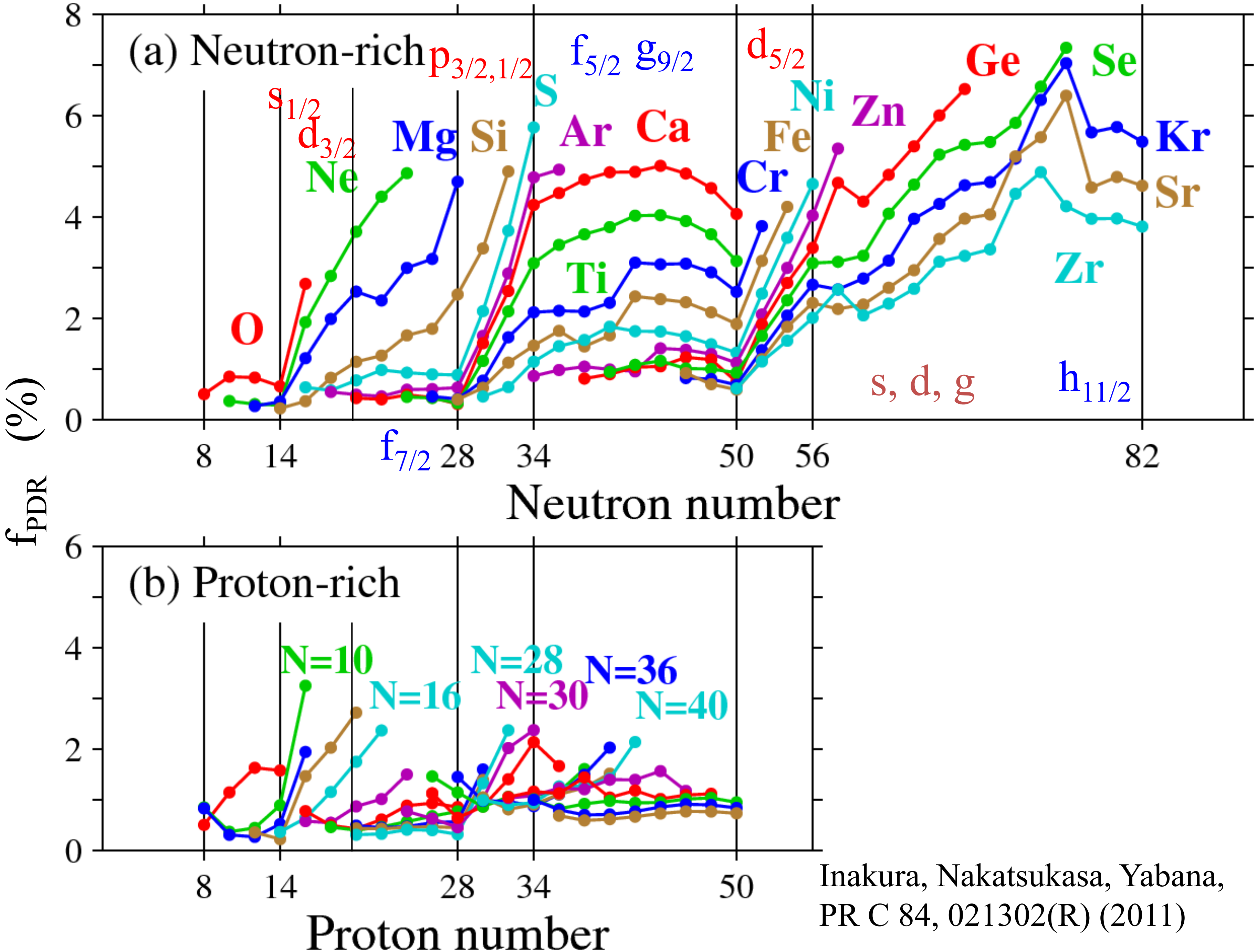
Ebata, Inakura, Nakatsukasa,
Phys. Rev. C 90, 024303 (2014)

“Magic numbers” for PDR emergence



Origin of "Magic numbers"

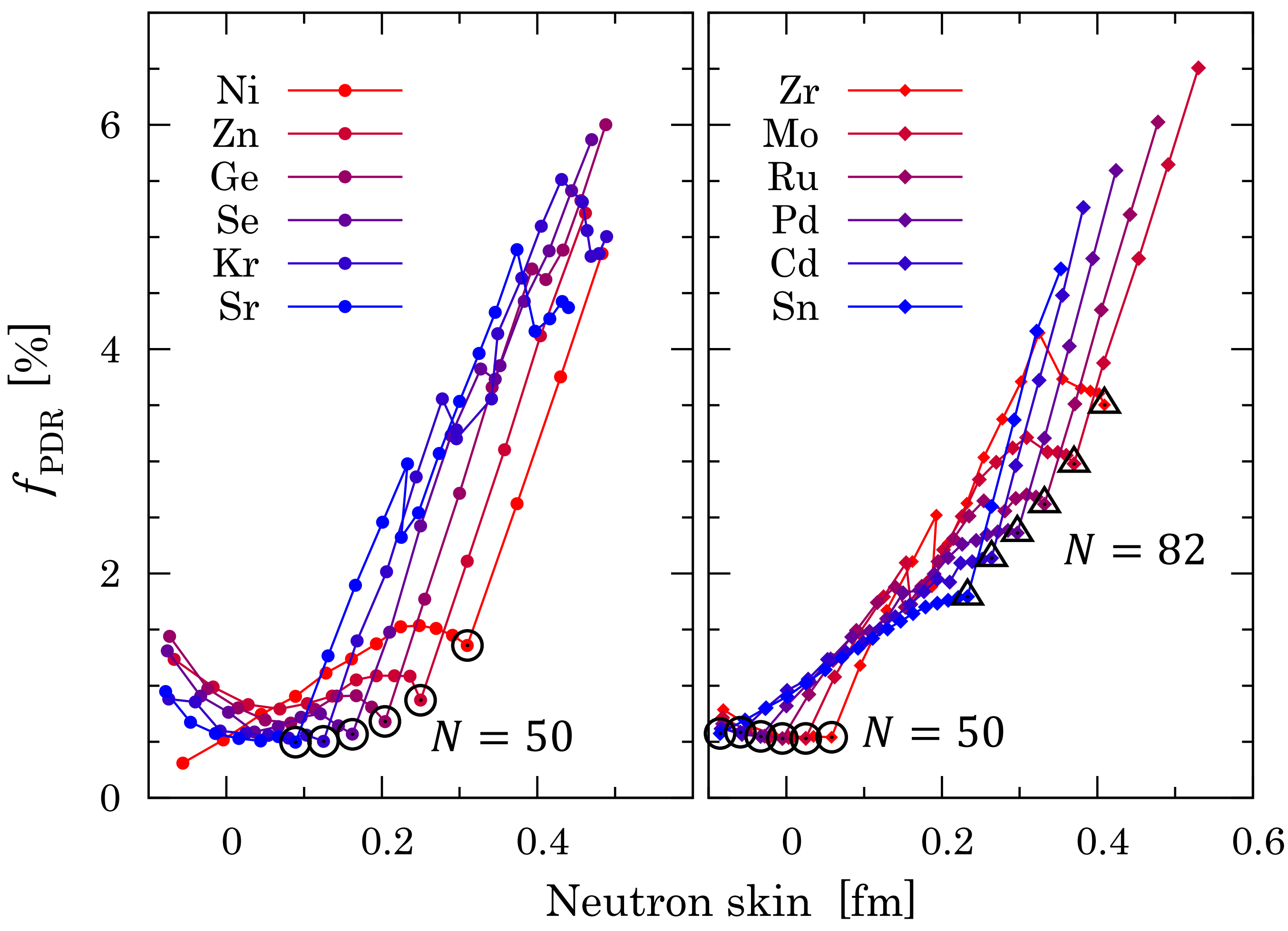
- Magic numbers: N(Z) =14, 28, 50, 82, …
- Extended distribution of neutrons(protons)
  - Weak binding orbitals
  - Low-*l* orbitals ($\ell = 0, 1, 2, \ldots$)



Inakura, Nakatsukasa, Yabana,
PR C 84, 021302(R) (2011)

Origin of “Magic numbers”

- Magic numbers: N(Z) =14, 28, 50, 82, …
- Extended distribution of neutrons(protons)
  - Weak binding orbitals
  - Low-*l* orbitals ($\ell = 0, 1, 2, ...$)
  - Skin thickness

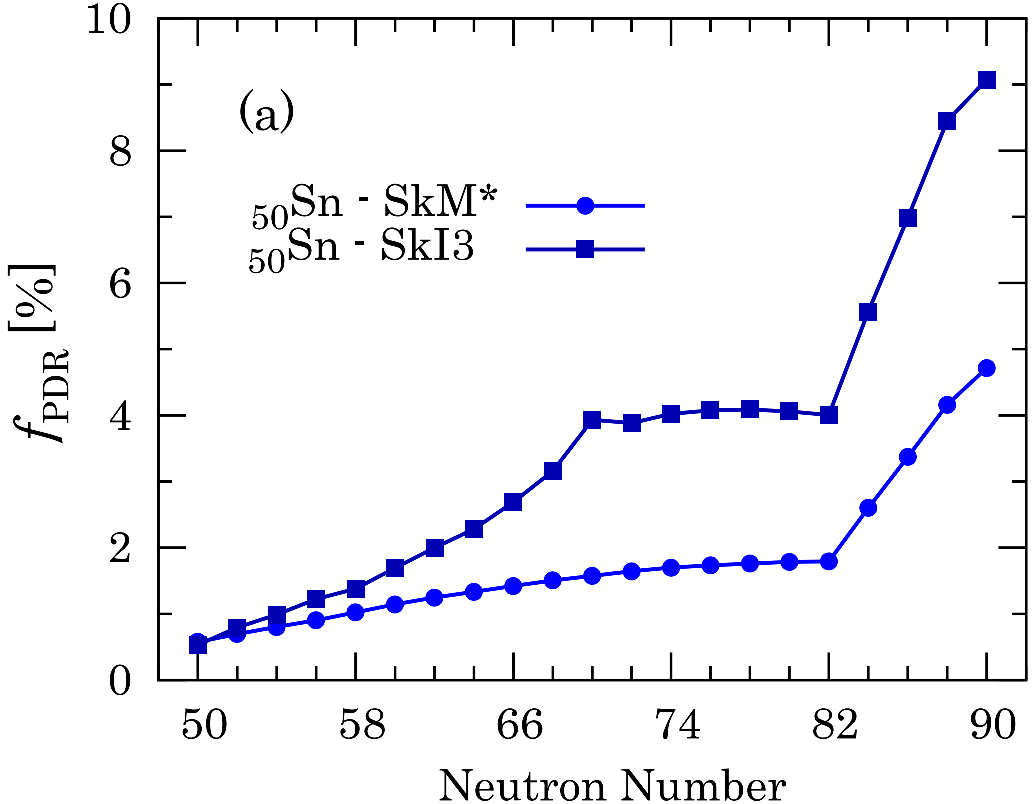

Ni
Zn
Ge
Se
Kr
Sr
Zr
Mo
Ru
Pd
Cd
Sn
$N = 50$
$N = 50$
$N = 82$
f_{PDR} [%]
Neutron skin [fm]
0
2
4
6
0
0.2
0.4
0
0.2
0.4
0.6

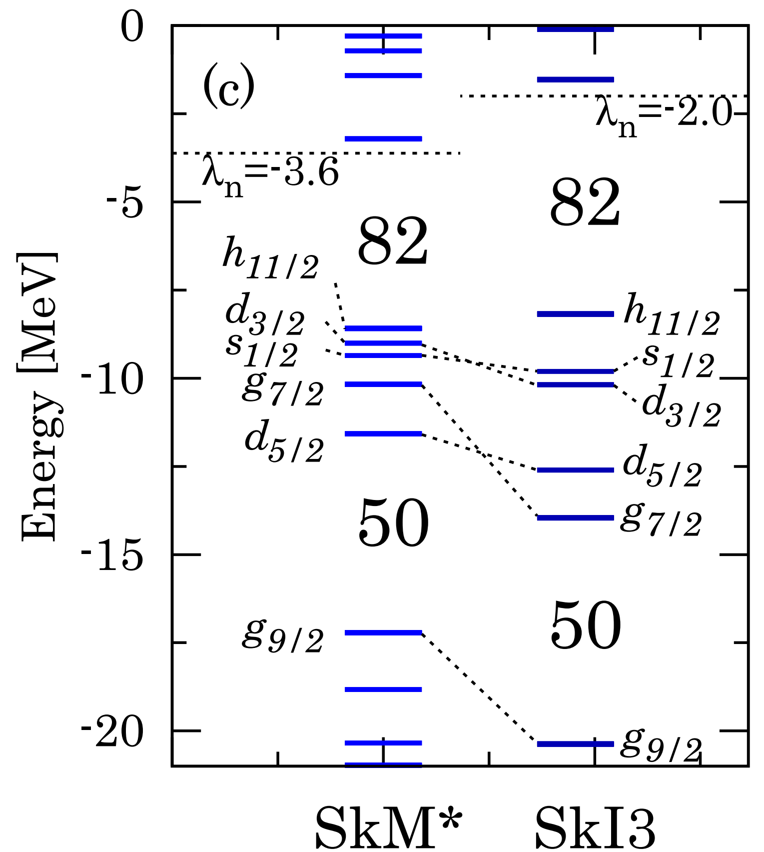
Origin of “Magic numbers”

- Magic numbers: N(Z) =14, 28, 50, 82, …
- Extended distribution of neutrons(protons)
  - Weak binding orbitals
  - Low-*l* orbitals ($\ell = 0, 1, 2, \ldots$)
  - Skin thickness
  - Shell structure / effective mass

Shell structure

- Effective mass: 0.6 (SkI3), 0.8 (SkM*)





Origin of "Magic numbers"

- Magic numbers: N(Z) =14, 28, 50, 82, …
- Extended distribution of neutrons(protons)
  - Weak binding orbitals
  - Low-*l* orbitals ($\ell = 0, 1, 2, \ldots$)
  - Skin thickness
  - Shell structure and effective mass
  - 3-body-induced (Density-dependent) spin-orbit interaction (Nakada's talk on Wednesday)

Summary

- Time-dependent approach to nuclear dynamics
  - Few-body model
  - Time-dependent density functional model
- E1 Giant resonance
  - Simple dynamics (p vs n)
  - Exotic nuclei: T-dep. of symmetry energy
- Low-energy E1 strength
  - Suitable for exotic nuclei
  - Sensitive to properties, such as skin, pairing, deformation, & shell effect